MODELING AND SIMULATION
FOR AIR DEFENSE FORCE ALLOCATION PLANNING

by

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ABSTRACT

The use of force ratios is an extremely valuable planning tool for the U.S. Army. Analysis of force ratios provides planning staffs and commanders an indication of the required combat power for successful operations. While the use of force ratios is prevalent for analyzing maneuver force allocation, little effort has been made to apply this technique to air defense operations. The initial attempt at force ratio analysis within the air defense domain, Correlation of Forces Air (COFA), achieves limited success. However, the COFA process suffers from several shortcomings.

This research develops a methodology that is a substantial improvement over COFA for air defense planning. Through the use of a designed experiment, the study demonstrates the possibility of modeling the modern air defense battle from simulation data. The experiment produced two distinct response equations modeling the effects of the various weapon systems on determining the remaining air defense and task force assets.

The development of the means to assess these two success criteria serves as the basis for the construction of an alternative force allocation planning tool. This study introduces a spreadsheet based air defense force allocation planning tool. The tool rapidly produces point estimates of the responses for multiple combinations of air

defense assets opposing an established air threat. Additionally, the tool generates the data needed to quickly construct prediction intervals for each predicted response. These capabilities mark a significant improvement in force ratio analysis for air defense operations. The spreadsheet planning tool equips the Air Defense Artillery commander with the information needed to make crucial force allocation decisions.

This research is limited by the scope of military operations and environment considered. Furthermore, the use of an unclassified simulation database precludes directly applying the variable relationships found here to actual combat situations. However, the most significant contribution of this research is the development of the new methodology for analyzing the battlefield. The framework developed in this research, is equally applicable to force ratio analysis for maneuver units. Additional research that accounts for the limitations of this effort could result in the development of useful force ratio planning tools for both Air Defense Artillery and maneuver units.

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TABLE OF CONTENTS

LIST OF TABLES viii

LIST OF FIGURES ix

INTRODUCTION 1
Background 1
Force Ratio Analysis 2
Correlation of Forces Air 4

REPRESENTING THE COFA PHENOMENON 9
Analysis of Historical Data 10
Analysis of Combat Training Center Data 10
Computer Simulation 13

RESEARCH SCOPE 16

STUDIED WEAPON SYSTEMS 20
ADA Weapon Systems 20
U.S. Army ADA Tactical Employment 22
Threat Aircraft 24
Typical Threat Helicopter Tactics 25

THE JANUS SIMULATION SYSTEM 27
Background 27
Description 28
Janus Output Measures 29
Aircraft Modeling 30
ADA Weapon Modeling 31

EXPERIMENTAL AND SCENARIO DESIGN 33
Scenario Design 33
Experimental Design 35

DATA ANALYSIS 40
Sensitivity Analysis 40
Final Experimental Design 42

APPLICATION AND LIMITATION 49
Application 49
Graphical Approach 49
Prediction Intervals and Spreadsheet Modeling 54
Limitations 61

CONCLUSION AND RECOMMENDATIONS 64

APPENDICES 68
Glossary 68
Experimental Design 73
Replication Calculation for the Sensitivity Analysis 76
Sensitivity Analysis Response Data 78
Estimated Effects for the Sensitivity Analysis 88
Replication Calculation for the Final Experiment 91
Final Experimental Design Response Response Data 93
Estimated Effects for the Final Experimental Design 103
Regression Analysis for the Final Experimental Design 106
Residuals Plots for the Final Experimental Design 114
Residuals for the Final Experimental Design 125
Air Defense Force Allocation Planning Tool 129

LIST OF REFERENCES 134

LIST OF TABLES

1. Notional Force Ratio Analysis 3

2. Notional COFA Analysis 5

3. Initial Factor Levels 37

4. Adjusted Factor Levels 42

5. COFA Analysis of Example Situation 60

LIST OF FIGURES

1. Contour Plot of the Percentage of Air Defense Assets Remaining 51

2. Contour Plot of the Percentage of the Task Force Remaining 51

3. Overlaid Contour Plots of Remaining Air Defense and Task Force Strength 52

4. Predicted Percentage of Air Defense Remaining... 55

5. Predicted Percentage of the Task Force Remaining 56

6. Half Width of the Prediction Interval for Air Defense Remaining 58

7. Half Width of the Prediction Interval for the Task Force Remaining 58

INTRODUCTION

Background

Since the end of the Cold War, the operational environment of the United States Army has changed drastically. The Cold War Army planned and trained to defend U.S. vital interests and allies abroad from a clearly defined enemy. Military planners developed detailed war plans against the back drop of established operational theaters. Units trained on the same ground they would fight.

All of that has changed. The fall of the Iron Curtain, the reunification of Germany, and the breakup of the former Soviet Union, have dramatically altered the geopolitical climate of the world. Countries traditionally considered to be our adversaries are now our allies. Once seen as a largely defensive force operating in mature theaters, the U.S. Army is being increasingly called upon to respond rapidly and perform a variety of missions around the world. Now, commanders and staffs must analyze missions and allocate combat forces for deployments within extremely tight time constraints without the luxury of previously developed detailed war plans.

Force Ratio Analysis

Critical to the success of any mission is ensuring that adequate combat power exists to counter the enemy threat. Analyzing "force ratios" is a widely used technique within the U.S. Army to allocate units for missions. The technique involves comparing the combat power of friendly and enemy forces by using numeric assessments of the relative strengths of units. Relative unit strength is dependent on both quality and quantity of those units. For example, three battalions of M60 tanks do not possess the same combat power as three battalions of M1A1 tanks simply because the M1A1 tank is a more modern and lethal weapon system. In order to make valid a comparison of combat power, a base unit is selected from the expected enemy. Planners then make a subjective evaluation of the combat power of all other types of units relative to the base unit. Once planners establish these values for all units germane to the mission, they multiply each unit's value by the number of units occurring. For the enemy side, the number of units is based on intelligence estimates. The new values are totaled for each side and multiplied by the expected unit percent strength. Finally, a force ratio comparison between the enemy and friendly side is made (United States Army Command and General Staff College [USACGSC], 1993). Table 1 illustrates this technique for battalion size units using a BTR battalion as the base unit. See the glossary (Appendix A) for descriptions of military equipment.

Table 1

Notional Force Ratio Analysis

Friendly Type	Value	Num.	Total	Enemy Type	Value	Num.	Total
M2 bn	2.00	1.00	2.00	BTR bn	1.00	6.00	6.00
M1A1 bn	3.15	1.00	3.15	BMP-2 bn	1.80	3.00	5.40
Cav sqdn	1.50	1.00	1.50	T-80 bn	2.90	4.00	11.60
Atk hel sqdn	4.00	1.00	4.00	Div recon bn	1.60	1.00	1.60
				AT bn	1.00	1.00	1.00
				Atk hel sqdn	3.00	1.00	3.00
Subtotal			10.65				28.60
x % strength			0.9				0.8
TOTAL			9.56				22.8
			Force ratio 9.56 : 22.8 or 1.0 : 2.38				

See the glossary (Appendix A) for an explanation of the unit descriptions.

Analyzing force ratios has proven useful for force allocation and comparing alternative courses of action for an operation. Historical evidence has shown that units are successful in conducting operations provided they possess an adequate force ratio. For example, a unit can expect to have at least a 50% success rate of defending from a fortified position against an attacking force, if the defending force possesses better than a 1:3 friend to foe force ratio (USACGSC, 1993). Based solely on this criteria, it follows that a course of action that provides for a more favorable force ratio throughout the operation would be the preferred solution. Although frequently accepted for analysis in the U.S. Army, the force ratio analysis technique is not without its detractors. In Army magazine, LTC Robert R. Leonhard (1996) argues that the numerically inferior force have won most historical battles due to increased maneuverability, better command and control, and technological advantages among others. This argument aside, force ratio analysis is still the accepted paradigm in U.S. Army operational planning. In the example in Table 1, the friendly force has a 1:2.58 force ratio. By the force ratio methodology, the friendly force could reasonably expect to successfully defend against the enemy force.

Correlation of Forces Air

While force ratios provided a useful tool for maneuver planning, no such tool existed to assist in the allocation of the supporting Air Defense Artillery (ADA) assets. In 1992, the 4th Battalion 3rd Air Defense Artillery Regiment (1993) of the 3rd Infantry Division (3 ID (M)) introduced a procedure, called Correlation of Forces Air (COFA), that integrated the concept of force ratios into the ADA planning process. Created by COL Michael Vane, COFA proposed numeric values of relative combat power for threat aircraft and friendly ADA systems. The ADA battalion's planners multiplied each of the threat aircraft values by the number of aircraft expected on each avenue of approach into the area of operation. The planner then developed alternative friendly ADA courses of action by arraying ADA systems along each avenue approach with enough combat power to counter the expected threat. One of the assumptions of the COFA technique was that a 1:3 force ratio of friendly ADA to threat aircraft was sufficient for successful ADA operations. Table 2 is an example of this technique being applied in a notional air defense situation for individual ADA weapons and aircraft. Utilizing the COFA technique, the ADA force specified, in the example, would be sufficient to defend against the expected threat aircraft attack.

Table 2

Notional COFA Analysis

	Friendly				Enemy		
Type	Value	Num.	Total	Type	Value	Num.	Total
Stinger	0.5	10	5	MI-8 HIP	1.8	20	36
BSFV	0.6	8	4.8	MI-24 HIND	2.4	10	24
Avenger	1.9	6	11.4				
TOTAL			21.2				60
			Force ratio	21.2 : 60 or 1.0 : 2.83			

See the glossary (Appendix A) for a description of ADA weapons and threat aircraft types.

While 4-3 ADA used COFA with good success, the technique does have some limitations and flaws as discussed below. For example, the numerical combat power values are the result of a subjective assessment of the ADA and threat aircraft systems by one person, the former battalion commander. They do not reflect any in depth analysis of the effectiveness of the systems. Thus, the validity of the numerical values is suspect.

Secondly, the simple adding of combat power values does not take into account any overlapping effects that may exist between weapon systems, friendly or enemy. U.S. Army ADA doctrine specifies the mixing of ADA weapon systems as an employment principle for operational planning. The doctrine is based on the belief that employing ADA systems with different capabilities together offsets the limitations of the individual systems; the net result is a more effective use of ADA assets than employing systems independently (Headquarters, Department of the Army [HQ DA], 1983). Likewise, using HIPs and HINDs (Appendix A) together may prove more effective for the enemy than employing them independently of one another. The COFA technique ignores the possibility of the overlapping effect of employing differing weapon systems together.

Additionally, the COFA technique does not provide for separate force ratios for differing operations. The heuristic is a 1:3 force ratio of ADA systems to threat aircraft is adequate for the successful conduct of any type of ADA operation. However, the battlefield disposition of ADA assets defending a maneuver force conducting a defensive operation is entirely different than ADA assets assigned to protect a maneuver unit conducting an offensive mission. In a defensive scenario, ADA weapons are usually emplaced in defilade positions which provide protection against enemy detection and fires. In an offensive situation, ADA systems are exposed a great deal more to enemy fire since maneuvering elements are far less likely to be able to utilize defilade positions. The increased vulnerability ADA assets encounter during offensive operations leads to an increase in battle damage sustained. Thus, it appears obvious that a single force ratio cannot be universally applied to all tactical situations.

Perhaps the largest flaw of COFA analysis is its failure to adequately address measure of success. The basic premise of COFA is that if a 1:3 ADA to air threat force ratio is achieved, the ADA units will succeed. The U.S. Army does not have an agreed upon definition of success for air defense operations. For example, a typical mission for a Stinger platoon is to provide air defense to a battalion task force conducting in a defensive posture. U.S. Army ADA doctrine calls for the platoon to provide air defense coverage "that allows the defended unit to retain sufficient combat power for follow-on missions" (HQ DA, 1992). On the other hand, the doctrinal manual for a battalion task force specifies that "the ratio of friendly vehicle to enemy aircraft losses (fixed-wing aircraft and attack helicopter) does not exceed 3:1" (HQ DA, 1988b). Another definition of success is provided in the doctrinal manual for the infantry battalion which specifies that "friendly losses to air attack are less than 10 percent" (HQ DA, 1988a). It is unclear if "success" to COFA is the nebulous ADA doctrine specification or the precise definitions that either the battalion task force or infantry battalion doctrines provide. Furthermore, none of the "success" criteria address the preservation of ADA combat power for future operations. Unless a commander is willing to accept Pyrrhic victories, maintaining both the task force and ADA combat power must be included as success criteria.

Although the COFA technique has flaws, it does provide the ADA battalion staffs a tool to quickly assess mission requirements and alternative courses of action. Furthermore, it is not known whether or not the simplified process that COFA uses to view force ratios is inadequate. As mentioned earlier, the COFA technique was used successfully although on a limited scale. COFA is limited to a "go/ no-go" decision. It does not provide the commander the means to assess the risk to his own force associated with destroying the enemy given a certain course of action. Commanders need a means to flexibly wargame courses of action and fully understand the impact of their decisions.

REPRESENTING THE COFA PHENOMENON

The increase in the number of unit deployments, coupled with the unpredictable nature of operational theaters, has had a tremendous impact on U.S. Army units. Commanders and staffs must quickly analyze the mission at hand and accurately allocate suitable resources for each contingency. In order to allocate resources for contingencies and other operations, many staffs utilize force ratio analysis.

While the use of force ratios is a well developed means for maneuver unit allocation, its corresponding air defense method, COFA, is in its infancy. Although already used with some success, the COFA technique suffers from several possible flaws: the validity of the relative combat power values, the exclusion of the possible overlapping effect of employing a mix of diverse weapon systems together, the failure to account for the differing dynamics of offensive and defensive operations, and the inadequate address of success criteria. While all of these areas warrant further consideration, this research will focuses on exploring overlapping effects of weapon systems employed together and success criteria.

Analysis of Historical Data

As with most research problems, there are several possible approaches. The most obvious approach is to gather historical data and draw conclusions about the capabilities of the systems from the outcomes of battles. In fact, the widely accepted 1:3 ratio for units in the defense was garnered from historical analysis (USACGSC). While this seems a logical choice, analysis of ADA weapon systems presents particular difficulties. Although the United States has engaged in several conflicts in recent years, there has been little to no significant ADA-to-air threat interaction. Furthermore, meaningful literature and data is classified and not appropriate for this forum for national security interests. In our most recent large scale combat deployment during the Persian Gulf War, the United States deployed both the Stinger and Avenger weapon systems. However, U.S. Forward Area Air Defense (FAAD) forces did not conduct a single engagement against an enemy aircraft during the war. Moreover, the Bradley Stinger Fighting Vehicle (BSFV) has never been deployed for combat operations.

Analysis of Combat Training Center Data

Another possible source of data is the U.S. Army's Combat Training Centers (CTCs). Army units conduct training at three CTCs: the National Training Center (NTC) at Ft. Irwin, California, the Joint Readiness Training Center (JRTC) at Ft. Polk, Louisiana, and the Combat Maneuver Training Center (CMTC) at Hohenfels, Germany.

Each of these training centers provides realistic force on force training against a highly trained and well equipped opposing force (OPFOR). OPFOR units, practicing typical threat tactics, engage and are engaged by friendly units using the Multiple Integrated Laser Engagement System (MILES). MILES system sensors are attached to soldiers' individual equipment, vehicles, and weapons, and provides both visual and audible cues to identify engaged or "destroyed" equipment and personnel. Trained Observer/Controllers (O/Cs) monitor all operations ensuring safety and provide training feedback to units. The CTCs also incorporate elaborate tracking and data collection systems that facilitate the production of detailed After Action Review products for the training units.

Although the CTCs arguably provide the best training Army units receive today, the data from CTCs is inadequate to analyze ADA to air threat force ratios. The use of MILES at the CTCs introduces several factors that invalidate any force ratio analysis. First, the MILES system only approximates the characteristics of the each weapon system. Ranges and the accuracy of MILES weapons differ from those they represent. For example, a Stinger missile has a range in excess of 4 kilometers where a MILES Stinger system's range is significantly less. Also, an actual Stinger is a "fire and forget" weapon system; once the missile is fired, nothing further is required of the Stinger crewman. With a MILES Stinger, crewman must hold the weapon system on target for approximately six seconds in order for the weapon to register a "kill" on an aircraft. Additionally, MILES devices are subject to degraded performance due to decreased visibility that are not consistent with actual weapon system performance. MILES weapons also require soldiers to have additional gunnery skills to ensure that each system is correctly boresighted to target. The inaccuracies of MILES also applies to OPFOR weapons. Rotary wing aircraft at the CTCs are represented by visually modified U.S. aircraft fitted with MILES. In addition to the size and shape discrepancies associated with this, CTC OPFOR aircraft do not possess a full complement of threat aircraft weapon systems. In effect, CTC engagement data do not show the effectiveness of soldiers using their assigned weapons, rather their MILES gunnery skills against a poorly simulated threat aircraft.

In addition to the problems that MILES presents, CTC data has other problems associated with it. The focus of the CTCs is to integrate all the components of land and air warfare into a realistic battlefield. As such, there are a tremendous amount of factors that influence any single battle. These can include terrain, weather, unit strength, morale, unit training proficiency, and the effects of multiple weapon systems. Hence, there are entirely too many factors that are not controlled that preclude an analyst from isolating the effects of individual weapons systems on one another and the enemy. At best, an analyst may be able to identify trends in unit performance based on weapon system use, but certainly no concrete mathematical relationships.

Computer Simulation

The use of computer simulation offers a viable solution for research of force ratios. Computer simulations estimate actual processes through the use of mathematical models. These models may be substantiated through engineering tests which yield the probability of hit (PH) and probability of kill (PK) values used for each weapon in the simulation. This approach can also extend to vehicle and personnel movement characteristics, the effects of terrain and weather, and other phenomena.

From studying the models, an analyst can draw conclusions about the behavior of the studied systems in the real world. One distinct advantage of simulation is that it provides a completely controlled experimental environment for the analyst studying weapon systems. An analyst can fully specify every detail of the scenario, the terrain, and the weapon systems employed. Because of this, he can develop a complete experimental design that will reveal the effects of changing weapon systems and any interaction effects he is interested in. Unlike the CTC cases, the simulation experimenter can control variables and interactions such as terrain, weather, morale, and unit training proficiency (Law & Kelton, 1991). Furthermore, many simulation models include faster than real time execution capability that greatly speeds data collection.

For all of its advantages, a computer simulation does have limitations. Since simulation is based on mathematical models, it is only an approximation of the real world, not an exact duplication (Law & Kelton, 1991). Weapon systems are represented by algorithms of the their characteristics. Simulation models typically only approximate a portion of the real world system characteristics, usually those most interesting to the model developers. Some of the most common effects not typically represented in simulation include the variability of human entity behavior stemming from morale or irrationality. Thus, conclusions drawn on the results of simulation are only as accurate as the simulation model represents them.

In order to estimate the inherent variability in real world processes, many simulations are stochastic in nature. By utilizing random variates, simulation models attempt to replicate processes that do not always produce the same result (Law & Kelton, 1991). For example, in order to represent weapon system engagements, simulations, such as Janus, incorporate previously established Probability of Hit (PH) and Probability of Kill (PK) data for given weapons. The simulation then utilizes a random variate, that conforms to the PH and PK data, in order to determine the outcome of the engagement (Titan, 1993). Because of the use of random variates, an experimenter must make use of multiple replications, of the same experimental conditions, in order to balance the effects of randomness. Furthermore, the use of random variables results in an estimation of system behaviors that preclude the use of simulation as a true optimization technique (Law & Kelton, 1991).

