CONFIGURATION AND THE EFFECTIVENESS OF AIR DEFENSE SYSTEMS IN SIMPLIFIED, IDEALIZED COMBAT SITUATIONS — A PRELIMINARY EXAMINATION

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ABSTRACT

The reported research is a product of a very limited, initial effort to develop configural theory for application to the design and assessment of ship self-defense and fleet air defense weapon systems and to illustrate its fundamental and substantial importance to analysis, modeling, and simulation in that context. Configural theory is a mathematical theory for quantifying the relationships between the behavior of weapons in use in combat and their individual characteristics. Its name is derived from its central concept, configuration, which is the mathematical expression of the fact that the disposition in space and time of the targets and weapons of the attacker and the defender is inseparable from the outcome of the engagement and the combat effectiveness of those weapons. Customary mathematical representations usually exclude much that is essential and thus are nonconfigural. Configural and nonconfigural assessments of the relative effectiveness of weapon systems compared in the report for situations that minimize the effects of configuration differ by factors from two to ten and more, and systems that are less effective in combat appear in nonconfigural assessments to be substantially superior to more effective systems. Configural theory can help produce more effective weapons and tactics for their use with no increase in required resources.

PREFACE

The research discussed in this report was performed under Phase I of a Small Business Innovation Research (SBIR) contract supported by the Office of Naval Technology (Topic N88-14, "Tactical Theory for Weapons Effectiveness"). The overall objective of both phases of the research is to develop the elements of a configural theory of the effectiveness of ship self-defense and fleet air defense weapons in tactical engagements. However, SBIR funding is too limited to proceed beyond defender configurations that are much more representative of land-based air defense screens than those characteristic of naval force dispositions.

In fact, in Phase I the principal objective was to develop the concepts and mathematics needed to assess configurally the relative effectiveness of multiple-round air defense weapon systems randomly distributed in a defended strip to protect a force beyond it from threat attackers using a single-axis attack corridor through it. That required generalizing configural theory and the (analytical) mathematical procedures which had been developed mainly for single-round weapons (the initial applicational focus) to multiple-round weapon emplacements in order to determine (among other things) the relationship between the number of emplacements of a given weapon and the number of rounds per emplacement that is necessary for equally effective deployments. As this report shows, those objectives were accomplished.

Because SBIR contracts permit only a very limited effort, this report is neither as comprehensive nor as detailed as the results merit (even though it was partly prepared and then revised with Phase II support). Since early 1990, when the draft was prepared, the concepts and mathematics have been extended to substantially more complex combat situations. Those results greatly strengthen and extend the conclusions presented in this report. However, although the draft was not revised for publication until late 1994, no new material from the subsequent research was incorporated.

The research reported makes extensive use of concepts and mathematical relationships that are products of mainly unsupported research by Horrigan Analytics in configural theory, which itself generalizes supported research by Horrigan Analytics in the mathematics of naval mine warfare, termed configured minefield theory. The formal, mathematical concept of configuration was introduced in the course of Horrigan Analytics' research in the mathematics of naval mine warfare in early 1970 under contract to the Office of Naval Research. That research continued with support from the Naval Material Command, Mine Warfare Project Office (PM-19), through the 1970s. Further research in configured minefield theory was performed under contracts with the Office of Naval Research and the Naval Surface Weapons Center for and with the support of the Office of Naval Technology and the Naval Sea Systems Command. The significance of configuration for weapons in general was first noted in the Horrigan Analytics report *Accomplishments in Configured Minefield Theory through 1977 — A Summary* (U) (HA 77-2), which was prepared under contract to the Office of Naval Research.

The conceptual and mathematical basis for configural theory for weapons in general was reviewed, initially in the context of naval mine warfare, in the early 1980s by the Naval Studies Board of the National Academy of Sciences, and it was endorsed in an 11 April 1983 letter from the chairman of the Naval Studies Board to the Chief of Naval Operations (OP 00). In particular, the letter states,

> . . . mines with characteristics that are favored by the present mathematical approach [nonconfigural assessments] may appear to be better than they actually are, and may even appear better than designs that are substantially superior. This can lead to the setting of stockpile levels that are significantly less than that actually required, and to the expectation of performance levels that will not be achieved in combat.
>
> . . . [T]he comments noted above can be made for all weapons—both strategic and tactical—in those areas where many weapons interact with many targets during some finite engagement time. (page 1)

The results discussed in this report illustrate that conclusion in the context of ship self-defense and fleet air defense.

Discussions with many Navy officers and civilians who are knowledgeable about ship self-defense and fleet air defense have helped greatly both in conceptualizing the problem and in preparing this report. Special thanks are due Dr. Philip A. Selwyn, then Director, Office of Naval Technology, and Mr. David S. Siegel, Scientific Officer, for the time and effort they contributed to reviewing the research in progress and to commenting on the results. Mr. Robert F. Obrochta, then of the Office of Naval Research, provided invaluable support in the course of the development of the basic concepts and several key computational procedures in the context of the mathematics of naval mine warfare. Many helpful observations were also made by Captain Gary W. Schnurrpusch, USN, and Professor George F. Carrier, Harvard University. Rear Admiral David R. Oliver, Jr., USN, then Director, Navy Programming (OP 80), who commented on the results discussed in this report and subsequent research results, provided important insights.

Special efforts of the staff at Horrigan Analytics should also be mentioned. In particular, Mr. William J. Clover, Jr., developed the computational procedures and associated computer programs to compute the configural casualty probability densities and to obtain numerical solutions to the nonlinear equations that are necessary to assess configurally the relative effectiveness of candidate air defense weapon systems. He also made many suggestions that improved the text, as did Dr. Cynthia L. Bathurst, Mr. William F. Macdonald, and Dr. Gerald P. Joyce II. Dr. Bathurst also prepared summary text and the introductory parts of the mathematical appendices and made the report more readable than it otherwise would have been. Ms. Eva H. Haussner prepared the graphics with thoroughness and care.

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EXHIBITS

SUMMARY

The research discussed in this report is a product of a very limited, initial effort to develop configural theory for application to the design and assessment of ship self-defense and fleet air defense weapon systems and to illustrate its fundamental and substantial importance to analysis, modeling, and simulation in that context.

Configural theory is a mathematical theory for quantifying the relationships between the behavior of weapons in use in combat and their individual characteristics. Its principal purpose is to provide concepts and mathematical relationships to improve our understanding of combat effectiveness. Its name is derived from its central concept, configuration, which is the mathematical expression of the fact that the disposition in space and time of the targets and weapons of the attacker and the defender is inseparable from the outcome of the engagement and the combat effectiveness of the weapons.

Although configuration in that technical sense is not in the military lexicon, the concept itself is of recognized military importance and is fundamental to strategy and tactics — for example, which weapons and combatants are placed where in a reverse slope defense to maximize its strength, which ships are placed where in a carrier battle group to best protect the carrier, which aircraft are placed in what wave and with what formation in an air strike to best neutralize the defender's communications. Weapons or platforms are assigned roles and relative positions to exploit their strengths, compensate for their weaknesses, and reinforce their combined effects. In that context, configuration is the mathematical expression of the physical realization of the governing strategic and tactical considerations.

Configuration is also widely believed to be accommodated in mathematical analyses, models, and simulations. However, customary mathematical representations usually exclude much that is essential. In configural theory configuration is a formal, mathematical concept that is the basis of the *configured encounter*, the mathematical representation of target-weapon encounters in combat. Configured encounters are necessarily constrained by the spatial *relationships* among the elements of the governing attacker-defender configuration as it evolves in the course of an engagement; and the associated random variables and processes, which are probabilistically dependent by association with the configuration, are treated as probabilistically dependent. Customary mathematical representations, even many that are used in Monte Carlo simulations, partly or wholly exclude those constraints or important consequences of them and treat many probabilistic dependencies as probabilistic independencies. In configural theory, an encounter that is "free" of such constraints or their consequences is termed a *free encounter*. Mathematically, it is a degenerate, limiting form of a configured encounter. As such, even though it greatly simplifies effectiveness assessments, inferences about combat effectiveness cannot be made from it.

The differences that accommodating configuration makes, both theoretically and practically, are fundamental, substantial, and important. The

results discussed in this report show the kinds of differences it makes for designing and assessing hypothetical weapon systems with operational characteristics that are representative of those used for fleet air defense, even in simplified combat situations that minimize the effects of configuration. They also show the kinds of differences it makes for predicting important characteristics of combat behavior, such as casualty production.

The formal concepts that appear to be needed in developing configural theory for application to ship self-defense and fleet air defense weapon systems (for instance, the attack corridor), as well as the probabilistic dependencies that must be preserved (for instance, functions that characterize weapon system capability, such as detection and damage functions), are discussed mainly in Sections I and II. The combat situation addressed is highly simplified: single-emplacement batteries of multiple-round air defense weapons are randomly distributed in a defended strip to protect a force beyond it from threat attackers using a single-axis attack corridor through it. The probabilistic characteristics of the behavior of such weapon systems in combat are discussed mainly in Section III. The key equations used in the report are established in the appendices.

More specifically, the report shows how assessments that adequately accommodate configuration differ from the customary, nonconfigural assessments (and how they are the same in the rare combat situations in which the encounters are equivalent to free encounters). Illustrations in Sections I and II compare alternative missile designs with operational characteristics that span a range of those used for fleet air defense. They show that the customary assessments of relative effectiveness of designs with characteristics favored by free-encounter assessments, because they implicitly postulate free encounters, can overstate combat effectiveness by factors of two to ten or more, and systems that are less effective in combat can appear to be substantially superior to more effective systems. As a result, such assessments understate the numerical requirements for systems with the characteristics favored in free-encounter assessments by similar factors.

Those illustrations also show how configuration affects the assessment of the relative importance of weapon characteristics as determined by their contribution to effectiveness. Illustrations in Section II show that (1) detection capability and missile range, which are indeed important, nonetheless contribute much less to combat effectiveness than free-encounter assessments state and (2) reliability and lethality contribute much more to combat effectiveness than free-encounter analyses state. Because configural assessments explicitly accommodate the scale of the encounters, they can also assess the relative importance of weapon characteristics in different scales of encounters. For instance, a configural assessment in Section II shows that the substantial improvement in free-encounter relative effectiveness that results from the overall capability of the hypothetical high-performance system to fire multiple rounds at a single attacker contributes nothing in heavy attacks.

The report also shows how configural theory leads to measures of effectiveness that capture important behavioral characteristics of weapons in use in combat that can be neither discerned nor assessed using free-encounter measures (and that include the customary free-encounter measures as particular cases). The measures addressed are based on casualty production. Customarily, the average number of casualties among the attackers or among the ships to be protected is used to quantify the effectiveness of an air defense deployment. However, that means of quantifying effectiveness masks important probabilistic characteristics. A better means, especially when high-value ships are considered, is the configural probability density of the random number of casualties among those ships that results from an attack.

Section III illustrates those probabilistic characteristics and their importance. In particular, in configured encounters there is typically greater (often much greater) uncertainty in the random numbers of casualties both among the attackers and among the ships to be protected than free-encounter casualty probability densities can show. Configural assessments of representative combat encounters show that, in contrast to free-encounter assessments for situations having the same average number of casualties, (1) the most probable number of casualties need not be the average number or even close to it, (2) the probability of a number of casualties close to the average can be considerably smaller than in a free encounter, and (3) small or large numbers of casualties can be more probable than numbers close to the average. Of especial importance are the higher probabilities of no survivors or of no casualties that free-encounter casualty probability densities mask.

Such comparisons of configural and free-encounter assessments make clear the contribution configural theory makes as well as the inadequacy and danger of relying on the customary, nonconfigural assessments. For instance, the configural assessments of the effectiveness for area defense of the air defense weapons that are discussed in Section III reveal large probabilities of no casualties among the ships to be protected and, simultaneously, substantial probabilities of large numbers. Not only do free-encounter assessments mask such bipolarities, but also they cannot provide the means to assess the contribution of point defenses, especially close-in weapon systems, to the survival of the ships to be protected. Configural assessments can.

Because configural theory accommodates configuration, factors and relationships that are manifestly important for weapon design, numerical requirements, relative-effectiveness assessments, weapon systems mixes, and operational deployment in appropriate quantities can be properly assessed. Because it more accurately quantifies how much particular operational characteristics contribute to combat effectiveness, it can better focus design effort and technological resources on those characteristics that can contribute most to combat effectiveness. In particular, it can provide the means to discern and to identify those characteristics that most need improvement or contribute most to combat effectiveness and to identify and to assess the design trade-offs that are most efficacious for overall mission success. Furthermore, it fosters the understanding and insight that can lead to innovative, more effective tactics. As a result, it can help produce more effective weapons with no increase in required resources.

I. CONFIGURATION, CONFIGURAL THEORY, AND THE EFFECTIVENESS OF AIR DEFENSE WEAPONS

Configural theory is a mathematical theory for quantifying the relationship between the behavior of weapons in use in combat and their individual characteristics. Its principal purpose is to provide concepts and mathematical relationships to improve our understanding of combat effectiveness. Its name is derived from its central concept, *configuration*, the mathematical expression of the fact that the disposition in space and time of the targets and the weapons of the attacker and the defender is inseparable from the outcome of the engagement and the combat effectiveness of those weapons. Mathematically accommodating configuration entails a probability space that comprises all possible trajectories and states of all the targets and all the weapons in the course of the maneuvers, the exchanges of fires, and the casualty production that could constitute an engagement.

