STRENGTH CRITERIA FOR COMPOSITE MATERIALS (A LITERATURE SURVEY)

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December 1982

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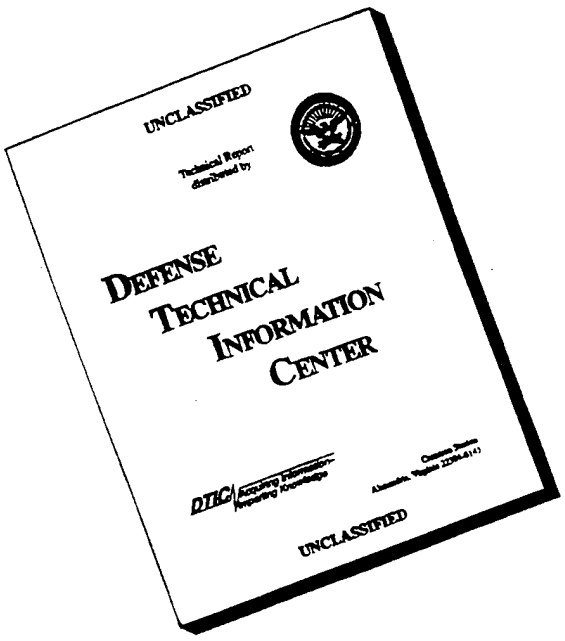

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Translation of "Sterktecriteria Voor Composietmaterialen (een Literatuur Overzicht)", Instituut TNO voor Werktuigkundige Constructies, Delft (Netherlands), Report IWECO 5071104-80-1; TDCK-75128, January 1980, pp. 1-46.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 DECEMBER 1982

STANDARD TITLE PAGE

1. Report No. NASA TM-77001	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle STRENGTH CRITERIA FOR COMPOSITE MATERIALS (A LITERATURE SURVEY)		5. Report Date December 1982 6. Performing Organization Code
7. Author(s) F. Roode		8. Performing Organization Report No. 10. Work Unit No.
9. Performing Organization Name and Address SCITRAN Box 5456 Santa Barbara, CA 93108		11. Contract or Grant No. NASw- 3542 13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Sterktecriteria Voor Composietmaterialen (een Literatuur Overzicht)", Instituut TNO voor Werktuigkundige Constructies, Delft (Netherlands), Report IWECO-5071104-80-1; TDCK-75128, January 1980, pp. 1-46. (N82-25333)

16. Abstract

Literature concerning strength (failure) criteria for composite materials is reviewed with emphasis on phenomenological failure criteria. These criteria are primarily intended to give a good estimation of the safety margin with respect to failure for arbitrary multiaxial stress states. The failure criteria do not indicate the types of fracture that will occur in the material. The collection of failure criteria is evaluated for applicability for the glass reinforced plastics used in mine detectors. Material tests necessary to determine the parameters in the failure criteria are discussed.

17. Key Words (Selected by Author(s))	18. Distribution Statement		
	Unclassified - Unlimited		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 51	22. Price

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NOMENCLATURE

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Fi, Fij	second and fourth order strength tensors (tensor polynomial strength criterion, Tsai and Mu)
X_C	compression strength in the direction of the fibers (warp)
X_t	tension strength in the direction of the fibers (warp)
Y_C	compression strength in the plane perpendicular to the direction of the fiber (woof)
Y_t	tension strength in the plane perpendicular to the direction of the fiber (woof)
Z_C	compression strength in the direction perpendicular to the plane
Z_t	tension strength in the direction perpendicular to the plane
σ1, σ2, σ3	normal stress in the direction of the fibers (1) in the plane perpendicular to the direction of the fibers (2) perpendicular to the plane (σ, σy, σz)
σ12, σ23, σ13	slip stresses in planes 1-2, 2-3, 1-3 respectively (τxy, τyz, τxz)
σ4, σ5, σ6	slip stress in planes 2-3, 1-3 and 1-2 respectively (Tsai, Mu)
Q,Q',R,R',S,S'	repulsion strength corresponding to σ4, σ5, σ6

1. INTRODUCTION /1

In the literature, different approaches have been found to study failure phenomena in composite materials. The two most important approaches may be found in [2].

1.1 Micromechanical approach

In this approach, the starting point is the study of the failure behavior of the components involved (matrix material fiber and finally the layers or lamellae).

The failure behavior of the different layers is subsequently combined into the failure behavior of the complete laminate.

The description of the failure in a composite material is a fairly complex task in which complete computer programs must be used to calculate the strength tension relation. To increase the practical applicability of such an approach, sometimes a simplified approach is used in which only two points on the stress-tension curve are calculated. These are the points at which the first failure occurs in the composite material (comparable to the fluid tension in a metal) and the final failure stress. The last point is determined by a so-called "netting theory" in which it is assumed that the fibers can only absorb normal stresses. Even in this simplified case, it is hardly possible to apply such failure analyses for practical engineering purposes. In practical engineering, it is always necessary to obtain, on the basis of simple relationships between mechanical parameters, an impression of the safety of a structure with regard to failure.

The inapplicability of the theory applies even more strongly to the approaches often found in the literature in which failure mechanics and static considerations are used. Especially for glass fibers (as used in mine detectors) a static approach for the brittle

failure behavior is inadmissible (because of the brittle failure behavior).

In [1], we find an overall survey of the study in the area of the micromechanical approach. The same publication also indicates that the usefulness of the micromechanical approach resides mainly in the possibility of choosing between different compositions of the composite materials.

Since for mine detectors the material must be considered basically firm, here actually this study loses much of its usefulness.

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The benefit of the micromechanical study must, within the framework of the study of the composite material for mine detectors be sought in the possibilities of achieving by means of these theories an estimate of the reliability of certain types of experiments.

In the micromechanical approach, the experiments are carried out in such a manner that only one form of failure occurs. In the macromechanical approach to be considered further on, much less attention is paid to this.

To make sure that a certain failure criterion gives conservative results in all cases, however, it is certainly recommended to conduct experiments also for one failure form so that there is a clear definition of the moment of appearance of the "failure".

1.2 Macromechanical approach

In the macromechanical approach, the primary purpose is to achieve a fairly simple criterion presenting the failure of the total laminate as a function of the load state. The number of criteria formulated in the course of time is very large. Reviews of such criteria may be found in [7,8,9].

A great drawback of most of the criteria (at least for the purpose of the study of mine detectors) is that one proceeds a priori from the hypothesis that the composite material is used in an optimal manner. This optimal use must be referred mainly to the stress state. Most failure hypotheses start from the assumption that the material returns to a plane stress state, and in this sense the failure hypotheses hardly differ from those formulated for the layers (lamellae).

The best known failure criteria in this area are:

a. Maximum stress theory (Stowell, Liu [19], Jenkins [23]

Here an arbitrary stress is decomposed into components along the different principal axes of the material.

Failure occurs when one of the stress components becomes higher than the failure limit corresponding to this direction.

In this connection, no difference is drawn between failures in tension or in compression, although the procedure itself suggests this. A problem arising in the maximum stress theory is also the fact that in the region in which transition takes place from one failure criterion (for example, tension strength in the direction of the fiber) to another failure criterion (for intance, slip) the strength is over-estimated (see Figures 1, 2 and 3). /3

b. Maximum tension theory

It is quite similar to the maximum stress theory in which now the tension in the different directions is considered the decisive factor. The theory which was proposed in 1966 by General Dynamics, Fort Worth Division [21], is nothing more than the application of the St. Venant maximum tension theory.

c. Deformation energy

The overwhelming majority of phenomenological failure criteria for composite materials are derived from deformation energy considerations and in particular the form changing energy.

The basis for this was obtained by Von Mises (1900) for isotropic materials with the formula

$$(\sigma x - \sigma y)^2 + (\sigma y - \sigma z)^2 + (\sigma z - \sigma x)^2 + 6(\tau xy^2 + \tau yz^2 + \tau zx^2) = 2\sigma_v^2 \qquad (1)$$

Although Von Mises had intended the criterion primarily for the flow of material, in the course of time for metals, it is only used to describe the flow.

In the area of composite materials, nevertheless, the application of deformation energy criteria is still maintained to describe the failure. As long as the materials considered are brittle, this is a reasonable starting point. A number of new criteria have been derived from the Von Mises criterion. Strictly speaking, most of the criteria do not give a real deformation energy, but rather a relation in stress variants. For convenience, these criteria are also called deformation energy because they are mostly an extension of the Von Mises formulation.

Hill [6] extended subsequently the Von Mises criterion to anisotropic materials in the form:

$$F(\sigma y - \sigma z)^2 + G(\sigma z - \sigma x)^2 + H(\sigma x - \sigma y)^2 + 2L\tau^2_{yz} + 2M\tau^2_{zx} + 2N\tau^2_{xy} = 1 \qquad /4$$

in which F, G, H, L, M and N are material parameters.

