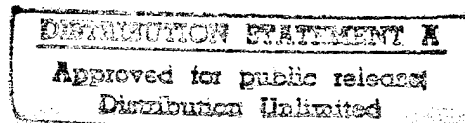# Computerized Design and Generation of Gear Drives With a Localized Bearing Contact and a Low Level of Transmission Errors

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COMPUTERIZED DESIGN AND GENERATION OF GEAR DRIVES WITH A LOCALIZED BEARING CONTACT AND A LOW LEVEL OF TRANSMISSION ERRORS

by

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Abstract

A general approach developed for the computerized simulation of loaded gear drives is presented. In this paper the methodology used to localize the bearing contact, provide a parabolic function of transmission errors, and simulate meshing and contact of unloaded gear drives is developed. The approach developed is applied to spur and helical gears, spiral bevel gears, face-gear drives, and worm-gear drives with cylindrical worms.

Introduction

Computerized simulation of meshing and contact of gear drives also known as TCA (Tooth Contact Analysis), has been a great achievement that has inspired many researchers to improve the gear technology and complement gear theory with new developments. A brief review of such developments of gear theory is presented in [1]. TCA computer programs for spiral bevel and hypoid gears, available commercially, have been developed by the engineers of the Gleason Works [2]. TCA computer programs for the same gear drives, based on the ideas that have been proposed in [3], have been represented in [4].

Initially, the TCA computer programs have been directed at the simulation of meshing and contact of unloaded spiral bevel and hypoid gear drives. A new Gleason's TCA computer program (available commercially) permits the simulation of meshing and contact of loaded gear drives mentioned above. An important contribution to the solution of this problem has been made by Krenzer [5,6] that has proposed to use the principle of gear rotation compatibility under load, and extended a TCA program for the discovery of the edge contact of the gear tooth surfaces. However, the elastic tooth deformations and stress analysis were determined using a simplified tooth model.

A powerful tool for the elastic tooth deformations and the stress analysis is the application of the finite element method. The application of this method for the previously mentioned program was contributed by Wilcox and Coleman [7-9].

A LTCA (Loaded Tooth Contact Analysis) computer program consists of two parts: (i) the geometric and kinematic analysis of the gear mechanism model that is accomplished as a regular TCA program, and (ii) the application of the finite element method for the determination of the load sharing, real contact ratio, and the stress analysis.

The LTCA method is still an approach applied only for the analysis of meshing of the gear drive under the load. It is assumed that the bearing contact of the gear tooth surfaces has been already localized, and the gear tooth surfaces are at every instant in point contact

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that is spread over an elliptical area under the load. Therefore, the LTCA computer program must be complemented with the synthesis of the gear drive.

Until now LTCA computer programs have been developed only for spiral bevel gear and hypoid gear drives. There is a great need to develop the LTCA programs for other types of gear drives (for example spur, helical gear drives and worm-gear drives), but synthesis of the gearing is needed first. Such synthesis must cover: (i) methods for the localization of the bearing contact, (ii) investigation of misalignment on the transmission errors, and (iii) compensation (at least partially) of transmission errors caused by misalignment.

The objective of this paper is to develop methods for the localization of bearing contact applicable for various types of gear drives, reduction of transmission errors caused by gear misalignment by application of a parabolic function of transmission errors, consideration of various approaches for the generation of a parabolic function of transmission errors, and to develop algorithms for the simulation of meshing and contact.

Examples are provided to demonstrate contact localization and predesigning of a parabolic function of transmission errors based on the research accomplished.

1   Localization of Bearing Contact and Predesigning of a Parabolic Function of Transmission Errors

The main goals for the synthesis of a gear drive are : (*i*) the localization of the bearing contact, and (*ii*) the predesigning of a parabolic function of transmission errors. Such synthesis must be accomplished before the development of the LTCA program.

The localization of the bearing contact is required to substitute an instantaneous line contact of surfaces by point contact, since a gear drive with line contact of surfaces is very sensitive to misalignment. If the gear tooth surfaces are in line contact, misalignment usually causes : edge contact, unacceptable shift of the bearing contact, transmission errors of high level and of a shape that may induce high vibrations.

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It was mentioned earlier that due to the elastic deformation of tooth surfaces the contact at a point is spread over an elliptical area. The dimensions of the contact ellipse can be controlled by the proper mismatch of the contacting surfaces.

The localization of the bearing contact must be complemented by the predesign of a parabolic function of transmission errors that is able to absorb the undesirable transmission errors caused by misalignment. This is the reason why any method of crowning of surfaces must be complemented by application of a predesigned parabolic function of transmission errors.

The Gleason Works pioneered the successful application of point contact for spiral bevel gears and hypoid gear drives. There is a great need now to localize the bearing contact in spur, helical gears and worm-gear drives with cylindrical worms, to provide a low level of transmission errors for such gear drives, and then to develop the LTCA computer programs. **Interaction of Parabolic and Linear Functions.** A predesigned parabolic function of transmission errors is designated for the absorption of almost linear functions of transmission errors caused by misalignment. The interaction of a parabolic and linear functions of transmission errors has been discussed in [10] and is illustrated by drawings figs. 1, 2 and 3. Fig. 1(a) shows that due to misalignment the transmission function $\phi_2(\phi_1)$ is a piecewise function. The function of transmission errors $\Delta\phi_2(\phi_1)$ (fig. 1(b)) is usually an almost linear function with the period of the cycle of meshing $(\frac{2\pi}{N_1})$, when one pair of teeth is changed for another one. The transfer of meshing is accompanied with a large acceleration if such a function of transmission errors exists. Fig. 2(a) shows the interaction of a predesigned parabolic function

$$\Delta\phi_2^{(2)} = -a\phi_1^2 \tag{1}$$

with a linear function

$$\Delta\phi_2^{(1)} = b\phi_1 \tag{2}$$

that is caused by misalignment.

The sum of functions $(\Delta\phi_2^{(1)} + \Delta\phi_2^{(2)})$ is again a parabolic function represented in coordinate system $(\Delta\psi_2, \psi_1)$ as

$$\Delta\psi_2 = -a\psi_1^2 \tag{3}$$

Axes of coordinate systems $(\Delta\psi_2, \psi_1)$ and $(\Delta\phi_2, \phi_1)$ are parallel, but the origins do not coincide. Function (3) represents a parabolic function with the same parabola parameter a. The location of points (A^*, B^*) of function $\Delta\psi_2(\psi_1)$ is asymmetrical in comparison with the symmetrical location of points (A, B) of function $\Delta\phi_2(\phi_1)$. Fig. 2(b) shows the resulting parabolic function $\Delta\psi_2(\psi_1)$ for several cycles of meshing. Fig. 2 illustrates that the linear function of transmission errors, $\Delta\phi_2^{(1)}(\phi_1)$, is absorbed by the predesigned parabolic function $\Delta\phi_2^{(2)}(\phi_1)$. As the result of such absorption, the transmission function for a misaligned gear drive will have the shape shown in fig. 3(a), and the acceleration and vibration at the transfer of meshing will be reduced. The application of this idea has been tested for spiral bevel gears that confirmed the reduction of the level of noise for the prototype up to 18 decibels at the spiral bevel gear meshing frequencies [11].

A predesigned parabolic function may be provided: (*i*) by mismatch of the contacting surfaces, (*ii*) by application of nonlinear relations between the motions of the generating and generated surface, and (*iii*) by controlled plunging of the generating surface.

The discovery of the type of transmission errors that are caused by gear misalignment must be a subject of special investigation for each type of the gear drive program. In the case of hypoid gear drives such an investigation has been accomplished for an imaginary process of gear generation that provides an ideal linear transmission function [12].

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The following is a brief description of the approach used for the localization of the bearing contact and the application of a predesigned parabolic function of transmission errors.

Application of Two Generating Surfaces in Point Contact. Two generating surfaces, Σ_c and Σ_t, are used for the generation of the pinion and gear tooth surfaces Σ_p and Σ_g, respectively. Surfaces Σ_c and Σ_t are mismatched, and in the process for generation each pair of surfaces Σ_c and Σ_p, Σ_t and Σ_g are in line contact. Surface Σ_p is generated as the envelope to the family of tool surfaces Σ_c. Respectively, Σ_g is generated as the envelope to the family of tool surfaces Σ_t. Lines of contact L_{cp} and L_{tg} do not coincide, and finally the point contact of generated tooth surfaces Σ_p and Σ_g will be provided. The discussed method of contact localization was applied for face-milled generation of spiral bevel tooth surfaces.

Note : There is a particular case when surface Σ_g is formate cut by the tool surface Σ_t. This means that Σ_g is a copy of the tool surface Σ_t. This method for generation is applied for face-milled formate cut hypoid gears.

The conjugation of surfaces Σ_p and Σ_g requires : (*i*) transformation of rotation with a predesigned parabolic function for absorption of transmission errors caused by gear misalignment (see section 2), and (*ii*) desired dimensions of the contact ellipse for a certain value of surface elastic approach δ. The choice of δ is based on experiments accomplished for unloaded gear drives. Fulfillment of these conditions of surface conjugation may be achieved by application of a local synthesis method that provides the proper mismatch of the contacting surfaces [10]. Direct relations between the principal curvatures and directions for the contacting surfaces [13] simplify the solution.

Application of Two Rigidly Connected Generating Surfaces Σ_c and Σ_t Being in Tangency Along a Line. This approach may be applied : (*i*) for generation of single and double circular arc helical gears, (*ii*) for modified involute helical gears, and for (*iii*) face-milled spiral bevel gears with uniform depth.

The schematic of generation is shown in fig. 4 and the generation is based on the following

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ideas :

(i) Two generating surfaces Σ_c and Σ_t are in tangency along a line (designated as L_{ct}), they are rigidly connected and move as a single rigid body in the process of imaginary generation.

(ii) Surface Σ_c generates the pinion tooth surface Σ_p as the envelope to the family of surfaces Σ_c, generated in coordinate system S_p, that is rigidly connected to the pinion. Surfaces Σ_c and Σ_p are in line contact at every instance, and their instantaneous line of tangency is designated as L_{cp}.