For the purposes of this research, computer simulation appears to be the best alternative for developing and prototyping a methodology for investigating for COFA force ratios. The intent of this research is to use a computer simulation, in a controlled experiment, in order to develop a methodology that will allow commanders to evaluate force allocation courses of action. For a range of force mix packages, the tool will provide the ability to estimate success against a projected enemy based on amount of the defended task force remaining and risk to ADA assets.

RESEARCH SCOPE

The consideration of the force ratios between ADA weapons and threat aircraft is a huge topic area. To fully consider all factors of this problem is well beyond the scope of this research effort. Eliminating several factors and controlling others reduces the size of the research area.

The modern battlefield is cluttered with numerous weapon systems, all with varying capabilities and limitations. Maneuver commanders combine the combat power of a variety of armored vehicles, artillery pieces, and individual weapons in the prosecution of each battle. The interaction of all of these weapon systems with an equally diverse enemy produces an enormously complex and dynamic battlefield. Although these weapons, both friendly and enemy, may indeed have an effect on the air battle, modeling and controlling all of these factors would produce a prohibitively large experiment. Furthermore, it is unclear if any degree of accuracy would accompany an estimate of an individual weapon's effects on various targets or its interactions with other weapons. Although many factors will influence the air battle, it is likely that the most significant interactions in the air battle will involve primarily ADA weapons and aircraft. Therefore, this limits the extent of the analysis to a defended battalion task force, static in location and composition. Air defense assets defend the task force from enemy helicopter attack. The task force does not encounter an opposing ground threat.

The number of ADA systems and aircraft considered must also be limited in any experimentation. Within the current U.S. Army, there are six fielded air defense systems: Patriot, HAWK, Chaparral, Stinger, Avenger, and BSFV. Inclusion of all of these systems would result in an extremely large experiment. The original COFA technique only specified values for Chaparral, Stinger, Avenger, and the BSFV. Therefore, there are no established comparison values for the Patriot and HAWK missile systems Chaparral system is no longer in use in the active Army and is being retired from service in the reserve components. Thus, this research focuses on the contributions of the Stinger, Avenger, and BSFV weapon systems to the air battle.

The changing world political climate has greatly altered perceptions of what can be considered "threat" or "friendly" aircraft. During the Cold War era, U.S. soldiers distinguished the weapons of the former Soviet Union as "threat" equipment. Soldiers now face the task of training to identify equipment as friendly or enemy based on the current mission. It is completely infeasible to consider all possible threat aircraft in this research. Since the original COFA technique established values for the HIND and the HIP, only these aircraft are included in the designed scenarios.

Another restriction on this analysis is the type of operation investigated. ADA units support maneuver forces conducting several types of operations. Generally, maneuver force operations can be classified as either offensive or defensive in nature. During offensive operations, ADA units typically move with the supported maneuver force.

Both the defended task force and the ADA assets are far more vulnerable to enemy fire than in defensive operations. Both offensive and defensive operations need to be considered separately in order to fully test the validity of COFA as a force ratio tool. This would necessitate the construction of two completely separate simulation experiments and a tremendous amount of time. Therefore, only ADA units supporting defensive operations are examined.

The dynamics of the modern military battlefield are further complicated by the effects of terrain, weather, and time of day. Heavily vegetated ground, hilly terrain, and urban areas all offer cover and concealment from enemy observation and fire. Since many battlefield systems are restricted to line-of-sight engagements, the terrain encountered often restricts weapons' engagement ranges. Associated with each unique terrain situation, a unique set of weapon system restrictions exists. To control the effects of terrain in this study, units conduct all operations in a southwest North American desert environment. Opposing sides encounter wide open spaces, however, weapon ranges are reduced in some cases by a series of mountain ranges.

As does terrain, weather and time of day also have significant impacts on military operations. Poor weather has severe implications for the trafficability of terrain. Terrain passable in fair weather often becomes impossible for units to operate in. Both weather and time of day have an obvious impact on visibility. Reduced visibility tremendously impacts the effectiveness of many line of sight weapon systems, particularly those not equipped with limited visibility siting devices. The maneuverability of vehicles, especially aircraft, is also greatly influenced by reduced visibility. As a control measure, all operations conducted in this investigation occur during daylight hours and fair weather.

The numerous limitations imposed on this experiment clearly limit the applicability of the results obtained beyond these restrictive conditions. A far more comprehensive analysis is required in order to adequately address the numerous factors influencing the modern battlefield. However, this analysis does address those factors that likely have the most significant effect on the air battle: the composition of the defending ADA force and the composition of the attacking enemy air threat. The greatest value of this research will be the analytical methodology developed for examining battlefield dynamics, in particular those influencing the air battle.

STUDIED WEAPON SYSTEMS

ADA Weapon Systems

The Stinger is a shoulder fired, anti-aircraft missile. The missile use an infrared guidance system making it a "fire and forget" weapon. The Stinger weapon includes an Identification Friend or Foe (IFF) transponder and weighs approximately thirty five pounds on the soldier's shoulder. The Stinger's accuracy and range, four kilometers, make it a formidable air defense weapon against enemy rotary wing and low flying, fixed wing aircraft (HQ DA, 1984).

A Stinger team consists of two soldiers: a team chief and a crewman. Stinger teams are equipped with a High Mobility Military Wheeled Vehicle (HMMWV) that transports up to six Stinger missiles. The team must remove the missiles from the vehicle in order to engage aircraft. In U.S. Army divisional ADA battalions, Stinger teams are organized into platoons, of ten teams each.

The Avenger weapon system is an advanced variant of the Stinger. The Avenger consists of a HMMWV with a weapon system turret mounted behind the cab on the vehicle bed. On the turret, two missile pods are mounted, one on either side. Each pod is capable of carrying four ready to fire Stinger missiles. From the turret or a remote launch station, Avenger crewmen fire the missiles, in rapid succession, from either a stationary position or while the vehicle is in motion. The Avenger also incorporates a Forward Looking Infrared (FLIR) sensor that gives the system the ability to detect, identify, and engage targets in darkness or conditions of limited visibility. The combination Avenger's FLIR, rapid firing rate, and fire on the move capability tremendously enhance the lethality of the weapon system (United States Army Air Defense Artillery School [USAADASCH], 1991).

Avenger crews consist of two soldiers comprising a squad: a squad leader and a crewman. The standard Avenger platoons consists of six Avenger squads. Avengers platoons have the same composition in both divisional and nondivisional units (USAADASCH, 1991).

The Bradley Stinger Fighting Vehicle (BSFV) is an evolving weapon system. The current system consists of a M2 Bradley Fighting Vehicle that carrying a traditional two man Stinger team in its rear passenger area. The basic infantry M2 Bradley Fighting Vehicle remains virtually unchanged maintaining its 25mm main gun, a TOW missile system, and a 7.62mm coaxial machine gun. In order to engage an aircraft, the vehicle stops and the Stinger team dismounts the vehicle. The BSFV concept was developed to provide the divisional Stinger teams the armored protection and firepower of the Bradley and increase the mobility of Stinger teams supporting maneuver operations. BSFVs are organized into platoons of four systems each. The platoon also includes a standard Bradley Fighting Vehicle for the platoon leader (HQ, DA, 1995).

The newest version of the BSFV is the Bradley Linebacker which is still in the acquisition phase. With its first possible fielding to occur in 1997, the Linebacker consists of a Bradley Fighting Vehicle without its organic TOW missile. In place of the TOW, the Linebacker has a modified Avenger missile pod, with a capacity of four Stingers ("Bradley Linebacker," 1996). The result is a highly mobile armored vehicle that can engage aircraft with an effectiveness similar to the Avenger. In addition to the Stinger and Avenger, this research includes the Linebacker system instead of the currently fielded BSFV.

U.S. Army ADA Tactical Employment

The scope of this research is limited to air defense in a defensive operational setting. Accordingly, the following discussion of air defense tactics is constrained to defensive operations. U.S. Army air defense tactics are based on four basic principles: mass, mix, mobility, and integration. Mass, in air defense terms, implies having sufficient air defense assets defeat the suspected enemy air threat (HQ DA, 1983). The principle of mass is the impetus for force ratio analysis.

To achieve mass when employing Stingers, BSFVs, and Avengers, unit commanders typically employ the systems close enough together that their engagement ranges overlap.

Ideally, they employ them within mutually supporting distance. Assets are considered to be in mutual support of one another if they can provide coverage for each other's "dead space" (HQ DA, 1992). A simple definition of dead space is an area, within the engagement range of a weapon, in which the weapon cannot engage targets due to terrain masking.

An additional measure to achieve mass, is weighting air defense coverage along the most likely enemy air avenues of approach (AAAs). In other words, moving assets to the area of expected air threat while accepting risk in areas less likely to encounter an air attack. Commanders can further attempt to augment the mass principle by placing ADA assets forward along the AAAs in order to achieve an early engagement on the enemy aircraft. This reduces the possibility that the aircraft can pose a threat to friendly units or assets (HQ DA, 1992).

The principle of mix prescribes mixing different air defense assets in order to offset the limitations of each system (HQ DA, 1983). In practical terms, mixing Stinger assets with BSFV units or Avengers would satisfy the principle of mix. Stinger units typically fight from individual fighting positions, or foxholes, constructed to provide cover from enemy fire (HQ DA, 1984). The Stinger team's ability to construct their own fighting positions allows it to be effectively employed in numerous tactical situations (HQ DA, 1992).

Because of their vulnerability to damage from enemy fire, Avengers are best suited for the defense of assets not involved in direct ground combat. If used in forward areas, Avengers should be placed along the flanks of units or in overwatch positions (USAADASCH, 1991). On the other hand, the armament of the BSFV provides a measure of protection to its crew and equipment. The BSFV was designed with the support of the forward maneuver force in mind. However, whenever possible, BSFV platoon leaders will attempt to coordinate for combat engineer assets to dig defilade positions for their BSFVs when supporting defensive operations (HQ DA, 1995).

Mobility calls for ADA assets to be sufficiently mobile to move about the battlefield in response to changing tactical conditions. Additionally, all ADA efforts must be fully integrated into the maneuver scheme of operations in order to insure success (HQ DA, 1983). Due to the scope of this research, the principles of mobility and integration are not fully addressed within this experiment.

Threat Aircraft

The Mi-8 HIP is a medium transport/assault helicopter. Available in numerous configurations, the Mi8T HIP E is the standard gunship version. It has a swivel mounted 12.7 mm nose machine gun, outrigger mounted, six 32 shot 57 mm rocket pods, four 250 kilogram bombs, and four AT2C SWATTER anti-tank guided missiles (ATGM). The helicopter is heavily armored and is capable of transporting up to 24 troops and 1,000 kilograms of ordinance simultaneously. The helicopter is also capable of transporting up to 24 troops for air assault missions. Produced by the former Soviet Union, the aircraft has been widely exported around the world (Headquarters, Department of the Army [HQ DA],1991).

The Mi-24 HIND is one of the most heavily armed attack helicopters in the world. The HIND is available in many variants, each with differing weapon systems. Dubbed the "flying tank", the HIND D features a four barrel 12.7mm Gatling type machine gun, turret mounted under the nose of the aircraft. It also has four 32 shot 57mm rocket pods, and four AT2C Swatter ATGMs. In lieu of the 57mm rocket pods, the HIND D is capable of carrying 250 or 500 kilogram bombs. The helicopter is primarily utilized in a close air support role especially against enemy tanks. The helicopter can also transport 8 troops for use in air assault operations (HQ DA, 1991). Produced by the former Soviet Union, the helicopter can be found in former Warsaw Pact nations and in several countries in the Middle East, Far East, Africa, and South America (Foreign Material Intelligence Battalion, 1990).

Typical Threat Helicopter Tactics

Defining threat helicopter tactics, in the post Cold war era, is an extremely situationally dependent venture. Since the scope of this study is limited to the consideration of HIPs and HINDs, included below is a description of the helicopter tactics of the former Soviet Union. In fact, the U.S. Army Training and Doctrine Command (TRADOC) utilizes the Soviet style tactics as the basic building block for developing Opposing Force (OPFOR) tactics for the Combat Training Centers. TRADOC believes that the countries of the former Soviet Union and its former allies will continue to pattern their doctrine and tactics on those of the former Soviet Union (United States Army Training and Doctrine Command [TRADOC], 1994).

The threat typically uses attack helicopters as an extension of the artillery. These "flying artillery" platforms usually attack in a flight of four aircraft but can also be employed in pairs. Attacks can be made from one or several directions simultaneously. Common attack profiles begin with a high speed, low level flight ingress while firing, running fire. Once near the target, the helicopters ascend to acquire and engage targets. Helicopters engage using either the running fire or hovering fire techniques. The helicopters then dive for a low level departure [TRADOC, 1994]. Attack helicopters ingress and egress along air avenues of approach (AAA). Typical AAAs include river beds and valleys that provide terrain masking from enemy observation. Helicopters will also fly behind ridge lines to avoid enemy fire (HQ DA, 1995).

Although there are numerous missions for threat attack helicopters, the most common use is in the direct air support role. The mission is normally flown by four helicopters attacking with rockets and guns or ATGMs in support of ground combat operations. These attacks emphasize limiting exposure time to about 20 seconds primarily using a running gun technique [TRADOC, 1994]. Because of its weight, HINDs cannot effectively hover while combat loaded and are therefore limited to the running gun technique (HQ DA, 90).

THE JANUS SIMULATION SYSTEM

Background

Janus 6.0, a constructive military simulation, is the simulation model for this research. Janus is suitable for this type of analysis since it accurately models both rotary wing aircraft and air defense weapon systems. Janus is also widely used and accepted throughout the military making it readily accessible those wishing to confirm, or expand on, the results of this study. Additionally, there exists validated data for the modeling of U.S. military and threat weapon systems. Because the validated data is classified, this experiment uses an unclassified training data base for analysis. However, the methodology will remain sound as well as transferable to an experiment using classified data.

Janus was named for the two faced Roman God of portals to reflect the ability to see the battlefield from both sides. Janus exists in several different versions. The Lawrence Livermore National Laboratory developed the initial version of Janus, Janus (L), as a means to model nuclear effects. Subsequently, another version, Janus (T), was developed for the U.S. Army's Training and Doctrine Command (TRADOC), Training and Analysis Center (TRAC) to assist in the combat development field. From Janus (T), a third version was fielded for use in the training community. The current version used in this research, Janus (Army), is commonly known as simply Janus (U.S. Army Simulation, Training and Instrumentation Command [STRICOM], undated).

Description

Janus is a stochastic, combat simulation. The current version, Janus 6.0, can accommodate up to six opposing sides. The simulation allows training personnel to interact with the systems modeled in the simulation during run time. Since it is primarily a training simulation, each opposing force only knows the disposition of other forces when its forces can detect the opposing force in the simulation. Detections between modeled systems, weapons effects and engagements are determined by stochastic processes. Janus models maneuver, artillery, air defense, and engineer units, as well as minefields, obstacles, chemical effects, and aircraft (STRICOM).

Janus uses digitized data from the Defense Mapping agency for its terrain representation. It models the effects of terrain and vegetation on line of sight and mobility. Janus also accounts for the impact of weather, visibility and light data (STRICOM).

Janus Output Measures

Janus supports the detailed analysis of simulations runs through two tools: the Janus Analyst Workstation (JAWS) and the Post Processor. Janus records all simulation data in recording files for each simulation run. JAWS accesses the recording files and allows playback of the entire simulation or selected events. The user can specify playback speeds equal to or exceeding real time. JAWS is particularly useful for conducting After Action Reviews (AARs) of unit training (STRICOM).

Janus also incorporates a Post Processor that retrieves information from the simulation recording files of single or multiple simulation runs. The Post Processor presents the requested information in report form. These reports include information regarding the effectiveness and time of engagement of indirect and direct fire weapons. Coroner's reports provide a detailed account of each battlefield kill in terms of time location and participants. Other reports show the impact of minefields, temperature, and chemical weapons on units. Game Analysis reports summarize engagement and detection ranges as well as indicate the contributions of selected weapon systems (STRICOM).

The Killer/Victim Scorecard is of particular value to this research. Presented in tabular form, the report displays all possible combinations of "killers" and "victims" in the simulation. The report identifies the number and type of weapon systems destroyed, "victims", by each weapon system type, "killers" (STRICOM). For example, the Killer/Victim Scorecard will identify the number of HIND helicopters destroyed by

Stinger weapon systems. Likewise, it will show the number of Stingers destroyed by HINDs.

Aircraft Modeling

Like all weapon systems in Janus, Janus represents aircraft through a series of mathematical algorithms. Weapon characteristics, such as weapon range, weapon effectiveness and survivability, are captured in database format. Scenario developers can manipulate the database in order to see the effect of changes in weapon characteristics on studied scenarios.

The Janus user can specify several movement parameters for aircraft. Janus aircraft movement routes can be established prior to runtime, in the scenario planning phase, or during scenario execution. The user may also dynamically alter routes during runtime. Besides direction, the user controls both aircraft altitude and speed through the use of two flight modes: "high and fast" or "low and slow". The settings for the altitude and speed, for each flight mode, are contained in the database. The user can manipulate these settings prior to runtime. During execution, the user can dynamically change the selection of flight mode. However, during runtime, he may not change the preset altitudes or speeds that correspond with each flight mode. Aircraft routes may consist entirely of "high and fast" or "low and slow" operations. Aircraft routes may also consist of multiple segments, each with a different flight mode, "high and fast" or "low and slow" (STRICOM).

Many combat aircraft also make use of "pop up" tactics. "Pop up" tactics consist of an aircraft hovering behind a terrain feature, out of the line of enemy fire. Aircraft will ascend to a "pop up" altitude, above the cover of the terrain, in order to identify and engage the enemy. Following the engagement, the aircraft will descend behind the terrain. Janus allows for the establishment of positions for use as "pop up" locations. As with flight modes, users can switch the aircraft from a hovering to a "pop up" altitude during runtime, however, the altitude values are set in the database and not alterable during scenario execution (STRICOM).

Janus models both helicopters and fixed wing aircraft through the use of the same parameters. Since in most cases fixed wing aircraft cannot hover, the database fields for "pop up" data are left blank. Although much has been said about poor fixed wing modeling in Janus, research has shown that Janus adequately captures helicopter effects (Daniels, 1994).

ADA Weapon Modeling

Due to the complexity of many military weapons, Janus models many weapon systems as compilations of several subsystems. For example, a U.S. M1A2 Abrams tank will consist of the carrier and two gun subsystems: the 120mm smooth bore gun and the coaxial machine gun. Since the gun systems may be types used by other weapon systems, they each have stand alone database entries.

Janus models the ADA weapon systems studied here using the same compilation technique. The Stinger missile is the common thread between all three weapon systems and has a separate entry in the weapon database. Although the actual missile is the same between all three weapon systems, some of its characteristics change when used in the Avenger and Linebacker systems. In the Janus database, values are specified for reload times and number rounds fired prior to reload. The values are for the Stinger weapon when utilized in a shoulder fired configuration, valid for a Stinger team. In the Avenger and Linebacker configurations, eight rounds and four rounds, respectively, can be fired before reload occurs compared to one round before reload for the shoulder fired Stinger. The difference in equipment and number of rounds to reload also changes the mean reload time values for the Avenger and Linebacker. Adjustments were made to the database to more accurately reflect these systems' capabilities while keeping the actual data and system behaviors unclassified yet realistic for the purpose of this research.

EXPERIMENTAL AND SCENARIO DESIGN

Identifying the relationships between the factors influencing the air battle is essential to the development of an alternative force ratio planning tool. This research focuses on exploring the impact that the amount of friendly ADA systems and the amount of attacking enemy aircraft have on the remaining combat strength of the defended task force and ADA assets. The research incorporates a standard factorial experiment that allows analysis of these variables within a tactical scenario.

