NATIONAL AIR INTELLIGENCE CENTER

A TWO-DIMENSIONAL NUMERICAL PROGRAM
SIMULATING THE EFFECTS OF FREE-ELECTRON LASER
NONROTATIONAL SYMMETRY

by

Qian Sihai and Shi Yijin

DTIC
SELECTED
JAN 1 9 1996
G

19960104 046

DTIC QUALITY INSPECTED 1

Accesion For		
NTIS CRA&I		☒
DTIC TAB		☐
Unannounced		☐
Justification		
By		
Distribution /		
Availability Codes		
Dist	Avail and / or Special	
A-1		

HUMAN TRANSLATION

NAIC-ID(RS)T-0366-95   31 October 1995

MICROFICHE NR: 95C000681

A TWO-DIMENSIONAL NUMERICAL PROGRAM
SIMULATING THE EFFECTS OF FREE-ELECTRON LASER
NONROTATIONAL SYMMETRY

By: Qian Sihai and Shi Yijin

English pages: 15

Country of origin: China
Translated by: Edward A. Suter
Requester: NAIC/TATD/Dr. James Newton
Approved for public release: distribution unlimited.

19960104 046

PAGES 3&11 ARE MISSING BECAUSE PAGES

WERE MISNUMBERED PER CHARLES

GUERRANT (513 257-3531) (DSN: 787-3531)

AT NATIONAL AIR INTELLIGENCE CENTER

(NAIC) WRIGHT-PATTERSON AFB, OH.

FEBRUARY 5, 1996

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this
translation were extracted from the best quality copy available.

A TWO-DIMENSIONAL NUMERICAL PROGRAM SIMULATING THE EFFECTS OF FREE-ELECTRON LASER NONROTATIONAL SYMMETRY

(Romanized title: "*Ziyou Dianzi Jiguang Feixuanzhuan Duichen Xiaoying De Erwei Shuzhi Moni*")

by Qian Sihai and Shi Yijin
(China Institute of Atomic Energy, P.O. Box 275 (18), Beijing 102413)

Abstract: A nonrotational symmetry 2D FEL numerical simulation program was set up with electron wiggling motion direction extended infinitely. This program was used to calculate light field evolution under conditions of nonideal incidence. Results showed that both shifting the center of the electron beam and tilting the beam axis caused the exponential degradation of luminous power output and excitation of higher-order transverse modes, as well as deviation of the light beam's barycenter. In addition to emittance, transverse phase space geometry of the electron beam is also an important parameter of FEL operation quality and has an influence on luminous power.

Key terms: nonrotational symmetry, nonideal incidence, phase space geometry

1. Introduction

The higher dimension effect of free-electron lasers [FELs] is a problem that [scientists] have wanted to understand for a long time but have not yet fully understood. It is not only important that its effects on transverse mode competition and light guide phenomena be understood for the field of physics, but the field of engineering design should

---

[1] Numbers on the extreme right indicate the pages of the original document.

also consider its effects on gain, extraction efficiency, and light beam quality. Because there is a larger amount of computation work for the FEL 3D [program], it is necessary to simplify the model even more and reduce the quantity of computations of individual plans in order to clearly understand the essence of this problem. Considering [this problem] as a two-dimensional approximation can decrease computation time by at least one magnitude. Thus, many people have obtained good results by using rotational symmetry approximations[1-5] and reducing the problem to two dimensions. Even though the geometrical structure of FELs is ordinarily close to rotational symmetry (and rotational symmetry approximations are thus important), there are still many situations that give rise to nonrotational symmetry. As far as the problem of collimation of the three axes in the FEL (the electron beam axis, the optical axis, and the oscillator[2] magnetic axis) is concerned, it is impossible for engineering design and physics debugging to completely avoid small deflections between the axes. We can only make demands for an allowable amount of deviation.