(iii) Similarly, surface Σ_t (of the couple of rigidly connected surfaces Σ_c and Σ_t) generates the gear tooth surface Σ_g as the envelope to the family of surfaces Σ_t, generated in coordinate system S_g, rigidly connected to the gear. Surfaces Σ_t and Σ_g are in line contact at every instant, and their instantaneous line of tangency is designated as L_{tg}.

(iv) The generated pinion and gear tooth surfaces are in contact at a point at every instant that is designated as L_{pg} (fig. 4). The instantaneous point of tangency L_{pg} of surfaces Σ_p and Σ_g is the point of intersection of instantaneous contact lines L_{cp} and L_{tg}. Point L_{pg} traces on Σ_p and (respectively, on Σ_g) the path of contact.

The described method for generation of pinion and gear tooth surfaces provides the desired localization of contact of the generated pinion-gear tooth surfaces. In addition, a predesigned parabolic function of transmission errors must be provided, and this can be achieved by the following two approaches :

Approach 1 for Execution of Predesigned Parabolic Function:

(1) The generating surface Σ_c and the pinion tooth surface Σ_p perform in the process of generation a related motion with constant velocity.

(2) Generating surface Σ_t performs its motion with the same constant velocity as Σ_c (recall that Σ_c and Σ_t are rigidly connected). However, the gear tooth surfaces Σ_g performs its motion with a varied velocity which is executed by the computer program of the CNC machine and provides the required transmission function

$$\phi_2 = \phi_1 \frac{N_1}{N_2} - a\phi_1^2 \tag{4}$$

Approach 2 for Execution of Predesigned Parabolic Function:

(1) In this case it is assumed that the generating surface Σ_t and the gear tooth surface Σ_g perform in the process of generation related motions with constant velocities.

(2) Generating surface Σ_c performs its motion with the same constant velocity as Σ_t. However, the pinion tooth surface Σ_p performs its motion with a varied velocity which is executed by the computer program of the CNC machine and provides the following transmission function

$$\phi_1 = \frac{N_1}{2aN_2} \pm \left[\left(\frac{N_1}{2aN_2} \right)^2 - \frac{\phi_2}{a} \right]^{0.5} \tag{5}$$

Application for Generation of Double Circular-Arc Helical Gears. The generation of pinion-gear tooth surfaces is based on application of two rigidly connected rack-cutters Σ_c and Σ_t (fig. 5). The rack-cutter surfaces are in tangency along two lines, $a - a$ and $b - b$. Two zones of meshing of the generated gears are provided since two lines of tangency ($a - a$ and $b - b$) of rack-cutters exist. Plane Π in fig. 5 is tangent to the pitch cylinders (they are axodes) of the gears. Vector **v** is the velocity of translational motion of the rack-cutter in the process for generation. The gears being generated perform related rotations about their axes when the rack-cutter is translated [10]. The generated pinion-gear tooth surfaces Σ_p and Σ_g are in point contact at every instant, and the bearing contact is localized. A hob or a grinding worm can also be used for gear generation instead of rack-cutters. The predesigned parabolic function can be provided as described above.

Application for Generation of Modified Involute Helical Gears. Application of the described ideas to involute helical gears will now be made [14]. Two rigidly connected rack-cutters Σ_c and Σ_t are applied as shown in fig. 6. The normal sections of the rack-cutters

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are shown in figs. 6(b) and 6(c). The profiles of the rack-cutters in the normal section are in tangency at points N_1 and N_2 (fig. 6(b)), and deviate each from other at all other points. The line of tangency of rack-cutter surfaces Σ_c and Σ_t passes through N_i $(i = 1, 2)$ and has the same orientation as the Z_a-axis. The generated pinion-gear tooth surfaces, Σ_p and Σ_g, are in point contact at every instant. Only one zone of meshing is provided in this case. A parabolic function of transmission errors is provided as described above.

Application for Generation of Face-Milled Spiral Bevel Gears with Uniform Tooth Depth. Two types of bearing contact of such gears can be provided : (i) across the tooth surface, and (ii) along the tooth surface. In the first case two rigidly connected generating cones are applied (fig. 7). The common generatrix $a - a$ is the line of tangency of the generating cones Σ_c and Σ_t. The path of points of contact between surfaces Σ_p and Σ_g, and the contact lines L_{jk} between the generating surface Σ_j $(j = c, t)$ and the generated surface Σ_k $(k = p, g)$ are shown in fig. 7.

In the second case (fig. 8), generating surfaces Σ_c and Σ_t represent a cone surface and a surface of revolution, respectively. Surfaces Σ_c and Σ_t are in tangency along a circle, and the arc of this circle is the path of contact $a - a$ of gear tooth surfaces Σ_p and Σ_g. The predesigned parabolic function of transmission errors is provided in both cases as described above.

Localization of Bearing Contact and Predesign of a Parabolic Function of Transmission Errors for Spur Involute Gears. The approach that is developed is based on the following ideas : the generation is accomplished by form-grinding and the contact localization is achieved by plunging the grinding cutting wheel while the pinion is generated.

In accordance with fig. 9, we have

$$E = E_0 - al^2 \tag{6}$$

where E and E_0 are the current and initial center distance, l is the axial displacement of the

tool, a is the parabola parameter of the parabolic function (al^2). Choosing the proper value of a, we may obtain the desired value of contact localization. The generation of the gear is accomplished without a plunging of the grinding wheel. A predesigned parabolic function for absorption of transmission errors caused by gear misalignment may be accomplished by modification of the profile of one of the pair of grinding wheels. The following alternative approaches may be applied for this purpose : (a) The profile of the grinding (cutting) wheel (used for the pinion or the gear) is represented as an involute profile generated by application of a base circle whose center is dislocated with respect to the center of the theoretical base circle (fig. 10). The diameter of the modified base circle differs from the diameter of the theoretical base circle. (b) The other two approaches are based on application of two imaginary rack-cutters, Σ_c and Σ_t, that are rigidly connected and are conjugated to the theoretical profiles of unmodified profiles of the spur gears. The profiles of the tools complement each other as a casting and model. The modification of the theoretical involute surface of the spur gear is achieved due to a nonlinear relation between the motions of rack-cutter surface Σ_t and the gear tooth surface Σ_g. The rack-cutter surface Σ_c and the pinion tooth surface Σ_p perform related motions determined by linear relations. In the case when the theoretical surface of the pinion is to be modified, nonlinear relations must be provided for the motions of the rack-cutter surface Σ_c and the pinion tooth surface Σ_p.

Localization of Bearing Contact in a Face-Gear Drive. The localization of contact when a face-gear is generated by a shaper is based on the following considerations : (i) Fig. 11 shows generation of a face-gear by a shaper. The meshing in the process for generation simulates the meshing of the pinion and the face-gear, if the shaper is identical to the pinion and has the same number of teeth as the pinion. However, since the pinion and the generated face-gear are in line contact, the face-gear drive is very sensitive to misalignment. (ii) The localization of the bearing contact may be achieved if a shaper with a larger number of teeth than the pinion is used [10].

Misalignment of a face-gear drive does not cause transmission errors and therefore the predesign of a parabolic function of transmission errors is not required. However, misalignment will cause a shift in the bearing contact, and therefore errors of alignment must be limited.

Localization of Bearing Contact in a Worm-Gear Drive with a Cylindrical Worm. There are various types of worm-gear drives with cylindrical worms depending on the type of geometry applied [10]. In the most common case, the generation of the worm-gear drive is based on the simulation of meshing of the worm with the worm-gear. This can be achieved if the hob is identical to the worm, and the settings and related motions of the hob and the worm-gear are the same as in the worm-gear drive. Then, the worm and the worm-gear tooth surfaces will be in line contact at every instant. However, such a worm-gear drive will be very sensitive to misalignment, and therefore it is necessary to localize the bearing contact.

Two types of localized bearing contact may be provided : (*a*) in the longitudinal direction, and (*b*) in the direction that is across the surface.

The localization of the contact in the longitudinal direction for a K worm-gear drive may be achieved as follows:

(i) The thread surface of the hob is generated by a cone (fig. 12(a)). The installment of the tool for the generation of the thread surface is illustrated in fig. 12(c).

(ii) The pitch diameters of the worm and the hob are the same, but the axial profiles deviate from each other (fig. 12(b)).

(iii) Due to such deviations, the thread surfaces of the hob and the worm are in tangency along a helix, which is the path of contact of the worm with the generated worm-gear (fig. 12(d)).

The disadvantage of such a method of localization of contact is that, due to misalignment of a worm-gear drive, the bearing contact may be shifted and become asymmetrical [15].

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Transmission errors will occur as well, and a predesigned parabolic function for absorption of transmission errors is required.

A more favorable method for the localization of the bearing contact is based on the application of an oversized hob. The meshing of the hob and the worm may be represented as the meshing of two crossed helical gears in internal tangency (fig. 13). The shortest distance ΔE and the crossing angle $\Delta \gamma$ depend on the chosen oversize that affects the deepness of localization. The investigation performed by the authors showed that the transmission errors caused by misalignment may be neglected. The bearing contact is directed across the worm-gear surface, and the resultant shift of the bearing contact due to misalignment is permissible.

2   Simulation of Meshing

We consider at this stage of the investigation that the bearing contact is already localized and the predesigned parabolic function for absorption of transmission errors caused by misalignment is provided. The tooth surfaces of the gear pair are in point contact at every instant, and the simulation of meshing is based on an algorithm that provides continuous tangency of the contacting surfaces. Henceforth, we have to consider two cases : (i) surface-to-surface contact, and (ii) curve-to-surface contact.

Surface-to-Surface Contact. Consider that the two contacting surfaces are represented in a fixed coordinate system S_f. The conditions of continuous tangency of two surfaces are represented as follows

$$\mathbf{r}_f^{(1)}(u_1, \ \theta_1, \ \phi_1) - \mathbf{r}_f^{(2)}(u_2, \ \theta_2, \ \phi_2) = 0 \tag{7}$$

$$\mathbf{n}_f^{(1)}(u_1, \ \theta_1, \ \phi_1) - \mathbf{n}_f^{(2)}(u_2, \ \theta_2, \ \phi_2) = 0 \tag{8}$$

Here, $\mathbf{r}_f^{(i)}$ and $\mathbf{n}_f^{(i)}$ are the position vector and the surface unit normal of the contact point on the surface 1 ($i = 1$) or surface 2 ($i = 2$); u_i and θ_i are the surface parameters ; ϕ_i is the rotational angle of the surface considering ϕ_1 is the known input parameter.