Scenario Design

This investigation is limited to the analysis of ADA weapon systems defending a unit in a defensive posture. In the scenario, a battalion task force consisting of three mechanized infantry companies and one tank company is arrayed in a hasty defense on a terrain database of the National Training Center at Fort Irwin, California.. The task force is arrayed along a likely enemy avenue of approach capable of supporting a regimental size threat advance. In order to validate the task force's defensive plan, two U.S. Army

Infantry officers were consulted: MAJ John McCarthy and CPT Brian Bedell. Each officer agreed that the task force defense design was both feasible and tactically sound.

ADA assets are emplaced in support of the task force defense in accordance with current U.S Army ADA doctrine. The actual enemy attacking the task force consists of the air threat assets previously mentioned. The aircraft follow predetermined flight patterns and routes. Each flight pattern and route adheres to the Soviet style attack helicopter doctrine described previously. The scenario concludes when all aircraft have completed their attack routes or are destroyed.

The task force does not encounter an attacking enemy ground force. The inclusion of an enemy ground force would introduce additional challenges for controlling the experiment. The added complexity of the experiment could cloud estimation of the critical ADA and air threat factors. It is important to understand, that it is inadvisable to make inferences regarding the performance of ADA or air threat assets to situations that include enemy ground forces. Any such inferences would be based on incomplete data and subject to considerable error.

Experimental Design

A traditional factorial experimental design provides the framework for analyzing the dynamics of the air battle. The experiment contains five factors or independent variables: the number of Stingers (S), the number of Bradley Linebackers (L), the number of Avengers (A), the number of HIPs (Hp), and the number of HINDs (Hd) (appendix B). The effects of changing the levels of these variables is measured against two responses or dependent variables: the percent of friendly ADA systems remaining and the percent of the friendly task force remaining.

The number of ADA systems at the start of each simulation run is variable based on the data point examined. In simulation runs where no ADA systems are destroyed, scenarios with higher initial ADA numbers would automatically result in a higher response, if the response is defined as the number of ADA weapons remaining. Scaling the response in terms of percentages eliminates this bias. The remaining number of defended task force vehicles is not subject to this bias. However for consistency, percentages are used for this response as well.

With a traditional factorial experiment, each independent variable is assigned a "high" and a "low" factor level. The factor levels correspond to the amount or value of each independent variable used in the experiment. The amounts of the independent variables used in the experiment are restricted to these factor levels. In this research, the factor levels correspond to numbers of ADA weapon system and number of enemy aircraft at the start of each simulation run. During experiment, the analyst examines the results of all possible combinations of the "high" and "low" factor levels on the selected response variables. By analyzing all possible combinations, the analyst can isolate the effect that each independent variable has on each response variable.

The selected experimental design consists of 16 factorial points and one center run. The factorial portion of the design is a resolution V, half fraction of the full factorial design for five factors, 2_V^{5-1}. In a resolution V design, no main effect is aliased with any other main effect or two factor interactions, nor are two factor interactions aliased with each other (appendix B). Thus, the design provides a relatively good estimate of the main effects and two factor interactions. Due to the sparsity of effects principle (Meyers & Montgomery, 1995) and the nature of the factors in this problem, it is likely that the responses will be dominated by the main effects and two factor interactions.

For the 2_V^{5-1} design, each factor is varied between two levels, a high and a low level. The high and low levels are coded in the design as +1 for high and -1 for low. The initial levels for each factor in the design are contained in Table 3. The selection of the initial factor levels is an arbitrary decision based on unit composition. For the ADA systems, the high levels corresponds to the number of ADA systems found in two platoons of each system type. For the aircraft, the high level corresponds to the typical number of aircraft found in two squadrons of each aircraft type.

Table 3

Initial Factor Levels

Factor	High	Low
Stingers (S)	20	0
Linebackers (L)	8	0
Avengers (A)	12	0
HIPs (Hp)	40	0
HINDs (Hd)	40	0

The 2_V^{5-1} design factorial portion of the design assumes that there are no quadratic effects in the model. In order to test that assumption, the design includes center point. The center point allows testing for curvature in the model. The Data Analysis chapter contains the results of this test.

As stated previously, Janus is a stochastic simulation. In order to account for the inherent variability of the model, the design replicates each design point several times to establish an estimate of the responses at that design point. Randomly selected seeds for each replication ensure independence between replications.

To determine the number of replications needed, the "specified precision" technique suggested by Law and Kelton (1991) is used. With the "specified precision" technique, the analyst specifies a maximum acceptable absolute error on the estimate of the mean, β. For an established β, the analyst projects the number of replications required based on the results of a few initial replications. Assuming that the actual population variance does not differ greatly from the estimate of population variance based on the sample, the required number of replications (n_a^*) is given by:

$$n_a^* = \min\left\{ i \ge n : t_{i-1,1-\alpha/2} \sqrt{S^2(n)/i} \le \beta \right\} \tag{1}$$

where:

n = number of replications in the sample
i = projected number of replications
$S^2(n)$ = estimate of population variance based on the sample
$\beta = |\bar{X} - \mu|$ = absolute error of the estimate
t = critical value from the Student's t distribution.

This experiment is exploratory in nature with the purpose of developing a methodology for more encompassing research. Furthermore, it is not within the intent of this research to identify valid relationships between the variables for direct application outside of this research; the election to use an unclassified training database precludes directly applying the analytical results of this research in a tactical setting. Understanding these intentions, 80% is a reasonable level of precision for confidence intervals on the estimate of the mean responses.

The selection of β for an experiment is an arbitrary decision of the analyst. In this experiment, the defended task force consists of slightly over 50 vehicles. The maximum number of ADA systems employed at any design point is 40. Based on the magnitude of these numbers, an estimate of the remaining task force vehicles or ADA systems that is within 1.5 vehicles of the actual mean seems accurate enough for this research. Therefore, for 80% of the confidence intervals on the means that are constructed, the expected absolute error is 1.5 vehicles.

When utilizing the "specified precision" technique, increased precision results in a corresponding cost of increased replications. Based on Equation 1 above, higher confidence levels or smaller acceptable absolute errors result in a greater number replications required in the experiment. In this experiment, a decrease in β to 1.0 vehicles, requires approximately doubling the number of replications of each design point. As is shown in the following chapter, the number of replications required for the sensitivity analysis and final experiment are 12 and 9 respectively; these seem appropriate for this investigation.

DATA ANALYSIS

Sensitivity Analysis

The selection of the factor levels is an arbitrary decision based on the composition of the military units in which each weapon system is found. High levels for the ADA weapons correspond to the number of systems in two ADA platoons. Likewise, the 40 aircraft, that constitute 2 attack helicopter squadrons, are the basis for the high factor level of the HIPs and HINDs.

In order to determine if the selected factor levels are adequate, this study incorporates a sensitivity analysis. The sensitivity analysis examines data collected from the experiment conducted at the initial factor level settings using coded values. The result of the "specified precision" technique determines the number of required replications for each data point. For the sensitivity analysis, two randomly selected points are subjected to the procedure: data points 14 and 16. Based on 5 initial replications of the two data points, a total of 12 replications of each point appears sufficient to provide an adequate estimate of the remaining ADA systems and the remaining vehicles in the task force. See Appendix C for a detailed explanation of the calculations for the required replications.

Appendix D contains a complete listing of the responses of each replication, for all data points in the sensitivity analysis experiment.

Utilizing coded factor levels, a factorial analysis is conducted on the data to determine the effects of each main factor on the response variables: the amount of ADA remaining and the amount of the task force remaining. With $\alpha = 0.05$, the analysis shows that the number of Linebacker weapon systems does not significantly contribute to determining the number of remaining ADA systems. Likewise, the number of Stingers does not have a significant effect on the amount of the task force remaining. It is also apparent that the effects of the HIPs and HINDs are much larger than that of any other factor, possibly overwhelming the contributions of the other factors. Appendix E contains the tabular factorial analyses for the amount of ADA remaining and the amount of the task force remaining.

To address the issues of main factor significance and factor dominance, the factor levels are adjusted for use in the final experiment. Because the HIPs and HINDs appear to be the dominant factors, their high levels are adjusted to correspond to one squadron of attack helicopters each. It must be understood that with the adjusted settings, the experimental region is significantly smaller than before. Accordingly, this restricts the applicability of the results found here to the same region. Any extrapolation of the experimental results outside the design space is subject to considerable error and must be approached with caution. Table 4 contains the adjusted settings for all five factors.

Table 4

Adjusted Factor Levels

Factor	High	Low
Stingers (S)	20	0
Linebackers (L)	8	0
Avengers (A)	12	0
HIPs (Hp)	20	0
HINDs (Hd)	20	0

Final Experimental Design

As in the sensitivity analysis, the method used to determine the number of replications required in the final experiment is the "specified precision". Once again two randomly selected data points, point 7 and point 10, are subjected to the procedure. Based on 5 initial replications, a total of 9 replications of each point appears sufficient to provide an adequate estimate of the responses. See Appendix F for a detailed explanation of the calculations for the required replications. Appendix G contains a complete listing of the responses of each replication, for all data points in the final experiment.

Using coded factor levels, a factorial analysis reveals the effects of all main factors and two factor interactions on the response variables. With $\alpha = 0.05$, it is apparent that with the new factor levels all of the main effects, as well as many of the interactions, are significant for both responses. Appendix H contains the tabular factorial analyses for the percent of ADA remaining and the percent of the task force remaining.

The percent of the ADA remaining is calculated by dividing the number of air defense weapon systems remaining at the conclusion of the simulation by the initial number of air defense weapons. It is important to remember that the initial number of air defense weapons changes based on the examined data point from the experimental design. Therefore, the initial number of air defense weapons is only constant among replications of the same data point. The percent of the task force remaining is calculated by dividing the remaining task force vehicles by the initial number of task force vehicles. The initial number of task force vehicles is 53 for every replication. The task force numbers do not include any air defense vehicles.

The analysis of variance portion of the factorial analysis for both responses includes a test for curvature. For both models, the test suggests the existence of curvature within the response region, $p = 0.000$ (Appendix H). Thus, a more complete model for both responses could include the quadratic terms of each of the five main factors. Ideally, axial points should be added to the existing design to allow estimation of these effects. In this experiment, 10 axial points could be added to transform the design into a face centered cube configuration. It must be understood that this research utilizes an unclassified training database. The database provides sufficient accuracy to not mislead trainees as to the performance of individual weapons. However, the database is not accurate enough to enable precise calculation of weapon system effects. The relationships between the factors identified here are for demonstration purpose only. Therefore, it is assumed that the benefit gained from adding these additional points to the experiment is not significant enough to warrant their inclusion.

Regression analysis furnishes a means for determining a functional form for each response: the percentage of ADA systems remaining and the percentage of the task force remaining. With an objective of minimizing the mean square error (MSE), the regression analysis renders functional forms for the relationships between the independent and dependent variables. Appendix I displays the regression results for each response, the percentage of ADA assets remaining and the percentage of the task force remaining. In order to reduce the susceptibility to round of errors, the regression analysis in this research was all performed using coded values. The resulting relationships appear as Equations 2 and 3 below.

$$\% \text{ ADA remaining} = 0.578 + 0.111\,S + 0.0872\,L + 0.156\,A - 0.212\,Hp - 0.194\,Hd - 0.0845\,S{*}L - 0.0532\,S{*}A + 0.0263\,S{*}Hp + 0.0304\,S{*}Hd - 0.0547\,L{*}A + 0.0234\,L{*}Hd + 0.0350\,A{*}Hp + 0.0361\,A{*}Hd + 0.0235\,Hp{*}Hd \qquad (2)$$

$$\% \text{ Task Force remaining} = 0.698 + 0.0759\,S + 0.0566\,L + 0.0777\,A - 0.182\,Hp - 0.146\,Hd - 0.0347\,S{*}L - 0.0291\,S{*}A + 0.00614\,S{*}Hp + 0.0235\,S{*}Hd - 0.0462\,L{*}A + 0.0334\,L{*}Hp - 0.00588\,L{*}Hd + 0.0529\,A{*}Hp - 0.00988\,A{*}Hd - 0.0158\,Hp{*}Hd \qquad (3)$$

The resulting R^2 values for the regression equations are 95.7% and 94.9% respectively. This shows that most of the variability in the responses is explained by these equations. The adjusted R^2 values are 95.3% and 94.4%. The small difference between the R^2 values and the adjusted R^2 values is an indication that the model does not contain unnecessary terms.

Residual analysis is a useful tool for identifying problem areas with regression models. Ordinary residuals are the difference between the fitted values of the response function and the actual observed values. Under the assumptions of regression, errors should follow a normal distribution (Fox, 1991). For the percentage of ADA and the task force remaining, the normal probability plots of the residuals do not provide any reason to doubt this assumption (Appendix J).

Typically, plots of the residuals against the fitted values of the response are used to detect nonconstant error variance (Cook & Weisberg, 1982). In this experiment, these plots do not provide any evidence that the assumption of constant error variance does not hold true in the center of the design space. However, at the extremes of the responses (0% and 100%), the plots show there is very little variance (Appendix J). This is a direct result of the definition of the responses. Logically, observed values of the responses cannot be lower than 0% or greater than 100%. Yet, the regression equations could predict values that violate these extremes. The actual observed values in these situations would lie on or near 0% or 100%. Thus, the error variance at the extreme responses will approach 0.

Residual plots can also reveal information regarding model misspecification. The test for curvature previously indicated the possibility of curvature in the response region (Cook & Weisberg). Plots of the residuals against the five factors of both response also support the existence of curvature in the experimental region (Appendix J). Due to the nature of the experimental design, it is unclear which factor or factor requires a quadratic term. Clearly, the nonlinearity of the main factors requires attention in future research efforts.

Another use of residual diagnostics is to assist in identifying points of high influence and outliers. Observations with high studentized residuals, $|t_i| \geq 2$, could be considered as possible outliers (Fox). Several observations of both responses have observations with high studentized residuals (Appendix K). An additional measure to help identify points of influence is using Cook's distance (Cook's D). Observations with large Cook's D values are considered possible points of high influence. Three observations with high studentized residuals also have large values for Cook's D lending strong support to the assertion that they are leverage points (Appendix K). It is inadvisable to remove the leverage points found here from this experiment. Although they influence the model, they help demonstrate the inherent variability of a dynamic battlefield.

The regression equations shown above clearly demonstrate the existence of interactions between the variables. However, the nature of the interactions does not support the notion of the existence of synergy between weapon systems, a long standing tenet of ADA. If synergy existed, the interaction effects between ADA weapons would be positive. For both the ADA and task force remaining equations, note that the interactions between ADA weapon systems are negative. In other words, the net effect of the weapons deployed in proximity with one another is less than if they deployed separately. A possible explanation for this is that it is due to the finite number of aircraft flown in the experiment. For nearly every replication at every design point, all of the enemy aircraft are destroyed, regardless of the amount of air defense deployed. With an increase in the amount of air defense assets, a similar number of opportunities to destroy aircraft are now shared among more air defense systems. In an experiment designed with a greater number of aircraft and an intelligent enemy, the nature of these interactions may be quite different.

In light of the question of the validity of the interaction terms, consideration of a regression model in the main effects only is warranted. Some argue that simplified regression models are more robust as they are less dependent on the contributions of the relationships between the independent variables. Equations 4 and 5 below are the simplified process equations for the amount of ADA and task force assets remaining (Appendix I).

$$\%\text{ADA remaining} = 0.578 + 0.111\ S + 0.0872\ L + 0.156\ A - 0.212\ Hp - 0.194\ Hd \tag{4}$$

$$\%\text{TFremaining} = 0.698 + 0.0759\ S + 0.0566\ L + 0.0777\ A - 0.182\ Hp - 0.146\ Hd \tag{5}$$

The resulting regression produce R^2 values of 83.7% (R^2 adj. = 83.1%) and 83.9% (R^2 adj. = 83.4%) respectively. This reveals that the inclusion of interaction terms explains over 10% more variability in the responses. An additional price to pay for process simplification is mean square errors that approximately triple those associated with the equations including (Appendix I). The lower R^2 values and corresponding increases in MSE, suggests that the inclusion of the interaction terms in the model is necessary.

With the experiment complete, the relationships between the variables must be integrated into the ADA planning process. Using the regression equations the ADA planner will understand the expected results of employing various ADA weapon system packages. However, manually calculating the results of the regression equations for multiple courses of action is time consuming and impractical. In order for them to be be useful, the regression equations must be incorporated into a planning tool that reduces the demands on the planner.

APPLICATION AND LIMITATION

Application

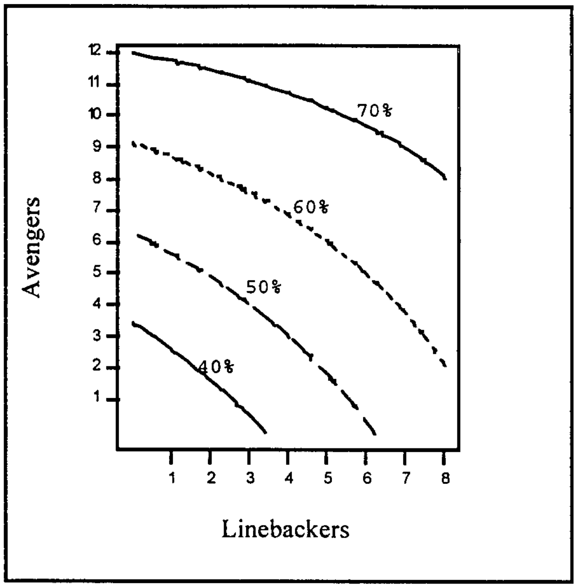
Graphical Approach

The development of the regression equations for the remaining strength of the defended task force and air defense weapons provides planners with the ability to analytically approach the air defense planning process. In a dynamic operational environment, the direct use of the regression equations, to assist in the force allocation process, would be unwieldy and cumbersome at best. However, techniques do exist for determining acceptable solutions for multiple response problems that can be readily applied in this situation.

In 1959, Arthur E. Hoerl suggested a relatively simple technique of overlaying response surface contour plots to determine regions of optimal response for more than one response variable. In general terms, the analyst selects two of the factors studied to plotted along the X and Y axes. Holding all other factors constant, the regression equation is used to plot contours of constant response. This process is repeated for all response criteria. The resulting contour plots overlaid on one another. From overlaid plots, the analyst can determine regions of optimality for response variables studied. For multiple factors, the process is repeated for all combinations of factors. The decision maker uses all sets of overlays to select the "best" point.

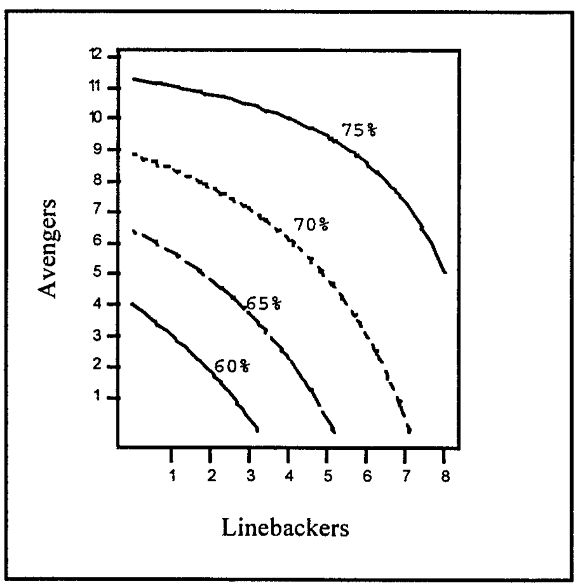
Consider this technique applied to the regressions equations developed for the remaining strength of the task force and air defense assets. From the Intelligence Preparation of the Battlefield (IPB) developed for the operation being planned, the air defense planner has a planning estimate of the amount and type of enemy aircraft expected. Provided the IPB estimate of the enemy aircraft is within the range of the data used to develop the regression models, the planner applies the graphical overlay technique. Holding the enemy aircraft factors constant at the estimated levels, only three factors remain for consideration: the numbers of Stingers, Linebackers, and Avengers. The planner can then select one of these weapon system to hold constant and develop contour plots of constant response. Figures 1 and 2 below show contour plots of constant response for the percentage of the task force remaining and the percentage of the air defense assets remaining, respectively. For each contour plot, the expected enemy threat consists of 10 HIPs and 10 HINDs. Additionally, the number of Stingers is held constant at 10.

