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Increment Thresholds for
Two Non-Identical Flashes

Bureau of Medicine and Surgery
MF12524004
Work Unit No. 2001D

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DEPARTMENT OF THE NAVY
NAVAL AIR DEVELOPMENT CENTER
WARMINSTER, PA. 18974

Crew Systems Department

NADC-CS-7120

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SUMMARY

Two flashes of 1° visual angle and 0.389 mL were superimposed upon a steady 1.19 mL background of the same size seen foveally. The second flash followed the first after delays ranging from 0 to 60 ms. The duration of the first flash was varied from 0 to 4.8 ms, and, for each duration of the first flash, the threshold duration of the second flash was determined. (When seen, the two flashes appeared as one.) At all delays below 25 ms, the threshold energy of the two flashes combined was constant, regardless of the duration of the first flash. At each delay between 25 and 50 ms, at short durations of the first flash, the threshold duration of the second flash was the same as that required when no first flash was presented. However, at longer durations of the first flash, the threshold average luminance provided by the two flashes during the total display time was constant. At the delay that required the maximum energy for threshold, 55 ms, for every duration of the first flash, the duration of the second flash was greater than that required when no first flash was presented. These findings, and others, are incorporated in a simple model.

TABLE OF CONTENTS

	Page
SUMMARY-----	ii
INTRODUCTION-----	1
METHOD-----	3
Apparatus-----	3
Procedure-----	3
RESULTS AND DISCUSSION-----	5
Intermediate Background Luminance-----	9
Derivation of Model-----	13
Model for Two Increments-----	20
Low Background Luminance-----	22
REFERENCES-----	23
INTRODUCTION TO APPENDIXES-----	24
APPENDIX A AND APPENDIX B-----	25

LIST OF FIGURES

Figure	Title	Page
1	Definitions of symbols used to describe experimental procedures-----	2
2	Threshold duration of the second flash as a function of the duration of the first flash-----	6
3	Threshold duration of the second flash as a function of the duration of the first flash-----	8
4	Threshold duration of the first flash as a function of the duration of the second flash-----	10
5	Threshold energy for two identical flashes as a function of total display time-----	11
6	Generalized model describing how two flashes interact to achieve threshold-----	18
7	Threshold duration of the second flash as a function of the duration of the first flash-----	21

LIST OF TABLES

Table	Title	Page
I	Average luminance (A_L) required for threshold for various combinations of \underline{i} and \underline{t}_1 .-----	12
II	Equations describing how two flashes interact to achieve threshold when $\underline{\Delta I}_1 = \underline{\Delta I}_2 = \text{constant}$ -----	19

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INTRODUCTION

Air Naval weapons systems often provide visual displays of signals that must be detected and interpreted by naval personnel. When two near-threshold signals occur in close temporal proximity they interact in ways that have not been described adequately.

Over the past forty years several experiments have been performed to determine how two, temporally-separated flashes, each by itself subliminal, interact to achieve threshold. (See Fig. 1.) To a background luminance, I , are added two flashes of durations t_1 and t_2 and luminances ΔI_1 and ΔI_2 . The question is: as a function of the time between the flashes, what durations or luminances of the flashes are required for detection?

Most of the experiments¹ on the problem have been limited to two identical flashes, i.e., where $t_1 = t_2$ and $\Delta I_1 = \Delta I_2$. Four empirical equations, referred to as the TEpee effect, have been suggested² to describe the results of such experiments.

A few studies have considered the more general problem of non-identical flashes. In general, the non-identical flash studies³⁻⁶ indicate that, at short i values, no matter how the energy is distributed between the two flashes, the total energy required for threshold is constant. The study of Granit and Davis⁷, however, does not agree with this finding for short i values.

At longer i values the picture is less clear. Ikeda,³ for example, reports that the two flashes aid each other when the first has less energy than the second, but interfere with each other when the first has more energy than the second. Rashbass's data,⁶ on the other hand do not suggest such relationships.

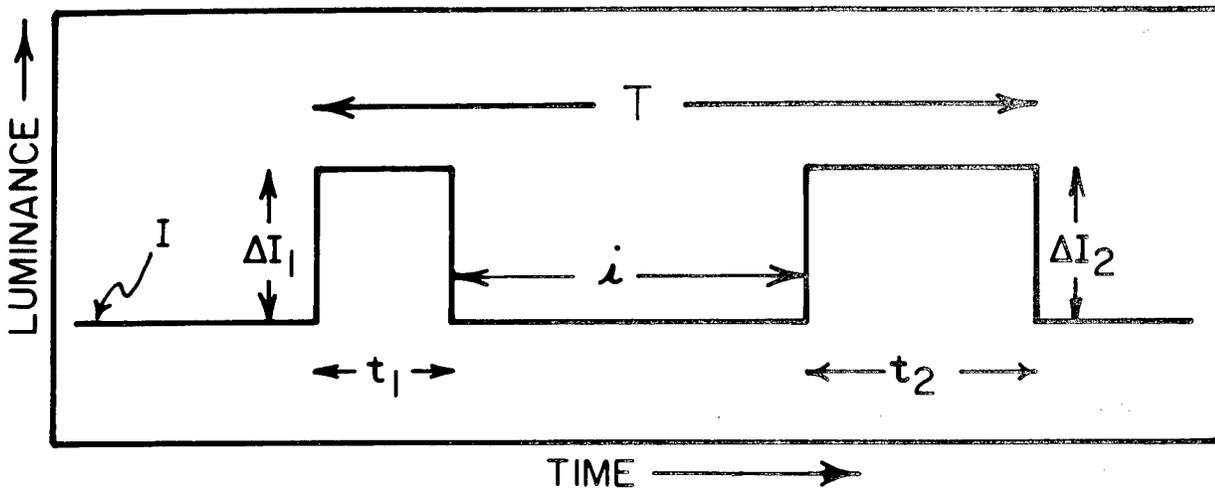


Figure 1. Definitions of symbols used to describe experimental procedures. \underline{I} = background luminance; \underline{t}_1 and \underline{t}_2 , durations of first and second flashes; $\underline{\Delta I}_1$ and $\underline{\Delta I}_2$, luminances of first and second flashes; \underline{i} , time between flashes; \underline{T} , total display time.

In addition to the problem of two flashes (increments), the studies reported examined other problems. For example, Ikeda³ and Rashbass⁶ also studied two decrements, an increment followed by a decrement, and a decrement followed by an increment. The present study, limited to the problem of two increments, examines this problem in greater detail. Thus, several \underline{i} values are used, and, at each \underline{i} value, \underline{t}_2 thresholds are determined for several values of \underline{t}_1 .

METHOD

Apparatus

The optical system and associated equipment, and the calibration procedures have been described earlier.² One arm of the system presented, in Maxwellian view, a continuously-exposed background in the form of a disk of white light, $1^\circ 7'$ visual angle in diameter, at an accommodative distance of 57 cm. A $5'$ black spot, centered in the $1^\circ 7'$ field, served as a fixation point for monocular viewing. The field was surrounded by complete darkness. The second arm of the optical system added two flashes of white light, successively, to the whole $1^\circ 7'$ background field. The durations of the flashes, \underline{t}_1 and \underline{t}_2 , as well as the interval between the flashes, \underline{i} , could be varied independently by a pulse generator that operated a galvanometer mirror system.

Procedure

The general procedure for each of two observers was as follows. In each session, before threshold determinations began, the observer dark adapted for 5 min, and then light adapted to the $1^\circ 7'$ background field for 5 min. On the observer's command, two flashes of the same luminance ($\underline{\Delta I}_1 = \underline{\Delta I}_2$) were added, successively, to the whole $1^\circ 7'$ background field, and the observer reported "Yes" if he detected any change in the

background field, or "No" if he detected no change. A modified method of limits was used, with adjacent steps of the variable differing by 0.40 ms. For each condition within a session, four thresholds were obtained.

Intermediate background luminance. The background luminance was 1.19 mL, and the luminance of each flash was 0.389 mL. (Added to a background of 1.19 mL, a single flash of 0.389 mL had to last about 5 ms to be detected.)

In each of 18 sessions, \underline{i} was set at some constant value, \underline{t}_1 was set at each of nine values, and, for each setting of \underline{t}_1 , the threshold value of \underline{t}_2 was determined. For example, in one session, \underline{i} was set at 25 ms. First, \underline{t}_1 was set at 2.4 ms and the value of \underline{t}_2 required for detection was found to be 3.0 ms. Then, with \underline{t}_1 still at 2.4 ms, the threshold \underline{t}_2 , determined again, was 2.6 ms. Next, with \underline{t}_1 set at 0.6 ms, the \underline{t}_2 threshold was determined twice. Then, with \underline{t}_1 set at 4.2 ms, the \underline{t}_2 threshold was determined twice; etc. The \underline{t}_1 values used were 0.0, 0.6, 1.2, 1.8, 2.4, 3.0, 3.6, 4.2, and 4.8 ms. The nine \underline{t}_1 values were given in a random order with two \underline{t}_2 thresholds determined at each \underline{t}_1 value, and then the random order was reversed, and two more \underline{t}_2 thresholds were obtained at each \underline{t}_1 value. Thus, in each session, for each \underline{t}_1 value, four \underline{t}_2 thresholds were obtained. (Occasionally, four thresholds could not be obtained because an observer detected a flash when \underline{t}_2 was zero.) The \underline{i} values used were 5, 15, 25, 30, 35, 40, 45, 50, and 60 ms, and each \underline{i} value was used in two sessions.

Data were also collected in sessions when (a) \underline{i} was constant, and, for different settings of \underline{t}_2 , the \underline{t}_1 threshold was determined, (b) for different combinations of \underline{t}_1 and \underline{i} , the \underline{t}_2 threshold was determined,

(c) for different combinations of $\underline{t_2}$ and \underline{i} , the $\underline{t_1}$ threshold was determined, and (d) for different settings of \underline{i} , with $\underline{t_1} = \underline{t_2}$, $\underline{t_1}$ and $\underline{t_2}$ were varied, concomitantly, to determine threshold.

Low background luminance. The background luminance was 0.118 mL, and the luminance of each flash was 0.112 mL.

In each of seven sessions, \underline{i} was set at some constant value (5, 25, 35, 45, 55, 65, or 75 ms), $\underline{t_1}$ was set at each of nine values, and, for each setting of $\underline{t_1}$, the threshold value of $\underline{t_2}$ was determined. In addition, in two sessions, for different settings of \underline{i} , with $\underline{t_1} = \underline{t_2}$, $\underline{t_1}$ and $\underline{t_2}$ were varied, concomitantly, to determine threshold.