Because of the special geometric structure of the linearly polarized oscillator, wiggler field transverse distribution is unavoidably elliptical. [We] cannot rule out carrying out investigations that fully use the elliptical nature of the wiggler field to inject oval cross-section electron beams. In addition, because of the introduction of glancing incidence lenses into the ring-shaped cavity, the light beam basically becomes an oval cross section (because the light waists of two directions do not coincide). Thus, asymmetry exists in two directions in the transverse direction. A geometrical approximation like this, that is, another kind of two-dimensional approximation relative to rotational symmetry, is to extend one direction infinitely so that it becomes translation-invariant. Here, the direction chosen is the one perpendicular to the wiggler field, which is called the x direction. The wiggler field [direction] is the y direction, and the electron longitudinal motion direction is the z direction.

In this article, we observed and studied two situations of nonideal electron beam incidence: in one, the electron beam has a slight deviation from the oscillator when injected towards the center, and in the other, the electron beam incidence direction has a small angle of deflection from the oscillator axis.

---

[2] The word *bozhenqi*, which I translated here as "oscillator," was not found in the dictionary. A more literal translation would be "wave vibrator." *Zhendangqi* is the most common word for oscillator.

Of these equations, numbers (1) and (2) are electron longitudinal motion equations, (3) and (4) are transverse motion equations, (5) is a complex light field $E(y, z) = E(y, z)\exp\{i\theta(y, z)\}$ Maxwell equation adaxial form, $Y = z/L_w$, L_w is the overall length of the oscillator, $k_w(z) = 2\pi/\lambda_w(z)$ and $k_s = 2\pi/\lambda_s$ are, respectively, oscillator wave number and light field wave number, and $a_w(y, z)$ is the reduced vector-potential of the oscillator.

$$a_w(y, z) = \frac{e}{mc^2} \frac{B_y(y, z)}{k_w(z)} \approx \frac{e}{mc^2} \frac{B_w(z)}{k_w(z)} [1 + \frac{1}{2}(k_w y)^2]$$

$$= a_w(z)[1 + \frac{1}{2}(k_w y)^2] \qquad (6)$$

$$F_w = J_0(\mu) - J_1(\mu), \qquad \mu = \frac{k_s a_w^2(z)}{8\gamma^2 k_w(z)} \qquad (7)$$

J_0 and J_1 are, respectively, zero- and first-level Bessel functions. $<...>$ indicates an average of the corresponding amount of electrons in the grids with difference grid coordinates of (y, z). Electric current intensity J in formula (5) is expressed in units of A/cm^2.

The electron motion equation was derived through a fourth-level Runge-Kutta method, and the light field equation was derived through a steady-state difference format.

The initial values of the electron motion equation (λ, ψ, y, and β_y) are usually sampled and given in four-dimensional phase space. Experience shows that the initial value of ψ is very sensitive to calculation, and that there are large discrepancies between different distribution sampling results. This is, of course, caused by there being an insufficient number of sampled electrons. Obviously, increasing the number of electrons N in the sample in order to stabilize the calculation is not a sound method. Because ψ brought about serious consequences in four variable quantities, we adopted a stationary initiation method[5]. We split up a number of samples N of electrons into m groups, each containing an equal number n of electrons. $N = mn$, and the number of electrons n in each group all had the same (γ, y, and β_y) initial sampling values, while each group's phase was fixed at $\psi_i = 2\pi i/n$, $i = 1, 2, ..., n$. By choosing $m = 512$, $n = 8$, we obtained stable results.

We used two methods to calculate sampling efficiency. One was from evolution of optical power $\eta_0 = (P(z) - P_{in})/\sum_i \gamma_i(0)$, and the other was from electron energy deficit $\eta_e = \sum_i [\lambda_i(0) - \gamma_i(z)]/\gamma_i(0)$. During the entire calculation, the relationship $\eta_0(z) = \eta_e(z)$ was strictly kept. Thus, the calculation was reliable and self-consistent.

