ARMY RESEARCH LABORATORY

# Wind Drift of Projectiles: A Ballistics Tutorial

Herbert A. Leupold

ARL-TR-1124 October 1996

# NOTICES

## Disclaimers

| **REPORT DOCUMENTATION PAGE** | | | Form Approved OMB No. 0704-0188 |
|---|---|---|---|
| Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate of any other aspect of this collection of information, including suggestions for reducing the burden, to Washington Headquarters Services, Directorate for Information Operations and Reports. 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0 188), Washington, DC 20503. | | | |
| 1. AGENCY USE ONLY *(Leave blank)* | 2. REPORT DATE<br>October 1996 | 3. REPORT TYPE AND DATES COVERED<br>Technical Report | |
| 4. TITLE AND SUBTITLE<br>WIND DRIFT OF PROJECTILES: A BALLISTICS TUTORIAL | | | 5. FUNDING NUMBERS |
| 6. AUTHOR(S)<br>Herbert A. Leupold | | | |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br>US Army Research Laboratory (ARL)<br>Physical Sciences Directorate (PSD)<br>ATTN: AMSRL-PS-PC<br>Fort Monmouth, NJ 07703-5601 | | | 8. PERFORMING ORGANIZATION REPORT NUMBER<br>ARL-TR-1124 |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) | | | 10. SPONSORING/MONITORING AGENCY REPORT NUMBER |
| 11. SUPPLEMENTARY NOTES | | | |
| 12a. DISTRIBUTION/AVAILABILITY STATEMENT<br>Approved for Public Release: Distribution is Unlimited | | | 12b. DISTRIBUTION CODE |
| 13. ABSTRACT *(Maximum 200 words)*<br>Projectile trajectories are altered in the presence of wind. This treatise discusses the lateral displacement of a projectile by a wind blowing at right angles to the direction of aim and explains the reasons for the same. The effect of bullet shape and muzzle velocity upon the trajectory and the resulting wind drift is included. | | | |
| 14. SUBJECT TERMS<br>wind drift, muzzle velocity, air resistance, projectile, sectional density, form factor | | | 15. NUMBER OF PAGES<br>10<br>16. PRICE CODE |
| 17. SECURITY CLASSIFICATION OF REPORT<br>Unclassified | 18. SECURITY CLASSIFICATION OF THIS PAGE<br>Unclassified | 19. SECURITY CLASSIFICATION OF ABSTRACT<br>Unclassified | 20. LIMITATION OF ABSTRACT<br>UL |

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-8
298-01

## CONTENTS



## FIGURES

# WIND DRIFT OF PROJECTILES: A BALLISTICS TUTORIAL

**Herbert A. Leupold**

## Derivation of Wind Drift Formula

The lateral displacement of $x_d$ of a projectile by a wind blowing at right angles to the direction of aim is given by the well known formula:

$$x_d = V_w(t_a - t_v) \tag{1}$$

where $V_w$ is the wind velocity, $t_a$ is the projectile's time of flight from the launcher to the target and $t_v$ its time of flight if fired in a vacuum.

At first glance, this formula is disturbing to many because of the suggestion of mysticism it seemingly presents, viz: How does the projectile "know" what its time of flight should be in a vacuum and why should it affect the projectile?