RESULTS AND DISCUSSION

The data obtained at the intermediate background luminance will be presented, and then a model, based on the data, will be derived. In the presentation of the data, two key definitions are used. The energy, \underline{E} , provided by the two flashes is defined as:

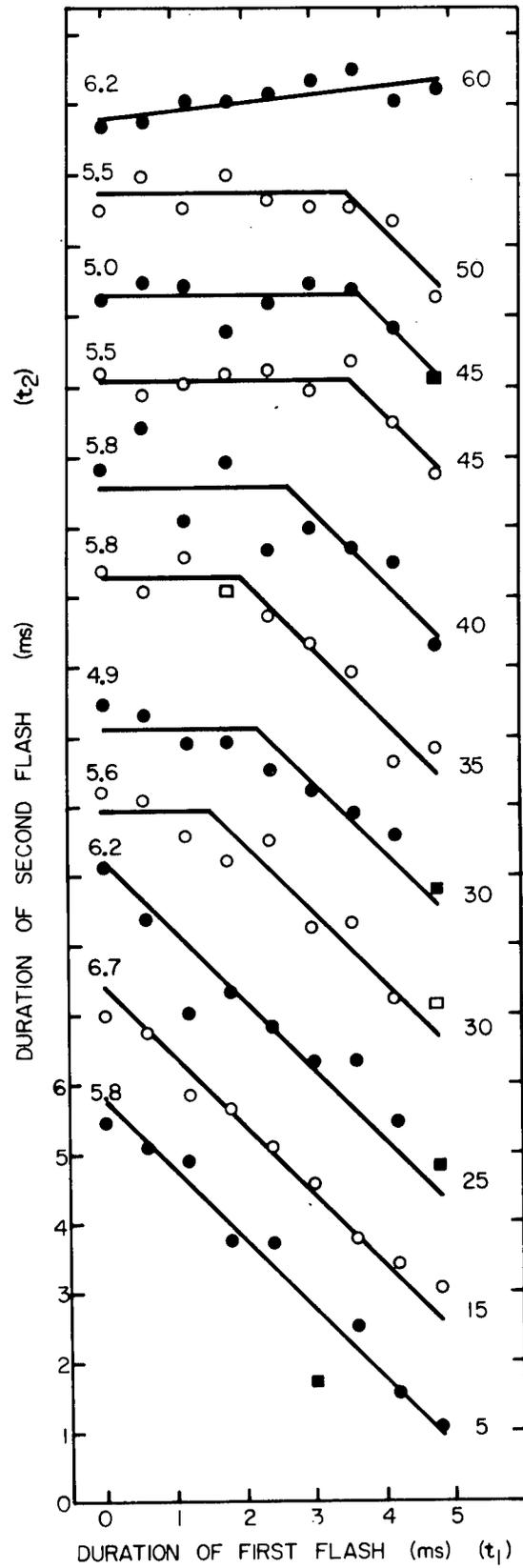
$$E = \Delta I_1 t_1 + \Delta I_2 t_2. \quad (1)$$

The average luminance, $\underline{A_L}$, provided by the two flashes during the time \underline{T} is:

$$A_L = \frac{\Delta I_1 t_1 + \Delta I_2 t_2}{t_1 + i + t_2} = \frac{E}{T} \quad (2)$$

With $\underline{\Delta I_1}$ and $\underline{\Delta I_2}$ in millilamberts, and $\underline{t_1}$, $\underline{t_2}$, and \underline{i} in milliseconds, the units of \underline{E} are millilambert milliseconds, and the units of $\underline{A_L}$ are millilamberts.

Figure 2. Threshold duration of the second flash as a function of the duration of the first flash. The number on the right of each curve gives the time between the two flashes, \underline{i} , in milliseconds. For clarity, all the curves except the lowest one are displaced upward. The number on the left of each curve gives the \underline{t}_2 intercept of the associated straight line, and thus locates each curve with respect to the vertical axis. Each circle represents the mean of four \underline{t}_2 thresholds; each square represents the mean of either two or three thresholds. $\underline{I} = 1.19$ mL; $\underline{\Delta I}_1 = \underline{\Delta I}_2 = 0.389$ mL. Each curve gives the data of one session of Observer C.T.



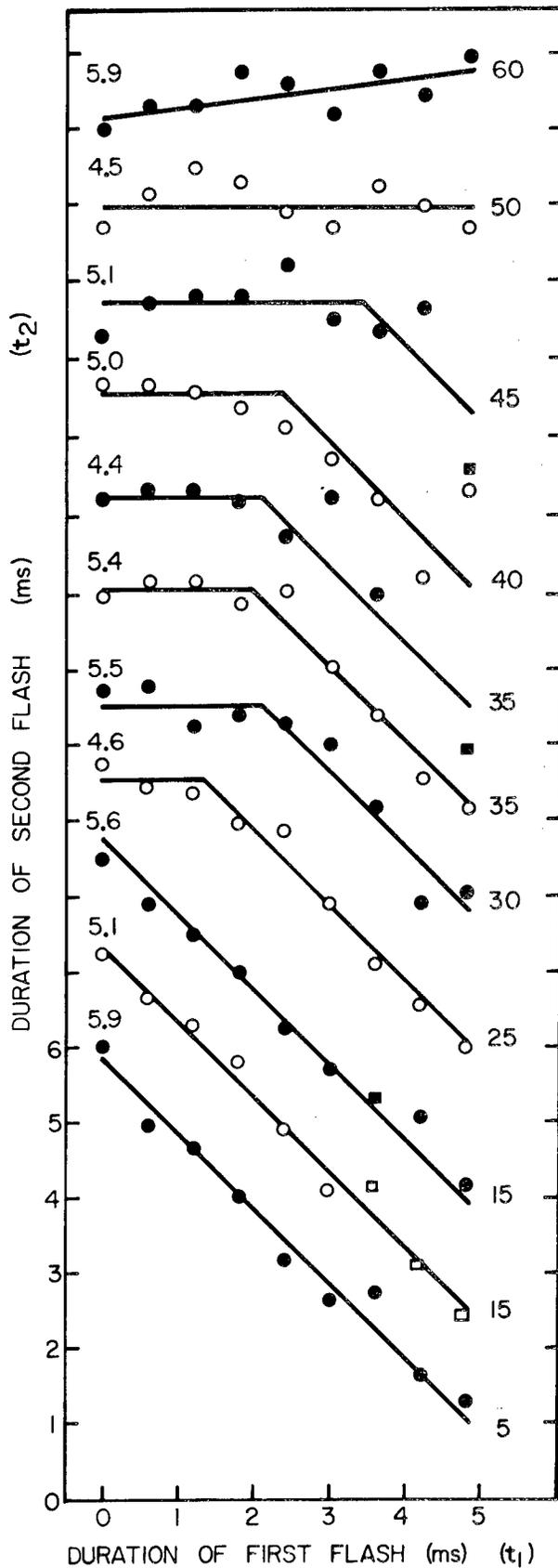


Figure 3. Threshold duration of the second flash as a function of the duration of the first flash. Notations are the same as those of Figure 2.

Data of Observer A. L.

Intermediate Background Luminance

Figures 2 and 3 show how two flashes interacted to achieve threshold. In Fig. 2, both of the sessions at $\underline{i} = 30$ ms and both of the sessions at $\underline{i} = 45$ ms are included to indicate session-to-session variability. Also, Fig. 3 gives both sessions for $\underline{i} = 15$ ms and both for $\underline{i} = 35$ ms.

Because the first point of each curve in Figs. 2 and 3 is at $\underline{t}_1 = 0$, this point represents the threshold duration for a single flash. For the 18 sessions, the distribution of 18 single-flash threshold durations gave a mean of 5.54 ms and a standard deviation of 0.451 ms for Observer C.T., and a mean of 5.04 ms and a standard deviation of 0.483 ms for Observer A.L.

Figure 4 shows data obtained when \underline{t}_2 was set at each of several values and the threshold value of \underline{t}_1 was determined. For two identical flashes, Fig. 5 shows how threshold energy varies as a function of \underline{T} .

Table I gives data obtained with different combinations of \underline{i} and \underline{t}_1 and transformed to the average luminance measure defined in Equation 2. For example, for Observer A.L., with $\underline{i} = 30$ ms and $\underline{t}_1 = 1.5$ ms, the mean threshold value of \underline{t}_2 was 4.50 ms. When these values are substituted in Equation (2), $\underline{A}_L = 0.065$ mL, as indicated in Table I. For another set of \underline{t}_1 and \underline{i} combinations used in another session, the data of Observer A.L. gave a mean $\underline{A}_L = 0.066$ mL with a standard deviation of 0.0042 mL, and the data of Observer C.T. gave a mean $\underline{A}_L = 0.069$ mL with a standard deviation of 0.0049 mL. Different combinations of \underline{t}_2 and \underline{i} were also used to obtain threshold \underline{t}_1 values, and the data were converted to the \underline{A}_L measure. For one session, for Observer A.L., the distribution of \underline{A}_L values had a mean of 0.065 mL, and a standard deviation of 0.0049; for one session for Observer C.T. the mean was