Although configuration in that technical sense is not in the military lexicon, the concept is of recognized military importance and is fundamental to strategy and tactics — for example, which weapons and combatants are placed where in a reverse slope defense to maximize its strength, which ships are placed where in a carrier battle group to best protect the carrier, which aircraft are placed in what wave and with what formation in an air strike to best neutralize the defender's communications. Assigning roles and positions to weapons and their platforms to exploit their strengths, compensate for their weaknesses, and reinforce their combined effects can make a decisive difference in combat. However, despite the recognized importance of configuration as a concept in military planning, its importance for the mathematical representation of the multiple-target, multiple-weapon encounters of combat and the constituent, individual one-target, one-weapon encounters that underlie casualty production and assessments of weapon system effectiveness is mainly neither realized nor understood.

The research discussed in this report is a product of a very limited, initial effort to develop configural theory for application to the design and assessment of ship self-defense and fleet air defense weapon systems. The results illustrate how strongly configuration affects both the effectiveness of such weapon systems and its assessment. In particular, they show that, for air defense missiles with operational characteristics representative of those of fleet air defense missiles, measures of effectiveness that are based on the customary conceptualization of weapons effectiveness are seriously misleading because they exclude configuration. Such measures

- can overstate the relative effectiveness of air defense systems with characteristics favored by the customary conceptualization by factors of two to ten or more and
- can make less effective air defense systems with characteristics favored by the customary conceptualization appear to be substantially superior to more effective systems.

More importantly, those results also show that the better understanding of how the behavior of such weapons in use in combat relates to their operational characteristics provides for the following:

- a better means to identify and to focus our technological resources on enhancing the characteristics that most need improvement or contribute most to combat effectiveness,
- a better means to identify and to assess the design trade-offs that are most efficacious for overall mission success, and
- the development of innovative, effective tactics.

In short, theory that accommodates the key configural elements of combat fosters the understanding and insight that can help produce more effective weapons and tactics for their use with no increase in required resources.

This section of the report introduces the general concepts of configural theory and briefly discusses a simple example in the context of fleet air defense that is discussed in more detail later in the report. Section II introduces formal concepts that appear useful for configural theory for application to the design and assessment of ship self-defense and fleet air defense systems in the context of this very limited, initial examination and shows, mainly by means of examples, how configuration affects the effectiveness of such weapon systems and its assessment. Section III uses the configural casualty probability densities associated with the combat situations discussed in Section II to illustrate how casualty production in a configured encounter differs probabilistically from the corresponding free encounter with the same average casualty production and discusses some operational consequences of that difference. Two mathematical appendices discuss the key equations used in the report: Appendix A derives the configural probability densities for casualties among the attackers and among the ships to be protected. Appendix B defines the relative effectiveness of alternative weapon systems and discusses its determination in both configured and free encounters.

1. Configural Theory and the Definition and Assessment of Weapons Effectiveness

The effectiveness of weapons is exclusively and ultimately determined in combat. However, before a weapon can be used in combat, its combat potential, real or supposed, must be identified, it must be developed, it must be procured, it must be fielded. Before that, its general nature and operating principles must be conceived. At each of those points its effectiveness is assessed by some means to some degree, and those assessments, including the quantitative assessments based on mathematical models of combat or on engineering or developmental tests, should it progress to that stage, are fundamentally mainly intuitive. There is no formal physical theory — scientific theory — that relates the behavior of weapons in use in combat to their individual characteristics, let alone to their "combat effectiveness". Nor is a need for such theory widely perceived. The engineering and developmental tests are taken to be sufficient, for what you see in those tests is widely believed to be what you get in combat.

Such models and tests, however, are not combat; and what is inferred about combat effectiveness from their results (sometimes despite their results) is a judgment or a belief, not a fact. Consequently, a weapon that is developed, procured, fielded, and, in the event of war, used, at least initially, is a weapon that is thought will be effective in combat. Many intuitive judgments, comparisons, and interpretations — naive (informal) theory, which is often mistaken for no theory — intervene. That is also true, and at least as important, in a field of many competing proposed or developmental weapons. Only those candidate weapons that are thought to be the most effective are developed, procured, and fielded. Which of them are, of course, depends both on their individual characteristics and how those characteristics are thought to affect combat effectiveness. Naive theory is not likely to provide a sound foundation. Furthermore, it is not likely to foster the design of weapons that are as effective as weapons designed with equivalent resources but in accordance with better theory. Worse, highly effective weapons have been conceived and even developed without benefit of any discernible formal theory only to be deployed in insufficient quantities and otherwise misused at times of great need because of incorrect theory. Sound theory for assessing weapons effectiveness manifestly is of great importance for the conception, design, and development of effective weapons and their proper deployment and use.

However, as numerous observers both inside and outside the defense analysis and military operations research community have noted, current mathematical models of combat — that is, models derived from the customary conceptualization of weapons effectiveness — although often replete with advanced mathematics, systematically overstate weapons effectiveness.[1] Assessments extrapolated from engineering or developmental tests overstate effectiveness as well. In the view of many weapons users, the overstatements are often great.

Configural theory is a mathematical conceptualization of the relationship between the behavior of weapons in use in combat and their individual characteristics. It makes explicit and accommodates configuration, a fundamental element of the multiple-target, multiple-weapon encounters typical of combat which is excluded by the customary conceptualization and the derivative mathematical models (particularly those used in weapons research and development, which usually address only one-target, one-weapon encounters). Among other configural elements, configuration includes the distribution in space of all the targets and all the weapons involved in an engagement at each instant during its course. For example, the positions and orientations of ships in a battle group and the locations of weapons on

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[1] For instance, see the Military Operations Research Society's workshop report *More Operational Realism in the Modeling of Combat (MORIMOC), 25-27 February 1986* (April 1991, AD-B154 505), UNCLASSIFIED LIMITED. It addresses three "usual deficiencies" in defense models, the first of which is "overestimating the lethality (damage effects) of almost everything".

those ships define a time-dependent distribution of targets and weapons in space — in the context of fleet air defense, the defender configuration. Similarly, the positions on their individual trajectories of an attacker's bombers within a wave as well as the positions of their antiship weapons on their individual trajectories after launch define a time-dependent distribution of targets and weapons in space — the attacker configuration. Together those configurations define another configuration, the attacker-defender configuration, and it strongly affects the combat effectiveness of the constituent weapons. As is illustrated in this report, configuration can change relative effectiveness, assessed and actual, by factors of two to ten or more.

a. Configuration in Combat, the Attacker-Defender Configuration, and Configured Encounters

Encounters in combat are generally multiple-target, multiple-weapon encounters. Actual encounters are always configured: Each target is always of a particular kind and, at each instant of time, is at a particular position and in a particular state, and each weapon is always of a particular kind and, at each instant of time, is at a particular position and in a particular state. Furthermore, the states of the targets and the weapons, which include the particular military role or mission of each, depend upon and constrain their locations. The targets and the weapons in the course of combat are inseparable from their disposition in space and time — and that, in turn, is inseparable from the outcome of the engagement and the combat effectiveness of the weapons.

The targets and weapons of the attacker as deployed or situated in space, whether by chance or plan, define (and in configural theory are defined by) the *attacker configuration*. Similarly, those of the defender define (and in configural theory are defined by) the *defender configuration*. Each entity in such a configuration has a corresponding *encounter region*, the set of locations of an enemy entity at which an interaction can occur with a positive probability. A *configured encounter* between two entities occurs at the epoch at which one entity first enters the encounter region of another. A configured encounter between two configurations occurs at the epoch at which an entity in either configuration first encounters an entity of the other.

The attacker configuration and the defender configuration, or parts of them, may be in relative motion; and elements of each may be in motion relative to their configurations as well. Moreover, both the motions of the attacker configuration and the defender configuration and the relative motions of their mobile elements are usually coordinated or directed by extensive command, control, and communications systems so as to avoid configural disadvantages and to exploit configural advantages. Once an encounter occurs, the attacker configuration and the defender configuration thus interact to form a single entity, the *attacker-defender configuration*, as each simultaneously adapts and reacts to both the assessed strengths and weaknesses of its and the enemy's configuration and to the actual and inferred effects of exchanges of fires on those configurations in accordance with the strategy, tactics, and training that define its military behavior. An *engagement* defines (and in configural theory is defined by) the stochastic development of an attacker-defender configuration.

Configuration neither implies nor excludes regularity in the positioning or in the arrangement of the targets or the weapons of the attacker or the defender. As noted, whether the positions of the targets and of the weapons as distributed or the kind of target or weapon at each position is the result of careful planning or a random process is not relevant. What is relevant is that neither a target nor a weapon in the attacker or the defender configuration can change its kind, its position can be changed only by natural forces (including propulsion systems), and its state can be changed only by command, internal processes, or combat damage.

Accommodating the attacker-defender configuration has a major mathematical consequence: Numerous stochastic processes and the associated random variables that in combat are probabilistically dependent but in the customary quantifications of weapons effectiveness and in derivative mathematical models and simulations are treated as probabilistically independent embody in configural quantifications the probabilistic dependencies customarily excluded. The target-weapon ranges and relative orientations are good examples: As the attacker configuration is shifted by a random amount relative to the defender configuration (or vice versa), the relative positions of the targets and the weapons change (mainly) in unison. Hence, randomness in the relative position of the attacker configuration and the defender configuration, regardless of its source (for example, the difference between the actual and expected location of an enemy force), introduces common random components into all the relative positions of the targets and weapons. That also applies to the orientations of the attacker and the defender configurations at the epoch of encounter. Thus, in a multiple-target, multiple-weapon configured encounter, all target-weapon ranges are dependent random variables. Similarly, the relative velocity and the orientations of a target and a weapon that encounter each other are dependent random variables (as are the *identities* of the target and the weapon). Because the ranges, velocities, and orientations that define a target-weapon encounter are dependent random variables, random events such as target acquisitions and target kills, which are customarily treated as independent events within their respective classes (and consequently exclude configuration), are dependent as well.

b. Free Encounters and Configured Encounters

More specifically, each of the detection probabilities, acquisition probabilities, hit probabilities, and damage probabilities associated with a particular target-weapon pair is always a function of the kind, state, and position of the target and the kind, state, and position of the weapon (as well as other variables). The positions of the target and the weapon are, of course, functions of the positions of the attacker configuration and the defender configuration. Consequently, the probability that any particular target is damaged by any particular weapon is a function of the positions of the attacker configuration and the defender configuration, as well as many other variables (including those that specify the configurations themselves). As the relative position of the configurations changes or the positions of the targets or the weapons relative to their configurations change, so, of course, do those probabilities as well as the particular targets that can be attacked by particular weapons.

Multiple-target, multiple-weapon configured encounters are thus very complex mathematically. Indeed, mathematical formulas for weapons effectiveness or casualty production that correctly accommodate configuration (*configural* formulas) usually differ markedly from their customary counterparts, which partly or entirely exclude configuration and, accordingly, are termed *nonconfigural*. (The formulas used to determine relative effectiveness in the configured and free encounters addressed in this report, which are discussed in Appendix B, are an example.) Except in a few simple cases, analytical models have never addressed configured encounters, and Monte Carlo simulations, although in principle they can more easily accommodate configuration than analytical models can, nonetheless typically exclude much of it.

Each possible target-weapon encounter in a multiple-target, multiple-weapon configured encounter, viewed by itself, is a *configured encounter*. The target-weapon range, their orientations, and other pertinent characteristics that define the encounter are all determined by the relative position of the attacker configuration and the defender configuration and the positions of the particular target and weapon relative to their respective configurations. Thus, even the one-target, one-weapon encounters that occur in the course of a multiple-target, multiple-weapon configured encounter can be very complex mathematically.

Encounters in combat situations in which there are at most a single target and a single weapon in the operating area of each are, of course, the simplest configured encounters. In such situations, the configural random variables that pertain to the target and the weapon respectively and define the initiation of the encounter are typically uniformly distributed. Also, simply because they define the initiation of an encounter between a lone target and a lone weapon that are not further constrained by a larger attacker-defender configuration, they are probabilistically independent. Such encounters are thus a special form (mathematically, a degenerate, limiting form) of a configured encounter. They are termed *free encounters*. A real multiple-target, multiple-weapon encounter that, to a good approximation, reduces to a free encounter can arise in an engagement with widely separated, identical targets in an area that contains widely separated, identical weapons, provided the targets and the weapons are so widely separated that the probability that any particular target is encountered by more than one weapon and the probability that any particular weapon encounters more than one target are both approximately zero.

For a multiple-target, multiple-weapon encounter to be a free encounter exactly, the configural random variables that define the initiation of each individual target-weapon encounter must be distributed as they are in a one-target, one-weapon free encounter, and, from individual encounter to individual encounter, must be independently distributed as well. The multiple-target, multiple-weapon free encounter is thus equivalent to a sequence of free encounters. However, such a sequence of free encounters is not likely to arise in combat, especially in the defense of a battle group.

Furthermore, the probabilities that characterize weapon capability (the detection, acquisition, hit, and damage probabilities), which are range- and orientation-dependent *functions* with values that generally differ substantially from encounter to encounter in the individual encounters that constitute a multiple-target, multiple-weapon configured encounter, are in free encounters unconditionalized into single numbers (such as the single-shot kill probability) that, as a result, are respectively constant from encounter to encounter. Consequently, those single numbers, because they are *constants* rather than configuration-dependent functions, characterize weapon capability only in free encounters.

Because such single numbers are not sufficient to determine combat effectiveness, using them to assess weapons effectiveness for the multiple-target, multiple-weapon encounters that typify combat, which are usually strongly configured, is by default assessing weapons effectiveness only for free encounters. Such assessments, of course, inherently favor weapons that excel in operational characteristics that contribute most to effectiveness in free encounters. As a result, as this report illustrates, using such single numbers to assess weapons effectiveness systematically overstates combat effectiveness in the multiple-target, multiple-weapon encounters that typify combat by the noted factors of two to ten or more.

