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The purpose of the research was to study the processes by which stochastic motion is self-generated in deterministic systems, and the consequences of the resulting stochasticity.

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1. This confirms our conversations of 27 Feb 97 and 11 Jul 97. Enclosed are a number of technical reports which were returned to our agency for lack of clear distribution availability statement. This confirms that all reports are unclassified and are "APPROVED FOR PUBLIC RELEASE" with no restrictions.

2. Please contact me if you require additional information. My e-mail is *silverr@onr.navy.mil* and my phone is (206) 625-3196.

ROBERT J. SILVERMAN

Final Report

ONR Contract N00014-89-J-1079

RESEARCH ON SELF-GENERATED STOCHASTIC MOTION

1 October 1988 – 30 September 1994

I. Summary of Technical Accomplishments

The purpose of the research under this contract was to study the processes by which stochastic motion is self-generated in deterministic systems, and the consequences of the resulting stochasticity. We consider representative problems, each of which incorporates one or more of the main underlying physical phenomena. Phenomena that we have studied under this contract include diffusion through stochastic webs, diffusion along resonances in more than two degrees of freedom (Arnold diffusion), equipartition in oscillator chains, and the dynamics of coupled dissipative systems, such as phase locking, chaos, synchronization, and synchronized chaos.

1. *Diffusion Through Stochastic Webs*

Diffusion has been explored in a two-dimensional phase space in which a connected separatrix layer (web) of intrinsic stochasticity bounds regions of regular motion (tiles). In the presence of weak extrinsic noise, if the web diffusion dominates, the noise slows the web diffusion rate; if the extrinsic diffusion dominates, the diffusion is enhanced. Analytic calculations agree well with numerical results.

2. *Arnold Diffusion*

When several standard maps are coupled together, KAM surfaces cannot isolate stochastic regions, and particles diffuse along stochastic layers by the process of Arnold diffusion. For the case of two coupled standard maps the rate of Arnold diffusion has been calculated both locally around a particular KAM curve and globally across many cells of the 2π periodic mapping. When more than two standard maps are coupled, the diffusion rate increases, depending on the total number of maps, N, and the number of phases in each coupling term, m, where $2 \leq m \leq N$. As N is increased, the diffusion rate increases as $N^{1/2}$, the length of the diagonal in the action space. As m is increased, the diffusion rate increases because the phase of the coupling term for a particular map becomes less correlated with the phase of the map itself. An analytic calculation of local diffusion for the m and K dependence has been developed, which is in good agreement with numerical results. The calculated local rate of diffusion and global phase space arguments are used to calculate the global diffusion.

3. *Equipartition in Coupled Oscillator Chains*

The energy transitions and time scales have been studied in the Fermi-Pasta-Ulam (FPU) oscillator chain and in a set of coupled pendula, for which the energy E, initially in a single or small group of low frequency modes, is distributed among modes. The energy transitions, with increasing energy, have been classified. At low energy the linear parts of the energies are distributed in a geometrically decreasing series. A transition occurs such that above this transition there is strong local coupling among neighboring modes with a characteristic resonant frequency. There is a second transition at a critical energy for which stochasticity among low-frequency resonances transfers energy into high-frequency resonances by the Arnold diffusion mechanism. Above this transition we determine a universal scaling for the time scale to approach equipartition among the modes.

4. *Chaos and Synchronization in Coupled Phase-Locked Loops*

A broad study has been undertaken to study the chaos, synchronization, and synchronized chaos in coupled dissipative mappings. The device used for this exploration was digital phase-locked loops (DPLL's). The study also concerned the use of synchronized chaos in communications applications, in collaboration with industry. During the past two years the basic work has continued with an AASERT grant administered by the ONR contract.

A single first order DPLL is topologically equivalent to a circle map, having the generic properties of phase-locking and chaos above a certain gain threshold. Coupled non-uniformly sampling DPLL's have interesting new dynamics which arise from a new class of coupled mappings. These have been studied theoretically, numerically, and experimentally. It was shown that chaotic signals can be synchronized, with a possible application to low-probability-of-intercept communication. The recent work under the AASERT grant has been directed mainly to theoretical and numerical studies of the behavior of many coupled loops, with various coupling configurations, and to the effect of noise.

II. Papers Published in Refereed Journals

(1) A. J. Lichtenberg and B. P. Wood, "Diffusion Through a Stochastic Web," *Phys. Rev. A* **39,** 2153 (1989).

(2) G. M. Bernstein, M. A. Lieberman, and A. J. Lichtenberg, "Nonlinear Dynamics of a Digital Phase Locked Loop," *IEEE Trans. on Communications* **37,** 1062 (1989).