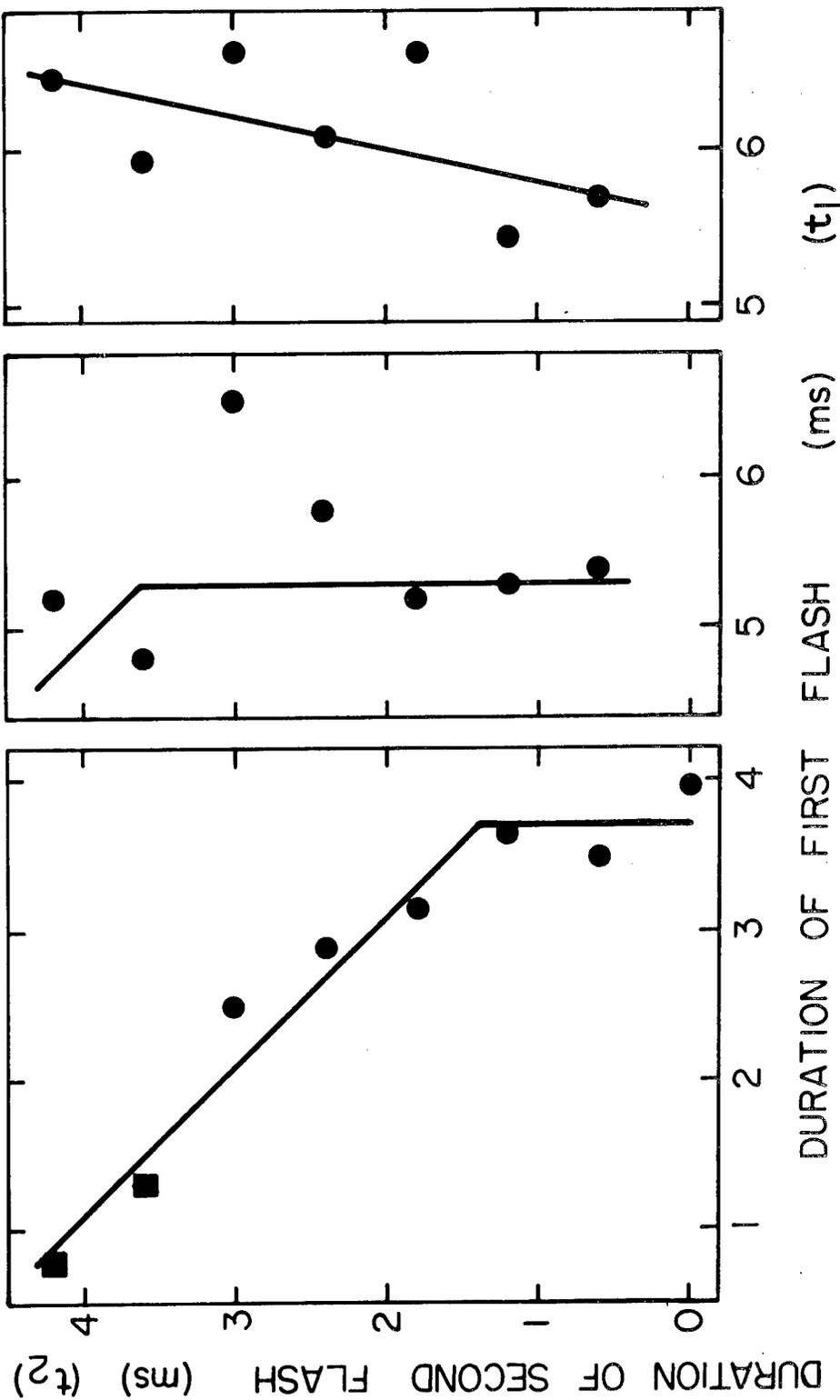


Figure 4. Threshold duration of the first flash as a function of the duration of the second flash. Left, $i = 35$ ms; center, $i = 45$ ms; right, $i = 60$ ms. $I = 1.19$ mL; $\Delta I_1 = \Delta I_2 = 0.389$ mL. Each circle represents the mean of four t_1 thresholds; each square represents the mean of either two or three t_1 thresholds. Each box gives the data of one session of Observer A. L.

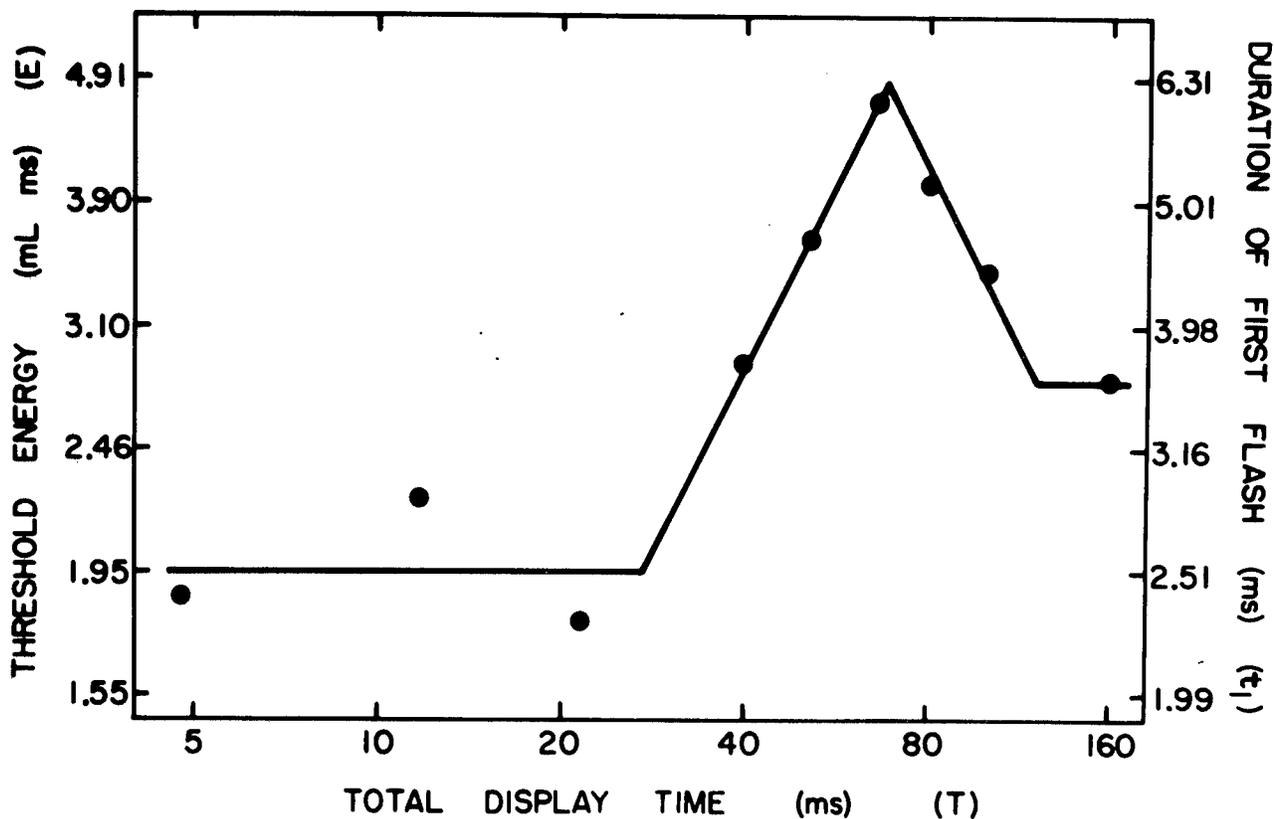


Figure 5. Threshold energy for two identical flashes as a function of total display time, on a log-log plot. Vertical markings spaced at 0.1 log unit; horizontal markings spaced at 0.3 log unit. From left to right, slopes of the four straight lines are 0.0, 1.0, -1.0, and 0.0. $\bar{I} = 1.19$ mL; $\Delta I_1 = \Delta I_2 = 0.389$ mL; $\bar{t}_1 = \bar{t}_2$. For the first point on the left, $\bar{i} = 0$. Data of one session of Observer A. L. TEpee effect.

Table I. Average luminance ($\underline{A_L}$) required for threshold for various combinations of \underline{i} and $\underline{t_1}$.^a

($\underline{I} = 1.19 \text{ mL}$; $\Delta I_1 = \Delta I_2 = 0.389 \text{ mL}$)

Time between flashes, \underline{i} (ms)	Duration of first flash, $\underline{t_1}$ (ms)	Threshold average luminance (mL)	
		Observer A.L.	Observer C.T.
30	1.5	.065	.069
30	2.7	.067	.074
30	2.8	.065	.076
40	2.0	.057	.061
40	3.2	.068	.067
40	3.8	.061	.067
40	4.2	.060	.073
50	3.8	.059	.064
Mean		.063	.069
Standard Deviation		.0040	.0052

^aSee Fig. 1 and Equation (2) for definitions of terms.

0.073, and the standard deviation was 0.0039.

Derivation of Model

On the basis of (a) the data presented above, (b) the data summarized by the TEpee effect,² and (c) the data of others, particularly those of Ikeda³ and Rashbass,⁶ a model was derived to describe the interaction of two flashes. In the development of the model, equations to describe the data were derived for low \underline{i} values, for intermediate \underline{i} values, and for the \underline{i} value at which the maximum threshold energy is required.

Low \underline{i} values. In Figs. 2 and 3, at \underline{i} values of 5, 15, and (for Observer C.T.) 25 ms, the data of each session were fitted with a straight line with a slope of -1.0. A line with a slope of -1.0 means that, for any value of \underline{t}_1 , $(\underline{t}_1 + \underline{t}_2)$ equals a constant. The value of the constant may be specified by the case when $\underline{t}_1 = 0$. When $\underline{t}_1 = 0$, $\underline{t}_2 = \underline{t}_S$, where \underline{t}_S is the threshold duration of a single flash. Thus, in general, at any low \underline{i} value, $(\underline{t}_1 + \underline{t}_2) = \underline{t}_S$. The first straight line of Fig. 5 also indicates that, for lower \underline{i} values, $(\underline{t}_1 + \underline{t}_2) = \underline{t}_S$. As noted above, the data of others³⁻⁶ also indicate that, at low \underline{i} values, it matters not how the energy is distributed between the two flashes.

Intermediate \underline{i} values. In Figs. 2 and 3, at the intermediate \underline{i} values, $\underline{i} = 30$ to $\underline{i} = 50$ ms, two straight lines were fitted to the data of each session. The first straight line, with a slope of zero, has, for any \underline{i} value, $\underline{t}_2 = \underline{t}_S$ when $\underline{t}_1 = 0$. Therefore, at any intermediate \underline{i} value, $\underline{t}_2 = \underline{t}_S$ for all \underline{t}_1 values from zero to some particular \underline{t}_1 value.

The second straight line has a slope of -1.0, which means that

$(\underline{t}_1 + \underline{t}_2)$ equals a constant. When $(\underline{t}_1 + \underline{t}_2)$ is constant, with $\underline{\Delta I}_1 = \underline{\Delta I}_2 = \text{constant}$, and \underline{i} constant, \underline{A}_L is constant. This deduction is perhaps more readily seen when Equation (2) is rearranged to the form:

$$\underline{A}_L = \frac{\underline{\Delta I}_1 (\underline{t}_1 + \underline{t}_2)}{(\underline{t}_1 + \underline{t}_2) + \underline{i}} = \frac{\underline{\Delta I}_1}{1 + \frac{\underline{i}}{(\underline{t}_1 + \underline{t}_2)}} \quad (3)$$

Thus, associated with each \underline{i} value, each line with a -1.0 slope, represents cases in which the average luminance is constant.

For the different intermediate \underline{i} values, do all these lines with a -1.0 slope represent the same average luminance? The answer, which is Yes, is based on three sources of evidence. First, from Equation (3), we see that, if \underline{A}_L remains constant, $(\underline{t}_1 + \underline{t}_2)$ must increase as \underline{i} increases. Such an increase in $(\underline{t}_1 + \underline{t}_2)$ is evident in the data of Figs. 2 and 3. Second, the data of Table I, and similar data mentioned in the text, indicate the constancy of \underline{A}_L for various combinations of \underline{t}_1 and \underline{i} that fall within the area described by the lines with the -1.0 slope. Third, in Fig. 5, the straight line with the +1.0 slope represents the equation, $\log \underline{E} = \log \underline{k} + \log \underline{T}$, or $\underline{E}/\underline{T} = \underline{k}$, where \underline{k} is a constant. Since $\underline{E}/\underline{T} = \underline{A}_L$, \underline{A}_L is constant for the different intermediate \underline{i} values.

