NAIC-ID(RS)T-0125-96

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by

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NAIC-ID(RS)T-0125-96

HUMAN TRANSLATION

NAIC-ID(RS)T-0125-96 11 June 1996

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English pages: 12

Source: Zhongguo Kongjian Kexue Jishu (Chinese Space Science and Technology), Vol. 14, Nr. 3, 1994; pp. 17-24

Country of origin: China
Translated by: SCITRAN
F33657-84-D-0165
Requester: NAIC/TASC/John L. Gass
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MATHEMATICAL PROCESSING OF RANGE AND RANGE RATE TRACKING DATA

Wang Zhengming Zhou Haiyin

ABSTRACT Employing radar or optical equipment to carry out tracking measurements with regard to spacecraft, it is possible to obtain observation data associated with ranges and their rates of change. Due to causes in many areas such as measurement equipment precision, environmental conditions, and so on, these measurement data include errors in all cases. Besides random errors associated with range finding and range change rates, there are also systemic errors associted with range finding. Through the setting up of appropriate mathematical models, spline functions and regression analysis methods are employed to give a type of method for estimating range, range rate, and range finding system errors. The methods in question are very easy to realize and calculate. Theoretical analysis and simulation calculations clearly show that the methods in question possess very high accuracy.

SUBJECT TERMS Spacecraft tracking Linear system Spline function Error analysis

1 INTRODUCTION

No matter what type of measurement system is used, carrying out tracking measurements with regard to spacecraft requires, in all cases, measurements [1] of ranges and their rates of change. Because of many types of complicated reasons, as far as measurement values are concerned--besides containing random errors--range finding also includes system errors[1-3]. These errors severely influence--even to the point of distorting--the true image of the facts. Also, because measurements are often indirect (for example, MISTRAM systems), when calculating parameter data which is necessary for use--during transmission processes determined by calculation formulae--errors are often enlarged from several times to several tens of times. This is not permissible in a number of situations where there is a need for high precision measurement data, for example, those in such things as missile accuracy analyses, spacecraft orbital predictions and so on [4].

With regard to making use of measurement data to give high precision estimates of ranges and their rates of change, systemic errors associated with estimated range finding are problems jointly concerned with measurement units (precisions of equipment concerned) and data utilization units (concerned with accuracies of range measurements and their rates of change).

This article considers problems in launch coordinate systems. We, first of all, demonstrate that--with respect to spacecraft in stable flight configurations (no stage separations)--observed ranges and their rates of change are capable, in all cases, of being expressed by the use of a cubic spline and its derived functions. On the basis of system error models--making use of spline function theory and regression analysis methods and through the setting up an appropriate mathematical model--mathematical methods are given which are not only able to estimate ranges and their rates of change but are also able to estimate range finding system errors. Theoretical analysis and simulation calculations both clearly show that the methods in question have trimmed errors which are small, and calculations are very convenient.

2 SPLINE FUNCTION DESCRIPTIONS ASSOCIATED WITH RANGES AND THEIR RATES OF CHANGE

Assuming oxyz to be the launch coordinate system, spacecraft orbit parameters are x(t), y(t), z(t), $\dot{x}(t),\dot{y}(t),\dot{z}(t)$. Here, x(t), y(t), z(t) express the position of spacecraft at

instant t. $\dot{x}(t),\dot{y}(t),\dot{z}(t)$ is the speed of spacecraft at

instant t . This article adds dots to the top of dependent /18 variables to stand for the derivatives of the functions in question with respect to time. The number of dots on top stands for the order of the derivative.

Assuming that the coordinates of observation stations in launch coordinate systems are (xo,yo,zo), then, observed ranges are:

$$R(t)=\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2} \tag{1}$$

Assuming that xt,y(t),z(t) are all fourth order continuous differentiable functions, then, from equation (1), it is possible to deduce that R(t) is also a fourth order differentiable function. Making use of x(t),y(t),z(t) as well as the first, second, and third order derivatives of special points, direct solution is made with respect to R(t). It is possible to demonstrate that R(4)(t) is a very small number.

