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A Note on Typing Variables and References

by

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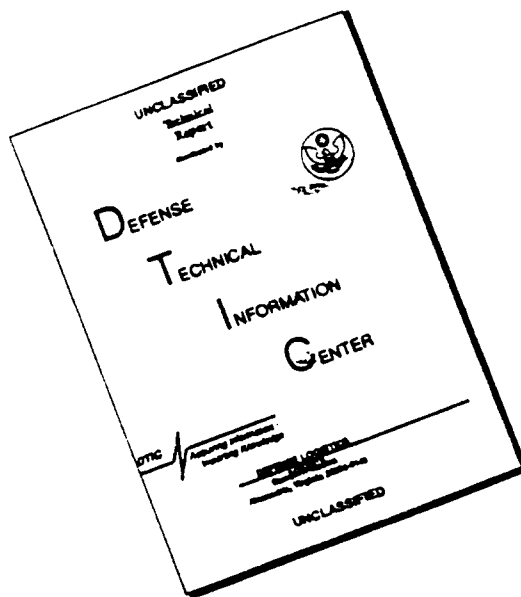
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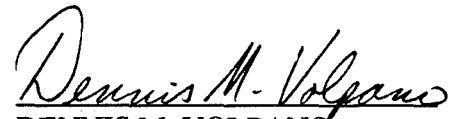
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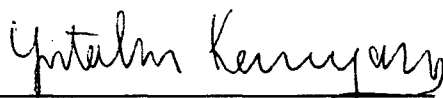
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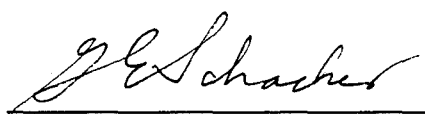

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<p>We consider the polymorphic typing of variables and references with C's address-of operator '&' in the context of nonweak types. A natural semantics and type system are given for a polymorphically-typed imperative language with first class functions. The type system is proved sound with respect to the natural semantics.</p>			
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A Note on Typing Variables and References

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Polymorphic typing of variables and references is considered in [1]. However, a treatment of the address-of operator '&' in the context of nonweak types is not given. The operator is treated in [2] but only in the context of weak types, since every type in that system is weak. In this note, the semantics and subject reduction theorem of [1] are reformulated in order to accomodate '&' in the presence of nonweak types.

The syntax of the language in [1] is extended as follows:

(Expressions) $e ::= \&e \mid l.1$

(Values) $v ::= l.0$

Meta-variable l ranges over locations. We say $l.1$ is a *variable* and $l.0$ is a *reference*. Unlike references, variables are not values. Variables and references replace variable locations and reference locations respectively in the syntax of [1].

Typing rules (REFLOC) and (VARLOC) of [1] are changed and a typing rule for '&' is added—see Figure 1. The domain of a location typing is no longer partitioned into variable and reference locations.

Some changes are needed in the evaluation rules. These changes are reflected in the new rules given in Figure 2.

We now turn to subject reduction. First, we introduce some lemmas:

Lemma 1 (Superfluosness) *If $\lambda; \gamma \vdash e : \tau$ and $l \notin \text{dom}(\lambda)$, then $\lambda[l : \tau']; \gamma \vdash e : \tau$.*

Lemma 2 (Substitution) *If $\lambda; \gamma \vdash v : \sigma$ and $\lambda; \gamma[x : \sigma] \vdash e : \tau$, then $\lambda; \gamma \vdash [v/x]e : \tau$. Also, if $\lambda; \gamma \vdash l.1 : \tau \text{ var}$ and $\lambda; \gamma[x : \tau \text{ var}] \vdash e : \tau'$, then $\lambda; \gamma \vdash [l.1/x]e : \tau'$.*

(REFLOC)	$\lambda; \gamma \vdash l.0 : \tau \text{ ref} \quad \lambda(l) = \tau$
(VARLOC)	$\lambda; \gamma \vdash l.1 : \tau \text{ var} \quad \lambda(l) = \tau$
(ADDRESS)	$\frac{\lambda; \gamma \vdash e : \tau \text{ var}, \quad \tau \text{ is weak}}{\lambda; \gamma \vdash \&e : \tau \text{ ref}}$

