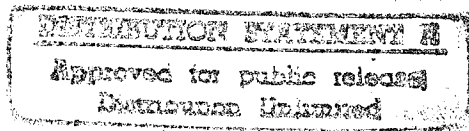Office of The Quartermaster General
Military Planning Division
Research and Development Branch

TEXTILE SERIES - REPORT NO. 60

# MECHANICS OF ELASTIC PERFORMANCE
# OF TEXTILE MATERIALS
Some Aspects of Stress Analysis of Textile Structures
Continuous Filament Yarns
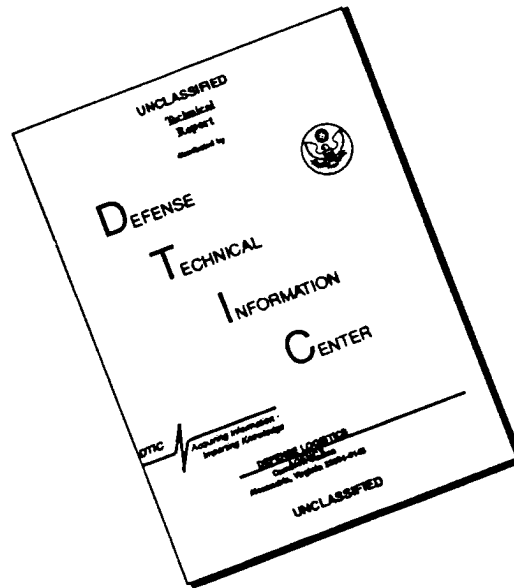
by
**MILTON M. PLATT**
Fabric Research Laboratories, Inc.

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# FOREWORD

Field requirements, which have served as the objective of Textile Section research during the past five years, call for full utilization of recent developments in textile technology. New and old fibers and their chemical modification, fiber blending, yarn construction, fabric design, and control of cloth structure during processing must be studied and related singly or in combination if significant progress is to be expected in meeting the ofttime conflicting needs of service apparel, tentage and equipage items.

Major research emphasis since 1945 has been placed on fabric construction and chemical finishing, with very promising results leading to improvement in wear resistance, water resistance, fireproofing, shrink resistance, and color fastness. More recently studies have been initiated in the area of fiber properties and translation of such properties into yarn and fabric structures. Work sponsored by the Quartermaster Corps at the Fabric Research Laboratories has centered on this latter phase. The paper presented here is illustrative of the approach of this group to the problem at hand. It involves a mechanical analysis of the stress-strain function at any point in a given yarn based upon yarn geometry and fiber behavior under tensile stresses, and integration of the relationship so derived to cover the mechanical performance of the total yarn. The implications of this work in the design of yarns to meet specific end requirements are evident.

STANLEY BACKER
Head, Textile Materials
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July 1949

MECHANICS OF ELASTIC PERFORMANCE OF TEXTILE MATERIALS

Some Aspects of Stress Analysis of Textile Structures*

Continuous Filament Yarns


## Introduction

Stress Analysis is that branch of physics concerned with the quantitative determination of internal stresses and strains produced in a body as the result of external loads and deformations. Methods of stress-analysis have found wide application in the fields of structural and mechanical engineering, where quantitative evaluation of internal forces and movements are an absolute requirement for useful, economical, and safe design. Used in conjunction with sound design principals, stress analysis has made possible the complete blueprinting of engineering structures where particular end-use properties of these structures are desired and can be formulated.

A moment's consideration will show the tremendous possibilities inherent in the use of stress analysis for the design of textile structures. Its application to mechanical fabrics, cords, ropes, etc., where load-carrying capacity and/or stability of size are important, is obvious. Just as important, however, would be its application to the many other types of textile structures which are used. Such end-use requirements as crush resistance of apparel fabrics, the combined performance of coated or laminated fabrics, etc., all depend to a degree on the inherent properties of the basic fibers and materials comprising the structure. However, the geometric form of these fibers, and the type of construction used, determine the manner in which these fibers react to the external forces applied in the course of use of the material, and hence influence the utility of the structure to a marked degree.

The need for studying the influence of geometry upon the manner in which stresses are distributed in textile structures has been emphasized by various workers in the field. The influence of geometry has been termed "Form Factor Effects." Work of both an empirical and theoretical nature has already been done along these lines, most of which is related to describing the fundamental geometry of certain textile structures. These workers include Backer, Hamburger, Hertel, Hotte, Peirce, Wormersly, and others. However, complete theoretical analysis has been limited chiefly by two properties of textile structures, one inherent and the other arising as a result of geometry, namely:

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(1) Every textile fiber possesses a nonlinear stress-strain curve for the ordinary rates of loading or straining. Hence, any textile structure in general will exhibit non-Hookian characteristics, making it impossible to describe textile performance by the direct use of well known engineering formulae. This does not eliminate the possibility of using engineering concepts as a fundamental guide in solving textile problems. Instead, it indicates that the complete solution of a textile problem involving stresses depends upon a quantitative knowledge of the fundamental stress-strain properties of the fiber.

(2) Every textile structure, beginning with the yarn, represents a statically indeterminate body,[1] meaning that stress distribution cannot be established from merely a consideration of the laws of static equilibrium, but that, in addition, the deformation of the structure must be considered. An illustration of static indeterminacy is furnished by comparing the determination of the end supporting forces of a two-span beam versus a single-span beam. These forces can always be found for a single-span beam by using the "law of moments" taken about either end support. A moment's trial will show such a procedure to be insufficient for the two-span beam. This latter case can only be solved by considering the deformations in the beam.

The Office of The Quartermaster General has been sponsoring a long-term research program aimed towards the understanding of the influence of form factors on the translation of inherent physical properties of fibers into textile structures, and the establishment in quantitative formulae of the relations between inherent properties and structure properties. The work herein reported is part of this program. Logical sequences of constructions of fiber length, including continuous filament, yarn twist, yarn size, fabric weave, and ends and picks per inch have been selected for experimental evaluation and theoretical interpretation and generalization.

This paper serves to present the influence of fiber type, yarn size, and yarn twist on the translation of inherent fiber properties into continuous filament singles yarns. Additional papers will be forthcoming on the effects of fiber length and twist on staple yarn properties and later, effects of weave and fabric geometry.

## Geometry of Singles Continuous Filament Yarns

The first step in the stress analysis of any structure consists in defining the geometry of the body, usually by assuming an ideal physical form. Theoretical analysis demands the assumption of no variability in form, and a compromise between duplicating actual average shape and the adoption of a geometry which is easy to

visualize and handle mathematically. The final check for the rationality of any assumptions is determined partially by judgment, and more convincingly by the agreement between the results arrived at using the assumptions and any experimental results.

The following initial assumptions will be made:

1. The yarn is uniform along its length, and its cross sectional outline is circular.

2. All fibers within a yarn possess equal properties and are circular in cross section.

3. The center line of each fiber lies in a perfect helix, with the center of the helix located at the center of the yarn cross section.

4. The fibers fall into a rotationally symmetric array in cross sectional view.

5. The diameter of the yarn is large compared with the fiber diameter.

These assumptions result in a yarn geometry illustrated by Figure 1. The helix angle, $\theta$, shown in (a) represents the angle between yarn and fiber axes of the outside fibers only. Assumption 4 describes this angle as equal for all fibers which fall into the outer sheath of the yarn. Any other layer of fibers, such as those shown in (b) to lie at a distance r from the yarn axis, possesses a helix angle designated by $\theta_r$.

A general equation relating helix angle, turns per inch, and diameter can be found in the literature[2] and will not be derived here. The relationship follows:

$$\text{Tan } \theta = N \pi KD \text{ - - - - - - - - - - - - - - - - - - - (1)}$$

$\theta$ = Helix angle

N = Yarn twist, turns per inch

D = 2r = Bounding diameter of layer of fibers, inches

K = Geometrical constant = $\dfrac{D - d}{D}$

d = Fiber diameter

FIBER AXIS

YARN AXIS

R

r

θ

(a)
LONGITUDINAL VIEW

(b)
CROSS-SECTIONAL VIEW

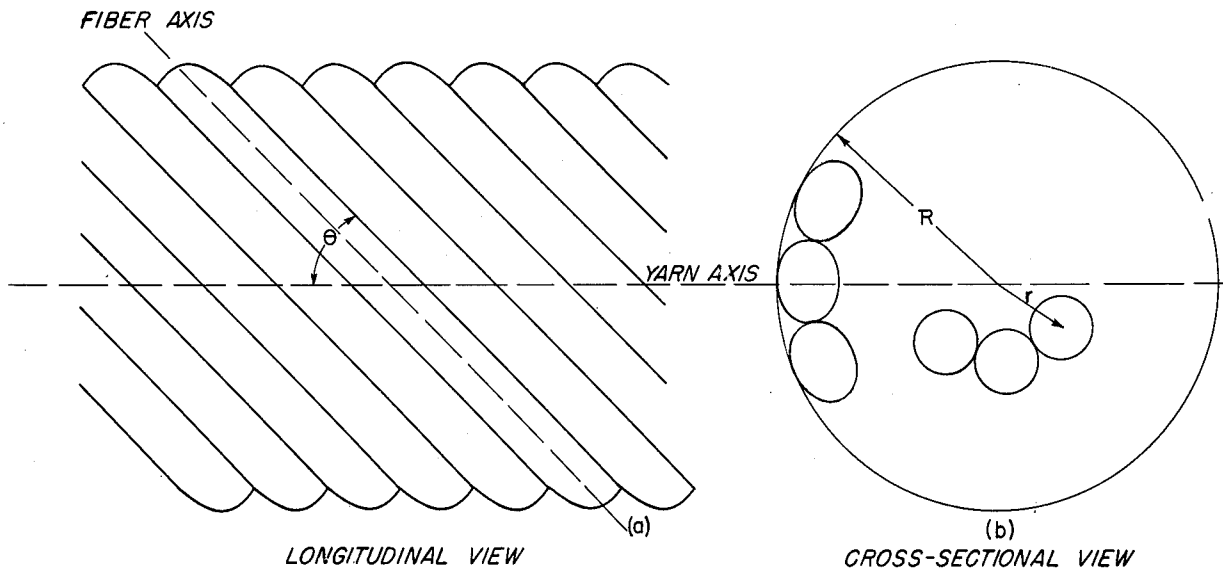FIG. I
IDEALIZED SINGLES YARN GEOMETRY

Equation (1) indicates that for a given yarn twist N, which is constant for all the fibers, the tangent of the helix angles of any layer of fibers is linearly proportional to the distance of the group from the yarn center, just so long as K is constant. By assumption 5, the ratio $d/D$ is considered small relative to unity and hence except for those fibers positioned close to the yarn axis, K will be approximately 1. In addition, for those fibers close to the yarn axis, the helix angle approaches 0, and hence, as will be seen later, variations in K will not influence the stress distribution to any significant degree.

Forces in Fibers

A.  Component Effect

The application of an external tensile load, P, along the axis of a yarn results in forces being applied to the various fibers. In general, the only stresses which can possibly act on the cross section of a fiber are:

1.  A tensile force, $p_r$, direction along the fiber axis; normal to the fiber cross section.

2.  A shear force, v, acting tangential to the fiber cross section.

- 4 -

3.  A bending moment, m.

4.  A torsional moment, t.

These stresses are illustrated in Figure 2.

It is now assumed that the fibers are perfectly flexible members, incapable of resisting any compressive forces. Because of the extremely low modulus of elasticity of textile fibers, in addition to the extremely large ratio of fiber length to fiber diameter, this is a reasonable assumption. As a result of this assumption, the stresses m, v, and t vanish, and the only fiber force acting is a direct tension, $p_r$. The concept of perfect flexibility is not an uncommon one in engineering. The classical design of the long cables of a suspension bridge depends upon the assumption of only tensile forces acting, as is the case for many other structures.
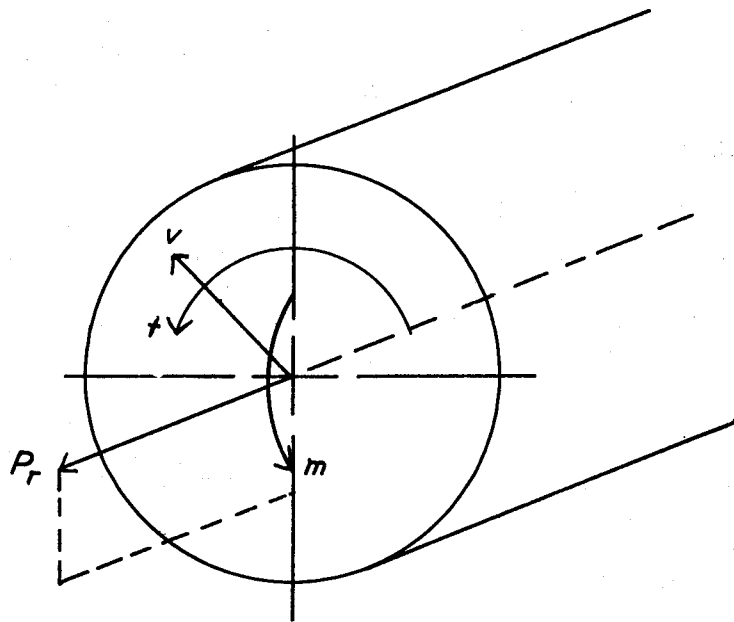
FIGURE   2

GENERAL   STRESSES   IN   FIBERS

$v, t, m$   assumed $= 0$.