4

Table 1 Simulation Parameters

electron beam		undulator	
Lorentz factor	$\gamma \approx 100.32$	Length	$L_* = 500\text{cm}$
Energy spread	$\delta\gamma/\gamma \approx 1\%$	Period	$\lambda_* = 8\text{cm}$
Emittance	$\varepsilon = \gamma\beta_\gamma = 4.4 \times 10^{-3}\text{cm}\cdot\text{rad}$		$k_* = \dfrac{2\pi}{\lambda_*} = 0.7854\text{cm}^{-1}$
Beam radius (adjustable)	$R_* = 0.21\text{cm}$		
Current	$I = 4.2\text{kA}/\text{cm}$	Peak magnetic field	$B_* = 2430\text{Gs}$
Optical field		Number of sampling electrons $N = 4096$	
Wavelength	$\lambda_r = 10.6\mu\text{m}$	Transverse width $y_{\_} = 3\text{cm}$	
Input power	$P_{in} = 1\text{MW}$		
Input waist size	$x_* = 0.35\text{cm}$		

To make a comparison with other calculations[1-5] and to test and verify the reliability of the program, see Table 1 for the parameters[2] used throughout the calculation. Electric current alone was found to be 4244A, which was $2r_e/\pi r_e^2$ times that of document [2] ($r = 0.3$cm). The subjective consideration that prompted seeking this ratio was to cause the electric current densities of the two to be equal. Therefore, for the output optical power between the two, [their] power densities should also be compared ($2r_e/\pi r_e^2$ is the ratio of the rectangular cross section to the circular cross section, and the X direction takes unit length). Figure 1 shows output optical power at different amounts of incident electron beam center energy γ. When $\gamma = 100$, density of output optical power is 1GW/cm². Thus, average power density is consistent with document [2].

3. Matching of electron beam emittance and oscillator output

Electron beam emittance is an important parameter of free electron laser operation quality. The emittance of short wavelength free electron lasers especially requires wavelength magnitude. Accordingly, the electron beam should realize full coincidence with the light beam to the greatest extent possible. This was suggested through experience and theoretical calculations. From the point of view of electron beam transport, so-called "full coincidence" means that the electron beam envelope evolves according to the evolution of

the light beam envelope. For electron beam transmission, there are not only requirements for beam waist size and position, the largest envelope radius, or other individual measurements, but for the entire transmission process. Thus, requirements must be proposed not only for emittance, but for the initial phase geometry of electron beams as well.

power P/W
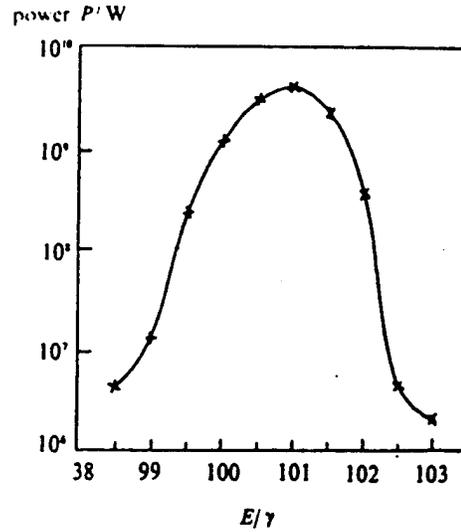
Fig. 1 Output power at different levels of incident electron beam energy γ

For the transport system of transverse motion determined in formulas (3) and (4) (including alternating quadrupole focusing), it is easy to establish the envelope evolution equation of the electron beam within the system and to solve it. The envelope radius expression formula for ideal injection conditions is

$$R^2(z) = R_0^2\cos^2(k_\beta z) + \frac{R_0 R_0'}{k_\beta}\sin(2k_\beta z) + (R_0'^2 + \frac{\varepsilon^2}{R_0^2}\,)\,\frac{1}{k_\beta^2}\,\sin^2(k_\beta z) \qquad (8)$$

/352

Here, ideal injection means that the electron beam positioning and orientation have no deviation, and that the injection phase oval center is coincident with the phase space coordinate center. (Nonideal injection is discussed in the following section.) Clearly, it is not only emittance ε that determines evolution $R(z)$, but also electron beam initial radius R_0 and slope $R_0' = dR_0/dz$, where $k_\beta = k_w a_w/\sqrt{2}r$ is the betatron motion wave number, which is determined by the intensity of the focusing field.