Figure 1: New Rules of the Type System

$$\begin{array}{l}
\text{(CONTENTS)} \quad \mu \vdash l.1 \Rightarrow \mu(l), \mu \\
\text{(BINDVAR)} \quad \frac{\begin{array}{l} \mu \vdash e_1 \Rightarrow v_1, \mu_1 \\ l \notin \text{dom}(\mu_1) \\ \mu_1[l := v_1] \vdash [l.1/x]e_2 \Rightarrow v_2, \mu_2 \end{array}}{\mu \vdash \text{letvar } x := e_1 \text{ in } e_2 \Rightarrow v_2, \mu_2} \\
\text{(UPDATE)} \quad \frac{\mu \vdash e \Rightarrow v, \mu'}{\mu \vdash l.1 := e \Rightarrow \text{unit}, \mu'[l := v]} \\
\text{(ADDROF)} \quad \frac{\begin{array}{l} \mu \vdash e_1 \Rightarrow l.0, \mu_1 \\ \mu_1 \vdash e_2 \Rightarrow v, \mu_2 \end{array}}{\mu \vdash *e_1 := e_2 \Rightarrow \text{unit}, \mu_2[l := v]} \\
\text{(ADDROF)} \quad \mu \vdash \&l.1 \Rightarrow l.0, \mu \\
\text{(ADDROF)} \quad \frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash \&*e \Rightarrow l.0, \mu'} \\
\text{(ALLOC)} \quad \frac{\begin{array}{l} \mu \vdash e \Rightarrow v, \mu' \\ l \notin \text{dom}(\mu') \end{array}}{\mu \vdash \text{ref } e \Rightarrow l.0, \mu'[l := v]} \\
\text{(DEREF)} \quad \frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash *e \Rightarrow \mu'(l), \mu'}
\end{array}$$

Figure 2: The New Evaluation Rules

Lemma 3 (\forall -intro) *If $\lambda; \gamma \vdash e : \sigma$ and α does not occur free in λ or in γ , then $\lambda; \gamma \vdash e : \forall \alpha. \sigma$.*

Lemma 4 *If $\lambda[l : \tau]; \gamma \vdash e : \tau'$ and l does not occur in e , then $\lambda; \gamma \vdash e : \tau'$.*

The preceding lemmas are straightforward variants of those in [1].

The subject reduction theorem now becomes:

Theorem 5 *Suppose that $\mu \vdash e \Rightarrow v, \mu'$, $\lambda \vdash e : \tau$, $\mu : \lambda$, and $\lambda(l)$ is weak if $l.1$ occurs in the range of μ or in a λ -abstraction in e , or $l.0$ occurs in the range of μ or in e . Then there exists a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash v : \tau$, and $\lambda'(l)$ is weak if $l.1$ or $l.0$ occurs in the range of μ' or in v .*

Proof. The proof is by induction on the structure of the derivation of $\mu \vdash e \Rightarrow v, \mu'$.

For brevity, we present only the interesting cases: (BIND), when e_1 is not a value, and the evaluation rules of Figure 2.

(BIND). Suppose e_1 is not a value. Then the evaluation must end with

$$\frac{\begin{array}{l} \mu \vdash e_1 \Rightarrow v_1, \mu_1 \\ \mu_1 \vdash [v_1/x]e_2 \Rightarrow v_2, \mu_2 \end{array}}{\mu \vdash \text{let } x = e_1 \text{ in } e_2 \Rightarrow v_2, \mu_2}$$

while the typing must end with

$$\frac{\begin{array}{l} \lambda \vdash e_1 : \tau_1 \\ \lambda; [x : \text{AppClose}_\lambda(\tau_1)] \vdash e_2 : \tau_2 \end{array}}{\lambda \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2}$$

Also, $\mu : \lambda$ and $\lambda(l)$ is weak if either $l.1$ occurs in the range of μ or in a λ -abstraction in e_1 or e_2 , or $l.0$ occurs in the range of μ or in e_1 or e_2 .

By induction, there exists a λ_1 such that $\lambda \subseteq \lambda_1$, $\mu_1 : \lambda_1$, $\lambda_1 \vdash v_1 : \tau_1$, and $\lambda_1(l)$ is weak if $l.1$ or $l.0$ occurs in the range of μ_1 or in v_1 .