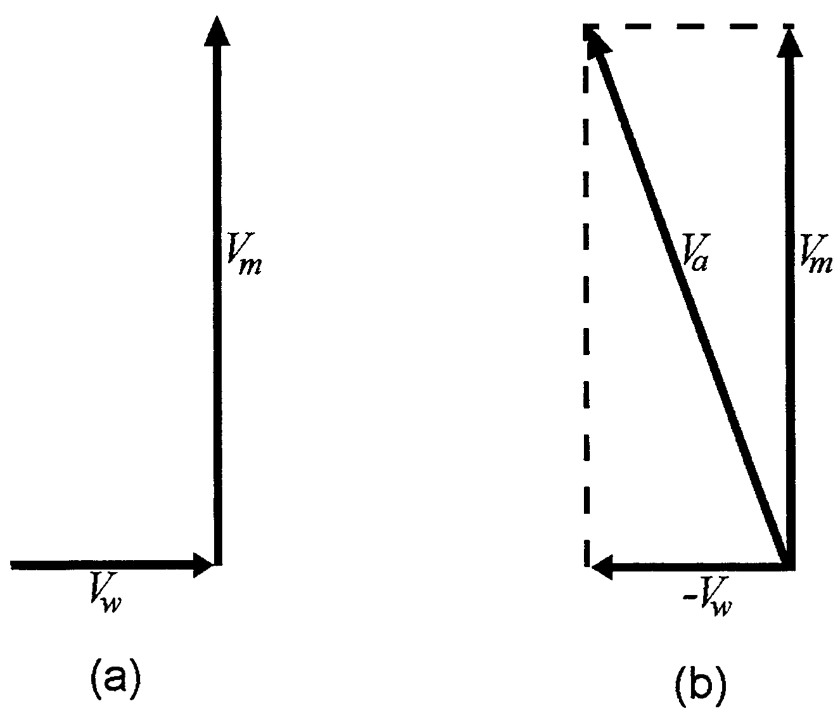


Figure 1: (a) Wind velocity, $V_w$, and projectile velocity, $V_m$, in ground reference frame, (b) Projectile velocity in air reference frame.

Fortunately the appearance of $t_v$ in (1) is only an accident of geometry and does not imply any causal link between $x_d$ and $t_v$.

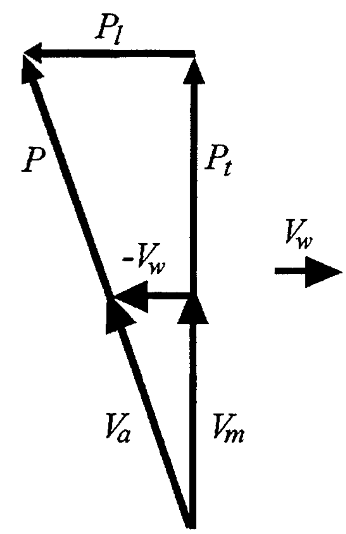


Figure 2: Projectile path and its components form triangles similar to those of the initial and all subsequent components of velocity.

The best way to illustrate this is to consider the motion of the projectile in the reference system in which the air is stationary. In that frame the force the air exerts upon the projectile is opposite to its direction of motion. Initially the components of projectile velocity $V_a$ with respect to the air are the muzzle velocity, $V_m$ and minus the wind velocity, $-V_w$. See Fig. 1.

The direction of flight in the air system does not change with time since the air exerts no force normal to the projectile's path in that reference frame. Therefore the projectile path and its components form triangles similar to those of the initial and all subsequent components of velocity as in Fig. 2. Here $P$ is the path taken by the projectile in the air reference frame, $P_l$ is the lateral displacement at the target of the projectile and $P_t$ the distance of the launch site to the target. From Fig. 2 we form the proportion:

$$\frac{P_l}{P_t} = \frac{V_w}{V_m}$$
$$P_l = \frac{V_w}{V_m} P_t \tag{2}$$

In the time the projectile has traversed path $P$ in the air system, the air has moved a distance $V_w t_a$ to the right. From this we must subtract the lateral displacement, $P_l$ of the projectile with respect to air to obtain the lateral drift, $x_d$, with respect to the ground. Doing this we obtain:

$$x_d = V_w t_a - P_l = V_w t_a - \frac{V_w P_t}{V_m} = V_w \left( t_a - \frac{P_t}{V_m} \right) \tag{3}$$

But $P_t / V_m$ is just the range $P_t$ divided by the muzzle velocity $V_m$, which is the time of flight $t_v$ in a vacuum since in a vacuum projectile velocity remains constant with the value $V_m$. Finally we obtain (1):

$$x_d = V_w(t_a - t_v) \tag{4}$$

Thus we see that the appearance of $t_v$ in (4) is a geometrical artifact that arises because the range $P_t$ can be written as $V_m t_v$.