Since $|\mathbf{n}_f^{(1)}| = |\mathbf{n}_f^{(2)}| = 1$, equations (7) and (8) yield a system of five independent nonlinear equations

$$f_i(u_1,\ \theta_1,\ \phi_1,\ u_2,\ \theta_2,\ \phi_2) = 0, \qquad (i = 1, 2, ..., 5) \tag{9}$$

in five unknowns; ϕ_1 is considered as the known input parameter. The solvability of equation system (9) can be verified with the Existence Theorem of Implicit Function System [16] as follows.

Assume that equations (9) are satisfied with the set of parameters

$$P = (u_1^0,\ \theta_1^0,\ \phi_1^0,\ u_2^0,\ \theta_2^0,\ \phi_2^0) \tag{10}$$

Equation (9) can be solved in the neighborhood of P by using the functions

$$\{u_1(\phi_1),\ \theta_1(\phi_1),\ u_2(\phi_1),\ \theta_2(\phi_1),\ \phi_2(\phi_1)\} \in C^1 \tag{11}$$

if

$$\frac{D(f_1, f_2, f_3, f_4, f_5)}{D(u_1, \theta_1, u_2, \theta_2, \phi_2)} = \begin{vmatrix} \dfrac{\partial f_1}{\partial u_1} & \dfrac{\partial f_1}{\partial \theta_1} & \dfrac{\partial f_1}{\partial u_2} & \dfrac{\partial f_1}{\partial \theta_2} & \dfrac{\partial f_1}{\partial \phi_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial f_5}{\partial u_1} & \dfrac{\partial f_5}{\partial \theta_1} & \dfrac{\partial f_5}{\partial u_2} & \dfrac{\partial f_5}{\partial \theta_2} & \dfrac{\partial f_5}{\partial \phi_2} \end{vmatrix} \neq 0 \tag{12}$$

The continuous solution of equations (9) is used to determine: (i) the paths of contact on surface Σ_1 and Σ_2, and (ii) the transmission function $\phi_2(\phi_1)$.

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Curve-to-Surface Contact. It will be shown below (see section 3) that curve-to-surface contact may occur in addition to surface-to-surface contact in a gear mechanism. To this end, let us consider the case when surface edge 1 is in tangency with surface 2. Thus,

$$\mathbf{r}_f^{(1)}(u_1(\theta_1),\ \theta_1,\ \phi_1) = \mathbf{r}_f^{(2)}(u_2,\ \theta_2,\ \phi_2) \tag{13}$$

$$\frac{\partial \mathbf{r}_f^{(1)}}{\partial \theta_1} \cdot \mathbf{n}_f^{(2)} = 0 \tag{14}$$

where $\mathbf{r}_f^{(1)}(u_1(\theta_1),\ \theta_1,\ \phi_1)$ represents edge E_1 on surface 1, and $\dfrac{\partial \mathbf{r}_f^{(1)}}{\partial \theta_1}$ is the tangent to E_1.

Equations (13) and (14) represent a system of four nonlinear equations in four unknowns considering that the input parameter ϕ_1 as known at each iteration. The solution of equations (13) and (14) is also based on the Existence Theorem for Implicit Function System [16]. The continuous solution of the systems of nonlinear equations (7) and (8), (13) and (14) is based on the application of the subroutine developed by [17].

3   DETERMINATION OF CONTACT ELLIPSE

The bearing contact is formed as the set of instantaneous contact ellipses. The center of a current contact ellipse is located at the point of tangency of the contacting surfaces. The determination of the contact ellipse requires the knowledge of the elastic approach of the contacting surfaces, the principal directions, and the principal curvatures. The contact ellipse is usually determined twice: (*i*) for the needs of the TCA program (for an unloaded gear drive), and in this case the elastic approach of the contacting surfaces is considered as known from the experimental data, and (*ii*) for the needs of the LTCA program (for a loaded gear drive), and in this case the elastic approach is related to the contact force.

The determination of principal curvatures and directions of the contacting surfaces is a complex problem, since the surfaces are usually represented in a three parameter form. The solution to this problem has been substantially simplified by the approach proposed in [18]. This approach enables us to represent the principal directions and curvatures of the generated surface by the principal directions and curvatures of the tool generating surface and the parameters of motion. Using this idea, direct relations between the curvatures of the generating and generated surfaces, and the dimensions of the instantaneous contact ellipse have been determined [10,19].

The determination of the orientation of the contact ellipse and its dimensions is based on the following procedures :

Step 1 : It is considered that the principal directions and principal curvatures of the meshing surfaces are known, and the elastic approach of the surface is given.

Step 2 : The meshing surfaces Σ_1 and Σ_2 are in tangency at current point P, and Π is the plane that is tangent to Σ_1 and Σ_2 at P (fig. 14). Unit vectors $\mathbf{e}_I^{(i)}$ and $\mathbf{e}_{II}^{(i)}$ ($i = 1, 2$) represent the principal directions of surfaces Σ_1 and Σ_2, respectively. The orientation of the contact ellipse is represented by angle α that is determined as follows,

$$\tan 2\alpha = \frac{g_2 \sin 2\sigma^{(12)}}{g_1 - g_2 \cos 2\sigma^{(12)}} \tag{15}$$

where

$$g_1 = \kappa_I^{(1)} - \kappa_{II}^{(1)} \tag{16}$$

$$g_2 = \kappa_I^{(2)} - \kappa_{II}^{(2)} \tag{17}$$

Here, $\kappa_I^{(i)}$ and $\kappa_{II}^{(i)}$ are the principal curvatures of surface 1 and surface 2 at the contact point, and $\sigma^{(12)}$ is the angle between $\mathbf{e}_I^{(1)}$ and $\mathbf{e}_I^{(2)}$ as shown in fig. 14.

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Step 3 : The magnitudes of the ellipse are determined by the equations

$$a = \left|\frac{\delta}{A}\right|^{\frac{1}{2}}, \quad b = \left|\frac{\delta}{B}\right|^{\frac{1}{2}} \tag{18}$$

where δ is the elastic approach; A and B are represented as

$$A = 0.25[\kappa_\Sigma^{(1)} - \kappa_\Sigma^{(2)} - (g_1^2 - 2g_1g_2cos2\sigma^{(12)} + g_2^2)^{\frac{1}{2}}] \tag{19}$$

$$B = 0.25[\kappa_\Sigma^{(1)} - \kappa_\Sigma^{(2)} + (g_1^2 - 2g_1g_2cos2\sigma^{(12)} + g_2^2)^{\frac{1}{2}}] \tag{20}$$

Here,

$$\kappa_\Sigma^{(i)} = \kappa_I^{(i)} + \kappa_{II}^{(i)}, \qquad i = 1, 2 \tag{21}$$

The largest and smallest values of a and b represent the major and minor of the contact ellipse.

4 CONCLUSION

Based on results of the studies reported herein the following conclusions can be drawn :

(1) Methods of localization of bearing contact have been developed for various types of gear drives.

(2) Interaction of a parabolic function with a linear function as the tool for the absorption of transmission errors caused by misalignment has been described.

(3) Approaches for the predesign of a parabolic function of transmission errors have been given.

(4) An Algorithm for simulation of meshing of a misaligned gear drive has been described.

(5) Determination of the instantaneous contact ellipse for an unloaded gear drive has been presented.

References

[1] Litvin, F.L., "Applied Theory of Gearing: State-of-the-Art," ASME *Journal of Mechanical Design*, in press, 1995.

[2] Gleason Works, "Understanding Tooth Contact Analysis," Publication No. 5D3139, Rochester, NY 14692, 1981.

[3] Litvin, F.L., *Theory of Gearing*, Nauka, Moscow, 1968 (in Russian).

[4] Litvin, F.L. and Gutman, Y., "Methods of Synthesis and Analysis for Hypoid Gear Drives of Formate and Helixform," Part 1-3. ASME *Journal of Mechanical Design*, 103:83-113, 1981.

[5] Krenzer, Theodore, J., "Tooth Contact Analysis of Spiral Bevel and Hypoid Gears under Load", S.A.E. Earthmoving Industry Conference, Peoria, IL. April 6-8 1981.

[6] Krenzer, Theodore, J., "Tooth Contact Analysis of Spiral Bevel and Hypoid Gears under Load," Gleason Publication No. SD3458, New York, April 1981.

[7] Wilcox, L. and Coleman, W, "Application of Finite Elements to the Analysis of Gear Tooth Stress," ASME Paper 92-PTG-30, San Francisco, CA, Oct. 1972.

[8] Wilcox, L. and Coleman, W, "Application of Finite Elements to the Analysis of Gear Tooth Stress," ASME *Journal of Engineering for Industry*, 95:1139-1148, 1973.

[9] Wilcox, Lowell E., "An Exact Analytical Method for Calculating Stresses in Bevel and Hypoid Gear Teeth," JSME Conference, Tokyo, Japan, Sept. 1981.

[10] Litvin, F.L., *Gear Geometry and Applied Theory*. Prentice Hall, Englewood Cliffs, NJ, 1994.

[11] Lewicki, D.G., Handschuh, R.F., Henry, Z.S. and Litvin, F.L., "Low-Noise, High-Strength, Spiral-Bevel Gears for Helicopter Transmissions", *Journal of Propulsion and Power*, 10:356-361, 1994.

[12] Litvin, F.L., Chen, J.S., Sep, T.M. and Wang, J.C., "Computerized Simulation of Transmission Errors and Shift of Bearing Contact for Face-Milled Hypoid Gear Drive," *Journal of Mechanical Design*, in press, 1995.

[13] Litvin, F.L. and Hsiao, C.-L., "Direct Relations Between Curvatures of Surfaces Being in Continuous Tangency," *Zeitschrift fur Angewandte Mathematik und Mechanik.* 74:225-234, 1994.

[14] Litvin, F.L., Chen, N.X., Lu, J. and Handschuh, R.F., "Computerized Design and Generation of Low-Noise Helical Gears with Modified Surface Topology," *Journal of Mechanical Design*, in press, 1995.