6

Considering the effects of the electronic envelope on the evolution of the light beam envelope and finding the conditions of fullest coincidence becomes a very complex nonlinear process, especially when gain is relatively large.

To emphasize the importance of incident phase space geometry, obtain a simpler situation, and obtain $R_0' = 0$ (the electron beam waist is obtained at the oscillator entrance), here, formula (8) can be simplified to

$$R^2(z) = R_0^2[1 + (\frac{\varepsilon^2}{k_\beta^2 R_0^4} - 1)\sin^2(k_\beta z)] \tag{9}$$

with the definition

$$R_{oe} \equiv \sqrt{\frac{\varepsilon}{k_\beta}} \tag{10}$$

In general, one can let $R_0 = \alpha R_{oe}$. Thus, formula (9) can be written as

$$R^2(z) = \alpha^2 R_{oe}^2[1 - (1 - \frac{1}{\alpha^4})\sin^2(k_\beta z)] \tag{11}$$

If an appropriate transport system is chosen that can ensure that the electron beam forms a waist at the oscillator entrance, and if the beam radius $R_0 = R_{oe}$, then, in later oscillator transports, the electron beam will retain an unchanging beam radius of

$$R^2(z) = R_{oe}^2 \tag{12}$$

As formula (11) demonstrates, the greater the deviation value of the incident beam radius R_{oe}, (when α is either much larger or smaller than 1), the larger the range in which the electron beam radius vibrates ($\alpha R_{oe} \sim \frac{1}{\alpha} R_{oe}$) will be.

We calculated the effects of different emittances on optical power where $R_0' = 0$ and $R_0 = R_{oe}$ (see Figure 3). When emittance is doubled, optical power decreases by more than twenty times.
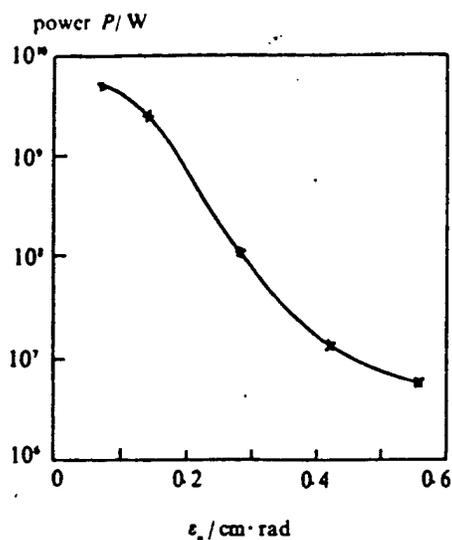
Fig. 2 Ouput power at different normalized electron beam emittances

4. Nonideal incidence of electron beams

Nonideal incidence is encountered most frequently in FEL nonlinear rotation. This has practical significance for the fields of physics and engineering. One situation of nonideal incidence is when electron beam injection deviates from the oscillator axis, and another is when the position of injection is correct, but there is a deviation in the direction of injection. Actually, nonideal incidence may be a combination of both.

4.1 Electron beam injection aiming position deviation (Deviation incidence)

Before the electron beam enters the oscillator, it must be guided through the beam transport system to an appropriate position and have a definite phase geometry (see the previous section). Because of transport system mechanical processing errors, electromagnetic wave motion and outside interference can bring about aiming position deviation. Physics and design cannot demand that the above-mentioned factor be zero, but can only propose a maximum allowable deviation. In addition, it is impossible for two-dimensional calculation of rotational symmetry to realize this. It can be predicted that electron beam deviation incidence will lead to nonrotational symmetry of light field