Below, we study data processing problems associated with sections where time length is [To,Tm]. Using subscripts To=t1, Tm=tn, Tj=To+h• j, j=-1,0...,m+1. $\| R^{(4)}(t) \|_\infty = \max\limits_{T_0<t<T_m} |R^{(4)}(t)|$.

Using Sj(t) to represent the jth spline, Bj(t) records the jth standard B spline.

Lemma 1 [5] Assuming R(t)∈C4[To,Tm] and S(t) satisfies the conditions

$$|R(t)-S_3(t)|\leqslant\frac{5}{384}\| R^{(4)}(t) \|_\infty\cdot h^4,\quad |\dot R(t)-\dot S_3(t)|\leqslant\frac{1}{24}\| R^{(4)}(t) \|_\infty\cdot h^3 \tag{2}$$

of cubic interpolative spline functions, one then has

$$\begin{cases}\dot S_3(T_0)=\dot R(T_0), & \dot S_3(T_m)=\dot R(T_m)\\ \dot S_3(T_j)=R(T_j), & j=0,1,\cdots,m\end{cases}$$

The first order and third order standard B splines which this article uses are respectively defined as:

$$B_1(\tau)=\begin{cases}0, & |\tau|\geqslant 1\\ 1-|\tau|, & |\tau|<1\end{cases}$$

$$B_3(\tau)=\begin{cases}0, & |\tau|\geqslant 2,\\ \dfrac{|\tau|^3}{2}-\tau^2+\dfrac{2}{3}, & |\tau|\leqslant 1\\ -\dfrac{|\tau|^3}{6}+\tau^2-2|\tau|+\dfrac{4}{3}, & 1<|\tau|<2\end{cases}$$

Lemma 2 There exists a unique set of coefficients (b-1, bo, b1,..., bm+1) so that

$$S_3(t)=\sum_{j=-1}^{m+1} b_j B_3 \frac{(t-T_j)}{h} \tag{3}$$

satisfies condition (2).

Synthesizing the two lemma above, it is possible to obtain

Theorem 1 During the normal flight phase of spacecraft (refers to phase with no stage separation), tracking range R(t) of spacecraft and the rate of change $R(t)$ can be described by the use of cubic spline function S3(t) as well as the derivative $\dot{S}_3(t)$. So long as m is sufficiently large, making h=(tn-t1)/m sufficiently small, it is then possible to guarantee trimmed errors being sufficiently small. In this, S3(t) is given by equation (3).

3 MATHEMATICAL MODELS OF OBSERVATION DATA

Observation data associated with R(t) can be represented as

$$y_i=R(t_i)+C(t_i)+\varepsilon_i \tag{4}$$

In this, R(ti) is the true value of R(t) for instant ti ; C(ti) is the observation system error associated with range finding at instant ti ; t_i is random error. /19

$\dot{R}(t)$ observation data can be represented as

$$p_i=\dot{R}(t_i)+\delta_i \tag{5}$$

In this, $\dot{R}(t_i)$ is the true value of $\dot{R}(t)$ at instant ti; δi is random error associated with speed measurements.

Below, we take problems associated with R(t), $\dot{R}(t)$, and C(t) and turn them into parameter estimation problems.