Since $p_r$, which parallels the fiber axis, represents the only force acting upon a fiber, then it is apparent that the contribution of $p_r$ toward the total load P acting on the yarn in tension is:

$$P/fiber = p_r \cos\theta_r \text{ - - - - - - - - - - - - - - - - - - - } (2)$$

$$\therefore P = \sum p_r \cos\theta_r \text{ - - - - - - - - - - - - - - - - - - } (3)$$

Equation (3) states that if the axial tension carried by each fiber be multiplied by the cosine of the helix angle of that fiber, and all such products summed, the total will represent the axial tensile load acting on the yarn. The completion of such a sum demands knowledge of $p_r$ for each fiber layer within which $\theta_r$ is constant. Such a method would be tedious and require a new solution for each yarn size considered, and hence a simpler mathematical form of equation (3) has been derived.

It is now assumed that the individual fibers in the yarn are considered as consolidated into an equivalent solid continuous medium rather than a group of discreetly individual fibers. However, it is still considered that at a distance r from the yarn center, the force, $p_r$, remains inclined to the yarn axis at the angle $\theta_r$ which the fiber at that radius possessed previously. In addition the total transformed yarn denier is assumed to be unchanged. The cross-sectional area of this new yarn will be discussed later.

As a result of this transformation it is now possible to treat the yarn as a continuous rather than discontinuous medium, with the use of differential elements of area as opposed to individual fibers. Figure 3 illustrates the element of force, $dp_r$, acting on the element of area defined by the shaded annular ring.

The magnitude of the force, $dp_r$, is equal to the product of the stress intensity (force per unit area), $f_r$, and the element of area, dA normal to the force. Reference to Figure 3 will show that this may be written as:

$$dp_r = f_r dA = f_r (2\pi r) dr' \text{ - - - - - - - - - - - - - - } (4)$$

However, it is apparent that:

$$dr' = dr \cos\theta_r$$

Therefore:

(a)
LONGITUDINAL VIEW

(b)
CROSS-SECTION
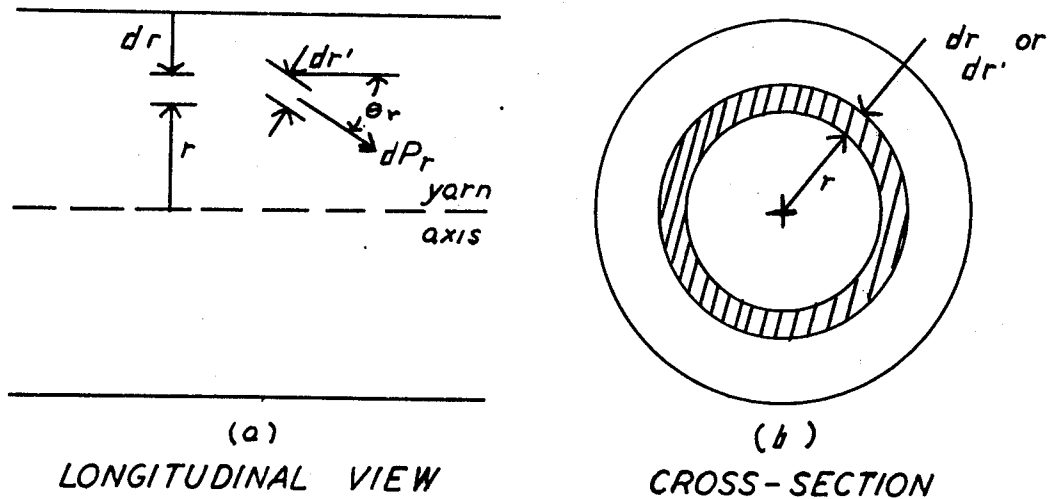
FIGURE 3

ELEMENTS OF FORCES IN SOLID YARN

$$dp_r = f_r 2\pi r \cos\theta_r dr \quad - - - - - - - - - - - - - - - - - \quad (5)$$

The component of the firce, $dp_r$, parallel to the yarn axis, $dP_r$, may be written:

$$dP_r = dp_r \cos\theta_r = f_r 2\pi r \cos^2\theta_r dr \quad - - - - - - - - - \quad (6)$$

The sum of all forces, $dP_r$, represents the total axial yarn load and is found from:

$$\underline{P} = \int_{r=0}^{r} d\underline{P}_r = 2\pi\int_{r=0}^{r} f_r r \cos^2\theta_r dr \quad - - - - - - - - - - - \quad (7)$$

In order to integrate equation (7) it is necessary to express both $f_r$ and $\theta_r$ as functions of r. From equation (1):

$$\tan\theta = N\pi KD$$

which becomes for the assumed solid yarn:

$$\tan\theta_r = N\pi\frac{(2r - dr)}{2r} 2r$$

$$\tan\theta_r = 2\pi Nr \quad - - - - - - - - - - - - - - - - \quad (8)$$

From the identity:

$$\cos^2\theta = \frac{1}{1 + \tan^2\theta}$$

It is apparent that:

$$\cos^2\theta_r = \frac{1}{1 + 4\pi^2 N^2 r^2} \quad ----- - - - - - - - (9)$$

Substituting (9) into (7) there results:

$$P = 2\pi \int_{r=0}^{r_f} f_r \frac{r\,dr}{(1 + 4\pi^2 N^2 r^2)} \quad - - - - - - - - (10)$$

The evaluation of the dependency of $f_r$ upon r is described later. Equation (10) was derived without any consideration of the inherent properties of the fibers comprising the yarn. It is dependent only upon yarn geometry and hence applies to continuous filament yarns made of any fibers which satisfy the assumptions previously given. Since the equation was derived from laws of static composition of forces, the effect is termed the "component effect." The component effect results in the important fact that the higher the helix angle, the greater the fiber tensile stresses when a given axial load acts on a continuous filament yarn, since from equation (10), the value of $f_r$ increases when N increases and P is held constant. Conversely, with equal fiber stresses, the most highly twisted yarn will have the lowest axial yarn load, since P decreases as $f_r$ is held constant and N increases. These effects are illustrated in Figures 4 and 5.

### B. Serigraph Effect

Equation (10) requires for its solution expressing $f_r$, the tensile stress intensity, as a function of r, the distance from the yarn center. A first approximation would be to assume f constant with r, i.e., equal stress intensity at all points in the yarn cross section. However, conditions of continuity of distortion must be satisfied, and as stated previously, the solution of statically indeterminate structures requires a study of the deformations involved. The "serigraph effect" is concerned with the effect of distortions upon the stress distribution. Whereas the component effect was general, dependent only upon yarn geometry, the serigraph effect, while caused by yarn geometry, depends for its magnitude upon inherent fiber properties.
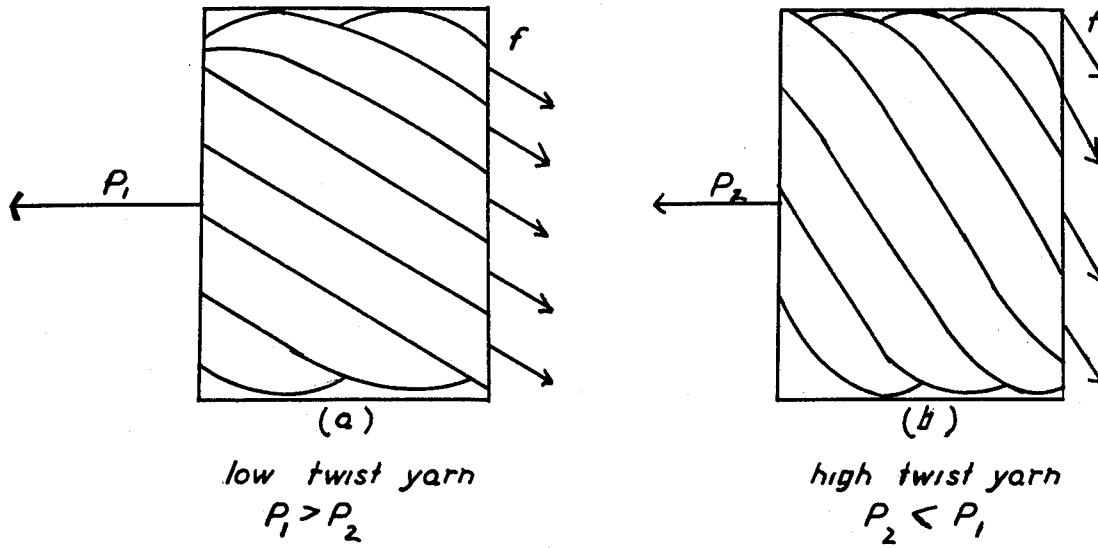
FIGURE 4

EFFECT OF HELIX ANGLE ON UTILIZATION
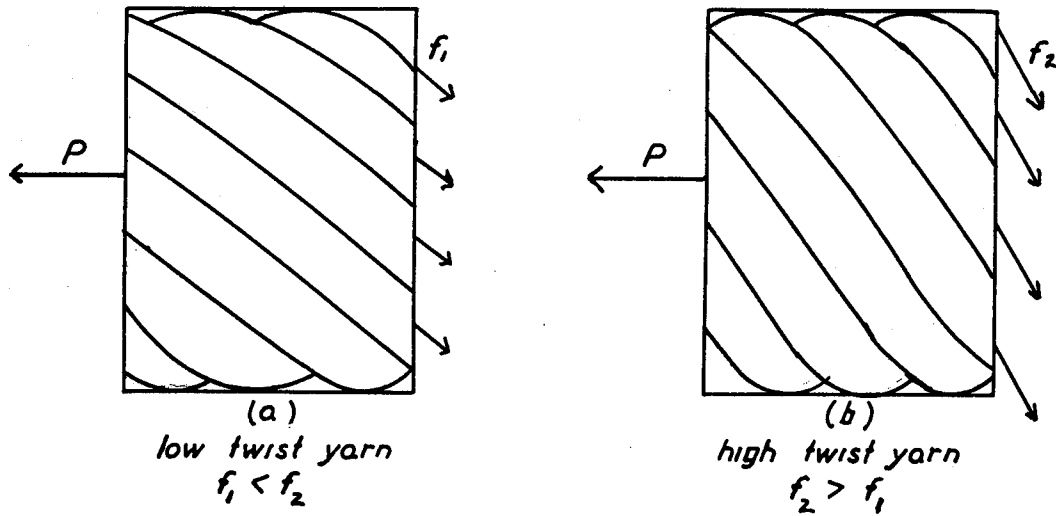OF FILAMENT STRENGTH (COMPONENT EFFECT)



FIGURE 5

EFFECT OF HELIX ANGLE ON FILAMENT
FORCES (COMPONENT EFFECT)

Because of the helix angle, the actual length of inclined fibers in a given length of yarn exceeds the yarn axial length, the excess being functionally related to the cosine of the helix angle. Figure 6 sketches a yarn with two filaments, one inclined at an angle $\theta$ to the yarn axis, and the other parallel to the yarn axis. The yarn is shown with an extension $\Delta L$, or an equivalent axial strain $\dfrac{\Delta L}{L}$. This yarn axial strain, $\dfrac{\Delta L}{L}$, also represents the unit strain of the center fiber which has a helix angle of 0. The inclined fiber possesses a length $\dfrac{L}{\cos \theta}$.

From Figure 6 it is apparent that the absolute elongation of the inclined fiber can be very closely approximated as:

$$\Delta L_\theta = \Delta L \cos \theta$$

Therefore, the unit strain, $\varepsilon_\theta$, in any fiber inclined at an angle $\theta$ to the yarn axis may be determined from:

$$\varepsilon_\theta = \frac{\Delta L \cos \theta}{L/\cos \theta} = \frac{\Delta L}{L} \cos^2 \theta \; - - - - - - - - (11)$$

or

$$\varepsilon(\text{fiber}) = \varepsilon(\text{yarn}) \cos^2 \theta \; - - - - - - - - - (12)$$

Since $\cos^2 \theta$ is equal to or less than unity, it is clear that for all helix angles greater than zero, the fiber strain is less than the yarn strain. In addition, as shown by equation (9), the further the fiber from the yarn axis, the greater the value of $\theta$, and therefore, the smaller the value of $\cos^2 \theta$. Hence, the further the fiber from the yarn axis, the smaller the fiber strain. After substituting equation (9) into (12) there results:

$$\varepsilon_\theta = \frac{\varepsilon_0}{1 + 4\pi^2 N^2 r^2} \; * \; - - - - - - - - - - - - - (13)$$

$\varepsilon_\theta$ = strain in fiber inclined at angle $\theta$ to yarn axis.