2. Configuration Can Change Relative Effectiveness, Actual and Assessed, by More Than a Factor of Ten

Combat is typified by multiple-target, multiple-weapon encounters between attackers and defenders that are respectively deployed so as to maximize configural advantages and minimize configural disadvantages, not by large or small numbers of isolated encounters between single targets and single weapons. How weapons effectiveness, both actual and assessed, depends on the attacker-defender configuration associated with the combat situations in which a weapon is to be used and how greatly it can differ between free encounters and representative configured encounters can be made concrete by comparing two hypothetical air defense systems over a range of simplified, idealized combat situations that include the key configural elements of a wide class of combat situations that are important in naval warfare. Those combat situations entail defeating enemy attackers (in particular, antiship missiles, which are the attackers considered in this report) that attempt to attack high-value targets after penetrating an area surrounding them that is defended by a particular air defense missile system.

To keep the comparison simple, the two missile systems with their missile complements are postulated to be of nearly the same weight and cubage. As a result, the relative effectiveness of the postulated air defense systems is quantified simply by the ratio of the numbers of the respective systems that must be deployed to produce essentially equal numbers of casualties among the attackers, as in this report, or to ensure essentially equal numbers of survivors among the ships to be protected.

The two hypothetical air defense missile systems, because of their postulated operational characteristics, differ greatly in effectiveness as it is customarily assessed, that is, as assessed in free encounters. Those characteristics are specified in the tables below. Specifically, as is consistent with the way the terms "low-performance" and "high-performance" are generally used, the (comparatively) low-performance system has a low detection probability and a short-range missile and the high-performance system has more than twice the detection probability and a missile with twice the range. Also, the high-performance system, as is usual for such systems, has a lower availability than that of the low-performance system. The conditional kill probabilities of the low- and high-performance missiles are equal.

SYSTEM	CHARACTERISTIC PROBABILITIES AVAILABILITY	DETECTION	DAMAGE	SINGLE-ENCOUNTER KILL	RANGE (YARDS)
HIGH-PERFORMANCE	0.7	0.8	0.5	0.280	20,000
LOW-PERFORMANCE	0.9	0.3	0.5	0.135	10,000

POSTULATED OPERATIONAL CHARACTERISTICS FOR SINGLE-ROUND EMPLACEMENTS

TABLE I-1

In multiple-round emplacements, as Table I-2 states, the hypothetical low-performance system, which has a ten-round magazine, has sufficient range to permit two rounds to be fired at an acquired attacker with a shoot-look-shoot fire discipline, but its launcher permits only one shot at an acquired attacker. The hypothetical high-performance system, which has an eight-round magazine, has sufficient range to permit four rounds to be fired with the shoot-look-shoot fire discipline, and its launcher permits that.

SYSTEM	ROUNDS IN MAGAZINE	FIRINGS PER DETECTED ATTACKER PERMITTED BY RANGE	LAUNCHER
HIGH-PERFORMANCE	8	4	4
LOW-PERFORMANCE	10	2	1

ADDITIONAL POSTULATED OPERATIONAL CHARACTERISTICS
FOR MULTIPLE-ROUND EMPLACEMENTS

TABLE I-2

All in all, including its lower availability, the high-performance system as customarily assessed is approximately nine times as effective as the low-performance system. In the simplified, idealized combat situations for which the two systems are configurally compared, the essential configural elements of important naval warfare situations are accommodated. In particular, small to large numbers of attackers in attack corridors of arbitrary width attempt to attack the high-value targets that are protected. To do so, the attackers must penetrate a defended area in which a particular missile system is deployed in a quantity that provides a specified casualty production among the attackers. The ratio of those quantities for a pair of alternative air defense systems and the same specified casualty production determines their free-encounter relative effectiveness in that situation.

The configural elements that characterize the combat situations examined in this report are few and simple and are summarized here; more details are given in Section II. In particular, the attacker configuration consists only of identical antiship missiles making a single-axis attack through the defended area in order to reach the targets being protected, which are themselves identical. The trajectories of the missiles, which are linear and parallel to the attack axis, are uniformly randomly and independently distributed across the attack corridor (at a constant, specified altitude), and the missiles themselves are sufficiently separated that at most one of them encounters any particular air defense emplacement at a time. The attackers do not attack the missile emplacements in the defended area. The attack corridor location is selected uniformly randomly and independently of the locations of the missile emplacements. The defender configuration consists only of the ships being protected and the missile emplacements, which are uniformly randomly and independently positioned in the defended area surrounding those ships.

Relative effectiveness graphs for emplacements of the two hypothetical multiple-round missile systems, which are discussed in more detail in Section II and the equations for which are discussed in Appendix B, are displayed in Exhibit I-1, following this page. The relative effectiveness of the hypothetical missile systems for producing a 0.95 configural attrition rate among the attackers (that is, the ratio of the configurally calculated average number of attackers that are destroyed to the number in the attack is 0.95) is displayed for sequences comprising 1, 2, 5, 10, 20, 50, 100, 200, and an unlimited number of attackers as a function of the width of the attack corridor, which ranges from zero to fifty nautical miles.

As the exhibit shows, the effectiveness of the high-performance system relative to the low-performance system for a free encounter with an arbitrary number of attackers or a configured encounter with a single attacker is a constant 9.03 as a function of the attack corridor width. At the other extreme, in configured encounters with a sequence comprising an unlimited number of attackers, the high-performance system is less effective than the low-performance system for attack corridors with widths of approximately seven nautical miles or greater. That is, for large numbers of attackers and wide attack corridors, *more* high-performance systems than low-performance systems must be deployed for equivalent casualty production — in the limit, for an unlimited number of attackers in an attack corridor of unlimited width, 1.61 times as many high-performance systems. Thus, free-encounter relative effectiveness can overstate the effectiveness

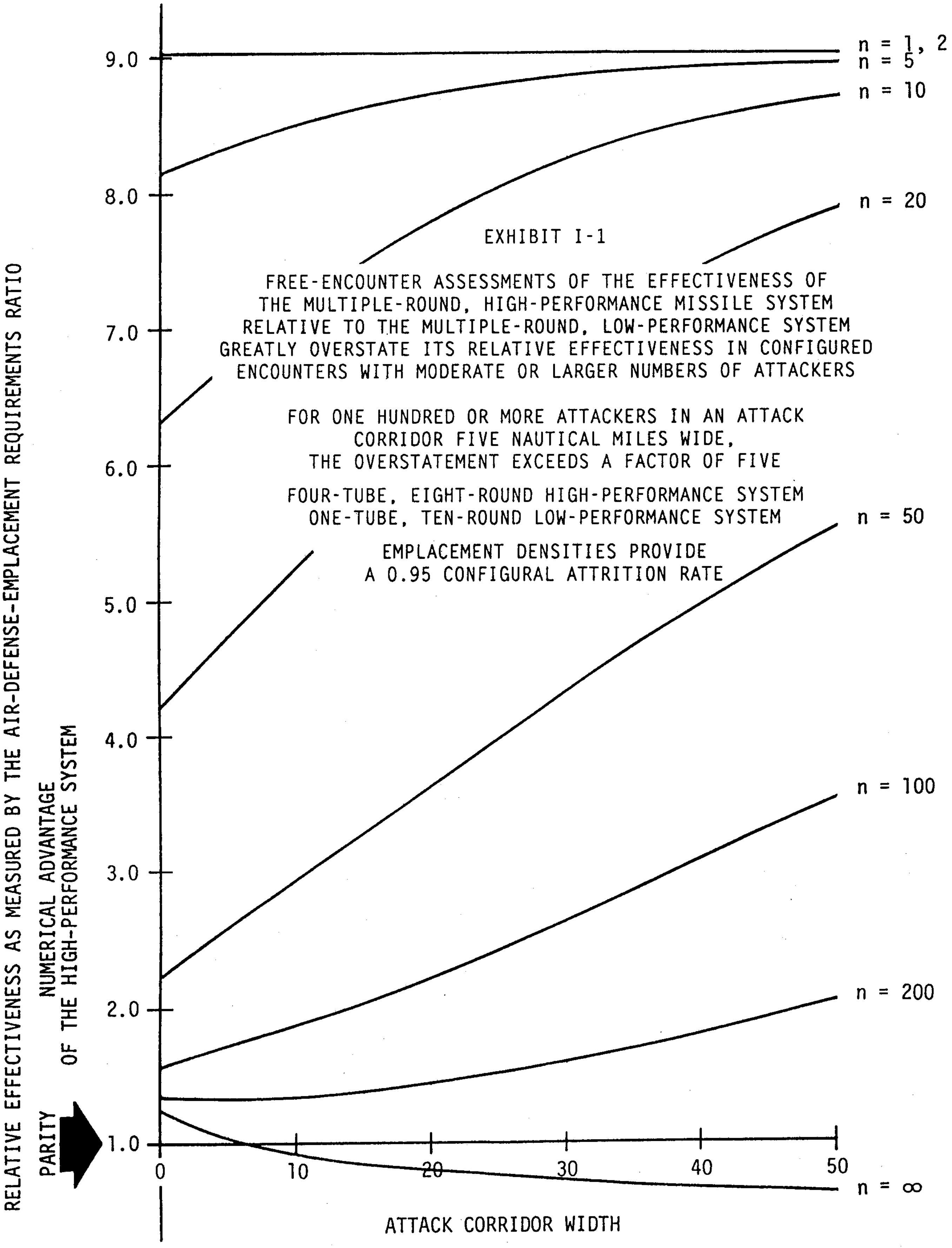


EXHIBIT I-1

FREE-ENCOUNTER ASSESSMENTS OF THE EFFECTIVENESS OF THE MULTIPLE-ROUND, HIGH-PERFORMANCE MISSILE SYSTEM RELATIVE TO THE MULTIPLE-ROUND, LOW-PERFORMANCE SYSTEM GREATLY OVERSTATE ITS RELATIVE EFFECTIVENESS IN CONFIGURED ENCOUNTERS WITH MODERATE OR LARGER NUMBERS OF ATTACKERS

FOR ONE HUNDRED OR MORE ATTACKERS IN AN ATTACK CORRIDOR FIVE NAUTICAL MILES WIDE, THE OVERSTATEMENT EXCEEDS A FACTOR OF FIVE

FOUR-TUBE, EIGHT-ROUND HIGH-PERFORMANCE SYSTEM
ONE-TUBE, TEN-ROUND LOW-PERFORMANCE SYSTEM

EMPLACEMENT DENSITIES PROVIDE
A 0.95 CONFIGURAL ATTRITION RATE

of the high-performance system relative to the low-performance system in configured encounters by more than a factor of fourteen.

In a situation with a sequence of one hundred attackers in an attack corridor five nautical miles wide, the relative effectiveness of the high-performance system is 1.73, as the exhibit shows. In such configured encounters, the 9.03 free-encounter relative effectiveness overstates the relative effectiveness of the high-performance system by a factor of more than five. An important consequence is that, to obtain the required 0.95 configural attrition rate among one hundred attackers in a five-mile-wide attack corridor, more than five times as many high-performance systems are needed than in the associated free encounters.

3. Configural Theory Reveals Important Behavioral Characteristics of Air Defense Weapons

Casualty production among the ships to be protected is an important factor in assessing the effectiveness of alternative fleet air defense and ship self-defense systems. Customarily, the average number of casualties among a group of ships is used to express quantitatively the results of an attack. A better means, especially for quantifying casualties among high-value ships, is the probability density of the random number of casualties that result from an attack. The configural casualty probability density (more precisely, the configural joint probability density of the individual-ship, indicator random variables for each number of attackers) provides the maximum possible information pertaining to the casualties that may result from an attack.

