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Modification of Spontaneous Emission and Applications to Semiconductor Lasers.

Report to the US Air Force under Contract SPC-93-4064

February 1994

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Abstract

In an earlier report we have shown for the first time how that zero lasing threshold is achieved between a pair of mirrors spaced at a distance of a half a wavelength of the lasing radiation mod,. while spontaneous emission is only partially suppressed. However, the residual spontaneous emission must be taken into account when considering laser characteristics other than the threshold pumping rate, for example, laser linewidth. The present report is concerned with a quantum mechanical model of the laser under conditions of suppressed spontaneous emission. For almost fully inhibited spontaneous emission, the accepted laser models are not applicable, since they are based on a statistical approach valid when a large number of modes participate in the spontaneous emission loss. For a partial suppression of spontaneous emission, the model is applicable, mainly with the modification of the atomic decay rate. It is found that when zero threshold is obtained between mirrors as mentioned above, the laser linewidth is reduced only by a modest factor of a quarter.

Our numerical simulation of a vertical cavity surface emitting DBR laser was continued, with the optimum design for channelling the spontaneous emission into the lasing mode, yielding a fraction of 60% of the total emission.

§1. Introduction

In an earlier report [1] we have shown for the first time that lasing without threshold can be achieved while spontaneous emission is only partially suppressed. For radiation between parallel mirrors with the distance between them equal to one half wavelength of the atomic transition, the number of modes of the electromagnetic field into which spontaneous emission is allowed is drastically reduced in comparison with the case of emission into free space; at the same time the transition probability into each allowed mode is slightly increased. As the ratio of the rates of stimulated and spontaneous emission is inversely proportional to the number of modes participating in spontaneous emission, the power of the latter becomes insignificant in comparison with stimulated emission power, and the lasing threshold is virtually reduced to zero. However, the residual spontaneous emission must be taken into account when considering laser characteristics other than the threshold current. When the electromagnetic field of the cavity sustains other modes in addition to the lasing mode, interaction of the lasing 'atoms' with these (unexcited) modes is responsible for a dissipation process which leads to diffusion of the phase and amplitude of the electromagnetic field, in other words, to loss of coherence. One of the most important consequences of this degree of incoherence is a finite linewidth for the single mode laser, which, in its turn, has practical implications on the transmission of the laser signal in optical fibres (due to dispersion).

In the quantum mechanical theory of dissipation, as developed by Lax [2] and Senitzky [3,4,] for a harmonic oscillator (such as a radiation mode) or a two-level oscillator [5], the process is described as due to the interaction of an oscillator with a loss mechanism, a system of many degrees of freedom and semi-continuous spectrum. In the description of spontaneous emission by a two-level atom, the loss mechanism is the free electromagnetic field, a collection of a large number of oscillators each of them weakly coupled to the two-level system. In the case of a damped radiation mode, the loss mechanism is a thermal reservoir of harmonic oscillators, representing cavity wall losses and other non coherent interactions. The equations of motion obtained for the atom or radiation mode are of the

Langevin type, with the action of the loss mechanism described by a combination of a damping force and a random force (with zero average). These Langevin type equations are employed in the fully quantum mechanical Laser model of Haken [6].

We have applied the Haken Laser model for the case of suppressed spontaneous emission. When spontaneous emission is fully inhibited, the model is not applicable since the very concept of the cavity electromagnetic field as a loss mechanism breaks down. For a partially suppressed spontaneous emission, the model is applicable with modifications that will be introduced below. The model employed, of a single lasing mode and a collection of two-level atoms interacting with a suppressed loss mechanism, and the Hamiltonian are described in §2, and the Heisenberg equations of motion for the electromagnetic field and the atomic variables are given in §3. An investigation of some of the laser characteristics, such as the laser linewidth is carried out in §4.

We have continued our work on the numerical simulation of semiconductor laser structures, similar to that of vertical cavity surface emitting DBR lasers . Varying the structure parameters to maximize the fraction of spontaneous emission into the lasing mode, we obtained for the optimum configuration a figure of 62% out the total. This work is described in §5.