$\varepsilon_0$ = yarn strain or strain in un-inclined fibers.

* A more precise expression, based on assuming Poisson's ratio = $\frac{1}{2}$, is

$$\varepsilon_\theta = \sqrt{\frac{(1 + \varepsilon_0)^3 + 4\pi^2 N^2 r^2}{(1 + \varepsilon_0)(1 + 4\pi^2 N^2 r^2)}} - 1.$$

This equation should be used if very high helix angles or fiber elongations are encountered.
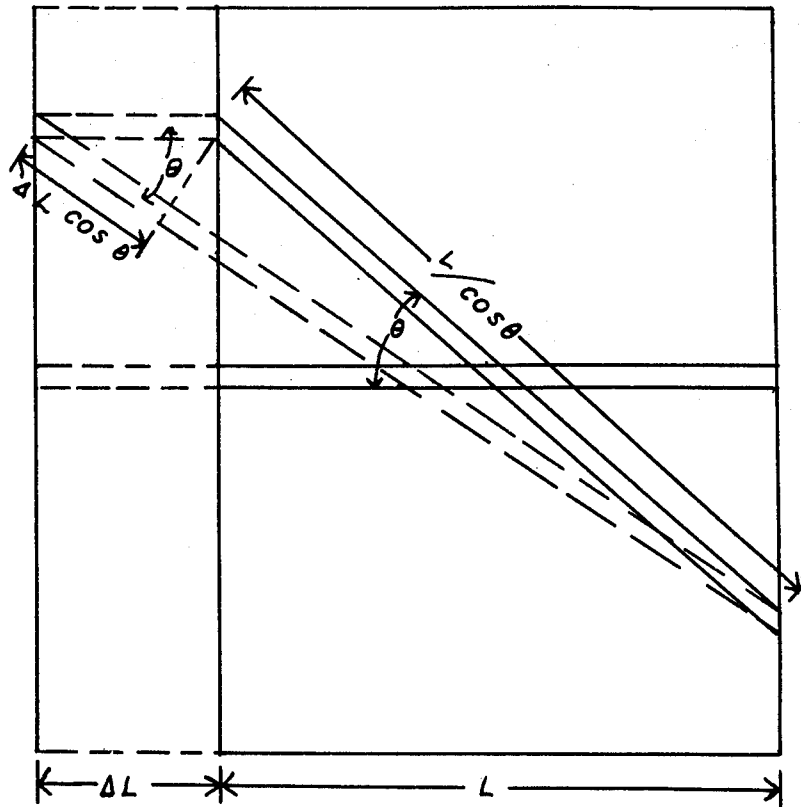
## FIGURE 6

## DEFORMATIONS IN TWISTED YARNS

For any textile fiber strain is functionally related to stress
for given testing conditions such as rate of loading or straining.
The magnitude of the stress being borne by a fiber when it is
strained a given amount may be found from the stress-strain diagram
for that particular fiber. Usually it is found that as the strain
decreases the stress also decreases, an exception being the dip in
a stress-strain curve beyond the yield point when tests are made
slowly at a constant rate of elongation. It has just been shown
that the strains present in a loaded yarn vary with the position
of the filament from the yarn axis. The center filaments ($\theta = 0^\circ$)
are always subjected to greater strains than are any of the other
filaments. Hence, the center filaments can be stressed to rupture
while the remaining filaments are stressed below their rupture
loads. As soon as the center filaments fail, there occurs a rapid
progressive rupturing of all remaining filaments and the yarn breaks.
As a result, the full breaking strength of the sum of the filaments
theoretically can never be attained. Instead, the maximum strength

of the yarn is limited by the sum of the components of the fiber
forces when the center fibers have reached their ultimate elonga-
tion. When this yarn elongation is reached, only a portion of the
strength of the inclined fibers contributes to yarn strength. The
precise amount of this contribution depends upon the stress-strain
curve of the material, in addition to the magnitude of the helix
angle, and will now be evaluated for 600/208 acetate yarn.

The stress-strain diagram of 600/208/0.3 acetate yarn is
shown in Figure 7. The curve was obtained from a constant rate
of load IP2 machine. Since the yarn twist is so slight, it is
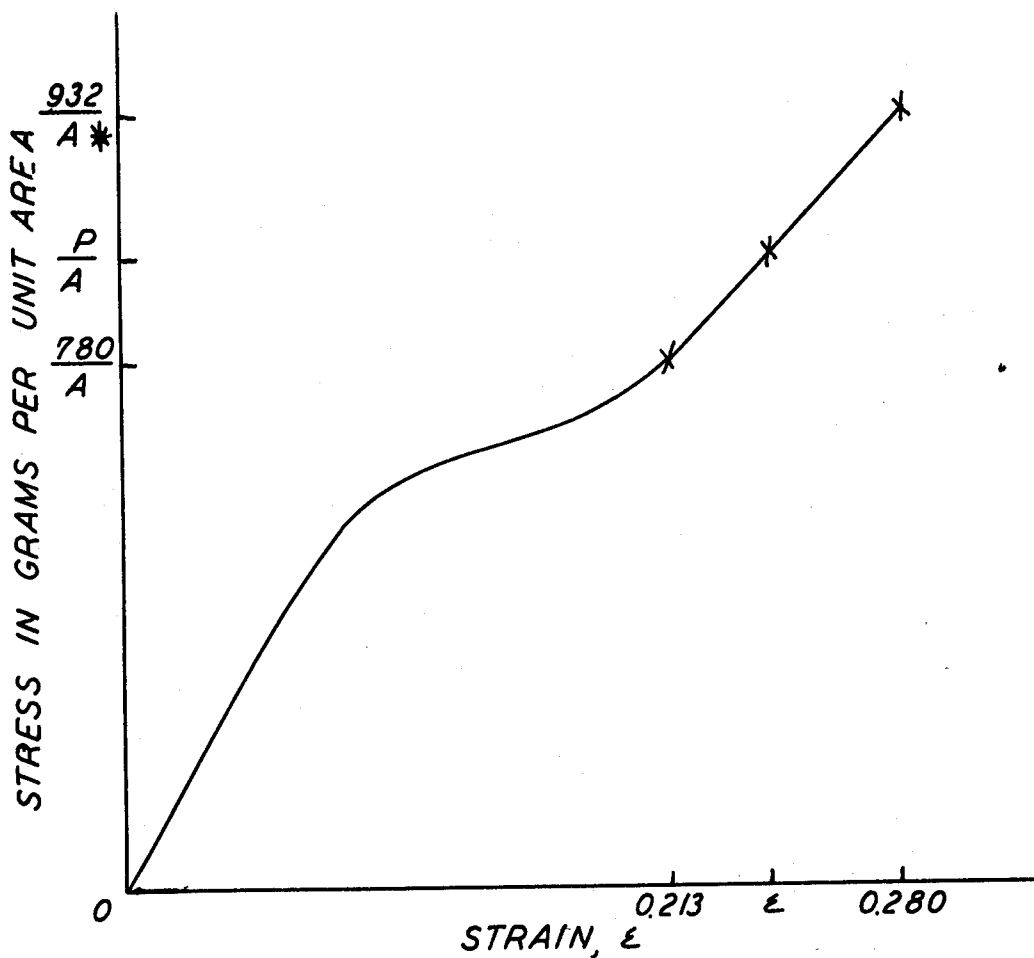assumed that the fiber inclination is negligible.



FIGURE 7

STRESS-STRAIN DIAGRAM, CONSTANT RATE
OF LOAD. 600/208/0.3 ACETATE YARN.

* A = CROSS SECTIONAL AREA OF 600 DENIER YARN

Now, at yarn rupture the stress intensity carried by the fibers is equal to $\frac{932}{A}$ grams per unit area (where $A$ is the cross-sectional area of the yarn) at a strain of 0.280. Fibers inclined to the yarn axis are subjected to a strain of:

$$\varepsilon = \frac{0.280}{1 + 4\pi^2 N^2 r^2} \quad ----------- (14)$$

and are carrying a stress of $P/A$ grams per unit area. From Figure 7 it is seen that the shape of the stress-strain curve close to rupture is essentially linear. The equation of this portion of the stress-strain curve may be written as:

$$\frac{P/A - 780/A}{\varepsilon - 0.213} = \frac{932/A - 780/A}{0.280 - 0.213} = \frac{2240}{A}$$

From which:

$$\frac{P}{A} = f = \frac{2240\varepsilon + 304}{A} \quad ------------- (15)$$

Substituting (14) into (15) there results:

$$f = \frac{628}{1 + 4\pi^2 N^2 r^2} + 304 \quad ----------- (16)$$

Equation (16) defines the stress distribution, $f$, at yarn rupture as a function of yarn twist, $N$, and distance of the fiber from the yarn center, $r$. It is apparent from this equation that for a given yarn twist, the further the fiber from the yarn center, the smaller the stress carried by that fiber at yarn rupture. In addition, for a given yarn size, the greater the yarn twist the less the fiber stress at a given distance from the yarn center. Figure 8 illustrates the variations of stresses within singles yarns in accordance with equation (16).

The determination of the variance of stress with position as determined by equation (16) now permits the integration of equation (10) for the 600/208 acetate yarn. Following substitution of (16) into (10) the following expression results after expansion of logarithmic and fractional terms into a power series and neglecting terms possessing powers of $R$ greater than 4.

$$P = 4(233 - 7450 \, N^2 R^2) \quad -------------- (17)$$

$P$ = tensile strength of yarn, gms
$N$ = yarn twist, turns/inch
$R$ = yarn radius, inches

$f = f$ rupture

$f$ (a)

yarn twist $= N_1$ turns
per inch

$f = f$ rupture

$f$ (b)

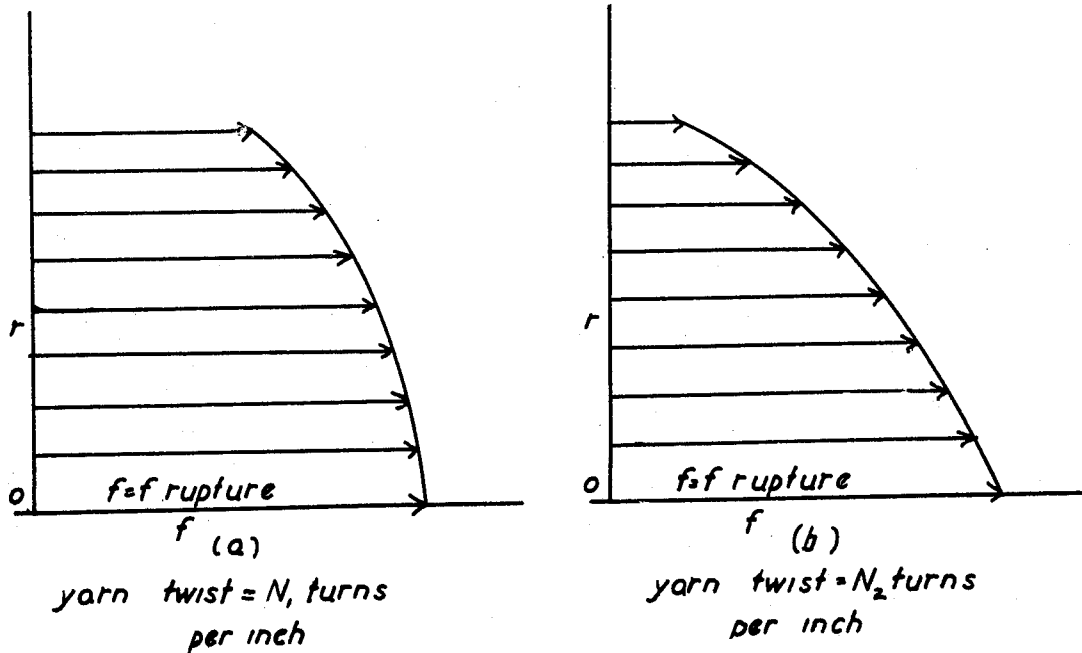yarn twist $= N_2$ turns
per inch

$N_2 > N_1$

## FIGURE 8

## TENSILE STRESS DISTRIBUTION
## IN SINGLES YARN

Equation (17) defines the breaking strength of the 600/208 acetate yarns in terms of the yarn radius and twist.  Similar equations have been derived for the various materials as listed in the table below.

TABLE 1
Equations Giving Effect of Twist on Yarn Strength

| Yarn | Equation * |
|---|---|
| 150/40 viscose | $P = 2(158 - 5600 \ N^2R^2)$ |
| 210/69 nylon | $P = 8100 \left[ \dfrac{1 + 6\pi^2 N^2 R^2}{(1 + 4\pi^2 N^2 R^2)^2} \right] - \dfrac{6650 \ \ln (1+4\pi^2 N^2 R^2)}{4\pi^2 N^2 R^2}$ |
| 300/104 acetate | $P = 2 (233 - 7450 \ N^2R^2)$ |
| 600/208 acetate | $P = 4 (233 - 7450 \ N^2R^2)$ |

* Equations for all materials simplified by expansion into power series except nylon, the series for which converges very slowly, and hence, the actual equation is given for this material.