Now to apply induction again we want to show that

$$\lambda_1 \vdash [v_1/x]e_2 : \tau_2.$$

By Lemma 1 we have

$$\lambda_1; [x : \text{AppClose}_\lambda(\tau_1)] \vdash e_2 : \tau_2,$$

so we can apply Lemma 2 to get what we want provided that we can show

$$\lambda_1 \vdash v_1 : \text{AppClose}_\lambda(\tau_1).$$

Now, applying Lemma 3 to $\lambda_1 \vdash v_1 : \tau_1$ we can get $\lambda_1 \vdash v_1 : \text{AppClose}_{\lambda_1}(\tau_1)$, but this is not good enough, because λ_1 may contain free strong type variables that are not free in λ . To proceed, we exploit our knowledge about what locations can occur in v_1 .

Let λ_1^- be formed by removing from λ_1 any typings $l : \tau$ such that τ is not weak. By the above use of induction, this process does not remove any typings of locations that occur in v_1 , as all such locations have weak types. So by Lemma 4, $\lambda_1^- \vdash v_1 : \tau_1$. Hence, by Lemma 3, $\lambda_1^- \vdash v_1 : \text{AppClose}_\lambda(\tau_1)$, since λ_1^- contains no strong type

variables. Lemma 1 then gives $\lambda_1 \vdash v_1 : AppClose_\lambda(\tau_1)$, and finally by Lemma 2 we get $\lambda_1 \vdash [v_1/x]e_2 : \tau_2$.

By the use of induction above, $\lambda_1(l)$ is weak if $l.1$ or $l.0$ occurs in the range of μ_1 . If a variable $l.1$ occurs in a λ -abstraction in $[v_1/x]e_2$, then either it occurs in v_1 or in a λ -abstraction in e_2 . In the first case, $\lambda_1(l)$ is weak by the above use of induction; in the second case, $\lambda(l)$ is weak by the hypothesis, and so $\lambda_1(l)$ is weak since $\lambda \subseteq \lambda_1$. Furthermore, if a reference $l.0$ occurs in $[v_1/x]e_2$, then either it occurs in v_1 or e_2 . In the former case, $\lambda_1(l)$ is weak by the above use of induction, and in the latter, $\lambda(l)$ is weak by the hypothesis, and so $\lambda_1(l)$ is weak.

Hence we can use induction a second time to show that there exists a λ' such that $\lambda_1 \subseteq \lambda'$, $\mu_2 : \lambda'$, $\lambda' \vdash v_2 : \tau_2$, and $\lambda'(l)$ is weak if $l.1$ or $l.0$ occurs in the range of μ_2 or in v_2 . Since $\lambda \subseteq \lambda_1 \subseteq \lambda'$, we are done.

(BINDVAR). The evaluation must end with

$$\frac{\begin{array}{l} \mu \vdash e_1 \Rightarrow v_1, \mu_1 \\ l \notin dom(\mu_1) \\ \mu_1[l := v_1] \vdash [l.1/x]e_2 \Rightarrow v_2, \mu_2 \end{array}}{\mu \vdash \text{letvar } x := e_1 \text{ in } e_2 \Rightarrow v_2, \mu_2}$$

while the typing must end with

$$\frac{\begin{array}{l} \lambda \vdash e_1 : \tau_1 \\ \lambda; [x : \tau_1 \text{ var}] \vdash e_2 : \tau_2 \\ \text{If } x \text{ occurs in a } \lambda\text{-abstraction in } e_2 \text{ then } \tau_1 \text{ is weak.} \end{array}}{\lambda \vdash \text{letvar } x := e_1 \text{ in } e_2 : \tau_2}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if either $l'.1$ occurs in the range of μ or in a λ -abstraction in e_1 or e_2 , or $l'.0$ occurs in the range of μ or in e_1 or e_2 .

By induction, there exists a λ_1 such that $\lambda \subseteq \lambda_1$, $\mu_1 : \lambda_1$, $\lambda_1 \vdash v_1 : \tau_1$, and $\lambda_1(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ_1 or in v_1 .

Since $l \notin dom(\lambda_1)$, $\lambda_1 \subseteq \lambda_1[l : \tau_1]$.

Since $\lambda_1[l : \tau_1] \vdash l.1 : \tau_1 \text{ var}$ and (by Lemma 1) $\lambda_1[l : \tau_1]; [x : \tau_1 \text{ var}] \vdash e_2 : \tau_2$, we can apply Lemma 2 to get

$$\lambda_1[l : \tau_1] \vdash [l.1/x]e_2 : \tau_2$$

Also, $\mu_1[l := v_1] : \lambda_1[l : \tau_1]$ by Lemma 1.

