MASTER COPY KEEP THIS COPY FOR REPRODUCTION PURPOSES

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	Jan. 24, 1995	Final (April 1, 1993 - November 30, 1994)

4. TITLE AND SUBTITLE

Wavelet Analysis and Its Applications

5. FUNDING NUMBERS

DAAH 04-93-G-0047

6. AUTHOR(S)

Charles K. Chui

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Charles K. Chui
Department of Mathematics
Texas A&M University
College Station, TX 77843-3368

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

Abstract. Time-frequency localization is one of the most essential features of the wavelet transform. It was shown that while high order Daubechies and Battle-Lemarie wavelets give poor time-frequency localizations, the Chui-Wang spline-wavelets provide asymptotically optimal time-frequency windows. On the other hand, we also showed that by using the scale 3 instead of 2, symmetry can be achieved by orthonormal wavelets with compact support. Multivariate wavelets, particularly those with matrix dilation, were studied, and the theory of oversampling frames was extended to this setting. Interpolating wavelets have distributional duals that lead to the notion of functional wavelet transform. Other extensions required a study of the stability issue and algorithmic construction in multivariate splines. Applications to systems theory lead to the study of Hankel approximation and localization of neural networks.

14. SUBJECT TERMS

Time-frequency localization, symmetry for compactly supported orthonormal wavelets, multivariate wavelets with matrix dilations, interpolating wavelets and functional wavelet transform, neural networks localized by splines and wavelets.

15. NUMBER OF PAGES 6

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

Wavelet Analysis and Its Applications

FINAL REPORT

Charles K. Chui
Department of Mathematics
Texas A&M University
College Station, TX 77843-3368

January 24, 1995

Sponsored by
SDIO/IST
U.S. Army Research Office

Contract Number DAAH 04-93-G-0047

Texas Engineering Experiment Station
Texas A&M University System

A. Statement of the problem studied

This research project was devoted to the study of the mathematical aspects of wavelet analysis and to investigate the various applications of wavelets and related areas. The original research plan was a three-year effort to fully develop the stationary theory of wavelets, both in the univariate and the multivariate settings, and to apply this theory to solve certain important engineering problems, including such applications as modeling the cochlea of the human ear. Unfortunately, due to budget difficulty, this research project was only funded for the first year. Although an additional fund greatly faciliate the six-month extension, there was not enough time nor manpower to complete the original somewhat ambitious goal. Nevertheless, a total of fourteen (14) papers were written and published (or accepted), and further research is being carried out, even without research support.

B. Summary of the most important results

The core of this research effort is the mathematical development of wavelet analysis. Hence, although the resutls obtained can be divided in the following three categories, the main results are in wavelet theory.

B.1. Wavelet Analysis

A time-frequency analysis of the wavelet transform was studied very carefully in [1]. It was shown that families of semi-orthogonal wavelets (including the Chui-Wang spline wavelets) provide asymptotically optimal time-frequency localization [1], while the width of the time-frequency window of orthogonal wavelets (incldуing the Daubechies and Battle-Lemarie wavelets) tends to infinity as the order increases. For applications to signal analysis, the complexity of the semi-orthogonal (particularly the Chui-Wang spline) wavelet decomposition and reconstruction schemes is given in [3]. Since orthonormal wavelets are commonly used, particularly by the engineers, we changed the scale from 2 to 3 in our paper [4] in order to yield compactly supported orthonormal wavelets with symmetry and anti-symmetry. It is important to remark that this requires the sacrifice of minimum support. To generalize to the multivariate setting, a general framework of multivariate wavelets together with their duals was developed in [5]. In particulay, when a matrix dilation is used, a general theory which includes that of oversampling frames was given in [6]. All of the wavelet theory mentioned above is a result of orthogonal projection. In [7], we considered wavelets obtained by interpolation projection and introudced the concept of functional wavelet transform induced by the distributional duals. A new method of construction of spline-wavelets was introduced in [8].