With respect to the use of such equations as given in Table I, the twist, N, and the radius, R, should represent both these quantities at yarn rupture.  When a yarn is being strained to rupture, the twist per unit length is continually decreasing as the elongation increases.  If $N_s$ is the yarn turns per inch at no load, then at yarn rupture the twist N is given as:

$$N = \frac{N_s}{1 + \epsilon_r} \quad - - - \quad - - - - - - - - - \quad (18)$$

where $\varepsilon_r$ is the unit strain at rupture. The yarn radius will also decrease with strain. Assuming, momentarily, that the yarn is composed of solid fibrous material (complete packing) then for most high polymers which extend at constant volume, the square of the yarn radius is inversely proportional to the total length of the sample while it is being stressed. Thus, if $F_s^2$ is the square of the radius at no load, then the square of the yarn radius at rupture, $R^2$, is given below:

$$R^2 = \frac{R_s^2}{1 + \varepsilon_r} \quad \text{-------------} \quad (19)$$

The original yarn radius, $R_s$, can be calculated. It is easily shown that for a solid yarn, the value $F_s^2$ in square inches is related to the yarn denier and the fiber specific gravity as follows:

$$R_s^2 = 0.055 \times 10^{-6} \times \frac{\text{Yarn Denier}}{\text{fiber specific gravity}} \quad \text{---} \quad (20)$$

If the yarn denier and fiber specific gravity are known, then equation (20) would give the value of $R_s^2$ for a solid yarn. The bulk factor of the yarn, however, must be considered before a realistic value of the radius can be calculated. This is best taken care of by use of a bulk specific gravity for the yarn. Peirce[3] has reported that for firm cotton and viscose yarns, a bulk specific gravity of 0.9 gives reasonably accurate results, and although determined for staple yarns, should apply closely to continuous filament yarns. Others have indicated acetate yarns to have a bulk specific gravity of about 0.86. While no data are available for nylon, the value of 0.9 has been used in the strength calculations.

Utilizing the equations given in Table I for a pedigreed set of yarns, the strength twist curves shown as Graphs 1, 2, 3, and 4 were determined. Experimental values of strength are also shown on the graphs. It is to be noted that reasonably good agreement has been obtained between the theoretical and the experimental strengths. The experimental data for all the materials follow in general the same type of parabolic strength-twist curve as the calculated curves do, except for the 300/104 acetate. This yarn is the only one of the group which was spun directly to different twists at the plant. The other yarns were all spun from the same lot of original low twist yarn. Hence, the relatively poor agreement for the data shown in Graph 3 may be the result of variation in fiber properties. However, when all of the calculated and actual strengths are plotted against each other, a linear agreement line having a slope close to 45° results, as shown in Graph 5.
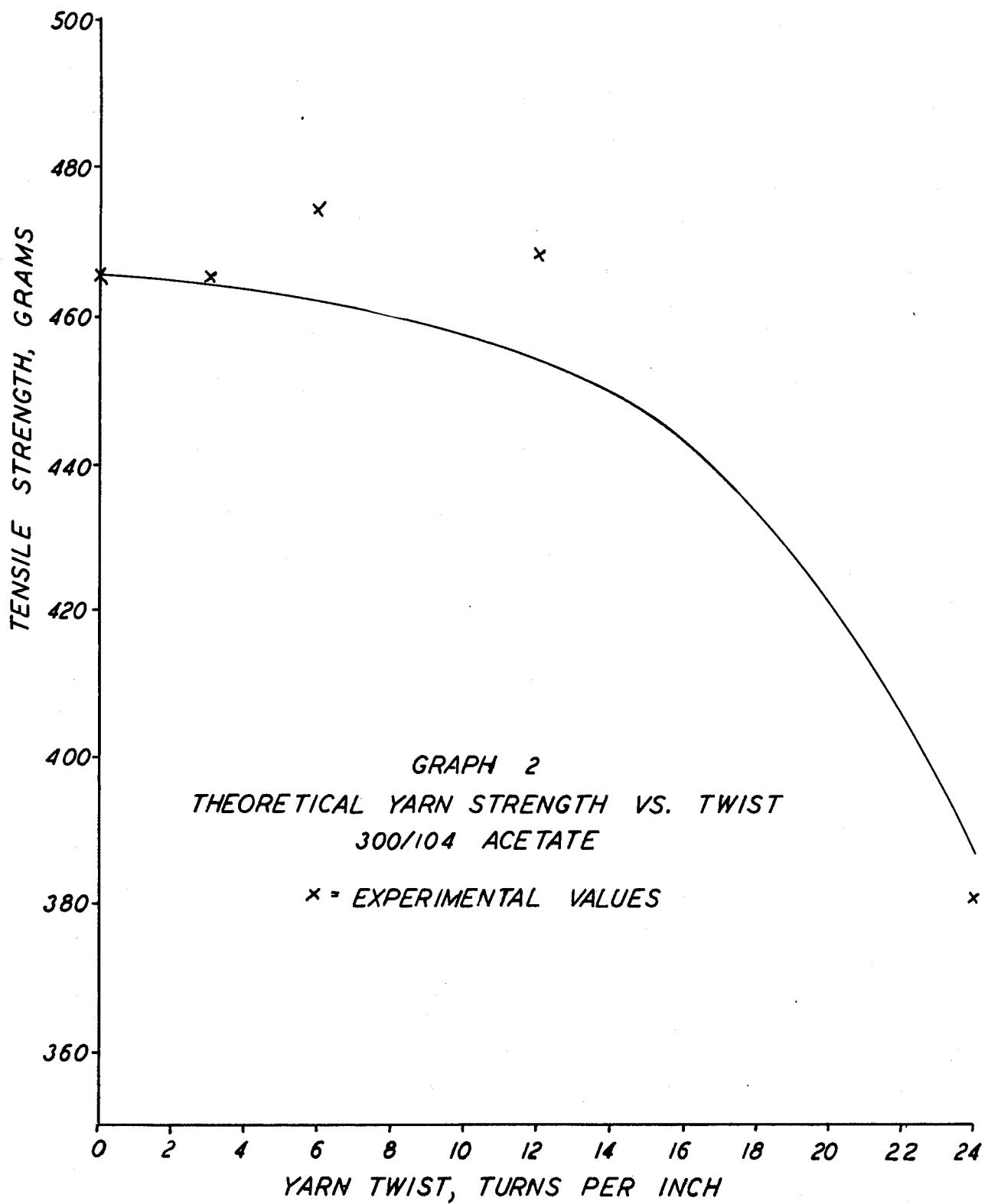
Influence of Fiber Properties

It has been indicated that the shape of the stress-strain curve is important in determining the magnitude of the serigraph effect. Each of the equations given in Table I, excepting that for nylon,
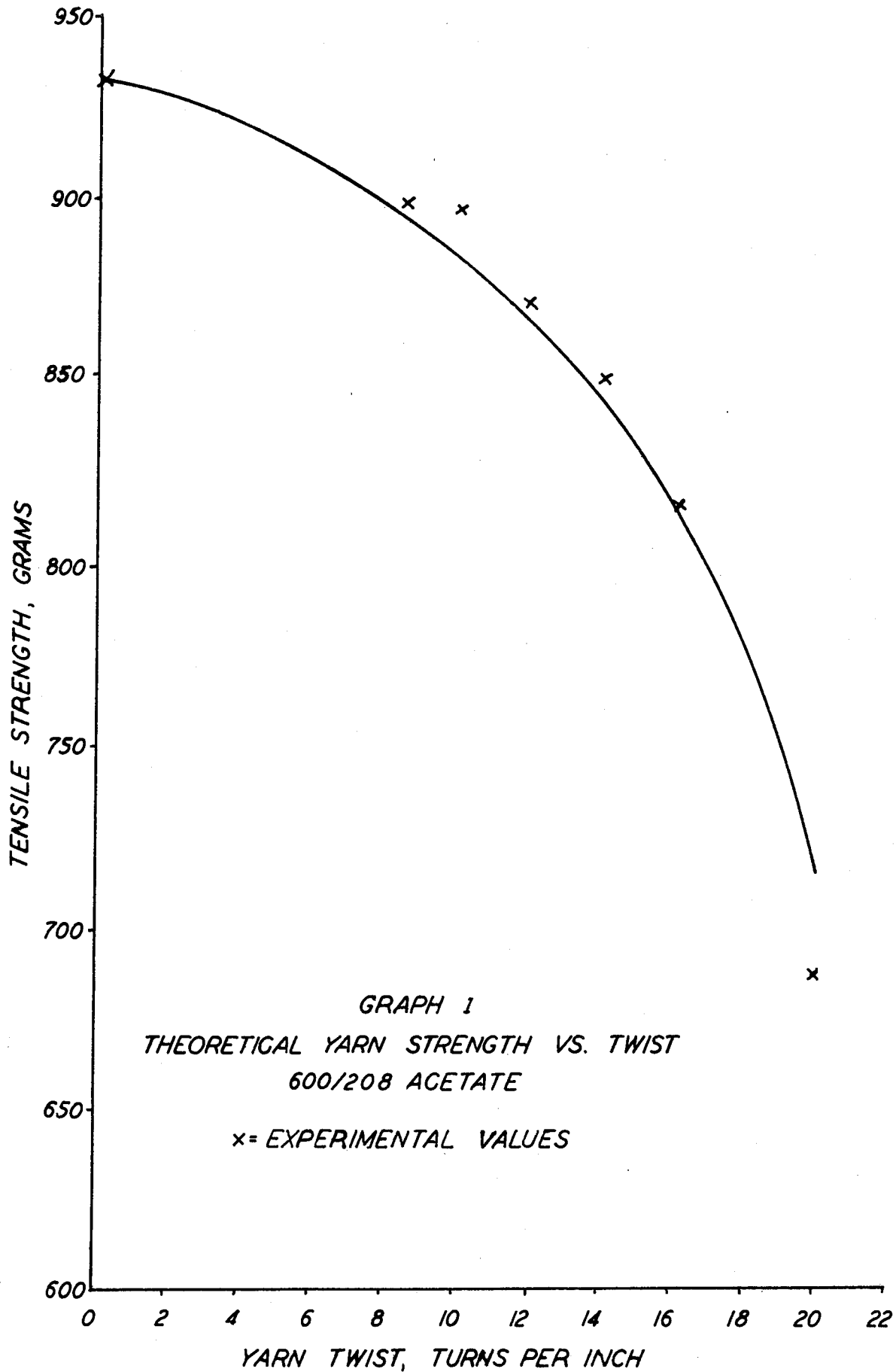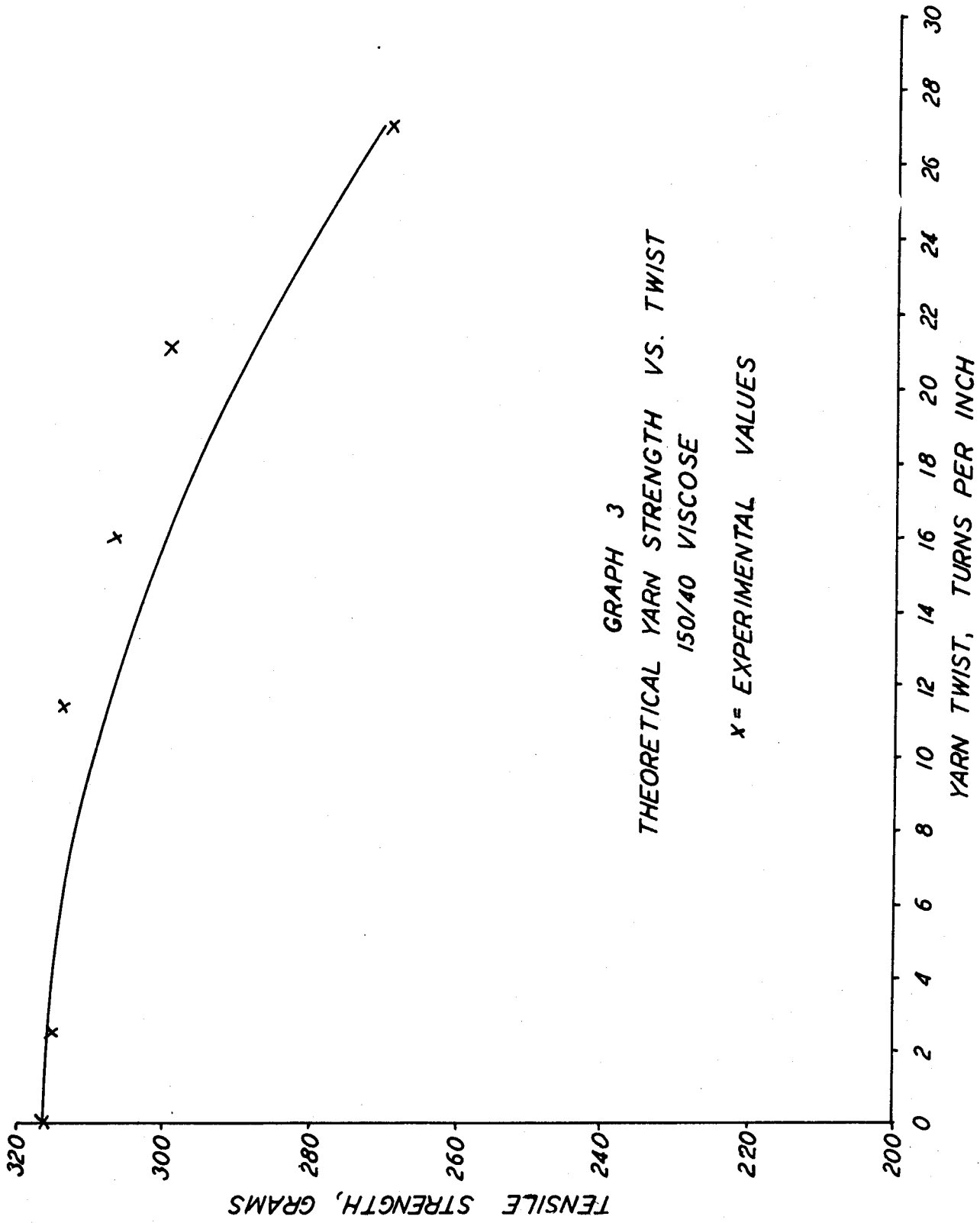
GRAPH 1
THEORETICAL YARN STRENGTH VS. TWIST
600/208 ACETATE

