National Defence — Research and Development Branch

Défense nationale — Bureau de recherche et développement

TECHNICAL MEMORANDUM 95/229

December 1995

# THE FORCED RESPONSE AND DAMPING CHARACTERISTICS OF A CANTILEVER BEAM WITH A THICK DAMPING LAYER

David C. Stredulinsky — Jeffrey P. Szabo





Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

Canada

**DEFENCE RESEARCH ESTABLISHMENT ATLANTIC**

9 GROVE STREET

P.O. BOX 1012
DARTMOUTH, N.S.
B2Y 3Z7

TELEPHONE
(902) 426-3100

**CENTRE DE RECHERCHES POUR LA DÉFENSE ATLANTIQUE**

9 GROVE STREET

C.P. BOX 1012
DARTMOUTH, N.É.
B2Y 3Z7

**National Defence** Research and Development Branch

**Défense nationale** Bureau de recherche et développement

# THE FORCED RESPONSE AND DAMPING CHARACTERISTICS OF A CANTILEVER BEAM WITH A THICK DAMPING LAYER

David C. Stredulinsky — Jeffrey P. Szabo

December 1995

Approved by C.W. Bright
Deputy Director General

Distribution Approved by C.W. Bright

Deputy Director General

TECHNICAL MEMORANDUM 95/229

**Defence Research Establishment Atlantic**

**Centre de Recherches pour la Défense Atlantique**

Canada

## Abstract

This technical memorandum describes the vibration analysis of a steel cantilever beam. The beam was 649 mm long, 204 mm wide and 9.5 mm thick with a free viscoelastic damping layer (27 mm thick EAR Isodamp C-1002) bonded to one surface. Predictions of composite loss factor and response were made at forcing frequencies between 10 Hz and 3000 Hz using a direct frequency response capability recently developed for the DREA in-house finite element code VAST. Analytical results were also obtained with the code PREDC, acquired from the University of Dayton, Ohio. The frequency dependent dynamic mechanical material properties, used in the VAST and PREDC analyses, were measured at DREA using a forced vibration non-resonant method. The VAST forced response vibration analysis was in good agreement with experiment. Above 500 Hz, the system composite loss factors predicted with the PREDC code were significantly below the measured loss factors and those predicted by the VAST code. This is attributed to the more complicated deformation of the damping layer, predicted by the finite element model, but not accounted for by the Euler-Bernoulli beam formulation used in the PREDC code.

## Résumé

Le présent document technique décrit l'analyse aux vibrations d'une poutre d'acier en porte-à-faux. Cette poutre, mesurant 649 mm de longueur sur 204 mm de largeur et 9,5 mm d'épaisseur, comporte une couche d'amortissement libre faite d'une substance viscoélastique (EAR Isodamp C-1002 de 27 mm d'épaisseur) et collée à l'une de ses faces. Les prédictions relatives au facteur d'affaiblissement composite et à la courbe de réponse ont été obtenues à des fréquences forcées comprises entre 10 et 3000 Hz, appliquées grâce à une installation d'analyse de réponse en fréquence par modulation directe, récemment mise au point pour le code VAST d'analyse par éléments finis, utilisé au CRDA. D'autres résultats d'analyse ont également été recueillis au moyen du code PREDC, obtenu de l'Université de Dayton, en Ohio, Les propriétés mécaniques dynamiques des matériaux en fonction de la fréquence, utilisées dans les analyses VAST et PREDC, ont été mesurées au CRDA au moyen d'une méthode de vibration forcée non résonante. Les résultats de l'analyse VAST concordaient avec ceux de l'expérience. Aux fréquences supérieures à 500 Hz, les facteurs d'affaiblissement composite du système, prédits par le code PREDC, étaient nettement inférieurs aux facteurs d'affaiblissement mesurés et aux facteurs prédits par le code VAST. Cet écart est attribuable à la déformation plus complexe de la coche d'amortissement, prédite par le modèle d'analyse par éléments finis, mais non prise en compte dans la formulation de poutre Euler-Bernoulli utilisée dans le code PREDC.

DREA TM/95/229

# THE FORCED RESPONSE AND DAMPING CHARACTERISTICS OF A CANTILEVER BEAM WITH THICK DAMPING LAYER

by

D. C. Stredulinsky and J. Szabo

## EXECUTIVE SUMMARY

**INTRODUCTION:** The control of hull radiated noise is very important for naval platforms. Both the DREA Ship Structural Mechanics Group (SM group) and the DREA Dockyard Laboratory (Dockyard Lab) have been conducting research on the application of elastomeric materials to vibration isolation and damping systems used for the reduction of machinery and hull vibrations. Over the past twenty years the SM group has developed, in-house and through contract, the general purpose finite element computer code VAST for vibration and strength analysis of complex structures. Recently the code capabilities have been extended to allow modelling of vibration isolation and damping systems which include "rubber-like" materials having frequency dependent damping and stiffness properties. The Dockyard Lab has developed methods for measuring these properties and as part of their work on development of vibration damping materials and decoupling tiles, they contracted the Centre for Cold Oceans Resources Engineering at Memorial University to measure the vibration response and acoustic radiation (10 - 3000 Hz frequency range) for a cantilever plate with and without a thick viscoelastic damping material bonded to one surface. The purpose of the present work was to compare the vibrating cantilever plate experiment with numerical predictions using the VAST code.

**PRINCIPAL RESULTS:** The VAST finite element predictions were in good agreement with the experiments up to 1800 Hz. Better results at higher frequencies may be achieved through mesh refinement and improved modelling of damping material dynamic shear properties.

**SIGNIFICANCE OF RESULTS:** This work has demonstrated that the SM group numerical prediction capability, together with Dockyard Lab material property measurement capability, can provide accurate prediction of the vibration response of structural systems incorporating commercially available elastomeric damping materials. The methods under development could be used to design damping systems to reduce vibration of the hull and other structures such as seabays and vibration isolation rafts, and to analyse machinery isolation systems. These tools provide improved capabilities for the evaluation and optimization of damping and vibration isolation systems to solve noise or vibration problems and for the evaluation of 'off-the-shelf' systems for use on existing and future naval platforms.

**FUTURE PLANS:** Further enhancements of the VAST finite element code are planned to improve modelling accuracy at higher frequencies and to improve the efficiency of the code to allow modelling of layered damping systems for more complex structures. Further work is planned to validate experimentally the application of the VAST code to prediction of vibration isolation mount dynamic characteristics and analysis of vibration damping systems for ship structures.

# Table of Contents

# 1 Introduction

The control of hull radiated noise is very important on naval platforms. Both the DREA Ship Structural Mechanics Group (SM group) and the DREA Dockyard Laboratory (Dockyard Lab) have been conducting research on the application of viscoelastic materials for hull vibration damping and machinery vibration control.

Over the past twenty years the SM group has developed, in-house and through contract, the general purpose finite element code VAST [1] for vibration and strength analysis of complex structures. The possibility of using this code for modelling machinery vibration isolation systems was investigated by Stredulinsky [2]. Based on this work, a direct frequency response method was incorporated in VAST [3] to allow frequency dependent stiffness and damping properties to be specified for individual element groups. With this capability the code can be used to model the steady-state forced response of structural systems incorporating viscoelastic damping materials.

Dockyard Lab has developed a forced vibration non-resonant method for measuring the dynamic mechanical properties of viscoelastic materials [4]. As part of Dockyard Lab work developing improved materials for vibration mounts, vibration damping and decoupling tiles, the Centre for Cold Oceans Resources Engineering at Memorial University [5] was contracted to measure the vibration response and acoustic radiation from a cantilever plate with and without a thick viscoelastic damping material bonded to one surface.

DREA has also recently obtained a data base of dynamic mechanical properties for commercially available viscoelastic damping materials and an associated computer code called PREDC [6] from the University of Dayton. This code can be used to predict the vibration damping loss factors for beams and rectangular plates with free or constrained layer damping treatments.

The forced vibration response experimental data for the cantilever beam and the independently measured material dynamic property data provided a good test case to check both the VAST and PREDC code predictions and indirectly verify the material property measurements.

# 2 Cantilever Beam Description

The cantilever beam is shown in Figure 1. The steel beam was 9.5 mm thick and clamped between steel blocks at one end. A 27 mm thick layer of EAR Isodamp C-1002 viscoelastic damping material [7] was bonded to the upper surface of the beam. Analysis of EAR C-1002 at Dockyard Lab showed that this material is a thermoplastic elastomer consisting of polyvinyl chloride heavily plasticized with dioctylphthalate (approximately 50 percent by weight). The material properties used in the analysis are given in Table 1.

Note that the Young's modulus of the damping material is not shown. The measured dynamic properties of the EAR material are frequency dependent and are presented in Section 3. The Poisson's ratio for the damping material was not measured. Viscoelastic polymers typically have a Poisson's ratio $\nu$ of 0.5 (incompressible) in the 'rubbery' region, decreasing to a value of 0.3 in the 'glassy' region [8]. The materials are usually used in the transition region between the 'rubbery' and 'glassy' regions where damping properties are greatest. The PREDC code assumes a Poisson's ratio of 0.5. The VAST finite element code presently will not run with an incompressible material so that a Poisson's ratio of 0.47 was used instead. Typical values for

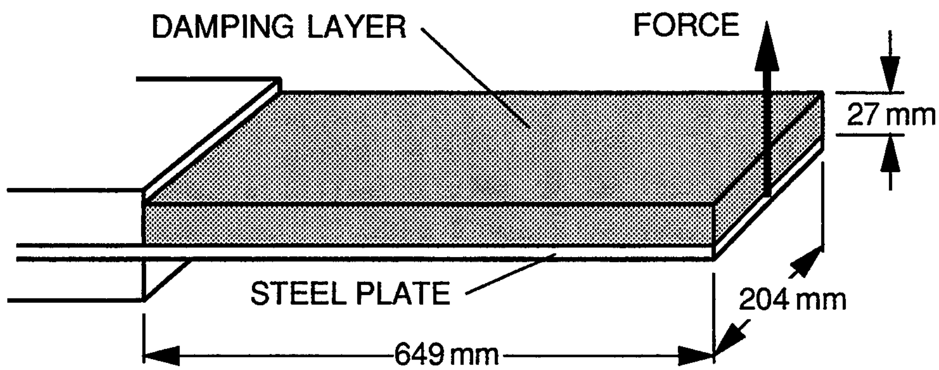



Figure 1: Steel cantilever with 27 mm thick free damping layer

the steel properties [9] were assumed since the measured properties were not available.