The data of the three intermediate \underline{i} values of Rashbass,⁶ and the data of two of the three intermediate \underline{i} values of Ikeda³ are well described by lines with slopes of 0 and -1.0.

In Figs. 2 and 3 the curves terminated when $\underline{t}_1 = \underline{t}_S$ because detection occurred when the first flash alone was presented. In Fig. 4, however, \underline{t}_2 was set at each of several values, and the threshold value of \underline{t}_1 was determined. The first two boxes of Fig. 4 represent intermediate

\underline{i} values. The data in these two boxes indicate that, following the line with a -1.0 slope, a vertical line is a fair fit. This vertical line represents the equation, $\underline{t}_1 = \underline{t}_S$ for various values of \underline{t}_2 . Similar findings are seen in the data of Ikeda³ and Rashbass⁶.

Value of \underline{i} requiring maximum energy. When the threshold is determined with two identical flashes, a maximum threshold energy is required at a particular \underline{i} value, \underline{i}_M . (See Fig. 5) In this section equations will be derived to relate \underline{t}_1 and \underline{t}_2 at \underline{i}_M whether the flashes are identical or not. First, equations will be derived that relate variables for identical flashes. Then, these equations will be used to derive equations for the case of non-identical flashes.

Consider the first two straight lines of Fig. 5, the figure for identical flashes. The first straight line represents the relation, $(\underline{t}_1 + \underline{t}_2) = \underline{t}_S$. The longest \underline{T} value at which this equation holds will be called the critical duration, \underline{T}_C . If we call the \underline{i} value at \underline{T}_C , \underline{i}_C , then $\underline{T}_C = \underline{t}_S + \underline{i}_C$. The second straight line, where \underline{A}_L is constant, extends from \underline{T}_C to a \underline{T} value, \underline{T}_M , at which the maximum energy is required for threshold. At \underline{T}_M , $\underline{t}_1 = \underline{t}_{1M}$, and $\underline{i} = \underline{i}_M$. Thus, with $\underline{t}_1 = \underline{t}_2$, $\underline{T}_M = 2\underline{t}_{1M} + \underline{i}_M$.

How are \underline{t}_{1M} and \underline{t}_S related? And how are \underline{i}_C and \underline{i}_M related? The \underline{A}_L is the same at \underline{T}_C and at \underline{T}_M . Therefore,

$$\underline{A}_L = \frac{\Delta I_1 \underline{t}_1 + \Delta I_2 \underline{t}_2}{\underline{t}_1 + \underline{i} + \underline{t}_2} = \frac{\Delta I_1 \underline{t}_S}{\underline{t}_S + \underline{i}_C} = \frac{\Delta I_1 \ 2\underline{t}_{1M}}{2\underline{t}_{1M} + \underline{i}_M}, \quad (4)$$

and, after simplification,

$$i_M = \frac{i_C(2t_{1M})}{t_S} \quad (5)$$

Based upon many sessions of the previous study,² the difference between $\log T_M$ and $\log T_C$ averaged 0.34; $\log T_M = \log T_C + 0.34$. Taking this average value of 0.34 as the best estimate, we can write $T_M = 2.1877 T_C$, or,

$$2t_{1M} + i_M = 2.1877(t_S + i_C) \quad (6)$$

Substituting, in Equation (6), the value of i_M given in Equation (5), and simplifying, indicates

$$t_{1M} = 1.0939 t_S, \quad (7)$$

and, substituting this value for t_{1M} in Equation (5) shows

$$i_M = 2.1877 i_C \quad (8)$$

In sum, when $\frac{\Delta I_1}{i_1} = \frac{\Delta I_2}{i_2} = \text{constant}$, and $t_1 = t_2$, the maximum threshold energy is required when $i = 2.1877 i_C$, and, at that i value, $t_1 = t_2 = 1.0939 t_S$.

[The earlier paper² indicated that two identical flashes at threshold appeared as one at i values below i_M . Above i_M , two identical flashes at threshold were seen, at least occasionally, as two. The i values of Figs. 2 and 3 were limited to i values at and below i_M . In the present experiment, for the two observers, from six sessions like

the one represented in Fig. 5, \underline{i}_M ranged from 55.18 to 59.59 ms. On the basis of these estimates of \underline{i}_M , for the sessions represented in Figs. 2 and 3, the longest \underline{i} value selected was 60 ms.]

In Figs. 2 and 3, at $\underline{i} = 60$ ms, which is approximately equal to \underline{i}_M , \underline{t}_2 increases linearly as \underline{t}_1 increases. If \underline{t}_2 increases linearly with \underline{t}_1 , the equation relating \underline{t}_1 and \underline{t}_2 may be derived from the point at $\underline{t}_1 = 0$ and the point at $\underline{t}_1 = \underline{t}_{1M}$. The coordinates of the first point are $\underline{t}_1 = 0$, $\underline{t}_2 = \underline{t}_S$; the coordinates of the second point are $\underline{t}_1 = 1.0939 \underline{t}_S$, $\underline{t}_2 = 1.0939 \underline{t}_S$. Using these two points, the slope of the straight line is $(1.0939 \underline{t}_S - \underline{t}_S)/1.0939 \underline{t}_S$, or 0.086. The intercept of the straight line is \underline{t}_S . Therefore, at \underline{i}_M ,

$$\underline{t}_2 = \underline{t}_S + 0.086 \underline{t}_1 . \quad (9)$$

The data at $\underline{i} = 60$ ms (see Figs. 2 and 3 for examples) were fitted with straight lines by the method of least squares. For Observer A.L., the two sessions gave lines with slopes of 0.172 and 0.128. For Observer C.T., for the two sessions, the slopes were 0.114 and 0.040. Bearing in mind \underline{i}_M varies somewhat from session-to-session, and the value of $(\log \underline{T}_M - \log \underline{T}_C)$, which determines the slope constant of Equation (9), also varies from session-to-session, we conclude that the slopes obtained experimentally agree quite well with the expected slope of 0.086.

The right box of Fig. 4 illustrates, for $\underline{i} = 60$ ms, the threshold \underline{t}_1 required for different \underline{t}_2 settings. For such data, by steps similar to those just given, \underline{t}_1 and \underline{t}_2 may be related by the equation

$$\underline{t}_1 = \underline{t}_S + 0.086 \underline{t}_2 \quad (10)$$

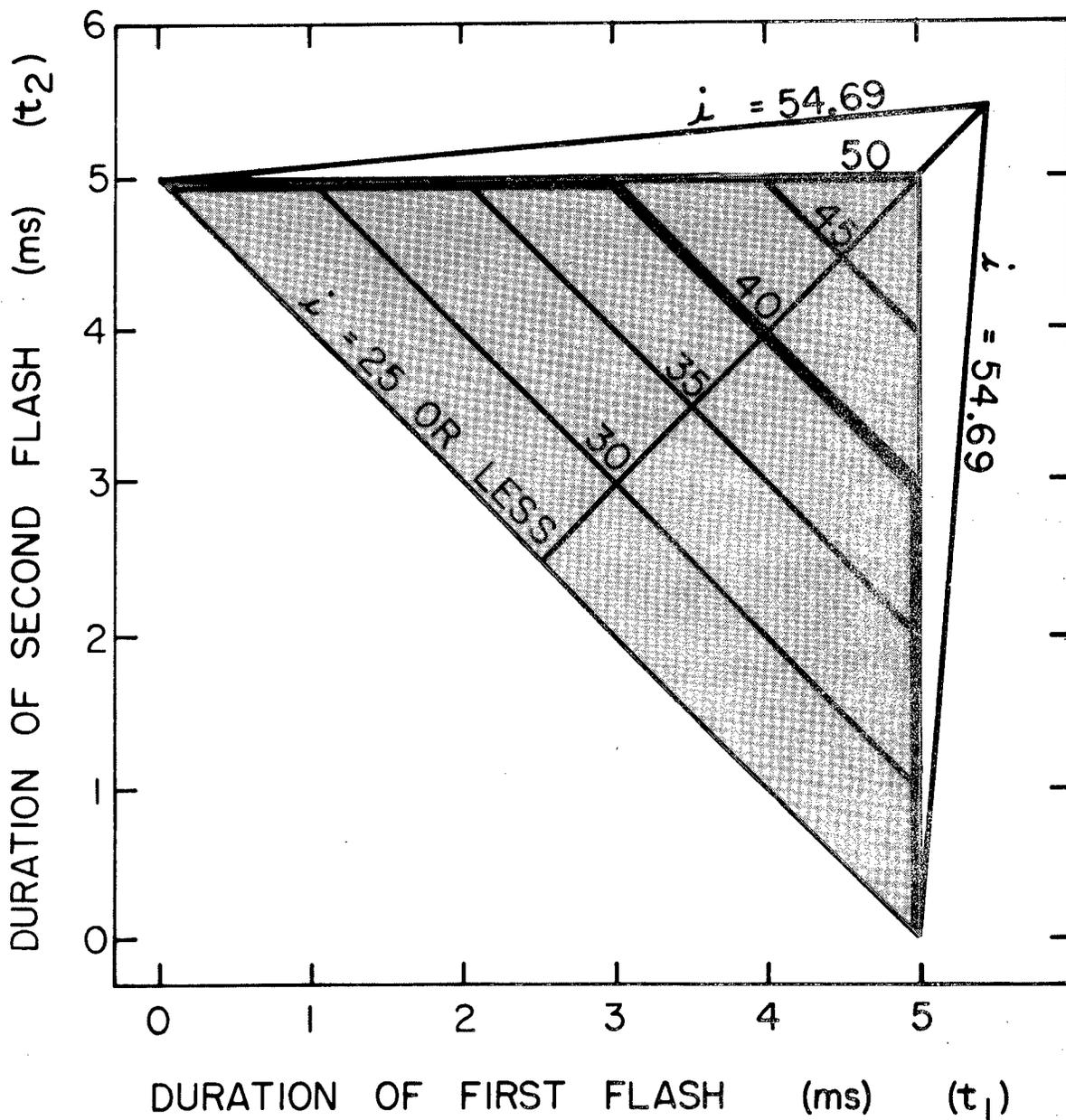


Figure 6. Generalized model describing how two flashes interact to achieve a threshold. The numbers give the time between the two flashes, \underline{i} , in milliseconds. The line with a slope of +1.0 represents cases when $\underline{t}_1 = \underline{t}_2$. Three heavy lines represent one particular \underline{i} value, $\underline{i} = 40$ ms. The shaded area enclosed by the right isosceles triangle represents cases in which the threshold average luminance, \underline{A}_L , contributed by the two flashes, during the period \underline{T} , is constant. In this plot, $\underline{\Delta I}_1 = \underline{\Delta I}_2$, $\underline{t}_S = 5.0$ ms, $\underline{i}_C = 25$ ms, $\underline{i}_M = 54.69$ ms.