x = EXPERIMENTAL VALUES

YARN TWIST, TURNS PER INCH

GRAPH 2
THEORETICAL YARN STRENGTH VS. TWIST
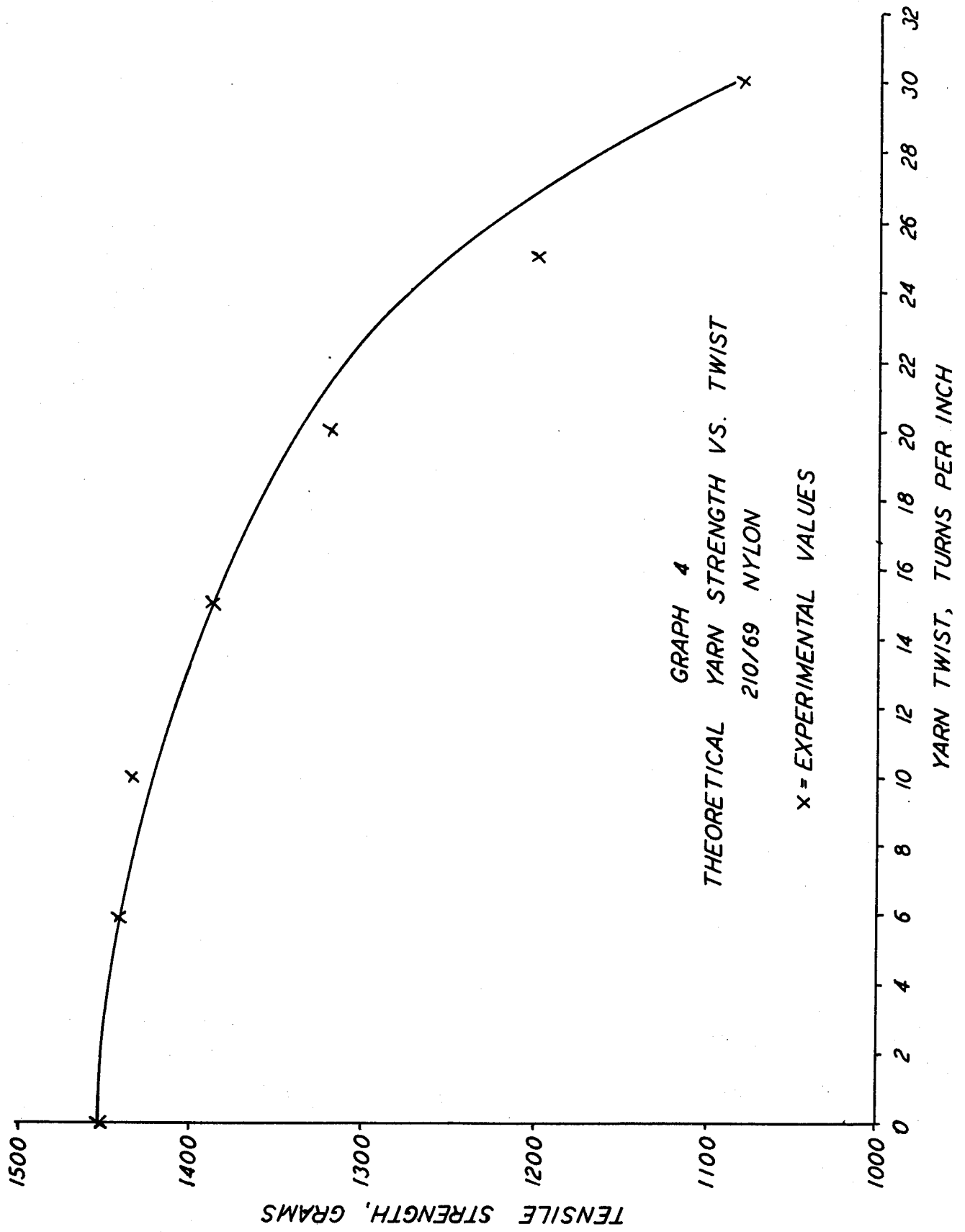300/104 ACETATE

× = EXPERIMENTAL VALUES

GRAPH 3

THEORETICAL YARN STRENGTH VS. TWIST

150/40 VISCOSE

x = EXPERIMENTAL VALUES

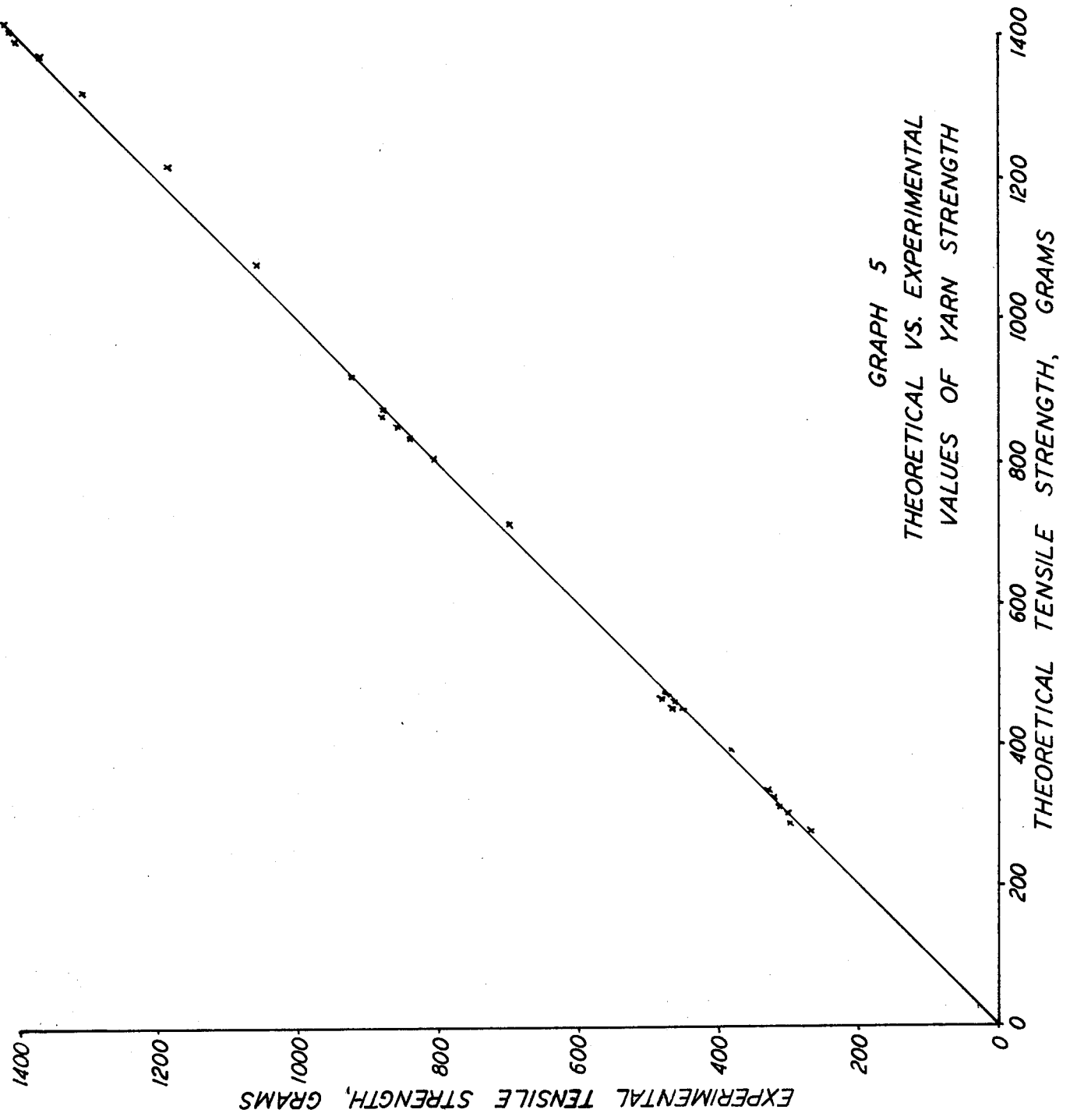YARN TWIST, TURNS PER INCH

TENSILE STRENGTH, GRAMS

GRAPH 4

THEORETICAL YARN STRENGTH VS. TWIST
210/69 NYLON

x = EXPERIMENTAL VALUES

YARN TWIST, TURNS PER INCH

TENSILE STRENGTH, GRAMS

GRAPH 5

THEORETICAL VS. EXPERIMENTAL

VALUES OF YARN STRENGTH

can be written in the form:

$$P = a - b\ N^2R^2 \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-} \ \text{-}\ (21)$$

where a represents the zero twist strength, while N and R are the yarn twist and yarn radius at rupture, respectively. The quantity b determines the extent to which a given geometry affects yarn strength. Actually, the ratio of b/a determines this extent. It can be shown that this ratio is functional with the stiffness of the fiber at high loads, as evidenced by the shape of the stress-strain diagram. Consider the hypothetical stress-strain diagrams of two fibers, A and B, as shown in Figure 9. Let it be assumed that both fibers A and B possess precisely equal properties, both geometric and inherent, except that the shapes of their stress-strain diagrams are different. Then, if yarns of similar geometry are made from each of these fibers, it is clear that the component effects would be alike. However, with respect to the serigraph effect, it is clear from Figure 9 that although at yarn rupture the center fibers in both yarns would be supporting equal stresses, $f_r$, the inclined fibers in yarn A would be supporting stresses $(f_\theta)_A$ while the inclined fibers in yarn B are subjected to stresses as low as $(f_\theta)_B$. From Figure 9 it can be seen that that the strength of yarn A will exceed the strength of yarn B.

In this connection, it is interesting to consider the work of Peirce[4] in discussing the loss in strength of cotton yarns after the optimum twist is exceeded. The general equation for strength fall-off is given by Peirce as the factor:

$$\frac{1}{1 + 0.0126\eta\ K^2}$$

where $\eta$ is a constant depending on zone of rupture diameter and yarn diameter, and K is the yarn twist multiplier, proportional to N R. This factor may be approximated as:

$$1 - a(0.0126\ \eta\ N^2R^2)$$

where a is the constant of proportionality relating twist multiplier to the product of yarn twist and yarn radius. This last equation expresses the influence of only the component effect. Actual experimental results check the above equation if the value of 0.0126 is replaced by 0.0140, which, according to Peirce, "is of the right order of magnitude but greater than would be expected of the purely geometric effect in a regular yarn."
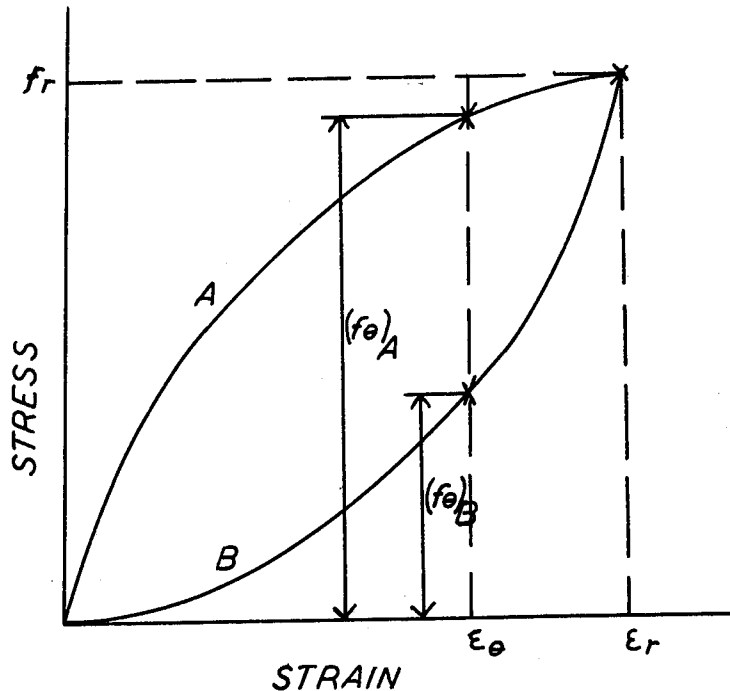
*FIGURE 9*

*HYPOTHETICAL STRESS-STRAIN DIAGRAMS OF TWO MATERIALS.*

It is clear, however, from what has been discussed already, that the value of 0.0140 could be significantly associated with the serigraph effect, since this factor has the ultimate result of producing a lower strength than would be expected from only the component effect. The elastic properties of the fiber can influence the effect of yarn geometry.