# 3 Damping Material Dynamic Properties

## 3.1 Measured Properties

A forced vibration non-resonant method [4] was used by DREA Dockyard Lab to measure the dynamic mechanical properties of the EAR material in tension and compression. Two experimental configurations were used since the frequency range accessible by tensile testing (0-1000 Hz) was complementary to that accessible by compression testing (100-10000 Hz). The tensile measurement involved mounting the specimen between a vibration shaker and a rigid support, as shown schematically in Figure 2. The amplitude of the force ($F$) transmitted to the support, the vibration acceleration ($\ddot{x}$) of the driven end of the sample and the phase angle ($\delta$) between force and acceleration were measured using 1/12 octave band resolution on a Bruel & Kjaer 2133 spectrum analyser. The real and imaginary parts of the complex Young's modulus and the material loss factor were calculated from

$$E' = -\frac{LF}{A\ddot{x}}\omega^2 \cos\delta \quad (1)$$

$$E'' = -\frac{LF}{A\ddot{x}}\omega^2 \sin\delta \quad (2)$$

Table 1: Material mechanical properties

| Material | Young's modulus $E$ (MPa) | Density $\rho$ (kg/m$^3$) | Poisson's ratio $\nu$ |
|---|---|---|---|
| Steel | $2.07 \times 10^5$ | 7870 | 0.30 |
| EAR C-1002 | — | 1280 | 0.50 |

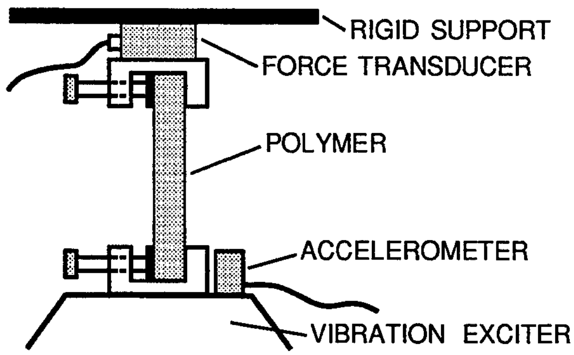



Figure 2: Schematic diagram of the set up for tensile excitation of polymer samples

$$\text{loss factor} \equiv \frac{E''}{E'} = \tan\delta \tag{3}$$

where $L$ and $A$ are the length and cross sectional area of the sample in tension. The experimental set up used for compression testing is not shown here but involved measuring the force and acceleration associated with dynamically compressing a 3 mm sheet of EAR between two rigid plates [4].

The measured storage moduli ($E'$) and material loss factors shown in Figure 3 are combined data from tensile and compression experiments. The data are in good agreement with results from other laboratories, as can be seen from examination of the round robin trial data on EAR C-1002 [10].

### 3.2 PREDC input

The PREDC vibration damping prediction code obtains input material properties from a data base of materials in the form of a 10-parameter model defining curves fitted to measured data. The fitting procedure also uses a temperature-frequency superposition which allows the data to be interpolated/extrapolated to arbitrary temperatures. It was necessary to fit the measured DREA data for EAR C-1002 to obtain the curve fitting parameters to use in the PREDC database. A second University of Dayton code MATPROP was used to access the database and plot the measured data points and fitted curves. The ten required parameters were adjusted through trial and error until an acceptable 'visual' fit was obtained. The curve fitting equations and the parameters used to fit the EAR C-1002 data are given in Appendix A. Note that the data base uses the shear modulus $G$ and that the PREDC program assumes that $G{=}E'/3$, consistent with $\nu = 0.5$. The measured Young's modulus $E'$ was converted to a shear modulus $G$ also by dividing by three to create the input data file for the MATPROP program. The fitted curves are compared to the measured data in Figure 4. A satisfactory fit was obtained up to 4000 Hz. This is above the highest forcing frequencies considered in the VAST and PREDC analyses. The 'peak' and 'valley' in the experimental values in the vicinity of 500 Hz are likely caused by some resonance in the experimental apparatus and not considered to be valid data.

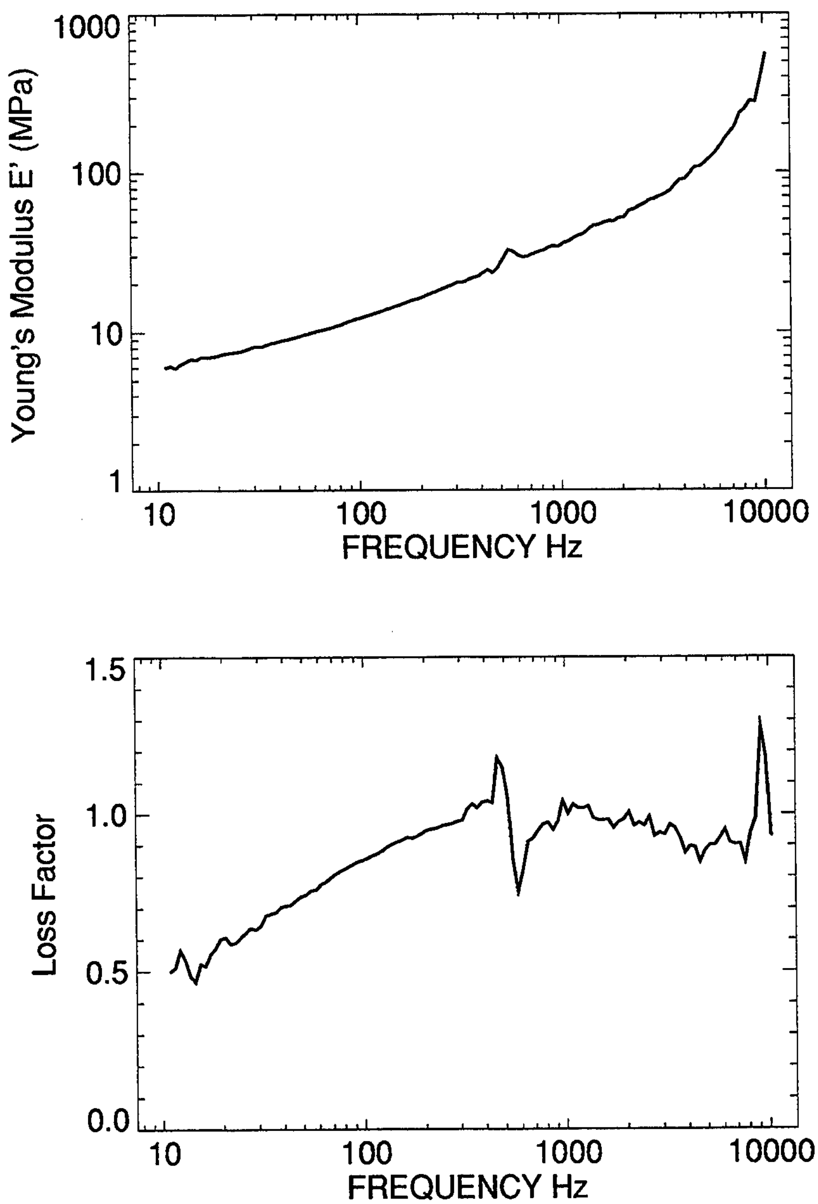



Figure 3: Measured dynamic mechanical properties for EAR C-1002

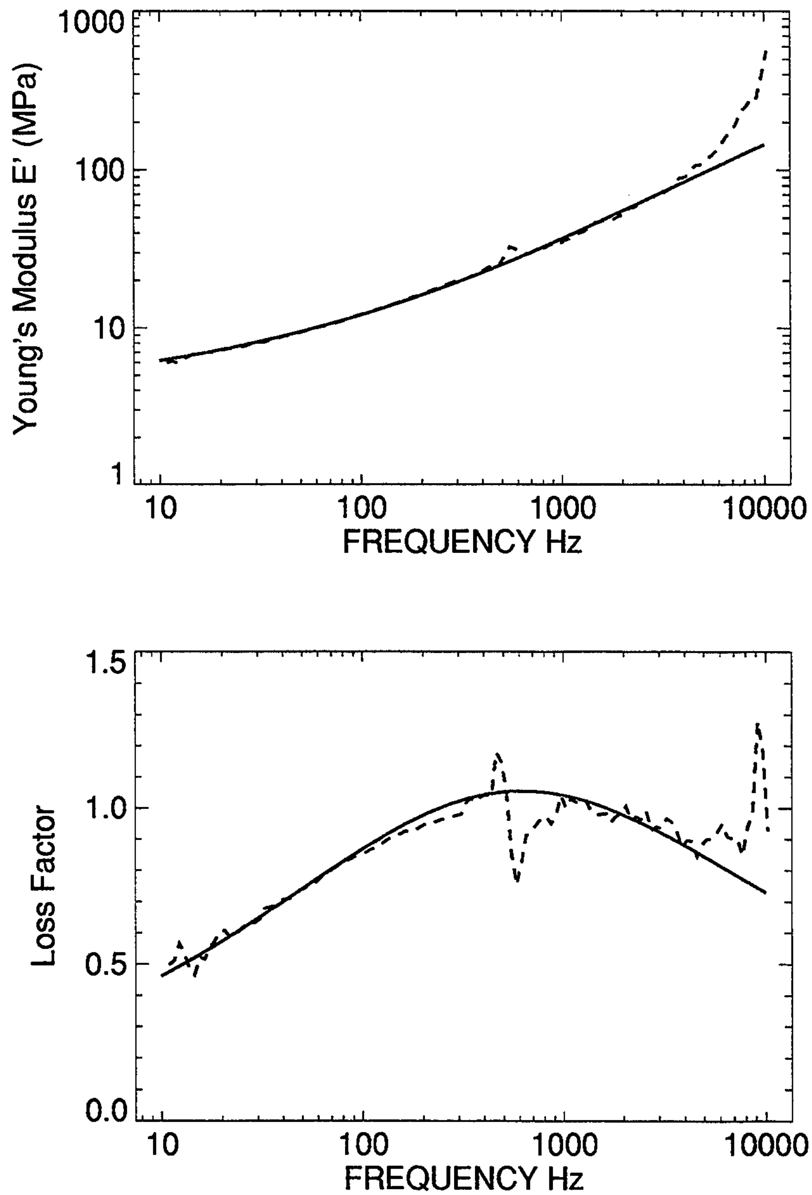



Figure 4: PREDC fitted dynamic mechanical properties for EAR C-1002 (—— Fitted curve, - - - Measured data)

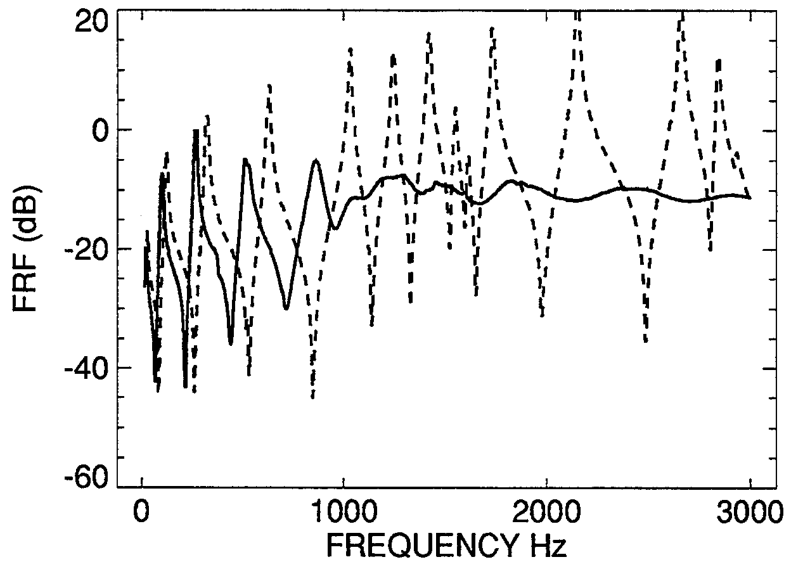



Figure 5: Measured frequency response function at tip of the cantilever (—— Damped beam, - - - Bare beam)

## 4 Cantilever Beam Response and Damping Measurements

The measurements of steady state forced response of a cantilever beam were reported in reference [5]. A vibration exciter was used to apply a transverse load on the centre-line at 9 mm from the tip on the bottom surface of the steel beam. A force transducer was used to measure the applied force and an accelerometer used to measure the acceleration of the top surface of the beam at several locations along the beam centre-line. A dual channel FFT analyser was employed to measure the frequency response function FRF (the transfer function between the applied force and the normal surface acceleration) over the frequency range from 0 to 3000 Hz. The frequency response function, measured on the centre-line near the tip, is shown in Figure 5 for both the bare steel beam and the beam with the damping layer bonded to it. Note that above 1000 Hz application of the damping layer dramatically reduced the level of the resonant peaks in the forced response.