8

transverse distribution (this can be an aspect of the light guide phenomenon) and will have an influence on optical power.
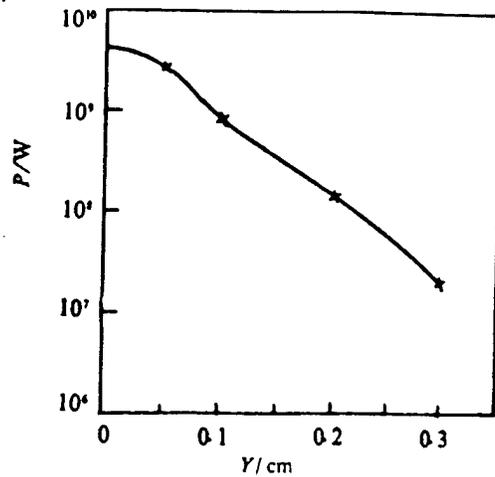
Fig. 3 Output power at different levels of electron beam incident deviation

We calculated light field output power at different levels of electron beam incident deviation[3] (see Figure 3). One can see that output power decreases exponentially as the amount of deviation increases. When deviation is an electron beam radius magnitude, light power decreases by two magnitudes. Thus, controlling deviation so that it is less than 1/10 of the beam radius, or about 0.3 mm (under the parameters we use) is a problem to which engineering designers, especially electron beam transport system designers, must pay attention.

Figure 4 describes the evolution of light field transverse distribution when deviation is 0.2 cm. In order to better understand the essence of the evolution diagram, an electron beam envelope evolution diagram with the same conditions is provided (see Figure 5). [Under] matching incidence conditions, $R_0 = R_{\alpha}$ is obtained. Despite [or] because of deviation incidence, the beam envelope oscillates and wiggles above and below the principal axis, but the envelope radius does not fundamentally change. Combining Figures 4 and 5 [shows that] beginning at the 1m section, the light field itself is smaller. This is the linear gain section. Although the electron beam deviates [at] the $y > 0$ area, the light field maintains left-right symmetry (axial symmetry). At 2m, a small error appears in the light

---

[3] *Piancha* (deviation) is translated by the Chinese authors as "displacement."

field, indicating that a higher level mode appears, even though the electron beam is already oscillating. Later, as the electron beam wiggles more toward the $y < 0$ area, the light field also deviates toward $y < 0$ because of exponential gain. Finally, a clear left-right asymmetry is formed. This shows that the light guide phenomenon is primarily located in the exponential gain area.
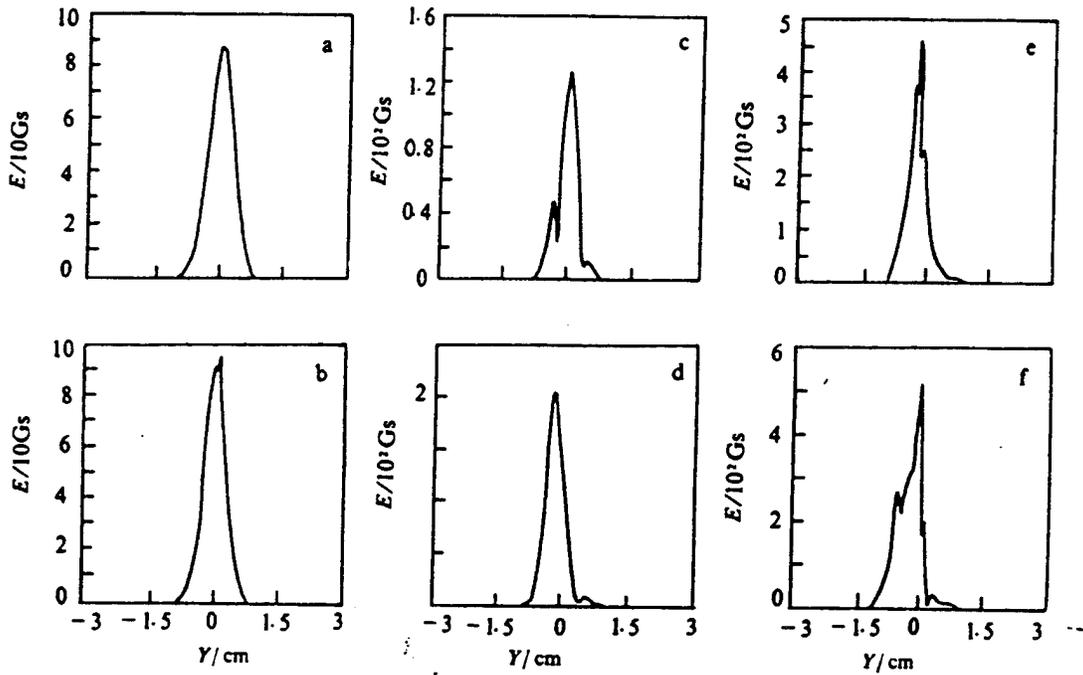
Fig. 4 Evolution of light field transverse distribution where electron beam incident deviation is 0.2 cm