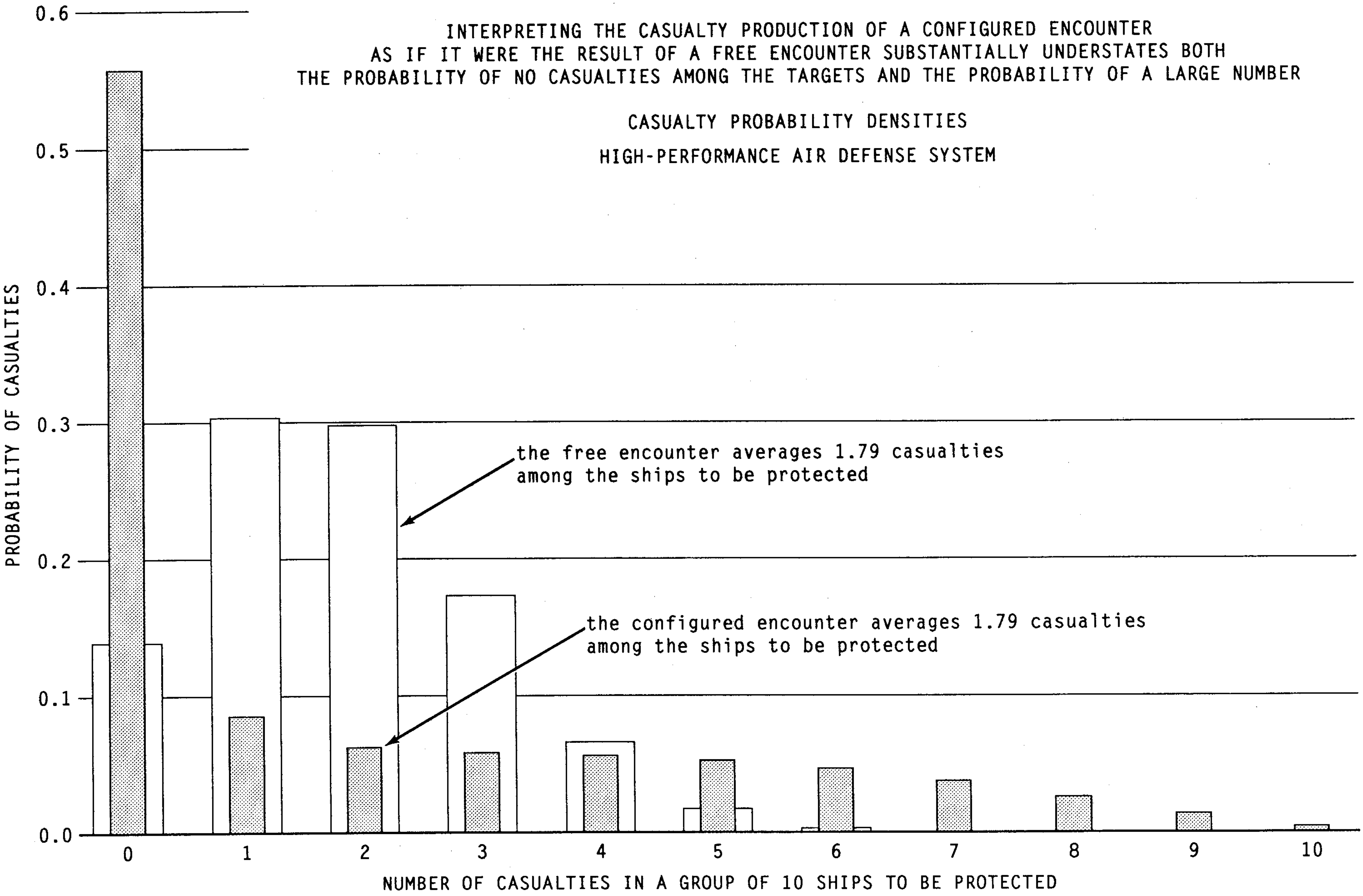
The configural casualty probability densities for the simplified, idealized combat situations explored in the initial research (the equations for which are discussed in Appendix A) are examined in Section III. That examination shows that air defense deployments that produce very high configural attrition rates against large groups of attackers can simultaneously result in small to moderate average numbers of casualties among the ships to be protected but large probabilities of no casualties among them and significant probabilities of large numbers. For the eight-round high-performance system deployed in the number required for an approximately 0.95 configural attrition rate against one hundred attackers in an attack corridor five nautical miles wide, Exhibit I-2, which follows this page, contrasts the configural probability density for casualties among the ten ships to be protected with the casualty probability density of the free encounter that produces the same average number of casualties. As the exhibit shows, in the configured encounter (dotted bars), the average number of casualties among the ten ships to be protected is 1.79, but the probability of no casualties is 0.56 and the probability of six or more, more than half the ships to be protected, is nearly 0.13, which is not operationally negligible. In contrast, as the exhibit also shows, in the corresponding free encounter (white bars), the probability of no casualties is 0.139 and the probability of six or more casualties is less than 0.004, which is operationally negligible. Free-encounter assessments thus can greatly understate both the probability of no casualties among the ships to be protected and the risk of a large number of casualties among them.

EXHIBIT I-2

INTERPRETING THE CASUALTY PRODUCTION OF A CONFIGURED ENCOUNTER AS IF IT WERE THE RESULT OF A FREE ENCOUNTER SUBSTANTIALLY UNDERSTATES BOTH THE PROBABILITY OF NO CASUALTIES AMONG THE TARGETS AND THE PROBABILITY OF A LARGE NUMBER

CASUALTY PROBABILITY DENSITIES

HIGH-PERFORMANCE AIR DEFENSE SYSTEM



the free encounter averages 1.79 casualties among the ships to be protected

the configured encounter averages 1.79 casualties among the ships to be protected

II. SIMPLIFIED, IDEALIZED AIR DEFENSE MISSILE BATTERIES AND THEIR RELATIVE EFFECTIVENESS IN MULTIPLE-TARGET, MULTIPLE-WEAPON CONFIGURED ENCOUNTERS

This section discusses how configuration affects the effectiveness of weapons with operational characteristics that are representative of missiles used for fleet air defense as well as how it affects the customary assessment of that effectiveness. After the formal concepts needed in this examination are discussed, a hypothetical missile with operational characteristics that, as the term is ordinarily used, define a "low-performance" weapon is compared in configured encounters that span a wide range of combat situations with a hypothetical missile with operational characteristics that, as the term is ordinarily used, define a "high-performance" weapon. The customary assessment of the relative effectiveness of the associated weapon systems, the elements of which are sketched in Appendix B, is shown to overstate the relative effectiveness of the high-performance system in configured encounters by more than a factor of fourteen in extreme situations and by more than a factor of five in representative situations. The comparison also illustrates how detection capability and missile range can contribute much less to combat effectiveness than customary analyses state.

Next, the high-performance system is compared with a system that is based on a "high-lethality" missile, which represents an alternative weapon improvement program for the low-performance system. Although in free encounters the high-performance system is more than twice as effective as the high-lethality system, in configured encounters essentially the opposite is true in extreme situations. In those situations, the high-lethality system is almost three times as effective as the high-performance system. In representative situations, it is still slightly more effective than the high-performance system. That comparison and others illustrate how reliability and lethality contribute much more to combat effectiveness than customary analyses state.

1. Concepts, Postulates, and Procedures

Concepts that have been developed for use in planning or controlling a military operation are usually neither simple enough nor fundamental enough to be useful directly either in quantitative theory itself or in discerning, identifying, and defining the military and physical elements of combat that are key to developing such theory. This subsection introduces several formal concepts — attack corridors, encounter regions, multitube emplacements, and multitube batteries — that appear useful for theory in the context of the very limited, initial examination reported. They are not expected to be comprehensive, and, after additional examination, they may prove unsatisfactory in that they lead to needless complexity or do not make important distinctions. However, in this report they provide a framework for discussing and assessing the relative effectiveness of ship air-defense missile emplacements. In this section they are used in examining the relative effectiveness of hypothetical, single-round and multiple-round missile emplacements in simplified, idealized configured encounters.

a. The Attacker and Defender Configurations

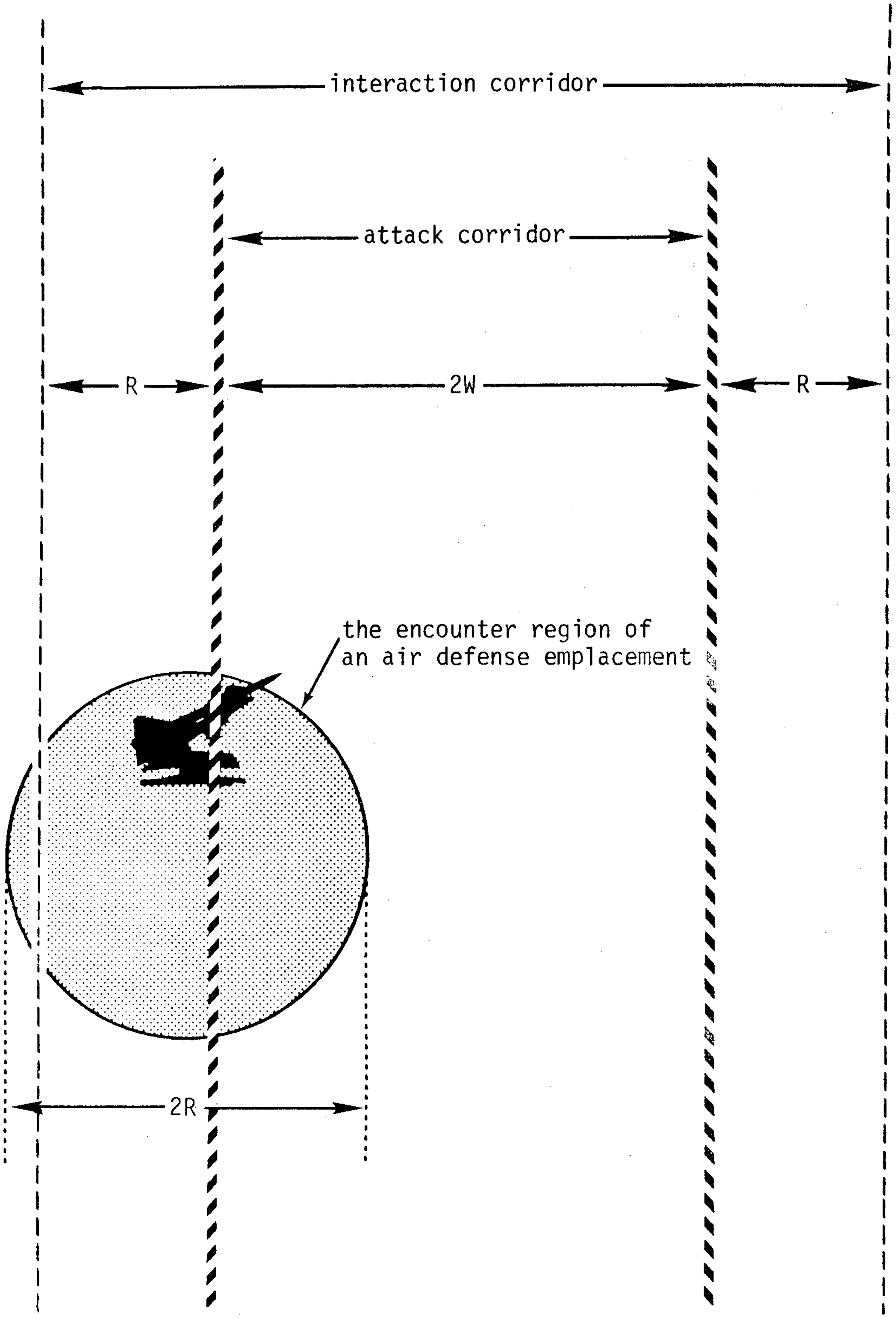
In the research discussed in this report, the defender configuration is postulated to have two principal parts: that comprising the ships to be protected — aircraft carriers, troop transports, essential supply ships — and that comprising the ships that provide the protection, which in this report is limited to the area defense provided by those ships. The ships that provide area defense are represented as multiple-round weapon emplacements (gun positions, missile launchers) or batteries of such emplacements. They are postulated to be uniformly randomly and independently distributed in the defended area (a postulate to be eliminated in future research), which constitutes the outer part of the defender configuration. The ships to be protected are postulated to constitute the inner part of that configuration. The attacker configuration, which is postulated not to fire on the emplacements that provide the area defense (a postulate also to be eliminated in future research), is a single-class, identical-signature threat using an attack corridor of arbitrary width that goes through the defended area along a randomly selected attack axis that intersects the inner part of the defender configuration. The attackers are on trajectories that are parallel to the attack corridor and distributed uniformly randomly and independently across it. They move with constant, equal velocities and at constant, equal altitudes (a restriction to be eliminated in subsequent research) through the defended area unless destroyed by fire from the defender configuration. The targeting information available to defender gunners and their command and control is perfect: No more than one emplacement at a time engages an attacker, and no more than one attacker at a time is in the encounter region of an emplacement. Destroyed attackers are instantly eliminated and draw no additional fire.

b. Attack Corridors, Interaction Corridors, Emplacements, and Batteries

An attack corridor, an encounter region for an air defense missile emplacement, and the associated interaction corridor are illustrated in Exhibit II-1, following this page. An *attack corridor* is a region centered about the threat attack axis within which the attackers confine themselves in attempting to penetrate a defended area. An *encounter region* (the shaded region in the exhibit) is the set of positions at which an attacker (with a trajectory parallel to the attack corridor) can be destroyed by a round fired from the emplacement at the instant an attacker is at any such position. (In the situation depicted, of course, the attacker trajectory must be in the attack corridor as well as intersect the encounter region of the emplacement.) An emplacement that is located anywhere in the attack corridor, of course, has a positive probability of being encountered by an attacker; that is, of having an attacker be at a position at which a round fired by the emplacement at the corresponding instant has a positive probability of destroying it. The encounter probability, of course, remains positive for an emplacement that is outside the attack corridor but within firing range of its boundary; that is, for an emplacement that has at least part of its encounter region within the attack corridor. As the exhibit shows, the *interaction corridor* is the smallest region containing the attack corridor and outside of which an emplacement has probability zero of being encountered by an attacker in the attack corridor.

EXHIBIT II-1

THE INTERACTION CORRIDOR OF AN ATTACK CORRIDOR IS THE LARGEST REGION THROUGHOUT WHICH AN EMPLACEMENT OF A PARTICULAR AIR DEFENSE SYSTEM HAS A POSITIVE PROBABILITY OF BEING ENCOUNTERED BY AN ATTACKER IN THE ATTACK CORRIDOR



Configurally, an encounter between an attacker and an emplacement takes place only while the attacker is so located that the emplacement has a positive probability of destroying it with a round fired at the corresponding instant. A configured encounter is illustrated in Exhibit II-2, following this page. As the exhibit suggests, the emplacement can fire at the attacker while the attacker is between the first possible firing point and the last possible firing point on its trajectory. The number of rounds that can be fired depends on the fire discipline, the attacker speed, its location in the attack corridor relative to the emplacement, its altitude, and the speed, range, and dynamics of the interceptor missile (or whatever kind of round is being considered).

A *multitube missile emplacement*, which is a generalization and a refinement of a multiple-round missile emplacement, is defined to be a missile launcher and a magazine. The launcher has a number of tubes or rails (more than one, except in the trivial case) from which all loaded, fault-free missiles can be successively launched. Initially, all tubes of a launcher at an available emplacement are loaded. Until all the missiles in the magazine have been used, new missiles are loaded into the cleared tubes after launches have been attempted from all tubes or the encounter ends. Such an emplacement is postulated to operate in conjunction with a means to detect and track attackers as well as to provide any guidance its interceptor missiles may require. By definition, it can control only one missile at a time (a restriction to be eliminated in future research). However, as Exhibit II-2 suggests and as previously noted, it can fire more than one missile at the same attacker in sequential firings as determined by the particulars of the encounter. Any multiple-round missile emplacement that can control only one missile at a time, such as those discussed in this report, is a (possibly trivial) multitube missile emplacement.