In this equation (2), it is also assumed implicitly that:

--the material is orthotropic

--there is no difference between tension and compression strength; with the relation $X_t = X_C = X$; $Y_t = Y_C = Y$, it was then simple to derive:

$$\left.\begin{aligned} 2F &= \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2} \qquad & 2L &= \frac{1}{Q^2} \\ 2G &= \frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2} & 2M &= \frac{1}{R^2} \\ 2H &= \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} & 2N &= \frac{1}{S^2} \end{aligned}\right| \qquad (3)$$

The six parameters of the failure criterion are thus determined entirely by the three tension strengths and the three slip strengths for the mutually perpendicular directions of the material.

To make it possible to compare with the following failure criteria, it is convenient to write the Hill theory in the following form:

$$F_{ij}\sigma_i\sigma_j = 1 \qquad i = 1,2 \ldots 6 \qquad (4)$$

The repetition of the sub-indices is reduced to the summation convention in which the sum is measured over all the values of the sub-indices (1 to 6 inclusively).

The term Fij can then be considered as a 6x6 matrix of the form:

$$F_{ij} = \begin{bmatrix} \frac{1}{X^2} & \frac{-1}{2}\left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right) & \frac{-1}{2}\left(\frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}\right) & 0 & 0 & 0 \\ & \frac{1}{Y^2} & \frac{-1}{2}\left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\right) & 0 & 0 & 0 \\ & & \frac{1}{Z^2} & 0 & 0 & 0 \\ & & & \frac{1}{Q^2} & & \\ & & & & \frac{1}{R^2} & \\ & & & & & \frac{1}{S^2} \end{bmatrix} \qquad (5)$$

The greatest drawback of the Hill criterion is that it is impossible to differentiate between tension and compression strength, while it was just established for composite materials that there can be a great difference between the two values. /5

One of the first attempts to include also the compression forces of the material in the failure criterion was made by Marin [10]. The latter used, to this end, the Hill criterion written in the main tensions (indicated for convenience here as σx^1, σy^1, σz^1

$$[Ka\ (\sigma x^1 - \sigma y^1)]^2 + [Kb(\sigma y^1 - \sigma z^1)]^2 + [Kc(\sigma z^1 - \sigma x^1)]^2 = 2\sigma xy^2$$

To eliminate the differences in tension and compression, he modified the relation into:

$$(\sigma x^1 - a)^2 + (\sigma y^1 - b)^2 + (\sigma z^1 - c)^2 + g\left\{(\sigma x^1 - a)(\sigma y^1 - b) + (\sigma y^1 - b)(\sigma z^1 - c) + (\sigma z^1 - c)(\sigma x^1 - a)\right\} = \sigma x^2 y \qquad (7)$$

The difference from the previous Hill relation is actually only that three terms have been added, specifically, σx^1, σy^1 and σz^1.

If the failure criterion is considered as a surface in the six-dimensional stress space, the addition of linear terms in the failure criterion implies that the origin of the rupture stress surface is shifted.

If we attempt to relate such a failure criterion (with shifted origin) with mechanical phenomena, this means that it is assumed that an internal stress is the cause of the difference in tension and compression force.

From the more micromechanically directed failure investigations, it is known [1] that the difference in the two strengths is mostly caused by a difference in the failure mechanism (in compression, it is not the material stress/strength which is decisive, but the danger of cracking the fiber).

The above illustrates the earlier remark that no attempt was made to describe or explain the failure mechanisms with the failure criteria.

A great drawback of the Marin theory is the fact that the failure criterion is given in terms of main stresses. Such a direction of the main stress does not have to coincide with the main directions (symmetry planes) of the material, see Grescszuk [2]. The problem is that then the tension/compression strengths must be known in other directions than the main directions of the material, to be able to determine the parameters of the failure criterion. Practically, this then raises many problems which the Marin theory had hardly touched. /6

The overwhelming majority of later authors recognized the problem in the Marin theory and have, therefore, deviated from the more general formulation of the Hill criterion [5].

Some of these theories are discussed below.

Tsai and Azzi proposed a simplification of the Hill criterion by assuming that the composite material is normally used in an optimal manner and is, therefore, in a flat stress state.

Assuming $\sigma 3 = \tau 13 = \tau 23 = 0$ (5) is converted into

$$\frac{1}{X^2}\sigma 1^2 - \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right)\sigma 1\sigma 2 + \frac{1}{Y^2}\sigma 2^2 + \frac{1}{T^2}\tau 12^2 = 1 \qquad (8)$$

A second hypothesis which is often put forward for composite materials is that a cross-section perpendicular to the fiber direction should behave isotropically, i.e., Y = Z.

It is apparent from the comparison that most pass through the Tsai-Azzi criterion.

$$\frac{\sigma 1^2}{X^2} - \frac{\sigma 1\sigma 2}{X^2} + \frac{\sigma 2^2}{Y^2} + \frac{\tau 12^2}{T^2} = 1 \qquad (9)$$

For the glass fabric composite materials considered here, this last designation cannot apply directly since the hypothesis of isotropism in the cross-section is not maintained.

The more general notation (8) should basically be defensible still were it not that bending any slip may practically also occur perpendicularly.

A method proposed by Tsai and Azzi [2] to solve this problem consists in applying the criterion (9/8) by layer (lamella).

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But it is very doubtful whether this approach can be implemented for practical purposes. It would specifically be necessary to establish the stress state per layer.

This might be done for composite materials with exact composition (winding techniques)(apart from the fact that it would involve an enormous amount of work). For composites with more arbitrary structure, this approach would hardly be reasonable. The reason why the application of the Tsai-Azzi criterion is not reasonable for the glass fabric considered here is the fact that it is not at all clear whether the failure in a layer is determined by a flat stress state. Specifically, the glass fibers in such a layer are not straight so that the third stress component may also have an effect.

The last drawback of the Tsai-Azzi criterion is the same as for the Hill criterion and concerns the fact that no differentiation is made between tension and compression strength.

For the sake of completeness, another simplification of the Tsai-Azzi criterion is indicated.

Indeed, in many investigations, it was found that the interaction term from (9) $\frac{\sigma_1\sigma_2}{x^2}$ may be eliminated in many cases so that

the Tsai-Azzi criterion is converted into the Norris-Puck criterion in the form

$$\frac{\sigma 1^2}{X^2} + \frac{\sigma 2^2}{Y^2} + \frac{\tau 12^2}{T^2} = 1 \qquad (10)$$

This aspect will be discussed in greater detail further on.

A failure criterion which can avoid most of the above-mentioned drawbacks was established by Hoffman [11].

Hoffman also started from the original Hill criterion (2) and added to this relation a number of linear stress terms to be able to eliminate the difference between tension and compression:

$$C1\,(\sigma 2 - \sigma 3)^2 + C2\,(\sigma 3 - \sigma 1)^2 + C3\,(\sigma 1 - \sigma 2)^2 \qquad (11)$$
$$+ C4\,\sigma 1 + C5\,\sigma 2 + C6\,\sigma 3 + C7\,\sigma 23^2 + C8\,\sigma 13^2 + C9\,\sigma 12^2 = 1$$

Such a failure criterion has thus nine material parameters /8 and therefore a large number of tests are needed to establish these material parameters.

With the results of

--three tension tests Xt, Yt, Zt

--three compression tests Xc, Yc, Zc and

--three slip tests Q, R, S the following relationships may be established:

$$C1 = \frac{1}{2}\left(\frac{1}{YtYc} + \frac{1}{ZtZc} - \frac{1}{XtXc}\right)$$

$$C2 = \frac{1}{2}\left(\frac{1}{ZtZc} + \frac{1}{XtXc} - \frac{1}{YtYc}\right)$$

$$C3 = \frac{1}{2}\left(\frac{1}{XtXc} + \frac{1}{YtYc} - \frac{1}{ZtZc}\right)$$

$$C4 = \frac{1}{Xt} - \frac{1}{Xc}$$

$$C5 = \frac{1}{Yt} - \frac{1}{Yc} \qquad (12)$$

$$C6 = \frac{1}{Zt} - \frac{1}{Zc}$$

$$C7 = \frac{1}{Q^2}$$

$$C8 = \frac{1}{R^2}$$

$$C9 = \frac{1}{S^2}$$

The failure criterion is established completely with these nine parameters/tests.

This criterion has a number of remarkable aspects: the fact that no difference is made between positive and negative slip strength. This possibility is left open in many of the criteria discussed below. It is also doubtful whether this extension is proper for the orthotropism considered here. /9

It may also be noted that the equation (12) is a quadratic equation so that the failure surface in the stress space is elliptical and convex (with origin not necessarily at zero).

