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Signal Processing in Subspaces

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**12a. DISTRIBUTION / AVAILABILITY STATEMENT**

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**13. ABSTRACT (Maximum 200 words)**

Reduced dimension or subspace signal processing algorithms are studied for several classes of signal processing problems. The approach consists of mapping data into a subspace with a rectangular matrix transformation prior to application of the signal processing algorithm. This approach reduces computational complexity, reduces the variability associated with quantities estimated from data, and generally introduces some asymptotic performance loss. However, in situations with limited data, the impact of reduced variance generally dominates the asymptotic performance loss and a net improvement in performance is obtained. Application of this principle and the corresponding performance and computational complexity tradeoffs are discussed for adaptive beamforming and detection, Volterra filtering, and estimation of higher order statistics.

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**14. SUBJECT TERMS** Partially adaptive beamforming, adaptive detection, Volterra filters, higher order statistic estimation, nonlinear system equalization, tensor product basis approximation, Kronecker products, generalized likelihood ratio, low rank estimates, subspace design.

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# FINAL REPORT

## A. Statement of the Problem Studied

The problem studied was mapping data into lower dimensional subspaces before application of signal processing algorithms. The purpose of such a mapping is to reduce the computational burden and increase the reliability of statistics estimated from data. Application of these concepts was studied in the areas of adaptive beamforming, adaptive detection, nonlinear filtering, and non-Gaussian signal processing problems.

## B. Summary of the Most Important Results

We built upon our previous work in the areas of beamforming and adaptive detection. Our most important new result in beamforming was showing how to use dimension reduction, that is, partially adaptive beamforming, to reduce the signal cancellation effects that occur when correlated interferers are present. The mean squared error at the beamformer output was expressed as the sum of an interference mean squared error term and a signal cancellation term due to correlated interference. Both of these terms depend on the adaptation space. Hence, we proposed choosing an adaptation space which results in very small signal cancellation while maintaining satisfactory interference cancellation. A design procedure was introduced for choosing an adaptation subspace that provided good performance over a range of interference scenarios.

In the area of adaptive detection we showed how a subspace mapping provides a mechanism for controlling the tradeoff between model bias and estimator variance for a class of generalized likelihood ratio detectors based on a generalized multivariate analysis of variance data model. Of particular note, we applied the subspace framework to study the detection performance of an array of vector sensors. Vector sensors measure all six electromagnetic field quantities at a point is space. The subspace framework was used to compare detection performance based on various subsets of the vector sensor output, such as various electrical field components or magnetic field components. We also examined optimum combinations of the vector sensor output.

The major emphasis of the work in the second and third years of the project was on subspace methods for nonlinear filtering and non-Gaussian problems. Here we studied Volterra filters and estimation of third and higher order statistics, respectively.

Our most important result in nonlinear filtering is the development of tensor product basis approximations (TPBA) for Volterra filters. Although the Volterra filter is relatively simple and general, its utility is limited because it is often highly overparameterized. This overparameterization creates difficulties for implementation of Volterra filters and using them in nonlinear system modeling. The TPBA greatly reduces the number of free parameters by mapping the input into a subspace before forming nonlinear combinations of the data. We have shown that the a large class of Volterra filters may be well approximated using the TPBA and developed several methods for designing the subspace, or approximation basis, that minimizes the approximated filter's mean square output error. These methods incorporate some prior knowledge concerning the filter or input to obtain a good subspace. In particular, we developed design methods based on knowledge of the filter's frequency support, knowledge of the desired filter parameters, or knowledge of the input statistics. We also demonstrated that the TPBA provides useful insight for understanding the nature of the Volterra filter response.

We applied the TPBA to the important problem of nonlinear system equalization. In this work we used a contraction mapping approach to design the equalizer. Such an equalizer is implemented as a cascade of identical Volterra filter stages. The degree of equalization increases as the number of stages increase for input signals satisfying certain norm conditions. We have shown that the class

of allowable inputs is increased and that the bounds on the quality of equalization are tighter when a reduced degree of freedom TPBA Volterra filter is used as an equalizer.

We recently developed a class of reduced degree of freedom Volterra filters for processing bandpass signals such as typically encountered in communication systems. We have shown how to implement the Volterra filter on baseband data, in such a way as to implement a desired Volterra filter on the bandpass or carrier based version of the data. In essence, this result indicates how to commute the order of carrier modulation and nonlinear filtering. This work has application to equalizing non linear communication channels.

Our most significant work in higher order statistics is development of tensor product based reduced degree of freedom or "low rank" estimates of higher order moment matrices. We map the raw data into a subspace, estimate the higher order moments of the subspace data, and then relate these moments to the higher order moments of the original data. This introduces a "bias", defined as the difference between the expected value of the low rank estimate and the true moment. Note that there is always a "variance" representing the variability of the estimated moments caused by limited sample sizes. We have shown that the low rank estimates have a larger bias but smaller variance than the corresponding conventional estimates. The tradeoff between bias and variance was analyzed and we demonstrated that the low rank estimates can offer significant reductions in the mean-squared estimation error. This makes the low rank estimates extremely useful for signal processing algorithms that are based on sample estimates of the higher order statistics. The low rank estimates also offer considerable reductions in the computational complexity of such algorithms. We provided several methods for designing subspaces that optimize the tradeoff between bias, variance, and computation. The utility of this approach was demonstrated with a noisy input, noisy output system identification problem.