a. $z = 0$; b. $z = 1$m; c. $z = 2$m; d. $z = 3$m; e. $z = 4$m; f. $z = 5$m

10
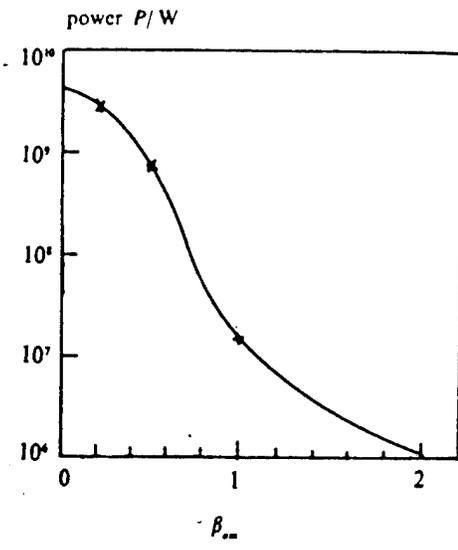
Figure 6. Output power at different electron beam incident angles (in units of β_{om})
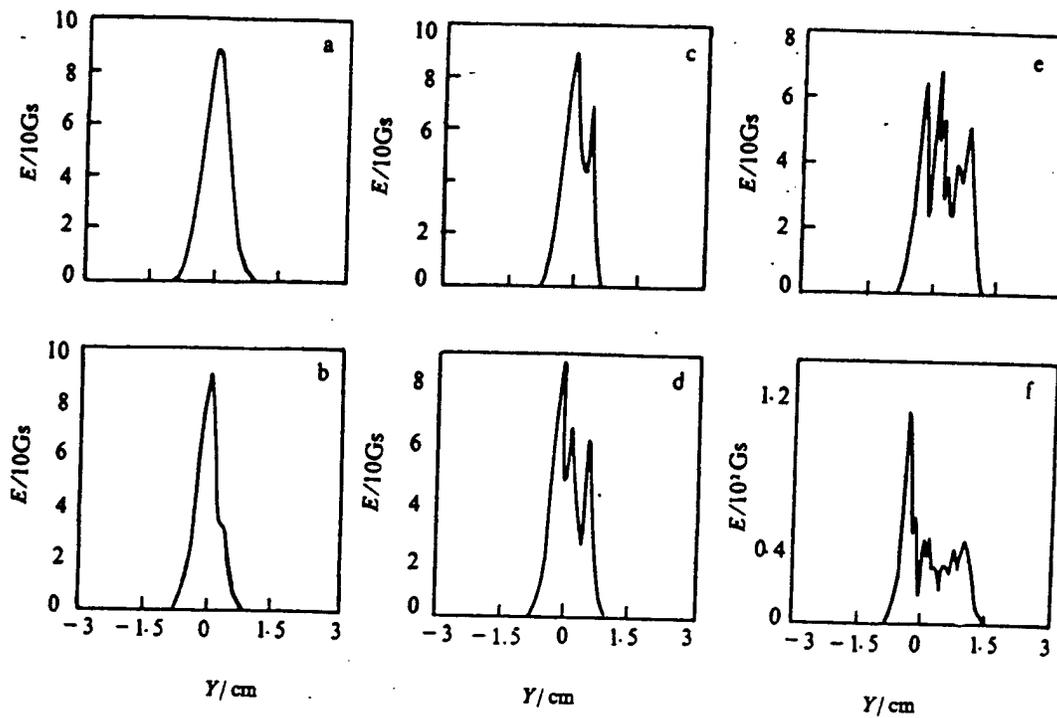
Figure 7. Evolution of light field transverse distribution where the electron beam incident angle is $2\beta_{om}$

