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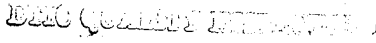
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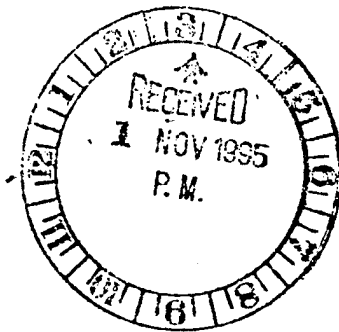
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Robert M'Closkey (now an Assistant Professor in the Mechanical Engineering Department at UCLA) worked in the general area of stabilization of strongly nonlinear systems. His thesis work addresses various control theoretic aspects of driftless control systems. Driftless systems area class of nonlinear problems that arise in physical systems with nonintegrable (nonholonomic) constraints and/or conservation laws. The stabilization of these systems present special problems no continuous dynamics or static function of the state can stablize the systems to a point. It is necessary to introduce explicit time variation into the control law for the continuous stabilization probe. Recently, several researchers have developed synthesis methods for generating stabilizing controllers. However, the closed-loop systems suffer from very slow convergence rates due to the fact that the control laws are Lipschitz functions.

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# Nonlinear Robust Control Theory and Applications

John C. Doyle, Principal Investigator

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## Overview of research program for Robert T. M'Closkey

Robert M'Closkey (now an Assistant Professor in the Mechanical Engineering Department at UCLA) worked in the general area of stabilization of strongly nonlinear systems. His thesis work addresses various control theoretic aspects of driftless control systems. Driftless systems are a class of nonlinear problems that arise in physical systems with nonintegrable (non-holonomic) constraints and/or conservation laws. The stabilization of these systems present special problems: no continuous dynamic or static function of the state can stabilize the systems to a point. It is necessary to introduce explicit time variation into the control law for the continuous stabilization problem. Recently, several researchers have developed synthesis methods for generating stabilizing controllers. However, the closed-loop systems suffer from very slow convergence rates due to the fact that the control laws are Lipschitz functions.

Our work in this area dramatically improves the existing convergence rates to guarantee exponential rates for all of the state variables. The control laws remain  $C^0$  but are not Lipschitz at certain points in the phase space. In addition to exploring new design methods, we have also extended the synthesis procedures of previous authors to the mathematical framework we have proposed. Control laws designed with these tools directly address performance limiting factors such as actuator slew rates and controller effort. The theory also allows for some rudimentary robustness analysis by identifying a class of perturbations which do not locally affect the closed-loop system stability. In addition, we have constructed an experimental mobile robot that is used as a testbed to compare the performance of various controllers. The device consists of a two-wheeled car that pulls several

carts. The rolling of the wheels imposes nonholonomic constraints. The control objective is to stabilize the car/trailer system with feedback to a desired position and orientation. The experiments have demonstrated the superior performance of the exponential stabilizers over the more traditional differentiable feedbacks.

M'Closkey's work provides a first step towards generating a more comprehensive set of tools for analysis and synthesis of nonlinear control laws. As part of this work, we have developed and extended tools for analyzing stability using homogeneous functions and vector fields (relative to a non-standard dilation). These results provide a powerful set of techniques which have broad application to more general control systems. In particular, we have been able to synthesize time-varying controllers which are only  $C^0$  at the origin without losing all of the standard analysis tools which one uses to prove stability (Lyapunov functions, averaging, etc). Recent work has extended the use of these methods to strongly nonlinear systems in which a drift term is present but the use of the the linearization is insufficient (or undesirable) for stabilization purposes.

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