As far as the consideration of problems within the time phase [To,Tm] is concerned, assuming that ti is the observation instant, k is the number of observation iterations each second, and ti=To+(i-1)/k, i=1,2,...,n, then,

Tj=tkhj+1, j=0,1,...,m; n=khm+1

On the basis of engineering background (see references [1-3]), within [To,Tm], C(t) can be described by the use of first order spline number S1(t):

(6)

From discussions in 2, we already have

$$R(t) \approx S_3(t) = \sum_{j=-1}^{m+1} b_j B_3\left(\frac{t-T_j}{h}\right), \quad \dot{R}(t) \approx \dot{S}_3(t) = \sum_{j=-1}^{m+1} \frac{b_j}{h} \dot{B}_3\left(\frac{t-T_j}{h}\right) \qquad (7)$$

Because of this, from equations (4) ~ (7), it is possible to obtain

$$\left.\begin{aligned} y_i &= \sum_{j=-1}^{m+1} b_j B_3\left(\frac{t-T_j}{h}\right) + \sum_{j=0}^{m} d_j B_1\left(\frac{t-T_j}{h}\right) + \varepsilon_i \\ p_i &= \sum_{j=-1}^{m+1} \frac{b_j}{h} \dot{B}_3\left(\frac{t-T_j}{h}\right) + \delta_i, \quad i=1,2,\cdots,n \end{aligned}\right\} \qquad (8)$$

If we are able to obtain from equation (8) estimates for (b-1,bo,...,bm+1) as well as (do,d1,...,dm), then, we
are able to respectively obtain estimates for C(t), R(t), and R(t) from equation (6) and equation (7).

4 ESTIMATION METHODS AND ERROR ANALYSIS

It is clearly shown by theoretical analyses and simulation calculations that, during relatively stable flight phases, h can be selected as 5 ~ 10s. It is possible to guarantee that $\| S_3(t) - R(t) \|_\infty$, $\| \dot{S}_3(t) - \dot{R}(t) \|_\infty$, $\| S_1(t) - C(t) \|_\infty$,
are all very small (primarily comparatively random errors as well as being within the ranges permited by engineering background). During orbital tracking, each second, it is possible to obtain over 20 observed data. Thus, n≥100m.

On the basis of the special characteristics of measurement equipment, in relation to random errors, it is possible to assume that

$$\left.\begin{aligned} &E\varepsilon_i = 0, E\delta_i = 0, E\varepsilon_i\delta_j = 0, \quad i,j=1,2,\cdots,n \\ &E\varepsilon_i^2 = \sigma_i^2, E\delta_i^2 = \theta_i^2, E\varepsilon_i\varepsilon_j = 0, E\delta_i\delta_j = 0 \quad i \neq j \end{aligned}\right\} \qquad (9)$$

Here, σi, θi (i=1,2,...,n) are already known positive numbers. It is possible to obtain them from appraisals of measurement equipment precisions.

Below, use is made of observation data and model (8) ot give estimates of parameters (do, d1,..., dm) as well as (b-1, bo,..., bm+1).

Lemma 3 [5] Assume A is any constant. Then, A can be expressed as:

$$A=\sum_{j=-1}^{m+1} AB_3\left(\frac{t-T_j}{h}\right)=\sum_{j=0}^{m} AB_1\left(\frac{t-T_j}{h}\right),\ t\in[T_0,T_m]$$

From Lemma 3, it is possible to know that parameters which await estimation in model (8) equations are not completely independent. We take model (8) and rewrite it as /20

$$\left.\begin{aligned} y_i&=\sum_{j=-1}^{m+1}(b_j+d_0)B_3\left(\frac{t_i-T_j}{h}\right)+\sum_{j=1}^{m}(d_j-d_0)B_1\left(\frac{t_i-T_j}{h}\right)+\varepsilon_i \\ p_i&=\sum_{j=-1}^{m+1}\frac{b_j+d_0}{h}\dot{B}_3\left(\frac{t_i-T_j}{h}\right)+\delta_i \end{aligned}\right\} \tag{10}$$

Here, use is made of Lemma 2 as well as $\sum_{j=-1}^{m+1} d_0\dot{B}_3\left(\frac{t_i-T_j}{h}\right)=0$.