With the definition of the Hoffman criterion practically, the maximum is retained of the original Hill criterion. But actually

of the Hoffman criterion it should be stated that this criterion has no physical basis but is rather a mathematical approximation. Many researchers have observed subsequently that a problem which arises with all the criteria considered here consists in the fact that the failure criteria are defined with regard to the principal axes of the material.

This implies that, in the calculation of an actual structure, the arbitrary stress state must be converted to the stress components in the main directions of the material.

It may also be established now that the problem is not so important for the orthotropic glass fabric reinforced composites considered here. If in this connection we refer to final element calculations, it happens in most cases that the main direction of the material coincides with the main direction of the elements.

This can also be a problem for other anisotropic materials. For the sake of completeness, we will also discuss below the approximations in which the conversion of the stress aces is resolved with respect to the tensorial algebra.

For the purpose of comparison with other failure criteria, consequently the Hoffman criterion is also written again the matrix form which like equation (4) can also be written as

$$Fi\,\sigma i + Fij\sigma i\sigma j = 1 \qquad (13)$$

with i = 1, 2...6. $\quad$ not i = 1,2 6

Here we have

$$Fi = \begin{vmatrix} \frac{1}{Xt} - \frac{1}{Xc} \\ \frac{1}{Yt} - \frac{1}{Yc} \\ \frac{1}{Zt} - \frac{1}{Zc} \\ 0 \\ 0 \\ 0 \end{vmatrix} \qquad (14)$$

/10

and

$$F_{ij} = \begin{bmatrix} \frac{1}{X_tX_c} & \frac{-1}{2}\left|\frac{1}{X_tX_c}+\frac{1}{Y_tY_c}-\frac{1}{Z_tZ_c}\right| & \frac{-1}{2}\left|\frac{1}{Z_tZ_c}+\frac{1}{X_tX_c}-\frac{1}{Y_tY_c}\right| & 0 & 0 & 0 \\ & \frac{1}{Y_tY_c} & \frac{-1}{2}\left|\frac{1}{Y_tY_c}+\frac{1}{Z_tZ_c}-\frac{1}{X_cX_t}\right| & 0 & 0 & 0 \\ & & \frac{1}{Z_tZ_c} & 0 & 0 & 0 \\ & & & \frac{1}{Q^2} & & \\ & & & & \frac{1}{R^2} & \\ & & & & & \frac{1}{S^2} \end{bmatrix}$$

d. Tensor polynomials /11

The following failure criteria are purposely no longer calculated to the deformation energy approximation.

Although a number of the theories can be reduced to deformation energy in the definition of the failure criteria, the starting point is a purely mathematical description of the failure criterion. The Hoffman criterion can be considered as a transition area (no tensors are used there yet).

One of the first theories in this area was formulated by Goldenblat and Kopnov [5] with the relation:

$$(F_i\sigma_i)^{\alpha} + (F_{ij}\sigma_i\sigma_j)^{\beta} + (F_{ijk}\sigma_i\sigma_j\sigma_k)^{\gamma} = 1 \qquad (15)$$

in which once again the summation convention is adopted with regard to the subindices:

Here it was also assumed:
Fi = strength tensor of the second order
Fij = strength tensor of the fourth order
Fijk = strength tensor of the sixth order

The conversions of the tensor in the rotations of the axes are known here from tensorial algebra. The great advantage of tensor polynomials is also the fact that the criterion is defined with respect to an arbitrary system. Goldenblat and Kopnov have considered in particular a special case of equation (15) with

$$\alpha = 1,\ \beta = \tfrac{1}{2},\ \gamma = -\infty$$

so that equation (15) is converted into:

$$F_i\sigma_i + \sqrt{F_{ij}\sigma_i\sigma_j} = 1 \qquad (16)$$

Tsai and Wu (4) have indicated that the square root in formula (16) is very impractical, since the result is a + sign. The Goldenblat and Kopnov criterion is, therefore, best applied in the quadratic form:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j - (F_i\sigma_i)^2 = 1 \qquad (17)$$

But even this form of the Goldenblat and Kopnov criterion is not much used practically. A problem which arises for this criterion refers to the definition of the interaction term Fij.

If these terms are determined directly with experiments, it may occur that the failure surface in the stress space is no longer closed (elliptical) but is converted into a parabolic or hyperbolic /12 surface which may lead to unrealistic theoretical strength properties.

This phenomenon was also indicated by Ashkenazi [20]. The above-mentioned problem becomes even greater if the third power term ($F_{ijk}\sigma_i\sigma_j\sigma_k$) is included.

Apart from the fact that in that case a very large number of interaction terms have to be determined, such a cubic equation may often lead to a nonclosed failure surface in the stress space.

To solve the above-mentioned problem Tsai and Wu [4] established a criterion which is more general than the Goldenblat-Kopnov criterion, simpler to apply and results in a closed (elliptical) failure surface in the stress space.

The Tsai-Wu criterion has the form:

$$F_{i\sigma i} + F_{i\sigma i\sigma j} = 1 \quad (i = 1,.....6) \tag{18}$$

Here, too, the summation convention is applied with regard to the sub-indices. To take into account the fact that the failure surface is elliptical (in the stress state), the following stability requirements are imposed:

$$F_{ii}\, F_{jj} - F_{ij}^2 \geq 0 \tag{19}$$

In this connection, the striking detail is that the original authors also accepted the equality signs in equation (19), while the later investigators established, on the basis of a more graphic interpretation of the failure surface (14), that the equality sign was not acceptable either.

It should be noted that the general equation (16) contains altogether (basically) 42° of freedom (unknowns). This number of unknowns may be reduced to a considerable extent by assuming that the Fij terms are symmetrical. Such an assumption may be made if we start from the hypothesis that there is a so-called F(σi) failure potential. Here the terms Fij are defined by:

$$F_{ij} = \partial^2 f / \partial\sigma_i \partial\sigma_j = \partial^2 f / \partial\sigma_j \partial\sigma_i = F_{ji} \tag{20}$$

The assumption of a failure potential implies nothing other than the assumption that the failure phenomenon is independent of the load path. Such a hypothesis is made essentially for all the

above-mentioned failure criteria. How far this assumption is justified depends both on the type of material and the phenomenon described as failure criterion. If the failure criterion is used /13 to describe a kind of fluid limit (or the first point at which a break occurs anywhere in the laminate), this assumption is generally valid.

If this criterion is used to describe the final total failure of the material, the assumption with regard to the independence of the load path is less valid. In such a case, specifically the final failure is preceded by plastic deformations which are to some extent path-dependent. Nevertheless, the failure criterion may still be valid for the so-called radial stress paths. In this connection, radial stress paths should be considered as paths in the stress space in which the corresponding ratio of the stress components would remain the same.

Since the failure criteria formulated in this report must be considered primarily as a design criterion and not so much a criterion in which very exact predictions must be made on the failure stresses occurring, such path-dependent effects may be left out of consideration preliminarily. The simplification taken then is that the path-dependent effects are included in the safety factors.

With the assumption of formula (20), the number of unknown parameters was reduced from 42 to 27 (6 for Fi and 21 for Fij). A still greater reduction in the number of degrees of freedom may be achieved by starting from orthogonal material properties which is directly permissible here for the material considered.

With such an isotropism, it may be stated directly that a connection between the normal and slip stresses may not arise so that terms such as F16 may be equal to zero. It may also be stated that if the reference system of axes coincides with the material (strength) main directions, we have [7]: $F4 = F5 + F6 = 0$.

In the matrix form, the relation (18) is then written as /14
follows:

$$Fi = \begin{bmatrix} F1 \\ F2 \\ F3 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad Fij = \begin{bmatrix} F11 & F12 & F13 & 0 & 0 & 0 \\ & F22 & F22 & 0 & 0 & 0 \\ & & F33 & 0 & 0 & 0 \\ & & & F44 & 0 & 0 \\ & & & & F55 & 0 \\ & & & & & F66 \end{bmatrix} \qquad (21)$$

The parameters in equation (21) must be obtained again from tension and compression tests. "Simple" single axis failure tests may be used for the terms of Fi and for the diagonal terms in Fij.

As an illustration: When loading in direction 1, the following values are found for tension and compression strength respectively $\sigma 1 = Xt$ and $\sigma 1 = -Xc$ (let on the minus sign).

For equation (18/21), we may write

$$Xt^2\ F11 + Xt\ Fi = 1 \text{ and}$$

$$Xc^2\ F11 - Xc\ Fi = 1$$

from which it follows that:

$$Fi = \frac{1}{Xt} - \frac{1}{Xc} \text{ and } F11 = \frac{1}{XtXc} \qquad (22a)$$

By a similar method, we may obtain for the other material directions

$$F2 = \frac{1}{Yt} - \frac{1}{Yc} \qquad F22 = \frac{1}{YtYc}$$

$$F3 = \frac{1}{Zt} - \frac{1}{Zc} \qquad F33 = \frac{1}{ZtZc} \qquad (22b)$$

$$F44 = \frac{1}{Q^2} \qquad F55 = \frac{1}{R^2} \qquad F66 = \frac{1}{S^2}$$

Here it may be assumed directly that for the orthotropic material there is no difference between the so-called positive and negative slip.