More detailed transfer functions, in 200 Hz wide frequency bands, were also measured in the vicinity of the lower resonant peaks. No measurements of damping were given in the original report [5]. The graphs of the detailed transfer functions for the tip of the beam were digitized and the loss factors measured based on the bandwidths of the resonant peaks in the digitized data. The measured loss factors are summarized in Table 2 for both the bare beam and the damped beam.

The application of the damping layer reduced the first four natural frequencies by 17 to 20 percent. In the vicinity of the fourth mode the damped beam exhibits a double peak (506 Hz and 521 Hz). The original report does not identify the second peak which has a higher loss

Table 2: Measured beam natural frequencies and loss factors

| Bending Mode No. | Frequency (Hz) Bare | Frequency (Hz) Damped | Loss Factor Bare | Loss Factor Damped |
|---|---|---|---|---|
| 1 | 17.8 | 14.3 | 0.069 | 0.063 |
| 2 | 113.3 | 94.5 | 0.009 | 0.034 |
| 3 | 315.3 | 262.0 | 0.003 | 0.016 |
| 4 | 620.5 | 506.0 | 0.002 | 0.019 |
| ? | — | 521.0 | — | 0.034 |
| 5 | — | 860 | — | 0.065 |

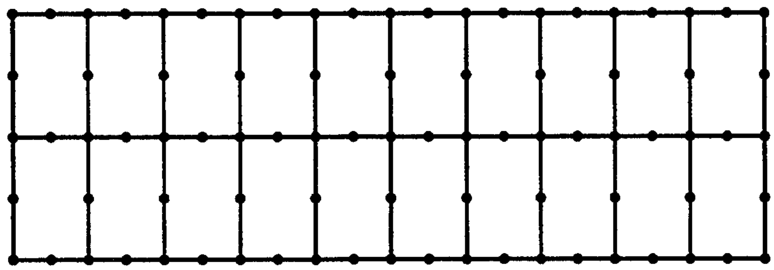
factor than the lower peak. The first mode shows a significantly higher value of loss factor than the higher modes. For the first damped mode and the four measured bare beam modes, the bandwidths were all approximately one Hz wide, suggesting the possibility that this was the bandwidth of the FFT filter. The measured bandwidths may be wider than the resonant peak bandwidths, causing artificially high loss factors for these cases. This is considered further in Section 5.2.

# 5 Bare Beam Analysis

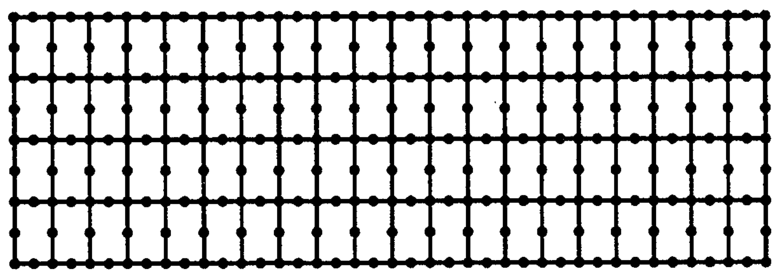
## 5.1 VAST Natural Frequency Predictions

Some analyses were conducted with the VAST finite element program for the bare beam before attempting the damped beam analysis. The most efficient VAST element for analysing the beam (essentially a flat plate) is the 8-noded thick/thin shell element (IEC 1). However in order to model the two layers (steel and damping layers), it was necessary to use 20-noded brick elements (IEC 2). The bare beam was modelled both with shell elements and with a single layer of brick elements to determine how well a thin layer of brick elements would work. A coarse mesh (shown in Figure 6a) having 10 elements along the length and 2 across the width was used initially. The convergence of the solution was then checked by considering a refined mesh (shown in Figure 6b) with 20 elements along the length and 4 across the width.

VAST natural frequency analyses were first conducted for four cases (combinations of the two element types and the two mesh refinements). The predicted natural frequencies are compared to the measured values in Table 3. The first ten predicted modes include five flexural bending modes, four torsional modes, and the 1st in-plane bending mode. Because of the symmetry of the loading from the vibration exciter, only the flexural modes were observed in the experiment. The shell element results show good convergence with differences between the coarse and refined meshes of less that 0.2 percent for the first four flexural modes and 0.8 percent for the 5th flexural mode. The differences in predicted natural frequencies between the coarse and fine meshes using brick elements were approximately ten times higher than for the shell elements and range from one percent for the 1st flexural mode to nine percent for the 5th flexural mode. This confirmed that the brick element meshes would not give as accurate a result as the shell elements, but should still provide results within two percent of the converged solution for the



a) Coarse mesh



b) Refined mesh

Figure 6: Finite element meshes for bare beam analysis

first five flexural modes when using the refined mesh. The predicted natural frequencies for the first four bending modes were 4.8 to 5.6 percent higher than measured. The VAST analysis assumed rigidly clamped nodes at the root of the cantilever. This boundary condition and/or errors in the assumed material properties and/or thickness dimension may account for this systematic difference.

## 5.2 VAST Frequency Response Analysis

In Section 4 it was suggested that the damping loss factors, calculated for the bare beam based on the measured peak band widths, may be too large. A second estimate of the damping was obtained based on the acceleration levels at the resonant peaks. The forced response of the cantilever beam was predicted at the first four natural frequencies using the VAST modal frequency response method with a transverse load of 1 N amplitude at the centre of tip. A loss factor of 0.002 was assumed for each mode and the predicted levels compared to the measured levels at each resonant peak. Since the response at the peak is inversely proportional to the loss factor, the values of loss factor needed to match the experimental peaks were determined. The loss factors measured in this manner, subsequently referred to as the 'level' method are compared to those obtained from the bandwidth measurements in Table 4. The percent difference between the two methods of obtaining loss factors decreases from 115 percent for the 1st mode to only 14 percent for the fourth mode. All values measured by the bandwidth method were higher than for the 'level' method, which supports the hypothesis that the bandwidths measured were of the FFT filter and not the true resonant peak bandwidths.

Table 3: VAST predictions of bare beam natural frequencies

| Mode No. | Natural Frequency (Hz) Shell Coarse | Shell Refined | Brick Coarse | Brick Refined | Measured | Mode Shape |
|---|---|---|---|---|---|---|
| 1 | 19.004 | 19.013 | 19.34 | 19.14 | 17.8 | 1st bending |
| 2 | 118.66 | 118.72 | 121.7 | 119.7 | 113.3 | 2nd bending |
| 3 | 123.4 | | | | | 1st torsion |
| 4 | 332.8 | 332.9 | 347.2 | 336.6 | 315.3 | 3rd bending |
| 5 | 375.9 | | | | | 1st in plane |
| 6 | 385.8 | | | | | 2nd torsion |
| 7 | 654.3 | 653.1 | 699.4 | 662.6 | 620.5 | 4th bending |
| 8 | 692.5 | | | | | 3rd torsion |
| 9 | 1067 | | | | | 4th torsion |
| 10 | 1085 | 1076 | 1193 | 1097 | | 5th bending |

Table 4: Comparison of loss factors based on resonant peak bandwidths to those based on peak levels

| Mode | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Frequency (Hz) | 17.8 | 113.3 | 315.3 | 620.5 |
| Bandwidth Method | 0.069 | 0.0089 | 0.0028 | 0.0016 |
| Level Method | 0.032 | 0.0062 | 0.0022 | 0.0014 |
| Percent difference | 115 | 44 | 27 | 14 |

This would apply only to the bare beam modes and the first damped beam mode, which had bandwidths of approximately one Hz. The higher damped beam modes had bandwidths which were greater than seven Hz.

Even with the 'level' method, the loss factors are higher for the lower frequencies and significantly higher than typical values for steel. Some damping could be attributed to the clamped end connection and possibly air movement around the beam. Since one of the goals of the work was to verify the prediction of the response of the finite element model of the beam with the damping layer, the values of loss factor predicted with the 'level' method were used for steel in the damped beam analysis in Section 6.2 to calibrate the finite element model to predict the correct response for the bare beam.

# 6 Damped Beam Analysis

## 6.1 PREDC Loss Factor and Natural Frequency Predictions

Before conducting a finite element analysis of the beam with the damping layer, the University of Dayton code PREDC was used to predict the damped beam natural frequencies and composite

Table 5: Comparison of the PREDC and measured natural frequencies and loss factors

| Bending | Frequency (Hz) | | Loss Factor | | |
|---|---|---|---|---|---|
| Mode No. | PREDC | Measured | PREDC | Measured | Meas. - Bare |
| 1 | 15.5 | 14.3 | 0.0027 | 0.063 | -0.006 |
| 2 | 97.3 | 94.5 | 0.0076 | 0.034 | 0.025 |
| 3 | 273.0 | 262.0 | 0.0138 | 0.016 | 0.013 |
| 4 | 536.5 | 506.0 | 0.0200 | 0.019 | 0.017 |
| ? | — | 521.0 | — | 0.034 | — |
| 5 | 889.5 | 860 | 0.0259 | 0.065 | 0.062 |
| 6 | 1330.5 | — | 0.0315 | — | — |
| 7 | 1868.1 | — | 0.0368 | — | — |

loss factors. The program used the fitted damping material loss factor curves discussed in Section 3.2. The results are compared to the measured data in Table 5.

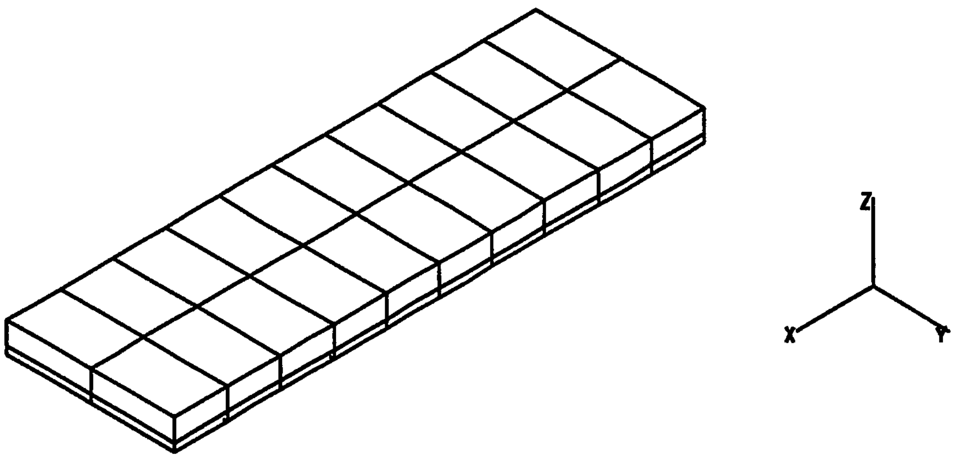
The predicted natural frequencies with PREDC were three to eight percent (averaging five percent) higher than the measured natural frequencies. This is consistent with the bare beam predictions with VAST which were five percent higher than the measured natural frequencies. Since the program only considers the damping material loss factor and does not include any other damping in the system, a column containing the differences between the measured damped beam and the measured bare beam loss factors has also been included. These differences are closer to the PREDC results but still two to three times higher than the PREDC loss factors for the second and fifth bending modes.