[15] Litvin, F.L., Seol, I.H. and Kim, K., "Computerized Design and Simulation of Meshing of Worm-Gear Drive with Longitudinally Localized Bearing Contact," ASME 21st Design Automation Conference, Boston, MA, Sept. 1995.

[16] Korn, G.A. and Korn, T.M., *Mathematical Handbook for Scientists and Engineering*, 2nd Ed., McGraw-Hill, 1968.

[17] Dongarra, J.J., Bunch, J.R., Moler, C.B. and Steward, G.W. 1979. LINPACK User's Guide, SIAM, Philadelphia.

[18] Litvin, F.L., "Die Beziehungen Zwischen den Krüummungen der Zahnoberflächen bei Räumlichen Verzahnungen," *Zeitschrift fur Angewandte Mathematik und Mechanik.* 49: 685-690, 1969 (in German).

[19] Litvin, F.L., Chen, N.X. and Chen, J.-S., "Computerized Determination of Curvature Relations and Contact Ellipse for Conjugate Surfaces," *Journal of Computers in Applied Mechanics and Engineering*, in press, 1995.
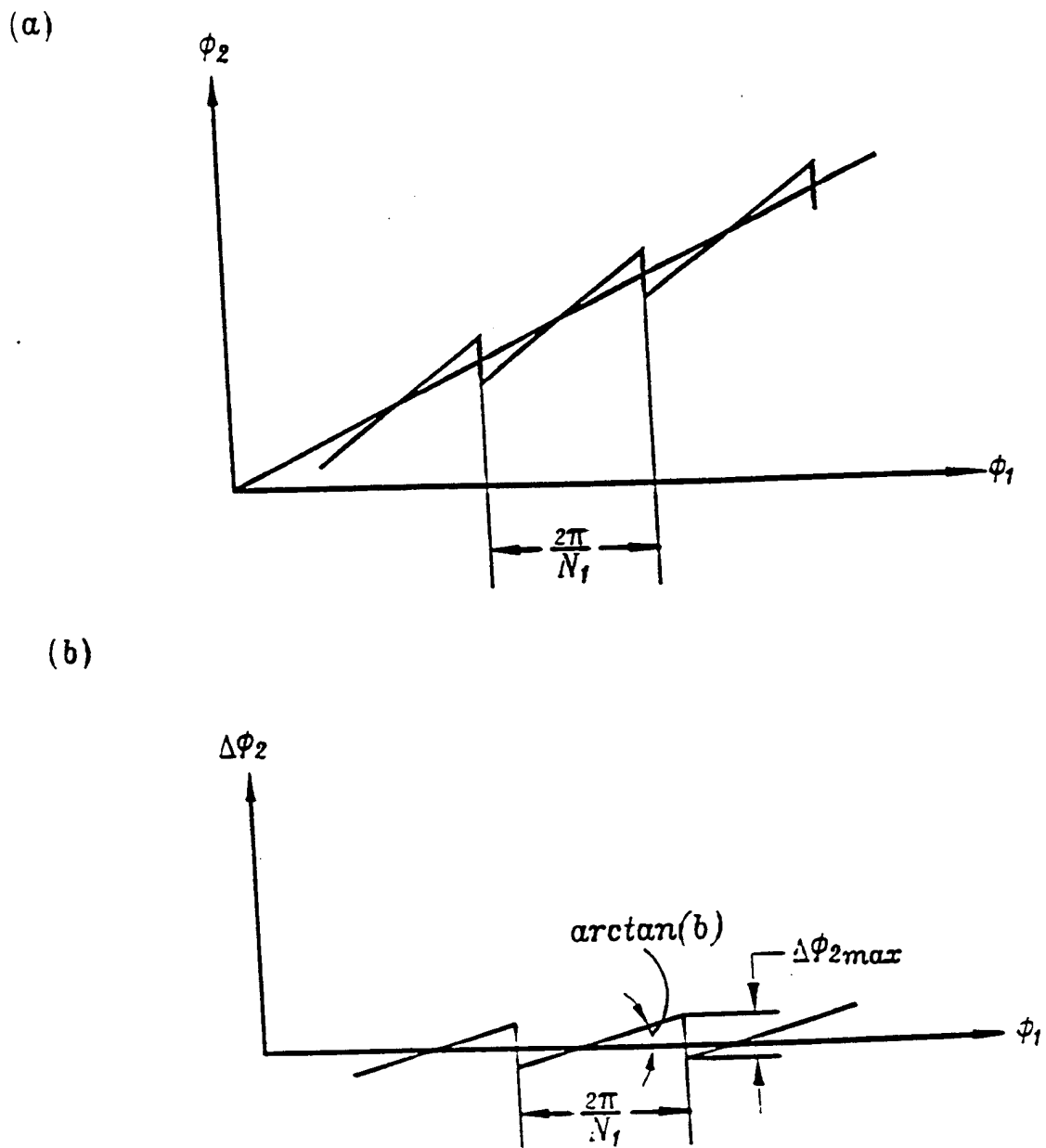
Figure 1: Transmission function and transmission errors for a misaligned gear drive.
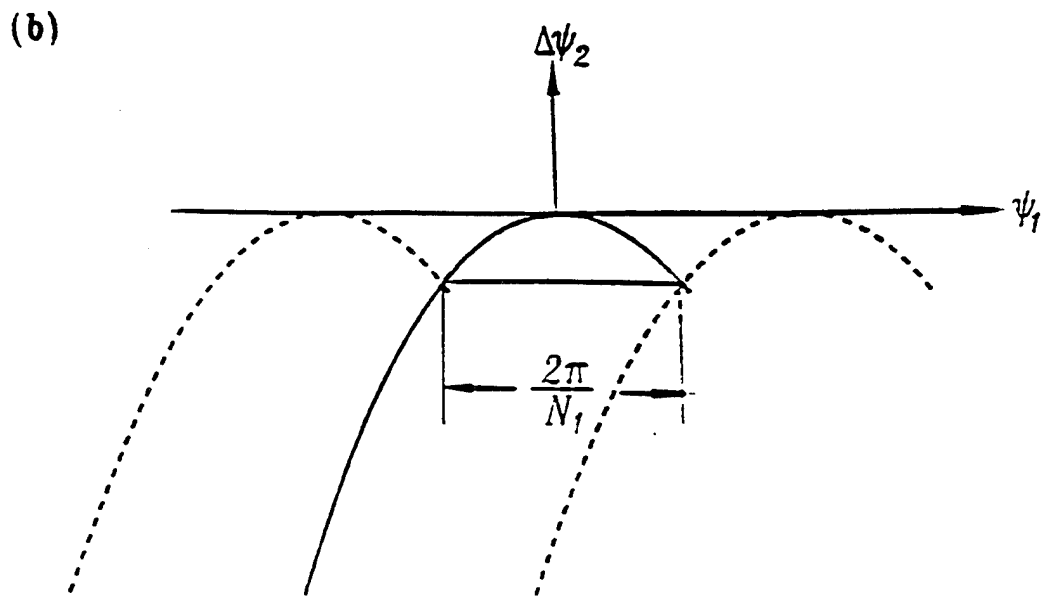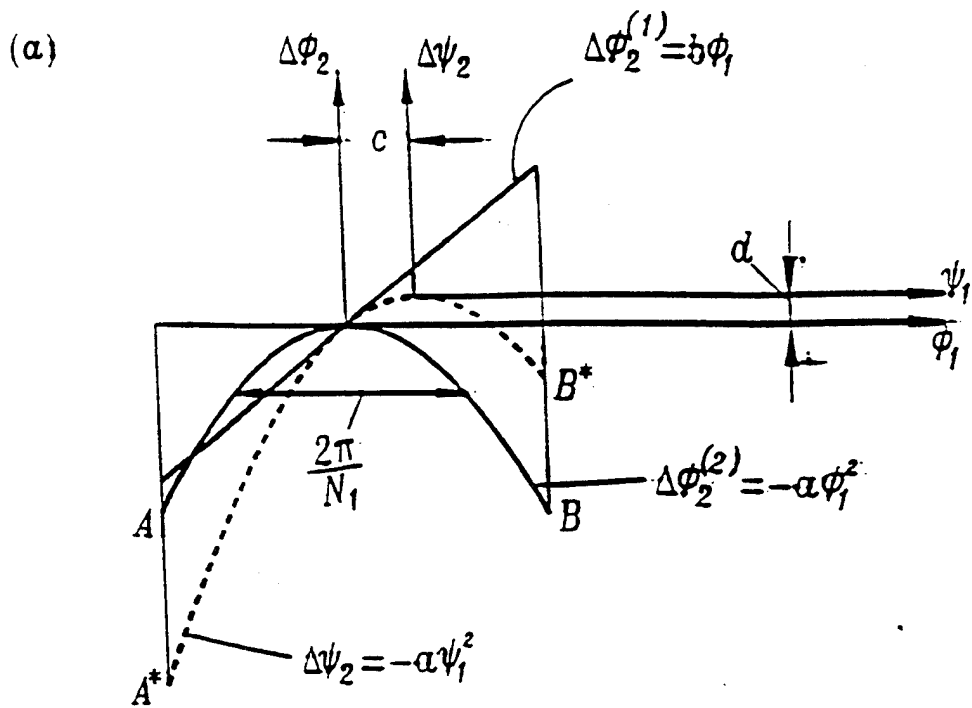
Figure 2: Interaction of parabolic and linear functions.
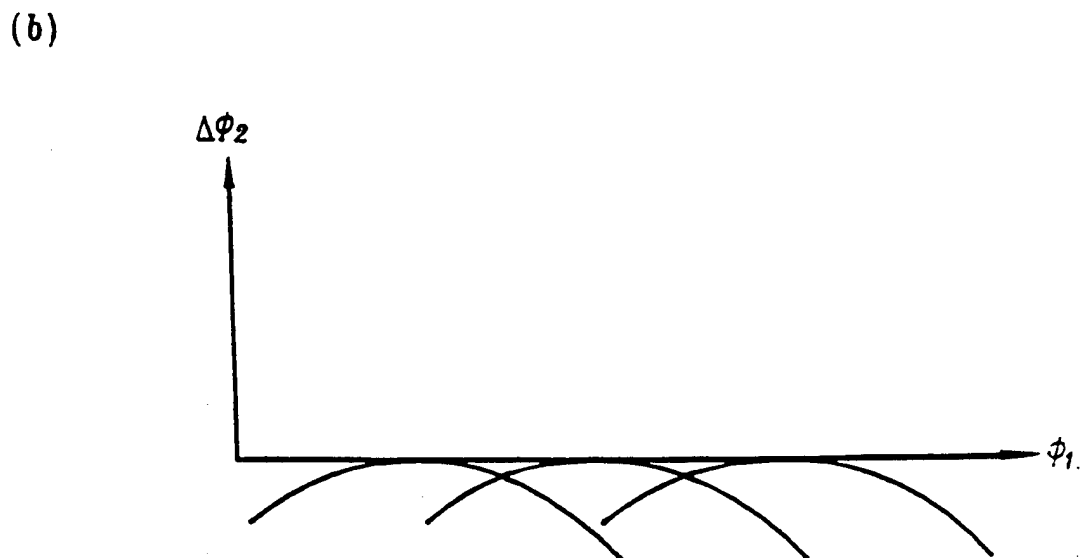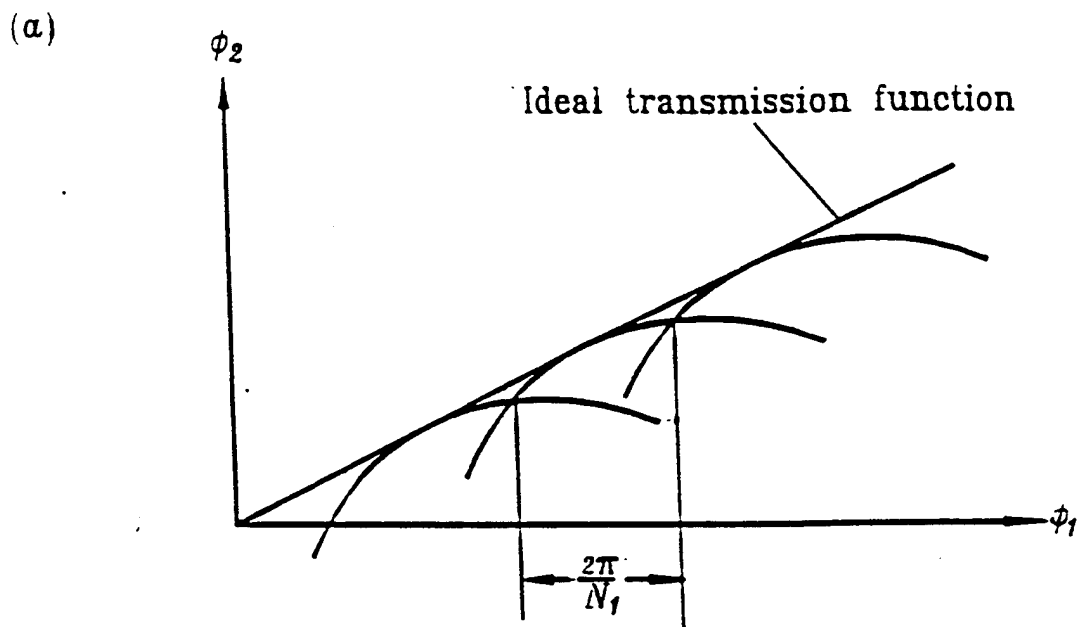
Figure 3: Transmission function and resulting parabolic function of transmission errors.
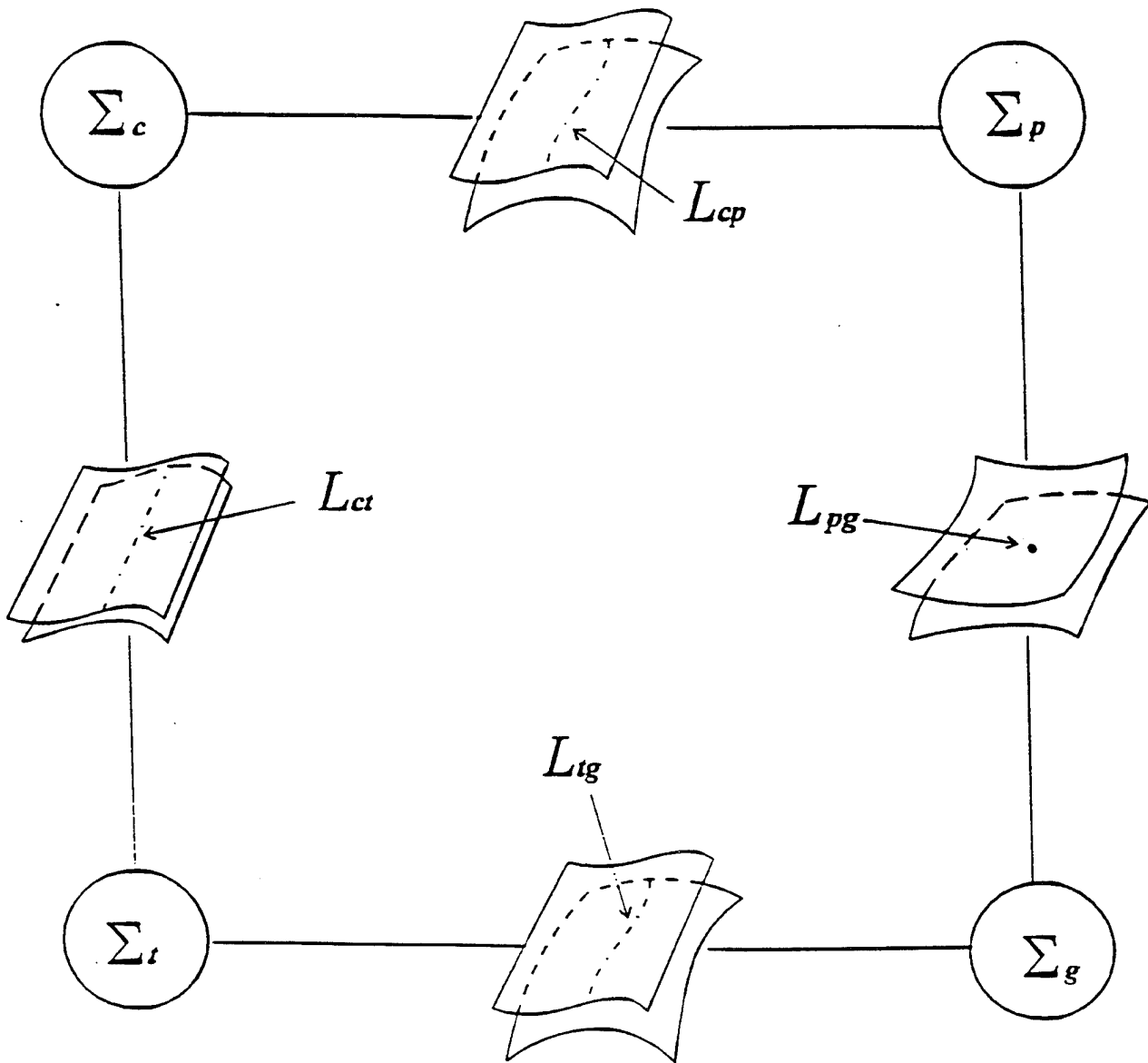