Figure 1



Contour Plot of the Percentage of Air Defense Assets Remaining

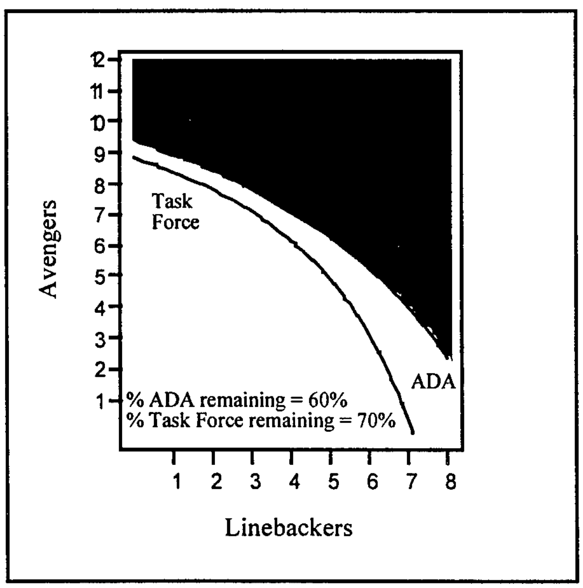
Figure 2



Contour Plot of the Percentage of the Task Force Remaining

The above plots can now be overlaid to assist in determining an acceptable allocation of air defense assets. Assume for this operation that 60% air defense assets remaining and 70% of the task force remaining is "acceptable". Figure 3 shows the combined contours of constant response, depicting the "acceptable" region as shaded. The planner can now visually identify force allocation packages of Stingers, Linebackers, and Avengers within the "acceptable" region. It is important to note that this technique does not constrain the planner to one "best" solution. Rather, it identifies an entire range of possibilities that meet the required constraints.

Figure 3.



Overlaid Contour Plots of Remaining Air Defense and Task Force Strength

Consider the following example. A unit expects an enemy air attack consisting of 10 HIPs and 10 HINDs. Holding the number of Stingers constant at 10, a planner evaluates the the combination of 10 Avengers and 4 Linebackers using Figure 3. The corresponding response for this combination of air defense assets lies within the shaded region of Figure 3. Therefore, this is an "acceptable" ADA employment package.

The contour plots above identify individual amounts of aircraft and air defense systems. Although the factor levels identified previously correspond to unit size configurations, enemy tactics and friendly force packages do not always conform to these configurations. Since HIPs and HINDs attack in pairs, typical threat packages are any combination of even numbers of HIPs and HINDs. For most operations, ADA elements fight as platoons. However, in some instances, platoons split into sections based on operational requirements. Furthermore, platoons often enter battle at less than 100% strength. For example, a platoon of Avengers may fight with only five Avengers due to maintenance problems or attrition. This situation frequently occurs during subsequent battles. Therefore, when selecting force allocation packages, ADA commanders must not consider squadrons of enemy aircraft and platoons of air defense weapons, rather the number of attacking aircraft and available ADA weapons.

Prediction Intervals and Spreadsheet Modeling

One shortcoming of the contour plot approach is that it does not address the variability associated with each of the responses. The plotted responses are mathematical estimations of their statistical mean. For each point on the response curves, there exists a region of variability that is not captured by the graphs. This could lead to the selection of a force allocation package with an unacceptable amount of expected variability in the response.

In order to account for variability, each predicted response of the regression models must be accompanied by a prediction interval. In the graphical approach, this would involve plotting "prediction bands" on either side of the contours of constant response. Adding two additional contour lines for each response contour could significantly increase the difficulty of analyzing the plots. Representing the response estimates and prediction intervals can be done easily and clearly through computer based spreadsheets.

First, modeling the responses in a spreadsheet is a feasible solution for this problem. In Microsoft Excel, data tables provide a means for determining the differences in the results of equations due to changing up to two variables. The data tables shown in the following discussion are the output of a prototype force allocation planning tool developed on MS Excel. The tool consists of a series of linked spreadsheets. Each data table used above is contained on a separate spreadsheet in the tool. The user specifies the expected enemy threat and the number of weapons for the ADA weapon system held constant on a separate data entry spreadsheet. The tool dynamically updates the remaining spreadsheets with each change on the data entry sheet. No additional user interaction is required beyond initial data entry. The tool's ease of use will advance its acceptance in the field Army. Appendix L provides examples of each spreadsheet in the tool as well as explains its construction.

For illustrative purposes, consider the same situation previously presented in the contour plot discussion. The expected enemy threat consists of 10 HIPs and 10 HINDs and Stingers are chosen to remain constant at 10 as well. Figures 4 and 5 are sample solution tables.

Figure 4

Avengers	Linebackers: 0	1	2	3	4	5	6	7	8
12	70%	71%	72%	73%	73%	74%	75%	76%	77%
11	67%	68%	69%	70%	71%	72%	73%	74%	75%
10	63%	64%	66%	67%	68%	69%	71%	72%	73%
9	60%	61%	63%	64%	66%	67%	69%	70%	72%
8	56%	58%	60%	61%	63%	65%	66%	68%	70%
7	53%	55%	56%	58%	60%	62%	64%	66%	68%
6	49%	51%	53%	56%	58%	60%	62%	64%	67%
5	46%	48%	50%	53%	55%	58%	60%	62%	65%
4	42%	45%	47%	50%	53%	55%	58%	61%	63%
3	39%	41%	44%	47%	50%	53%	56%	59%	61%
2	35%	38%	41%	44%	47%	50%	54%	57%	60%
1	32%	35%	38%	41%	45%	48%	51%	55%	58%
0	28%	32%	35%	39%	42%	46%	49%	53%	56%

Predicted Percentage of Air Defense Remaining

Figure 5

Avengers										
	12	77%	77%	77%	77%	78%	78%	78%	78%	79%
	11	74%	75%	75%	76%	76%	77%	77%	78%	78%
	10	72%	73%	74%	74%	75%	76%	76%	77%	78%
	9	70%	71%	72%	73%	74%	75%	75%	76%	77%
	8	68%	69%	70%	71%	72%	73%	74%	75%	77%
	7	66%	67%	69%	70%	71%	72%	74%	75%	76%
	6	64%	66%	67%	68%	70%	71%	73%	74%	75%
	5	62%	64%	65%	67%	69%	70%	72%	73%	75%
	4	60%	62%	64%	65%	67%	69%	71%	73%	74%
	3	58%	60%	62%	64%	66%	68%	70%	72%	74%
	2	56%	58%	60%	62%	65%	67%	69%	71%	73%
	1	54%	56%	59%	61%	63%	66%	68%	70%	73%
	0	52%	54%	57%	59%	62%	65%	67%	70%	72%
		0	1	2	3	4	5	6	7	8
						Linebackers				

Predicted Percentage of the Task Force Remaining

As done with contour plot method previously, assume that 60% air defense assets remaining and 70% of the task force remaining is "acceptable". As before, the planner can easily identify a range of combinations of air defense assets that satisfy these conditions. For example, from Figures 3 and 4 above, 10 Stingers (held constant), 8 Avengers and 4 Linebackers appears to be an adequate force structure to defend against 10 HIPs and 10 HINDs (held constant). Likewise, the tabular results of the data table analysis, in Figures 4 and 5, also indicate that this air defense mix is "sufficient", pointing to an average of 63% air defense assets remaining and 72% of the task force remaining. However, if the planner considers the variation associated with this allocation of assets, he may come to a very different conclusion.

Secondly, predictions intervals constructed for each point estimate of the response will provide the decision maker an understanding of the variability associated with the regression models. Prediction intervals are determined using Equation 6 below.

$$\hat{y}(x_0) \pm t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2(1 + x'_0\,(X'X)^{-1}x_0} \tag{6}$$

where:

MSE is the estimate of $\hat{\sigma}^2$
n = the number of observations in the sample
p = the number of parameters in the model
[note: $(X'X)^{-1}$ for response is contained in Appendix I]

It is important to realize that the width of the prediction interval is dependent on the location in the response region. Incorporating this formula into a similar data to that used for the predicted response, will enable the decision maker to more fully understand the possible consequences of his decision. As before, holding the number of Stingers, HIPS, and HINDs constant at 10 each, Figures 6 and 7 show the half width of the prediction intervals for each response at each combination of air defense weapons. The prediction interval half widths are the numbers that must be added to and subtracted from the predicted responses to form complete prediction intervals, α = 0.10. A 90% confidence interval is selected as it seems a reasonable degree of precision for operational planning. In practice, the confidence used should reflect the degree of precision desired by the commander. Modeling in a spreadsheet allows for easy adjustment of the confidence level.

Figure 6

Avengers \ Linebackers	0	1	2	3	4	5	6	7	8
12	13.7%	13.7%	13.7%	13.7%	13.7%	13.7%	13.7%	13.7%	13.7%
11	13.7%	13.7%	13.7%	13.6%	13.6%	13.6%	13.7%	13.7%	13.7%
10	13.7%	13.7%	13.6%	13.6%	13.6%	13.6%	13.6%	13.7%	13.7%
9	13.7%	13.7%	13.6%	13.6%	13.6%	13.6%	13.6%	13.7%	13.7%
8	13.7%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.7%
7	13.7%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.7%
6	13.7%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.7%
5	13.7%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.7%
4	13.7%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.6%	13.7%
3	13.7%	13.7%	13.6%	13.6%	13.6%	13.6%	13.6%	13.7%	13.7%
2	13.7%	13.7%	13.6%	13.6%	13.6%	13.6%	13.6%	13.7%	13.7%
1	13.7%	13.7%	13.7%	13.6%	13.6%	13.6%	13.7%	13.7%	13.7%
0	13.7%	13.7%	13.7%	13.7%	13.7%	13.7%	13.7%	13.7%	13.7%

Half Width of Prediction Interval for Air Defense Remaining

Figure 7

Avengers \ Linebackers	0	1	2	3	4	5	6	7	8
12	11.1%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.1%
11	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
10	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
9	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
8	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
7	11.0%	11.0%	11.0%	11.0%	10.9%	11.0%	11.0%	11.0%	11.0%
6	11.0%	11.0%	11.0%	10.9%	10.9%	10.9%	11.0%	11.0%	11.0%
5	11.0%	11.0%	11.0%	11.0%	10.9%	11.0%	11.0%	11.0%	11.0%
4	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
3	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
2	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
1	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
0	11.1%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.1%

Half Width of Prediction Interval for the Task Force Remaining

For the identical acceptable conditions as before (at least 60% ADA remaining and at least 70% of the task force remaining), now consider the same application of the 10 Stingers, 8 Avengers, and 4 Linebackers versus 10 HIPs and 10 HINDs. From Figure 6, the prediction interval factor for the percentage of ADA remaining is 13.6%, rendering the interval 49.4% to 86.6%. Similarly figure 7 gives 11% as the half width of the prediction interval for the percentages of the task force remaining resulting in the interval 61% to 83%. Although the predicted responses for the amount of ADA assets remaining (63%) and the task force remaining (72%) are within permissible bounds, the low end of prediction intervals of both responses extend well below the "acceptable" region. Thus considering the variability inherent in the models, the decision maker may be drawn to an entirely different allocation of ADA resources.

As indicated previously, the inclusion of the prediction interval on the graphical application is possible. However, the additional of two contours for each existing contour on the plot could greatly decrease the readability of the graphs. One possible solution is to only plot the contours of constant response that correspond to the upper ranges of the prediction intervals. For example, a point on the contour at 70% would be interpreted as the combination of variables whose mean response added to the half width of the prediction interval sums to 70%. The resulting graphs would be as easy to interpret as the plots illustrated in the figures in this research. Additional research is needed to determine if the tabular spreadsheet presentation or the graphical presentation is preferable.

A significant contribution of this research is that it demonstrates a means to analytically model the battlefield. The resulting regression models are readily incorporated into graphical and spreadsheet applications. To illustrate the improvement that this method makes in force ratio analysis, consider the same planning situation analyzed with COFA. COFA provides planning values for all of the weapon systems considered here except the Linebacker. The Linebacker is variant of the BSFV which is widely considered improvement in firepower and survivability. Lacking an established COFA value for the Linebacker, the BSFV COFA value will be used here as a conservative estimate. Table 5 displays the COFA analysis for 10 Stingers, 4 Linebackers, and 8 Avengers opposing 10 HIPs and 10 HINDs.

Table 5

COFA Analysis of Example Situation

Type	Friendly Value	Num.	Total	Type	Enemy Value	Num.	Total
Stinger	0.5	10	5	MI-8 HIP	1.8	10	18
Linebacker	0.6	4	2.4	MI-24 HIND	2.4	10	24
Avenger	1.9	8	15.2				
TOTAL			22.6				42
			Force ratio	22.6 :42 or 1.0 : 1.85			

Based on the 1:3 force ratio convention, COFA plainly indicates that the ADA assets allocated are more than enough to combat the expected enemy air threat. Moreover, COFA suggests that the Avengers and Stingers are more than enough air defense without any contribution from the Linebackers. Without the Linebackers and three fewer Avengers, the resulting force ratio of 1:2.9 is still better than the 1:3 COFA guideline. From Figures 4 and 5 above for this level of air defense, the predicted amount of air defense remaining is 46% with 62% of the task force remaining. Most likely, few commanders would accept predicted losses of this magnitude. Even without considering any prediction intervals, both of these predicted values fall well below the previously established thresholds of 60% for the ADA and 70% for the task force. One could argue that the 60% and 70% thresholds are arbitrary and could be adjusted. However, COFA provides no measures of performance to adjust the values. COFA merely offers the 1:3 ratio as a planning guide with only a binary indication of the expected outcome.

Limitations

Unfortunately, this research, as well as the resulting planning tool, suffers from limitations that preclude its immediate introduction as a planning aid. Many of the limitations are a result of the initial scope of the research. The disposition, mission, and composition of the task force in the experiment impose severe limitations. The scenario arrays a battalion task force in a defensive posture, protected by various combinations of air defense assets. All task force and air defense assets are stationary and occupy positions in partial defilade. Since defilade provides a measure of protection from observation and enemy fire, this research provides no insight into the differences that may be encountered by units in full defilade. Likewise, the increased exposure and vulnerability, characteristic of offensive operations, are not addressed. Additionally, the task force composition and activities remained static throughout the simulation. Clearly, differing types and amounts of tanks and Infantry Fighting Vehicles would influence the battle as did differing types and amounts of air defense vehicles. This work does not capture these influences.

Limitations also exist on the enemy side. The task force is attacked by various configurations of enemy aircraft, however, no enemy ground threat accompanies the air attack. The effect an enemy ground attack on the air defense and task force remaining as well as any interaction between enemy ground forces and air attack remain unknown. Additionally, the task force does not fight an "intelligent or thinking" enemy. All of the enemy attack routes are preplanned. This results in an enemy that attacks relentlessly, without regard to its losses or successes on the battlefield. It is highly likely, that this artificiality introduces a disproportionate number of friendly and enemy losses when compared to battles conducted with knowledgeable opponents.

In the experiment, the effects of terrain, time of day, and weather are controlled. The mountainous desert terrain, daylight hours, and fair weather encountered in the simulation favor all play an influential role in determining visibility and trafficability. A different combination of terrain, time, and weather could drastically impact the results obtained here. Applying the relationships established here to situations with different environmental conditions must be approached with extreme caution.

The experimental factor levels selected for the friendly and enemy units constrain the study further. The relationships established between the variables can only be considered valid within the response region. Thus, when utilizing the developed regression equations, an analyst must insure that the number of aircraft and air defense systems do not exceed the high factor levels used in the experiment. Using values, for any of the air defense weapons or the enemy aircraft greater than high factor levels, could result in significant error.

A final shortcoming of this work is that the Janus suite used incorporates an unclassified "training" database. The U.S. Army considers actual performance data for many weapon systems to be "classified". For unit level training, the Army typically utilizes "unclassified" weapons system performance data for training use, reserving the use of "classified" data for detailed analysis and operational planning. For academic and publishing considerations, "unclassified" data were used here.

CONCLUSION AND RECOMMENDATIONS

The use of force ratios is an extremely valuable planning tool for the U.S. Army. Analysis of force ratios provides planning staffs and commanders an indication of the required combat power for successful operations. While the use of force ratios is prevalent for analyzing maneuver force allocation, little effort has been made to apply this technique to air defense operations. The initial attempt at force ratio analysis within the air defense domain, Correlation of Forces Air, achieves some success, however, the process suffers from several shortcomings. These shortcomings include: the questionable validity of the relative combat power values, the exclusion of the possible synergistic effect of employing a mix of diverse weapon systems together, the failure to account for the differing dynamics of offensive and defensive operations, and the inadequate address of success criteria. With the increase in the number of unit deployments, the need arises for a development of an air defense force ratio planning tool that addresses the short falls of Correlation of Forces Air, COFA. This study addresses two of these shortcomings: the consideration of synergistic effects of weapon systems and success criteria.

This research develops a methodology that is a substantial improvement over COFA for air defense planning. Through the use of a designed experiment, the study unquestionably demonstrates the possibility of modeling the modern air defense battle.

Using Janus, an established military training simulation, a battlefield scenario simulates a representative air defense battle. Multiple replications of the simulation at numerous design points fixed the relationships existing among the air defense and air threat weapon systems. The experiment produced two distinct response equations modeling the effects of the various weapon systems on determining the remaining air defense and task force assets. Unlike with traditional force ratio techniques, now the decision maker has quantifiable measures of effectiveness for air defense operations.

The development of the means to assess these two success criteria provides a solid foundation for the construction of alternative force allocation planning tools. Using a graphical technique of comparing contours of constant response, an operational planner can identify a range of feasible force allocation courses of action for air defense units, at a glance. For each course of action, the decision maker will understand the expected values of both success criteria: the percentage of ADA and the task force remaining.

Although an attractive solution, the simple graphical approach fails to consider the inherent variability associated with each of the responses. On the other hand, examining the predicted responses and associated variability of regression equations readily lends itself to spreadsheet data table analysis. Developed for this research, a spreadsheet based air defense force allocation planning tool rapidly produces point estimates of the responses for multiple combinations of air defense assets opposing an established air threat. Additionally, the tool generates the data needed to quickly construct prediction intervals for each predicted response. The spreadsheet planning tool equips the ADA commander with the information required to make crucial force allocation decisions.

Moreover, the tool's simple design and ease of use enable it to be utilized with minimal instruction.

While this research has resulted in a new methodology for force ratio analysis for the air defense battle, the research is limited in several ways. The scope of the friendly and enemy missions, force compositions, and the constant environmental conditions limit the applicability of the findings. Extrapolations of the results found here to situations outside the experimental region are subject to considerable error. Further, since the scenario did not include an enemy ground force, it is not prudent to assume that the relationships found in this experiment apply when a ground threat is present. Additionally intelligent interaction did not exist on either the enemy or friendly side. Thus, combatants were restricted to preset plans that do not account for the adjustments that intelligent agents engaged battle might make. Finally, it must be understood that the unclassified database used provides valuable insight into the battlefield. However, it is not sufficiently accurate for establishing precise analytical relationships between the variables.