(3) A. J. Lichtenberg and A. Ujihara, "Application of Nonlinear Mapping Theory to Commodity Price Fluctuations," *J. of Math. Econ. and Control* **13,** 225 (1989).

(4) A. J. Lichtenberg and B. P. Wood, "Diffusion on Two Space and Time Scales," *Phys. Rev. Lett.* **62,** 2213 (1989).

(5) G. M. Bernstein and M. A. Lieberman, "A Method for Obtaining a Hamiltonian for Nonlinear LC Circuits," *IEEE Trans. Circuits and Systems* **36,** 411 (1989).

(6) C. G. Goedde, A. J. Lichtenberg, and M. A. Lieberman, "Parametric Instabilities in the Discrete Sine-Gordon Equation," *Physica D* **41,** 341 (1990).

(7) M. de Sousa Vieira, A. J. Lichtenberg, and M. A. Lieberman, "Nonlinear Dynamics of Self-Synchronizing Systems," *Int. J. of Bifurcations and Chaos* **1,** 691 (1991).

(8) A. J. Lichtenberg, K. Itoh, S.-I. Itoh, and A. Fukuyama, "The Role of

Stochasticity in Sawtooth Oscillations," *Nucl. Fusion* **32,** 495 (1992).

(9) S. E. Parker, X. Q. Xu, A. J. Lichtenberg, and C. K. Birdsall, "Evidence of Stochastic Diffusion Across a Cross-Field Sheath," *Phys. Rev. A* **45,** 3949 (1992).

(10) A. J. Lichtenberg, "The Application of Mappings to Physical Systems," *Proc. Physics Summer School on Nonlinear Dynamics and Chaos*, R. L. Dewar and B. I. Henry, Eds., Australian National Univ., World Scientific, pp. 277–319 (1991).

(11) J. Gullicksen et al., "Secure Communication by Synchronization to a Chaotic Signal," *Proc. 1st Exp. Chaos Conf.*, S. Vohra et al, Eds., World Scientific, pp. 137–144 (1991).

(12) A. J. Lichtenberg and M. A. Lieberman, *Regular and Chaotic Dynamics*, 2nd Ed., Springer-Verlag, *Appl. Math. Sci.* **38,** New York (1992).

(13) M. de Sousa Vieira et al., "Numerical and Experimental Studies of Self-Synchronization and Synchronized Chaos," *Int. J. Bifur. and Chaos* **2,** 645 (1992).

(14) M. de Sousa Vieira, A. J. Lichtenberg, and M. A. Lieberman, "Synchronization of Regular and Chaotic Signals," *Phys. Rev. A, Rapid Comm.* **46,** 7359 (1992).

(15) C. G. Goedde, A. J. Lichtenberg, and M. A. Lieberman, "Chaos and the Approach to Equilibrium in the Sine-Gordon Equation," *Physica D* **59,** 200 (1992).

(16) A. J. Lichtenberg, "Arnold Diffusion in a Torus with Time-Varying

Fields," *Phys. Fluids B* **4,** 3132 (1992).

(17) A. Fukuyama, K. Itoh, S.-I. Itoh, S. Tsuji, and A. J. Lichtenberg, "Stochasticity-Driven Disruptive Phenomena in Tokamaks," *14th Int'l. Conf. on Controlled Fus. Res.*, IAE CN 56/D-4-2 (October 1992).

(18) B. P. Wood, A. J. Lichtenberg, and M. A. Lieberman, "Arnold and Arnold-Like Diffusion in Many Dimensions," *Physica D* **71,** 132 (1994).

(19) W. E. Wonchoba, M. A. Lieberman, and A. J. Lichtenberg, "The Dynamics of a Class of Digital Oscillators," *Nonlinearity* **7,** 1695 (1994).

(20) J. DeLuca, A. J. Lichtenberg, and M. A. Lieberman, "Time Scale to Ergodicity in the Fermi-Pasta-Ulam System," *Chaos* **5,** 283 (1995).

(21) M. de Sousa Vieira, A. J. Lichtenberg, and M. A. Lieberman, "Nonlinear Dynamics of Digital Phase-Locked Loops with Delay," *Int. J. of Bifur. and Chaos* **4,** 715 (1994).

(22) M. de Sousa Vieira, A. J. Lichtenberg, and M. A. Lieberman, "Self-Synchronization of Many Coupled Oscillators," *Int. J. of Bifur. and Chaos* **4,** 1563 (19994).

III. Graduate Students

G. M. Bernstein, PhD, 1989

C. G. Goedde, PhD, 1990

B. P. Wood, PhD, 1991

J. DeLuca, PhD, 1994

W. E. Wonchoba, PhD, 1995

P. Khoury, PhD, 1996 (Expected)