Figure 4: Schematic of generation by two rigidly connected generating surfaces.
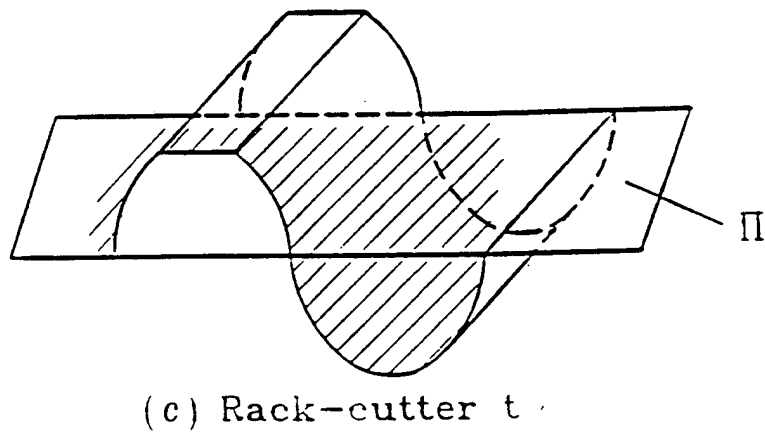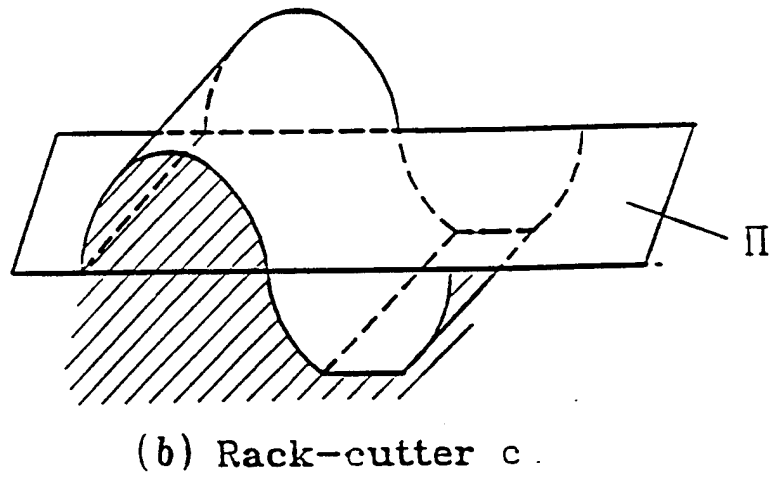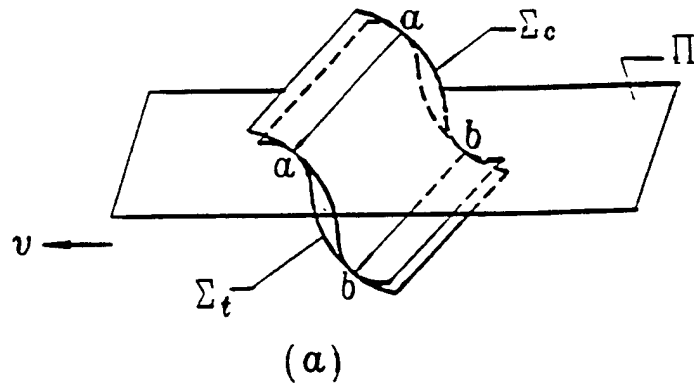
Figure 5: Surfaces of imaginary rack-cutters for generation of double circular-arc helical gears.
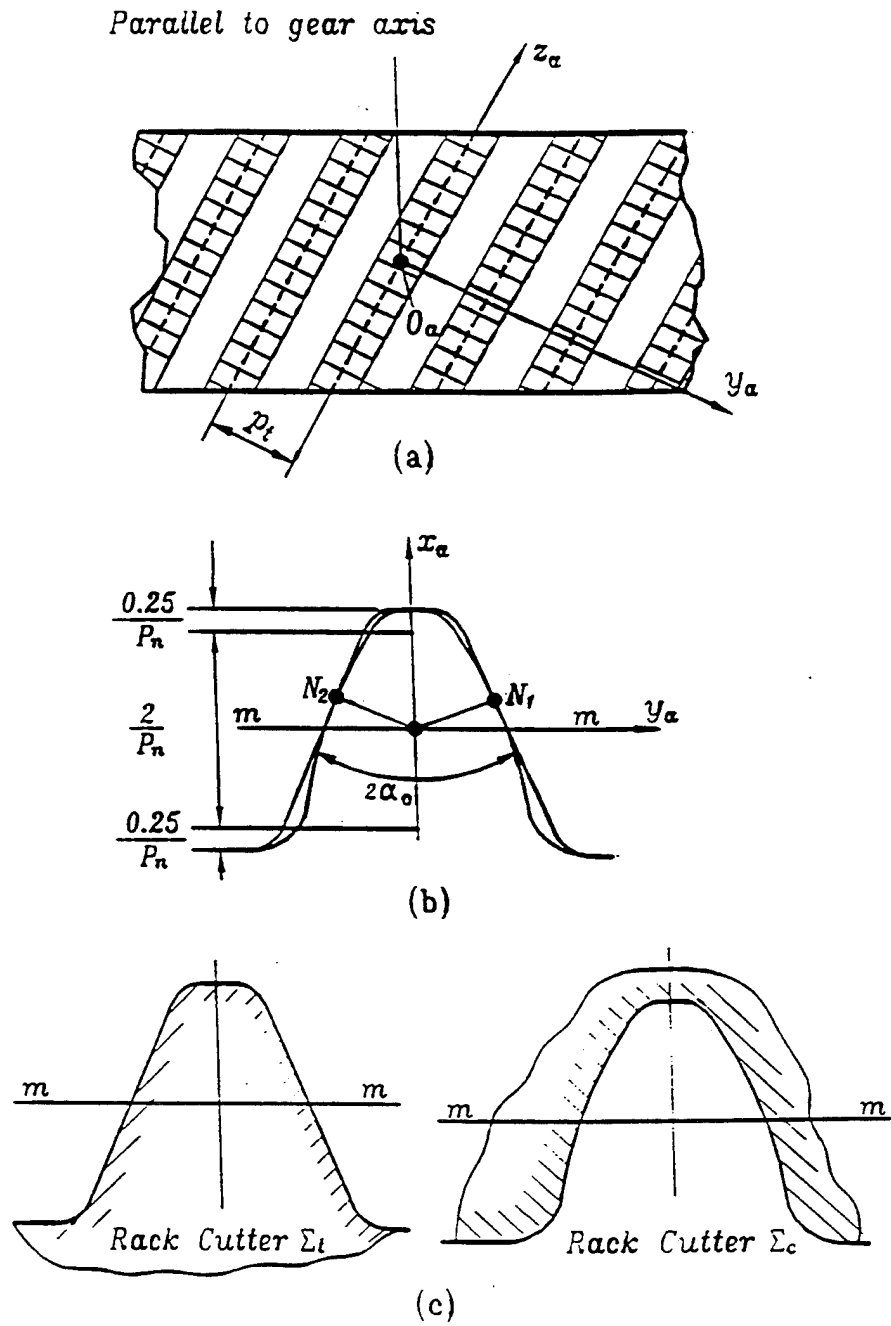
Figure 6: Schematic of rack-cutters for generation of helical modified involute gears.
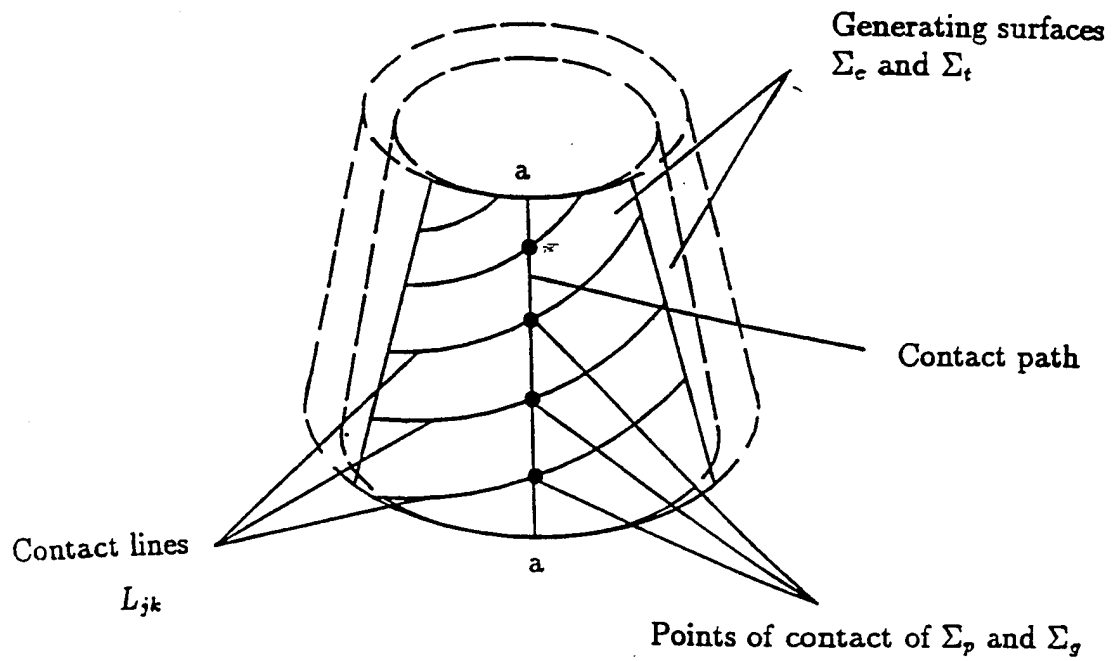
Figure 7: Contact path and contact lines for spiral bevel gears with across surface bearing contact.
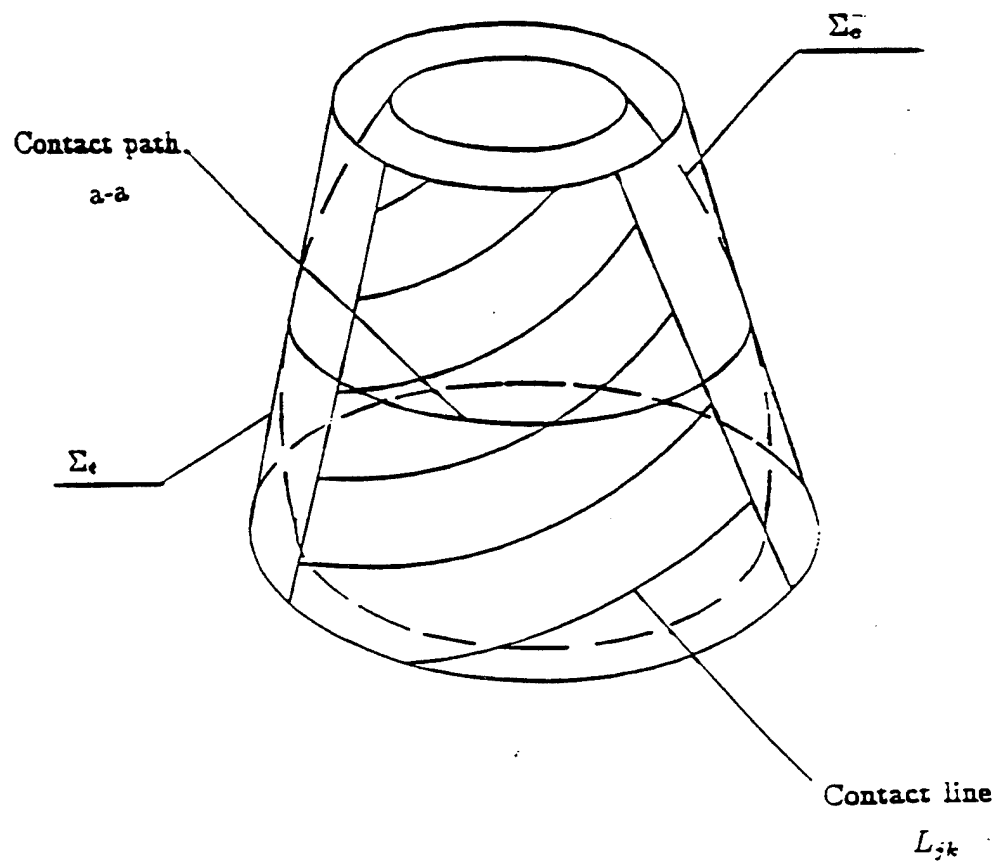