As stated previously, the most significant contribution of this research is that it provides a new methodology to analytically model the air defense battlefield. Using this methodology as a framework, it is recommended that expanded experiments are conducted that account for this research's limitations. Separate regression models could be developed for different unit missions, environmental conditions, and enemy tactics. While difficult to control, it is critical to include human interaction into the experiment. The addition of intelligent combatants would significantly heighten the realism of the scenarios and help align the results of the experiment with reality. The utilization of a classified database would ensure that the variable relationships obtained would be as close to valid as possible.

Once the variable relationships from the expanded experiments are obtained, they must be incorporated into planning tool. In this research, both graphical and tabular approaches are explored. Additional research is required to determine which technique is best suited for development into an integrated air defense force ratio planning tool.

The significance of this research is not limited to the domain of air defense artillery. Like COFA, the force ratio process for maneuver forces does not clearly define measures of effectiveness or success criteria. Utilizing the same methodology applied here, maneuver force planning could be equally improved. Expanded experiments considering the range of factors influencing the battle could certainly produce analytical models of the maneuver battlefield. As with the models developed for the air defense battle, the maneuver models would provide the basis for force allocation planning tools that provide commanders a clear understanding of the consequences of their decisions.

Appendix A

Glossary

ADA - Air Defense Artillery.

AT6 Spiral - Anti-tank missile produced by the former Soviet Union. It has a maximum range of approximately 6000 m (HQ DA,1991).

AT bn - Anti Tank Battalion.

ATGM - Anti-tank guided missile.

Atk hel sqdn - Attack helicopter squadron.

Avenger - A U.S. Army air defense weapon. Consists of a turret mounted on the bed of a HMMWV containing 8 ready to fire Stinger missiles (USAADASCH, 1991).

BFV - Bradley Fighting Vehicle - M2. A U.S. Army, light armored, infantry vehicle capable of carrying six soldiers in addition to a three man crew. The BFV is armed with a 25 mm chain gun, a 7.62 mm coaxial machine gun, and a TOW missile launcher (Janes Armour and Artillery [Janes], 1992).

BMP-2 - A lightly armored variant of the BMP-1 infantry combat vehicle produced by the former Soviet Union. It's armament includes a 30 mm main gun, a 7.62 mm coaxial machine gun, and an anti-tank missile launcher (HQ DA,1991).

Bn - battalion.

BSFV - Bradley Stinger Fighting Vehicle. A U.S. Army air defense vehicle based on the BFV (HQ DA, 95). The newest version is called the Linebacker (USAADASCH, 1996).

BTR - A lightly armored wheeled personnel carrier manufactured by the former Soviet Union. Several variants exist with the most common armament including a 14.5mm machine gun and a 7.62mm coaxial machine gun (HQ DA,1991).

Cav sqdn - Cavalry squadron.

COFA - Correlation of Forces Air.

Div recon - Division reconnaissance element.

HMMWV - High Mobility Military Wheeled Vehicle.

Linebacker - The latest version of the BSFV. Replaces the TOW missile launcher with a Stinger missile pod with four ready to fire missiles (USAADASCH, 1996).

M1A1 - The current U.S. Army main battle tank named the Abrams. The tank is armed with a 120 mm smooth bore gun, a 7.62 mm coaxial machine gun, and a 12.7 mm machine gun. The tank is an improvement over the M60 series tanks in protection, mobility, and firepower (Janes, 1992).

M60 - A U.S. Army main battle tank.. The tank is armed with a 105 mm rifled gun, a 7.62 mm coaxial machine gun, and a 12.7 mm machine gun. The tank has been almost entirely replaced in the active army by the M1 Abrams tank (Janes, 1992).

Mi-8 HIP - A troop carrying helicopter manufactured by the former Soviet Union and exported to its allies. The helicopter is available with several weapon system configurations that include a variety of rockets, bombs, anti-tank guided missiles and machine guns (HQ DA,1991).

Mi-24 HIND - An attack helicopter, also capable of troop transport, manufactured by the former Soviet Union and exported to its allies. Dubbed the "flying tank", the helicopter is heavily armored and is available with several weapon system configurations that include a variety of rockets, bombs, anti-tank guided missiles, and machine guns (HQ DA,1991).

OPTEMPO - Operational Tempo

Stinger - An U.S. Army, shoulder fired, short range, air defense missile (HQ DA,1985). The missile is also incorporated in other weapon systems such as the BSFV and Avenger.

TOW - Tube lauched, Optically tracked Wire guided missile. A U.S. Army wire guided anti-tank missile (HQ DA, 1995).

USACGSC - The United States Army Command and General Staff School.

Appendix B

Experimental Design

The following table depicts the experimental design in coded variable form.

Run	S	L	A	Hp	Hd	Remarks
1	-1	-1	-1	-1	1	
2	1	-1	-1	-1	-1	no run
3	-1	1	-1	-1	-1	no run
4	1	1	-1	-1	1	
5	-1	-1	1	-1	-1	no run
6	1	-1	1	-1	1	
7	-1	1	1	-1	1	
8	1	1	1	-1	-1	no run
9	-1	-1	-1	1	-1	
10	1	-1	-1	1	1	
11	-1	1	-1	1	1	
12	1	1	-1	1	-1	
13	-1	-1	1	1	1	
14	1	-1	1	1	-1	
15	-1	1	1	1	-1	
16	1	1	1	1	1	
Center	0	0	0	0	0	

(Data points with the remark "no run" indicate that no simulation run was required due to the lack of enemy aircraft. The following page shows the design in natural variable form.)

Resolution V, 1/2 fraction.

Design Generators: E = ABCD
Defining Relation: I = ABCDE

Alias Structure

A + BCDE	AB + CDE	BD + ACE
B + ACDE	AC + BDE	BE + ACD
C + ABDE	AD + BCE	CD + ABE
D + ABCE	AE + BCD	CE + ABD
E + ABCD	BC + ADE	DE + ABC

The following table depicts the experimental design in coded variable form.

Run	S	L	A	Hp	Hd	Remarks
1	0	0	0	0	20	
2	20	0	0	0	0	no run
3	0	8	0	0	0	no run
4	20	8	0	0	20	
5	0	0	12	0	0	no run
6	20	0	12	0	20	
7	0	8	12	0	20	
8	20	8	12	0	0	no run
9	0	0	0	20	0	
10	20	0	0	20	20	
11	0	8	0	20	20	
12	20	8	0	20	0	
13	0	0	12	20	20	
14	20	0	12	20	0	
15	0	8	12	20	0	
16	20	8	12	20	20	
Center	10	4	6	10	10	

(Data points with the remark "no run" indicate that no simulation run was required due to the lack of enemy aircraft.)

Appendix C

Replication Calculation for the Sensitivity Analysis

To determine the number of required replications, the observed values from the initial 5 replications of two randomly selected data points, are used to calculate an estimate of the variance.

Data Point 14

	Systems Remaining	
Replicate	ADA	Task Force
1	21	34
2	17	27
3	16	25
4	19	28
5	12	25
Variance	11.5	13.7

Data Point 16

	Systems Remaining	
Replicate	ADA	Task Force
1	38	42
2	33	41
3	40	41
4	40	39
5	40	40
Variance	9.2	1.3

Equation 1, listed below, utilizes the calculated variances to estimate the required number of replications (n_a^*). For this experiment $\beta = 1.5$ systems and $\alpha = 0.02$.

$$n_a^* = \min\left\{ \geq n : t_{i-1,1-\alpha/2}\sqrt{S^2(n)/i} \leq \beta \right.$$

(1)

where:

n = number of replications in the sample

i = projected number of replications

$S^2(n)$ = estimate of population variance based on the sample

$\beta = |\bar{X} - \mu|$ = absolute error of the estimate.

Data Point 14

i	ADA error	Task Force error
5	2.3249	2.5376
6	2.0434	2.2303
7	1.8457	2.0145
8	1.6965	1.8517
9	1.5792	1.7236
10	***1.4831***	1.6188
11	1.4028	1.5312
12	1.3343	***1.4563***

Data Point 16

i	ADA error	Task Force error
5	2.0795	***0.7817***
6	1.8277	0.6870
7	1.6508	0.6206
8	1.5174	0.5704
9	***1.4124***	0.5309
10	1.3265	0.4986
11	1.2547	0.4717
12	1.1934	0.4486

At 12 replications, it is projected that the errors on the estimates are all below $\beta = 1.5$ systems.

Appendix D

Sensitivity Analysis Response Data

Point 1

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	0.0%	41	21.2%
2	0	0.0%	45	13.5%
3	0	0.0%	43	17.3%
4	0	0.0%	43	17.3%
5	0	0.0%	41	21.2%
6	0	0.0%	43	17.3%
7	0	0.0%	40	23.1%
8	0	0.0%	44	15.4%
9	0	0.0%	41	21.2%
10	0	0.0%	44	15.4%
11	0	0.0%	40	23.1%
12	0	0.0%	41	21.2%

Point 2 (No aircraft in experimental design - no simulation runs performed.)

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	100.0%	0	100.0%
2	0	100.0%	0	100.0%
3	0	100.0%	0	100.0%
4	0	100.0%	0	100.0%
5	0	100.0%	0	100.0%
6	0	100.0%	0	100.0%
7	0	100.0%	0	100.0%
8	0	100.0%	0	100.0%
9	0	100.0%	0	100.0%
10	0	100.0%	0	100.0%
11	0	100.0%	0	100.0%
12	0	100.0%	0	100.0%

Point 3 (No aircraft in experimental design - no simulation runs performed.)

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	100.0%	0	100.0%
2	0	100.0%	0	100.0%
3	0	100.0%	0	100.0%
4	0	100.0%	0	100.0%
5	0	100.0%	0	100.0%
6	0	100.0%	0	100.0%
7	0	100.0%	0	100.0%
8	0	100.0%	0	100.0%
9	0	100.0%	0	100.0%
10	0	100.0%	0	100.0%
11	0	100.0%	0	100.0%
12	0	100.0%	0	100.0%

Point 4

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	18	35.7%	21	59.6%
2	20	28.6%	34	34.6%
3	21	25.0%	29	44.2%
4	18	35.7%	22	57.7%
5	25	10.7%	37	28.8%
6	20	28.6%	24	53.8%
7	18	35.7%	29	44.2%
8	22	21.4%	36	30.8%
9	22	21.4%	27	48.1%
10	21	25.0%	24	53.8%
11	19	32.1%	20	61.5%
12	20	28.6%	33	36.5%

Point 5 (No aircraft in experimental design - no simulation runs performed.)

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	100.0%	0	100.0%
2	0	100.0%	0	100.0%
3	0	100.0%	0	100.0%
4	0	100.0%	0	100.0%
5	0	100.0%	0	100.0%
6	0	100.0%	0	100.0%
7	0	100.0%	0	100.0%
8	0	100.0%	0	100.0%
9	0	100.0%	0	100.0%
10	0	100.0%	0	100.0%
11	0	100.0%	0	100.0%
12	0	100.0%	0	100.0%

Point 6

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	8	75.0%	16	69.2%
2	19	40.6%	25	51.9%
3	9	71.9%	17	67.3%
4	7	78.1%	13	75.0%
5	8	75.0%	15	71.2%
6	16	50.0%	28	46.2%
7	9	71.9%	18	65.4%
8	9	71.9%	16	69.2%
9	9	71.9%	13	75.0%
10	6	81.3%	22	57.7%
11	9	71.9%	32	38.5%
12	14	56.3%	19	63.5%

Point 7

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	18	10.0%	29	44.2%
2	7	65.0%	13	75.0%
3	11	45.0%	20	61.5%
4	6	70.0%	15	71.2%
5	5	75.0%	19	63.5%
6	14	30.0%	21	59.6%
7	5	75.0%	14	73.1%
8	10	50.0%	17	67.3%
9	12	40.0%	13	75.0%
10	8	60.0%	13	75.0%
11	5	75.0%	9	82.7%
12	12	40.0%	13	75.0%

Point 8 (No aircraft in experimental design - no simulation runs performed.)

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	100.0%	0	100.0%
2	0	100.0%	0	100.0%
3	0	100.0%	0	100.0%
4	0	100.0%	0	100.0%
5	0	100.0%	0	100.0%
6	0	100.0%	0	100.0%
7	0	100.0%	0	100.0%
8	0	100.0%	0	100.0%
9	0	100.0%	0	100.0%
10	0	100.0%	0	100.0%
11	0	100.0%	0	100.0%
12	0	100.0%	0	100.0%

Point 9

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	0.0%	39	25.0%
2	0	0.0%	39	25.0%
3	0	0.0%	39	25.0%
4	0	0.0%	39	25.0%
5	0	0.0%	39	25.0%
6	0	0.0%	39	25.0%
7	0	0.0%	39	25.0%
8	0	0.0%	39	25.0%
9	0	0.0%	39	25.0%
10	0	0.0%	39	25.0%
11	0	0.0%	39	25.0%
12	0	0.0%	39	25.0%

Point 10

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	20	0.0%	43	17.3%
2	20	0.0%	42	19.2%
3	20	0.0%	45	13.5%
4	20	0.0%	44	15.4%
5	20	0.0%	48	7.7%
6	20	0.0%	49	5.8%
7	20	0.0%	50	3.8%
8	20	0.0%	44	15.4%
9	20	0.0%	46	11.5%
10	20	0.0%	46	11.5%
11	20	0.0%	48	7.7%
12	20	0.0%	45	13.5%

Point 11

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	8	0.0%	46	11.5%
2	8	0.0%	47	9.6%
3	8	0.0%	47	9.6%
4	8	0.0%	44	15.4%
5	8	0.0%	49	5.8%
6	8	0.0%	50	3.8%
7	8	0.0%	43	17.3%
8	8	0.0%	47	9.6%
9	8	0.0%	45	13.5%
10	8	0.0%	48	7.7%
11	8	0.0%	45	13.5%
12	8	0.0%	45	13.5%

Point 12

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	28	0.0%	38	26.9%
2	26	7.1%	39	25.0%
3	28	0.0%	37	28.8%
4	26	7.1%	37	28.8%
5	26	7.1%	36	30.8%
6	27	3.6%	39	25.0%
7	28	0.0%	39	25.0%
8	28	0.0%	37	28.8%
9	26	7.1%	39	25.0%
10	28	0.0%	39	25.0%
11	28	0.0%	39	25.0%
12	28	0.0%	39	25.0%

Point 13

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	12	0.0%	42	19.2%
2	12	0.0%	45	13.5%
3	12	0.0%	43	17.3%
4	12	0.0%	43	17.3%
5	12	0.0%	41	21.2%
6	12	0.0%	39	25.0%
7	12	0.0%	39	25.0%
8	12	0.0%	43	17.3%
9	12	0.0%	45	13.5%
10	12	0.0%	42	19.2%
11	12	0.0%	42	19.2%
12	12	0.0%	42	19.2%

Point 14

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	21	34.4%	34	34.6%
2	17	46.9%	27	48.1%
3	16	50.0%	25	51.9%
4	19	40.6%	28	46.2%
5	12	62.5%	25	51.9%
6	13	59.4%	28	46.2%
7	11	65.6%	26	50.0%
8	19	40.6%	35	32.7%
9	13	59.4%	25	51.9%
10	11	65.6%	25	51.9%
11	21	34.4%	29	44.2%
12	22	31.3%	29	44.2%

Point 15

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	15	25.0%	30	42.3%
2	18	10.0%	33	36.5%
3	9	55.0%	21	59.6%
4	8	60.0%	19	63.5%
5	19	5.0%	34	34.6%
6	15	25.0%	25	51.9%
7	13	35.0%	18	65.4%
8	18	10.0%	37	28.8%
9	20	0.0%	35	32.7%
10	13	35.0%	28	46.2%
11	12	40.0%	31	40.4%
12	13	35.0%	26	50.0%

Point 16

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	38	5.0%	42	19.2%
2	33	17.5%	41	21.2%
3	40	0.0%	41	21.2%
4	40	0.0%	39	25.0%
5	40	0.0%	40	23.1%
6	40	0.0%	43	17.3%
7	40	0.0%	40	23.1%
8	40	0.0%	43	17.3%
9	38	5.0%	39	25.0%
10	40	0.0%	40	23.1%
11	40	0.0%	40	23.1%
12	40	0.0%	41	21.2%

Center Point

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	6	70.0%	8	84.6%
2	9	55.0%	15	71.2%
3	9	55.0%	12	76.9%
4	10	50.0%	10	80.8%
5	9	55.0%	16	69.2%
6	7	65.0%	15	71.2%
7	4	80.0%	8	84.6%
8	5	75.0%	13	75.0%
9	9	55.0%	8	84.6%
10	5	75.0%	10	80.8%
11	12	40.0%	17	67.3%
12	8	60.0%	6	88.5%

Appendix E

Estimated Effects for the Sensitivity Analysis

Estimated Effects and Coefficients for ADA Remaining

Term	Effect	Coef	Std Coef	t-value	P
Constant		0.4068	0.01309	31.08	0.000
S	0.0859	0.0429	0.01349	3.18	0.002
L	-0.0050	-0.0025	0.01349	-0.19	***0.853***
A	0.2128	0.1064	0.01349	7.89	0.000
Hp	-0.5827	-0.2914	0.01349	-21.59	0.000
Hd	-0.4117	-0.2058	0.01349	-15.25	0.000

Analysis of Variance for ADA Remaining

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	26.9578	26.9578	5.39157	154.23	0.000
Residual Error	198	6.9217	6.9217	0.03496		
Curvature	1	0.5393	0.5393	0.53929	16.65	0.000
Lack of Fit	10	4.9002	4.9002	0.49002	61.82	0.000
Pure error	187	1.4822	1.4822	0.00793		
Total	203	33.8795				

Estimated Effects and Coefficients for the Task Force Remaining

Term	Effect	Coef	Std Coef	t-value	P
Constant		0.5183	0.00097	53.29	0.000
S	0.0333	0.0166	0.01003	1.66	***0.099***
L	0.0457	0.0228	0.01003	2.28	0.024
A	0.1554	0.0777	0.01003	7.75	0.000
Hp	-0.4864	-0.2432	0.01003	-24.26	0.000
Hd	-0.3554	-0.1777	0.01003	-17.72	0.000

Analysis of Variance for the Task Force Remaining

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	18.7299	18.7299	3.74598	194.12	0.000
Residual Error	198	3.8209	3.8209	0.0193		
Curvature	1	0.8656	0.8656	0.8656	57.7	0.000
Lack of Fit	10	2.199	2.199	0.2199	54.37	0.000
Pure error	187	0.7563	0.7563	0.00404		
Total	203	22.5509				

Appendix F

Replication Calculation for the Final Experiment

To determine the number of required replications, the observed values, from the initial 5 replications of two randomly selected data points, are used to calculate an estimate of the variance.

Data Point 1

Replicate	Systems Remaining ADA	Systems Remaining Task Force
1	15	49
2	16	49
3	16	43
4	16	51
5	17	50
Variance	0.5	9.8

Data Point 10

Replicate	Systems Remaining ADA	Systems Remaining Task Force
1	49	42
2	49	41
3	43	41
4	51	39
5	50	40
Variance	1.8	1.2

Equation 1, listed below, utilizes the calculated variances to estimate the required number of replications (n_a^*). For this experiment $\beta = 1.5$ systems and $\alpha = 0.02$.

$$n_a^* = \min\left\{\geq n : t_{i-1,1-\alpha/2}\sqrt{S^2(n)/i} \leq \beta\right. \quad (1)$$

where:

n = number of replications in the sample
i = projected number of replications
$S^2(n)$ = estimate of population variance based on the sample
$\beta = |\bar{X} - \mu|$ = absolute error of the estimate.