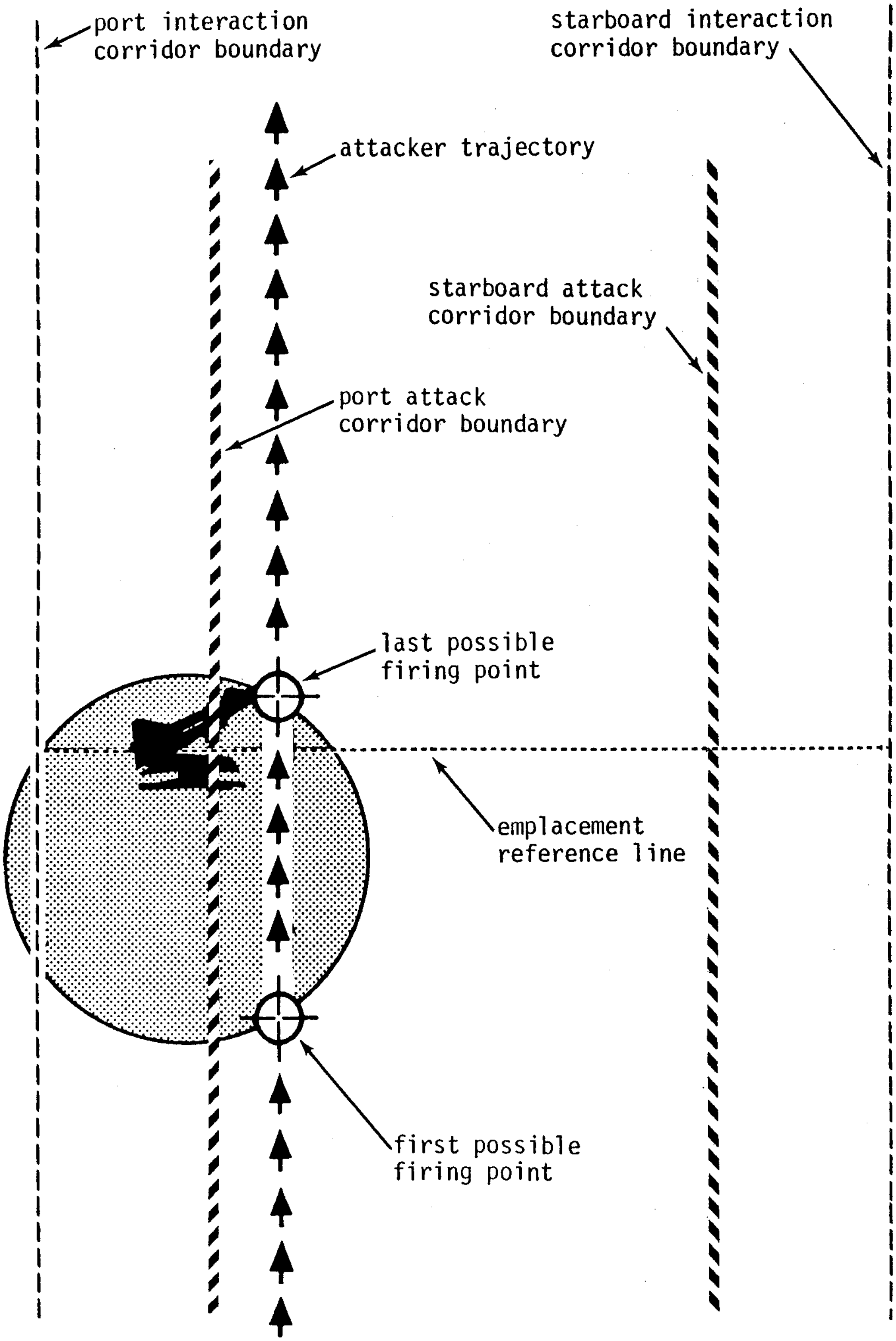
A *multitube missile battery* is defined to be a number of identical multitube missile emplacements that (relative to the range of their weapons) are essentially co-located, have a common magazine, have a common means for search and detection, can instantaneously redirect their fire among targets acquired by the battery, and are controlled by a single commander. The encounter region for a missile battery is identical to that of any of its emplacements. Each available emplacement in a battery can launch and control a missile simultaneously with the other available emplacements of the battery.

c. Fire Discipline, Missile Failure Modes, and Reloads for Multitube Emplacements

Given a successful firing from a particular multitube emplacement, a shoot-look-shoot fire discipline is postulated to govern. The emplacement, by definition, engages no other attackers until its assigned attacker either is destroyed or leaves its encounter region. If a missile does not destroy the attacker after reaching the vicinity of the attacker and the attacker is still in the encounter region, the emplacement is postulated instantaneously to fire the missile in the next tube in the firing sequence at that emplacement. Missiles that are faulty and fail at launch or immediately thereafter, termed *launch aborts*, are distinguished from missiles

EXHIBIT II-2

A CONFIGURED ENCOUNTER BETWEEN AN ATTACKER ON A LINEAR TRAJECTORY IN THE ATTACK CORRIDOR AND AN AIR DEFENSE EMPLACEMENT IN THE INTERACTION CORRIDOR



that are faulty but reach the vicinity of the target before failing. Launch aborts are postulated instantaneously to result in firing the missile in the next tube in the firing sequence at that emplacement until a successful launch occurs or launches have been attempted from all loaded tubes at that emplacement. Should launches be attempted from all tubes before a successful launch occurs, an unengaged emplacement in the same battery, if available, is postulated instantaneously to commence firing. That process continues until one of the following happens: a successful launch occurs; all missiles that had been loaded in available emplacements at the start of the encounter are used; the attacker leaves the encounter region; or the attacker is destroyed.

If a missile that is successfully launched is the last missile in the launcher at that emplacement or the last missile aborts at launch, the emplacement immediately initiates a reload cycle. In a reload cycle, any nonfires are postulated to be extracted from their tubes and missiles from the magazine are postulated to be loaded until all tubes are reloaded or the magazine becomes empty. A reload cycle is postulated to take a fixed time, and during a reload cycle an emplacement cannot fire upon attackers.

d. Mathematical Procedures for Calculating Relative Effectiveness

Mathematical procedures that permit the numbers of multitube emplacements or the numbers of multitube batteries that are necessary for essentially equivalent effectiveness to be calculated for alternative weapon systems are a product of the research discussed in this report. They are discussed in the mathematical appendices. (Appendix A addresses casualties produced among the attackers by multitube batteries and casualties produced among the targets to be protected by attackers that successfully penetrate the defended area. Appendix B addresses how the relative effectiveness of alternative weapon systems is determined in both free and configured encounters.) Combat effectiveness of the defensive weapons may be identified with either the random number of casualties produced among the attackers that attempt to penetrate the defended area or the random number of survivors among the ships to be protected; that is, casualty production among the attackers or casualty prevention among the ships to be protected. (In this report, only effectiveness defined by essentially equivalent casualty production among the attackers is examined although the mathematics developed in Appendix A addresses casualty production among the ships to be protected.) Under such circumstances, the relative effectiveness of systems that are comparable in terms of weight, cubage, and other pertinent physical factors can be measured by the ratio of the numbers of the respective systems that are necessary to accomplish the specified objective. That ratio, which is termed *numerical advantage*, is defined to be the ratio of the number required of the system that requires the larger number (the less effective system) to the number required of the system that requires the smaller number (the more effective system) for essentially equivalent results in a specified combat situation.

For a specified, required configural attrition rate among a specified number of attackers (that is, fractional casualty production), the *attacker casualty procedure* (the procedure used in this report) configurally calculates the respective number of each specified system that is required to produce an attacker casualty distribution of which the average is the required number of casualties. For a specified, required configural probability of no casualties among a specified number of ships to be protected or for a specified, required configural average fraction of survivors, the *defender casualty procedure* (which is not further discussed in this report) configurally calculates the required number of each specified system. The relative effectiveness of any particular systems for the specified objective is the ratio of the larger of those two numbers to the smaller, the corresponding numerical advantage.

2. The Hypothetical Low-Performance and High-Performance Missiles and Their Relative Effectiveness in Single-Round Emplacements and Multiple-Round, Multitube Emplacements in Configured Encounters

The concepts discussed in the preceding subsections are illustrated by determining the relative effectiveness of a multitube emplacement (mathematically, a multitube battery that comprises only a single multitube emplacement) using a hypothetical low-performance missile and a multitube emplacement using a hypothetical high-performance missile to provide a 0.95 configural attrition rate over a range of attack corridor widths for sequences of hypothetical attackers with as few as one attacker and as many as one hundred attackers as well as an unlimited number. The relative effectiveness is determined by the mathematical procedures discussed on pages B-1 and B-2 of Appendix B and the values of the postulated operational characteristics specified in the following subsections.

Both hypothetical interceptor missiles have a postulated speed of one thousand knots, and the hypothetical attackers, which are antiship missiles that maintain a fifty-foot altitude over the relevant part of the attack, have a postulated speed of six hundred knots. The other postulated operational characteristics of the hypothetical air defense missiles in single-round emplacements are displayed in the following table:

	CHARACTERISTIC PROBABILITIES				
SYSTEM	AVAILABILITY	DETECTION	DAMAGE	SINGLE-ENCOUNTER KILL	RANGE (YARDS)
HIGH-PERFORMANCE	0.7	0.8	0.5	0.280	20,000
LOW-PERFORMANCE	0.9	0.3	0.5	0.135	10,000

POSTULATED OPERATIONAL CHARACTERISTICS FOR SINGLE-ROUND EMPLACEMENTS

TABLE II—1

The free-encounter relative effectiveness of the missiles — or, more accurately, the corresponding single-round emplacements — as measured by numerical advantage is given, as Appendix B notes, by the ratio of the product of the single-encounter kill probability and range of the high-performance system to that of the low-performance system, as evaluating equation (B-6.1) using equation (B-5.1) from that appendix makes clear. For the postulated values, the single-round, high-performance missile emplacement is therefore 4.15 times more effective in free encounters than the single-round, low-performance missile emplacement.

a. Relative Effectiveness of Single-Round Emplacements of the Low- and High-Performance Missiles in Configured Encounters

Relative effectiveness graphs for single-round emplacements of the two hypothetical missiles considered as systems are displayed in Exhibit II-3, following this page. The relative effectiveness of the hypothetical missiles in single-round emplacements for producing a 0.95 configural attrition rate among the attackers is displayed for sequences comprising 1, 2, 5, 10, 20, 50, 100, and an unlimited number of attackers as a function of the width of the attack corridor, which ranges from zero to fifty nautical miles. As the exhibit shows, the relative effectiveness for a free encounter with an arbitrary number of attackers or a configured encounter with a single attacker is a constant 4.15 as a function of the attack corridor width. At the other extreme, in configured encounters with a sequence comprising an unlimited number of attackers, the high-performance system is less effective than the low-performance system for attack corridors with widths of approximately twenty-five nautical miles or greater. That is, for large numbers of attackers and wide attack corridors, the high-performance system requires more single-round emplacements than the low-performance system — in the limit, almost 1.29 times more single-round emplacements. Thus, free-encounter assessments can overstate the effectiveness of the high-performance missile relative to the low-performance missile in configured encounters in extreme situations by more than a factor of five.

In a representative situation with a sequence of one hundred attackers in an attack corridor five nautical miles wide, the relative effectiveness of the high-performance missile is 1.37, as the exhibit shows. The 4.15 free-encounter relative effectiveness thus overstates the relative effectiveness of the high-performance system in such configured encounters by slightly more than a factor of three.