§2. The Model and its Hamiltonian

The system under consideration consists of a collection of two-level atoms, a nearly resonant lasing mode and a 'suppressed' loss mechanism. A two-level atom has lower and upper energy eigenstates $|1>$, and $|2>$, with energies $(-\frac{1}{2} h\omega, \frac{1}{2} h\omega)$, respectively, and may be described by angular momentum operators proportional to the Pauli spin matrices, with the commutation relation and total given by:

(1) $[l_1, l_2] = il_3$, and cyclic permutations,

(2) $(l_1)^2 + (l_2)^2 + (l_3)^2 = l_0(l_0 + 1)$, $l_0 = \frac{1}{2}$,

In place of (l_1, l_2), it is useful to introduce the angular momentum combinations:

(3) $l_+ = l_1 + i\, l_2$, $l_- = l_1 - i\, l_2$,

which act as step-up, step-down operators:

(4) $l_+ = |2><1|$, $l_- = |1><2|$.

The lasing mode with frequency ω_0 is described by creation annihilation operators $a^\dagger$, a , and the Hamiltonian of the mode and the two-level atoms, using j as an index for different atoms, is given by:

(5) $\quad H_0 = h\, \omega_0 (a^\dagger a + \frac{1}{2}) + h\, \omega\, \Sigma_j\, l_3(j) + H_C$,

and the coupling between atoms and mode through dipole interaction is:

(6) $\quad H_C = h\, \Sigma_j g\,(j)\, (a^\dagger l_-(j) + a\, l_+(j))$,

where $g\,(j)$ is a coupling constant proportional [7] to the atomic transition dipole moment and the amplitude of the electric field of the unexcited mode at the position of the j-th atom. The form of H_C suggests it is convenient to introduce the collective operators:

(7) $\quad S_z = 2\, \Sigma_j l_3(j)\,,\;\; g\, S_- = \Sigma_j\, g\,(j)\, l_-(j)\,,\;\; g\, S_+ = \Sigma_j\, g\,(j)\, l_+(j)$,

and obtain:

(8) $\quad H_0 = h\, \omega_0 (a^\dagger a + \frac{1}{2}) + h\, \omega\, \frac{1}{2}\, S_z + h\, g\, (a^\dagger S_- + a\, S_+)$.

Taking the loss mechanism into account, the total Hamiltonian is given by:

(9) $\quad H = H_0 + H_{LM} + H_{INT}$,

where the second term on the right hand side is the Hamiltonian of the free loss mechanism, and the last term is its interaction with the mode and the atoms. This interaction is given by:

(10) $\quad H_{INT} = h\, (F^\dagger a + F\, a^\dagger) + h\, \Sigma_j\; (\Gamma^\dagger(j)\, l_-(j) + \Gamma(j)\, l_+(j))$,

where F and $\Gamma\,(j)$ denote a coordinate of the loss mechanism through which it interacts with the lasing radiation mode and the j-th two-level atom, respectively, and any coupling coefficient is absorbed in the definition of the coordinate. With the help of collective operators for the two-level atoms, the last equation can be rewritten in the form:

(11) $\quad H_{INT} = h\, (F^\dagger a + F\, a^\dagger) + h\, (\Gamma^\dagger S_- + \Gamma\, S_+)$.

We have taken the same loss mechanism interacting with the radiation mode and all the atoms without loss of generality, as different constituent parts can be combined into a single mechanism which by definition has a very large number of degrees of freedom. In the design of thresholdless laser it is the aim to obtain a structure where the perturbation by H_{INT} is minimized. This is most clearly illustrated in the case of atoms in a cavity whose fundamental frequency is higher than the atomic transition; in such a case the interacting part of the loss mechanism is the electric field which its spectrum excludes any terms in H_{INT} which gives rise to non-zero matrix elements between the relevant atomic states. Usually, one lacks detailed information about the spectrum of F or Γ, but certain statistical properties, such as relations between the expectation values of the coordinate and its moments, have been deduced [3,5] under quite general assumptions. These assumptions, however, may not be valid for micro-cavity configurations considered for the suppression of spontaneous emission. To see how the statistical properties of F and Γ may be different for micro-cavities, it is best to look at the Heisenberg equations of motion derived from the Hamiltonian given above, as is done in the next section.