Data Point 7

i	ADA error	Task Force error
5	***0.4848***	2.1462
6	0.4261	1.8864
7	0.3849	1.7038
8	0.3538	1.5661
9	0.3293	***1.4578***

Data Point 10

i	ADA error	Task Force error
5	***0.9198***	***0.7510***
6	0.8084	0.6601
7	0.7302	0.5962
8	0.6712	0.5480
9	0.6248	0.5101

At 9 replications, it is projected that the errors on the estimates are all below $\beta = 1.5$ systems.

Appendix G

Final Experimental Design Response Response Data

Point 1

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	0.0%	25	51.9%
2	0	0.0%	25	51.9%
3	0	0.0%	19	63.5%
4	0	0.0%	36	30.8%
5	0	0.0%	25	51.9%
6	0	0.0%	22	57.7%
7	0	0.0%	26	50.0%
8	0	0.0%	28	46.2%
9	0	0.0%	26	50.0%

Point 2 (No aircraft in experimental design - no simulation runs performed.)

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	100.0%	0	100.0%
2	0	100.0%	0	100.0%
3	0	100.0%	0	100.0%
4	0	100.0%	0	100.0%
5	0	100.0%	0	100.0%
6	0	100.0%	0	100.0%
7	0	100.0%	0	100.0%
8	0	100.0%	0	100.0%
9	0	100.0%	0	100.0%

Point 3 (No aircraft in experimental design - no simulation runs performed.)

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	100.0%	0	100.0%
2	0	100.0%	0	100.0%
3	0	100.0%	0	100.0%
4	0	100.0%	0	100.0%
5	0	100.0%	0	100.0%
6	0	100.0%	0	100.0%
7	0	100.0%	0	100.0%
8	0	100.0%	0	100.0%
9	0	100.0%	0	100.0%

Point 4

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	9	67.9%	3	94.2%
2	13	53.6%	9	82.7%
3	9	67.9%	4	92.3%
4	12	57.1%	5	90.4%
5	12	57.1%	8	84.6%
6	12	57.1%	8	84.6%
7	11	60.7%	7	86.5%
8	4	85.7%	7	86.5%
9	7	75.0%	7	86.5%

Point 5 (No aircraft in experimental design - no simulation runs performed.)

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	100.0%	0	100.0%
2	0	100.0%	0	100.0%
3	0	100.0%	0	100.0%
4	0	100.0%	0	100.0%
5	0	100.0%	0	100.0%
6	0	100.0%	0	100.0%
7	0	100.0%	0	100.0%
8	0	100.0%	0	100.0%
9	0	100.0%	0	100.0%

Point 6

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	5	84.4%	5	90.4%
2	4	87.5%	10	80.8%
3	4	87.5%	2	96.2%
4	12	62.5%	5	90.4%
5	12	62.5%	8	84.6%
6	5	84.4%	4	92.3%
7	5	84.4%	3	94.2%
8	3	90.6%	10	80.8%
9	1	96.9%	8	84.6%

Point 7

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	5	75.0%	15	71.2%
2	4	80.0%	16	69.2%
3	4	80.0%	16	69.2%
4	4	80.0%	16	69.2%
5	3	85.0%	17	67.3%
6	5	75.0%	15	71.2%
7	3	85.0%	17	67.3%
8	5	75.0%	15	71.2%
9	5	75.0%	15	71.2%

Point 8 (No aircraft in experimental design - no simulation runs performed.)

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	100.0%	0	100.0%
2	0	100.0%	0	100.0%
3	0	100.0%	0	100.0%
4	0	100.0%	0	100.0%
5	0	100.0%	0	100.0%
6	0	100.0%	0	100.0%
7	0	100.0%	0	100.0%
8	0	100.0%	0	100.0%
9	0	100.0%	0	100.0%

Point 9

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	0	0.0%	39	25.0%
2	0	0.0%	38	26.9%
3	0	0.0%	39	25.0%
4	0	0.0%	36	30.8%
5	0	0.0%	39	25.0%
6	0	0.0%	37	28.8%
7	0	0.0%	39	25.0%
8	0	0.0%	39	25.0%
9	0	0.0%	39	25.0%

Point 10

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	19	5.3%	37	28.8%
2	19	5.3%	39	25.0%
3	16	25.0%	40	23.1%
4	19	5.3%	39	25.0%
5	19	5.3%	39	25.0%
6	19	5.3%	37	28.8%
7	19	5.3%	36	30.8%
8	20	0.0%	39	25.0%
9	16	25.0%	39	25.0%

Point 11

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	8	0.0%	39	25.0%
2	8	0.0%	39	25.0%
3	8	0.0%	39	25.0%
4	8	0.0%	38	26.9%
5	8	0.0%	40	23.1%
6	8	0.0%	39	25.0%
7	8	0.0%	36	30.8%
8	8	0.0%	40	23.1%
9	8	0.0%	39	25.0%

Point 12

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	8	71.4%	11	78.8%
2	16	42.9%	22	57.7%
3	15	46.4%	17	67.3%
4	13	53.6%	11	78.8%
5	9	67.9%	16	69.2%
6	12	57.1%	11	78.8%
7	13	53.6%	17	67.3%
8	12	57.1%	11	78.8%
9	8	71.4%	14	73.1%

Point 13

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	10	16.7%	38	26.9%
2	9	25.0%	36	30.8%
3	11	8.3%	40	23.1%
4	9	25.0%	38	26.9%
5	10	16.7%	31	40.4%
6	12	0.0%	36	30.8%
7	12	0.0%	35	32.7%
8	10	16.7%	32	38.5%
9	9	25.0%	34	34.6%

Point 14

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	9	71.9%	13	75.0%
2	8	75.0%	11	78.8%
3	6	81.3%	6	88.5%
4	7	78.1%	8	84.6%
5	4	87.5%	5	90.4%
6	6	81.3%	8	84.6%
7	6	81.3%	11	78.8%
8	6	81.3%	15	71.2%
9	5	84.4%	6	88.5%

Point 15

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	5	75.0%	9	82.7%
2	3	85.0%	7	86.5%
3	8	60.0%	9	82.7%
4	8	60.0%	7	86.5%
5	4	80.0%	2	96.2%
6	3	85.0%	4	92.3%
7	4	80.0%	8	84.6%
8	7	65.0%	10	80.8%
9	6	70.0%	8	84.6%

Point 16

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	14	65.0%	26	50.0%
2	19	52.5%	20	61.5%
3	20	50.0%	27	48.1%
4	25	37.5%	27	48.1%
5	14	65.0%	19	63.5%
6	12	70.0%	14	73.1%
7	25	37.5%	25	51.9%
8	27	32.5%	32	38.5%
9	20	50.0%	23	55.8%

Center Point

	ADA		Task Force	
Run	Destroyed	Remaining	Destroyed	Remaining
1	6	70.0%	10	80.8%
2	8	60.0%	6	88.5%
3	9	55.0%	15	71.2%
4	4	80.0%	3	94.2%
5	5	75.0%	10	80.8%
6	4	80.0%	4	92.3%
7	8	60.0%	9	82.7%
8	4	80.0%	5	90.4%
9	5	75.0%	6	88.5%

Appendix H

Estimated Effects for the Final Experimental Design

Estimated Effects and Coefficients for ADA Remaining

Term	Effect	Coef	Std Coef	t-value	P
Constant		0.5782	0.00665	87.01	0.000
S	0.2228	0.1114	0.00686	16.27	0.000
L	0.1745	0.0872	0.00686	12.74	0.000
A	0.3112	0.1556	0.00686	22.72	0.000
Hp	-0.4242	-0.2121	0.00686	-30.97	0.000
Hd	-0.3883	-0.1941	0.00686	-28.34	0.000
S*L	-0.1691	-0.0845	0.00686	-12.34	0.000
S*A	-0.1064	-0.0532	0.00686	-7.77	0.000
S*Hp	0.0526	0.0263	0.00686	3.84	0.000
S*Hd	0.0608	0.0304	0.00686	4.44	0.000
L*A	-0.1094	-0.0547	0.00686	-7.99	0.000
L*Hp	0.0213	0.0106	0.00686	1.55	0.123
L*Hd	0.0468	0.0234	0.00686	3.42	0.001
A*Hp	0.07	0.035	0.00686	5.11	0.000
A*Hd	0.0722	0.0361	0.00686	5.27	0.000
Hp*Hd	0.0471	0.0235	0.00686	3.44	0.001

Analysis of Variance for ADA Remaining

Source	DF	Seq SS	Asj SS	Adj MS	F	P
Main Effects	5	18.2675	18.2675	3.65531	535.62	0.0000
2-Way Interact.	9	2.6224	2.6224	0.29138	42.7	0.0000
Residual Error	138	0.9418	0.9418	0.00682		
Curvature	1	0.1522	0.1522	0.15517	27.03	0.0000
Lack of Fit	1	0.0163	0.0163	0.01628	2.87	0.0920
Pure Error	136	0.7703	0.7703	0.00566		
Total	152	21.8408	21.8408			

Estimated Effects and Coefficients for the Task Force Remaining

Term	Effect	Coef	Std Coef	t-value	P
Constant		0.6978	0.00537	130.08	0.000
S	0.1514	0.0759	0.00553	13.72	0.000
L	0.1132	0.0566	0.00553	10.24	0.000
A	0.1554	0.0777	0.00553	14.06	0.000
Hp	-0.3638	-0.1819	0.00553	-32.89	0.000
Hd	-0.2917	-0.1458	0.00553	-26.37	0.000
S*L	-0.0694	-0.0347	0.00553	-6.28	0.000
S*A	-0.0582	-0.0291	0.00553	-5.26	0.000
S*Hp	0.0123	0.0061	0.00553	1.11	0.269
S*Hd	0.047	0.0235	0.00553	4.25	0.000
L*A	-0.0924	-0.0462	0.00553	-8.36	0.000
L*Hp	0.0668	0.0334	0.00553	6.04	0.000
L*Hd	-0.0118	-0.0059	0.00553	-1.06	0.290
A*Hp	0.1058	0.0529	0.00553	9.56	0.000
A*Hd	-0.0198	-0.0099	0.00553	-1.79	0.076
Hp*Hd	-0.0315	-0.0158	0.00553	-2.85	0.005

Analysis of Variance for the Task Force Remaining

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	9.9868	9.9868	1.99737	453.63	0.000
2-Way Interact.	10	1.3062	1.3062	0.13062	29.66	0.000
Residual Error	137	0.6032	0.6032	0.004		
Curvature	1	0.2353	0.2353	0.23529	86.97	0.000
Pure Error	136	0.3679	0.3679	0.0071		
Total	152	11.8962	11.8962			

Appendix I

Regression Analysis for the Final Experimental Design

Regression Analysis for ADA Remaining - full model

Predictor	Coef	Stdev	t-ratio	p
Constant	0.578169	0.006645	87.01	0.000
S	0.111411	0.006849	16.27	0.000
L	0.087233	0.006849	12.74	0.000
A	0.155603	0.006849	22.72	0.000
Hp	-0.212122	0.006849	-30.97	0.000
Hd	-0.194140	0.006849	-28.34	0.000
S*L	-0.084526	0.006849	-12.34	0.000
S*A	-0.053193	0.006849	-7.77	0.000
S*Hp	0.026304	0.006849	3.84	0.000
S*Hd	0.030397	0.006849	4.44	0.000
L*A	-0.054710	0.006849	-7.99	0.000
L*Hp	0.010634	0.006849	1.55	***0.123***
L*Hd	0.023406	0.006849	3.42	0.001
A*Hp	0.034980	0.006849	5.11	0.000
A*Hd	0.036097	0.006849	5.27	0.000
Hp*Hd	0.023531	0.006849	3.44	0.001

s	0.08219
R-sq	95.8%
R-sq(adj)	95.3%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	15	20.9153	1.3944	206.41	0.000
Error	137	0.9255	0.0068		
Total	152	21.8408			

(L*Hp is a candidate for elimination)

Regression Analysis for ADA Remaining - reduced model

Predictor	Coef	Stdev	t-ratio	p
Constant	0.578169	0.006679	86.57	0.000
S	0.111411	0.006884	16.18	0.000
L	0.087233	0.006884	12.67	0.000
A	0.155603	0.006884	22.6	0.000
Hp	-0.212122	0.006884	-30.81	0.000
Hd	-0.194140	0.006884	-28.2	0.000
S*L	-0.084526	0.006884	-12.28	0.000
S*A	-0.053193	0.006884	-7.73	0.000
S*Hp	0.026304	0.006884	-3.82	0.000
S*Hd	0.030397	0.006884	4.42	0.000
L*A	-0.054710	0.006884	-7.95	0.000
L*Hd	0.023406	0.006884	3.4	0.001
A*Hp	0.034980	0.006884	5.08	0.000
A*Hd	0.036097	0.006884	5.24	0.000
Hp*Hd	0.023531	0.006884	3.42	0.001

s	0.08261
R-sq	95.7%
R-sq(adj)	95.3%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	14	20.899	1.4928	218.74	0.000
Error	138	0.9418	0.0068		
Total	152	21.8408			

The $(X'X)^{-1}$ matrix for the reduced model of ADA Remaining

0.006536	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.006944	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.006944	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.006944	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.006944	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.006944	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.006944	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.006944	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.006944	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.006944
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.006944	0.000000	0.000000	0.000000	0.000000
0.000000	0.006944	0.000000	0.000000	0.000000
0.000000	0.000000	0.006944	0.000000	0.000000
0.000000	0.000000	0.000000	0.006944	0.000000
0.000000	0.000000	0.000000	0.000000	0.006944

Regression Analysis for ADA Remaining - main factors only

Predictor	Coef	Stdev	t-ratio	p
Constant	0.57817	0.01259	45.93	0.000
S	0.11141	0.01298	8.59	0.000
L	0.08723	0.01298	6.72	0.000
A	0.15560	0.01298	11.99	0.000
Hp	-0.21212	0.01298	-16.35	0.000
Hd	-0.19414	0.01298	-14.96	0.000

s	0.1557
R-sq	83.7%
R-sq(adj)	83.1%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	5	18.2765	3.6553	150.76	0.000
Error	147	3.5642	0.0242		
Total	152	21.8408			

Regression Analysis for the Task Force Remaining - full model

Predictor	Coef	Stdev	t-ratio	p
Constant	0.697838	0.005365	130.08	0.000
S	0.075855	0.005530	13.72	0.000
L	0.056624	0.005530	10.24	0.000
A	0.077724	0.005530	14.06	0.000
Hp	-0.181891	0.005530	-32.89	0.000
Hd	-0.145833	0.005530	-26.37	0.000
S*L	-0.034722	0.005530	-6.28	0.000
S*A	-0.029113	0.005530	-5.26	0.000
S*Hp	0.006143	0.005530	1.11	0.269
S*Hd	0.023504	0.005530	4.25	0.000
L*A	-0.046207	0.005530	-8.36	0.000
L*Hp	0.033387	0.005530	6.04	0.000
L*Hd	-0.005876	0.005530	-1.06	0.290
A*Hp	0.052885	0.005530	9.56	0.000
A*Hd	-0.009882	0.005530	-1.79	0.076
Hp*Hd	-0.015759	0.005530	-2.85	0.005

s	0.06636
R-sq	94.9%
R-sq(adj)	94.4%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	15	11.29298	0.75287	170.98	0.000
Error	137	0.60323	0.00440		
Total	152	11.89621			

(Deletion of any model terms results in an increase in MSE.)

The $(X'X)^{-1}$ matrix for the full model of the Task Force Remaining

0.006536	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.006944	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.006944	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.006944	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.006944	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.006944	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.006944	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.006944	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.006944	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.006944
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.006944	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.006944	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.006944	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.006944	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.006944	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.006944

Regression Analysis for the Task Force Remaining - main factors only

Predictor	Coef	Stdev	t-ratio	p
Constant	0.697838	0.009214	75.74	0.000
S	0.075855	0.009497	7.99	0.000
L	0.056624	0.009497	5.96	0.000
A	0.077724	0.009497	8.18	0.000
Hp	-0.181891	0.009497	-19.15	0.000
Hd	-0.145833	0.009497	-15.36	0.000

s	0.114
R-sq	83.9%
R-sq(adj)	83.4%

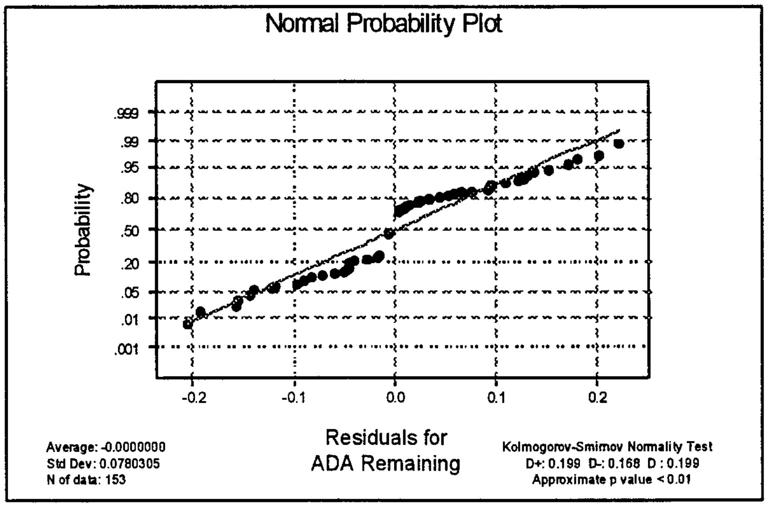
Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	5	9.9868	1.9974	153.77	0.000
Error	147	1.9094	0.0130		
Total	152	11.8962			

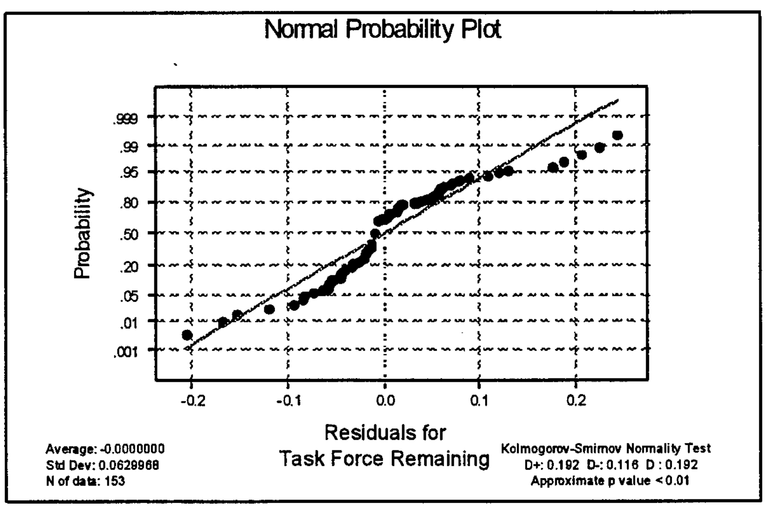
Appendix J

Residuals Plots for the Final Experimental Design

Normal Probability Plot of Residuals for the ADA Remaining



Normal Probability Plot of Residuals for the Task Force Remaining



Both plots show that the residuals generally fall along the line of normal probability. Each plot also includes the results of a Kolmogorov-Smirnov (K-S) test for goodness of fit on the normal distribution. Neither the plots or the K-S tests provide sufficient evidence to doubt the assumption of normality. See below for a discussion of the K-S test.