a. $z=0$; b. $z=1\text{m}$; c. $z=2\text{m}$; d. $z=3\text{m}$; e. $z=4\text{m}$; f. $z=5\text{m}$

13
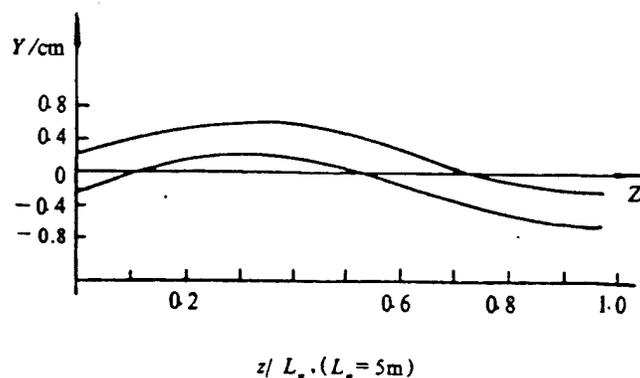
Figure 8. Electron beam locus where incident angle is $2\beta_{om}$ and $R_0 = R_{oc}$

5. Conclusions

This work is some of the results of our nonaxial symmetry 2D FEL calculations. There is still much work to be carried out in this area. But in light of the results that have already been obtained, this kind of nonaxial symmetry 2D approximation model not only has value for physics, but also has great significance for engineering design. At least we have suggested some requirements for pre-oscillator electron beam transport systems. Judging from the calculated results, the calculation method and program organization of this [computer] program are reasonable and reliable. This program can provide all of the information about light fields and electron beam evolution, (including light fields' transverse amplitude, phase distribution electron beam energy spectra, and phase diagrams), and thus is beneficial for physics research. Its greatest difference from ordinary programs[1-5] is that it can process nonaxial symmetry effects, such as nonideal injection of electron beams. Of course, this program can also be used for variable-parameter oscillators.

Up to now, we have only made some amplifier computations, and oscillator operations required approximately 190 CPU hours (using an AST 386/25 microcomputer). Thus, further reduction work remains to be done.

14

Reference Documents

[1]   Fawley, W.M. et al. *Phys. Rev.*, 1984, **A30**:2472.

[2]   Scharlemann, E.T. *J. Appl. Phys.*, 1985, **58**:2154.

[3]   Scharlemann, E.T. et al. Proc. 7th Int. FEL Conf. *Nucl. Instr. and Meth.*, 1986, **A250**:150.

[4]   Faatz, B. et al. *Nucl. Instr. and Meth.*, 1990, **A296**:654.

[5]   Beijing Applied Physics and Computational Mathematics Institute. *Collected Works of the Fourth FEL Conference.* Beijing: Beijing Applied Physics and Computational Mathematics Institute, 1989.105.

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

ORGANIZATION	MICROFICHE
BO85 DIA/RTS-2FI	1
C509 BALLOC509 BALLISTIC RES LAB	1
C510 R&T LABS/AVEADCOM	1
C513 ARRADCOM	1
C535 AVRADCOM/TSARCOM	1
C539 TRASANA	1
Q592 FSTC	4
Q619 MSIC REDSTONE	1
Q008 NTIC	1
Q043 AFMIC-IS	1
E404 AEDC/DOF	1
E410 AFDTC/IN	1
E429 SD/IND	1
P005 DOE/ISA/DDI	1
1051 AFIT/LDE	1
PO90 NSA/CDB	1

Microfiche Nbr: FTD95C000681
NAIC-ID(RS)T-0366-95