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Final Report

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3. TITLE OF PROPOSAL: Modified Eigenvalue Problems, with Applications to Structural Dynamic Re-Analysis on Parallel Computers
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Problem Statement:

Matrix eigenvalue problems play a significant role in dynamic analysis of structures. The natural frequencies and the modes of a free vibrating system are intimately related to the eigenvalues and the eigenvectors of the generalized system of characteristic equations corresponding to that vibrating system. To calculate even a few of the natural frequencies and modes of a large scale structure is computationally intensive. A frequently encountered problem in structural dynamics is how to take into account analysis and design changes introduced after the structural dynamic analysis has been completed and the natural frequencies and modes have been computed. The problem is how the natural frequencies and modes obtained from a previous analysis can be used to derive the response of the new modified structure, without extensive additional computations. In particular, we focus on the structural dynamic re-analysis problem that the system is modified in a few small localized regions – the incremental stiffness matrix and the incremental mass matrix are low rank matrices, but their magnitude need not be small relative to the original matrices. Dynamic solution procedures that are suitable for parallel computer environments are also investigated.

Summary of Results

In this project, we have developed and implemented a number of parallel algorithms related to matrix factorization and generalized eigenvalue problems. Furthermore, a Lanczos based method for the modified eigenvalue problem has been developed. The following summarizes the results of this project:

1. Development and implementation of a new parallel sparse factorization method(Ref. 1,3). Specific contribution includes an efficient procedure for the solution of triangular systems, which is generally known as one of the most difficult problems for the parallelization of direct solution methods. The parallel procedure for the solution of triangular systems is particularly important for generalized eigenvalue problem which requires solution of linear systems at each Lanczos iteration step.
2. A parallel implementation of a robust Lanczos algorithm for generalized eigenvalue problems (Ref. 1,10). The parallel algorithm has been implemented in a finite element structural analysis program. To ensure the robustness and accuracy, the parallel procedure implemented includes spectral shifts, partial and selective re-orthogonalizations. We have also tested various approaches and developed a robust implementation so that the finite element matrices can be in the form of either element matrices or assembled system matrices. Tradeoffs between spectral shifts, factorization and re-orthogonalization have been studied.

3. The development of a new method for solving perturbed symmetric eigenvalue problems (Ref. 3,5,9). This method is based on an extension of the block Lanczos method for standard and generalized eigenvalue problems. This new method is capable of updating the modified eigenvalues and eigenvectors even when the modification made to the structures may cause significant changes to the mode shapes of the system.

In addition, we have also studied the following problems related to eigenvalue problems and the Lanczos and conjugate gradient methods for structural analysis:

1. A study of the solution methods for the minimization of the generalized Rayleigh Quotient subject to linear and quadratic constraints (Ref. 7). Applications of the methods for the evaluation of thin shell finite elements are illustrated.
2. Development of a robust and stable incomplete Cholesky (IC) factorization (Ref. 2,8). In particular, we have examined in detail the tradeoffs between the storage requirement and the effectiveness (in terms of the total CPU time for the solution) of the IC preconditioners for the conjugate gradient method. A technique has been developed to manage both the storage space and to optimize the "goodness" of the preconditioning matrix.
3. Development of a simple but effective procedure to enhance the block conjugate gradient method for the solution of a linear system of equations with multiple right hand sides (Ref. 2,6). This development is important since engineering structures are often designed to resist multiple load cases.
4. A study of the use of the conjugate gradient method for nonlinear analysis of structures (including snap through and simple bifurcation of structures) (Ref. 2). Based on the relationship between the conjugate gradient and the Lanczos methods, we have examined how to take advantage of the information (such as the eigenvalues, the residual and search direction vectors) in solving nonlinear problems. For example, by simply keeping track of the inertia of the (tangential) stiffness matrix, we can determine easily the secondary load-displacement path related to simple nonlinear bifurcation of structures.
5. A comparison study on a variety of preconditioners (including polynomial preconditioners, incomplete Cholesky factors, domain decomposition and a combination of the preconditioning methods) for conjugate gradient methods on a distributed memory parallel computer (Ref. 2). Both linear and nonlinear structural problems have been examined in this study.

All parallel solution procedures have been implemented and tested on an Intel i860 parallel computer.

Scientific Personnel and Advanced Degrees Awarded

1. Gene H. Golub, Professor of Computer Science
2. Kincho H. Law, Associate Professor of Civil Engineering
3. Cheryl M.M. Carey, Post-Doctoral Research Scientist, Scientific Computing and Computational Mathematics Program (Dr. Carey is now a research scientist at the Center for Research on Parallel Computation, California Institute of Technology.)
4. David R. Mackay, Ph.D., Department of Civil Engineering (Dr. Mackay is now a computational scientist, Supercomputer Systems Division, Intel Corporation, Oak Ridge National Laboratory, Oak Ridge, TN.)
5. Made Suarjana, Ph.D., Department of Civil Engineering (Dr. Suarjana is now a faculty member at the Civil Engineering Department, Bandung Institute of Technology, Indonesia.)

Publications:

1. David R. Mackay. *Solution Methods for Static and Dynamic Structural Analysis on Distributed Memory Computers*. Ph.D. Thesis. Department of Civil Engineering. Stanford University, August 1992.
2. Made Suarjana. *Conjugate Gradient Method for Linear and Nonlinear Structural Analysis on Sequential and Parallel Computers*. Ph.D. Thesis. Department of Civil Engineering, Stanford University, February 1994.
3. Cheryl Carey, Hsin-Chu Chen, Gene Golub and Ahmed Sameh, "A New Approach for Solving Perturbed Symmetric Eigenvalue Problems." *Computing Systems in Engineering*, 3(6):671-679, 1992.
4. Kincho H. Law and David R. Mackay, "A Parallel Row-Oriented Sparse Solution Method for Finite Element Structural Analysis." *International Journal for Numerical Methods in Engineering*, 36:2895-2919, 1993.
5. Cheryl Carey, Gene Golub and Kincho H. Law, "A Lanczos-based Procedure for Structural Dynamic Re-Analysis." *International Journal for Numerical Methods in Engineering*, (accepted for publication, 1994).
6. Made Suarjana and Kincho H. Law, "Successive Conjugate Gradient Method for Structural Analysis with Multiple Load Cases." *International Journal for Numerical Methods in Engineering*, (accepted for publication, 1994).
7. Cheryl Carey, "A Class of Constrained Generalized Rayleigh Quotient Problems," (submitted for publication, 1994).
8. Made Suarjana and Kincho H. Law, "A Robust Incomplete Cholesky Factorization Based on Value and Space Constraints." *International Journal for Numerical Methods in Engineering*, (submitted for publication, 1994).
9. K.H. Law, C.M.M. Carey, D.R. Mackay and G.H. Golub, "Numerical Methods for Dynamic Re-Analysis of Structures with Localized Modifications." Second International Conference on Computational Structures Technology, Athens, Greece (submitted for consideration, 1994).
10. D.R. Mackay and K.H. Law, "An Implementation of a Generalized Lanczos Procedure for Structural Dynamic Analysis on Distributed Memory Computers." Second International Conference on Computational Structures Technology, Athens, Greece (submitted for consideration, 1994).