§3. Equations of Motion

In this section we will look at the derivation of the Langevin type equations of motion for the mode and atomic operators, investigate the applicability of the assumptions for the case of microcavity structure, and the required modifications.

The loss mechanism coordinate F entering H_{INT} is written as a sum:

(12) $F(t) = \Sigma_\omega \, g_\omega F_\omega(t)\, e^{-i\omega t}$,

with a similar summation for Γ, while the Hamiltonian for the free loss mechanism is given by

(13) $H_{LM} = h\,\Sigma_\omega \,(F_\omega^\dagger F_\omega + \Gamma_\omega^\dagger \Gamma_\omega)$.

The Heisenberg equations of motion for F_ω, a, and their Hermitean conjugates are then immediately available with the substitution of Eq. (13) in the total Hamiltonian, Eq.(9); the equations for the loss mechanism operators are formally integrated to give:

(14) $F_\omega^\dagger = F_\omega^\dagger(t{=}0) + i\int_0^t a^\dagger(t')\, g_\omega\, e^{-i\omega t'} dt'$,

which is substituted in the equations for the mode operators obtaining:

(15) $da^\dagger/dt = i\,\omega_0\, a^\dagger + i\ \Sigma_\omega\, g_\omega^*\, F_\omega^\dagger(t{=}0)\, e^{i\omega t} - \int_0^t dt'\, a^\dagger(t')\, \Sigma_\omega \, |g_\omega|^2\, e^{i\omega(t-t')}$.

To evaluate the last term in the equation above, some assumptions are to be made on the nature of the loss mechanism. The cavity finite Q factor is due to such non-resonance processes as Ohmic resistance and losses due to scattering centres. In the design of thresholdless lasers the aim is to have a very high Q by minimizing these losses (for example by using highly polished mirrors and cooling them) but the spectrum of the losses is not drastically influenced. The summation over ω in Eq.(15) may be therefore replaced by integration and the largest contribution to the integral over t' will come from an area near $t' = t$. Subsequent integration of Eq.(15) over t to obtain $a(t)$ on the left hand side, will tend to select frequencies ω near ω_0 if the integration is carried for times long enough that $\omega_0 t \gg 1$, since $a^\dagger(t)$ which appears under the integral on the right hand side oscillates essentially with the same frequency, ω_0, as the free operator (it is assumed that the nature of the loss mechanism is such that the interaction with it introduces slow changes , that is only over many periods of the ω_0 oscillation). Thus, one may replace $a^\dagger(t')$ with $a^\dagger(t)$ and take it outside the integral sign on the left hand side, and describe the system as having a short 'memory', or a Markovian system. With these approximation, one obtains finally

(16) $da^\dagger/dt = i\,\omega_0\, a^\dagger + i\ \Sigma_\omega\, g_\omega{}^*\, F_\omega^\dagger(t=0)\, e^{i\omega t} - \kappa\, a^\dagger$,

where the last term on the right hand side presents a damping force, with κ defined as an average of $|g_\omega|^2$ over a small neighbourhood of ω_0, and can be identified with the reciprocal of the cavity mode lifetime. The second term on the right hand side of this equation depends only on the free loss mechanism operators and represents a fluctuating force with zero expectation value. For the usual case of cavity with dimensions large in comparison with the mode wavelength, where again the interaction with the loss mechanism involves a summation over many frequencies, a similar treatment for the atomic operators, defined in Eq.(7), is applicable, leading to Markovian behaviour. In the case of exact resonance of the lasing mode with the atomic transition, $\omega_0 = \omega$, it is useful to introduce slowly varying operators,

$$a^\dagger \to a^\dagger\ e^{i\omega t}\ ,\ S_+ \to S_+\ e^{i\omega t}\quad \textit{(and Hermitean conjugates)}\ ,$$

and denote the fluctuating forces acting on the radiation mode and the atomic operators by f and Γ_+, etc. , respectively. Then the laser equations are:

$$da^\dagger/dt = -\kappa\ a^\dagger\ + i g\, S_+ + f^\dagger$$

(17) $$dS_+/dt = -\gamma\, S_+ - i g\, a^\dagger\ + \Gamma_+$$

$$dS_z/dt = (S_z - S_0)/T + 2ig(S_-\, a^\dagger - a\ S_+)\ ,$$

where γ in the second equation is a decay constant obtained in an averaging process over coupling coefficients with different frequency components of the loss mechanism coordinate similar to that leading to the definition of κ in the first equation; the first term on the right hand side of the last equation represents contribution due to pumping and other non coherent relaxation, S_0 is the stationary value of S_z under the action of these processes alone, and T is the relaxation time.

As is clear from the discussion above, the interaction of a microcavity with the loss mechanism can be still described as Markovian; reduced losses are reflected in decreased values of κ. It is a different case for the atomic equations of motion. In a structure where spontaneous emission is ideally restricted almost completely into the lasing mode, for example, when all cavity dimensions are comparable with the atomic transition wavelength, there are only a small number of modes of the electromagnetic field within an appropriate frequency range to couple with the two level atoms, the loss mechanism model is not applicable, and the Markovian approximation is no longer valid. Instead, one has to solve the coupled equations of motion for the atoms and each of the non-lasing modes coupled to it. This may be carried out only numerically except for a small number of such modes. In the work by DeMartini et. al. [8], the only configuration so far where lasing without threshold was achieved, spontaneous emission was only partially suppressed, as we have shown in an earlier report [1] . Only one mode propagating in a normal direction to the pair of mirrors forming the cavity participated in spontaneous emission. However, emission into modes propagating parallel to the mirrors was not inhibited. Interaction of the atomic system with this large number of modes will be of a short memory, and the Markovian approximation will be valid. The only modification will be in the value of the atomic decay constant, γ in Eq.(17). Instead of the reciprocal of the (free space) radiative lifetime, an adjusted parameter must be taken. We had calculated directly the value of atomic decay time for the DBR laser configurations employed in our numerical modelling, discussed in §5. In the next section, we apply the modified Eq.(17) to the calculation of laser parameters.

§4. Laser Characteristics

The fact that spontaneous emission is only partially suppressed has two consequences, loss of coherence and energy loss, which affect different characteristics differently.

One way to assess the loss of coherence is through consideration of the laser linewidth. For the case of partial suppression of spontaneous emission, Eq.(17) is applicable, with the only requirement that the atomic decay constant γ be modified accordingly. The atomic variables can be eliminated from the coupled Eq.(17) and the time dependence of $a^\dagger$ and a obtained in terms of the fluctuating forces. The mode operators are therefore described only statistically, as a superposition of well defined variable and a stochastic variable, relatively small for a large average number of photons. This leads to a diffusion in the phase and amplitude of the laser. The phase diffusion is responsible for the laser linewidth which is given [6] by:

(18) $\Delta\omega = \{(\kappa\gamma)^2 / (\gamma + \kappa)^2\}(h\omega / P)(N_1 + N_2) / (N_2 - N_1)$,

where κ , γ are the decay constants for the cavity and atomic system, respectively, in Eq.(17) , P is the laser output power; N_1, N_2 are the numbers of atoms at the lower and upper lasing mode, respectively, with their sum , the total number of atoms, a constant, and their difference clamped at the threshold value, and so the last quotient in the right hand side is independent of the laser power above threshold.

A suppression of spontaneous emission rate by a certain factor leads to a reduction of the laser linewidth by the square of that factor, since when the passive cavity bandwidth is larger than the atomic gain width, $\kappa > \gamma$, the curly brackets in the right hand are proportional to γ^2. We have shown [1] that in the configuration [7] employed to obtain zero lasing threshold, spontaneous emission rate is reduced by a half, and thus laser linewidth is reduced only by a modest factor of a quarter, relative to a less compact configuration.