## 6.2 Damped Beam Finite Element Analysis

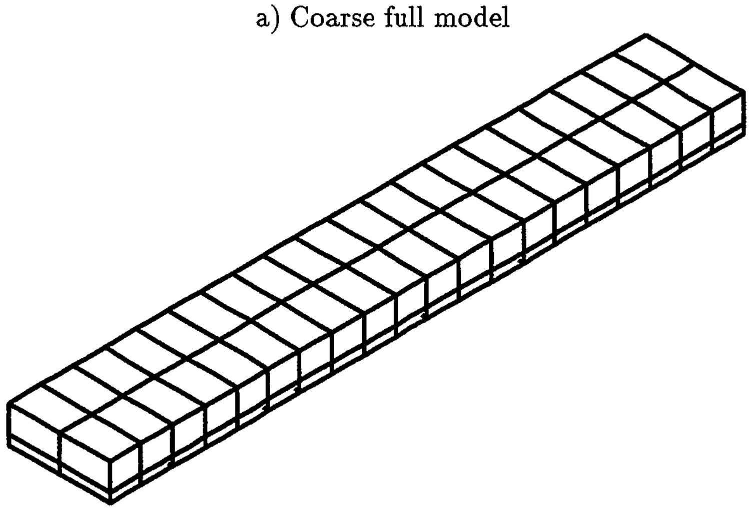
The forced response of the beam with a damping layer was predicted using the VAST Version #7.1 direct frequency response method [2, 3]. This code allows different frequency dependent Young's moduli and loss factors to be specified for each element group. The frequency dependence of the modulus and loss factor can be chosen by specifying coefficients of quadratic polynomial weighting functions or by using linear interpolation between points in a table of frequencies and corresponding weighting values. In the present work the tabular method was employed.

The table of frequency dependent loss factors for the steel beam was produced based on the predictions for the bare beam using the 'level' method (see Table 4). Tables of frequency weighting factors for the Young's modulus and loss factors for the damping material were produced using the PREDC code, based on the fitted curves shown in Figure 4. The tables were included in the VAST input files CANT4.USE and CANT5.USE which are listed along with the other VAST input files for these analyses in Appendix B.

The finite element models were constructed using 20-noded brick elements (IEC 2). Two meshes were used; a coarse mesh of the entire beam which is shown in Figure 7a and a refined 'half' model shown in Figure 7b. The half model was used to reduce the problem size and could be employed in this case because of the symmetry of the geometry and loading.




a) Coarse full model



b) Refined half model

Figure 7: Finite element meshes of beam constructed with 20-noded brick elements

The forced response was predicted for 300 frequency values, equally spaced on a logarithmic scale over the frequency range from 10 Hz to 3000 Hz. The program is limited to considering 100 points in one run, so that three runs were required to cover the entire frequency range.

The displacement amplitudes and phases for each forcing frequency and each node in the finite element model were stored by VAST in a binary file PREFX.T52, where the PREFX is a five character label identifying all vast files associated with a given model and analysis. The postprocessing program for VAST Version #7.1 is still under development so a FORTRAN code T52RESP was created which would read the .T52 files and extract the displacement amplitudes for a given node and all forcing frequencies. These data were stored in an ASCII file which was used in PV-WAVE to create the frequency response graphs. Within PVWAVE the displacement amplitudes $D$ (mm) were converted to acceleration amplitudes and reduced to a frequency response function magnitude, FRF in dB, at each forcing frequency $f$ in Hz, using the equation

$$\mathrm{FRF} = 20 \log \left( 4\pi^2 f^2 D \right). \tag{4}$$

The predicted FRFs for the coarse and refined meshes (CANT4 and CANT5 respectively) are compared to the measured FRFs [5] in Figure 8 . The graph is plotted again in Figure 9 on a logarithmic frequency axis to show more clearly the low frequency response. The predicted phase plot is also shown although no experimental phase measurements were available for comparison.

Below 1800 Hz there is reasonably good agreement between the refined mesh curves and the experimental curve. The predicted frequency of the first resonant peak was ten percent higher than measured. The next four resonant peaks ranged from four to seven percent higher than measured, consistent with the five percent difference between the bare beam measurements and predictions. Over this frequency range the experiment and refined finite element model response differed by less than 5 dB.

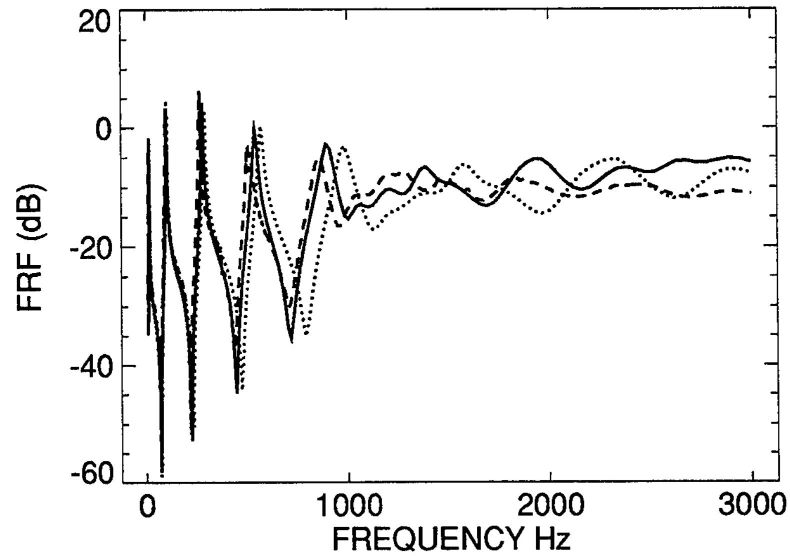
Refining the mesh caused the FRF curve to shift to the left with the amount of the shift increasing at higher frequencies. Above 1000 Hz there were significant differences between FRFs for the coarse and the refined meshed suggesting that a further refinement of the mesh should be used in this region. The predicted FRF values over the frequency range from 1800 to 3000 Hz were up to 8 dB higher than the measured values.

The VAST direct frequency response module also predicts the system composite loss factor for each forcing frequency. This composite damping loss factor $\eta$ is given by

$$\eta = \left( \sum_{i=1}^{n_g} W_i \eta_i \right) / W \tag{5}$$

where $W_i$ is the strain energy and $\eta_i$ is the loss factor for element group $i$, $W$ is the total strain energy and $n_g$ is the number of element groups in the finite element model. The system composite loss factor for each forcing frequency was also extracted from the .T52 file using the T52RESP code and graphed using PVWAVE. The predicted system composite loss factors are compared to the measured loss factors in Figure 10. The composite loss factors predicted using the University of Dayton code PREDC are shown also.

There is reasonable agreement between the VAST loss factor predictions and the measured points. Above 1000 Hz there is a significant shift between the curves for the coarse and refined



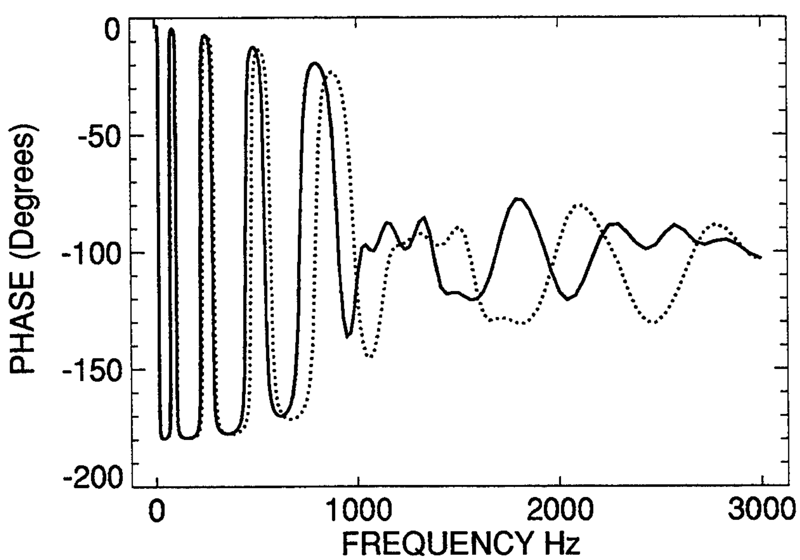




Figure 8: Comparison of measured and predicted frequency response functions at the center of the tip of the cantilever; ( —— CANT5 refined FE mode, ······ CANT4 coarse FE model, - - - measurement)

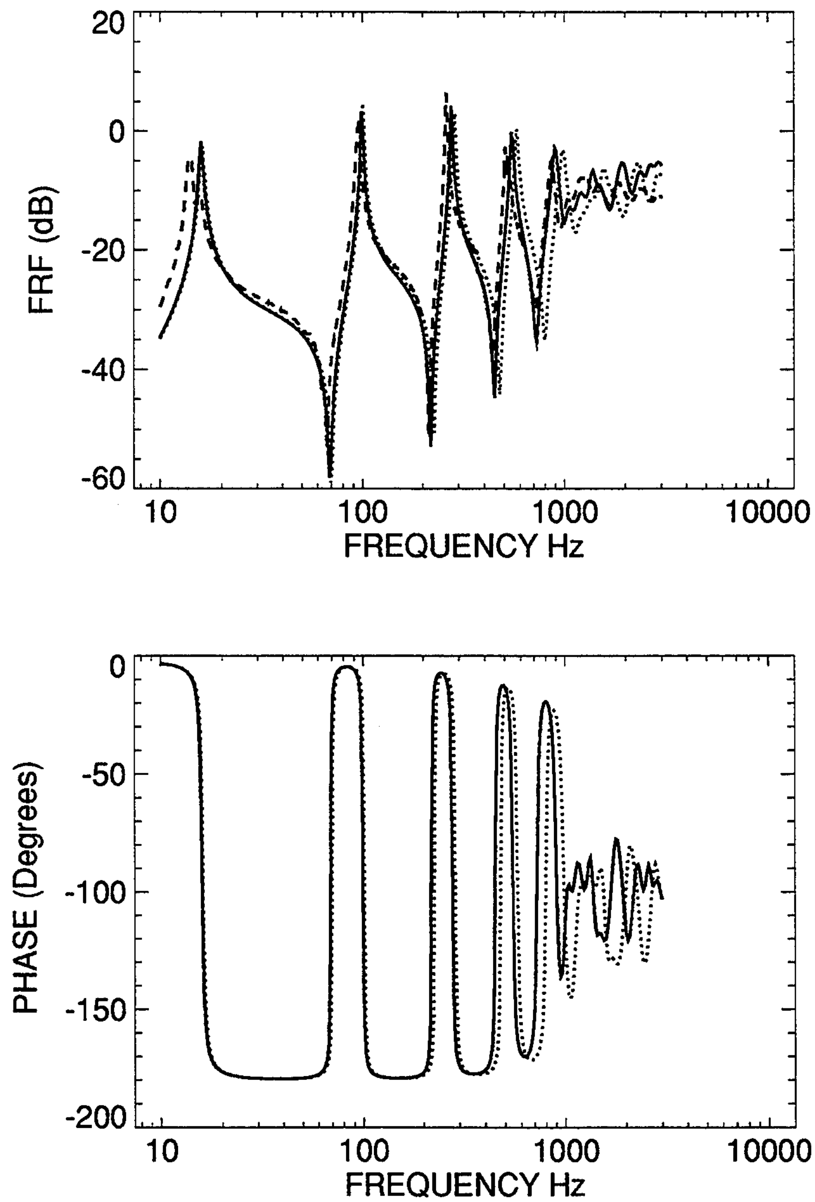



Figure 9: Comparison of measured and predicted frequency response functions at the center of the tip of the cantilever; (—— CANT5 refined FE mode, ······ CANT4 coarse FE model, - - - measurement)