Next, by the use of induction above, $\lambda_1(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of $\mu_1[l := v_1]$. Thus, $\lambda_1[l : \tau_1](l')$ is weak since $\lambda_1 \subseteq \lambda_1[l : \tau_1]$. Now suppose that a variable $l'.1$ occurs in a λ -abstraction in $[l.1/x]e_2$. Then either $l'.1$ occurs in a λ -abstraction in e_2 , or else $l' = l$ and x occurs in a λ -abstraction in e_2 . In the first case, by the hypothesis, $\lambda(l')$ is weak and so $\lambda_1[l : \tau_1](l')$ is weak. In the second case, by the restriction on the (LETVAR) rule, τ_1 is weak, and so $\lambda_1[l : \tau_1](l')$ is weak. Finally, if $l'.0$ occurs in $[l.1/x]e_2$ then it occurs in e_2 . Thus, by the hypothesis, $\lambda(l')$ is weak and so $\lambda_1[l : \tau_1](l')$ is weak.

So by a second use of induction, there exists a λ' such that $\lambda_1[l : \tau_1] \subseteq \lambda'$, $\mu_2 : \lambda'$, $\lambda' \vdash v_2 : \tau_2$, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ_2 or in v_2 . Since $\lambda \subseteq \lambda_1 \subseteq \lambda_1[l : \tau_1] \subseteq \lambda'$, we are done.

(ADDROF). Suppose the evaluation ends with

$$\mu \vdash \&l.1 \Rightarrow l.0, \mu$$

while the typing ends with

$$\frac{\lambda \vdash l.1 : \tau \text{ var}, \tau \text{ is weak}}{\lambda \vdash \&l.1 : \tau \text{ ref}}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if either $l'.1$ or $l'.0$ occurs in the range of μ . Since $\lambda \vdash l.1 : \tau \text{ var}$, we have $\lambda(l) = \tau$ by rule (VARLOC). Thus, $\lambda \vdash l.0 : \tau \text{ ref}$ by (REFLOC). Furthermore, by the restriction on rule (ADDRESS), τ , or $\lambda(l)$, is weak.

Now suppose the evaluation ends with

$$\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash \&*e \Rightarrow l.0, \mu'}$$

while the typing ends with

$$\frac{\lambda \vdash e : \tau \text{ ref}}{\frac{\lambda \vdash *e : \tau \text{ var}, \tau \text{ is weak}}{\lambda \vdash \&*e : \tau \text{ ref}}}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ -abstraction in e , or $l'.0$ occurs in the range of μ or in e .

By induction, there is a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash l.0 : \tau \text{ ref}$, $\lambda'(l)$ is weak, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ' . And, we're done.

(CONTENTS). The evaluation must end with

$$\mu \vdash l.1 \Rightarrow \mu(l), \mu$$

while the typing must end with

$$\frac{\lambda \vdash l.1 : \tau \text{ var}}{\lambda \vdash l.1 : \tau}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if either $l'.1$ or $l'.0$ occurs in the range of μ . From $\mu : \lambda$, we have $\lambda \vdash \mu(l) : \lambda(l)$. Since $\lambda \vdash l.1 : \tau \text{ var}$, we have $\lambda(l) = \tau$, so $\lambda \vdash \mu(l) : \tau$.

(UPDATE). Suppose the evaluation ends with

$$\frac{\mu \vdash e \Rightarrow v, \mu'}{\mu \vdash l.1 := e \Rightarrow \text{unit}, \mu'[l := v]}$$

while the typing ends with

$$\frac{\lambda \vdash l.1 : \tau \text{ var}}{\frac{\lambda \vdash e : \tau}{\lambda \vdash l.1 := e : \text{unit}}}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ -abstraction in e , or $l'.0$ occurs in the range of μ or in e .

By induction, there exists a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash v : \tau$, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ' or in v .