b. Additional Operational Characteristics for the Multiple-Round Emplacements

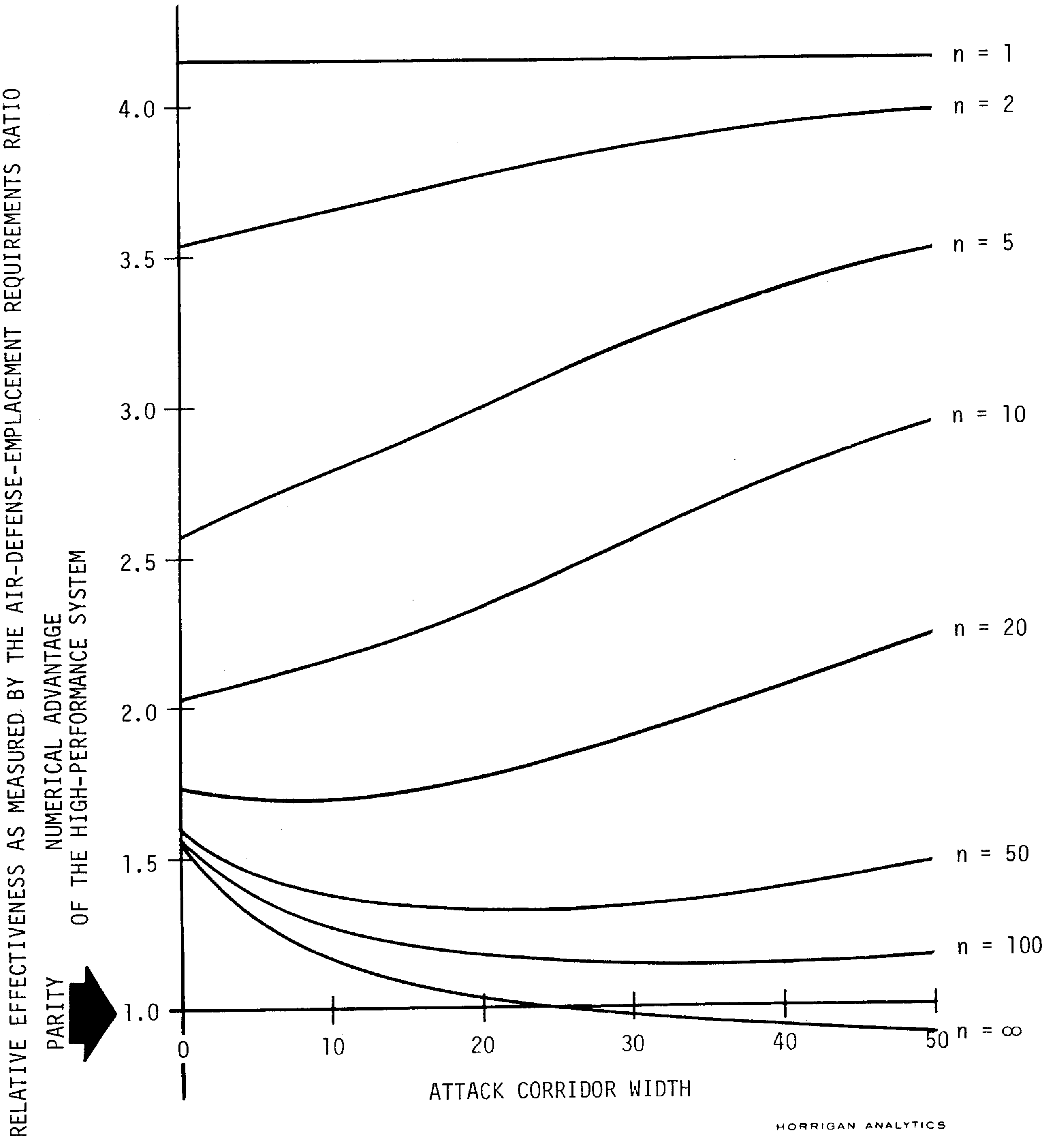
As long as only single-round emplacements are considered, there is no need explicitly to assign components of the availability probability or the detection probability to the missile and to the launcher, for only the products of the respective components are relevant. For a multiple-round emplacement (and for a battery), however, an explicit assignment is necessary. The values of the availability of the launcher and the (successful-launch) reliability of the missile of the high- and low-performance, multitube systems, along with the resulting values of the single-round availability, are given in the following table:

EXHIBIT II-3

FREE-ENCOUNTER EFFECTIVENESS ASSESSMENTS SUBSTANTIALLY OVERSTATE THE EFFECTIVENESS OF THE SINGLE-ROUND, HIGH-PERFORMANCE SYSTEM RELATIVE TO THE SINGLE-ROUND, LOW-PERFORMANCE SYSTEM IN MULTIPLE-ATTACKER CONFIGURED ENCOUNTERS

FOR ONE HUNDRED OR MORE ATTACKERS IN AN ATTACK CORRIDOR FIVE NAUTICAL MILES WIDE, THE OVERSTATEMENT EXCEEDS A FACTOR OF THREE

EMPLACEMENT DENSITIES PROVIDE A 0.95 CONFIGURAL ATTRITION RATE



HORRIGAN ANALYTICS

SYSTEM	LAUNCHER AVAILABILITY	MISSILE RELIABILITY	SINGLE-ROUND AVAILABILITY
HIGH-PERFORMANCE	0.875	0.8	0.7
LOW-PERFORMANCE	0.95	0.947	0.9

POSTULATED COMPONENTS OF THE PROBABILITY OF AVAILABILITY

TABLE II-2

In the multitube emplacement, the detection probability is assigned to the emplacement (in effect, to the launcher equipments), not to the missiles, for once an emplacement detects an attacker, no further detections of that attacker are necessary for successive launches against it from that emplacement. The conditional damage probability is assigned to the individual missiles.

In the multitube emplacement, the high-performance missile system has an advantage over the low-performance missile system that is also not explicit in the table displaying their single-round operational characteristics: its longer range at least doubles the duration of similar encounters and thereby permits more high-performance missiles than low-performance missiles to be fired at an attacker. As the following table specifies, for the combat situations considered in this report, the range of the low-performance missile is postulated to be sufficient to permit two rounds to be fired against any attacker that encounters its emplacement and the range of the high-performance missile is postulated to be sufficient to permit four rounds to be fired. (In future research, such numbers are to be dynamics-determined functions of the relative positions of the emplacements and the attackers.)

SYSTEM	ROUNDS IN MAGAZINE	FIRINGS PER DETECTED ATTACKER PERMITTED BY	
		RANGE	LAUNCHER
HIGH-PERFORMANCE	8	4	4
LOW-PERFORMANCE	10	2	1

ADDITIONAL POSTULATED OPERATIONAL CHARACTERISTICS
FOR MULTIPLE-ROUND EMPLACEMENTS

TABLE II-3

Furthermore, the low-performance system has a single-tube launcher that permits only one missile to be fired at an attacker, as the table also states. The high-performance system has a four-tube launcher that permits four missiles sequentially to be fired at a single attacker. Whether any rounds in addition to the first are fired from a multitube launcher and how many, of course, depend on whether the assigned attacker is destroyed by the first round or any subsequent round that is in the launcher at the time of assignment.

The multitube emplacement gives the missiles, in effect, a higher reliability than they have, for the multitube emplacement instantaneously replaces a launch abort with a new missile, provided one is loaded, an operation that enhances the system based on the high-performance missile more than the system based on the low-performance missile, which has a higher reliability but only a single-tube launcher. Similarly, a multitube emplacement gives the missiles, in effect, a higher kill probability, for more than one round can be fired at a single attacker when necessary.

c. Relative Effectiveness of Multiple-Round Emplacements of the Low- and High-Performance Missiles in Configured Encounters

Relative effectiveness graphs for emplacements of the two hypothetical multiple-round missile systems, which are displayed in Exhibit II-4, following this page, as well as in Exhibit I-1, following page 9, reflect the enhanced capability of the high-performance emplacement in free encounters that results from its four-tube launcher. The free-encounter effectiveness of the high-performance system relative to the low-performance system increases from 4.15 in single-round emplacements to 9.03 in the multitube emplacements, as evaluating equation (B-6.1) using equation (B-4.2) with the appropriate values of the postulated operational characteristics shows. As noted in Section I, the relative effectiveness of the hypothetical missile systems for producing a 0.95 configural attrition rate among the attackers is displayed for sequences of 1, 2, 5, 10, 20, 50, 100, 200, and an unlimited number of attackers as a function of the width of the attack corridor, which ranges from zero to fifty nautical miles. As the exhibit shows, the effectiveness of the high-performance system relative to the low-performance system for a free encounter with an arbitrary number of attackers or a configured encounter with a single attacker is a constant 9.03 as a function of the attack corridor width. The increase in the free-encounter relative effectiveness of the multiple-round, high-performance system, which exceeds the 4.15 of its single-round version by a factor of two, is solely a result of its being able to fire four rounds at a single attacker.

At the other extreme, in configured encounters with a sequence of an unlimited number of attackers, the effectiveness of the high-performance system is not affected by the capability to fire multiple rounds at a single attacker and is less than that of the low-performance system for attack corridors with widths of approximately seven nautical miles or greater. That is, for large numbers of attackers and wide attack corridors, more high-performance systems than low-performance systems must be deployed for equivalent casualty production — in the limit, 1.61 times as many high-performance systems.

Thus, as noted in Section I, free-encounter effectiveness assessments can overstate the effectiveness of the high-performance system relative

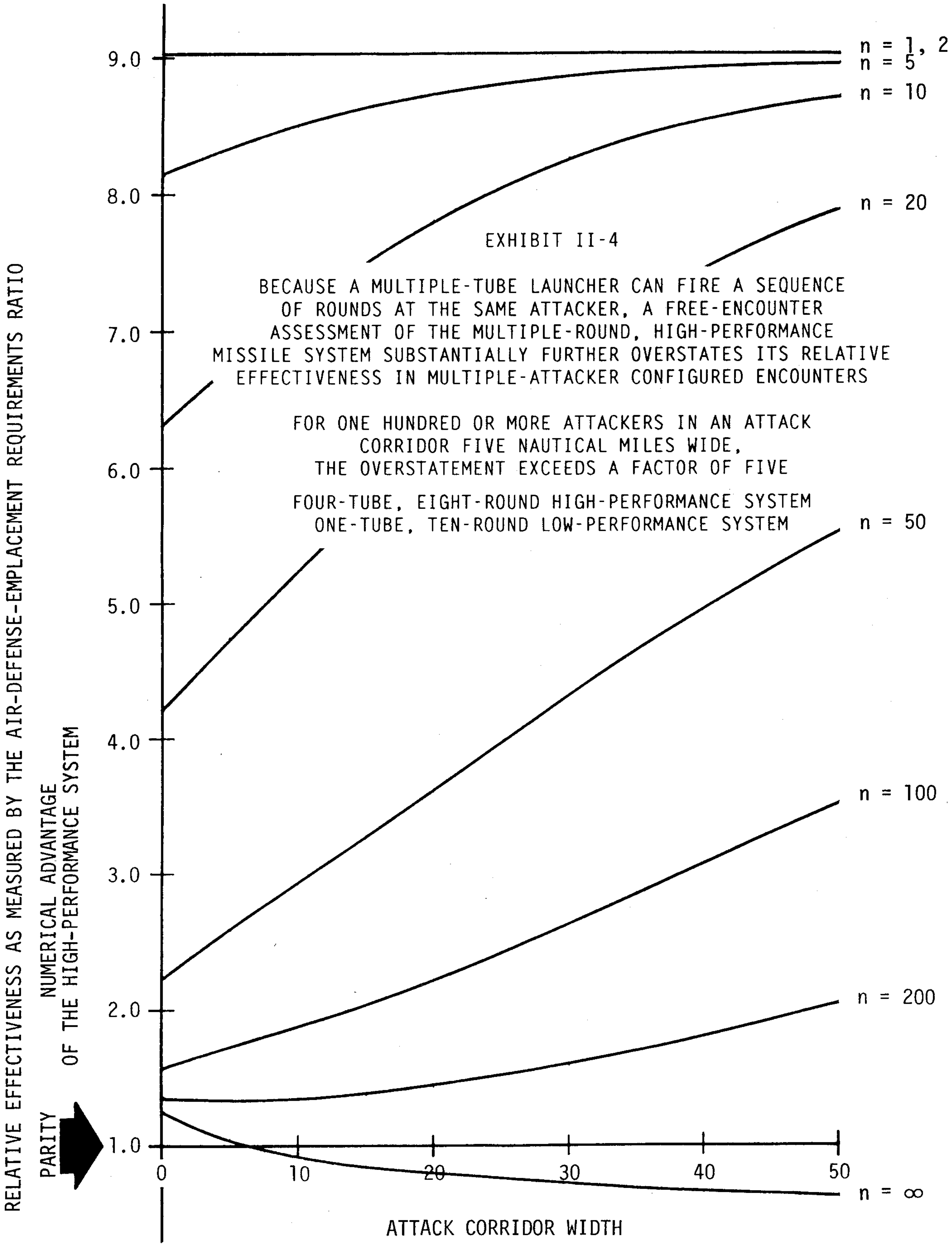


EXHIBIT II-4

BECAUSE A MULTIPLE-TUBE LAUNCHER CAN FIRE A SEQUENCE OF ROUNDS AT THE SAME ATTACKER, A FREE-ENCOUNTER ASSESSMENT OF THE MULTIPLE-ROUND, HIGH-PERFORMANCE MISSILE SYSTEM SUBSTANTIALLY FURTHER OVERSTATES ITS RELATIVE EFFECTIVENESS IN MULTIPLE-ATTACKER CONFIGURED ENCOUNTERS

FOR ONE HUNDRED OR MORE ATTACKERS IN AN ATTACK CORRIDOR FIVE NAUTICAL MILES WIDE, THE OVERSTATEMENT EXCEEDS A FACTOR OF FIVE

FOUR-TUBE, EIGHT-ROUND HIGH-PERFORMANCE SYSTEM
ONE-TUBE, TEN-ROUND LOW-PERFORMANCE SYSTEM

to the low-performance system in configured encounters with an unlimited number of attackers by more than a factor of fourteen. In a representative situation with a sequence of one hundred attackers in an attack corridor five nautical miles wide, the relative effectiveness of the high-performance system is 1.73, as the exhibit shows. In such configured encounters, the 9.03 free-encounter relative effectiveness overstates the relative effectiveness of the high-performance system by a factor of more than five. In heavy attacks in attack corridors five nautical miles wide, more than five times as many high-performance missile emplacements are required than free-encounter assessments state. The substantial improvement in free-encounter relative effectiveness that results from the overall capability of the high-performance system to fire multiple rounds at a single attacker clearly contributes nothing in heavy attacks.

3. The Relative Effectiveness of Multitube Emplacements of the Hypothetical High-Lethality Missile and the Hypothetical High-Performance Missile in Configured Encounters

Usually there are at least several ways in which a weapon system can be improved. The high-performance system can be viewed as the result of one way of improving the low-performance system: increase its detection probability and increase its range. Those increases, which alone would increase the relative effectiveness of the low-performance system by a factor of 5.33, are obtained in part, however, through a twenty-two percent decrease in availability. The combined effect is that the high-performance improvement option is only 4.15 times more effective than the low-performance system. Increasing lethality (the conditional kill probability) is another way of improving the low-performance system. An alternative, hypothetical improvement program for the low-performance system that requires equivalent resources can increase its lethality from 0.5 to 0.9 without changing the values of the other parameters. The resulting system, the "high-lethality" system, is accordingly 1.80 times more effective than the low-performance system, as evaluating equations (B-6.1) and (B-5.1) with the appropriate values of the postulated operational characteristics shows.