A comparison of relations (22) with those from the Hoffman criterion (14) shows that the characteristics discussed up to now are exactly the same.

The great difference between the Hoffman criterion and the Tsai-Wu (tensor polynomial) criterion lie in the definition of the /15
cross-terms F12, F13 and F23.

For the Hoffman criterion, the cross-terms are dependent parameters which are established completely if the parameters in equation (22) are determined.

In the Tsai-Wu criterion, the cross-terms are independent material parameters which have also to be determined by separate experiments.

The authors of the criterion (Tsai and Wu) consider that the advantage of this independence in the cross-terms resides mainly in the greater flexibility of the criterion to achieve a proper prediction for the failure strength for multiaxial stress states also. In this connection, Tsai and Wu [7] state that most failure criteria describe well the uniaxial failure strengths, but raise problems in the multiaxial stress state. The similarity of the form of the Hoffman and Tsai-Wu criteria with regard to the uniaxial failure strength seems to confirm this view. But even the more flexible formulation of the Tsai-Wu criterion leads rather to a shift of the problem than to its solution. Specifically the problem which now has to be solved for the Tsai-Wu criterion is the question as to which experiment is most suitable for determining the cross-terms. The historical developments in this connection are sufficiently illustrative.

The Russian investigators used primarily tension and compression tests on a so-called 45° blank (the material axes form an angle of 45° with regard to the main axes of the blank).

If for plane 1-2 the experimental results are indicated as Ut and Uc, we find for F12

$$F12 = \frac{2}{Uc^2}\left[1 + \frac{Uc}{2}\left(\frac{1}{Xt} - \frac{1}{Xc} + \frac{1}{Yt} - \frac{1}{Yc}\right) - \frac{Uc^2}{4}\left(\frac{1}{XtXc} + \frac{1}{YtYc} + \frac{1}{S^2}\right)\right]$$

or (23)

$$F12 = \frac{2}{Ut^2}\left[1 - \frac{Ut}{2}\left(\frac{1}{Xt} - \frac{1}{Xc} + \frac{1}{Yt} - \frac{1}{Yc}\right) - \frac{Ut^2}{4}\left(\frac{1}{XtXc} + \frac{1}{YtYc} + \frac{1}{S^2}\right)\right]$$

In [12], Tsai and Wu also indicated that these experiments were hardly sensitive for a variation in F12 (see Figure 4). In /16 this publication, the authors also say that much better results may be expected for a positive slip test on the 45° blank. For an experimentally determined value V, F12 is defined by:

$$F12 = \frac{-1}{2V^2}\left[1 - V\left(\frac{1}{Xt} - \frac{1}{Xc} - \frac{1}{Yt} + \frac{1}{Yc}\right) - V^2\left(\frac{1}{XtXc} + \frac{1}{YtYc}\right)\right] \quad (24)$$

But the practical results seem to be very disappointing, even for such an experiment.

In [14], Collins and Crane explained with a purely graphic interpretation of the Tsai-Wu criterion that the positive slip experiments on 45° blanks probably do not provide the desired results. This type of slip experiment is indeed hardly used any longer.

An additional problem in the experimental determination of the cross-terms depends on the fact that the stability criterion (19) has always to be satisfied. Thus, it can happen very often that the experimentally determined value cannot be applied to the cross-terms.

This is illustrated by the results of Pipes and Cole [13] when the cross-terms F12 are determined with off-axis experiments (experiments in which the material forms an angle with the main axis of the blank).

Of the four experimentally determined values of F12, only one value seems to satisfy the stability criteria. On the basis of these results, the conclusion may, therefore, be drawn immediately that it is impossible to determine the cross-terms with these experiments.

In later publications, especially by Wu [7 and 17] alternative procedures are proposed to determine the values of the cross-terms. In these procedures we start from a really biaxially stressed blank (stress $\sigma 1$, $\sigma 2$). In Wu's procedure, there must be an optimal biaxiality ratio B B = $\sigma 1/\sigma 2$) determined for which the value F12 can be defined.

Unfortunately, the optimal value of B depends on the value of F12, so that an iteration process must be used (with the corresponding number of tests).

In the same publications, it is also indicated that a decision may be taken to include terms of the higher order (Fijk, Fijkl, /17 etc.) in the failure criterion. This decision depends on the (expermentally determined) value of Fij with regard to the precision of the solution (determined on the basis of the hypothesis that the spread in experimental failure experiments is, for instance, approximately 10%). If the value of Fij is greater than the precision of the solution, it will be necessary to include additional higher terms. This is not related to the fact that the situation becomes even more complicated when these terms of higher order must be included. Even for these terms of higher order, optimal multiaxial experiments must be defined with the necessary interaction work concerned. Moreover, the terms of higher order (Fijk) still depend on the lower order terms Fij. According to Wu, the Fij terms can be determined first, after which the determination of the Fijk terms no longer affects the values of Fij.

The practical calculations in [15] also show that the values of Fij must be adjusted to a great extent after the determination

of Fijk. Tennyson, McDonald and Nanyare used in [15] an actual hybrid computation technique to be able to describe properly the interaction between the different cross-terms. For the purpose of the intended design (for a material not considered here), such an effort is totally unwarranted. Therefore, it may be stated immediately that terms such as Fijk must not be included in the failure criterion. This becomes even more apparent if we recall that the use of terms such as Fijk implies immediately that the failure surface in the stress space is no longer convex with all the related problems. To sum up, it may be stated that the use of a (Tsai-Wu) tensor polynomial approximation does give greater flexibility but that this is achieved to a great extent in the form of more complex experiments. In the experimental determination of cross-terms such as F12, one should also consider thoroughly the benefit achieved in the sense of a more exact description of failure under a multiaxial stress state, as compared with the much more complicated experiments. The next chapter will discuss this in greater detail.

/18

2. CHOICE OF A FAILURE CRITERION

In the last chapter, a large number of failure criteria were described. For the sake of clarity nevertheless, the number of failure criteria discussed in this report is limited to the most important. The literature contains countless variants of these failure criteria.

Radenkovic and Boschat [8] have, for instance, converted the Tresca criterion by defining the slip strength as a function depending on the direction.

Griffith and Baldwin [8,24] have attempted to reformulate the deformation energy criterion for general orthotropid materials by the main stress axes coinciding with the main axes of the material. Regarding most of the variants of the failure criteria, it may be stated that only a more complex mathematical formulation is used

without achieving a gain in flexibility. The overwhelming majority of these criteria are hardly used except for very special composite materials.

But there are still two exceptions to this rule:

--Franklin [8] proposed extending the Hoffman criterion by multiplying the cross terms F12, F23, F13 in the Fij matrix by an extra parameter (α, β, γ). (Also see Appendix A). This parameter must then again be determined with a multiaxial test and the basic philosophy is then essentially the same as for the Tsai-Wu criterion.

Shu and Rosen [18] have followed to determined the slip strengths an approach which is actually no longer part of the macromechanical but rather the micromechanical approach. In this approach, we use a limit load analysis as known from the theory of plasticity. By defining subsequently a kinematically permissible displacement field, an upper and lower limit are found, respectively, for the failure load.

The more consistent with reality are the displacement and stress fields, the smaller the differences between the lower and upper limits.

In [18], the above-indicated theory is applied to a unidirectional material. For the slip strength in plane 1-2 ($\tau 12$), it is apparent that the lower and upper limits can differ at maximum by 27% (see Figure 5) which seems to be a very reasonable approximation in view of the measurement precision of failure tests. The /19
same theory seems to furnish less good solutions for the slip strength in plane 2-3 (see Figure 6) and the applicability of the theory to this case must be considered rather doubtful.

How far the results for τ_{12} are applicable for a glass fabric is not yet quite clear. It should be possible to use basically the same stress and displacement fields, so that the possibility of determining one of the failure strengths (S) directly from the properties of the component sections (glass fiber content, fluid limit of the matrix material) should remain open. It would seem interesting to test this in the future for a practical case.

In choosing a failure criterion, it must be realized that it is impossible to establish a failure criterion which applies to all composite materials.

This phenomenon is actually known also in the "composite world", and the Tsai-Wu criterion (in which the failure criterion is the measure) is, for example, a direct consequence of this. This choice of the failure criterion must then be associated directly with the type of composite material. A number of general requirements can, in each case, be associated directly with the failure criterion:

1. The criterion must be invariant with respect to the coordinate transformation,
2. it should be flexible enough to be able to describe the experimental results,
3. the criterion must provide a solution for a certain load path,
4. the criterion must be mathematically operational.