The K-S test is a goodness of fit test to determine if sample data fits a hypothesized distribution (Law & Kelton). The test takes several forms based on the knowledge of the hypothesized distribution. For the case here, we reject the hypothesis that the sample data approximates the normal distribution when:

$$\left(\sqrt{n} - 0.01 + \frac{0.85}{\sqrt{n}}\right) D_n > c'_{1-\alpha}$$

where:

$c'_{1-\alpha} = 0.895$ at $\alpha = 0.05$.

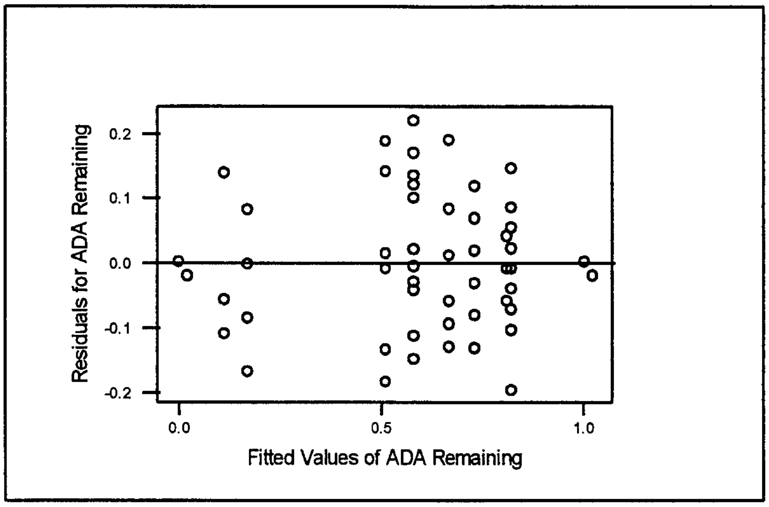
$D_n = \max\left\{D_n^+, D_n^-\right\}$

$D_n^+ = \max_{1 \le i \le n}\left\{\frac{i}{n} - F(X_i)\right\} D_n^- = \max_{1 \le i \le n}\left\{F(X_i) - \frac{(i-1)}{n}\right\}$

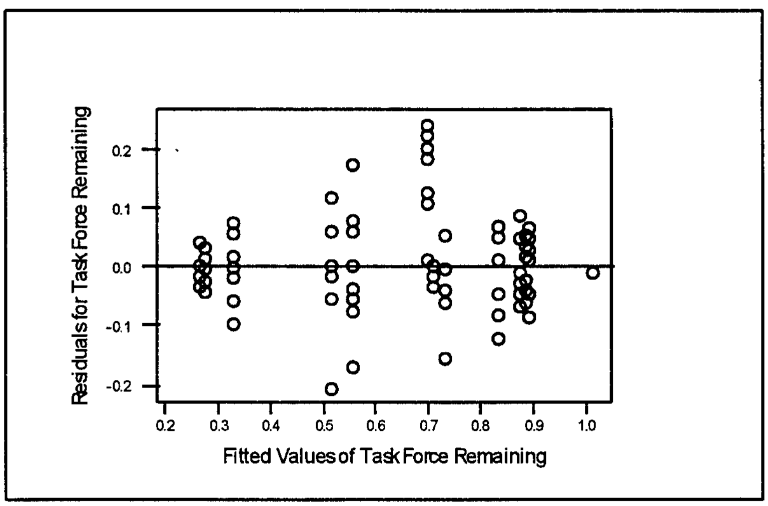
i = the rank order position of the observation (lowest to greatest)

$F(X_i)$ = the value of the cumulative probability distribution of the hypothesized (normal) distribution. X_i here is the value of the residual. The residual sample mean and variance are used as estimates for the distribution mean and variance.

Plot of Residuals versus the Fitted Values of ADA Remaining



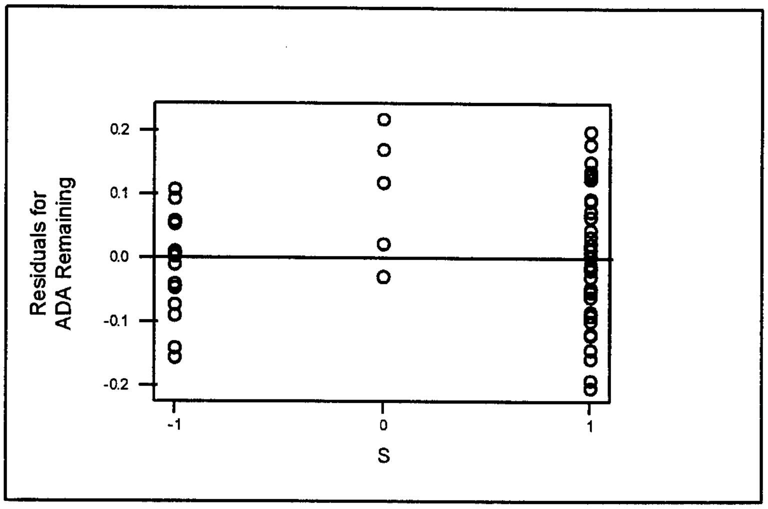
Plot of Residuals versus the Fitted Values of Task Force Remaining

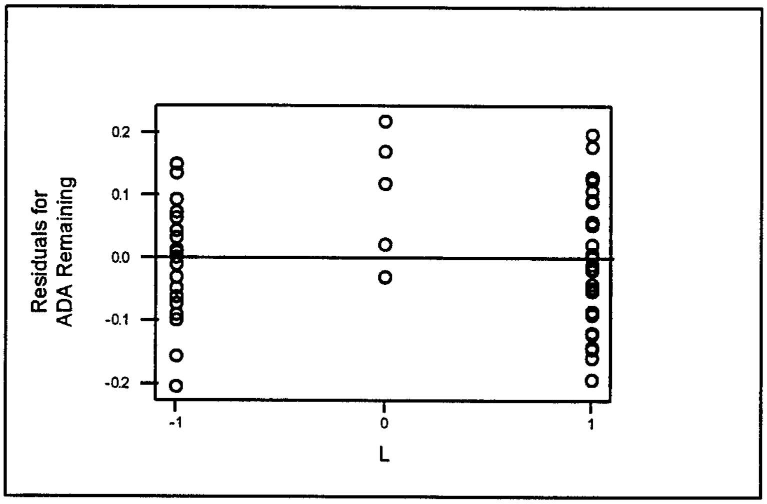


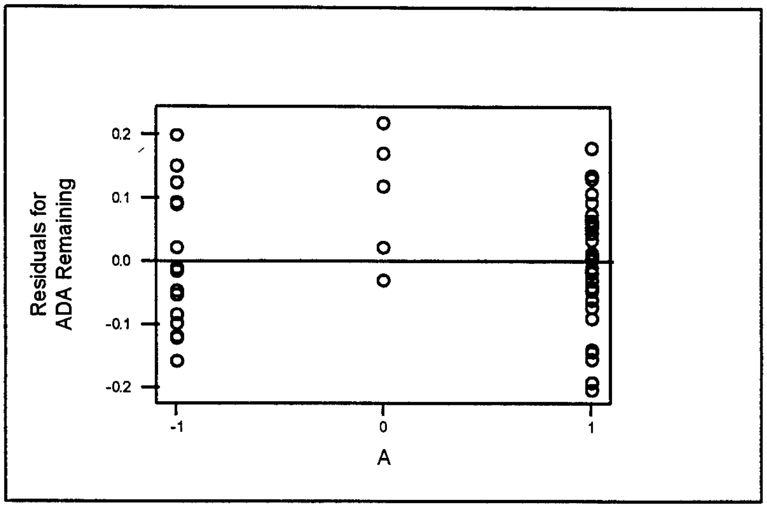
Both of the above plots, indicate that the assumption of constant error variance appears to hold true for the center of the design space. The small variances shown at the "tails" of the responses are the result of the experimental design and the definitions of the responses. It is impossible for actual response values to exceed 100% or be lower than 0%. However, the regression equations could conceivably predict values that would violate these logical extremes. In these situations, the larger predicted values would correspond to observed values that lie on or near 0% or 100%. Hence, this negates the possibility of large variances at the extremes.

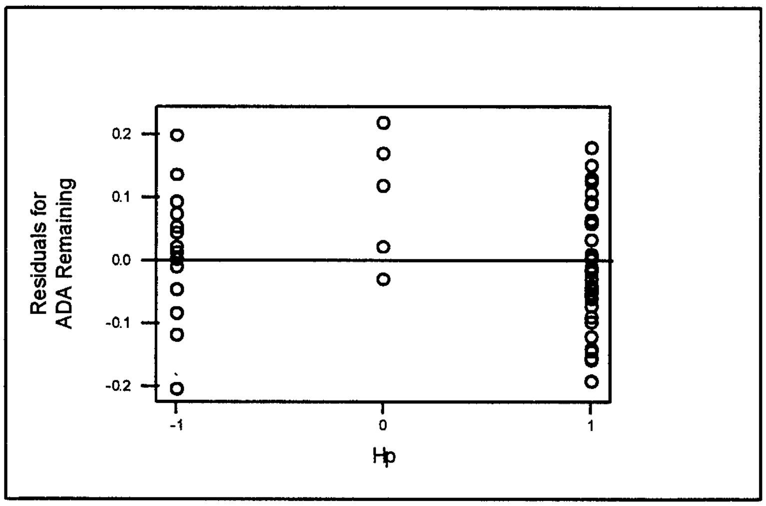
Plots of residuals versus factors provide information regarding model specification. A plot showing a "curve" points toward the nonlinearity of the factor plotted versus the residuals (Fox). The following pages contain plots of the residuals versus each of the five main factors of the two response models.

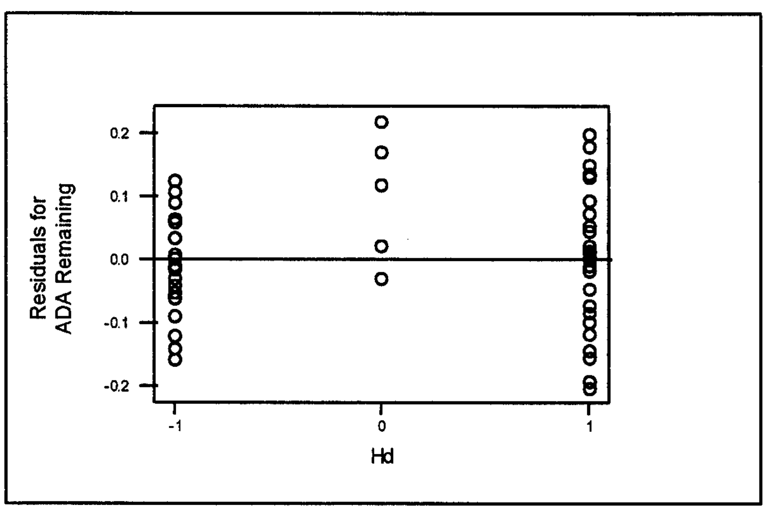
Plots of Residuals versus Each Main Factor for ADA Remaining





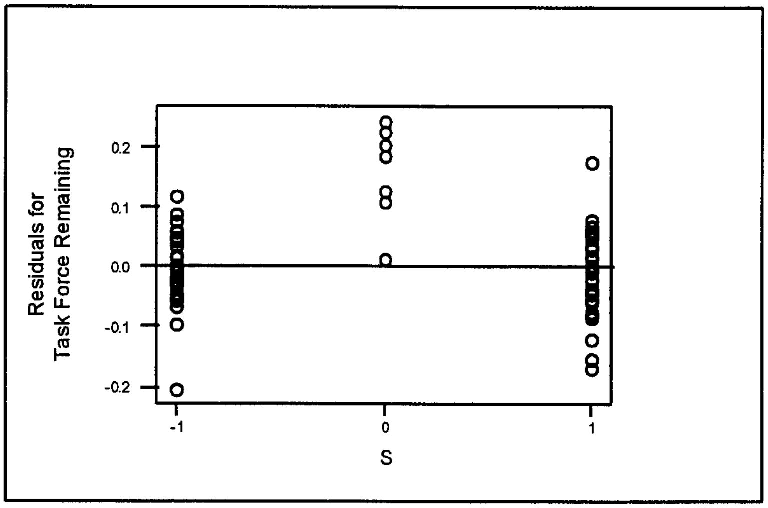

Residuals for
ADA Remaining
0.2
0.1
0.0
-0.1
-0.2
-1
0
1
A

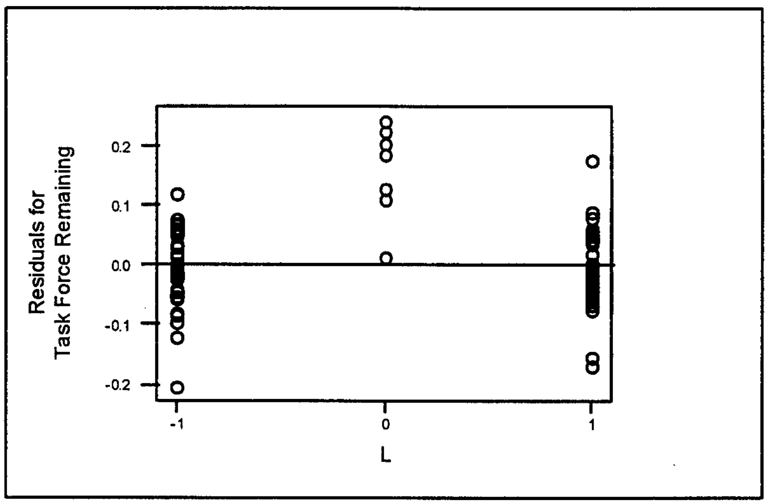

Residuals for
ADA Remaining
0.2
0.1
0.0
-0.1
-0.2
-1
0
1
Hp

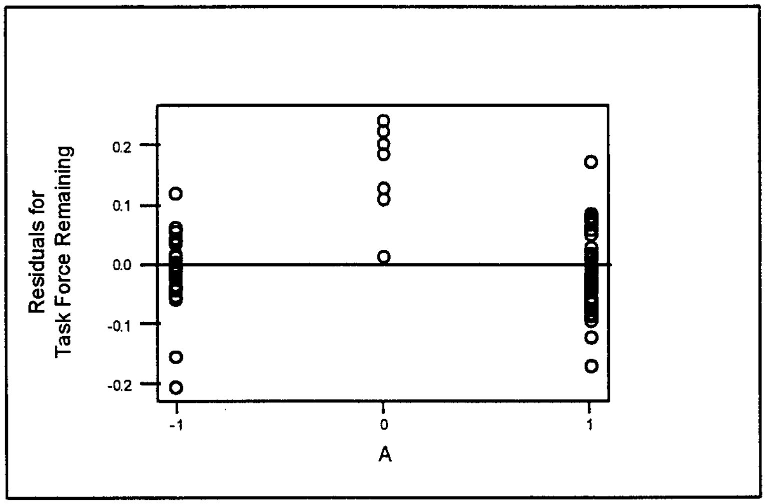

Residuals for
ADA Remaining
0.2
0.1
0.0
-0.1
-0.2
-1
0
1
Hd

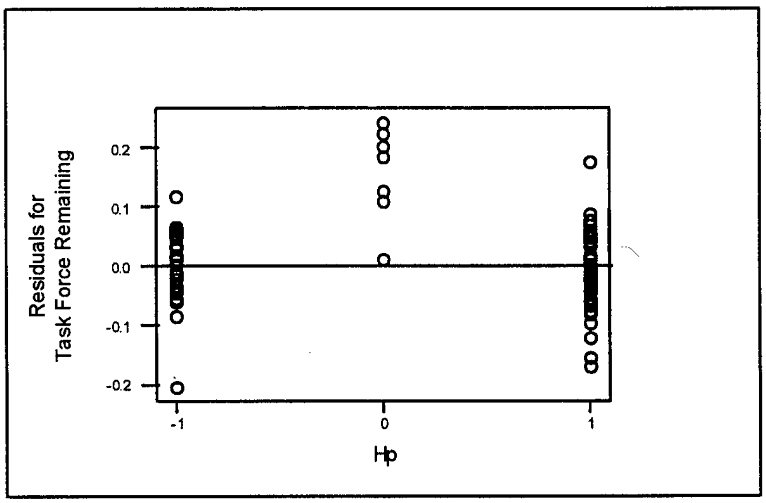
Each of the above residual plots, for the Air Defense Remaining, suggest nonlinearity in the main factors. It is difficult to determine which factor or factors are truly nonlinear as there is only one center point in the design.

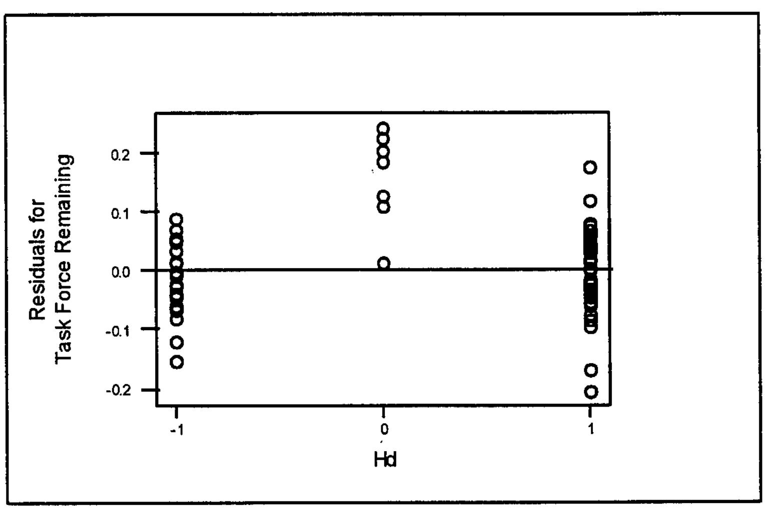
Plots of Residuals versus Each Main Factor for the Task Force Remaining






Residuals for
Task Force Remaining
0.2
0.1
0.0
-0.1
-0.2
-1
0
1
A


Residuals for
Task Force Remaining
0.2
0.1
0.0
-0.1
-0.2
-1
0
1
Hp


Residuals for
Task Force Remaining
0.2
0.1
0.0
-0.1
-0.2
-1
0
1
Hd

As with the plots for the Air Defense Remaining, each of the residual plots, for the Task Force remaining, suggest nonlinearity in the main factors. It is difficult to determine which factor or factors are truly nonlinear as there is only one center point in the design.

Appendix K

Residuals for the Final Experimental Design

For identifying possible outliers and points of high influence (leverage points), the following relationships are used:

1. Residual - the difference between the observed value and predicted response.

2. St. Residual (r_i) - the internally studentized residual, sometimes referred to as the standardized residual.

$$r_i = \frac{e_i}{\sqrt{\sigma^2(1-h_{ii})}}$$

where:

σ^2 is estimated by MSE

e_i is the ordinary residuals

h_{ii} is the ith diagonal of **H** (the hat matrix)

Observations with $|r_i| \geq 2$ are listed in the tables below.

3. R-Student (t_i) - the externally studentized residual

$$t_i = \frac{e_i}{\sqrt{S_i^2(1-h_{ii})}}$$

where:

e_i is the ordinary residuals

h_{ii} is the ith diagonal of **H** (the hat matrix)

$$S_i^2 = \frac{(n-p)MSE - e_i^2/(1-h_{ii}}{n-p-1}$$

Observations with $|t_i| \geq 2$ are possible outliers (Fox).