The postulated operational characteristics of the alternative, improved missiles as single-round emplacements are compared in the following table:

	CHARACTERISTIC PROBABILITIES				
SYSTEM	AVAILABILITY	DETECTION	DAMAGE	SINGLE-ENCOUNTER KILL	RANGE (YARDS)
HIGH-PERFORMANCE	0.7	0.8	0.5	0.280	20,000
HIGH-LETHALITY	0.9	0.3	0.9	0.243	10,000

POSTULATED OPERATIONAL CHARACTERISTICS FOR SINGLE-ROUND EMPLACEMENTS

TABLE II-4

As before, the free-encounter relative effectiveness of the missiles, as measured by numerical advantage, is given by the ratio of the product of the single-encounter kill probability and range of the high-performance system to that for the high-lethality system. For the postulated values, the high-performance missile in a single-round emplacement is therefore 2.30 times more effective in free encounters than the high-lethality missile in a single-round emplacement. Consequently, the high-performance option appears to be the better choice. Of course, in free encounters the high-performance system is the more effective system, and by a substantial margin.

In multitube emplacements, the high-performance system appears to be even a better choice, for it has the same advantages over the high-lethality system as over the low-performance system that are discussed in Section II.2b. Specifically, in free encounters (as equations (B-4.2) and (B-6.1) in Appendix B show), the eight-round, high-performance system is 5.01 times more effective than the ten-round, high-lethality system. In configured encounters with a moderate and larger number of attackers and emplacement densities that provide a 0.95 configural attrition rate, however, that margin rapidly decreases, as an application of the procedure discussed on pages B-1 and B-2 of Appendix B shows. As Exhibit II-5, which follows this page, illustrates, the effectiveness of the high-performance system relative to the high-lethality system for a combat situation with only twenty attackers in an attack corridor five nautical miles wide is smaller than its free-encounter value by almost a factor of two. Again, for an unlimited number of attackers and very wide attack corridors, free-encounter relative effectiveness assessments can overstate the relative effectiveness of the high-performance system in configured encounters by more than a factor of fourteen. Almost three times as many high-performance systems as high-lethality systems are required in that extreme situation for equal configural attrition rates among the attackers.

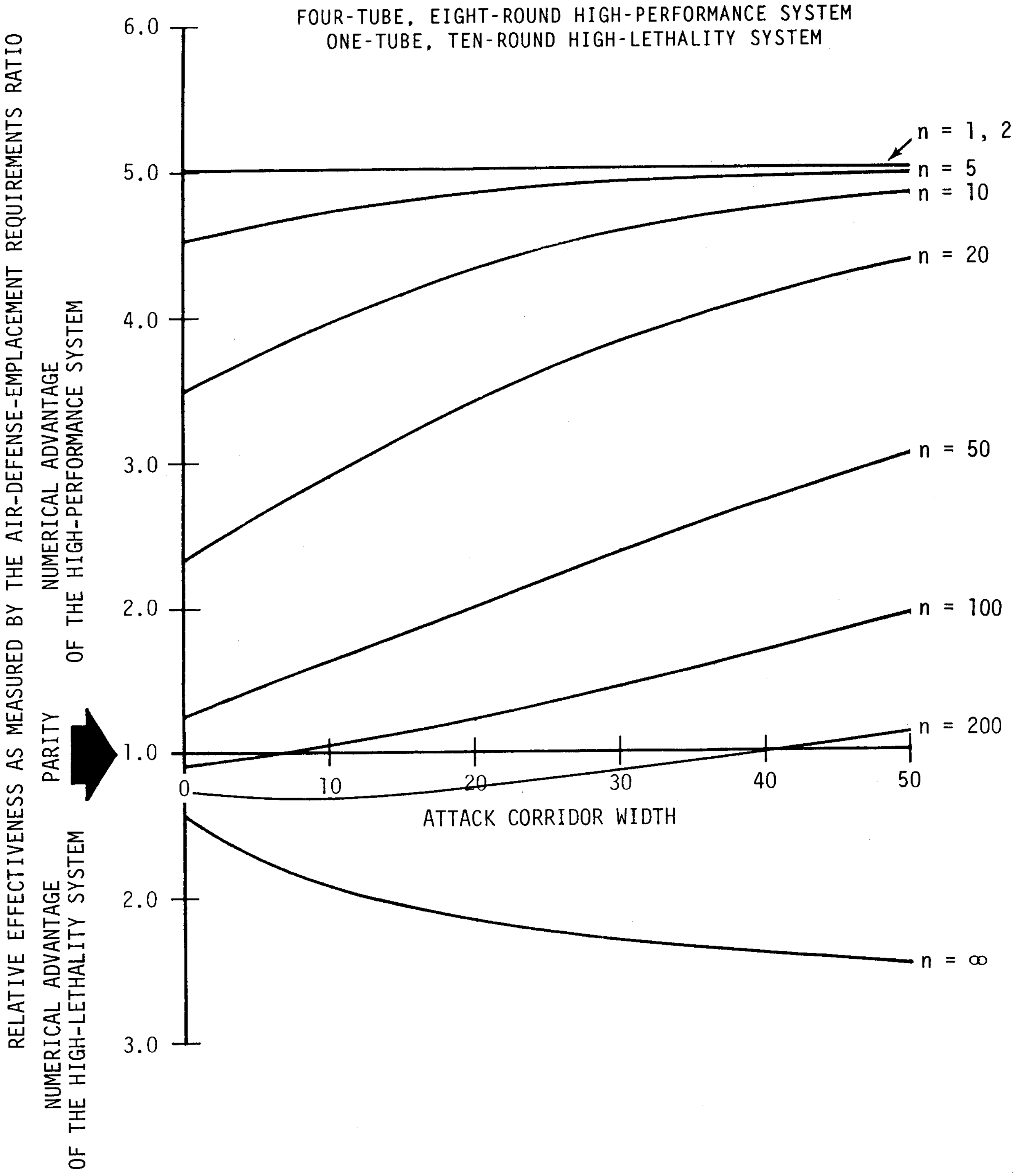
For a representative situation with one hundred attackers in an attack corridor five nautical miles wide, the high-lethality system is slightly more effective than the high-performance system. For two hundred attackers in the five-mile-wide corridor, the high-lethality system is substantially more effective. Specifically, thirty percent more high-performance emplacements than high-lethality emplacements are needed to produce the required 0.95 configural attrition rate among the attackers. For such multiple-target, multiple-weapon configured encounters, the free-encounter assessment overstates the relative effectiveness of the high-performance system by a factor of more than six.

4. Why Assessing Effectiveness in Configured Encounters Is Important for Weapons Research and Development

In examining relative effectiveness graphs, including those in the preceding subsections, it is important to note that all the individual graphs displayed in a particular exhibit show the relative effectiveness of the same two weapon systems. All the defining parameter values of the high-performance system, for instance, are the same regardless of the

EXHIBIT II-5

IN CONFIGURED ENCOUNTERS WITH TWO HUNDRED OR MORE ATTACKERS IN ATTACK CORRIDORS UP TO FORTY NAUTICAL MILES WIDE, THE MULTIPLE-ROUND, HIGH-LETHALITY SYSTEM IS AT LEAST SLIGHTLY MORE EFFECTIVE THAN THE MULTIPLE-ROUND, HIGH-PERFORMANCE SYSTEM


FOUR-TUBE, EIGHT-ROUND HIGH-PERFORMANCE SYSTEM
ONE-TUBE, TEN-ROUND HIGH-LETHALITY SYSTEM

number of attackers and the width of the attack corridor that configurally define each encounter. All the defining parameter values of the weapon systems are also the same regardless of whether the encounters are free encounters. That is essential to understanding why the relative effectiveness of the high-performance system as assessed in a configured encounter with a large number of attackers can be smaller by factors of five or ten and more than its relative effectiveness as customarily assessed in a free encounter with the same number of attackers.

What causes the dramatic decrease in the relative effectiveness of the high-performance system in an encounter with a large number of attackers from that assessed in a free encounter to that assessed in a configured encounter is not just the difference in the values of the operational characteristics of the two systems being compared, as it is sometimes mistakenly thought to be. In comparing the effectiveness of the high-performance system and the high-lethality system, for instance, the values of the reliability and the lethality of the high-lethality system are respectively larger than those of the high-performance system. However, those values are the same in both the free encounter and the configured encounter. What causes the relative effectiveness of the high-performance system in an encounter with a moderate or large number of attackers to be so much less in a configured encounter than in a free encounter is configuration. Put simply, an encounter with multiple attackers in an attack corridor of any particular width is not a free encounter. Thus, even though weapon parameters have the same values regardless of whether the encounter is free or configured, those values matter very differently — they have very different effects — in a configured encounter than in a free encounter. Moreover, how they matter in a configured encounter depends upon the number of attackers.

Even for a high-performance system with lethality and availability values that respectively equal those of the high-lethality system, the relative effectiveness of that high-performance system would be substantially less in representative configured encounters than in free encounters. Of course, it is the same for one attacker in a configured encounter and the corresponding free encounter; but it decreases by more than a factor of six in configured encounters with a large number of attackers. That decrease results from the configuration that is implicit in the number and positions of the emplacements in the defended area and in the multiplicity of attackers that are confined within the attack corridor — configuration that is excluded by free encounters — in conjunction with the values of the operational characteristics. Because the encounters that typify combat are not only multiple-target, multiple-weapon encounters but also configured encounters, an effectiveness assessment based on free encounters imputes to the weapons being assessed an effectiveness that is greater than they can ever realize in combat, often far greater.

As a consequence, free encounters simply are not suitable for assessing combat effectiveness or for quantifying how much particular operational characteristics can contribute to combat effectiveness. More specifically, as the comparisons of the hypothetical air defense systems examined in this section illustrate, effectiveness assessments derived from free encounters, because they exclude the configuration inherent in combat,

- overstate the effectiveness of what are ordinarily considered high-performance systems relative to what are ordinarily considered low-performance systems, often greatly;
- overstate the relative contribution to combat effectiveness of detection probability and range, often greatly; and
- understate the relative contribution to combat effectiveness of reliability and lethality, often greatly.

Configural theory, because it more accurately quantifies how much particular operational characteristics can contribute to combat effectiveness — reliability and lethality, for instance — can better identify those characteristics that most need improvement and can better focus design effort and technological resources on those characteristics that can contribute most to combat effectiveness.

III. THE CONFIGURAL DISTRIBUTIONS OF CASUALTIES AMONG THE ATTACKERS ATTEMPTING TO PENETRATE THE DEFENDED AREA AND OF CASUALTIES AMONG THE SHIPS TO BE PROTECTED

Probabilistically, weapons behave differently, often very differently, in configured encounters with even small numbers of targets than in free encounters. In particular, as Section II illustrates, the number of weapons required for the casualty production in a configured encounter to be essentially equivalent to that in a free encounter with the same number of targets can be factors of two to ten or more larger than for the free encounter. In addition, the probability density for the random number of casualties in a configured encounter can differ greatly in several important ways, even from that for a free encounter with equal average casualty production. Evaluating the equations derived in Appendix A makes that clear. This section illustrates and discusses those differences.

Several characteristics of the probabilistic behavior in configured encounters of the random number of casualties among the attackers and of the random number of casualties among the targets to be protected that are not present in the behavior of the same weapons in such free encounters are important for understanding the contribution of the operational characteristics of the individual weapons to their effectiveness. Those behavioral characteristics also apparently have significance for understanding better the evolution of engagements in combat situations in which the attackers, to reach their primary targets, must penetrate an area defended by air defense emplacements that themselves are not attacked. Such situations, of course, are not likely in fleet air defense, but nonetheless they need to be understood even in that context to understand the more general (and important) situations in which the emplacements not only are attacked but also are protected by their own close-in weapon systems, situations which are to be addressed in subsequent research.

To distinguish the behavioral characteristics of casualty production in configured encounters that are specific to a particular weapon from those associated with free encounters, the casualty production of a weapon in a configured encounter can be contrasted with that in a free encounter in which the average casualty production is the same, that is, with the customary conceptualization of casualty production. For comparisons of weapon systems in which parity in relative effectiveness is defined to be essentially equivalent casualty production, either among the attackers, as in the simplified, idealized combat situations considered in this report, or among the ships to be protected, the appropriate configural casualty probability density is compared to the casualty probability density of a free encounter with equal average casualty production.

Because that multiple-target, multiple-weapon free encounter is a sequence of independent attempts by those attackers, one by each attacker, to penetrate a defended area (which, because the encounters are free, presents the same kill probability to each attacker) — a sequence of identical and independent trials — the corresponding casualty probability density is the binomial probability density in which the number of trials equals the number of attackers and the success probability, which is the free-encounter attrition rate, equals the attrition rate of the configured encounter.

Thus, for example, for a configured encounter of one hundred attackers attempting to penetrate a defended area in which the emplacement density provides a 0.95 configural attrition rate for an attack corridor of the specified width, the corresponding free encounter consists of each of those one hundred attackers freely encountering a defended area that has a 0.95 free-encounter attrition rate. (Note that a defended area with an emplacement density that provides a particular attrition rate for a configured encounter has an emplacement density at least that of a defended area that provides an equal attrition rate for a free encounter with the same number of attackers, and often much greater.) The free-encounter casualty probability density is, of course, the binomial probability density for one hundred independent trials with a 0.95 success probability at each trial.