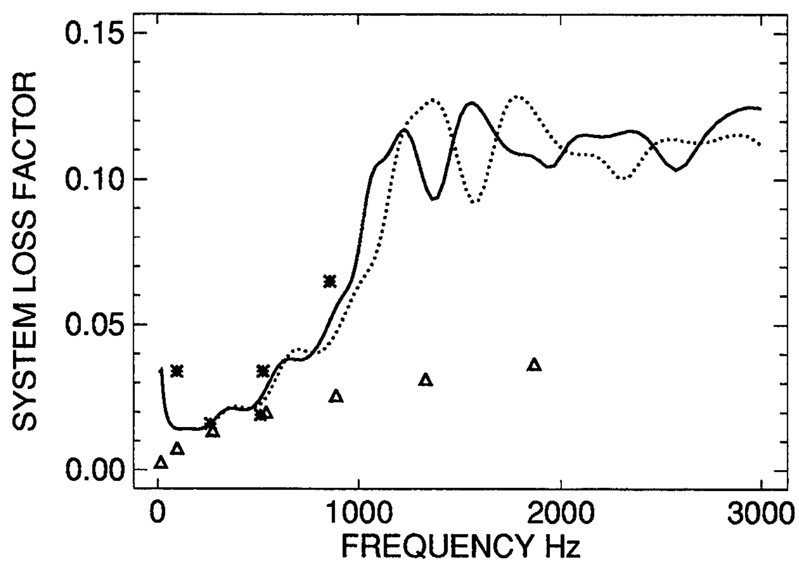



Figure 10: Composite loss factors for damped cantilever; (—— CANT5 refined FE mode, ······ CANT4 coarse FE model, * measurement, △ PREDC code)

meshes, suggesting that a further refinement of the mesh should be used in this frequency range. The loss factors predicted with the PREDC code prediction were significantly lower than the measured points and the VAST prediction, both at very low frequencies and at frequencies above 500 Hz. The PREDC code only considers the damping material loss factor. In the VAST analysis, the bare beam loss factors were used as the material loss factor for the steel beam. If no damping was assumed for the steel, then the VAST prediction was close to the PREDC points in the lower frequency range.

The PREDC code uses an Euler-Bernoulli beam formulation which considers only flexural bending of the steel beam and damping layer. The deformation of the beam and damping layer predicted by the VAST FE model shows much more complex motion of the damping layer at higher frequencies. This may account for the greater damping above 500 Hz which was not predicted with the PREDC code. An example of the deformed shape of the beam and damping layer, at a forcing frequency of 1440 Hz is shown in Figure 11. This is the refined half model. The upper left hand edge is the center-line of the beam. The free end, where the load was applied, is at the lower left hand corner of the figure.

The VAST finite element predictions of the forced vibration response of the steel cantilever beam with a thick damping layer, were in good agreement with experimental results up to 1800 Hz. Above this frequency the analysis indicated that a further refinement of the FE mesh should be considered. The FE analysis also showed that there was significant axial and transverse shear deformation in the damping layer at higher frequencies. Modification of the VAST code, to include frequency dependent values of Poisson's ratio, should improve modelling of the shear

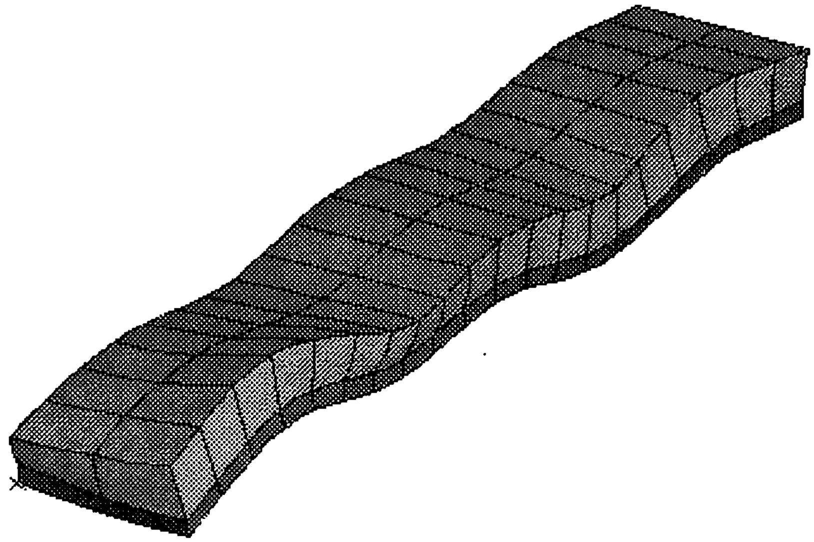


Figure 11: Deformed shape of the damped cantilever at 1440 Hz

properties which may improve response predictions for higher frequencies. Characterization of the complex Poisson's ratio of a polymeric material requires accurate measurement of two complex moduli. Recent experiments at Dockyard Lab have yielded information about the complex Poisson's ratio of various polymers including EAR C-1002, from a combination of acoustic and dynamic mechanical experiments [11]. This information could be used in future calculations.

The PREDC code considered only flexural bending. The VAST FE analysis predicted much more complicated wave motion in the damping layer at higher frequencies. This may explain why the measured loss factors and those predicted by VAST were significantly larger than those obtained with the PREDC code at forcing frequencies above 500 Hz.

# 7 Summary and Conclusions

An analysis has been conducted at DREA of a steel cantilever plate with a thick free layer of EAR Isodamp C-1002 viscoelastic damping material bonded to one surface. Predictions obtained from a finite element analysis using the DREA in-house finite element code VAST, and an analytical code PREDC, have been compared to experimental data. Predictions of composite loss factor and response were made at forcing frequencies between 10 and 3000 Hz using the recently developed direct frequency response capability in VAST. Loss factor predictions were also obtained with the vibration damping prediction code PREDC, acquired from the University of Dayton, Ohio. The frequency dependent dynamic mechanical material properties, used in the VAST and PREDC analyses, were measured at the DREA Dockyard Laboratory using a forced vibration non-resonant method.

The VAST finite element predictions of the forced vibration response of the steel cantilever beam with a thick damping layer were in good agreement with experimental results up to 1800 Hz. Above this frequency the analysis indicated that a further refinement of the FE mesh and possibly a frequency dependent Poisson's ratio should be considered. Above 500 Hz, the system composite loss factors predicted with the PREDC code were significantly below the measured loss factors and those predicted by the VAST code. This is attributed to the more complicated deformation of the damping layer, predicted by the finite element model, but not accounted for by the Euler-Bernoulli beam formulation used in the PREDC code. The differences between the VAST and PREDC loss factor predictions may not be as large for a continuous application of a thick damping layer over a hull plate. The plate would likely provide greater constraint of in-plane motion of the damping layer than the cantilever beam.

# 8 Recommendations for Further Work

This investigation has lead to several recommendations for further work.

1. Further enhancements of the VAST frequency response module are recommended. This includes the modelling of a complex frequency dependent Poisson's ratio, the modelling of incompressible materials and development of single and multiple layer, shear deformable, finite elements which will allow efficient modelling of free and constrained layer viscoelastic damping systems for application to plate and shell structures. A contract to conduct this work has recently been awarded.

2. It is desirable to conduct a more controlled experiment, possibly in-house, which would include measurement of all material properties, more accurate measurement of the system damping characteristics, and measurement of the forced response phase information. These data would then be compared to more refined finite element models including the modelling enhancements listed above. Plate structures, more representative of shipboard applications, would likely be considered.

3. Further experimental work could be conducted to validate the application of the VAST finite element code to more complicated shipboard damping systems, for example, to hull structures, seabays or machinery isolation rafts and mounts.

4. The fitting of the Dockyard Lab dynamic mechanical property data to the ten-parameter University of Dayton model was done manually by selecting approximate parameters, editing the ASCII material data file and then comparing the fitted curves to the data using the MPROP program. This process was repeated until an acceptable fit was obtained. It would be desirable to have a more efficient method to fit the experimental data to the material model. An interactive code is presently under development which employs a user friendly graphical interface to allow viewing of changes to the fitted modulus and loss factor curves as mouse-controlled sliders are moved to vary each parameter.

# Appendix A PREDC database curve fitting

The following equations are used in the University of Dayton code PREDC and damping material data base.

The reduced frequency $F_r$ is defined by the following equation

$$\log(F_r) = \log(F) - 12\frac{(T - T_0)}{(\frac{525}{B} + T - T_0)}. \tag{6}$$

where $F$ is the frequency in Hz, $T$ is the temperature (degrees F. or degrees C.), $T_0$ is a reference temperature and $B = 1$ for British units or $B = 1.8$ for metric units.

The shear modulus $G$ is defined by the equation

$$\log(G) = \log(G_l) + 2\frac{\log\left(\frac{G_{rom}}{G_l}\right)}{1 + \left(\frac{F_{rom}}{F_r}\right)^n} \tag{7}$$

and the loss factor defined by

$$\log(\eta) = \log(\eta_{frol}) + \frac{C}{2}\left[A(S_h + S_l) + \left(1 - \sqrt{1 + A^2}\right)(S_l - S_h)\right] \tag{8}$$

where

$$A = \frac{\log(F_r) - \log(F_{rol})}{C} \tag{9}$$

Ten parameters are required to fit the modulus and loss factor curves. For the EAR C-1002 material the following parameters were used to obtain a good fit to the DREA measured data.

| Param: | $T_0$ | $F_{rom}$ | $G_{rom}$ | $n$ | $G_l$ | $\eta_{frol}$ | $S_l$ | $S_h$ | $F_{rol}$ | $C$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Value: | 93.33 | 4E+07 | 2.67E+07 | 0.38 | 1.13E+06 | 1.05 | 0.4 | -0.33 | 5E+06 | 1.3 |

where the shear modulus $G$ is in Pascals and the temperature is in degrees C.

The measure data was assumed to be at a temperature $T = 20$ degrees C with the reference temperature $T_0$ arbitrarily selected, thus the curve fitting parameters are not likely to be valid for other temperatures in this case. Also the DREA experimental data was for the Young's modulus $E'$ and, as assumed in the PREDC code, was converted to a shear modulus using $G = E'/3$.

# Appendix B VAST input data files

The analysis was conducted on the VAX 6400 system using Version 7.1 of the VAST finite element code. The input files needed for the analysis were a USE file which controls the analysis and includes the damping loss factor information, a GOM file which contains node and element geometry information, an SMD file which contains the nodal displacement boundary conditions and a LOD file which contains the loading information.