Note

$$a_j=b_j+d_0,\ j=-1,0,\cdots,m+1$$

$$g_j=d_j-d_0,\ j=1,2,\cdots,m$$

$$X=(x_{i,j})_{(2n)\times(2m+3)}$$

$$x_{i,j}=\begin{cases}\dfrac{1}{\sigma_i}B_3\left(\dfrac{t_i-T_{j-2}}{h}\right), & i=1,2,\cdots,n,\quad j=1,2,\cdots,m+3\\[2ex] \dfrac{1}{\theta_{i-n}}\dfrac{1}{h}\dot{B}_3\left(\dfrac{t_{i-n}-T_{j-2}}{h}\right), & i=n+1,n+2,\cdots,2n,\ j=1,2,\cdots,m+3\\[2ex] \dfrac{1}{\sigma_i}B_1\left(\dfrac{t_i-T_{j-m-3}}{h}\right), & i=1,2,\cdots,n,\ j=m+4,\cdots,2m+3\\[2ex] 0, & i=n+1,\cdots,2n,\quad j=m+4,\cdots,2m+3\end{cases}$$

$$\beta=(a_{-1},a_0,\cdots,a_{m+1},g_1,g_2,\cdots,g_m)'$$

$$Y=\left(\frac{y_1}{\sigma_1},\cdots,\frac{y_n}{\sigma_n},\ \frac{p_1}{\theta_1},\cdots,\frac{p_n}{\theta_n}\right)',\qquad \varepsilon=\left(\frac{\varepsilon_1}{\sigma_1},\cdots,\frac{\varepsilon_n}{\sigma_n},\frac{\delta_1}{\theta_1},\cdots,\frac{\delta_n}{\theta_n}\right)'$$

In this way, we obtain linear regression model:

$$\left.\begin{array}{l} Y=X\beta+\varepsilon \\ E\varepsilon=0, \mathrm{COV}(\varepsilon)=I_{2n\times 2n} \end{array}\right\} \tag{11}$$

Theorem 2 Assume that random errors associated with range finding and its rate of change ε1 and δ1 respectively satisfy the conditions of equation (9). Then, it is possible to obtain linear regression model (11). Moreover, square matrix X'X is a positively defined matix. Thus, from formula

$$\beta_{LS}=(X'X)^{-1}X'Y \tag{12}$$

it is possible to give linear unbiased estimates associated with β variances which are uniformly minimal.

Proof From equation (9), it is possible to know that the second form of equation (11) is set up. From discussions in 2, trimmed errors associated with using S3(t) to approximate R(t), using $\tilde{S}_3(t)$ to approximate $\tilde{R}(t)$ and using S1(t) to approximate C(t) can be ignored in calculations. Thus, the first form in equation (11) is set up.

If X'X is a positively defined matrix, then, by the Gauss-Markov theorem (see Reference [6]), it is possible to know that estimates β_{LS} given by equation (11) are minimum linear unbiased estimates consitent with parameter β variances associated with linear regression model (11).

Thus, we only need to prove that matrix X'X is postively defined. Note

$$X=\begin{pmatrix} X_1 & X_2 \\ X_3 & 0 \end{pmatrix}, \quad \tilde{X}=\begin{pmatrix} X_1 & X_2 \\ X_3 & 0 \end{pmatrix}_{(2m+4)\times(2m+3)}$$

In this, $X_1=(x_{i,j})_{1\le i\le m,\ 1\le j\le m+3}$; $X_2=(x_{i,j})_{1\le i\le m,\ m+4\le j\le 2m+3}$;

$X_3=(x_{i,j})_{m+1\le i\le 2m,\ 1\le j\le m+3}$

Moreover, $\tilde{X}$ is a (2m+4) row by (2m+3) column matrix formed from the first row of X, the kh/2 row, the khj row (j=1,2,...,m), the mkh+1 row, the mkh+(kh/2) row, and the kn(m+j) row (j=1,2,...,m). X_1 is the (m+2)x(m+3) matrix composed of elements located at the intersection of the previous (m+2) row /21 and the previous (m+3) column. X_2 is an (m+2)xm matrix composed of elements located at the intersection of the (m+2) row in front of $\tilde{X}$ and the m column behind. X_3 is a matrix composed of elements at the point of intersection between $\tilde{X}$ rear (m+2) row and front (m+3) column.