By rule (LIT), $\lambda' \vdash \text{unit} : \text{unit}$. Since $\lambda \vdash l.1 : \tau \text{ var}$, $\lambda(l) = \tau$ by (VARLOC). So $l \in \text{dom}(\lambda')$ since $\lambda \subseteq \lambda'$, and thus $\text{dom}(\mu'[l := v]) = \text{dom}(\lambda')$. If l' is a location such

that $l' \neq l$, then $\lambda' \vdash \mu'(l') : \lambda'(l')$ since $\mu' : \lambda'$. If $l' = l$ then $\mu'[l := v](l') = v$. So $\lambda' \vdash \mu'[l := v](l') : \tau$ since $\lambda' \vdash v : \tau$. Thus, $\mu'[l := v] : \lambda'$. Finally, by the above use of induction, $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of $\mu'[l := v]$.

Now suppose the evaluation ends with

$$\frac{\begin{array}{l} \mu \vdash e_1 \Rightarrow l.0, \mu_1 \\ \mu_1 \vdash e_2 \Rightarrow v, \mu_2 \end{array}}{\mu \vdash *e_1 := e_2 \Rightarrow \mathbf{unit}, \mu_2[l := v]}$$

while the typing ends with

$$\frac{\begin{array}{l} \lambda \vdash *e_1 : \tau \text{ var} \\ \lambda \vdash e_2 : \tau \end{array}}{\lambda \vdash *e_1 := e_2 : \mathbf{unit}}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ -abstraction in e_1 or e_2 , or $l'.0$ occurs in the range of μ or in e_1 or e_2 .

By rule (L-VAL), $\lambda \vdash e_1 : \tau \text{ ref}$. By induction, there exists a λ_1 such that $\lambda \subseteq \lambda_1$, $\mu_1 : \lambda_1$, $\lambda_1 \vdash l.0 : \tau \text{ ref}$, $\lambda_1(l)$ is weak, and $\lambda_1(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ_1 . By Lemma 1, $\lambda_1 \vdash e_2 : \tau$. Suppose that a variable $l'.1$ occurs in a λ -abstraction in e_2 . Then by the hypothesis, $\lambda(l')$ is weak and so is $\lambda_1(l')$ since $\lambda \subseteq \lambda_1$. Likewise, if $l'.0$ occurs in e_2 , then $\lambda(l')$ is weak and thus so is $\lambda_1(l')$.

So by a second use of induction, there is a λ' such that $\lambda_1 \subseteq \lambda'$, $\mu_2 : \lambda'$, $\lambda' \vdash v : \tau$, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ_2 or in v . The proof is now similar to the first (UPDATE) case above.

(ALLOC). The evaluation must end with

$$\frac{\begin{array}{l} \mu \vdash e \Rightarrow v, \mu' \\ l \notin \text{dom}(\mu') \end{array}}{\mu \vdash \mathbf{ref} e \Rightarrow l.0, \mu'[l := v]}$$

while the typing ends with

$$\frac{\lambda \vdash e : \tau, \tau \text{ is weak}}{\lambda \vdash \mathbf{ref} e : \tau \text{ ref}}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ -abstraction in e , or $l'.0$ occurs in the range of μ or in e .

By induction, there exists a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash v : \tau$, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ' or in v .

Now $\lambda' \subseteq \lambda'[l : \tau]$ since $l \notin \text{dom}(\mu')$.

By Lemma 1 and the above use of induction, $\mu'[l := v] : \lambda'[l : \tau]$. Furthermore, $\lambda'[l : \tau] \vdash l.0 : \tau \text{ ref}$ by rule (REFLOC). Again by the above use of induction, $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of $\mu'[l := v]$, and hence $\lambda'[l : \tau](l')$ is weak since $\lambda' \subseteq \lambda'[l : \tau]$. Finally, $\lambda'[l : \tau](l) = \tau$ and τ is weak by the restriction on rule (REF).

(DEREF). The evaluation must end with

$$\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash *e \Rightarrow \mu'(l), \mu'}$$

while the typing ends with

$$\frac{\frac{\lambda \vdash e : \tau \text{ ref}}{\lambda \vdash *e : \tau \text{ var}}}{\lambda \vdash *e : \tau}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ -abstraction in e , or $l'.0$ occurs in the range of μ or in e .

By induction, there exists a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash l.0 : \tau \text{ ref}$, $\lambda'(l)$ is weak, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ' .

Since $\lambda' \vdash l.0 : \tau \text{ ref}$, $\lambda'(l) = \tau$ by rule (REFLOC). Now $\lambda' \vdash \mu'(l) : \lambda'(l)$, since $\mu' : \lambda'$, so $\lambda' \vdash \mu'(l) : \tau$. \square

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