This means that the criterion must have a simple conversion between stress space and tension space.

The criterion must also be applicable to strength analyses and in particular to the method of finite elements.

With these general requirements, a number of marginal notes may be made with regard to the glass fabric reinforced material considered here.

For 1: For the orthotropic material considered here and /20
with regard to the application of the criterion to the finite element methods, the requirement that the criterion should be invariant cannot be so important.

For 2: The requirement of sufficient flexibility for the failure criterion must be related mainly to the question of whether the criterion must be able to describe differences in tension and compression properties. Since no compression tests have yet been carried out on the present material, no definite answer may be given to this question, but the results in Tables 1 and 2 for comparable materials indicate that the differences in tension and compression properties are fairly significant. It is, therefore, stated also that the failure criteria to be chosen should also be able to describe differences in tension and compression: The failure criteria described in the previous paragraph should now be tested for the remaining requirements 2, 3 and 4.

2.1 Maximum stress theory and maximum tension theory

Apart from the problem already indicated that the maximum stress theory gives an overestimate of the strength properties, both criteria raise very great problems with regard to the conversion of stress to tension space and inversely (requirement no. 4).

If, for instance, a maximum tension theory is converted to the stress theory, wrong results may occur as shown in Figure 7 (with arrows). The same thing may happen if a maximum stress theory is converted to the tension space (Figure 8).

Such phenomena are only to be attributed to the partly linear nature of the failure criterion.

This effect can already occur for flat stress states.

On the whole, the multiaxial stress states are even more complex.

Both criteria can be practically much more complicated than apparent at a first glance and additional stability requirements must be imposed on the criterion.

For a flat stress state, altogether six stability criteria are needed for the maximum tension theory in the form [7] /21

$$\frac{S16}{S12} \times \frac{Xt6}{Xt1} - \frac{S11}{S12} \geq \frac{Xc2}{Xt1} \qquad \frac{S26}{S12} \times \frac{Xc6}{Xt2} - \frac{S22}{S12} \geq \frac{Xt1}{Xc2} \qquad (25)$$

$$\frac{S16}{S12} \times \frac{Xc6}{Xc1} - \frac{S11}{S12} \geq \frac{Xt2}{Xc1} \qquad \frac{S65}{S16} \times \frac{Xt6}{Xt2} - \frac{S26}{S16} \geq \frac{Xt1}{Xt2}$$

$$\frac{S26}{S12} \times \frac{Xt6}{Xt2} - \frac{S22}{S12} \geq \frac{Xc1}{Xt2} \qquad \frac{S66}{S16} \times \frac{Xt6}{Xt2} - \frac{S26}{S16} \geq \frac{Xc1}{Xc2}$$

These are the terms of the compliance matrix (flexibility matrix).

For the maximum stress theory, also stability requirements must be imposed in the form:

$$\frac{C16\ C22}{C12\ C66} \times \frac{Xc6}{Xc1} - \frac{C22}{C12} \geq \frac{Xc2}{Xt1} \quad \text{etc.} \qquad (26)$$

Since their relations are no longer used, however, they are not written out in greater detail here. Further information may be found in the literature [7] page 381.

It is apparent that the number of stability requirements for a real three-dimensional failure criterion becomes so large that there is no practical possibility of applying the criterion.

The maximum tension and stress theories must, therefore, be described as practically inapplicable.

The Hill criterion and the criteria of Tsai-Azzi and Norris-Puck derived from it are not applicable, since here the differences

between tension and stress cannot be discounted. In the flat stress state, this problem is solved to some extent by formulating the criterion concerned per quadrant in the stress space, while the corresponding strength numbers are used for each quadrant. In the bi-dimensional case, this operation can still be considered, but for the three-dimensional case, this leads to very complex formulae, and there is also the problem that the surface is no longer convex so that both requirements 3 (clear solution for a load path) and 4 (clear conversion from stress to tension theory) are no longer satisfied.

For these reasons, we must also abandon the inapplicable criteria of Hill, Tsai-Azzi and Norris-Puck. /22

The great drawback of the Marin criterion is that the direction of the main stress may coincide with the main directions of the material which for a structure need absolutely not be the case. For this reason, the Marin criterion does not apply either.

There remain the criteria of Hoffman, Franklin and Tsai-Wu.

The only difference between these criteria is the definition of the cross-terms Fij ($i \neq j$). For the Hoffman criterion, the cross-terms satisfy immediately the stability requirements.

For Franklin and Tsai-Wu, extra attention must be paid to the stability criteria (19).

Moreover, in the last two cases, rather complicated biaxial experiments are needed to determine the parameter values of the cross-terms.

Before beginning such complicated experiments, we must naturally examine the gain in precision which may be expected with these criteria (Franklin, Tsai-Wu). This will be considered in particular

on the basis of the term F12. To simplify the plan to some extent, the parameter picture of the F12 values is subdivided into two partial regions while the value of F12 as follows from the Hoffman criterion is used as separation (designated hereafter as $F12_H$).

I $0 \leqq Fij \leqq F12_H$

This region was studied thoroughly by Narayanaswami [16].

In this investigation in [16], two failure criteria are studied, specifically the Tsai-Wu criterion with F12 = 0 and the Hoffman criterion. The author determined for different composite materials and for different load states the failure strengths with the two different criteria.

On the basis of the results, it was possible to establish that the difference between the two criteria was never more than 10% in the extreme case. Since this 10% level is taken in the literature as a sort of magic limit with regard to measurement precision in failure experiments, in the publication in question the conclusion is also drawn that for practical purposes it makes no difference as to which criterion is applied.

/23

$Fij > F12_H$
$Fij < 0$

In this connection, no investigations are known in which the effect of the cross-terms on the precision of failure strengths was estimated. But it is quite possible to estimate quantitatively the effect of the cross terms, if we limit ourselves to the composite material to be used in the mine detectors.

In the first place, one may study the parameter region which is permissible at maximum for the composite material in question here. This attempt is made in Appendix A. The latter formulates the cross-terms Fij as a function of the Hoffman parameters in the form:

$$F12 = \alpha F12_H$$
$$F23 = \gamma F23_H$$
$$F13 = \beta F13_H \qquad (27)$$

By applying the stability criteria, it may be found that:

$$|\alpha| < \sim 1.5 \text{ to } 2$$
$$|\beta| < \sim 1.1$$
$$|\gamma| < \sim 1.1$$

From these numerical values, the conclusion may be drawn that the boundary values of the cross-terms F13 and F23 are approximately equal to the values of the Hoffman parameters (as long as Fij has the same sign as the Hoffman parameters). For practical purposes, the limiting values for the parameters F13 and F23 can be taken as equal to the Hoffman parameters (or $|\gamma| = |\beta| = 1$). The exact experimental determination of the parameters F13 and F23 (in accordance with the Tsai-Wu or Franklin concept) should imply the biaxial experiments must be carried out in plane 1-3 or 2-3. These experiments are very difficult (see the problems in the determination of the interlaminate tension strength in [25]) and, therefore, proportionately inaccurate (probably inaccuracy more than 10%).

In view of the results of the study by Narayanaswami [15], it can actually be stated also that there is no benefit in determining experimentally the parameter values of F13 and F23, and that it is best to use for these parameters the Hoffman formulation (14).

For the parameter F12, there is somewhat more latitude with regard to the Hoffman criterion ($|a| < 1.5$ to 2) and in this plane experiments may be conducted with somewhat higher precision. /24

But in this case also we must expect very spectacular differences. Indeed, Franklin [8] established that the application of the Hoffman criterion may give an over-estimation of the strength in the order of 50% (for the case described by him), but Franklin corrected thereafter the value of F12 with a value $a = -0.53$, which is larger by factors than the possible values for the present glass fiber material. On the basis of the results of [2] and [16],

the conclusion can be drawn actually that the application of a Hoffman formulation gives deviations of 20-25% for the cross-term F12 in the most unfavorable case.

Thereby, this precision may, if desired, be fairly simply doubled by carrying out a slip test on a material sample under 45° in the plane 1-2 (see Figure 9). This is then a positive slip test, of which it was already stated earlier that the test is probably not exact enough to determine exactly F12. The test should be amply sufficient to establish the sign of the F12 term.

To correct the Hoffman parameter F12 for this sign (this does not affect the stability criteria), the precision is brought back to within 10%.

To summarize, it may be stated that the stability criteria impose strength limitations on the cross-terms, such that the inclusion of the test precision is amply sufficient to use the Hoffman criterion.