It is interesting to consider the effect of increasing the yarn size while maintaining the yarn twist (turns per inch) constant. Equation (21) can be written in more general terms to take into consideration differences in initial yarn radius:

$$P = R^2(a' - b'N^2R^2) - - - - - - - - - - - (22)$$

The maximum value of P produced by varying R can be calculated from equation (22) by:

$$\frac{\partial P}{\partial R} = 0$$

or:

$$\frac{\partial P}{\partial R} = 2a'R - 4b'N^2R^3 = 0$$

$$\therefore R = 0 \text{ (zero strength)}$$

or:

$$N^2 R^2 = \frac{a'}{2b'}$$

$$NR = \sqrt{\frac{a'}{2b'}} \quad \text{-- -- -- -- -- -- -- -- -- -- --} \quad (23)$$

Equation (23) indicates that for a given fiber (a' and b' are material constants), yarn strength at a given twist can be increased by increasing yarn weight up to the point where the product of yarn twist and yarn radius satisfy equation (23). Further increases in yarn weight will result in a decreased strength. Moreover, independent of what yarn twist is considered, the product of this yarn twist, N, and yarn radius, R, will be the same at the maximum strength point. The value (N x R) can be shown to be proportional to the "twist multiplier" as used in staple yarn terminology)* since the yarn radius varies inversely as the square root of the yarn count (cotton system).

The effect of yarn size on yarn strength at different twists is indicated graphically in Figure 10.

## Yarn Stress-Strain Characteristics

### A. Elongation to Rupture

In the discussion relating to the magnitude of the "Serigraph Effect" it was shown that at yarn rupture the strains present among the various fibers in a yarn varied with the helix angle. Yarn rupture occurred at the point where the shortest fibers, i.e., those fibers which lie close to the yarn axis and hence possess zero helix angle, reach their limiting extension. This limiting extension is also the extension of the yarn to rupture. The effect of yarn twist or yarn size is merely to alter the distribution of extensions among the various fibers.

Since the fibers close to the yarn axis are the first to fail as a result of tension, it can be concluded that the yarn elongation to rupture should be constant, independent of the yarn twist or yarn size. It is true that exceedingly high yarn twists can produce initial spinning tensions in the fibers which are of sufficient magnitude to alter their elastic properties by, say, cold-working the fiber. Such a yarn would not be expected to exhibit the same elongation to rupture as a low twist yarn. However, where spinning does not influence the elastic properties of the fiber, the yarn elongation should not be altered by changing either the yarn twist or yarn size.

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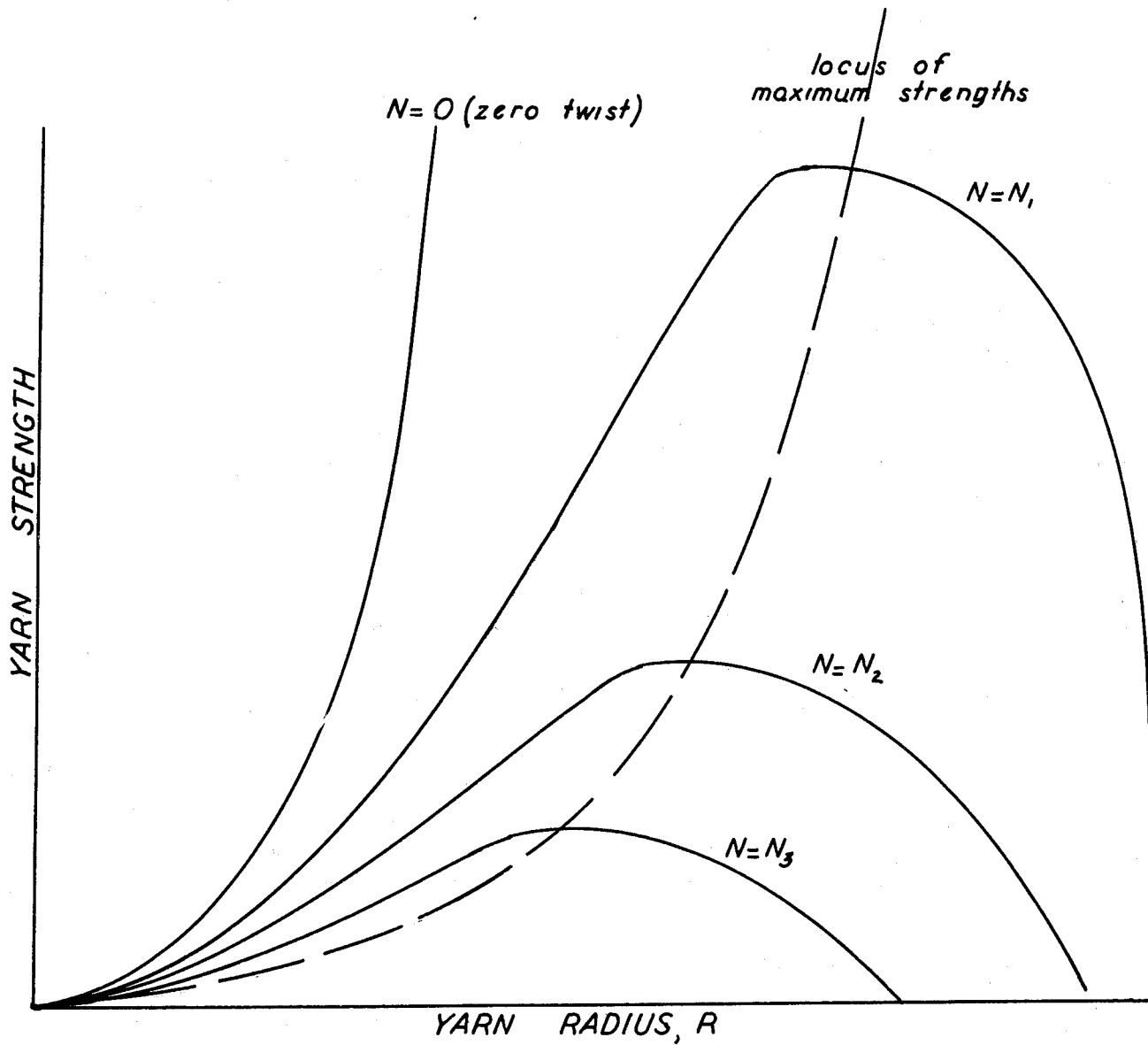* Twist Multiplier = $\dfrac{N}{\sqrt{C}}$, where C, the cotton count, varies inversely as the yarn denier.

FIGURE 10

EFFECT OF YARN SIZE (R) ON YARN STRENGTH
AT VARIOUS TWISTS, N.

$N_0 < N_1 < N_2 < N_3$

LOCUS OF MAXIMUM STRENGTHS SUCH THAT $N \times R$
IS CONSTANT.

Graph 6 plots the elongation to rupture of a series of singles yarns with various twists and yarn weights. It can be seen from this Graph that changing the twist has not produced any substantial changes in the elongation to rupture. In addition, for the two acetate yarns, the yarn elongation is substantially the same as the fiber elongation.

B. **Stress-Strain Curves**

It has been shown that twist produces the following effects on the properties of continuous filament singles yarns:

1. The higher the twist, the lower the yarn strength.

2. Yarn elongation is independent of yarn twist just so long as spinning does not alter the mechanical properties of the fibers.
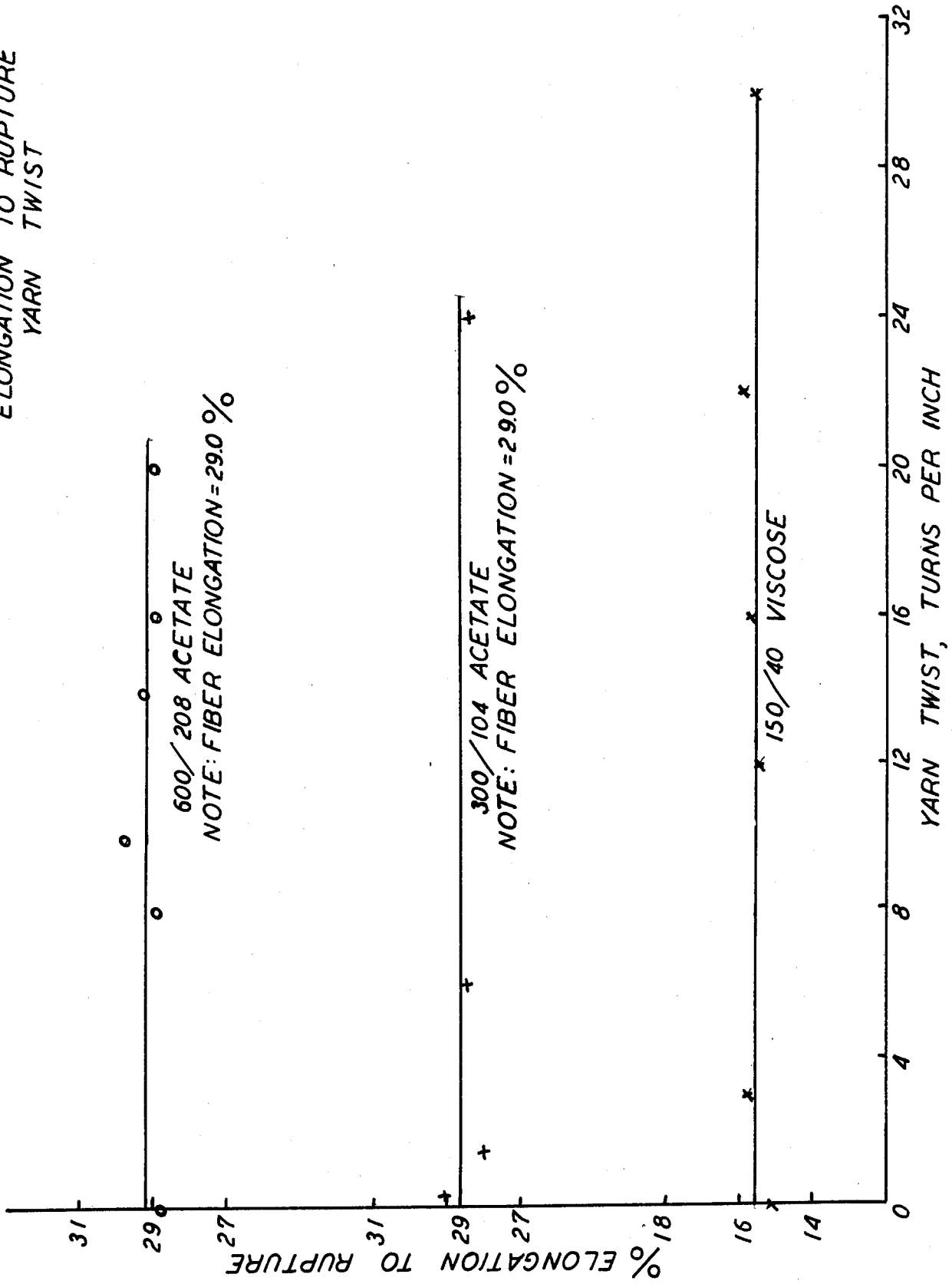
Figure 11 shows the theoretical relationship that might be expected between the stress-strain diagrams of a group of yarns with successively higher twists. These curves have been drawn so as to satisfy (1) and (2) above. In addition, however, they have been drawn so that at any load below rupture, the elongation of the higher twist yarns exceeds that of the lower twist yarns. This must be so since, for the same total load on the yarns, the fibers in the high twist yarns are subjected to a higher axial tension than are those in a lower twist yarn. Both the component and serigraph effects produce this difference in fiber loads. Since higher fiber tensions produce greater fiber extensions, the trends shown in Figure 11 are logical.

The change in shape as well as the change in endpoints of such curves, as shown in Figure 11, are capable of mathematical analysis and prediction. The procedure is a laborious one involving first the determination of the empirical formula of the fiber (or zero twist yarns) stress-strain curve. The component and serigraph effects are then written analytically for the whole range of loads from zero to rupture, in terms of yarn twist, N, and yarn size, R. The final expression would be the stress-strain equation for a given material in terms of both N and R.