Laser linewidth has some consequences on considering transmission through dispersive medium (as optical fibre), but more important are dynamic parameters defining its response to modulation. Laser relaxation , due to the finite time needed for population to reach threshold, leads to oscillations and strictly affects the laser frequency passband. Thus, in a thresholdless laser, relaxation oscillation is eliminated.

5. Numerical Simulation

We have continued our numerical modelling of vertical cavity surface emitting DBR lasers, employing the two dimensional code developed by on the investigators [9] with modifications to deal with spontaneous emission, which can not be described in a dielectric constant formalism, as described in an earlier report [1].

The configurations investigated were all of the vertical cavity surface emitting laser type structure, cylindrical in form with a circular cross section. The structure constituted of a

DBR at the top and bottom of the cylinder, and the active material, the quantum well, sandwiched in the middle, inside a 'cavity' (substrate). The material is characterized by its dielectric constant, in its turn dependent on the carrier concentration which is a two dimensional function (owing to cylindrical symmetry there is no dependence on the azimuthal angle) obtained from a solution of the diffusion function for a given injection current.

Knowing the material gain lineshape, for each structure the mode spectrum was obtained as the structure gain dependence on frequency, below threshold. For a given injection current a search is made for self oscillating modes, that is solutions of the Maxwell equations with boundary conditions of zero radiation falling on one edge of the cylinder and non zero field at the other edge, and the threshold current is found for each mode. For lasing at the mode with the lowest threshold current, spontaneous emission is calculated for all modes (knowing the spatial variation of the modes). The coupling coefficient C, defined as the fraction of spontaneous emission into the lasing mode out of total emission into all modes, is obtained, and used to correct the threshold current in a linear approximation.

We obtained the highest coupling coefficient $C = 0.62$ for a structure with the following characteristics. The quantum well gain lineshape was assumed to have a peak at energy of 0.8 eV (1.55 μ wavelength), and halfwidth of 50 meV. The DBR coupling coefficient, K, the depth of modulation of the dielectric constant [9] , which controls the coupling between radiation propagating in opposing directions along the cylinder axis and determines the structure's frequency stopgap, was taken to be $K = 9.2 \quad 10^{-3}$. The cylinder radius was 12 μ , the QW was placed in cavity with one wavelength (in the substrate) height, and the DBR above and below the cavity were of 12 μ height each. The threshold current obtained was 33mA. The top power obtained was 3.5 mW at a current of 59 mA; at higher pumping, population hole burning limits the output power.

For this high spontaneous emission coupling, some linear assumptions made in the calculations are no longer valid. Different radiation modes compete for the same carrier population, and one may no longer solve the Maxwell and diffusion equation (for the carriers) for each frequency independently. The threshold current also can not be obtained in a linearized approximation. For this structure also , the laser spectrum becomes a function of injection current, moving to higher frequencies with increasing current, at the rate of $10^{-3}\,\mu^{-1}$ for current increase of 5 mA . Even for this optimum coupling, the structure sustained 4-5 side modes, in addition to lasing mode. Further work will require improvements in the numerical analysis, and better treatment of the coupling between the radiation and material equations.

References

1. A. Shalom, 'Modification of spontaneous emission and applications to semiconductor lasers', Report to the USAF under contract SPC-93-4064, September 1993.
2. M. Lax, Phys. Rev. **129,** 2342 (1963).
3. I. R. Senitzky, Phys. Rev. **119**, 670, (1960); **124**, 642 (1961).
4. I. R. Senitzky, Phys. Rev. A **48**, 4629 (1993).
5. I. R. Senitzky, Phys. Rev. **131**, 2827 (1963).
6. H. Haken, *Laser Theory* (Springer-Verlag, Berlin, 1984).
7. For a derivation, see e.g., A. Shalom, Phys. Rev. A **45**, 443 (1992).
8. F de Martini and G. R. Jacobovitz, Phys. Rev. Lett. **60**, 1711 (1988); F. de Martini, F. Cairo, P. Mataloni, and F. Verzegnassi, Phys. Rev. A **46**, 4220 (1992).
9. M. Federighi, J. Appl. Phys. **65**, 2620, (1989).