It is easy to prove that, with regard to any number set r =

$(r_1, r_2, \cdots, r_{2n+3})'$ all have r'X'Xr >(illegible) r' X ' X r.
As
far as another definition from X is concerned, we have:

$$X_1=\begin{pmatrix} \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & 0 & 0 & \cdots & 0 & 0 & 0 \\ \frac{1}{48} & \frac{23}{48} & \frac{23}{48} & \frac{1}{48} & 0 & \cdots & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{pmatrix}, \quad X_2=\begin{pmatrix} 0 & 0 & \cdots & \cdots & 0 \\ \frac{1}{2} & 0 & \cdots & \cdots & 0 \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & \cdots & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & \cdots & \cdots & 1 \end{pmatrix}$$

$$X_3=\begin{pmatrix} -\frac{1}{2h} & 0 & \frac{1}{2h} & 0 & \cdots & 0 & 0 & 0 \\ -\frac{1}{16h} & -\frac{5}{8h} & \frac{5}{8h} & \frac{1}{16} & \cdots & 0 & 0 & 0 \\ 0 & -\frac{1}{2h} & 0 & \frac{1}{2h} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -\frac{1}{2h} & 0 & \frac{1}{2h} \end{pmatrix}$$

Making use of elementary transformation methods, it is possible to prove that X is a full column ordered matrix. Thus, X ' X is positively defined. Because X ' X is positively defined, the theorems are proven.

Below, we come to discussion of estimation errors associated with R(t)+C(To), $\dot{R}(t)$, as well as C(T)-C(To). Assuming

$$R(t)+C(T_0)=S_3(t)=\sum_{j=-1}^{m+1} a_j B_3\left(\frac{t-T_j}{h}\right)$$

$$\dot{R}(t)=\dot{S}_3(t)=\sum_{j=-1}^{m+1} \frac{a_j}{h} \dot{B}_3\left(\frac{t-T_j}{h}\right)$$

$$C(t)-C(T_0)=S_1(t)=\sum_{j=1}^{m} g_j B_1\left(\frac{t-T_j}{h}\right)$$

note $\beta_{LS}=(\hat{a}_{-1}, \hat{a}_0, \cdots, \hat{a}_{m+1}, \hat{g}_1, \cdots \hat{g}_m)'=(\hat{a}', \hat{g}')'$

Use $\hat{S}_3(t)=\sum_{j=-1}^{u+1} \hat{a}_j B_3\left(\frac{t-T_j}{h}\right)$,

to act, respectively, as estimates of S3(t), $\dot{S}_3(t)$, and S1(t).

Define $E(S_3)=E\sum_{i=1}^{n}|\hat{S}_3(t_i)-S_3(t_i)|^2,\ E(\dot{S}_3)=E\sum_{i=1}^{n}|\hat{\dot{S}}_3(t_i)-\dot{S}_3(t_i)|^2,$

$$E(S_1)=E\sum_{i=1}^{n}|\hat{S}_1(t_i)-S_1(t_i)|^2,\ a=(a_{-1},a_0,\cdots,a_{m+1})',\ g=(g_1,g_2,\cdots,g_m)',$$

/22

$$Z=(z_{i,j})_{n\times(m+3)},\ W=(w_{i,j})_{n\times(m+3)},\ U=(u_{i,j})_{n\times m}.$$

In this,

$$z_{i,j}=B_3\left(\frac{t_i-T_{j-2}}{h}\right),\ w_{i,j}=\frac{1}{h}\dot{B}_3\left(\frac{t_i-T_{j-2}}{h}\right),\ i=1,2,\cdots,n,\ j=1,2,\cdots,$$

$$u_{i,j}=B_1\left(\frac{t_i-T_j}{h}\right),\ i=1,2,\cdots,n,\ j=1,2,\cdots,m.$$

Because of this, $E(S_3)=E\|Z(\hat{a}-a)\|^2,\ E(\dot{S}_3)=E\|W(\hat{a}-a)\|^2,\ E(S_1)=E\|U(\hat{g}-g)\|^2,$

