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13. ABSTRACT *(Maximum 200 words)*

This work focused on a comparison of three methods for evaluating weapon scores/importances in combat models. We also analyzed the ATCAL (attrition calibration) process in use at the U. S. Army Concepts Analysis Agency. This included a reformulation of the attrition equations at the heart of ATCAL in dimensionless form and certain approximations that reduce them to a system of non-linear polynomial equations. Finally, we investigated stochastic attrition models which use stochastic differential equations. COSAGE (Combat Sample Generator) data from a historically based Ardennes study was used to validate some of our results.

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FINAL REPORT

1. ARO PROPOSAL NUMBER: 31495-MA

2. PERIOD COVERED BY REPORT: 1 August 1993 – 31 January 1995

3. TITLE OF PROPOSAL: Differential Geometric, Stochastic, and Computational Methods for DOD Simulations

4. CONTRACT OR GRANT NUMBER: DAAH04-93-G-0310

5. NAME OF INSTITUTION: Texas A&M University

6. AUTHORS OF REPORT: Dr. Peter F. Stiller, Professor of Mathematics and Computer Science, Texas A&M University

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:

   Technical Reports to be published by the U.S. Army Concepts Analysis Agency

   1) Stiller, P., "An Analysis of Three Methods for Computing Weapon Scores" preprint attached.
   2) Stiller, P., "Attrition Modeling – Theory and Practice", in preparation.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

   Dr. Peter F. Stiller
   Mr. Merrill Heddy, Graduate Student
   Mr. James Warren, Undergraduate Student

9. REPORT OF INVENTIONS (BY TITLE ONLY): none

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Final Report

Grant Number DAAH 04-93-G-0310

"Differential Geometric, Stochastic, and Computational Methods for DOD Simulations"

Summary: During the period of performance (1 August 1993 to 31 January 1995) our work focused on the various tasks outlined in our proposal. The results are described in summary below. Two papers "An Analysis of Three Methods for Computing Weapon Scores" (preprint attached) and "Attrition Modeling – Theory and Practice (in preparation), explain our results in detail. We have also presented our findings at a "Target Allocation/Attrition and Modeling Workshop" held at the U.S. Army Concepts Analysis Agency (CAA) on 19 January 1994 and a similar Workshop held on 20 October 1994.

Data: The COSAGE data we are using was provided by Dr. John Warren and Mr. Gerry Cooper of CAA. The data consists of sixteen COSAGE replications for each of six postures, broken down into four hour intervals over the course of a twenty-four hour engagement. The data is drawn from a historically based CAA study of the Ardennes campaign.

Details: The first task we worked on was a comparative study of three methods for assigning weapons values and/or importances to weapons types: the classical eigenvalue (potential/anti-potential) method, a non-linear method analogous to the eigenvalue method which we call the fire allocation method, and finally the method (also non-linear) used by CAA in their ATCAL (attrition calibration) routine. In all cases, we studied the sensitivity of the method to changes in entries in the KV-scoreboard and the nature of the distributions of the values when dealing with stochastic attrition rates.

We also were able to prove uniqueness (under some restrictions) and existence of solutions for the two non-linear methods mentioned above. Interestingly, despite being non-linear, both the fire allocation method and the ATCAL weapon importance method have much in common mathematically with the linear eigenvalue method. In principal, it appears that both non-linear methods could suffer from defects similar to those possessed by the linear eigenvalue method. At least we have not yet been able to rule out effects similar to those that occur when the matrices become reducible in the eigenvalue method. We were able to prove a result similar to the Perron-Frobenius Theorem (which guarantees the existence and uniqueness of non-negative solutions to certain simultaneous eigenvalue problems) for the ATCAL method. It also would be nice to have an analogous result for the fire allocation case, but this remains an open question. This result is based on a fixed point type argument and is of considerable theoretical interest.

Our second task involved work on the ATCAL attrition model itself, which is used at CAA to assess attrition in a high (theater) level deterministic model, notably CEM. ATCAL uses a set of non-linear (and non-dynamic!) equations which are calibrated to the results of a detailed low level stochastic simulation (COSAGE).

We began by reformulating the ATCAL attrition equations (for direct fire only) in non-dimensional terms. This allows one to see more clearly the various interactions among the many variables involved. By making use of two simple and very accurate approximations, we were able to approximate the ATCAL attrition equations by a system of non-linear polynomial equations. While not practical for the very large systems that arise in CAA's models, Gröbner bases techniques offer a way to analyze ATCAL's behaviour for small numbers of weapons types on each side. Finally, we compared some of the assumptions built into ATCAL with actual COSAGE data taken over time. The assumption of exponential decline in numbers for a fixed weapons system is not borne out by the COSAGE data. This calls into question the exponential averaging method which permeates much of ATCAL.

Since time dependent data can be collected from COSAGE, we have recommended that more realistic "fractional loss versus fractional average present during the engagement" curves be used.

Finally, ATCAL is neither dynamic nor stochastic. Our third task involved looking at methods which are both dynamic and stochastic and which might serve as the basis for an ATCAL-like attrition assessor. Such a model, when calibrated to COSAGE output, would presumably give a better transfer of the essential parameters of the detailed lower level model to the high level theater model. Our focus has been on stochastic differential/diffusion equation models and other differential geometric/dynamical system type models.

The calibration of such a model generally requires more COSAGE replications than are currently being run. For that reasons, the data available to us could only be used in a limited way to test our approach. As a result, further research on replacing the attrition equations in ATCAL with something that is both stochastic and dynamic, and capable of capturing the variation in COSAGE runs, will be required.