However, it is possible to deduce the general results of such an involved calculation by reference to Figure 11:

1. The higher the yarn twist, the lower either the secant modulus or tangent modulus of the stress-strain curve.

2. The higher the yarn twist, the lower the energy absorption to rupture. This is so since the stress-strain curve for a high twist yarn lies below that of a low twist yarn and the ultimate elongations are equal.

GRAPH 6

ELONGATION TO RUPTURE vs.
YARN TWIST

600/208 ACETATE
NOTE: FIBER ELONGATION = 29.0 %

300/104 ACETATE
NOTE: FIBER ELONGATION = 29.0 %

150/40 VISCOSE

YARN TWIST, TURNS PER INCH

0    4    8    12    16    20    24    28    32

% ELONGATION TO RUPTURE
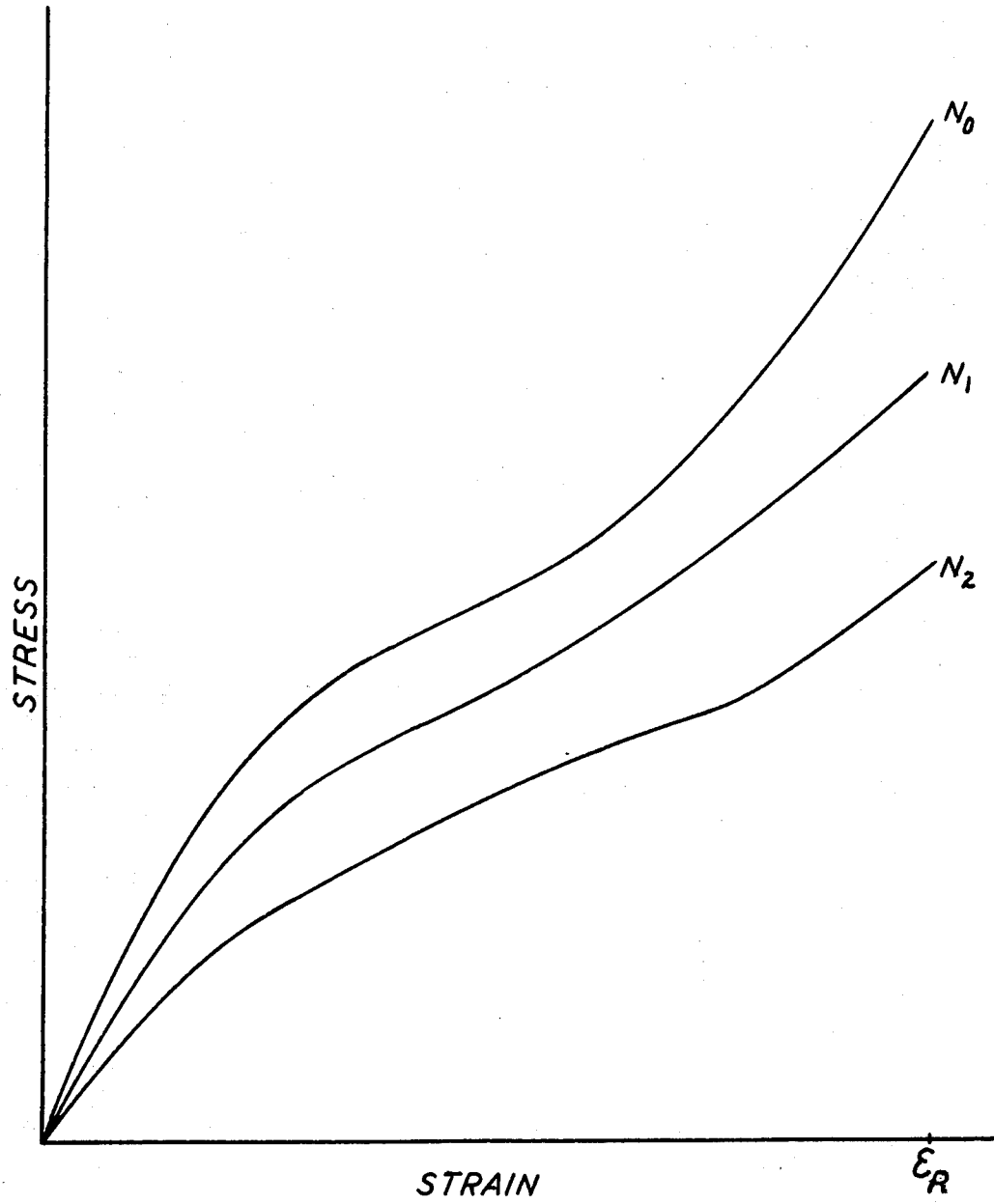
31   29   27   31   29   27   18   16   14

FIGURE II
THEORETICAL EFFECT OF YARN TWIST, N
ON STRESS STRAIN DIAGRAMS

$$N_2 > N_1 > N_0$$

## C. Elastic Characteristics of Yarns

The performance of textile structures in end use is very rarely a one-time loading to rupture proposition. Instead, most textile products are usually subjected to a continual repeated stress action, the magnitude of the stresses being below that rupture load which the specimen is capable of withstanding, previous to use. Mechanical failure of a material in end use occurs when, as a result of repeated loading and unloading, the structure is incapable of absorbing and returning the imparted energy without the occurrence of failure as either a permanent deformation or an actual rupture of the component parts, depending upon the end-use requirement. One peculiar physical characteristic of textile materials which heretofore has identified them as unpredictable engineering materials, has been the fact that the elastic properties depend upon the previous loading history. Most textiles exhibit the delayed components of deflection; primary and secondary creep, the magnitude of each being a function of load, time, and inherent properties.

More specifically than actual load, the internal stresses govern the magnitude of the delayed deflections for a given material, and since the stress distribution of filamentous singles yarns changes with twist, it is to be expected that the repeated stress properties would also depend upon twist. Three criteria of repeated stress performance have been considered: (a) secondary creep, (b) corrected residual elongation, and (c) elastic performance.

The data given in this section on elastic properties have already been reported by Hamburger.[5] The graphs expressing the data are repeated for convenience in following the discussion.

### 1. Secondary Creep

Secondary creep is that portion of the delayed deflection exhibited by a material which is not recoverable with time following load removal. The removal of secondary creep results in a change in the load-extension diagram, with an attendant loss in energy absorption capacity. It has been found that poor abrasion resistance characterizes a material most of whose deflection is attributable to secondary creep. Both poor crease resistance and poor dimensional stability also follow when most of the elongation possessed by a material can be ascribed to secondary creep. On the other hand, in order to resist rupture due to stress concentrations, secondary creep can often be a valuable asset to a material. Hence, it can easily be seen that the determinations of this property is important to a textile evaluation.

The effect of both load and twist on the secondary creep of 300/104 continuous filament acetate rayon is given in

GRAPH 7
% SECONDARY CREEP vs. YARN TWIST
300/104 ACETATE
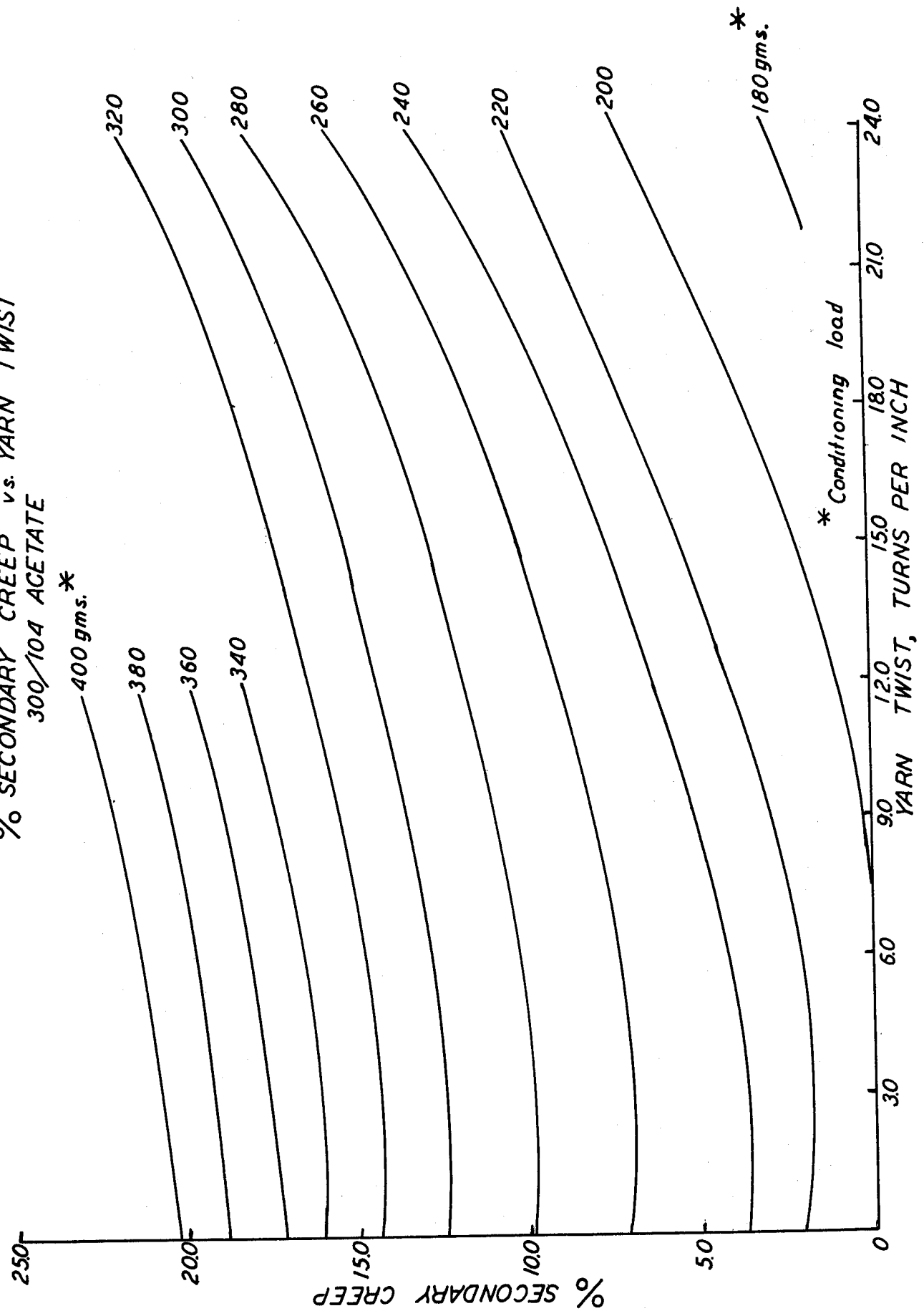
Graph 7, which illustrates two points: (1) the secondary creep is a function of conditioning load, the greater the load acting on the untwisted elements, the greater the secondary creep; (2) at a given conditioning load, the greater the twist, the greater the secondary creep.