Table II. Equations describing how two flashes interact to achieve threshold when $\underline{\Delta I}_1 = \underline{\Delta I}_2 = \text{constant}$.^a

Limits of \underline{i}	Limits of \underline{t}_1 or \underline{t}_2	Equation
$0 \leq \underline{i} \leq \underline{i}_C$	$0 \leq \underline{t}_1 \leq \underline{t}_S$	$\underline{t}_1 + \underline{t}_2 = \underline{t}_S$
	$0 \leq \underline{t}_1 \leq \underline{t}_S \left(\frac{\underline{i}}{\underline{i}_C} - 1 \right)$	$\underline{t}_2 = \underline{t}_S$
$\underline{i}_C \leq \underline{i} \leq 2 \underline{i}_C$	$\underline{t}_S \left(\frac{\underline{i}}{\underline{i}_C} - 1 \right) \leq \underline{t}_1 \leq \underline{t}_S$	$\underline{t}_1 + \underline{t}_2 = \left(\frac{\underline{t}_S}{\underline{i}_C} \right) \underline{i}$ ($A_L = \text{constant}$)
	$0 \leq \underline{t}_2 \leq \underline{t}_S \left(\frac{\underline{i}}{\underline{i}_C} - 1 \right)$	$\underline{t}_1 = \underline{t}_S$
$\underline{i} = 2.188 \underline{i}_C$	$0 \leq \underline{t}_1 \leq 1.094 \underline{t}_S$	$\underline{t}_2 = \underline{t}_S + .086 \underline{t}_1$
	$0 \leq \underline{t}_2 \leq 1.094 \underline{t}_S$	$\underline{t}_1 = \underline{t}_S + .086 \underline{t}_2$

^a See Fig. 1 for definitions of $\underline{\Delta I}_1$, $\underline{\Delta I}_2$, \underline{t}_1 , \underline{t}_2 , and \underline{i} . \underline{t}_S is the time a single flash must last to achieve threshold. \underline{i}_C is the longest time between two flashes for which the threshold energy is constant.

Model for Two Increments

All of the relationships described above are brought together in a numerical example given in Fig. 6. The equations describing the model in general terms are summarized in Table II. Most of the equations given were derived in the preceding section. The limits within which each equation holds are given in Table II. [In Fig. 6 and Table II, it is assumed that $\log \frac{T_M}{T_C} - \log \frac{T_C}{T_C} = 0.34$.] Note that the model requires only two constants, $\underline{t_S}$ and $\underline{i_C}$, and that the model is symmetrical with respect to the line representing cases where $\underline{t_1} = \underline{t_2}$.

An application of the Table II equations to the numerical example of Fig. 6 is as follows. Say $\underline{i} = 40$ ms. With $\underline{i} = 40$ ms, \underline{i} falls between $\underline{i_C}$ and $2 \underline{i_C}$, so the equations given in the 2nd, 3rd, and 4th rows of Table II apply. In the 2nd row, $t_S [(i/i_C) - 1] = 5 [(40/25) - 1] = 3.0$ ms. Thus, when $0 \leq t_1 \leq 3.0$ ms, $t_2 = t_S = 5.0$ ms. The 3rd row of Table II indicates that when $3.0 \leq t_1 \leq 5.0$ ms, $t_1 + t_2 = [t_S/i_C]i = [5/25]40 = 8.0$ ms. The 4th row of Table II indicates that when $0 \leq t_2 \leq 3.0$ ms, $t_1 = 5.0$ ms.

The model of Fig. 6 represents the case where $\underline{\Delta I_1} = \underline{\Delta I_2} = \text{constant}$, and $\underline{t_1}$ and $\underline{t_2}$ are varied. An analogous model can be derived for the case where $\underline{t_1} = \underline{t_2} = \text{constant}$, and $\underline{\Delta I_1}$ and $\underline{\Delta I_2}$ are varied, as in the studies of Ikeda³ and Rashbass.⁶ Also, the equations of Table II may be rewritten in units of energy. The model applies to cases where either the luminances of both flashes are equal or the durations of both flashes are equal. Whether the model applies to cases where both luminance and duration differ ($\underline{\Delta I_1} \neq \underline{\Delta I_2}$ and $\underline{t_1} \neq \underline{t_2}$) is an experimental question yet to be answered.

The model applies only to \underline{i} values from 0 to $\underline{i_M}$. Beyond $\underline{i_M}$, the two flashes, at threshold, are sometimes seen as two, and this suggests

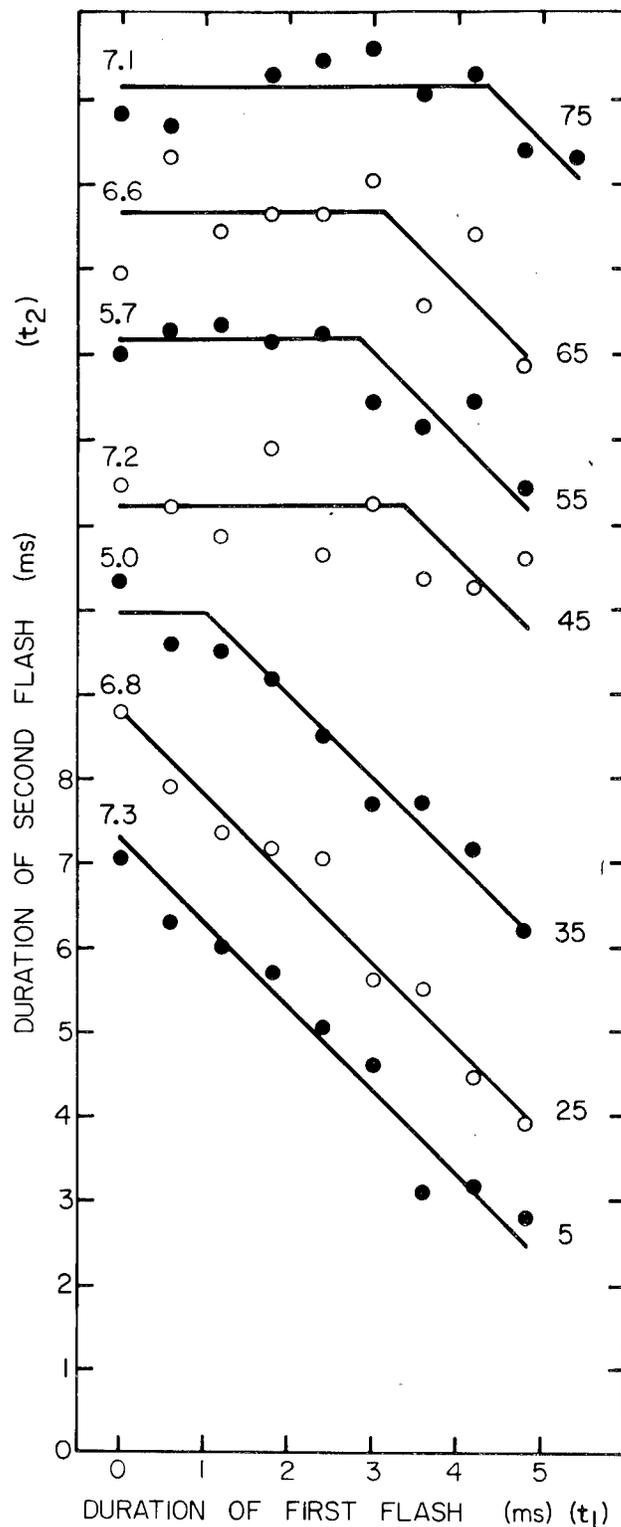


Figure 7. Threshold duration of the second flash as a function of the duration of the first flash. The number on the right of each curve gives the time between the two flashes, \underline{i} , in milliseconds. For clarity, all the curves except the lowest one are displaced upward. The number on the left of each curve gives the $\underline{t_2}$ intercept of the associated straight line. Each experimental point represents the mean of four $\underline{t_2}$ thresholds. $\underline{I} = 0.118$ mL; $\underline{\Delta I_1} = \underline{\Delta I_2} = 0.112$ mL. Each curve gives data of one session of Observer C.T.

that beyond \underline{i}_M , some other mechanisms may play a role in determining experimental results.

For \underline{i} values beyond \underline{i}_M , Fig. 5 provides information for identical flashes. The lines with the +1.0 and -1.0 slopes in Fig. 5 indicate that at $\log \underline{T}$ values equally below or above $\log \underline{T}_M$, the same threshold energy is required. This is a general finding of the earlier study² with identical flashes. Consider this finding with respect to Fig. 6. In Fig. 6, $\underline{T}_C = 30$ ms, so $\log \underline{T}_M = (\log 30) + 0.34 = 1.8171$. $\log \underline{T}_M \pm 0.0389$ gives $\log \underline{T}$ values of 1.7782 and 1.8560, or \underline{T} values of 60 and 71.78 ms. Thus, the threshold energy is the same at $\underline{T} = 60$ ms and at $\underline{T} = 71.78$ ms; i.e., when \underline{i} equals either 50 or 61.78 ms, $\underline{t}_1 = \underline{t}_2 = 5$ ms. Similarly, when \underline{i} equals either 40 or 81.75 ms, $\underline{t}_1 = \underline{t}_2 = 4.0$ ms.

An alternative to the model given here has been provided by Rashbass⁶. According to his model, each set of data points of Figs. 2 and 3 is fitted by a segment of an ellipse, with the center of the ellipse at the origin ($\underline{t}_1 = \underline{t}_2 = 0$) and the \underline{t}_1 and \underline{t}_2 intercepts equal. Rashbass's model and the present model fit the data of the present experiment and the data of Rashbass's experiment equally well. (A simplification of Rashbass's model can be achieved by interrelating the family of ellipses by the equations of the TEpee effect.)

Low Background Luminance

The data of Fig. 7, and similar data for the other observer, suggest that the model is applicable to low as well as to intermediate background luminances.