4. Cook's D (D_i) - Cook's distance

$$D_i = \left(\frac{r_i^2}{p}\right)\left(\frac{h_{ii}}{(1-h_{ii})}\right)$$

where:

r_i^2 is the square of the ordinary residual

p is the number of parameter in the model

h_{ii} is the ith diagonal of **H** (the hat matrix)

Observations with $D_i > 4/(n - p - 1)$ are possible leverage points (Fox).

Residuals of the Final Model with Interactions for ADA Remaining

Observation	Data Point	Residual	St. Resid.	R-Student	Cook's D
35	4	0.19172	2.45147	2.49756	0.04638
49	6	-0.19525	-2.49651	-2.54560	0.04810
50	6	-0.19525	-2.49651	-2.54560	0.04810
114	13	-0.16674	-2.13207	-2.16021	0.03508
115	13	-0.16674	-2.13207	-2.16021	0.03508
141	16	0.19156	2.44940	2.49536	0.04630
143	16	-0.18344	-2.34556	-2.38507	0.04246
148	center	0.22183	2.69409	2.75782	0.00318
149	center	0.17183	2.08685	2.11288	0.00191
150	center	0.22183	2.69409	2.75782	0.00318
152	center	0.22183	2.69409	2.75782	0.00318
153	center	0.17183	2.08685	2.11288	0.00191

The D_i cutoff for the ADA Remaining response model is 0.02899. Observations 35, 49, 50, 114, 115, 141, and 143 all exceed the cutoff.

Residuals of the Final Model with Interactions for the Task Force Remaining

Observation	Data Point	Residual	St. Resid.	R-Student	Cook's D
4	1	-0.20639	-3.29819	-3.42491	0.08463
101	12	-0.15510	-2.47866	-2.52691	0.04780
141	16	0.17609	2.81410	2.88854	0.06161
143	16	-0.17006	-2.71769	-2.78384	0.05746
146	center	0.18678	2.82402	2.89935	0.00328
148	center	0.24447	3.69632	3.88143	0.00562
150	center	0.22524	3.40555	3.54654	0.00477
152	center	0.20601	3.11478	3.21949	0.00399
153	center	0.18678	2.82402	2.89935	0.00328

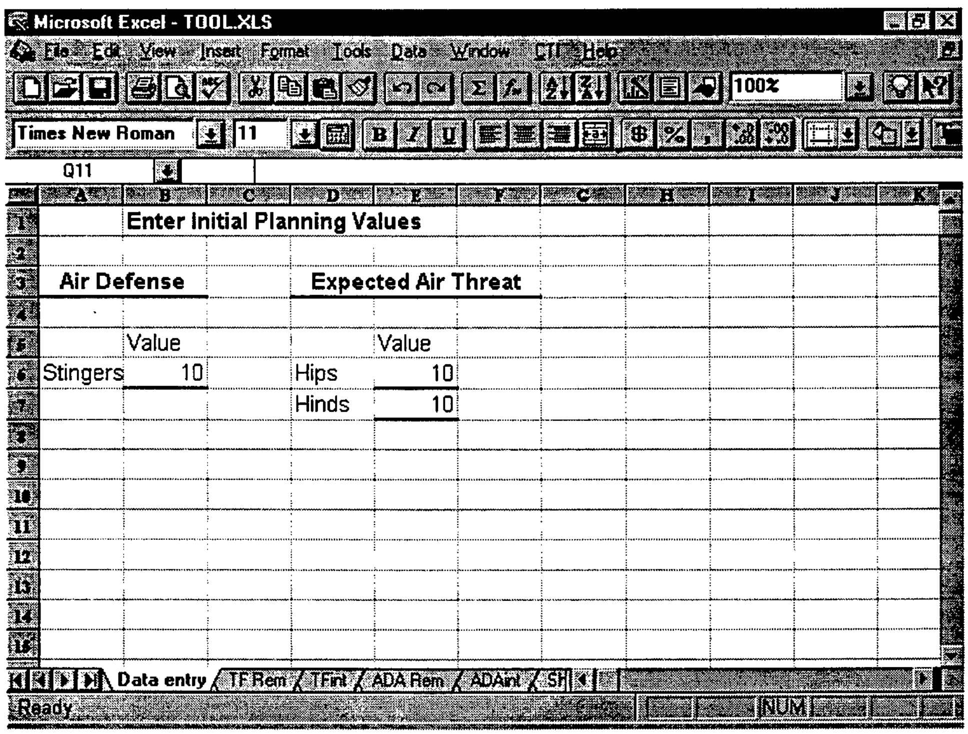
The D_i cutoff for the Task Force Remaining response model is 0.02920. Observations 4, 101, 141, and 143 exceed the cutoff.

Observations 101, 141, and 143 are the only three observations that are leverage points for both response models. Although these leverage points are also possible outliers, the data reflects the stochastic nature of the simulation. After verifying all simulation parameters, it is determined that the observations must remain included in the experiment to account for variability in the model.

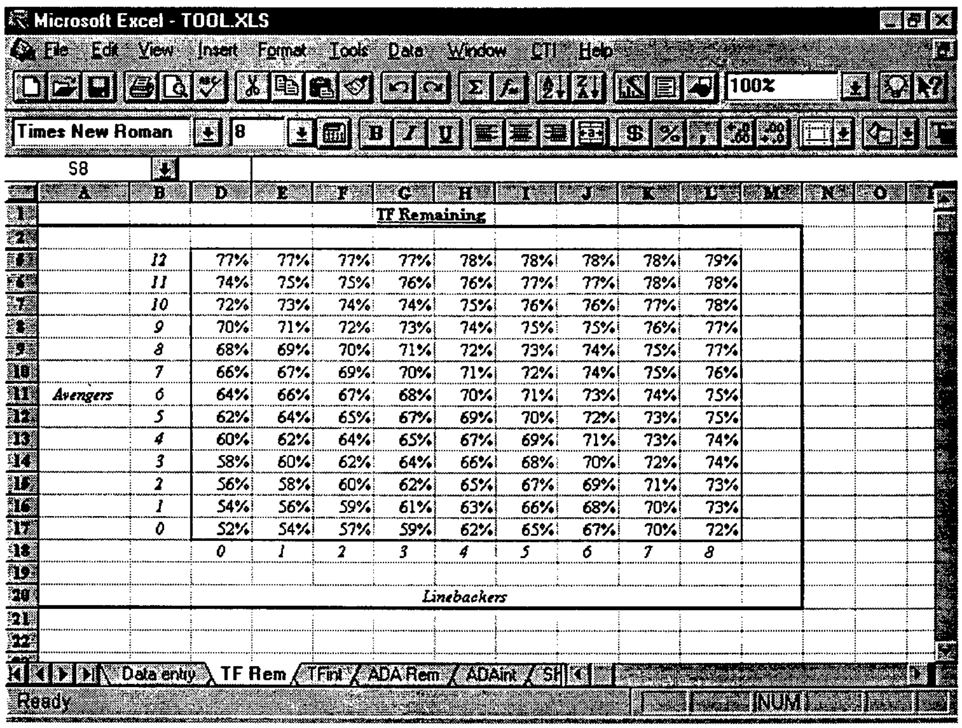
Appendix L

Air Defense Force Allocation Planning Tool

The Air Defense Force Allocation Planning Tool consists of a series of linked spreadsheets. Built in Microsoft Excel, the tool has a very simple design and easy to use interface. The Data Entry screen, shown below, requires the user to enter values for the expected enemy threat. For this prototype tool, the Stinger weapon system is selected to remain constant throughout the analysis. The uncomplicated design of the tool facilitates expanding this selection to include the other two ADA weapon systems considered, Linebackers and Avengers.



The Task Force Remaining screen (TF Rem) and the ADA Remaining screen (ADA Rem) show the predicted percentages of the amounts remaining of the task force and ADA assets. The predictions are based on the regression equations developed for each response model, Equations 2 and 3 in the Data Analysis chapter. The TF Rem screen appears below. The ADA Rem screen is nearly identical in appearance to the TF Rem screen.

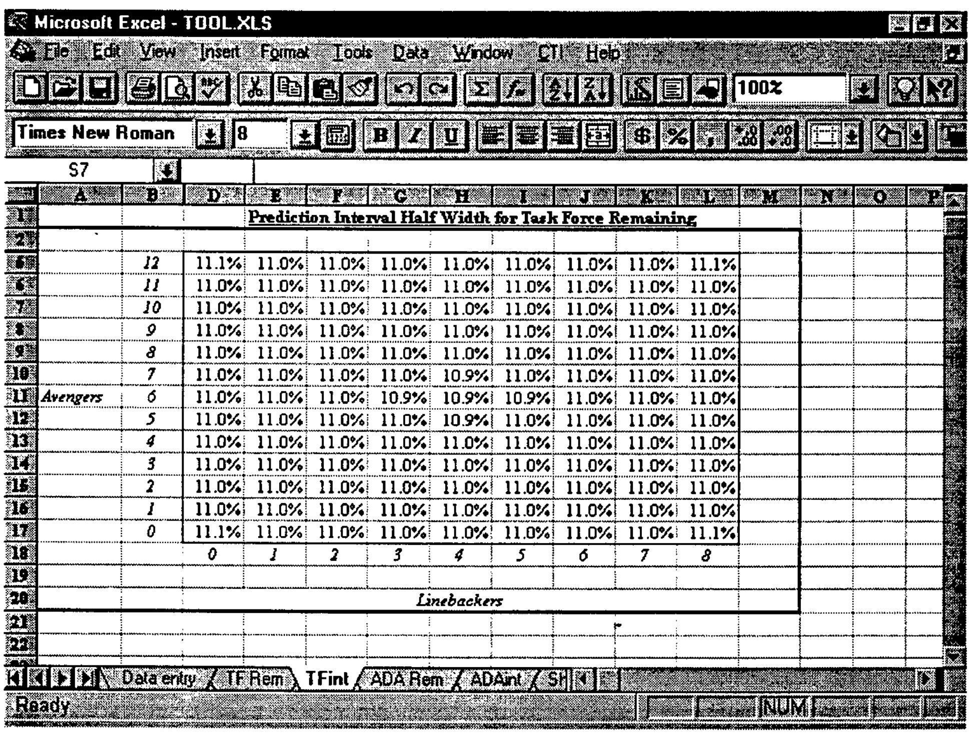


TF Remaining

	12	77%	77%	77%	77%	78%	78%	78%	78%	79%
	11	74%	75%	75%	76%	76%	77%	77%	78%	78%
	10	72%	73%	74%	74%	75%	76%	76%	77%	78%
	9	70%	71%	72%	73%	74%	75%	75%	76%	77%
	8	68%	69%	70%	71%	72%	73%	74%	75%	77%
	7	66%	67%	69%	70%	71%	72%	74%	75%	76%
Avengers	6	64%	66%	67%	68%	70%	71%	73%	74%	75%
	5	62%	64%	65%	67%	69%	70%	72%	73%	75%
	4	60%	62%	64%	65%	67%	69%	71%	73%	74%
	3	58%	60%	62%	64%	66%	68%	70%	72%	74%
	2	56%	58%	60%	62%	65%	67%	69%	71%	73%
	1	54%	56%	59%	61%	63%	66%	68%	70%	73%
	0	52%	54%	57%	59%	62%	65%	67%	70%	72%
		0	1	2	3	4	5	6	7	8

Linebackers

The Task Force Half Width of the Prediction Interval screen (TFint) and the ADA Half Width of the Prediction Interval screen (ADAint) show the factors that must be added and subtracted to the predicted responses to form 90% prediction intervals. The prediction interval half widths are based on the prediction interval equation found in the Application and Limitations chapter, Equation 6. For easy reference, the set up the prediction interval half width screens is identical to the set up of the screens for the predicted responses. The TFint screen is shown below; the ADAint screen appears nearly identical.

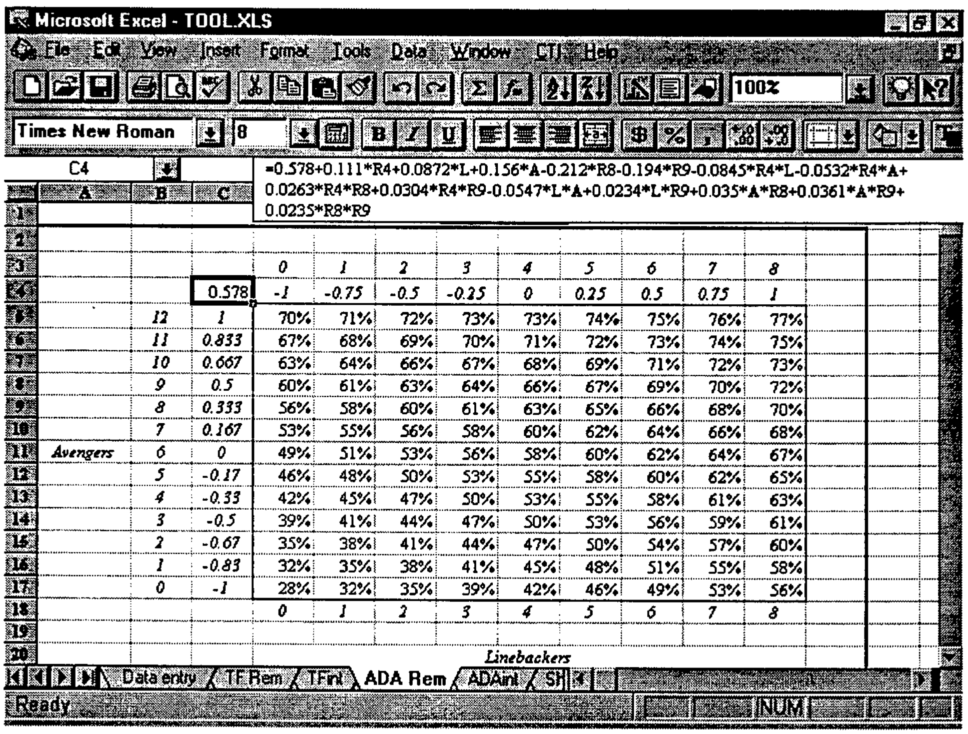


Prediction Interval Half Width for Task Force Remaining

	12	11.1%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.1%
	11	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
	10	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
	9	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
	8	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
	7	11.0%	11.0%	11.0%	11.0%	10.9%	11.0%	11.0%	11.0%	11.0%
Avengers	6	11.0%	11.0%	11.0%	10.9%	10.9%	10.9%	11.0%	11.0%	11.0%
	5	11.0%	11.0%	11.0%	11.0%	10.9%	11.0%	11.0%	11.0%	11.0%
	4	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
	3	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
	2	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
	1	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%
	0	11.1%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.0%	11.1%
		0	1	2	3	4	5	6	7	8

Linebackers

Each prediction and interval factor sheet contains links to the Data Entry sheet. The links make available the Stinger, HIP and HIND values needed for each calculation. At upper left hand corner of each data table, an equation cell contains the equation for the regression model or appropriate prediction interval factor. This corner cell is usually hidden from view when the tool is in use. The ADA Rem screen below shows all hidden cells and the calculations contained in the equation cell. The other spreadsheet screens are similar in design.



A	B	C									
			0	1	2	3	4	5	6	7	8
		0.578	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
	12	1	70%	71%	72%	73%	73%	74%	75%	76%	77%
	11	0.833	67%	68%	69%	70%	71%	72%	73%	74%	75%
	10	0.667	63%	64%	66%	67%	68%	69%	71%	72%	73%
	9	0.5	60%	61%	63%	64%	66%	67%	69%	70%	72%
	8	0.333	56%	58%	60%	61%	63%	65%	66%	68%	70%
	7	0.167	53%	55%	56%	58%	60%	62%	64%	66%	68%
Avengers	6	0	49%	51%	53%	56%	58%	60%	62%	64%	67%
	5	-0.17	46%	48%	50%	53%	55%	58%	60%	62%	65%
	4	-0.33	42%	45%	47%	50%	53%	55%	58%	61%	63%
	3	-0.5	39%	41%	44%	47%	50%	53%	56%	59%	61%
	2	-0.67	35%	38%	41%	44%	47%	50%	54%	57%	60%
	1	-0.83	32%	35%	38%	41%	45%	48%	51%	55%	58%
	0	-1	28%	32%	35%	39%	42%	46%	49%	53%	56%
			0	1	2	3	4	5	6	7	8
							Linebackers				

LIST OF REFERENCES

Bradley Linebacker. (1996, April). Air Defense Artillery Highlights, 2, 3.

Cook, R. D. & Weisberg, S. (1982). Residuals and Influence in Regression. New York: Chapman and Hall.

Daniels, C. L. (1994). Comparison of Fixed Wing Algorithms for Janus. Unpublished master's thesis, Naval Post Graduate School, Monterey, CA.

4th Battalion 3rd Air Defense Artillery Regiment. (1993). 4-3 ADA Planner's Guide. Kitzingen, Germany: Author.

Foreign Material Service Battalion. (1990). Consolidated Technical Intelligence Bulletin. Aberdeen Proving Grounds, MD: Author.

Fox, J. (1991). Regression Diagnostics. Newbry Park, CA: Sage Publications.

Headquarters, Department of the Army. (1983). US Army Air Defense Artillery Employment (Field Manual 44-1). Washington, DC: Author.

Headquarters, Department of the Army. (1984). Air Defense Artillery Reference Handbook (Field Manual 44-1-2). Washington, DC: Author.

Headquarters, Department of the Army. (1988a). Mission Training Plan for the Infantry Battalion. (ARTEP 7-20-MTP). Washington, DC: Author.

Headquarters, Department of the Army. (1988b). Mission Training Plan for the Tank and Mechanized Infantry Battalion Task Force. (ARTEP 71-2-MTP). Washington, DC: Author.

Headquarters, Department of the Army. (1990). Tactics, Techniques and Procedures for Avenger Squad Operations (Field Manual 44 - 31). Washington, DC: Author.

Headquarters, Department of the Army. (1991). The Soviet Army: Troops, Organization, and Equipment (Field Manual 100-2-3). Washington, DC: Author.

Headquarters, Department of the Army. (1992). Mission Training Plan for an ADA Platoon, Stinger (ARTEP 44-117-11-MTP). Washington, DC: Author.

Headquarters, Department of the Army. (1995). Bradley Stinger Fighting Vehicle Platoon and Squad Operations (Field Manual 44-43). Washington, DC: Author.

Hoerl, A. E. (1959, November). Optimum Solution of Many Variable Equations. Chemical Engineering Progress, 55 (11) 69 - 78.

Foss, C. F. (Ed.). (1992). Janes Armour and Artillery 1992-1993. Coulsdon, Surrey, UK: Janes Information Group Limited.

Law, A. M. & Kelton, D. W. (1991). Simulation Modeling and Analysis. New York: McGraw-Hill.

Leonhard, R. R. (1996, June). Force XXI and the Theory of Winning Outnumbered. Army, 46, 56 - 64.

Meyers, R. H. & Montgomery, D. C. (1995). Response Surface Methodology: Process and Product Optimization Using Designed Experiments. New York: John Wiley and Sons, Inc.

Titan Inc. (1993). Janus 3.X/VMS System Design Manual. Fort Leavenworth, KS: Author.

United States Army Air Defense Arttillery School. (1991). Avenger Platoon/Squad - preliminary draft (Field Manual 44-44). Fort Bliss, TX: Author.

United States Army Command and General Staff College. (1993). The Tactical Decision Making Process (Student Text 100-9). Fort Leavenworth, KS: Author.

United States Army Simulation, Training and Instrumentation Command. (undated). Software User's Manual: Janus Version 6.0. Orlando, FL: Author.

United States Army Training and Doctrine Command. (1994). Heavy Opposing Force (OPFOR) Tactics Handbook - draft (TRADOC Pamphlet 350-16). Fort Monroe, VA: Author.