Lemma 4 [6] Under the assumptions of model (11), one has

$X)^{-1}$, $COV(\hat{a})=V_1$, $COV(\hat{g})=V_2$, $E(\beta_{LS})=\beta$, $E(\hat{a})=a$, $E(\hat{g})=g$, $COV\ (\beta_{LS})=(X'$

Here, (X'X)-1 is a 2m+3 order square matrix. V1 is an m+3 order matrix. It is composed of elements at locations where (X'X)-1 front m+3 lines and forward m+3 columns intersect. V2 is an m order matrix. It is composed of elements at locations where (X'X)-1 rear m lines and rear m columns instersect.

Theorem 3 Under the assumptions of model (11), we have E(S3)=tr(Z'ZV1), E($\dot{S}_3$)=tr(W'WV1), and E(S1)=tr(U'UV). Here, $tr(\cdot)$ stands for the sum of the elements of primary diagonals of square matrices.

Proof Only proof is of the expression for E(S3). The rest are similar.

$$\begin{aligned}E(S_3)&=E\|Z(\hat{a}-a)\|^2=E(\hat{a}-a)'Z'Z(\hat{a}-a)\\&=Etr[(\hat{a}-a)'Z'Z(\hat{a}-a)]=Etr[Z'Z(\hat{a}-a)'(\hat{a}-a)]\\&=tr(Z'ZCOV(\hat{a}))=tr(Z'ZV_1)\end{aligned}$$

What is worth pointing out is that the conclusions described above are obtained under the presupposition that trimmed errors can be assumed to be capable of being ignored in calculations. As far as how to guarantee this point is concerned, it is possible to make use of simulation methods--simulating the production of an orbit (for example, using actual orbit design tracks) as well as system errors. Assuming C(t)=at(a is a constant), calculations are done of R(t). For cases where random errors are not added, calculations are done of S3(t) and S1(t). In conjunction with this, solutions are gotten for

$$\sum_{i=1}^{n} |R(t_i)-S_3(t_i)|^2, \sum_{i=1}^{n} |\dot{R}(t_i)-\dot{S}_3(t_i)|^2, \sum_{i=1}^{n} |C(t_i)-S_1(t_i)|^2,$$

appropriately selecting h to guarantee the above three summations are, respectively, smaller than

$$\frac{1}{100}\sum_{i=1}^{n} \sigma_i^2, \frac{1}{100} \sum_{i=1}^{n} \theta_i^2, \frac{1}{100} \sum_{i=1}^{n} \sigma_i^2.$$

Through large amounts of simulation calculations--during stable flight phases--selecting h=5s, it is possible to guarantee the requirements described above being satisfied.

5 SIMULATION CALCULATION EXAMPLES

Example 1 Assuming that orbital parameters $x(t)$, $y(t)$, $x(t)$,x(t), $\dot{x}(t)$, $\dot{y}(t)$, $\dot{z}(t)$ satisfy orbital movement equations

$$\left.\begin{aligned} \frac{d\mu}{dt} &= \dot{\mu} \\ \frac{d\dot{\mu}}{dt} &= c_\mu + g_\mu + a_{e\mu} + a_{e\mu} \end{aligned}\right\} \quad . \tag{13}$$

In these, μ is one of x(t), y(t), or z(t). c_μ, g_μ, $a_{e\mu}$, $a_{e\mu}$ are, respectively, resistance acceleration, acceleration of gravity, drag acceleration, and Ke acceleration.

Given initial values associated with the instant To [x(To,..., $\dot{z}$ (To)]', from equation (13), it is possible to produce /23 orbits. Thus, it is possible to calculate true values R(ti) and $\dot{R}$ (ti) associated with measured data for various instants ti.