REFERENCES

- ¹ M.A. Bouman and G. van den Brink, *J. Opt. Soc. Am.* 42, 617 (1952); E. Davy, *J. Opt. Soc. Am.* 42, 937 (1952); G. van den Brink and M.A. Bouman, *J. Opt. Soc. Am.* 44, 616 (1954); W.C. Clark, Ph.D. thesis, Univ. of Mich., (1958), University Microfilm No. MIC 58-7807; H.R. Blackwell, *J. Opt. Soc. Am.* 53, 129 (1963); H. Schuckman and J. Orbach, *Science*, 150, 1623 (1965); T. Uetsuki and M. Ikeda, *J. Opt. Soc. Am.* 60, 577 (1970); M. Grossberg, *Percept. Psychophys.* 7, 377 (1970).
- ² R.M. Herrick, *J. Opt. Soc. Am.* (in press).
- ³ M. Ikeda, *J. Opt. Soc. Am.* 55, 1527 (1965).
- ⁴ R.T. Kintz and R.M. Boynton, *J. Opt. Soc. Am.* 59, 212 (1969).
- ⁵ W.S. Battersby and G.L. Defabaugh, *Vision Res.* 9, 757 (1969).
- ⁶ C. Rashbass, *J. Physiol. (London)* 210, 165 (1970).
- ⁷ R. Granit and W.A. Davis, *Am. J. Physiol.* 98, 644 (1931).

Appendixes

Appendix A, pages 25-40, provides tables of all the data collected in the experiment.

Appendix B, pages 41-51, describes the logic and the results of statistical analysis performed on the data.

Table 1A

Mean threshold duration of each flash (msec) when $t_1 = t_2$.

Log I (mL) = 0.075. Log ΔI_1 (mL) = Log ΔI_2 = -0.41. Observer A.L.

Time between flashes, i (msec)	Session				
	1	2	3	4	5
0	2.57	2.76	-----	2.40	2.75
6	-----	-----	-----	2.90	3.20
8	-----	3.58	-----	-----	-----
17	2.74	3.35	-----	2.30	3.45
30	-----	-----	3.20	-----	-----
32	-----	-----	-----	3.75	3.40
35	4.10	4.47	3.80	-----	-----
40	-----	-----	4.65	-----	-----
42	4.07	5.31	-----	4.70	5.15
48	-----	-----	4.90	-----	-----
54	4.75	5.61	-----	6.05	5.35
55	-----	-----	5.10	-----	-----
70	5.00	6.99	5.40	5.20	5.10
85	-----	-----	4.45	-----	-----
92	4.00	5.10	-----	4.40	4.30
100	-----	-----	3.90	-----	-----
110	4.33	-----	-----	-----	-----
150	-----	-----	-----	3.60	4.15

Note.--In sessions 1 and 2 data were collected by the double-randomized, up-and-down method, and each table entry is the mean of at least 17 "crossings". In sessions 3, 4, and 5 data were collected by a modified method of limits, and each table entry is the mean of four thresholds.

Table 2A

Mean threshold duration of each flash (msec) when $\underline{t}_1 = \underline{t}_2$.

Log I (mL) = 0.075. Log ΔI_1 (mL) = Log ΔI_2 = -0.41. Observer C.T.

Time between flashes, \underline{i} (msec)	Session				
	1	2	3	4	5
0	2.30	3.12	—	3.25	2.85
6	—	—	—	2.80	3.05
8	—	3.21	—	—	—
17	3.37	3.35	—	3.05	3.10
30	—	—	3.75	—	—
32	—	—	—	4.00	4.15
35	4.23	4.21	4.00	—	—
40	—	—	4.35	—	—
42	4.45	4.37	—	4.45	4.50
48	—	—	5.40	—	—
54	7.29	5.33	—	5.15	5.15
55	—	—	6.15	—	—
70	5.47	5.99	5.85	5.50	5.15
85	—	—	5.00	—	—
92	5.27	5.45	—	4.90	4.70
100	—	—	4.90	—	—
110	5.00	—	—	—	—
150	—	—	—	4.95	4.65

Note.--In sessions 1 and 2 data were collected by the double-randomized, up-and-down method, and each table entry is the mean of at least 20 "crossings". In sessions 3, 4, and 5 data were collected by a modified method of limits, and each table entry is the mean of four thresholds.

TABLE 3 A

Mean Threshold Duration of Second Flash, t_2 , in Milliseconds
 When $\text{Log } I \text{ (mL)} = 0.075$, $\text{Log } \Delta I_1 \text{ (mL)} = \text{Log } \Delta I_2 = -0.41$. Observer A.L.

Time between flashes i (msec)	Duration of first flash, t_1 , in milliseconds								
	0.0	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8
5	4.85	4.10	4.05	3.50	2.80	2.25	2.30	1.35	0.73 ³
5	6.05	5.11	4.70	4.05	3.20	2.65	2.75	1.65	1.30
15	5.00	4.40	4.05	3.55	2.65	1.85	1.93 ³	0.87 ³	0.20 ²
15	5.30	4.70	4.30	3.80	3.05	2.50	2.13 ³	1.85	0.95
25	4.80	4.50	4.40	4.00	3.90	2.95	2.15	1.60	1.05
25	4.95	4.70	4.90	3.75	3.50	2.40	2.05	1.80 ³	0.87 ³
30	4.90	4.25	4.35	3.85	4.35	3.00	2.65	1.45	—
30	5.70	5.75	5.20	5.35	5.25	4.95	4.10	2.85	3.00
35	4.40	4.50	4.50	4.25	3.80	4.30	3.05	—	1.00 ²
35	5.30	5.50	5.50	5.20	5.35	4.35	3.70	2.85	2.45
40	5.70	5.55	5.30	4.85	5.25	4.75	4.70	4.45	3.95
40	5.10	5.10	5.00	4.80	4.55	4.15	3.65	2.65	3.75
45	5.30	5.30	4.90	5.00	5.00	4.35	4.65	3.90	4.30 ²
45	4.65	5.05	5.15	5.15	5.55	4.85	4.70	5.00	2.93 ³
50	4.20	4.65	5.00	4.80	4.40	4.20	4.75	4.50	4.20
50	5.25	5.30	5.95	5.60	5.05	5.30	6.00	5.10	5.15
60	4.45	5.45	5.20	5.40	6.40	5.30	6.25	5.25	5.65
60	4.75	5.05	5.05	5.50	5.35	4.95	5.50	5.20	5.70

Note: Each row of table gives data of one session. Each table entry is the mean of four thresholds, unless noted otherwise.

^{2,3}Occasionally, four thresholds were not obtained because the observer detected a flash when t_2 equalled zero; i.e., when only the first flash was presented. Superscript 3 indicates the mean is based on 3 thresholds; superscript 2 indicates the mean is based on 2 thresholds. An omission indicates one or no t_2 thresholds could be determined.

TABLE 4 A

Mean Threshold Duration of Second Flash, t_2 , in Milliseconds
 When $\text{Log } I \text{ (mL)} = 0.075$, $\text{Log } \Delta I_1 \text{ (mL)} = \text{Log } \Delta I_2 = -0.41$. Observer C.T.

Time between flashes i (msec)	Duration of first flash, t_1 , in milliseconds								
	0.0	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8
5	4.95	5.10	3.30	3.10	3.00	2.10	2.55	1.65	1.30
5	5.45	5.10	4.90	3.75	3.70	1.70 ²	2.50	1.55	1.05
15	5.00	4.50	4.30	4.05	3.35	2.85	2.25	1.20	1.55
15	6.30	6.05	5.15	4.95	4.40	3.85	3.05	2.70	2.35
25	5.70	4.95	5.15	4.50	4.55	4.00	2.90	2.40 ³	1.67 ³
25	6.10	5.35	4.00	4.30	3.80	3.30	3.30	2.45	1.80 ³
30	5.85	5.75	5.25	4.85	5.15	3.90	3.95	2.85	2.80 ³
30	5.25	5.10	4.70	4.70	4.30	4.10	3.65	3.35	2.53 ³
35	5.90	5.60	6.10	5.60 ³	5.25	4.85	4.45	3.15	3.35
35	5.20	6.05	5.65	5.00	6.25	4.50	4.50	4.15	2.05
40	5.45	4.95	4.95	5.45	5.10	5.10	4.60	4.55	4.90
40	6.05	6.65	5.30	6.15	4.90	5.20	4.90	4.70	3.50
45	4.95	5.15	5.10	4.45	4.85	5.15	5.05	4.50	3.80 ³
45	5.60	5.30	5.45	5.60	5.65	5.35	5.75	4.90	4.15
50	4.90	4.95	4.80	5.10	5.45	4.65	5.05	4.75	4.40
50	5.25	5.75	5.30	5.75	5.40	5.30	5.30	5.10	4.00
60	6.10	6.15	6.45	6.45	6.55	6.75	6.90	6.45	6.60
60	5.75	5.70	5.90	6.90	5.65	5.95	6.00	6.50	5.70

Note: Each row of table gives data of one session. Each table entry is the mean of four thresholds, unless noted otherwise.

^{2,3}Occasionally, four thresholds were not obtained because the observer detected a flash when t_2 equalled zero; i.e., when only the first flash was presented. Superscript 3 indicates the mean is based on 3 thresholds; superscript 2 indicates the mean is based on 2 thresholds. An omission indicates one or no t_2 thresholds could be determined.

TABLE 5A

Mean Threshold Duration of First Flash, t_1 , in Milliseconds
 When $\text{Log } I \text{ (mL)} = 0.075$, $\text{Log } \Delta I_1 \text{ (mL)} = \text{Log } \Delta I_2 = -0.41$. Observer A.L.

Time between flashes i (msec)	Duration of second flash, t_2 , in milliseconds						
	0.6	1.2	1.8	2.4	3.0	3.6	4.2
45	5.40	5.30	5.20	5.80	6.55	4.80	5.20
60	5.70	5.45	6.65	6.10	6.65	5.95	6.50

Note: Each row of table gives data of one session. Each table entry is the mean of four thresholds.

TABLE 6A

Mean Threshold Duration of First Flash, t_1 , in Milliseconds
 When $\text{Log } I \text{ (mL)} = 0.075$, $\text{Log } \Delta I_1 \text{ (mL)} = \text{Log } \Delta I_2 = -0.41$. Observer C.T.

Time between flashes i (msec)	Duration of second flash, t_2 , in milliseconds						
	0.6	1.2	1.8	2.4	3.0	3.6	4.2
45	5.90	5.90	6.10	5.60	5.70	6.15	5.85
60	6.20	6.35	6.70	6.65	5.20	6.30	5.35

Note: Each row of table gives data of one session. Each table entry is the mean of four thresholds.

Table 7A

Mean threshold duration of second flash, t_2 , in milliseconds.

Log I (mL) = 0.075. Log ΔI_1 (mL) = Log ΔI_2 = -0.41. Observer A.L.