## Appendix B.1 CANT4 coarse mesh model of beam and damping layer

The VAST input files for the coarse finite model of the beam and damping layer are listed as follows:

**The VAST control file CANT4.USE:**

```
Entire cantilever structure forced response - coarse mesh
    0    0
    1    0    1    1    1    1    0    1    6    0    0
IELEMA
    1    0    1    1    1
IASSEM
    0    1    1
IASEM4
    2    0
    1    0    1
    2    1    2
TABLE2
    1    7    0
 1.000e+01 0.032e-00 1.910e+01 0.032e-00 1.197e+02 0.620e-02 3.365e+02 0.220e-02
 6.626e+02 0.140e-02 1.097e+03 0.120e-02 1.000e+04 0.100e-02
TABLE1
    1   27    0
 1.000e+01 6.202e-01 2.000e+01 7.273e-01 3.000e+01 8.113e-01 4.000e+01 8.835e-01
 5.000e+01 9.483e-01 6.000e+01 1.008e+00 7.000e+01 1.063e+00 8.000e+01 1.116e+00
 9.000e+01 1.166e+00 1.000e+02 1.214e+00 2.000e+02 1.620e+00 3.000e+02 1.955e+00
 4.000e+02 2.252e+00 5.000e+02 2.525e+00 6.000e+02 2.779e+00 7.000e+02 3.020e+00
 8.000e+02 3.249e+00 9.000e+02 3.468e+00 1.000e+03 3.679e+00 2.000e+03 5.496e+00
 3.000e+03 6.999e+00 4.000e+03 8.317e+00 5.000e+03 9.507e+00 6.000e+03 1.060e+01
 7.000e+03 1.162e+01 8.000e+03 1.257e+01 9.000e+03 1.346e+01
TABLE2
    2   27    0
 1.000e+01 4.618e-01 2.000e+01 5.734e-01 3.000e+01 6.452e-01 4.000e+01 6.982e-01
 5.000e+01 7.400e-01 6.000e+01 7.742e-01 7.000e+01 8.029e-01 8.000e+01 8.274e-01
 9.000e+01 8.488e-01 1.000e+02 8.675e-01 2.000e+02 9.759e-01 3.000e+02 1.021e+00
 4.000e+02 1.043e+00 5.000e+02 1.052e+00 6.000e+02 1.055e+00 7.000e+02 1.055e+00
 8.000e+02 1.052e+00 9.000e+02 1.047e+00 1.000e+03 1.042e+00 2.000e+03 9.782e-01
 3.000e+03 9.238e-01 4.000e+03 8.803e-01 5.000e+03 8.447e-01 6.000e+03 8.150e-01
 7.000e+03 7.896e-01 8.000e+03 7.674e-01 9.000e+03 7.479e-01
ISTIFM
    1    1
IMASSM
```

```
   0     0     0
   0
IDECOM
   1     0
ILOAD1
   0     1     1     0
IDISP6
   4     1     2     0     3     1     1     1     0
 198
   0
 100     2
     448.0   3000.0
IREACT
   0     0
```

The VAST geometry file CANT4.GOM:

```
Cantilever beam with damping layer
   0     0     3     3
 321   321     2
   1     0.0000    0.0000    9.5000                                                 0
   2     0.0000    0.0000    4.7500                                                 0
   3     0.0000    0.0000    0.0000                                                 0
   4     0.0000   51.0000    9.5000                                                 0
   5     0.0000   51.0000    0.0000                                                 0
   6     0.0000  102.0000    9.5000                                                 0
   7     0.0000  102.0000    4.7500                                                 0
   8     0.0000  102.0000    0.0000                                                 0
   9     0.0000  153.0000    9.5000                                                 0
  10     0.0000  153.0000    0.0000                                                 0
  11     0.0000  204.0000    9.5000                                                 0
  12     0.0000  204.0000    4.7500                                                 0
  13     0.0000  204.0000    0.0000                                                 0
  14    32.4500    0.0000    9.5000                                                 0
  15    32.4500    0.0000    0.0000                                                 0
  16    32.4500  102.0000    9.5000                                                 0
  17    32.4500  102.0000    0.0000                                                 0
  18    32.4500  204.0000    9.5000                                                 0
  19    32.4500  204.0000    0.0000                                                 0
  20    64.9000    0.0000    9.5000                                                 0
  21    64.9000    0.0000    4.7500                                                 0
  22    64.9000    0.0000    0.0000                                                 0
  23    64.9000   51.0000    9.5000                                                 0
  24    64.9000   51.0000    0.0000                                                 0
  25    64.9000  102.0000    9.5000                                                 0
  26    64.9000  102.0000    4.7500                                                 0
  27    64.9000  102.0000    0.0000                                                 0
  28    64.9000  153.0000    9.5000                                                 0
  29    64.9000  153.0000    0.0000                                                 0
  30    64.9000  204.0000    9.5000                                                 0
***** lines for nodes 31 to 291 removed from listing
```

```
 292  519.2001    0.0000   36.5000                                                      0
 293  519.2001    0.0000   23.0000                                                      0
 294  519.2001   51.0000   36.5000                                                      0
 295  519.2001  102.0000   36.5000                                                      0
 296  519.2001  102.0000   23.0000                                                      0
 297  519.2001  153.0001   36.5000                                                      0
 298  519.2001  204.0001   36.5000                                                      0
 299  519.2001  204.0001   23.0000                                                      0
 300  551.6501    0.0000   36.5000                                                      0
 301  551.6501  102.0000   36.5000                                                      0
 302  551.6501  204.0000   36.5000                                                      0
 303  584.1000    0.0000   36.5000                                                      0
 304  584.1001    0.0000   23.0000                                                      0
 305  584.1000   51.0000   36.5000                                                      0
 306  584.1001  102.0000   36.5000                                                      0
 307  584.1002  102.0001   23.0000                                                      0
 308  584.1000  153.0001   36.5000                                                      0
 309  584.1000  204.0001   36.5000                                                      0
 310  584.1001  204.0001   23.0000                                                      0
 311  616.5505    0.0000   36.5001                                                      0
 312  616.5505  102.0003   36.5001                                                      0
 313  616.5505  204.0006   36.5001                                                      0
 314  649.0000    0.0000   36.5000                                                      0
 315  649.0000    0.0000   23.0000                                                      0
 316  649.0000   51.0000   36.5000                                                      0
 317  649.0000  102.0000   36.5000                                                      0
 318  649.0000  102.0000   23.0000                                                      0
 319  649.0000  153.0000   36.5000                                                      0
 320  649.0000  204.0000   36.5000                                                      0
 321  649.0000  204.0000   23.0000                                                      0
    2   20    0
 0.207E+06 0.300E+00 0.787E-08
***** numbers reduced from I5 to I4 format on next 20 lines of listing
   1  20  25   6  14  23  16   4   3  22  27   8  15  24  17   5   2  21  26   7
  20  39  44  25  33  42  35  23  22  41  46  27  34  43  36  24  21  40  45  26
  39  58  63  44  52  61  54  42  41  60  65  46  53  62  55  43  40  59  64  45
  58  77  82  63  71  80  73  61  60  79  84  65  72  81  74  62  59  78  83  64
  77  96 101  82  90  99  92  80  79  98 103  84  91 100  93  81  78  97 102  83
  96 115 120 101 109 118 111  99  98 117 122 103 110 119 112 100  97 116 121 102
 115 134 139 120 128 137 130 118 117 136 141 122 129 138 131 119 116 135 140 121
 134 153 158 139 147 156 149 137 136 155 160 141 148 157 150 138 135 154 159 140
 153 172 177 158 166 175 168 156 155 174 179 160 167 176 169 157 154 173 178 159
 172 191 196 177 185 194 187 175 174 193 198 179 186 195 188 176 173 192 197 178
   6  25  30  11  16  28  18   9   8  27  32  13  17  29  19  10   7  26  31  12
  25  44  49  30  35  47  37  28  27  46  51  32  36  48  38  29  26  45  50  31
  44  63  68  49  54  66  56  47  46  65  70  51  55  67  57  48  45  64  69  50
  63  82  87  68  73  85  75  66  65  84  89  70  74  86  76  67  64  83  88  69
  82 101 106  87  92 104  94  85  84 103 108  89  93 105  95  86  83 102 107  88
 101 120 125 106 111 123 113 104 103 122 127 108 112 124 114 105 102 121 126 107
 120 139 144 125 130 142 132 123 122 141 146 127 131 143 133 124 121 140 145 126
 139 158 163 144 149 161 151 142 141 160 165 146 150 162 152 143 140 159 164 145
```

```
 158 177 182 163 168 180 170 161 160 179 184 165 169 181 171 162 159 178 183 164
 177 196 201 182 187 199 189 180 179 198 203 184 188 200 190 181 178 197 202 183
    2   20    0
 0.100E+02 0.470E+00 0.128E-08
***** numbers reduced from I5 to I4 format on next 20 lines of listing
 204 215 218 207 212 217 213 206   1  20  25   6  14  23  16   4 205 216 219 208
 215 226 229 218 223 228 224 217  20  39  44  25  33  42  35  23 216 227 230 219
 226 237 240 229 234 239 235 228  39  58  63  44  52  61  54  42 227 238 241 230
 237 248 251 240 245 250 246 239  58  77  82  63  71  80  73  61 238 249 252 241
 248 259 262 251 256 261 257 250  77  96 101  82  90  99  92  80 249 260 263 252
 259 270 273 262 267 272 268 261  96 115 120 101 109 118 111  99 260 271 274 263
 270 281 284 273 278 283 279 272 115 134 139 120 128 137 130 118 271 282 285 274
 281 292 295 284 289 294 290 283 134 153 158 139 147 156 149 137 282 293 296 285
 292 303 306 295 300 305 301 294 153 172 177 158 166 175 168 156 293 304 307 296
 303 314 317 306 311 316 312 305 172 191 196 177 185 194 187 175 304 315 318 307
 207 218 221 210 213 220 214 209   6  25  30  11  16  28  18   9 208 219 222 211
 218 229 232 221 224 231 225 220  25  44  49  30  35  47  37  28 219 230 233 222
 229 240 243 232 235 242 236 231  44  63  68  49  54  66  56  47 230 241 244 233
 240 251 254 243 246 253 247 242  63  82  87  68  73  85  75  66 241 252 255 244
 251 262 265 254 257 264 258 253  82 101 106  87  92 104  94  85 252 263 266 255
 262 273 276 265 268 275 269 264 101 120 125 106 111 123 113 104 263 274 277 266
 273 284 287 276 279 286 280 275 120 139 144 125 130 142 132 123 274 285 288 277
 284 295 298 287 290 297 291 286 139 158 163 144 149 161 151 142 285 296 299 288
 295 306 309 298 301 308 302 297 158 177 182 163 168 180 170 161 296 307 310 299
 306 317 320 309 312 319 313 308 177 196 201 182 187 199 189 180 307 318 321 310
```

**The VAST boundary condition file CANT4.SMD:**

```
 13 0.100E+26
  1 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  2 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  3 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  4 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  5 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  6 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  7 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  8 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  9 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
 10 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
 11 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
 12 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
 13 1 1 1 1 1 1 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
  0
  0
```

**The VAST load file CANT4.LOD:**

```
cant4 - unit load in z direction at node 198 - center of end
    0    0    0
    0    1
    1    0
  198 0 0.000E+00 0.000E+00 1.000E+00 0.000E+00 0.000E+00 0.000E+00
```

## Appendix B.2 CANT5 refined mesh half-model of beam and damping layer

The VAST input files for the refined mesh half-model of the beam and damping layer are listed as follows:

**The VAST control file CANT5.USE:**

```
Cantilever structure forced response - 1/2 model - refined mesh
    0    0
    1    0    1    1    1    1    0    1    6    0    0
IELEMA
    1    0    1    1    1
IASSEM
    0    1    1
IASEM4
    2    0
    1    0    1
    2    1    2
TABLE2
    1    7    0
 1.000e+01 0.032e-00 1.910e+01 0.032e-00 1.197e+02 0.620e-02 3.365e+02 0.220e-02
 6.626e+02 0.140e-02 1.097e+03 0.120e-02 1.000e+04 0.100e-02
TABLE1
    1   27    0
 1.000e+01 6.202e-01 2.000e+01 7.273e-01 3.000e+01 8.113e-01 4.000e+01 8.835e-01
 5.000e+01 9.483e-01 6.000e+01 1.008e+00 7.000e+01 1.063e+00 8.000e+01 1.116e+00
 9.000e+01 1.166e+00 1.000e+02 1.214e+00 2.000e+02 1.620e+00 3.000e+02 1.955e+00
 4.000e+02 2.252e+00 5.000e+02 2.525e+00 6.000e+02 2.779e+00 7.000e+02 3.020e+00
 8.000e+02 3.249e+00 9.000e+02 3.468e+00 1.000e+03 3.679e+00 2.000e+03 5.496e+00
 3.000e+03 6.999e+00 4.000e+03 8.317e+00 5.000e+03 9.507e+00 6.000e+03 1.060e+01
 7.000e+03 1.162e+01 8.000e+03 1.257e+01 9.000e+03 1.346e+01
TABLE2
    2   27    0
 1.000e+01 4.618e-01 2.000e+01 5.734e-01 3.000e+01 6.452e-01 4.000e+01 6.982e-01
 5.000e+01 7.400e-01 6.000e+01 7.742e-01 7.000e+01 8.029e-01 8.000e+01 8.274e-01
 9.000e+01 8.488e-01 1.000e+02 8.675e-01 2.000e+02 9.759e-01 3.000e+02 1.021e+00
 4.000e+02 1.043e+00 5.000e+02 1.052e+00 6.000e+02 1.055e+00 7.000e+02 1.055e+00
 8.000e+02 1.052e+00 9.000e+02 1.047e+00 1.000e+03 1.042e+00 2.000e+03 9.782e-01
 3.000e+03 9.238e-01 4.000e+03 8.803e-01 5.000e+03 8.447e-01 6.000e+03 8.150e-01
 7.000e+03 7.896e-01 8.000e+03 7.674e-01 9.000e+03 7.479e-01
ISTIFM
    1    1
IMASSM
    0    0    0
    0
IDECOM
    1    0
ILOAD1
    0    1    1    0
IDISP6
    4    1    2    0    1    1    1    1    0
  383
```

```
   0
 100    2
    448.0  3000.0
   0    0
IREACT
   0    0
```

The VAST geometry file CANT5.GOM:

```
cantilever beam with damping layer - 1/2 model
   0    0     3     3
 621  621     2
   1    0.0000    0.0000    9.5000                                        0
   2    0.0000    0.0000    4.7500                                        0
   3    0.0000    0.0000    0.0000                                        0
   4    0.0000   25.5000    9.5000                                        0
   5    0.0000   25.5000    0.0000                                        0
   6    0.0000   51.0000    9.5000                                        0
   7    0.0000   51.0000    4.7500                                        0
   8    0.0000   51.0000    0.0000                                        0
   9    0.0000   76.5000    9.5000                                        0
  10    0.0000   76.5000    0.0000                                        0
  11    0.0000  102.0000    9.5000                                        0
  12    0.0000  102.0000    4.7500                                        0
  13    0.0000  102.0000    0.0000                                        0
  14   16.2250    0.0000    9.5000                                        0
  15   16.2250    0.0000    0.0000                                        0
  16   16.2250   51.0000    9.5000                                        0
  17   16.2250   51.0000    0.0000                                        0
  18   16.2250  102.0000    9.5000                                        0
  19   16.2250  102.0000    0.0000                                        0
  20   32.4500    0.0000    9.5000                                        0
  21   32.4500    0.0000    4.7500                                        0
  22   32.4500    0.0000    0.0000                                        0
  23   32.4500   25.5000    9.5000                                        0
  24   32.4500   25.5000    0.0000                                        0
  25   32.4500   51.0000    9.5000                                        0
  26   32.4500   51.0000    4.7500                                        0
  27   32.4500   51.0000    0.0000                                        0
  28   32.4500   76.5000    9.5000                                        0
  29   32.4500   76.5000    0.0000                                        0
  30   32.4500  102.0000    9.5000                                        0
***** lines for nodes 31 to 591 removed from listing
 592  584.1002    0.0000   36.5000                                        0
 593  584.1002    0.0000   23.0000                                        0
 594  584.1002   25.5000   36.5000                                        0
 595  584.1002   51.0000   36.5000                                        0
 596  584.1002   51.0000   23.0000                                        0
 597  584.1002   76.5000   36.5000                                        0
 598  584.1002  102.0000   36.5000                                        0
 599  584.1002  102.0000   23.0000                                        0
```

```
  600  600.3254    0.0000   36.5000                                                 0
  601  600.3254   51.0001   36.5000                                                 0
  602  600.3254  102.0001   36.5000                                                 0
  603  616.5498    0.0000   36.4999                                                 0
  604  616.5498    0.0000   23.0000                                                 0
  605  616.5498   25.5000   36.4999                                                 0
  606  616.5498   50.9999   36.4999                                                 0
  607  616.5498   50.9999   23.0000                                                 0
  608  616.5498   76.4999   36.4999                                                 0
  609  616.5498  101.9998   36.4999                                                 0
  610  616.5498  101.9999   23.0000                                                 0
  611  632.7751    0.0000   36.5000                                                 0
  612  632.7751   51.0001   36.5000                                                 0
  613  632.7751  102.0001   36.5000                                                 0
  614  649.0000    0.0000   36.5000                                                 0
  615  649.0000    0.0000   23.0000                                                 0
  616  649.0000   25.5000   36.5000                                                 0
  617  649.0000   51.0000   36.5000                                                 0
  618  649.0000   51.0000   23.0000                                                 0
  619  649.0000   76.5000   36.5000                                                 0
  620  649.0000  102.0000   36.5000                                                 0
  621  649.0000  102.0000   23.0000                                                 0
    2   40    0
 0.207E+06 0.300E+00 0.787E-08
***** numbers reduced from I5 to I4 format on next 40 lines of listing
   1  20  25   6  14  23  16   4   3  22  27   8  15  24  17   5   2  21  26   7
  20  39  44  25  33  42  35  23  22  41  46  27  34  43  36  24  21  40  45  26
  39  58  63  44  52  61  54  42  41  60  65  46  53  62  55  43  40  59  64  45
  58  77  82  63  71  80  73  61  60  79  84  65  72  81  74  62  59  78  83  64
  77  96 101  82  90  99  92  80  79  98 103  84  91 100  93  81  78  97 102  83
  96 115 120 101 109 118 111  99  98 117 122 103 110 119 112 100  97 116 121 102
 115 134 139 120 128 137 130 118 117 136 141 122 129 138 131 119 116 135 140 121
 134 153 158 139 147 156 149 137 136 155 160 141 148 157 150 138 135 154 159 140
 153 172 177 158 166 175 168 156 155 174 179 160 167 176 169 157 154 173 178 159
 172 191 196 177 185 194 187 175 174 193 198 179 186 195 188 176 173 192 197 178
 191 210 215 196 204 213 206 194 193 212 217 198 205 214 207 195 192 211 216 197
 210 229 234 215 223 232 225 213 212 231 236 217 224 233 226 214 211 230 235 216
 229 248 253 234 242 251 244 232 231 250 255 236 243 252 245 233 230 249 254 235
 248 267 272 253 261 270 263 251 250 269 274 255 262 271 264 252 249 268 273 254
 267 286 291 272 280 289 282 270 269 288 293 274 281 290 283 271 268 287 292 273
 286 305 310 291 299 308 301 289 288 307 312 293 300 309 302 290 287 306 311 292
 305 324 329 310 318 327 320 308 307 326 331 312 319 328 321 309 306 325 330 311
 324 343 348 329 337 346 339 327 326 345 350 331 338 347 340 328 325 344 349 330
 343 362 367 348 356 365 358 346 345 364 369 350 357 366 359 347 344 363 368 349
 362 381 386 367 375 384 377 365 364 383 388 369 376 385 378 366 363 382 387 368
   6  25  30  11  16  28  18   9   8  27  32  13  17  29  19  10   7  26  31  12
  25  44  49  30  35  47  37  28  27  46  51  32  36  48  38  29  26  45  50  31
  44  63  68  49  54  66  56  47  46  65  70  51  55  67  57  48  45  64  69  50
  63  82  87  68  73  85  75  66  65  84  89  70  74  86  76  67  64  83  88  69
  82 101 106  87  92 104  94  85  84 103 108  89  93 105  95  86  83 102 107  88
 101 120 125 106 111 123 113 104 103 122 127 108 112 124 114 105 102 121 126 107
```

```
 120 139 144 125 130 142 132 123 122 141 146 127 131 143 133 124 121 140 145 126
 139 158 163 144 149 161 151 142 141 160 165 146 150 162 152 143 140 159 164 145
 158 177 182 163 168 180 170 161 160 179 184 165 169 181 171 162 159 178 183 164
 177 196 201 182 187 199 189 180 179 198 203 184 188 200 190 181 178 197 202 183
 196 215 220 201 206 218 208 199 198 217 222 203 207 219 209 200 197 216 221 202
 215 234 239 220 225 237 227 218 217 236 241 222 226 238 228 219 216 235 240 221
 234 253 258 239 244 256 246 237 236 255 260 241 245 257 247 238 235 254 259 240
 253 272 277 258 263 275 265 256 255 274 279 260 264 276 266 257 254 273 278 259
 272 291 296 277 282 294 284 275 274 293 298 279 283 295 285 276 273 292 297 278
 291 310 315 296 301 313 303 294 293 312 317 298 302 314 304 295 292 311 316 297
 310 329 334 315 320 332 322 313 312 331 336 317 321 333 323 314 311 330 335 316
 329 348 353 334 339 351 341 332 331 350 355 336 340 352 342 333 330 349 354 335
 348 367 372 353 358 370 360 351 350 369 374 355 359 371 361 352 349 368 373 354
 367 386 391 372 377 389 379 370 369 388 393 374 378 390 380 371 368 387 392 373
    2   40    0
 0.100E+02 0.470E+00 0.128E-08
***** numbers reduced from I5 to I4 format on next 40 lines of listing
 394 405 408 397 402 407 403 396   1  20  25   6  14  23  16   4 395 406 409 398
 405 416 419 408 413 418 414 407  20  39  44  25  33  42  35  23 406 417 420 409
 416 427 430 419 424 429 425 418  39  58  63  44  52  61  54  42 417 428 431 420
 427 438 441 430 435 440 436 429  58  77  82  63  71  80  73  61 428 439 442 431
 438 449 452 441 446 451 447 440  77  96 101  82  90  99  92  80 439 450 453 442
 449 460 463 452 457 462 458 451  96 115 120 101 109 118 111  99 450 461 464 453
 460 471 474 463 468 473 469 462 115 134 139 120 128 137 130 118 461 472 475 464
 471 482 485 474 479 484 480 473 134 153 158 139 147 156 149 137 472 483 486 475
 482 493 496 485 490 495 491 484 153 172 177 158 166 175 168 156 483 494 497 486
 493 504 507 496 501 506 502 495 172 191 196 177 185 194 187 175 494 505 508 497
 504 515 518 507 512 517 513 506 191 210 215 196 204 213 206 194 505 516 519 508
 515 526 529 518 523 528 524 517 210 229 234 215 223 232 225 213 516 527 530 519
 526 537 540 529 534 539 535 528 229 248 253 234 242 251 244 232 527 538 541 530
 537 548 551 540 545 550 546 539 248 267 272 253 261 270 263 251 538 549 552 541
 548 559 562 551 556 561 557 550 267 286 291 272 280 289 282 270 549 560 563 552
 559 570 573 562 567 572 568 561 286 305 310 291 299 308 301 289 560 571 574 563
 570 581 584 573 578 583 579 572 305 324 329 310 318 327 320 308 571 582 585 574
 581 592 595 584 589 594 590 583 324 343 348 329 337 346 339 327 582 593 596 585
 592 603 606 595 600 605 601 594 343 362 367 348 356 365 358 346 593 604 607 596
 603 614 617 606 611 616 612 605 362 381 386 367 375 384 377 365 604 615 618 607
 397 408 411 400 403 410 404 399   6  25  30  11  16  28  18   9 398 409 412 401
 408 419 422 411 414 421 415 410  25  44  49  30  35  47  37  28 409 420 423 412
 419 430 433 422 425 432 426 421  44  63  68  49  54  66  56  47 420 431 434 423
 430 441 444 433 436 443 437 432  63  82  87  68  73  85  75  66 431 442 445 434
 441 452 455 444 447 454 448 443  82 101 106  87  92 104  94  85 442 453 456 445
 452 463 466 455 458 465 459 454 101 120 125 106 111 123 113 104 453 464 467 456
 463 474 477 466 469 476 470 465 120 139 144 125 130 142 132 123 464 475 478 467
 474 485 488 477 480 487 481 476 139 158 163 144 149 161 151 142 475 486 489 478
 485 496 499 488 491 498 492 487 158 177 182 163 168 180 170 161 486 497 500 489
 496 507 510 499 502 509 503 498 177 196 201 182 187 199 189 180 497 508 511 500
 507 518 521 510 513 520 514 509 196 215 220 201 206 218 208 199 508 519 522 511
 518 529 532 521 524 531 525 520 215 234 239 220 225 237 227 218 519 530 533 522
 529 540 543 532 535 542 536 531 234 253 258 239 244 256 246 237 530 541 544 533
 540 551 554 543 546 553 547 542 253 272 277 258 263 275 265 256 541 552 555 544
```

```
551 562 565 554 557 564 558 553 272 291 296 277 282 294 284 275 552 563 566 555
562 573 576 565 568 575 569 564 291 310 315 296 301 313 303 294 563 574 577 566
573 584 587 576 579 586 580 575 310 329 334 315 320 332 322 313 574 585 588 577
584 595 598 587 590 597 591 586 329 348 353 334 339 351 341 332 585 596 599 588
595 606 609 598 601 608 602 597 348 367 372 353 358 370 360 351 596 607 610 599
606 617 620 609 612 619 613 608 367 386 391 372 377 389 379 370 607 618 621 610
```