B.2. Spline modeling

Due to the fact that spline-wavelets are simple, explicit, and flexible, and that near-optimality for time-frequency localization is achieved, we returned to further develop the algorithmic and stability aspects of multivariate, and particularly bivariate, spline functions on arbitrary triangulatons. When vertex splines are used to achieve the optimal order of approximation from the spline apaces of degree $3r + 2$ with rth order of smoothness, it

is well-known that instability occurs as a result of near-singularity at a vertex of degree 4. In our paper [9], the supports of vertex splines are enlarged, to include an interior edge if necessary, to assure stability. In order to lower the degree from $3r+2$ to $3r+1$ while maintaining the optimal order of approximation, it is necessary (even for $r=1$ and the regular three-directional mash) to refine the triangulations. In [10], we focussed on the case $r=1$ but allowing an arbitrary triangulation, and developed an algorithm for some minimum refinement.

B.3. Systems theory

Recently, there has been some interest in the investigation of the relevance of wavelets in the theory of linear systems. In an attempt to extend the notion of the continuous (or integral) wavelet transform (CWT) to the half-plane in our work [11], we related this generalized CWT to the Hankel transform in systems analysis. In [12], we proved that the Hankel approximation is continuous even when the s-numbers (or singular values) are not simple, and hence, assured the convergence of the corresponding rational approximants. A comprehensive study of rational approximation in system engineering was given in [13]. For neural systems, we studied networks with one hidden layer in [14] and incorporated splines and spline-wavelets for local approximation and analysis.

C. List of publications

1. C. K. Chui and J. Z. Wang, A study of compactly supported scaling functions and wavelets, in *Curves and Surfaces*, P. J. Laurent, A. LeMéhauté, and L. L. Schumaker (eds.), A. K. Peters Publ., Wellesley, Mass. 1994, 121–140.
2. C. K. and J. Z. Wang, High-order orthonormal scaling functions and wavelets give poor time-frequency localization, *J. Fourier Anal. and Appl.*, accepted for publication.
3. C. K. Koc, G. Chen, and C. K. Chui, Complexity analysis of wavelet signal decomposition and reconstruction, *IEEE Trans. Aerospace and Electronic Systems* **30** (3) (1994), 910–918.
4. C. K. Chui and J. A. Lian, Construction of compactly supported symmetric and antisymmetric orthonormal wavelets, *Appl. and Comp. Harmonic Anal.* **2** (1) (1995), 21–51.
5. C. K. Chui and C. Li, A general framework of multivariate wavelets with duals, *Appl. and Comp. Harmonic Anal.* **1** (4) (1994), 368–390.
6. C. K. Chui and X. L. Shi, Affine operators and frames of multivariate wavelets, in *Advances in Computational Mathematics*, H. P. Dikshit and C. A. Micchelli (eds.), World Scientific Publ., 1994, 139–155.
7. C. K. Chui and C. Li, Dyadic affine decompositions and functional wavelet transforms, accepted for publication in *SIAM Math. Analysis*.
8. C. K. Chui and J. M. de Villiers, Applications of optimally local interpolation to constructing interpolatory approximants and compactly supported wavelets, accepted for publication in *Math. Comp.*
9. C. K. Chui, D. Hong, and R. Q. Jia, Stability of optimal order approximation by bivariate splines over arbitrary triangulations, accepted for publication in *Trans. American Math. Soc.*

10. C. K. Chui and D. Hong, Construction of local C^1 quartic spline elements for optimal order approximation, submitted to *Math. Comp.*
11. C. K. Chui and X. Li, Generalized wavelet decompositions of bivariate functions, *Proc. of Amer. Math. Soc.* **121** (10) (1994), 125–131.
12. C. K. and X. Li, Continuity of best Hankel approximation and convergence of near-best approximants, *SIAM J. Constrol & Optimization* **32** (6) (1994), 1769–1781.
13. C. K. Chui and G. Chen, Rational approximation theory and its applications in system engineering: An overview, *Proc. 33rd Conf. on Decision and Control*, Lake Buena Vista, FL, December 1994, 1487–1492.
14. C. K. Chui, X. Li, and H. N. Mhaskar, Neural networks for localized approximation, *Math. Comp.* **63** (1994), 607–623.

D. Participating scientific personnel

Charles K. Chui (P. I.)
Andy Chan
Steve Liu
Jörg Hanisch
Dong Hong (Ph. D. granted, Aug. 1993)
Jian-ao Lian (Ph. D. granted, Aug. 1993)