A second important contribution of our work in higher order statistics was the development of a tensor product algebra for dealing with high order moments and cumulants. This algebra offers a powerful degree of analytic tractability.

C. List of Publications

Journal Articles

R. Sharma and B. Van Veen, "Modular Structures for Adaptive Beamforming and the Gram-Schmidt Preprocessor," *IEEE Transactions on Signal Processing*, vol. 42, pp. 448-451, February 1994.

F. Qian and B. Van Veen, "Partially Adaptive Beamformer Design Subject to Worst Case Performance Constraints," *IEEE Transactions on Signal Processing*, vol. 42, pp. 1218-1221, May 1994.

R. Nowak and B. Van Veen, "Efficient Methods for Identification of Volterra Filter Models," *Signal Processing* Special Issue on Nonlinear Signal Processing, vol. 38, pp. 417-428, August 1994. Invited Paper.

R. Nowak and B. Van Veen, "Random and Pseudorandom Excitation Sequences for Discrete Time Volterra System Identification," *IEEE Transactions on Signal Processing*, vol. 42, pp. 2124-2135, August 1994.

F. Qian and B. Van Veen, "Partially Adaptive Beamforming for Correlated Interference Rejection," *IEEE Transactions on Signal Processing*, vol. 43, pp. 506-515, February 1995.

R. Nowak and B. Van Veen, "Invertibility of Higher Order Moment Matrices," *IEEE Transactions on Signal Processing*, vol. 43, pp. 705-708, March 1995.

F. Qian and B. Van Veen, "Quadratically Constrained Adaptive Beamforming for Coherent Signals and Interference," *IEEE Transactions on Signal Processing*, vol. 43, pp. 1890-1900, August 1995.

R. Nowak and B. Van Veen, "Tensor Product Basis Approximations for Volterra Filters," *IEEE Transactions on Signal Processing*, vol. 44, pp. 36-50, January 1996.

K. Burgess and B. Van Veen, "Subspace Based Adaptive Generalized Likelihood Ratio Detection," *IEEE Transactions on Signal Processing*, vol. 44, pp. 912-927, April 1996.

R. Nowak and B. Van Veen, "Volterra Filter Equalization: A Fixed Point Approach," *IEEE Transactions on Signal Processing*, vol. 45, pp. 377-388, February 1997.

T. Andre, R. Nowak, and B. Van Veen, "Low Rank Estimation of Higher Order Statistics," *IEEE Transactions on Signal Processing*, to appear March 1997.

G. Raz and B. Van Veen, "Baseband Volterra Filters for Implementing Carrier Based Nonlinearities," *IEEE Transactions on Signal Processing*, submitted.

## Invited Conference Papers

K. Burgess and B. Van Veen, "Vector Sensor Detection Using a Subspace GLRT," *IEEE Workshop on Statistical Signal and Array Processing Proceedings*, pp. 109-112, June 1994.

K. Burgess and B. Van Veen, "GLRT Detection of Broadband Signals Using an Array of Vector Sensors," *Twenty-Eighth Annual Asilomar Conference on Signals, Systems, and Computers Conference Record*, (5 pages), November 1994.

R. Nowak and B. Van Veen, "Volterra Filter Approximations," *Proceedings IEEE Workshop Nonlinear Systems and Signal Processing*, Special session on Polynomial Filtering, June 1995 (4 pages).

## Contributed Conference Papers

K. Burgess and B. Van Veen, "A GLRT for Narrowband Vector Sensor Array Detection," *IEEE 1994 International Conference on Acoustics, Speech, and Signal Processing*, vol. 4, pp. 253-256, April 1994.

R. Nowak and B. Van Veen, "Volterra Filtering with Spectral Constraints," *IEEE 1994 International Conference on Acoustics, Speech, and Signal Processing*, vol. 4, pp. 137-140, April 1994.

K. Burgess and B. Van Veen, "Robustness of the Subspace GLRT to Signal Mismatch," *IEEE 1995 International Conference on Acoustics, Speech, and Signal Processing*, pp. 2048-2051, May 1995.

R. Nowak and B. Van Veen, "Reduced Parameter Volterra Filters," *IEEE 1995 International Conference on Acoustics, Speech, and Signal Processing*, pp. 1569-1572, May 1995.

T. Andre, R. Nowak, and B. Van Veen, "Low Rank Estimators of Higher Order Statistics," *IEEE 1996 International Conference on Acoustics, Speech, and Signal Processing*, pp. 3026-3029, May 1996.

D. List of Reportable Inventions

E. List of Participating Scientific Personnel

   Barry Van Veen - Principal Investigator

   Graduate Students (some of these did not receive direct financial support, but worked on problems within the scope of the project)

   Qian Feng:  received Ph.D. degree in August 1993
   Keith Burgess:  received Ph.D. degree in August 1994
   Rob Nowak:  received Ph.D. degree in August 1995
   Thomas Andre:  received M.S. degree in August 1995
   Gil Raz:  expecting Ph.D. degree in May 1998