**The VAST boundary condition file CANT5.SMD:**

```
 170
   1 1 1 1 1 1 1
   2 1 1 1 1 1 1
   3 1 1 1 1 1 1
   4 1 1 1 1 1 1
   5 1 1 1 1 1 1
   6 1 1 1 1 1 1
   7 1 1 1 1 1 1
   8 1 1 1 1 1 1
   9 1 1 1 1 1 1
  10 1 1 1 1 1 1
  11 1 1 1 1 1 1
  12 1 1 1 1 1 1
  13 1 1 1 1 1 1
  20 0 1 0 1 0 1
  21 0 1 0 1 0 1
  22 0 1 0 1 0 1
  33 0 1 0 1 0 1
  34 0 1 0 1 0 1
  39 0 1 0 1 0 1
  40 0 1 0 1 0 1
  41 0 1 0 1 0 1
  52 0 1 0 1 0 1
  53 0 1 0 1 0 1
  58 0 1 0 1 0 1
  59 0 1 0 1 0 1
  60 0 1 0 1 0 1
  71 0 1 0 1 0 1
  72 0 1 0 1 0 1
  77 0 1 0 1 0 1
  78 0 1 0 1 0 1
  79 0 1 0 1 0 1
  90 0 1 0 1 0 1
  91 0 1 0 1 0 1
  96 0 1 0 1 0 1
  97 0 1 0 1 0 1
  98 0 1 0 1 0 1
***** similar lines for the following nodes along the beam centerline
        have been removed from the file listing:
  109, 110, 115, 116, 117, 128, 129, 134, 135, 136, 147, 148, 153, 154, 155,
  166, 167, 172, 173, 174, 185, 186, 191, 192, 193, 204, 205, 210, 211, 212,
  223, 224, 229, 230, 231, 242, 243, 248, 249, 250, 261, 262, 267, 268, 269,
```

```
280, 281, 286, 287, 288, 299, 300, 305, 306, 307, 318, 319, 324, 325, 326,
337, 338, 343, 344, 345, 356, 357, 362, 363, 364, 375, 376, 381, 382, 383,
405, 406, 413, 416, 417, 424, 427, 428, 435, 438, 439, 446, 449, 450, 457,
460, 461, 468, 471, 472, 479, 482, 483, 490, 493, 494, 501, 504, 512, 516
523 0 1 0 1 0 1
526 0 1 0 1 0 1
527 0 1 0 1 0 1
534 0 1 0 1 0 1
537 0 1 0 1 0 1
538 0 1 0 1 0 1
545 0 1 0 1 0 1
548 0 1 0 1 0 1
549 0 1 0 1 0 1
556 0 1 0 1 0 1
559 0 1 0 1 0 1
560 0 1 0 1 0 1
567 0 1 0 1 0 1
570 0 1 0 1 0 1
571 0 1 0 1 0 1
578 0 1 0 1 0 1
581 0 1 0 1 0 1
582 0 1 0 1 0 1
589 0 1 0 1 0 1
592 0 1 0 1 0 1
593 0 1 0 1 0 1
600 0 1 0 1 0 1
603 0 1 0 1 0 1
604 0 1 0 1 0 1
611 0 1 0 1 0 1
614 0 1 0 1 0 1
615 0 1 0 1 0 1
  0
  0
```

**The VAST load file CANT5.LOD:**

```
cant5 - unit load in z direction at node 383 - center of end
   0     0     0
   0     1
   1     0
  383 0 0.000E+00 0.000E+00 1.000E+00 0.000E+00 0.000E+00 0.000E+00
```

# References


[1] "Vibration and Strength Analysis Program (VAST), Version #7.0", Martec Limited, Halifax, Nova Scotia, December 1993.

[2] Stredulinsky, D.C., "Isolation of Structure-Borne Sound from Ship Machinery: A Literature Review and Possibilities for Finite Element Modelling", DREA Technical Memorandum 91/208, May 1991.

[3] Orisamolu, I.R., Palmeter, M.F., Jiang, L., and Chernuka, M.W., "Enhancement of the Frequency Response Module of the VAST Finite Element Program for Vibration Isolation and Structural Acoustic Analysis", DREA Contractor Report 94/408, October 1993.

[4] Szabo, J.P., "A Forced Vibration Non-Resonant Method for the Determination of the Complex Modulus in the Audio Frequency Range", DREA Technical Memorandum 92/201, 1992.

[5] Klein, K., and Guigné, J., "An Experimental Study of a Damping Material Applied to a Vibrating Cantilever", DREA Contractor Report 92/461, April 1993.

[6] Drake, M.L., "PREDC/MPROP Vibration Damping Program User's Manual", Vibrations Group, Aerospace Mechanics Division, Research Institute, The University of Dayton, Dayton, Ohio, 1994.

[7] EAR Isodamp C-1002 damping material, EAR Specialty Composites Division of Cabot Corporation, Indianapolis, IN.

[8] Nashif, A.D., Jones, D.I.G, and Henderson, J.P., "Vibration Damping", John Wiley and Sons, New York, 1985.

[9] Smithells Metals Reference Handbook, Eric A. Brandes, ed., Buttersworth, Toronto, 1983.

[10] Jones, D.I.G., "Results of a Round Robin Test Program: Complex Modulus Properties of a Polymeric Damping Material", Wright Patterson Air Force Base, FWAL/TR-92/3104 (1992).

[11] Taylor, E.L. and Szabo, J.P., "From Mechanics to Acoustics: Modelling Polymer Properties", DREA Technical Memorandum 95/211, May 1995.

## DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

| | |
|---|---|
| 1. ORIGINATOR (The name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Establishment sponsoring a contractor's report, or tasking agency, are entered in section 8.)<br>Defence Research Establishment Atlantic<br>P.O. Box 1012, Dartmouth, N.S. B2Y 3Z7 | 2. SECURITY CLASSIFICATION (Overall security of the document including special warning terms if applicable.)<br>Unclassified |

3. TITLE (The complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S,C,R or U) in parentheses after the title.)

The Forced Response and Damping Characteristics of a Cantilever Beam with a Thick Damping Layer

4. AUTHORS (Last name, first name, middle initial. If military, show rank, e.g. Doe, Maj. John E.)

Stredulinsky, David C. and Szabo, Jeffrey P.

| | | |
|---|---|---|
| 5. DATE OF PUBLICATION (Month and year of publication of document.)<br>December 1995 | 6a. NO. OF PAGES (Total containing information. Include Annexes, Appendices, etc.)<br>35 | 6b. NO. OF REFS. (Total cited in document.)<br>11 |

6. DESCRIPTIVE NOTES (The category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)

DREA Technical Memorandum

8. SPONSORING ACTIVITY (The name of the department project office or laboratory sponsoring the reseach and development. include the address.)

Defence Research Establishment Atlantic
P.O. Box 1012, Dartmouth, N.S. B2Y 3Z7

| | |
|---|---|
| 9a. PROJECT OR GRANT NUMBER (If appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant.)<br>1.g.3 1.f.1 | 9b. CONTRACT NUMBER (If appropriate, the applicable number under which the document was written.) |
| 10a. ORIGINATOR'S DOCUMENT NUMBER (The official document number by which the document is identified by the originating activity. This number mrst be unique to this document.)<br>DREA Technical Memorandum 95/229 | 10b. OTHER DOCUMENT NUMBERS (Any other numbers which may be assigned this document either by the originator or by the sponsor.) |

11. DOCUMENT AVAILABILITY (Any limitations on further dissemination of the document, other than those imposed by security classification)

( X ) Unlimited distribution
( ) Distribution limited to defence departments and defence contractors; further distribution only as approved
( ) Distribution limited to defence departments and Canadian defence contractors; further distribution only as approved
( ) Distribution limited to government departments and agencies; futher distribution only as approved
( ) Distribution limited to defence departments; further distribution only as approved
( ) Other (please specify):

12. DOCUMENT ANNOUNCEMENT (Any limitation to the bibliographic annoucement of this document. This will normally correspond to the Document Availability (11). However, where futher distribution (beyond the audience specified in 11) is possible, a wider announcement audience may be selected.)

UNLIMITED DISTRIBUTION

13. ABSTRACT (a brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), (R), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual).

This technical memorandum describes the vibration analysis of a steel cantilever beam. The beam was 649 mm long, 204 mm wide and 9.5 mm thick with a free viscoelastic damping layer (27 mm thick EAR Isodamp C-1002) bonded to one surface. Predictions of composite loss factor and response were made at forcing frequencies between 10 Hz and 3000 Hz using a direct frequency response capability recently developed for the DREA in-house finite element code VAST. Analytical results were also obtained with the code PREDC, acquired from the University of Dayton, Ohio. The frequency dependent dynamic mechanical material properties, used in the VAST and PREDC analyses, were measured at DREA using a forced vibration non-resonant method. The VAST forced response vibration analysis was in good agreement with experiment. Above 500 Hz, the system composite loss factors predicted with the PREDC code were significantly below the measured loss factors and those predicted by the VAST code. This is attributed to the more complicated deformation of the damping layer, predicted by the finite element model, but not accounted for by the Euler-Bernoulli beam formulation used in the PREDC code.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus-identified. If it not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title).

Vibration
Damping
Finite Element
Polymer