Time between flashes, i (msec)	Duration of first flash, t_1 , in milliseconds							
	0.5	1.5	2.0	2.7	2.8	3.2	3.8	4.2
30	4.45	4.50	—	3.55	3.25	—	—	—
40	—	—	4.90	—	—	5.30	3.70	3.05
50	—	—	—	—	—	—	5.10	—

Note.--All data collected in one session. Each entry is the mean of four thresholds.

Table 8A

Mean threshold duration of second flash, t_2 , in milliseconds.

$\text{Log } I \text{ (mL)} = 0.075$. $\text{Log } \Delta I_1 \text{ (mL)} = \text{Log } \Delta I_2 = -.041$. Observer C.T.

Time between flashes, i (msec)	Duration of first flash, t_1 , in milliseconds							
	0.5	1.5	2.0	2.7	2.8	3.2	3.8	4.2
30	5.05	4.95	—	4.40	4.50	—	—	—
40	—	—	5.40	—	—	5.10	4.50	5.10
50	—	—	—	—	—	—	6.10	—

Note.--All data collected in one session. Each entry is the mean of four thresholds.

Table 9A

Mean threshold duration of second flash, \underline{t}_2 , in milliseconds.

Log I (mL) = 0.075. Log ΔI_1 (mL) = Log ΔI_2 = -0.41. Observer A.L.

Time between flashes, \underline{i} (msec)	Duration of first flash, \underline{t}_1 , in milliseconds	
	2.0	4.0
25	4.50	2.07
30	4.20	2.75
35	4.70	3.80
40	5.85	4.30
45	—	5.25
50	—	5.00

Note.--All data collected in one session. At $\underline{i} = 25$ and $\underline{t}_1 = 4.0$ msec the entry is the mean of three thresholds; each other entry is the mean of four thresholds.

Table 10A

Mean threshold duration of second flash, $\underline{t_2}$, in milliseconds.
 Log I (mL) = 0.075. Log ΔI_1 (mL) = Log ΔI_2 = -0.41. Observer C.T.

Time between flashes, \underline{i} (msec)	Duration of first flash, $\underline{t_1}$, in milliseconds	
	2.0	4.0
25	4.30	2.70
30	4.60	3.30
35	4.75	4.00
40	5.70	5.50
45	—	5.70
50	—	6.35

Note.--All data collected in one session. Each entry is the mean of four thresholds.

Table 11A

Mean threshold duration of first flash, \underline{t}_1 , in milliseconds.

Log I (mL) = 0.075. Log ΔI_1 (mL) = Log ΔI_2 = -0.41. Observer A.L.

Time between flashes, \underline{i} (msec)	Duration of second flash, \underline{t}_2 , in milliseconds	
	2.0	4.0
25	4.00	—
30	3.80	3.15
35	4.85	—
40	5.15	4.00
45	—	4.90
50	—	6.35

Note.--All data collected in one session. Each entry is the mean of four thresholds.

Table 12A

Mean threshold duration of first flash, \underline{t}_1 , in milliseconds.

Log I (mL) = 0.075. Log ΔI_1 (mL) = Log ΔI_2 = -0.41. Observer C.T.

Time between flashes, \underline{i} (msec)	Duration of second flash, \underline{t}_2 , in milliseconds	
	2.0	4.0
25	4.60	2.85
30	5.25	3.50
35	5.85	3.95
40	7.15	5.50
45	—	6.55
50	—	6.05

Note.--All data collected in one session. Each entry is the mean of four thresholds.

Table 13A

Mean Threshold Duration of Each Flash (msec) When $t_1 = t_2$.
 Log I (mL) = -0.933. Log ΔI_1 (mL) = Log ΔI_2 = -0.952. Observer A.L.

Time between flashes i (msec)	Session	
	1	2
0	2.80	3.35
20	3.20	3.30
30	—	3.50
35	3.75	—
40	—	3.90
45	4.20	—
50	—	4.70
55	4.75	—
60	—	5.20
70	5.10	5.85
90	5.40	5.90
110	5.15	5.20
150	5.10	—

Note: Each entry is the mean of four thresholds.

Table 14A

Mean Threshold Duration of Each Flash (msec) When $t_1 = t_2$.
 Log I (mL) = -0.933. Log ΔI_1 (mL) = Log ΔI_2 = -0.952. Observer C.T.

Time between flashes i (msec)	Session	
	1	2
0	2.75	2.70
20	2.60	3.00
30	—	3.30
35	3.90	—
40	—	3.40
45	4.40	—
50	—	4.25
55	5.00	—
60	—	4.65
70	5.45	5.25
90	4.80	4.80
110	5.30	5.15
150	5.25	—

Note: Each entry is the mean of four thresholds.

TABLE 15A

Mean Threshold Duration of Second Flash, t_2 , in Milliseconds

When $\text{Log } I \text{ (mL)} = -0.933$, $\text{Log } \Delta I_1 \text{ (mL)} = \text{Log } \Delta I_2 = -0.952$. Observer A.L.

Time between flashes i (msec)	Duration of first flash, t_1 , in milliseconds									
	0.0	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4
5	6.35	5.55	4.95	4.90	3.85	4.85	3.55	3.40	3.20	—
25	7.10	6.75	6.30	5.50	7.10	6.10	4.35	4.85	3.80	—
35	9.10	9.10	8.25	8.25	7.15	7.60	8.25	6.70	6.85	—
45	5.95	6.75	6.20	6.30	5.75	6.40	5.70	5.65	5.10	—
55	6.65	6.90	6.25	6.70	6.90	6.15	5.70	6.30	4.50	—
65	7.95	8.45	8.00	7.95	7.60	8.25	7.80	7.90	7.25	—
75	6.90	7.40	—	7.50	7.60	7.65	7.30	7.85	6.90	7.20

Note: Each row of table gives data of one session. Each table entry is the mean of four thresholds.

TABLE 16A

Mean Threshold Duration of Second Flash, t_2 , in Milliseconds

When $\text{Log } I \text{ (mL)} = -0.933$, $\text{Log } \Delta I_1 \text{ (mL)} = \text{Log } \Delta I_2 = -0.952$. Observer C.T.

Time between flashes t_1 (msec)	Duration of first flash, t_1 , in milliseconds									
	0.0	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4
5	7.05	6.30	6.00	5.70	5.05	4.60	3.10	3.15	2.80	—
25	6.80	5.90	5.35	5.15	5.05	3.60	3.50	2.45	1.90	—
35	5.35	4.60	4.50	4.20	3.50	2.70	2.70	2.15	1.20	—
45	7.40	7.15	6.80	7.85	6.60	7.20	6.30	6.20	6.55	—
55	5.50	5.80	5.85	5.65	5.75	4.95	4.65	4.95	3.95	—
65	5.60	7.25	6.35	6.55	6.55	6.95	5.45	6.30	4.75	—
75	6.80	6.65	—	7.25	7.40	7.55	7.00	7.25	6.35	6.25

Note: Each row of table gives data of one session. Each table entry is the mean of four thresholds.

Appendix B

After the decision had been made that the data were best fitted by one or two straight lines, an objective method was sought to fit such lines to the data. Dr. R. J. Wherry, Jr. suggested an analysis of variance (ANOVA) program to accomplish this fitting and to provide an estimate of the variance explained by the lines fitted to the data.

In each session $\underline{t_2}$ values were obtained at each of nine $\underline{t_1}$ settings from 0.0 to 4.8 milliseconds. Figure 1B gives examples of straight lines fitted to illustrative data.

In Figure 1Ba the data are fitted with a single straight line of zero slope. This hypothesis is testable by the traditional ANOVA in which one determines if there are significant differences between any of the nine mean $\underline{t_2}$ values. If the F-ratio is significant then the hypothesis cannot be accepted, and some other hypothesis must be true. One testable hypothesis is that the nine $\underline{t_1}$ settings could be divided into two "Blocks," the first of which (Block A) is composed of the shorter $\underline{t_1}$ settings while the second (Block B) is composed of the longer $\underline{t_1}$ settings. This hypothesis is illustrated in Figure 1 Bb. In this illustration, Block A contains the $\underline{t_1}$ values to the left of the break point and Block B contains the $\underline{t_1}$ values to the right of the break point. The reason for a division into two "Blocks" is to test the general hypothesis that there is no difference between Block A and Block B (i.e., no difference in $\underline{t_2}$ values for shorter $\underline{t_1}$ settings than for longer $\underline{t_1}$ settings). If the F-ratio for "Blocks" is significant, a zero slope line could be fitted to Block A while a -1 slope line would fit Block B.

To support the hypothesis that there is no differences in the means of the data from the different t_1 settings in Block A, a non-significant F-ratio must be obtained for the Block A term. To support the hypothesis that a single straight line would fit all the means in Block B, a linear term in Block B should have a significant F-ratio while a non-linear term should not have a significant F-ratio.

For each session ten ANOVAs were run, and, for each ANOVA, the split of the t_1 settings between Block A and Block B is given in Table B1. It will be noted that for the first ANOVA all nine t_1 settings are included in Block A and therefore this ANOVA tests the adequacy of the fit of a single straight line of zero slope to all the data. Similarly in the last ANOVA all the t_1 settings are in Block B. The partitioning of the degrees of freedom for each ANOVA is given in Table B2.

Table B3 is a typical printout of the ANOVAs for a single session. In ANOVAs 2-8 (from Table B1) the data were fitted with two lines. It was specified that these lines had to intersect at some t_1 value between the highest t_1 setting included in Block A and the lowest t_1 setting included in Block B. A subroutine tested if this intersection fell within these bounds.

