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13. ABSTRACT *(Maximum 200 words)*

This final report summarizes a three year project in the area of robust H_∞ control, based mostly on the time domain / game theoretic approach, previously developed by the author. Areas covered included: (1) A framework for sampled data control, including optimization of sample and hold components. (2) Investigating fundamental ties between the time and the frequency domain / Hardy space approaches, including control based proofs and variants of the Nehari and Beurling-Lax theorems, factorization techniques, dichotomy, etc. (3) Robust identification and control of periodic and closed to periodic systems. (4) Computationally feasible Robust control of delay systems. (5) Robust system identification. (6) H_∞ based design for frequency varied passivity. (7) Miscl. The research resulted both in novel design methods for robust control and in solidifying the theoretical basis of this methodology.

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This final report summarizes research covered by the ARO grant listed above. The research concerned the development of novel design and analysis methods in robust H_∞ control, as well as exploration of the theoretical foundation of this methodology. In the most part, the work is focused on the time domain / game theoretic approach that was developed by the author in the late 1980's. Following is an itemized list of areas covered and references to articles documenting this research.

Robust Sampled Data Control. Sampled data systems generically comprise an analog (continuous time) plant and a digital (discrete time) control mechanism. When the plants bandwidth of operation is wide relative to the sampling frequency the intersample dynamics is significant. Detailed time domain analysis through the technic that came to be known as "lifting" allows to incorporate in one model both the discrete and the continuous dynamics and account for the impact of both on the I/O induced L_2 norm. Our work includes both design in the standard setting of predicated samplers and hold mechanisms, as well as tools to design customized samplers and hold functions. Also included is a differential Riccati equation tool to precisely quantify the tradeoff between the controller sampling frequency and achievable H_∞ performance. This work that was included in the original proposal was performed mostly during the review process and is documented in [21, 24, 23].

Relations between the time domain and Hardy space approaches. Here my goal was to illuminate on the fundamental ties between the seemingly very different approaches to H_∞ control: the time domain / game theoretic approach, on the one hand, and the factorization based / frequency domain / operator theoretic / Hardy space approach, on the other hand. This effort included the development of control oriented, time domain counterparts of the Nehari and the Beurling-Lax Theorems, as documented in [35, 37, 36, 31, 26, 25]. Preliminary results in this effort included also a a time domain based exploration of isometries and J-isometries that are related to LQ optimizations and relations to factorization theory (spectral, normalized, inner denominators, etc.) [36], implications on related two player games [22] and classical interpolation problems [31].

Robust system identification. The standard premise in robust H_∞ control is that models are associated with a quantification of uncertainty, or model error, in the induced L_2 norm. This called for the development of identification algorithms that produce both models and induced norm error bounds, and indeed, aim at minimizing those errors. This effort included the development that combines induced norm (hence worst case) error bounds in a probabilistic setting and a unified approach that robust system identification and control. The research is documented in [17, 18, 19].

Robust control and identification of periodic systems. Periodic and close to periodic models are appropriate when the plant in question includes any of several common mechanisms. Most common are mechanical rotations, such as in electric drives, or slow vibrations (that occur at a frequency that is well within the range of other significant dynamics). Other areas of potential applications include switching power electronic devices, when the frequency band of operation approaches the switching frequency. Typical to such applications is that while the rate of time variation may rule out the use of a time invariant model, the drift in the system dynamics from one period to another is negligible or very small and measurement / estimates of the underlying period are relatively easy to obtain. One direction pursued was a combination of the "lifting" method with the classical Schur algorithm. A second approach was based on a factorization of (closed to) periodic system as a cascade of a memoryless periodic system and an LTI (or slowly time varying) system. Results include error bounds when fast components of the model are dropped. Related publications are [1, 2, 3, 4, 5, 6, 7, 8, 9]

Robust control of systems with delays. State space methods in H_∞ control reduce design and analysis problems to solving (algebraic) Riccati equations. When the system involves delays, state space models are infinite dimensional and so are the Riccati equations that have to be solved. Operator Riccati equations that arise in LQ optimization are generally notoriously hard to solve. The present study concern the development of a computationally viable solution and finite dimensional compensator realization in the relatively simple case where delays are restricted to a single lag at the input (or output) port. The time domain / game theoretic approach is utilized in a reduction of the original problem to a set of LQ optimization problems and differential games that involve, each a finite dimensional system. This reduces the associated operator Riccati equations to a set of algebraic and differential matrix equations. Results are documented in [27, 32, 28, 33, 29, 30]. These results utilized an independent work by this author on state space models for delay and neutral functional differential equations [34].

Frequency weighted passivity. Physically meaningful concepts of energy and power supply arise naturally in many engineering contexts and can be used as a basis for robust control design. Passive systems are systems that consume energy along processes. A counterpart of the small gain theorem states that an interconnection of two passive system is stable. This allows to replace the small gain restriction on model errors by a passivity restriction. Geometrically, this allows the uncertainty set to be a half plane, rather than a small disk, and motivates strict passivity as a design goal. This goal is not feasible in strictly proper plants. The reported project extended known ideas concerning the reduction of the passivity objective to an allied H_∞ problem that can be solved using available, effective design tools. It began with an extension of these results to a general quadratic dissipativity framework and then included frequency weighting. A typical objective, enabled in this framework, would be to render the closed loop system passive over a designated band and having the gain rolled off over a complimentary band. The basic results are reported in [13]. This effort was motivated by some concrete problems in power generation and distributions. Such applications are discussed in [14, 20, 16, 15].

Nonlinear perturbations. The note [10] concerns a design of robust H_∞ controllers for a cascade of a linear system and a nonlinear element, satisfying a sector-like condition. Result include both internal (state space) and I/O BIBO stability.

Reduced order controllers for LTV systems. Results in [11, 12] concern the design of LQ optimal controller of a predetermined order for LTV systems. The results extend the Bernstein-Haddad approach to this class.

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