A possible exception to this is the cross-term F12, but for this term the precision can be brought rapidly within 10% limits through a slip test on a 45° blank. The following tests are needed to determine the failure criterion:

Hoffman criterion: tension tests)
compression tests) in directions 1, 2 and 3
slip tests in directions 1, 2 and 3

determination of the sign of F12: slip tests on 45° blanks in the plane 1-2

/26

3. EXPERIMENTS

A number of researchers have applied for purely theoretical reasons boundary conditions on the type of experiments needed to determine the failure criteria [1,3,7]. For the sake of completeness

a number of these boundary conditions are indicated. In [4], Wu imposes two main requirements on the experiments:

- the stresses in the blank must be calculated under the boundary conditions taken in the experiment
- the stresses in the blank must be uniform.

Here Wu stated that in the determinations of the parameters which are defined by overall material properties, this second requirement is not so important. But if the parameters are determined by local properties as is the case for the failure, this second requirement must immediately be satisfied. This implies practically that experiments with notched blanks are not permissible.

Another aspect which must be considered in the determination of the failure criterion is the fact that the criteria to be determined are valid only for radial stress paths (if there are of course inelastic deformations before the failure, which should very certainly be the case here). But this implies that the stress in the structure must remain the same in regard to the form until the moment of the total failure, since otherwise a too favorable picture would be obtained with regard to the failure strength. Practically, this is due to the fact that one has to test one type of failure per experiment. For example, it is not desirable that when a failure occurs in a test-bar, the stress distribution should change in such a manner that another failure type is indicated (where the material is for example much more resistant). It is then useful also after conducting the test to check whether a type of failure has indeed occurred. In this connection, tests in the form of bending tests are advised against most strongly.

The number of possible types of experiments is limited too strongly by the previous boundary conditions. Lenoe [26] and Whitney [27] have published extensive reviews on the possibility of accomplishment and the limitations of the different types of

experiments. Although most of the authors arrived at the conclusion that the cylindrical blank is the only one which gives reasonable results, this conclusion is nevertheless inspired too much by the desire to be as flexible as possible in the choice of multiaxial /26
stress states. As is apparent from the above, this is also vital in the application of the Tsai-Wu criterion. But if we limit ourselves to a Hoffman criterion, this requirement is much less significant.

A last aspect to be discussed here is the thickness effect mentioned by a number of authors (see for example [3]). This thickness effect is explained by the fact that for a plate material the outermost fibers experience much less support from the matrix material than the central fibers. This effect should occur whenever the fibers are curved (just as for a fabric). The effect should be clearly noticeable when the plate thicknesses are lower (less fibers in the thickness direction) and will lead from thinner plates to a reduction in the failure strength (see Figure 10). Although in the mine detector research will be damped for the plate thicknesses, it can be important if thinner plates are removed from the original plate to undergo tests subsequently.

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Table 1 Tensile and Compressive Properties Normal to the Interlaminar Plane[22]

Material			Tension		Compression	
Resin	Reinforcement	Resin Content (Wt%)	Modulus of Elasticity (psi)	Maximum Stress (psi)	Modulus of Elasticity	Maximum Stress (psi)
Epoxy	181 fabric (Volan finish)[a]	31.8	1.35×10^5	2230	1.42×10^5	59,000
Epoxy	Unidirectional glass filaments	35.2	1.42×10^5	3590	1.33×10^5	21,600
Epoxy	Crossplied (90°) glass filaments	31.1	1.91×10^5	3380	1.84×10^5	93,800
Phenolic	181 fabric (A1100 finish)[b]	27.5	1.13×10^5	710[c]	2.07×10^5	80,400

[a] Trade name for methacryatochromic chloride.
[b] Trade name for γ-aminopropyltriethoxysilane.
[c] These values may be too low because of machining of tensile specimen. For unnecked specimens the modulus was 2×10^5 psi and the strength was 2790 psi.

Table 2 Mechanical Properties of Fabric Laminates

Laminate Description					Tensile Properties				Compression Properties				Interlaminar Shear	
Glass Type	Glass Surface Treatment	Fabric Style	Resin	Resin Content Percent by Weight	Strength (10^3 psi) Warp Direction	Strength (10^3 psi) Fill Direction	Modulus (10^6 psi) Warp Direction	Modulus (10^6 psi) Fill Direction	Strength (10^3 psi) Warp	Strength (10^3 psi) Fill	Modulus (10^6 psi) Warp	Modulus (10^6 psi) Fill	Warp	Fill
S	HTS	181[a]	Epoxy	28.5–37.6	97.7	95.3	3.15	3.09	67.4	64.0	4.60	[illegible]	3040	3120
S	HTS	143[a]	Epoxy	30	139.5	31.8	5.52	0.74	79.8	34.2	5.85	[illegible]	4950	2230
E	Volan[b]	181	Epoxy	28	55.8	52.6	3.16	2.89	59.2	52.5	4.22	3.94	3330	3110
D	HTS	181	Epoxy	33.9	35.0	33.3	2.69	2.57	52.7	49.2	2.85	2.79	3710	3720
E	Volan[b]	181	Polyester	35	48.0	45.3	2.80	—	38.0	36.3	—	—	—	—

[a] Fabric Construction: Style 181: 57 × 54 ends and picks/inch; 0.0085 inches thick; 8 harness satin weave; warp and fill yarn, 225½.
Style 143: 49 × 30 ends and picks/inch; 0.009 inches thick; crowfoot satin weave; warp yarn, 225½; fill yarn, 450½.
[b] Trade name for methacryatochromic chloride.

TABLES 1 and 2: Strength values in tension and compression for different composite materials [3]

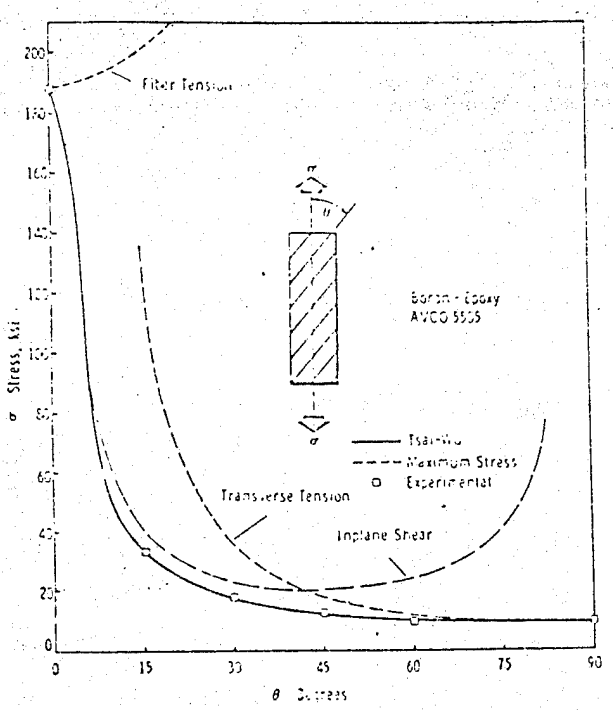

Figure 1. Comparison between the maximum stress theory and experimental results [13]

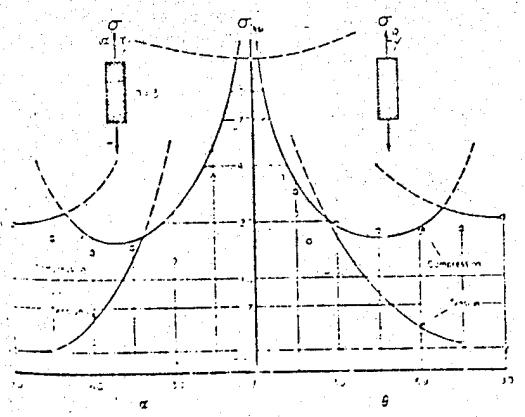

Figure 6.18 Maximum Stress Theory for Failure of Fibrous Unidirectional and Angle-ply Composites. Experimental points are shown for compression and tension strengths of nonwoven-glass epoxy composites (Ref: 44)

Figure 2. Comparison between maximum stress theory and experimental results reference [3]

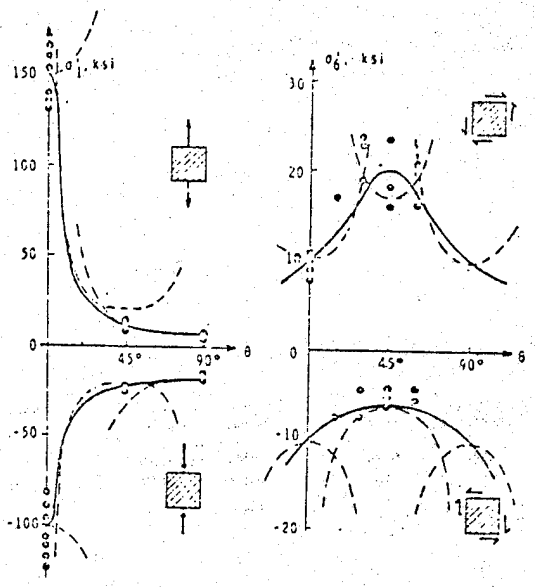

Figure 3. Off-axis uniaxial and shear strengths of graphite-epoxy composite. Solid lines represent our theory; dashed lines, the maximum stress theory; and dots, experimental data from tubular specimens

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Figure 3. Comparisons between maximum stress theory Tsai-Wu theory and experimental results, reference [4]

/33

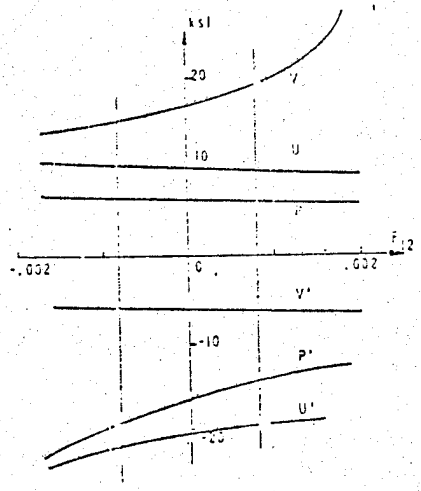

Figure Effect of F_{12} on the combined-stress test data for graphite-epoxy composite. The bounds of F_{12} are shown. Quadratic approximations by Hill and Hoffman correspond to essentially zero value for F_{12} in this Figure.