**Effect of Projectile Shape**

If the projectile is a sphere it will always present the same cross section normal to its path in the air frame of reference, i.e., a circular one (Fig. 3).

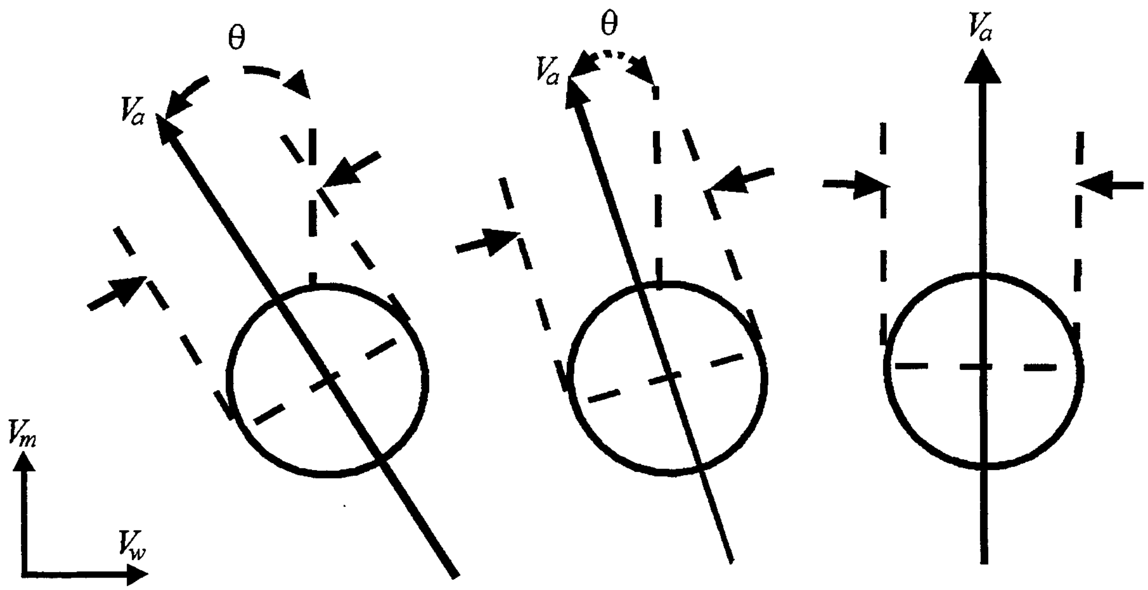


Figure 3: The angles, $\theta$, have been exaggerated for clarity. In actual cases $V_m$ is always much larger than $V_w$ and hence $\theta$ is small. (A) high wind velocity, (B) medium wind velocity, (C) zero wind velocity.

If, however, the projectile is of the usual pointed cylinder form it would seem that the cross section impacted by the air's relative motion varies with wind velocity as in Fig. 4.