Point (1) states an inherent property of acetate rayon. Other materials may or may not possess a similar characteristic. For example, nylon shows a much smaller change in secondary creep with load than does acetate rayon. However, considering acetate rayon, it becomes apparent why twisting increases the secondary creep at a given conditioning load. For a given yarn load, the internal filament forces are greater for a twisted than for an untwisted yarn. This has been explained as being caused by both the component and serigraph effects. Since secondary creep increases with load on an untwisted yarn, the increase in secondary creep with twist at a given yarn load is explained. For the nylon mentioned above, one should find secondary creep to be less influenced by yarn twist than the acetate rayon. Except for such effects as yarn slippage due to twist removal, this should certainly be the case.
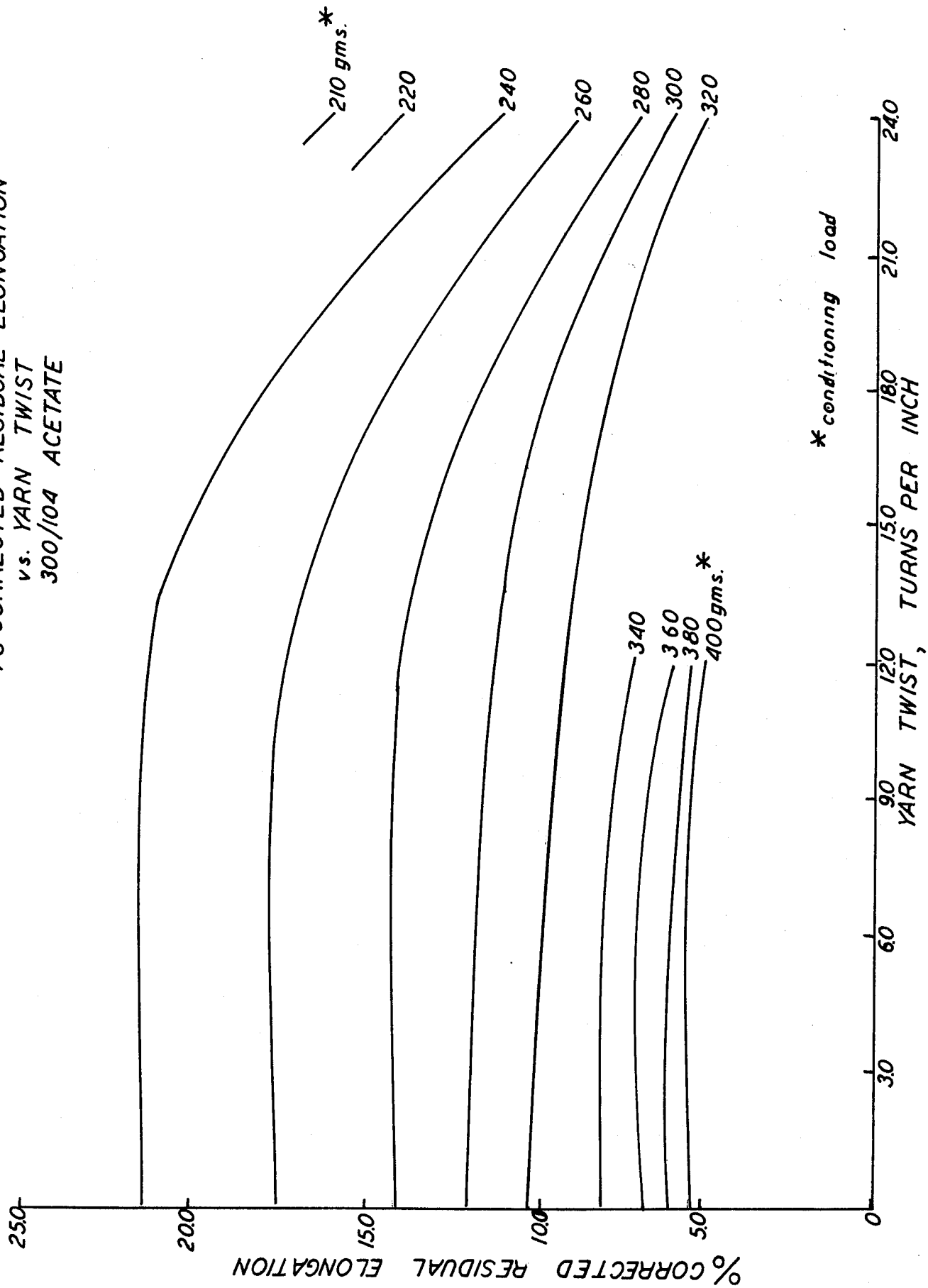
2. Corrected Residual Elongation

If a material possesses secondary creep, the process of mechanical conditioning generally causes a decrease in the extensibility of the specimen. It is of value to know the extensibility of such a mechanically conditioned material, since a large decrease in the elongation after conditioning reflects a large decrease in the energy-absorbing capacity of the material following repeated stressing in use. A material may show a high elongation to rupture on a one-time loading basis, and may be selected for a particular end use whenever such a large elongation is desirable. However, if this elongation is not attainable again, and the end-use requirement is not a one-time use, the choice of this material may not have been the best.

The per cent corrected residual elongation delineates the elongation to rupture following the removal of secondary creep. It is termed "residual" since it represents the remaining elongation. The expression "per cent corrected" is used since the elongation is expressed as a per cent of the new gage length, i.e., the sum of the original gage length before stressing plus the added gage length due to secondary creep removal.

Dependent as it is upon secondary creep removal, the per cent corrected residual elongation should be expected to vary with both load and twist. Graph 8 plots the effects of load and twist on the per cent corrected residual elongation.

GRAPH 8
% CORRECTED RESIDUAL ELONGATION
vs. YARN TWIST
300/104 ACETATE

210 gms.*
220
240
260
280
300
320

340
360
380
400 gms.*

*conditioning load

YARN TWIST, TURNS PER INCH

% CORRECTED RESIDUAL ELONGATION

Two points are worthy of note from examination of this Graph:

(1) Inherently, acetate rayon shows a decrease in the corrected residual elongation with load. This decrease parallels the increase in secondary creep with load.

(2) At a given load, the greater the twist, the less the per cent corrected residual elongation. The explanation for this effect is similar to that given for the increase in secondary creep with twist at a given load.

It is interesting to note that the shapes of all the curves in Graphs 7 and 8 are essentially parabolic with twist at constant load. This same general shape resulted for the effects of twist on tensile strength. The deviation of the stress distribution from uniformity, as twist changes, is also parabolic with twist. Hence, the tie-in between stress distribution, as produced by yarn geometry, and repeated stress performance appears justified.

### 3. Elastic Performance

The preceding paragraphs have discussed certain of the changes in elastic properties exhibited by textiles following repeated stressing. The extent to which a material duplicates its original stress-strain characteristics, taking into account the effects of both primary and secondary creep, is indicated by the Elastic Performance Coefficient. This coefficient can be applied to all types of loading, tension, bending, torsion, bulk compression, etc., and is determined from the repeated stress data. The method for calculation has been described by Hamburger in "Mechanics of Elastic Performance."(5) Qualitatively, the coefficient should be regarded as a numerical index which expresses, as a fraction of unity, the degree of perfect spring-like behaviour exhibited by a material.
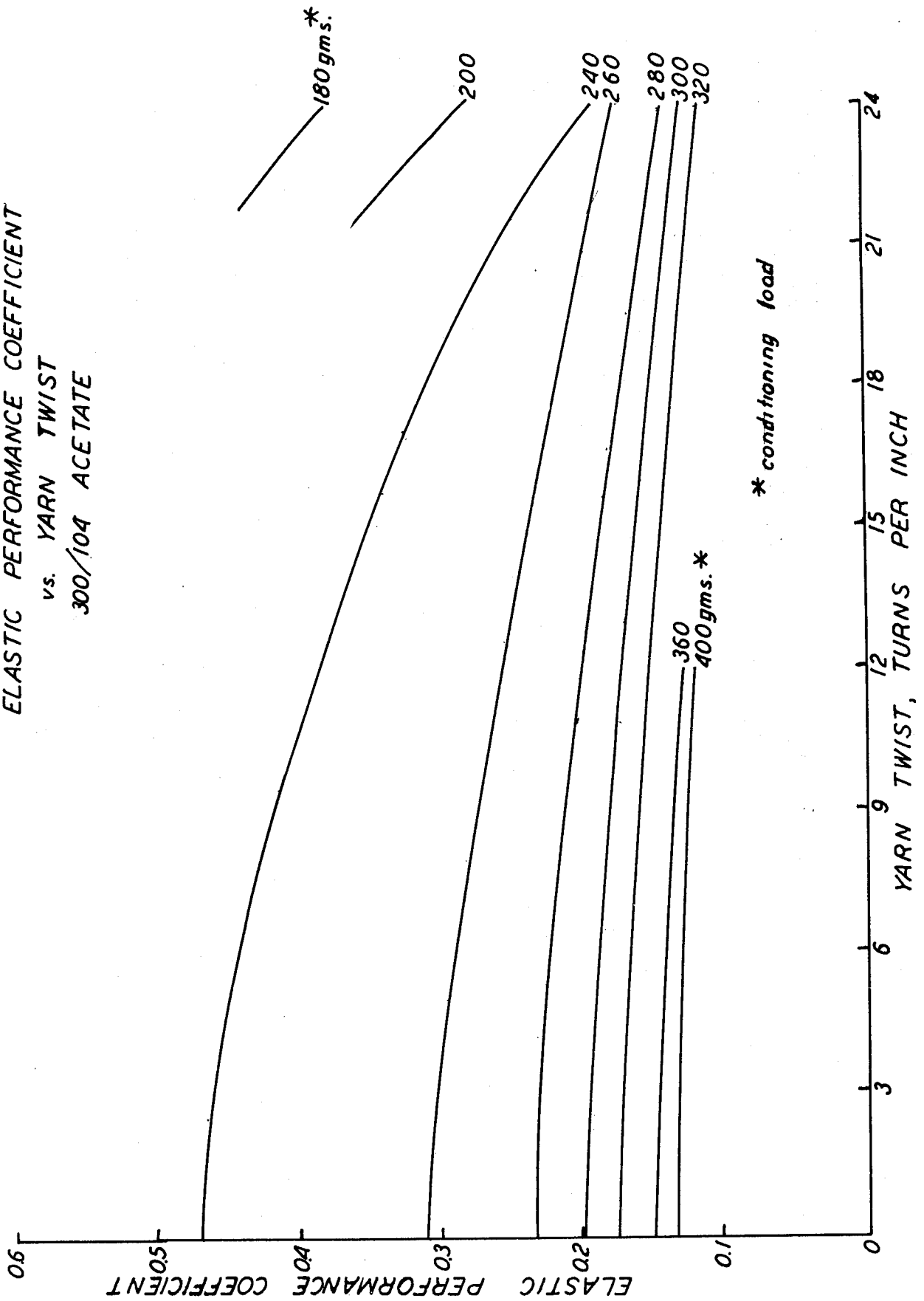
Graph 9 plots values of the Elastic Performance Coefficient as a function of both load and twist. Once again, it is clear that two predominant effects are evidenced:

(1) The higher the load, the smaller the elastic performance coefficient.

(2) At a given load, the greater the twist, the poorer the elastic performance.

Also, it should be apparent that (2) above depends upon the inherent property indicated by (1), since were it not for the fact that higher loads reduce the elastic performance,

GRAPH 9
ELASTIC PERFORMANCE COEFFICIENT
vs. YARN TWIST
300/104 ACETATE

180 gms. *
200
240
260
280
300
320
360
400 gms. *

* conditioning load

YARN TWIST, TURNS PER INCH

ELASTIC PERFORMANCE COEFFICIENT

49 15634

the effects of higher filament stresses in highly twisted yarns would not produce reductions in the elastic performance coefficient.

## Conclusion

The concepts developed in this paper represent only a part of the type of quantitative "applied mechanics" of textile structures, the complete establishment of which is the overall long-range objective of the research being done for the Quartermaster Corps. The work has been concerned with the development of rational laws describing the influence of geometrical factors on textile performance, beginning with the smallest textile units with respect to size but the most complex from a technical point of view, namely, the fiber and the yarn. It is felt that the yarn form factors isolated and described in the main body of this paper form a necessary part of the logical development of geometrical effects, and just as fiber and yarns are the fundamental units in the construction of a textile product, so must their mechanical properties be regarded in relation to the mechanical properties of the finished structure. It has been shown that stress analysis permits quantitative explanation and prediction of the following effects:

1. Rupture properties of singles continuous filament yarns. Both rupture load and rupture elongation are capable of physical analysis and correlation with the properties of the fiber.

2. Energy absorption. The shape of the stress-strain diagram of yarns can be quantitatively predicted from the fiber stress-strain diagram and the influence of yarn geometry upon the energy absorption properties determined.

3. The dependency of the elastic properties of yarns upon yarn geometry has been described and explained. In particular, it has been shown how stress distribution is responsible for changes in the visco-elastic characteristics exhibited by yarns comprised of known fibers and how these changes may be calculated.

4. Optimum yarn sizes for different twist yarns have been shown to be capable of quantitative determination. Rational laws governing the relation between yarn size and twist have been developed.

5. It has been shown that the extent to which yarn geometry influences yarn properties is partly functional with the inherent fiber properties. In particular, it has been shown that the rupture properties can be described in terms of constants of the fiber as determined by its stress-strain diagram. Future work on this program is concerned with the determination and classification of the constants for a wide variety of textile fibers.

In addition to yielding information on the performance of singles, continuous filament yarns, the broad ideas developed have led to the extension of certain of the concepts to other higher order textile structures, the results of which will be published in later works, but will be briefly mentioned here.

1. Similar performance characteristics of staple fiber yarns.(6) It has been found that when fiber slippage is properly taken into account, many of the properties of singles staple fiber yarns are capable of evaluation by the same general methods as presented herein. The influence of staple-fiber yarn geometry on the translation of strength, elongation, and visco-elastic characteristics are all capable of analytic description.

2. Characteristics of plied continuous filament yarns. It can be shown that the elastic characteristics of plied yarns, for example, are critically dependent upon the presence or absence of a core which lies in the center of the yarn. For example, for a seven-ply yarn, one can show that increase in ply twist, maintaining yarn twist as it lies in the ply constant, causes a parabolic decrease in the energy to rupture. On the other hand, for a three-ply yarn, increases in the ply twist would show a parabolic increase in energy absorption to rupture.

3. Certain of the methods developed in this work are capable of being translated directly into fabric geometry effects. For example, the effects of high fabric crimp in tightly woven fabrics, both on tensile strength and elongation, are capable of being analysed in much the same manner as given for the serigraph effect.

From the examples it is clear that the mechanical properties of textile structures are capable of being analyzed and explained, and hence, predicted. The ability to blueprint and design fabrics for mechanical end uses is possible. It is felt that other end uses dependent upon the so-called "obscure" properties of textiles, such as wear resistance, drape, etc., are also capable of engineering design, once the physical analogy of the tactile property is established.

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