Assuming that range system error is C(t)=at--in situations where random errors are not added--one obtains observation data yi = R(ti) + μti, pi = $\dot{R}(t_i)$. Table 1 below gives the differences between estimated values and real values obtained by the use of the methods in this article.

Trimmed error square summations are, respectively,

$$\sum_{i=1}^{601} |\hat{R}(t_i)-R(t_i)|^2=1.455\,66D-2, \sum_{i=1}^{601} |\hat{C}(t_i)-C(t_i)|^2$$

$$\sum_{i=1}^{601} |\dot{R}(t_i) - \hat{\dot{R}}(t_i)|^2=4.352\,95D-3$$

Table 1 Trimmed Errors Displayed by R(t), C(t), and R(t)Splines

i	$\hat{R}(t_i)-R(t_i)$	$\hat{C}(t_i)-C(t_i)$	$\hat{\dot{R}}(t_i)-\dot{R}(t_i)$
1	7.574 85D-5	0	5.305 57D-3
101	5.172 02D-3	-2.570 15D-3	-3.166 62D-4
201	6.442 70D-3	-3.967 80D-3	1.282 69D-4
301	4.707 85D-3	-2.830 46D-3	-3.003 38D-5
401	5.650 41D-3	-5.002 75D-3	3.677 13D-5
501	4.512 03D-3	-4.405 23D-3	3.422 08D-4
601	-1.033 14D-4	-2.304 72D-3	-4.240 71D-3

Example 2 On the foundation of Example 1, there will be produced R(ti) and $\dot{R}$ (ti) with random errors added to obtain observation values. yi = R(ti) $+C(t_i)+\varepsilon_i$, $p_i=\dot{R}(t_i)+\delta_i$,

In these, $\varepsilon_i\sim$ N(0,0.012); $\delta_i\sim N(0,0.03^2)$.032); ; $E\varepsilon_i\varepsilon_j=0$, Eδiδj = 0, $(i\neq j)$, $E\varepsilon_i\delta_j=0$, 0, (i,j=1,2,...,n). As related to random error assumptions, it is possible to consult reference [1]. We obtained Table 2.

Table 2 Estimation Errors Associated with the Methods of This Article

i	$\hat{R}(t_i)-R(t_i)$	$\hat{C}(t_i)-C(t_i)$	$\hat{\dot{R}}(t_i)-\dot{R}(t_i)$
1	1.877 43D-2	0	-8.308 79D-3
101	2.155 90D-2	-2.040 03D-2	-3.992 08D-3
201	3.293 63D-2	-5.126 47D-2	5.864 7 D -4
301	3.059 65D-2	-1.426 44D-2	-4.633 22D-4
401	2.539 76D-2	-1.948 12D-2	-2.134 26D-3
501	8.181 96D-3	-1.262 67D-3	-3.871 58D-3
601	-4.683 51D-3	-2.238 30D-2	1.89023D-4

Estimation error square summations are, respectively,

$$\sum_{i=1}^{601}|\hat{R}(t_i)-R(t_i)|^2=0.284\,950,\quad \sum_{i=1}^{601}|C(t_i)-\hat{C}(t_i)|^2=0.327\,256\,;$$

$$\sum_{i=1}^{601}|\hat{\dot{R}}(t_i)-\dot{R}(t_i)|^2=1.031\,883D-2$$

The methods of this article are not only suitable for use in forms where system errors are μt. Moreover, so long as it is assumed that range findings have systemic errors, and measurements of speed do not have systemic errors, then, the methods of this article are appropriate for use in any other situation associated with unusual value system errors. At this time, all that is required is to take spline function S1(t) and change it into an /24 appropriately high order spline function in order to guarantee $\|C(t)-S_1(t)\|_\infty$ being sufficiently small and estimation precisions not being influenced. Realizations of theoretical analyses and actual calculations are similar.

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