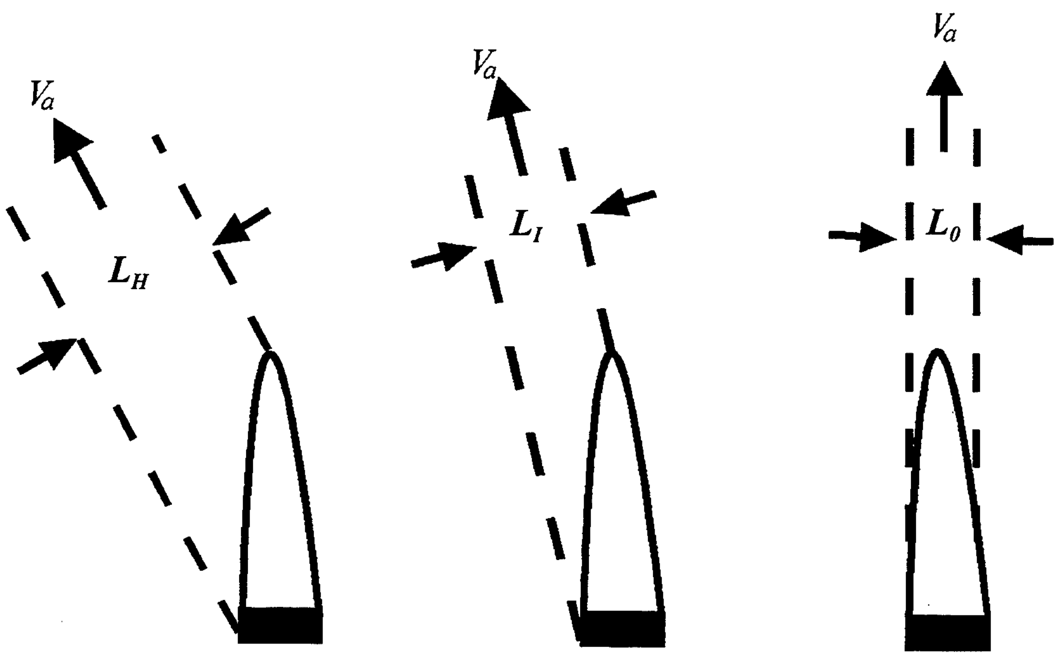


Figure 4: (A) high wind velocity, (B) intermediate wind velocity, (C) zero wind velocity.

This means that the time of flight would be affected by wind velocity since the three relative airspeed orientations, Fig. 4(A), (B), (C), result in different air resistance and, hence, in different times of flight. The cases (A) to (C) result in decreasing times of flight because the cross-sectional areas, $L$, presented to the oncoming air decrease in that sequence.

This complication does not occur in practice because a spinning projectile such as that from a rifle will always precess about the direction of air-resistance, i.e., in the direction opposite to the projectile's air speed thus automatically orienting itself point onward. The vanes of non-spinning dart-like projectiles produce the same effect.

**Effect of Muzzle Velocity on Wind Drift**

If the muzzle velocity is increased, both $t_a$ and $t_v$ decrease. At most muzzle velocities and ranges actually used this results in a decrease in $(t_a - t_v)$ itself, indicating less wind drift. But at low velocities and for some ballistic shapes it is actually possible for the decrease in $t_v$ to exceed the drop in $t_a$ with increased muzzle velocity so that the wind drift becomes greater. This is a problem for projectiles from 22 calibre rim fire target rifles wherein target ammunition is actually loaded to velocities below the standard to minimize wind effects.

**Effect of Bullet Shape and Mass on Wind Drift**

A sharply pointed bullet that is aerodynamically shaped loses velocity much more slowly than a flat or round nosed bullet because it encounters less air resistance and so has a higher average velocity over a given range. This means that at equal muzzle velocity $t_a$ will be shorter than for a less well shaped bullet, and since $t_v$ is the same for both projectiles, $\Delta t$, and hence the wind drift, $x_d$, will also be less.

The deceleration of $a$ of the bullet due to air resistance $f$ is given by Newton's law:

$$a = f / m \tag{5}$$

where $m$ is the bullet mass. The force $f$ is proportional to $A$, the cross-sectional area of the bullet presented to the air. So from (5) we have:

$$a = k\ A / m \tag{6}$$

where $k$ is a constant of proportionality equal to an average pressure on the projectile. The quantity $m/A$ is called the sectional density and since the bullet velocity loss is inversely proportional to it, it should be as large as possible to minimize wind drift. Finally, equation (6) takes on a particularly simple form if expressed in terms of the average density of the bullet, $\sigma$. Then bullet mass is bullet volume, $V$, multiplied by $\sigma$ or:

$$m = \sigma V = \sigma Al \tag{7}$$

where $l$ is the average length of the bullet and the sectional density, $s$, becomes:

$$s = m / A = \sigma Al / a = \sigma l \tag{8}$$

So the sectional density is the average length of a bullet times its density, and if all bullets are made of the same material, lead, a bullet's efficiency in cutting through air can be measured by its mean length.