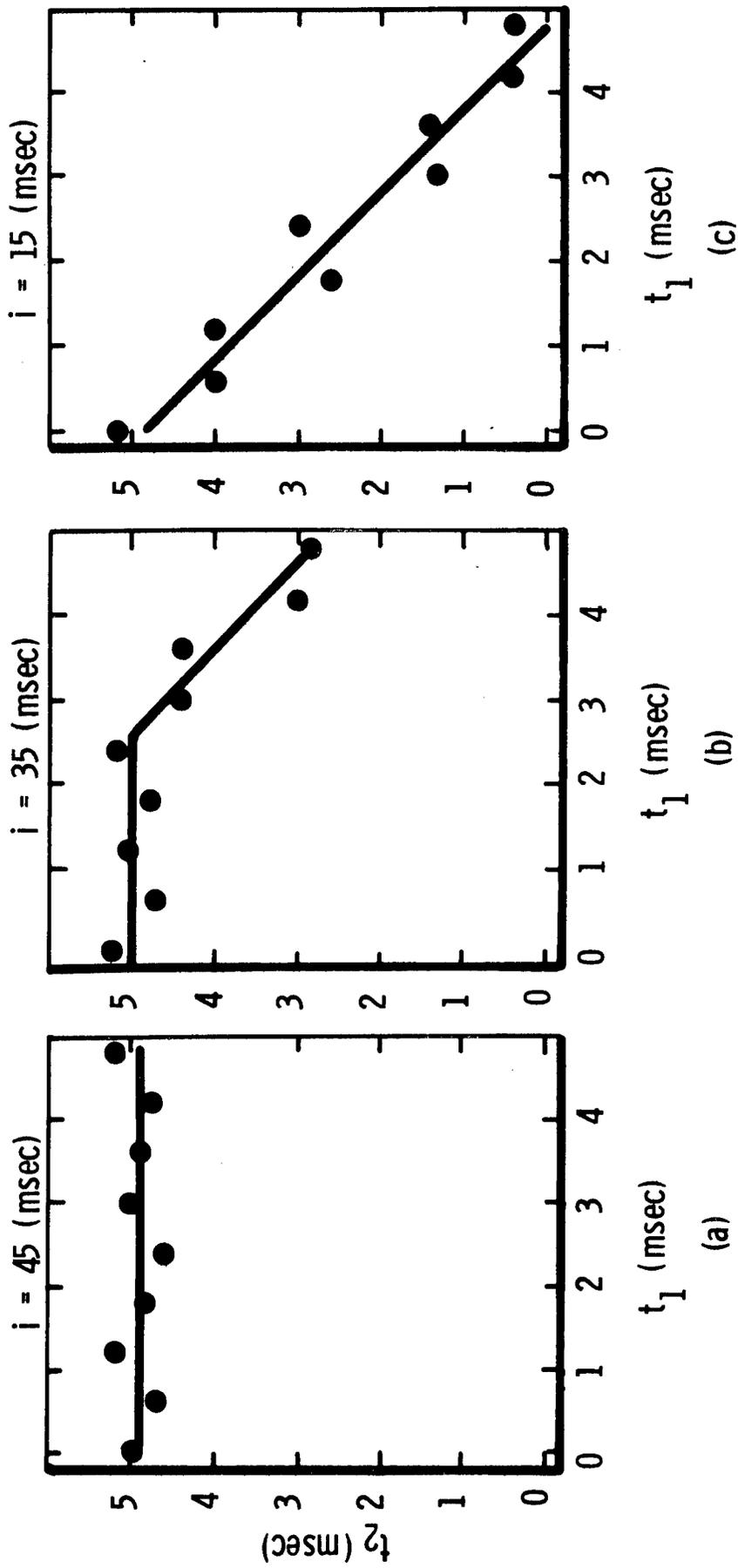


Figure 1B. Illustrations of Hypothetical lines Fit to Typical Data.

Table B1

Assignment of t_1 Settings to Block A and Block B
for Each Analysis of Variance

<u>Analysis of variance number</u>	<u>Block A t_1 settings</u>	<u>Block B t_1 settings</u>
1	1-9*	—
2	1-8	9
3	1-7	8-9
4	1-6	7-9
5	1-5	6-9
6	1-4	5-9
7	1-3	4-9
8	1-2	3-9
9	1	2-9
10	—	1-9

*Table entries refer to the ordered t_1 settings employed in the experiment, i.e., (1)0.0, (2)0.6, (3)1.2, (4)1.8, (5)2.4, (6)3.0, (7)3.6, (8)4.2, (9)4.8 msec.

Table B2

Partitioning of Degrees of Freedom to Sources
of Variance in Each Analysis of Variance

Analysis of variance number	Source of Variance			
	<u>Block A</u>	<u>Blocks</u>	<u>Linear B</u>	<u>Non-Linear B</u>
1	8*	0	0	0
2	7	1	0	0
3	6	1	1	0
4	5	1	1	1
5	4	1	1	2
6	3	1	1	3
7	2	1	1	4
8	1	1	1	5
9	0	1	1	6
10	0	0	1	7

*Table entries are the number of degrees of freedom assigned.

Table B3

Typical Printout From Analysis of Variance Program

Analysis of variance number	Source	Sums of Squares	Degrees of Freedom	Mean Square	F-ratio
1	BLOCK A	51.22	8	6.40	14.19
	WITHIN	11.28	25	.45	
	TOTAL	62.50	33	1.89	
2-5*	-	-	-	-	-
6	BLOCKS	29.08	1	29.08	64.46
	BLOCK A	2.97	3	.99	2.19
	LIN.B	18.90	1	18.90	41.89
	NONLIN.B	.28	3	.09	.20
	WITHIN	11.28	25	.45	
	TOTAL	62.50	33	1.89	
	<u>Block A Mean</u>	<u>Break Point</u>	<u>E-Value</u>	<u>Proportion of Variance Explained</u>	
	5.075	1.998	.7676	.9366	
7	<u>Source</u>	<u>SS</u>	<u>DF</u>	<u>MS</u>	<u>F-ratio</u>
	BLOCKS	25.50	1	25.50	56.53
	BLOCK A	1.21	2	.60	1.34
	LIN.B	22.99	1	22.99	50.98
	NONLIN.B	1.52	4	.38	.84
	WITHIN	11.28	25	.45	
TOTAL	62.50	33	1.89		
	<u>A-Mean</u>	<u>BP</u>	<u>E-Value</u>	<u>PVE</u>	
	5.267	1.423	.7759	.9467	

*No fit, therefore omitted.

Table B3 (continued)

Analysis of
variance number

8	<u>Source</u>	<u>SS</u>	<u>DF</u>	<u>MS</u>	<u>F-ratio</u>
	BLOCKS	15.85	1	15.85	35.14
	BLOCK A	1.13	1	1.13	2.49
	LIN.B	32.47	1	32.47	71.99
	NONLIN.B	1.77	5	.35	.79
	WITHIN	11.28	25	.45	
	TOTAL	62.50	33	1.89	
	<u>A-Mean</u>	<u>BP</u>	<u>E-Value</u>	<u>PVE</u>	
	5.325	1.209	.7732	.9434	
9	<u>Source</u>	<u>SS</u>	<u>DF</u>	<u>MS</u>	<u>F-ratio</u>
	BLOCKS	11.69	1	11.69	25.92
	LIN.B	35.66	1	35.66	79.05
	NONLIN.B	3.87	6	.65	1.43
	WITHIN	11.28	25	.45	
	TOTAL	62.50	33	1.89	
	<u>A-Mean</u>	<u>BP</u>	<u>E-Value</u>	<u>PVE</u>	
	5.700	.353	.7576	.9244	
10	<u>Source</u>	<u>SS</u>	<u>DF</u>	<u>MS</u>	<u>F-ratio</u>
	LIN.B	47.15	1	47.15	104.52
	NONLIN.B	4.08	7	.58	1.29
	WITHIN	11.28	25	.45	
	TOTAL	62.50	33	1.89	
		<u>A-Mean</u>	<u>BP</u>	<u>E-Value</u>	<u>PVE</u>
	0	0	.7543	.9204	

In order to select that split of the data which provided the best fit to the hypotheses illustrated in Figure B2 , the following criteria were employed:

ANOVA Source

BLOCK A	F-ratio must be non-significant to support hypothesis of zero slope line through means in Block A.
BLOCKS	F-ratio must be significant to support hypothesis that two lines would fit data.
LIN.B	F-ratio must be significant to support hypothesis that a straight line of some slope will explain means in Block B.
NONLIN.B	F-ratio must be non-significant to support hypothesis that best fitting line for Block B means is a straight line.

Thus, in Table B3 the seventh ANOVA provided the optimum split of the data.

The "E-value" is an estimate of the proportion of the total variance which is explainable. That is if:

$$\sigma_{TOTAL}^2 = \sigma_{WITHIN}^2 + (\sigma_{BLOCKS}^2 + \sigma_{LIN B}^2 + \sigma_{BLOCK A}^2 + \sigma_{NONLIN B}^2)$$

then
$$E = \frac{\sigma_{BLOCKS}^2 + \sigma_{LIN B}^2}{\sigma_{TOTAL}^2}$$

The right-most entry under the ANOVA table is the proportion of explainable variance explained by the straight line (or lines) and is computed:

$$\frac{\sigma_{\text{BLOCKS}}^2 + \sigma_{\text{LIN B}}^2}{\sigma_{\text{TOTAL}}^2 - \sigma_{\text{WITHIN}}^2}$$

Table B4 and B5 summarizes, for each observer, the E-values and the proportion of explainable variance value selected for each of the constant i sessions.

Table B4

Summary of E Values and Proportion
of Variance Explained Values for A. L.

Time between flashes i (msec)	<u>Original E value</u>	<u>Proportion of variance explained</u>	<u>Replication E value</u>	<u>Proportion of variance explained</u>
5	.7737	.9240	.8663	.9747
15	.8365	.9793	.8326	.9725
25	.7980	.9806	.7630	.9749
30	.6587	.8936	.6635	.9299
35	.6383	.8391	.6861	.9721
40	.2752	.6736	.3850	.6470
45	.1512	.4478	.3127	.5950
50	—	—	—	—
Average	.5902	.8197	.6442	.8666

Table B5

Summary of E Values and Proportion
of Variance Explained Values for C. T.

Time between flashes i (msec)	Original E value	Proportion of variance explained	Replication E value	Proportion of variance explained
5	.7274	.8069	.7969	.9489
15	.8028	.9025	.7405	.9655
25	.7757	.9464	.7602	.8563
30	.7903	.8980	.4714	.8855
35	.6207	.9360	.6626	.8049
40	—	—	.5087	.6790
45	.2647	.7282	.4223	.9128
50	—	—	.4882	.8074
60	—	—	—	—
Average	.6636	.8697	.6064	.8575

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13. ABSTRACT
Two flashes of 1° visual angle and 0.389 mL were superimposed upon a steady 1.19 mL background of the same size seen foveally. The second flash followed the first after delays ranging from 0 to 60 ms. The duration of the first flash was varied from 0 to 4.8 ms, and, for each duration of the first flash, the threshold duration of the second flash was determined. (When seen, the two flashes appeared as one.) At all delays below 25 ms, the threshold energy of the two flashes combined was constant, regardless of the duration of the first flash. At each delay between 25 and 50 ms, at short durations of the first flash, the threshold duration of the second flash was the same as that required when no first flash was presented. However, at longer durations of the first flash, the threshold average luminance provided by the two flashes during the total display time was constant. At the delay that required the maximum energy for threshold, 55 ms, for every duration of the first flash, the duration of the second flash was greater than that required when no first flash was presented. These findings, and others, are incorporated in a simple model.

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