Figure 4. Sensitivity of the cross-terms with respect to different types of experiments

u)
) tension/compression on 45° blank
u')

v)
) slip on 45° blank
v')

p)
) hydrostatic experiments
p')

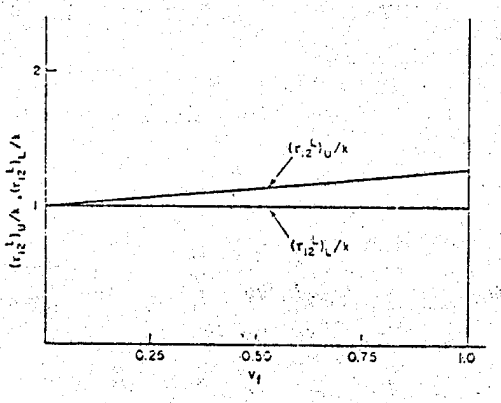

Figure 5. Lower (l) and upper (u) limits of the slip strength τ12 (by limit load analysis) as a function of the glass fiber content and the flow limit k of the matrix material

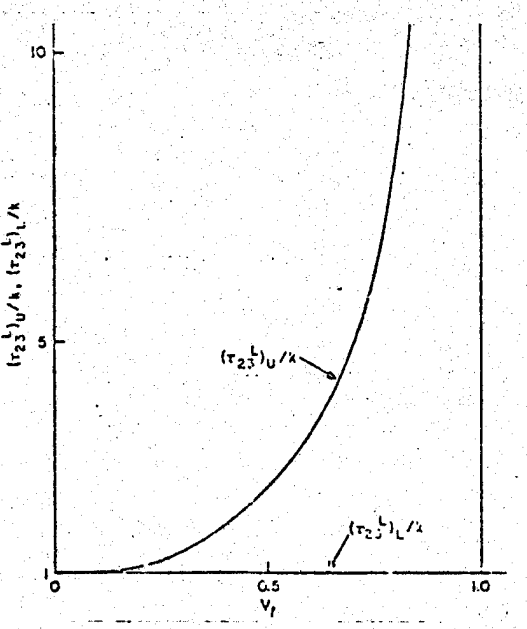

Figure 6. Lower (l) and upper (u) limit of the slip strength τ23 (by load limit analysis) as a function of the glass fiber content Vf and the flow limit k of the material of the matrix

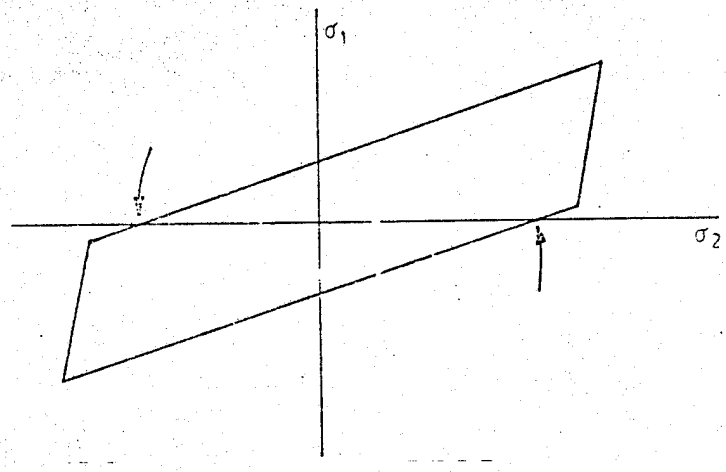

Figure 7. Maximum tension theory in the stress space

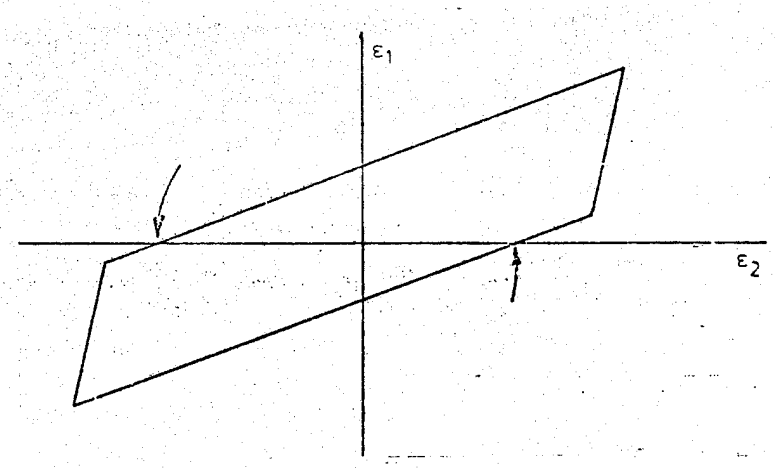

Figure 8. Maximum stress theory in the tension space

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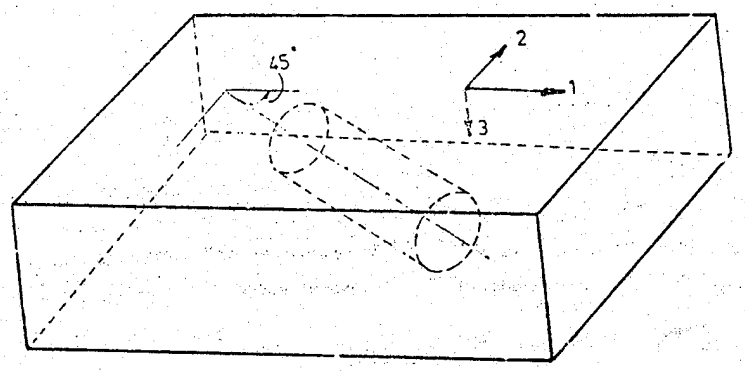

Figure 9. Material sample (in the plane 1-2 plane) to determine the F12 cross-terms with regard to the slip test

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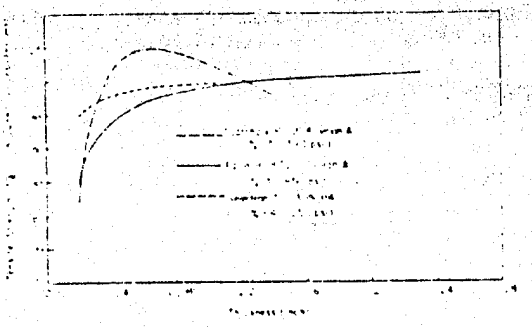

Figure 6.15 The Effect of Thickness on the Tensile Strengths of Epoxy (Scotchply and Epon 828) and Polyester (Structron) Laminates Expressed as a Percentage of the 1-in.-thick Control Values (Ref. 39)

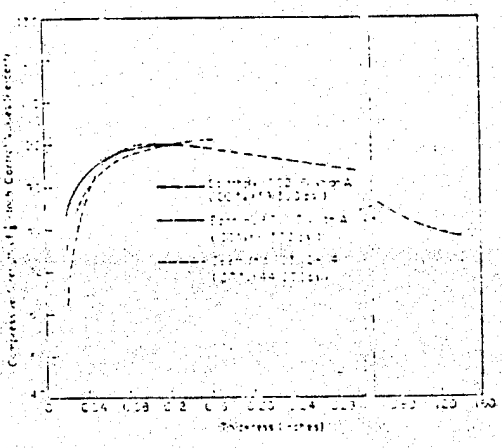

Figure 6.16 Effect of Thickness on the Compressive Strengths of Epoxy (Scotchply and Epon 828) and Polyester (Structron) Laminates Expressed as a Percentage of the 1-in.-thick Control Values (Ref. 39)