Figure 8: Generating surfaces Σ_c and Σ_t that provide contact path along the surface.
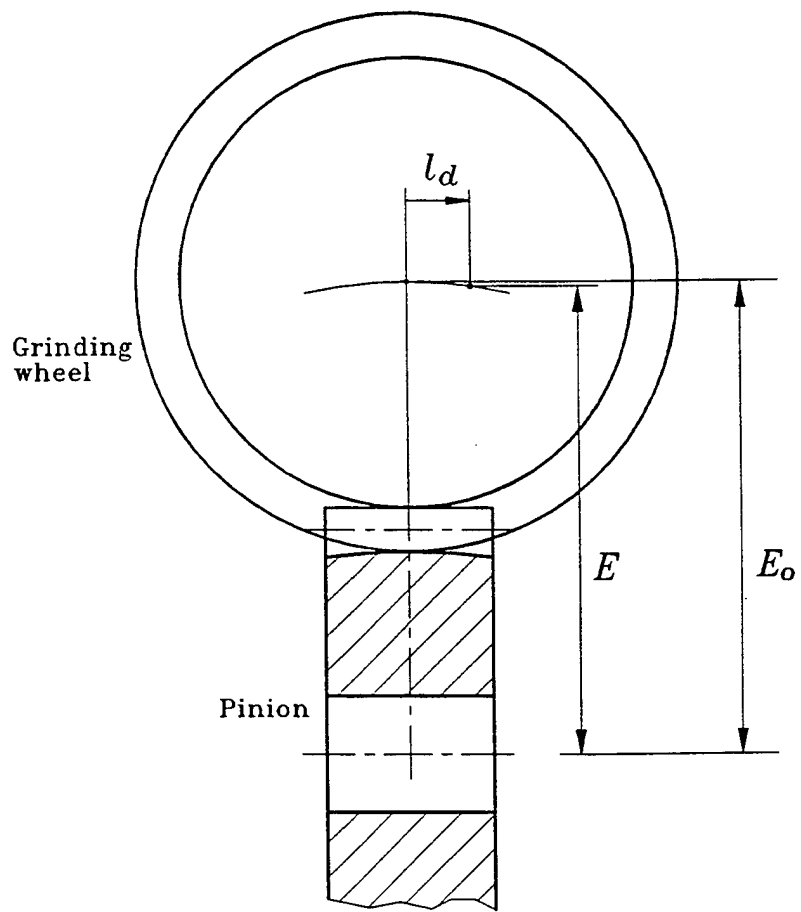
Figure 9: Form-grinding of the pinion by a plunging grinding wheel : $E = E_0 - al^2$.
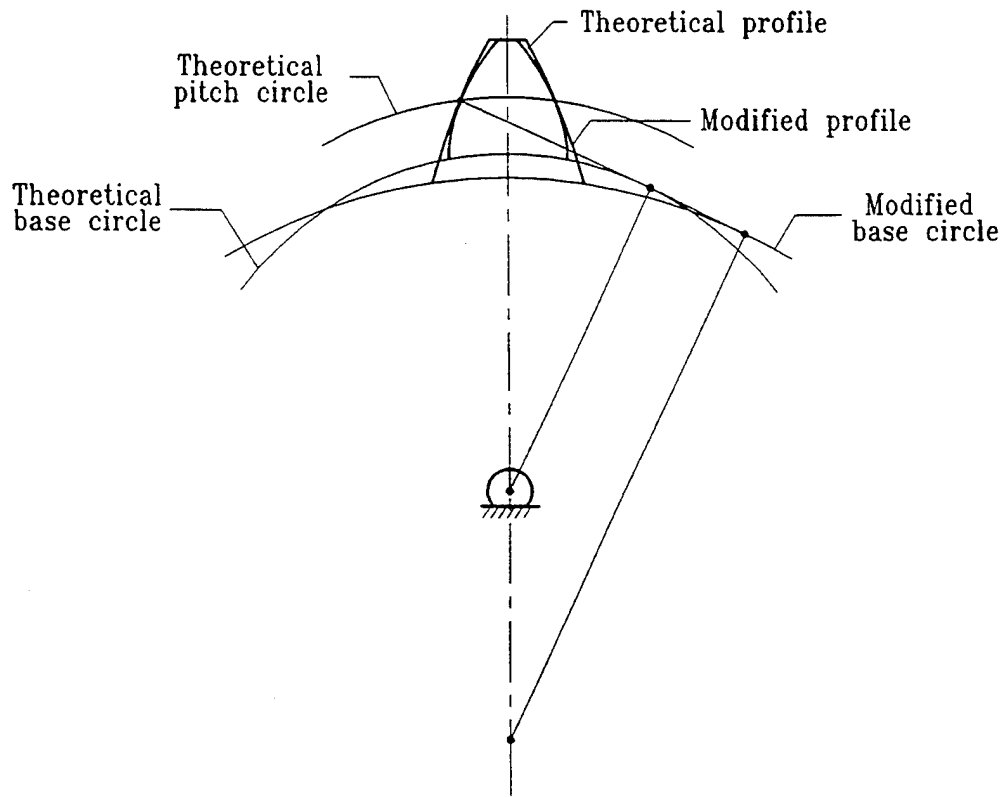
Figure 10: Modification of profile of a spur pinion or gear.
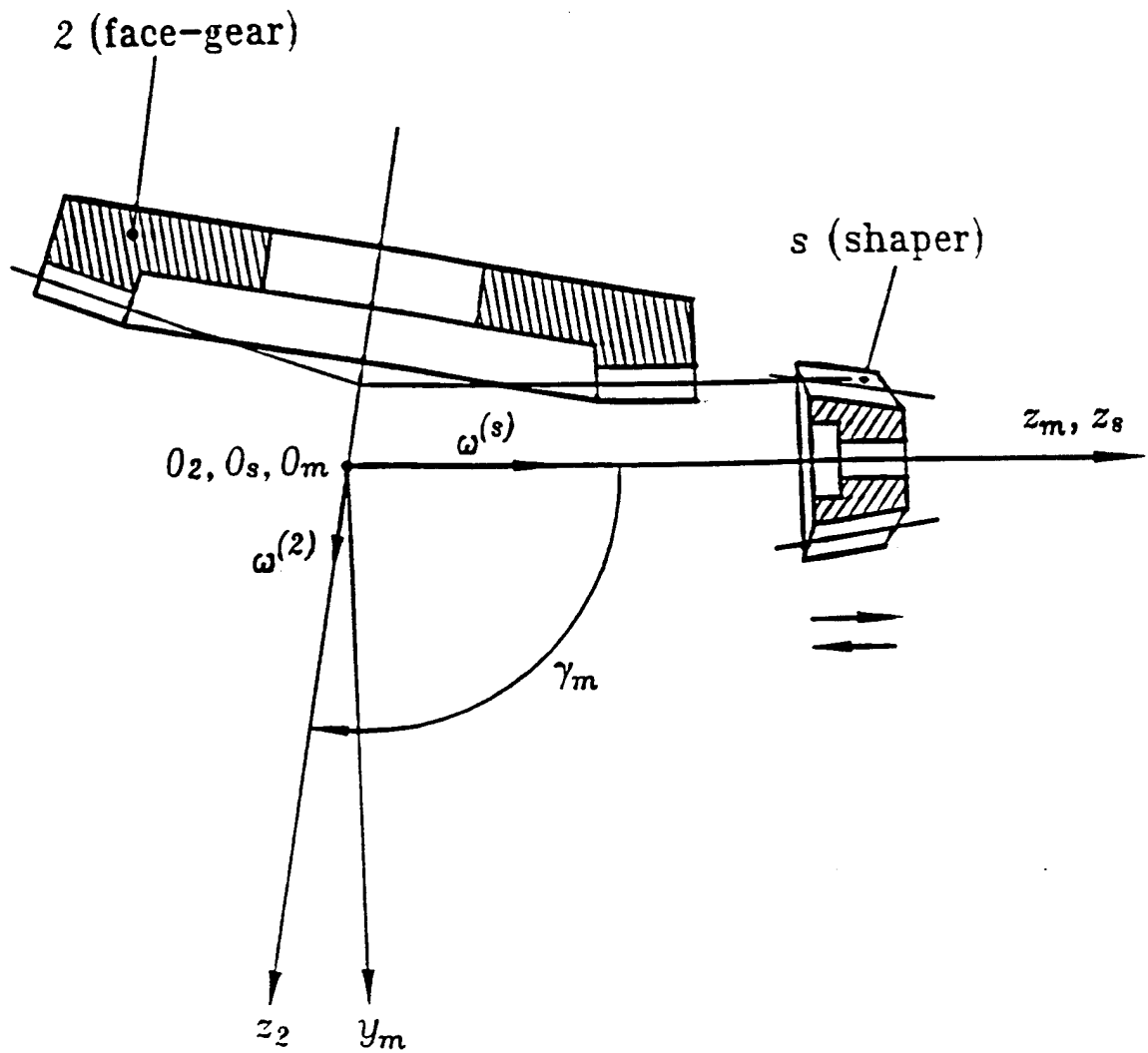
Figure 11: Schematic of generation of face-gear by shaper.
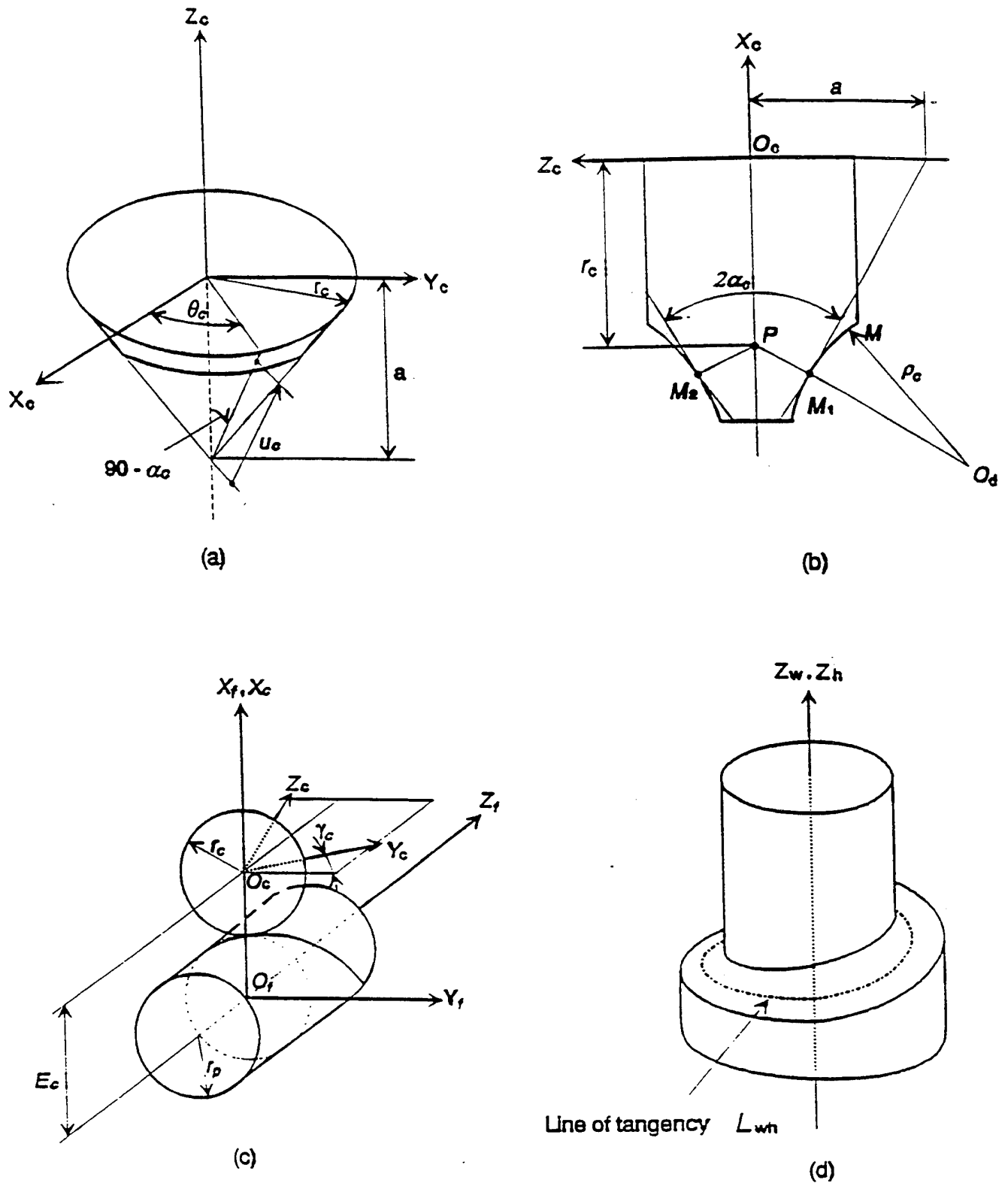
Figure 12: (a) Generating cone for hob: (b) Modified cone for worm: (c) Generation of hob and worm: (d) Helix (tangent line between hob and worm) on worm-surface.
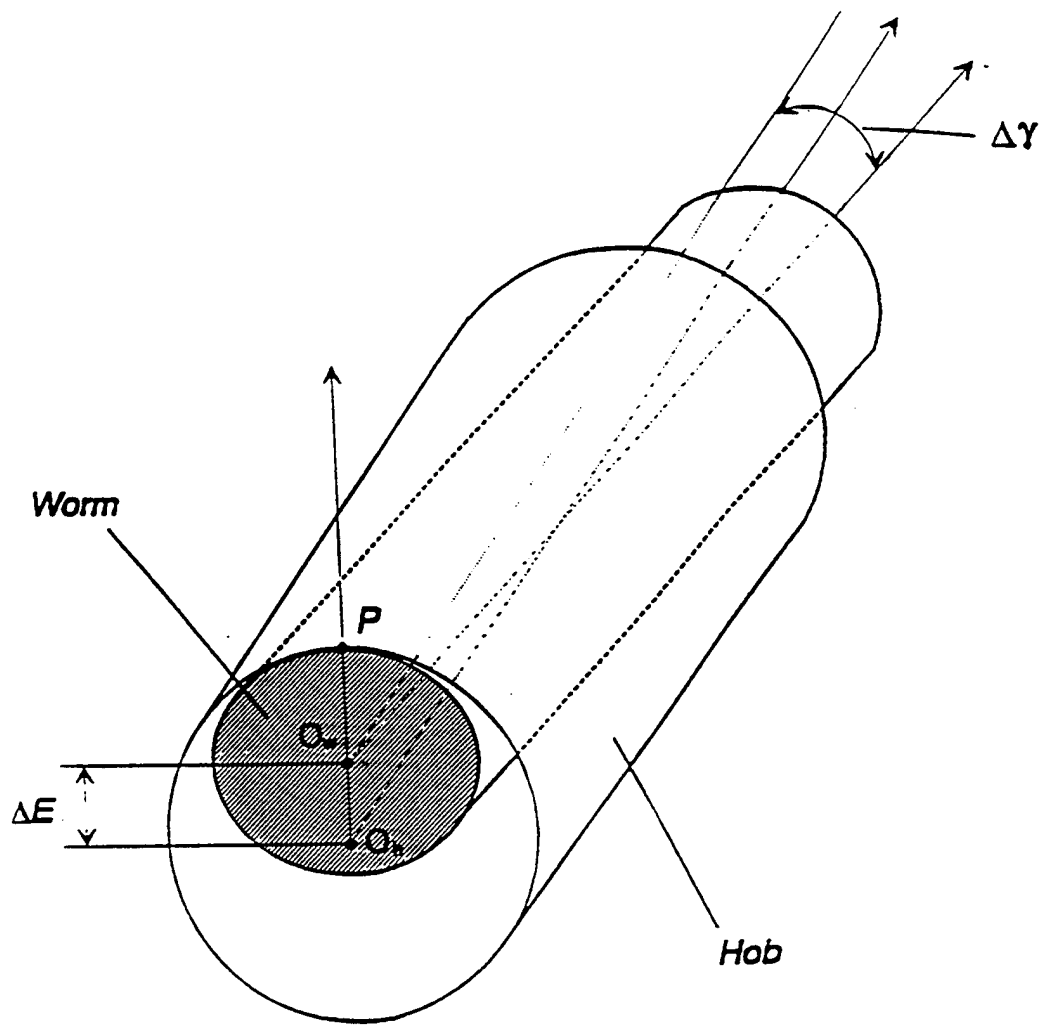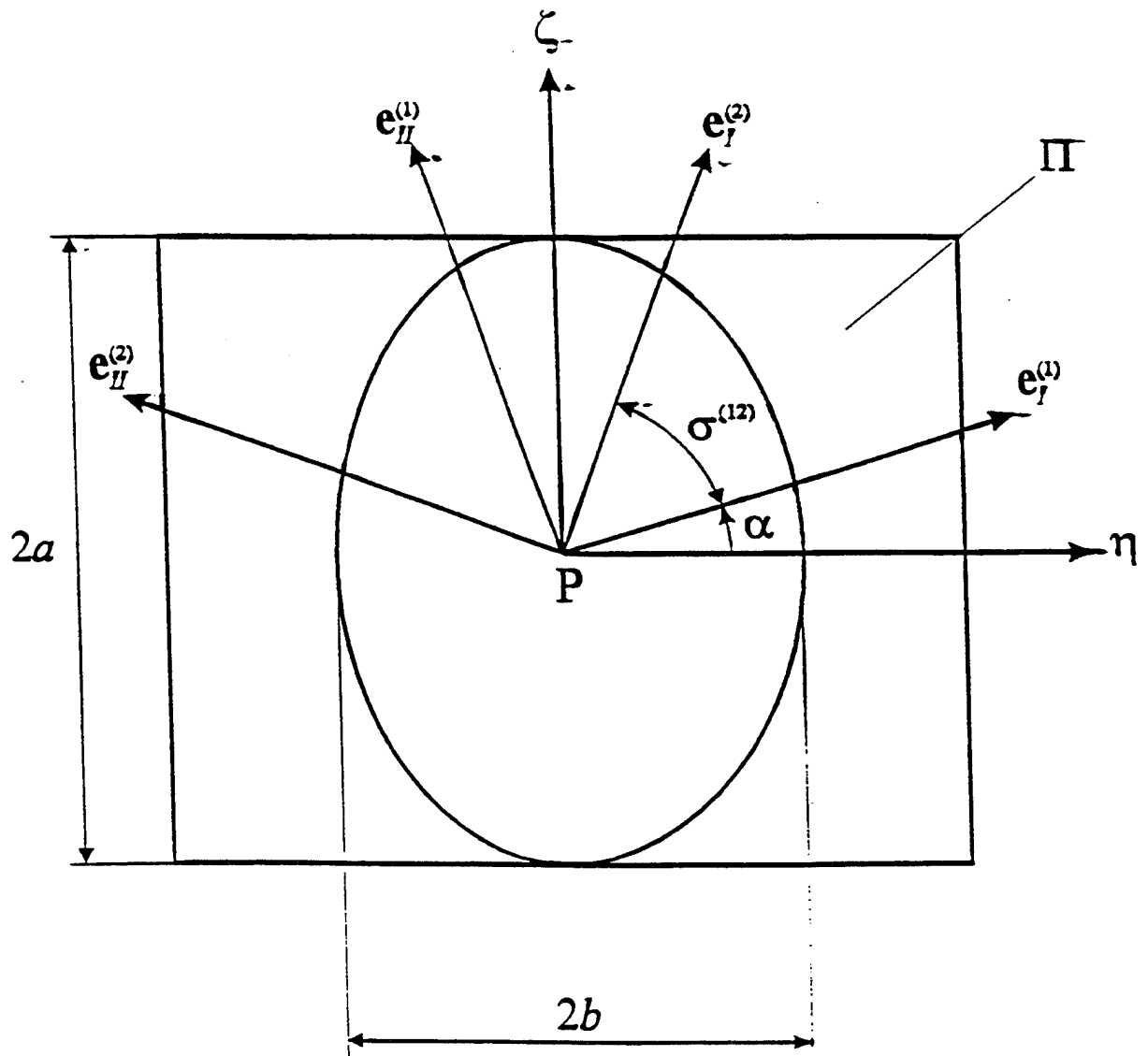
Figure 13: Pitch cylinders of over-size hob and worm.

Figure 14: Contact ellipse.

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13. ABSTRACT (Maximum 200 words)

A general approach developed for the computerized simulation of loaded gear drives is presented. In this paper the methodology used to localize the bearing contact, provide a parabolic function of transmission errors, and simulate meshing and contact of unloaded gear drives is developed. The approach developed is applied to spur and helical gears, spiral bevel gears, face-gear drives, and worm-gear drives with cylindrical worms.

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