In summary, we note that to minimize wind drift one should:

(1) Use a bullet of the best possible aerodynamic shape or form factor.

(2) Maximize sectional density consistent with bullet stability and attainable velocity.

(3) Use the highest possible muzzle velocity for a bullet best satisfying (1) and (2) at the longer ranges with high velocity centerfire rifles.

This is because the time of flight $t_a$ is a complex function of these bullet quantities and of the initial velocity. In practice $t_a$ is usually obtained from ballistic tables where it is tabulated for various form factors, sectional densities and muzzle velocities.

ARMY RESEARCH LABORATORY
PHYSICAL SCIENCES DIRECTORATE
MANDATORY DISTRIBUTION LIST

Defense Technical Information Center*
ATTN: DTIC-OCC
8725 John J. Kingman Rd STE 0944
Fort Belvoir, VA 22060-6218
(*Note: Two DTIC copies will be sent from STINFO office, Ft. Monmouth, NJ)

Director
US Army Material Systems Analysis Actv
ATTN: DRXSY-MP
(1) Aberdeen Proving Ground, MD 21005

Commander, AMC
ATTN: AMCDE-SC
5001 Eisenhower Ave.
(1) Alexandria, VA 22333-0001

Director
Army Research Laboratory
ATTN: AMSRL-D (John W. Lyons)
2800 Powder Mill Road
(1) Adelphi, MD 20783-1197

Director
Army Research Laboratory
ATTN: AMSRL-DD (COL Thomas A. Dunn)
2800 Powder Mill Road
(1) Adelphi, MD 20783-1197

Director
Army Research Laboratory
2800 Powder Mill Road
Adelphi, MD 20783-1197
(1) AMSRL-OP-SD-TA (ARL Records Mgt)
(1) AMSRL-OP-SD-TL (ARL Tech Library)
(1) AMSRL-OP-SD-TP (ARL Tech Publ Br)

Directorate Executive
Army Research Laboratory
Physical Sciences Directorate
Fort Monmouth, NJ 07703-5601
(1) AMSRL-SE
(1) AMSRL-SE-C (V. Rosati)
(1) AMSRL-SE-C (M. Hayes)
(1) AMSRL-OP-FM-RM
(22) Originating Office

Advisory Group on Electron Devices
ATTN: Documents
Crystal Square 4
1745 Jefferson Davis Highway, Suite 500
(2) Arlington, VA 22202

Commander, CECOM
R&D Technical Library
Fort Monmouth, NJ 07703-5703
(1) AMSEL-IM-BM-I-L-R (Tech Library)
(3) AMSEL-IM-BM-I-L-R (STINFO Ofc)

Deputy for Science & Technology
Office, Asst Sec Army (R&D)
(1) Washington, DC 20310

HQDA (SARDA-TR)
Dr. Richard Chait
(1) Washington, DC 20310

Director
Naval Research Laboratory
ATTN: Code 2627
(1) Washington, DC 20375-5000

USAF Rome Laboratory
Technical Library, FL2810
ATTN: Documents Library
Corridor W, STE 262, RL/SUL
26 Electronics Parkway, Bldg. 106
Griffiss Air Force Base
(1) NY 13441-4514

Dir, ARL Battlefield
Environment Directorate
ATTN: AMSRL-BE
White Sands Missile Range
(1) NM 88002-5501

Dir, ARL Sensors, Signatures,
Signal & Information Processing
Directorate (S3I)
ATTN: AMSRL-SS
2800 Powder Mill Road
(1) Adelphi, MD 20783-1197

Dir, CECOM Night Vision/
Electronic Sensors Directorate
ATTN: AMSEL-RD-NV-D
(1) Fort Belvoir, VA 22060-5806

Dir, CECOM Intelligence and
Electronic Warfare Directorate
ATTN: AMSEL-RD-IEW-D
Vint Hill Farms Station
(1) Warrenton, VA 22186-5100

Cdr. Marine Corps Liaison Office
ATTN: AMSEL-LN-MC
(1) Fort Monmouth, NJ 07703-5033