Figure 10. Thickness effect for composite materials

APPENDIX A Stability for the Franklin criterion /A1

Stability for the Franklin Criterion

The starting point may be the Hoffman criterion, which may be written in its simplest form as follows (as regards the quadratic terms)

$$\begin{bmatrix} C2+C3 & -C3 & -C2 & 0 & 0 & 0 \\ & C1+C3 & -C1 & 0 & 0 & 0 \\ & & C1+C2 & 0 & 0 & 0 \\ & & & m & 0 & 0 \\ & & & & n & 0 \\ & & & & & p \end{bmatrix} = Fij \qquad (a\text{-}1)$$

Franklin attempted to achieve a better consistency for a multiaxial stress state by introducing three additional parameters α β γ through which the relation (a-1) is converted into:

$$\begin{bmatrix} C2-C3 & -\gamma C3 & -\beta C2 & 0 & 0 & 0 \\ & C1+C3 & -\alpha C1 & 0 & 0 & 0 \\ & & C1+C2 & 0 & 0 & 0 \\ & & & m & 0 & 0 \\ & & & & n & 0 \\ & & & & & p \end{bmatrix} = Fij \qquad (a\text{-}2)$$

To have a closed convex failure surface in the stress state, the Franklin theory must also satisfy the stability criteria as defined in the Wu theory (tensor polynomials)

$$Fii\ Fjj - Fij^2 \geq 0 \qquad (a\text{-}3)$$

This stability criterion is used to have an estimate of the magnitude of the new parameters introduced

1 parameter α

$$F11\,F22 - F12^2 \geq 0$$
$$(C2 + C3)(C1 + C3) - \alpha^2 C3^2 \geq 0$$
$$C3^2 + C2C3 + C1C3 + C1C2 - \alpha^2 C3^2 \geq 0$$
$$(1 - \alpha^2)\,C3^2 + C2C3 + C1C3 + C1C2 \geq 0 \qquad \text{(a-4)}$$

To be able to state something more about the above criterion is tested on the composite material used in mine detectors. Here we may state that

$$\frac{1}{XtXc} \quad \frac{1}{YtYc} \qquad \text{(a-5)}$$

/A2

so that

$$\text{Zodat}$$
$$C1 = \frac{1}{ZtZc} = C2$$
$$C3 = \frac{1}{2}\left|\frac{2}{XtXc} - \frac{1}{ZtZc}\right| \qquad \text{(a-6)}$$

With (a-6), relation (a-4) may be written as:

$$(1-\alpha^2)\,C3^2 + 2C1C3 + C1^2 \geq 0 \qquad \text{(a-7)}$$

The order of magnitude now depends on the relationship between C1 and C3, it may be established directly from equation (a-7) that α must be limited at the top by the stability requirement.

Since no compression tests have been carried out yet in the present circumstances, it is still difficult to estimate realistically the real maximum value of α. To obtain a first idea, it is further stated that Zc (illegible) Xc (also see Tables 1 and 2) and Xt (illegible) 10 Zt (the latter is derived from the measurements by Tegelaar).

Thus $C1 = \frac{1}{ZtZc}$

$$C3 = \frac{1}{2}\left|\frac{2}{10ZtZc} - \frac{1}{ZtZc}\right| = \frac{1}{2}\left|\frac{-4}{5} \quad \frac{1}{ZtZc}\right| = \frac{-2}{5}C1 \qquad \text{(a-8)}$$

Substituting (a-8) in (a-7), we find

(a-8) substitueren in (a-7) levert

$$(1-\alpha^2)\,C3^2 + -2.\frac{5}{2}\,C3^2 + \frac{25}{4}\,C3^2 > 0 \qquad (a\text{-}9)$$

$$1 - \alpha^2 - 5 + \frac{25}{4} > 0$$

$$\frac{9}{4} - \alpha^2 > 0 \rightarrow \alpha^2 < \frac{9}{4} \qquad \boxed{|\alpha| < \frac{3}{2}}$$

When choosing the relation between Xt and Zt, we must start from the maximum value of Zt as shown in the paper by Tegelaar (Experimental Determination of the Material Properties of Glass Fiber Reinforced Polyester; IWECO Report no. 5072020-78-1). To study the effect of these relations on the value of α, a second case is considered:

$Xt \approx 20\ Zt$

so that $C1 = \frac{1}{ZtZc}$ and $C3 = \frac{1}{2}\left|\frac{2}{20ZtZc} - \frac{1}{ZtZc}\right| = \frac{-9}{20}C1$ (a-9)

The substitution of (a-9) in (a-7) gives

$$(1-\alpha^2)\,C3^2 + 2\,\frac{-20}{9}\,C3^2 + \frac{400}{81}\,C3^2 > 0$$

or ofwel $1 - \alpha^2\,\frac{-360}{81} + \frac{400}{81} > 0$

$$1 + \frac{40}{81} - \alpha^2 > 0$$

$$\alpha^2 < \frac{121}{81} \qquad \boxed{|\alpha| < \frac{11}{9} = 1{,}22} \qquad (a\text{-}10)$$

Thus the value of α is lower in this case.

In the preceding, very little attention was paid (necessarily) to the compression strengths.

It may be established directly that Xc<Zc since the pressure in the X-direction possibly causes the failure mechanism to be

determined by the cracking of the fibers, while for the pressure in the Z direction, the failure of the matrix material will predominate. The consequence of this difference between Xc and Zc is that the maximum permissible value of α is again somewhat higher.

But it may well be doubted whether Xc and Zc show a very great difference and in this sense, it may be expected that the shifts in the maximum will not be spectacular for α. It may be said preliminarily that $-2 < \alpha < 2$ seems to be most applicable for the material in question here for α.

II Parameter β

$$F11\,F33 - F13^2 > 0$$

$$(C2 + C3)(C1 + C2) - \beta^2 C2^2 > 0 \qquad \text{(a-11)}$$

$$C2^2 + C2C3 + C1C3 + C1C2 - \beta^2 C2^2 > 0$$

$$(1 - \beta^2) C2^2 + C2C3 + C1C3 + C1C2 > 0$$

If we use once again the relations (a-6) equation (a-11) is converted into

(a-11) over in

$$(1 - \beta^2) C2^2 + 2C2C3 + C2^2 > 0$$

$$(2 - \beta^2) C2^2 + 2C2C3 > 0 \qquad \text{(a-12)}$$

/A4

The substitution of (a-8) gives

$$(2 - \beta^2) C2^2 + 2\,\frac{-2}{5}\,C2^2 > 0$$

$$2 - \beta^2 - \frac{4}{5} > 0$$

$$\beta^2 < \frac{6}{5} \qquad \boxed{|\beta| < \left|\sqrt{\frac{6}{5}}\right|} \qquad \text{(a-13)}$$

The substitution of (a-9) gives

$$(2 - \beta^2)\, C2^2 + 2\,\frac{-9}{20}\, C2^2 > 0$$

$$2 - \beta^2 - \frac{9}{10} > 0 \qquad \text{(a-14)}$$

$$\beta^2 < \frac{11}{10} \qquad \boxed{\beta < \sqrt{\frac{11}{10}}}$$

From (a-13) and (a-14), it is apparent that α may hardly be much larger than 1.

III Parameter γ

$$\begin{aligned} &F22\,F33 - F23^2 > 0 \\ &(C1 + C3)(C1 + C2) - \gamma^2 C1^2 > 0 \\ &C1^2 + C1C3 + C2C3 + C1C2 - \gamma^2 C1^2 > 0 \qquad \text{(a-15)} \\ &(1 - \gamma^2) C1^2 + C1C3 + C2C3 + C1C2 > 0 \end{aligned}$$

The substitution of the relations (a-6) gives

$$\begin{aligned} &(1-\gamma^2) C1^2 + 2C1C3 + C1^2 > 0 \\ &(2 - \gamma^2) C1^2 + 2C1C3 > 0 \quad \text{or also} \\ &(2-\gamma^2) C2^2 + 2C2C3 > 0 \end{aligned} \qquad \text{(a-16)}$$

A comparison of relations (a-16) and (a-12) shows that γ has the same value as β, specifically:

$$|\gamma| < \left|\sqrt{\frac{6}{5}}\right| \quad \text{and} \quad |\gamma| < \left|\sqrt{\frac{11}{10}}\right| \qquad \text{(a-17)}$$

Consequently, it cannot be stated that for the Hoffman criterion $\alpha = \beta = \gamma = 1$.

These values thus satisfy directly the stability criteria. It may also happen that the maximum values of β and γ are $\frac{6}{5}$:1.095. When it is recalled also that most of the experimenters state that the strength values have a 10% spread (Wu also uses this percentage in [7] to determine the precision of the tensor polynomial), it may be stated a priori that the maximum values of β and γ can be established as 1 just as well.