As this section illustrates, for small and large numbers of attackers, the configural probability densities of the random numbers of attackers that are destroyed by the defender configuration differ greatly from the binomial casualty probability densities of the corresponding free encounters. Specifically, for small numbers of attackers, the configural casualty probability density can be bimodal with a much larger probability of no casualties among them than the binomial casualty probability density of the corresponding free encounter. For a large number of attackers, the configural casualty probability density can have numerous modes or a much larger probability of no survivors than the binomial casualty probability density of the corresponding free encounter. In either case, the standard deviations of the configural casualty probability densities appear systematically to be appreciably larger than those of the random numbers of casualties in the corresponding free encounters.

Likewise, for different numbers of attackers and emplacement densities that correspond to different configural attrition rates, the configural probability densities of the random numbers of casualties among the ships to be protected also behave very differently than the binomial casualty probability densities of the corresponding free encounters. They can be bimodal and, depending on the configural attrition rate among the attackers, can have much larger probabilities of no casualties or of no survivors than are consistent with the corresponding binomial casualty probability densities. Also, the configural probability densities of the random numbers of casualties among the ships to be protected are much less concentrated about their means than is possible for the binomial casualty probability densities of their free-encounter counterparts and have, accordingly, larger standard deviations.

As this section illustrates, important probabilistic characteristics of casualty production in configured encounters that strongly affect weapon design considerations are masked in assessments based on free encounters. The situations examined in this section make plain in a different way than the free-encounter overstatements of effectiveness discussed in Section II the fundamental unsuitability of using the behavior of weapon systems in free encounters for assessing design trade-offs or relative effectiveness.

1. How the Random Number of Casualties among the Attackers Behaves Probabilistically

Generally, the configural casualty probability density associated with a configured encounter in which a moderate or larger number of attackers attempt to penetrate the defended area differs strikingly from the binomial casualty probability density of the corresponding free encounter. For instance, it can be multimodal. For the emplacement densities that correspond to low configural attrition rates, it can have much larger probabilities of no casualties among the attackers than is possible in the corresponding binomial casualty probability density; and for emplacement densities that correspond to high configural attrition rates, it can have much larger probabilities of no survivors than is possible in the corresponding binomial casualty probability density. Also, for moderate and larger numbers of attackers, those configural casualty probability densities, even for the emplacement densities that correspond to very high configural attrition rates, have long sections in which the probability of any particular number of casualties is slowly increasing, almost flat, or slowly decreasing. In all cases, their standard deviations can be much larger than is possible for the binomial casualty probability densities of the corresponding free encounters. The following subsections provide specific examples that are the results of evaluating equations (A-18.2) and (A-19.1), which are the equations specified on pages A-18 and A-19 of Appendix A, for the postulated operational characteristics of the weapon systems.

a. Substantial Probabilities of Catastrophic Failure Associated with Low-Emplacement-Density Area Defense Are Concealed by Comparisons Based on Average Casualties

For emplacement densities of the hypothetical, multiple-round, high-performance system that produce low (0.2) to moderate (0.5) configural attrition rates among small numbers of attackers, the probability that the area air defense fails catastrophically (that is, all attackers successfully penetrate the defended area) for representative attack corridor widths is substantial. It ranges from somewhat less than 0.2 to more than 0.5 for combat situations in which there are as many as twenty attackers in an attack corridor five nautical miles wide. Moreover, such large probabilities that the area defense fails catastrophically are concealed by the corresponding average numbers of casualties. For instance, the catastrophic failure probability of the high-performance system can exceed 0.4 even though, as deployed, it averages the same number of casualties as the low-performance system that, as deployed, has a catastrophic failure probability more than ten times smaller. Consequently, using equal average casualty production to establish parity in the relative effectiveness of those systems in configured encounters with small to moderate numbers of attackers implicitly and artificially increases the assessed effectiveness of the high-performance system.

b. The Variation in the Random Numbers of Casualties Produced among the Attackers Can Be Great

Furthermore, for any number of attackers, the corresponding configural casualty probability densities are much less concentrated about their means than the binomial probability densities of the random number of casualties in the corresponding free encounters. For configural attrition rates of 0.5 or so and even as many as one hundred or more attackers in a five-mile-wide attack corridor, the configural probability density of casualties among the attackers is better approximated by a uniform probability density between twenty-five and seventy-five casualties, for instance, than the binomial casualty probability density of the corresponding free encounter.

Specifically, in an attack by a sequence of one hundred attackers, almost any number of attackers between twenty-five and seventy-five can penetrate the defended area with probabilities that vary from uniformity by little more than forty percent (whereas, in the corresponding free encounter, that variation is many orders of magnitude). That is illustrated by the configural probability density of the random number of casualties among one hundred attackers in an attack corridor five nautical miles wide attempting to penetrate an area defended by emplacements with the high-lethality system and a single-tube launcher with a ten-round magazine, which is displayed (black bars) in Exhibit III-1, following this page. As is shown in the exhibit, the probability of any particular number of casualties among the attackers between twenty-five and seventy-five casualties, for instance, ranges from slightly less than 0.01 (at seventy-five) to slightly more than 0.02 (between forty-two and fifty-four). The variation below and above 0.015 is not much more than forty percent for any particular number of casualties between twenty-five and seventy-five. The contrast with the corresponding binomial probability density (white bars) is great.

The variation of the casualty production associated with the two probability densities is, of course, similarly great. The standard deviation of the random number of casualties among the attackers in the configured encounter is 17.62, which is more than three times greater than the 5.00 standard deviation of the random number of casualties among the attackers in the corresponding free encounter — the greatest possible standard deviation of *any* free encounter with one hundred attackers.

The greater uncertainty in the random number of casualties that result from the configured encounter is also clear from other comparisons of the configural casualty probability density in the exhibit and the binomial probability density of casualties for the corresponding free encounter. In particular, the probability of fewer than thirty-five casualties or more than sixty-five, which is the probability that the random number of casualties is outside the "three sigma" range, is 0.002 for the free encounter but 0.405 for the configured encounter. Furthermore, in the free encounter, fewer than thirty-five casualties among the attackers, the probability of which is less than 0.0009, are operationally essentially impossible, but in the configured encounter that event has a probability of 0.201 and consequently is likely to occur often.

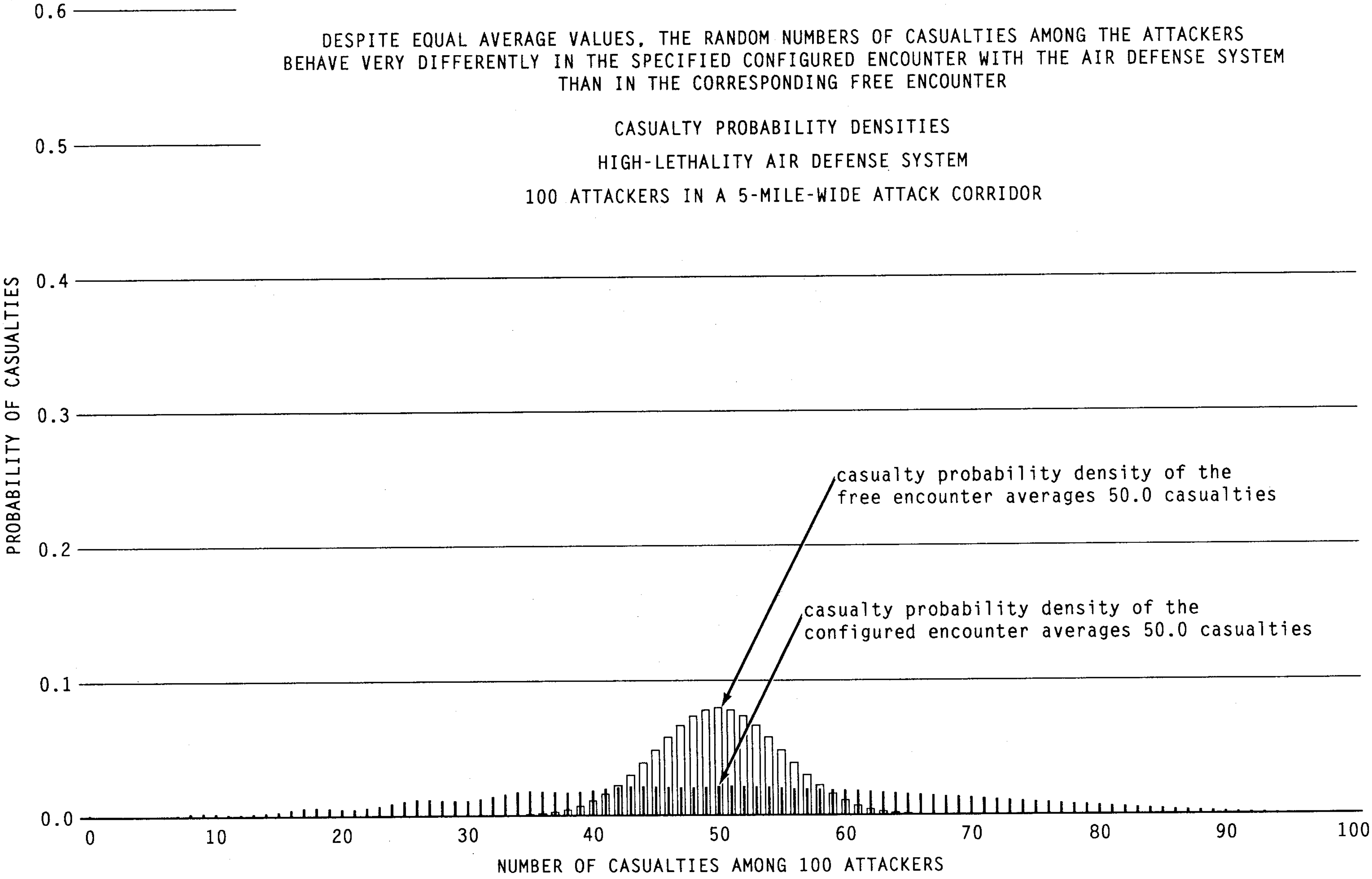
EXHIBIT III-1

DESPITE EQUAL AVERAGE VALUES, THE RANDOM NUMBERS OF CASUALTIES AMONG THE ATTACKERS BEHAVE VERY DIFFERENTLY IN THE SPECIFIED CONFIGURED ENCOUNTER WITH THE AIR DEFENSE SYSTEM THAN IN THE CORRESPONDING FREE ENCOUNTER

CASUALTY PROBABILITY DENSITIES

HIGH-LETHALITY AIR DEFENSE SYSTEM

100 ATTACKERS IN A 5-MILE-WIDE ATTACK CORRIDOR



casualty probability density of the free encounter averages 50.0 casualties

casualty probability density of the configured encounter averages 50.0 casualties

c. High Emplacement Densities Provide Large Probabilities of Destroying All the Attackers

Attacker casualty probability densities for emplacement densities that correspond to high (0.9 or greater) configural attrition rates among the attackers in a heavy attack differ greatly from the corresponding free-encounter casualty probability densities and mainly in a way that is essentially the opposite of that for attacker casualty probability densities for emplacement densities that correspond to low to moderate attrition rates. Specifically, the probability that no attacker in a heavy attack survives the attempt to penetrate a defended area in which sufficient emplacements are deployed to provide a high configural attrition rate is much larger in a configured encounter than in the corresponding free encounter, even though the average number of casualties among the attackers is the same.

For instance, the probabilities of no survivors among 1, 2, 5, 10, 20, 50, and 100 attackers in an attack corridor five miles wide through areas respectively defended by high-lethality emplacements and high-performance emplacements with the densities needed to provide the 0.95 attrition rate among each of the different-sized attacker configurations are compared in the following table for free encounters and configured encounters:

	NUMBER OF ATTACKERS						
	1	2	5	10	20	50	100
EITHER SYSTEM, FREE ENCOUNTERS	0.95	0.90	0.77	0.60	0.36	0.08	0.01
HIGH-LETHALITY SYSTEM, CONFIGURED ENCOUNTERS	0.95	0.90	0.79	0.65	0.47	0.24	0.13
HIGH-PERFORMANCE SYSTEM, CONFIGURED ENCOUNTERS	0.95	0.92	0.85	0.78	0.71	0.62	0.54

PROBABILITY OF NO SURVIVORS AMONG THE SPECIFIED
NUMBER OF ATTACKERS FOR A 0.95 CONFIGURAL ATTRITION RATE
IN AN ATTACK CORRIDOR FIVE NAUTICAL MILES WIDE

TABLE III-1

For one attacker attempting to penetrate a defended area with a 0.95 attrition rate, the average number of casualties must be 0.95 and, regardless of the kind of weapon or encounter, the casualty probability density consequently must have a 0.95 probability of no survivors or, equivalently, of one casualty. For large numbers of attackers, however, no such "forcing" occurs. For instance, for fifty and for one hundred attackers, the corresponding average numbers of casualties are 47.50 and 95.00, and the probabilities of no survivors among the attackers are 0.08 and 0.01

respectively in free encounters, as the table states. In contrast, for the high-lethality system in the corresponding configured encounters, the probabilities of no survivors are respectively much larger, 0.24 and 0.13 . The difference is still greater for the high-performance system. As the table shows, the corresponding probabilities for the high-performance system in configured encounters are 0.62 and 0.54 .

Despite those large concentrations of probability at one point in the configural casualty probability densities, the associated standard deviations are also much larger than those of the corresponding free-encounter binomial casualty probability densities. For the high-performance system and one hundred attackers, for instance, despite the 0.54 probability of no survivors, the standard deviation of the random number of casualties is 8.47 . That is almost four times greater than the standard deviation (2.18) of the binomial casualty probability density for the corresponding free encounter, the mode of which is only 0.18 and occurs at 95 casualties. The associated uncertainty in casualties among the attackers consequently introduces a corresponding uncertainty in casualties among the ships to be protected.

2. How the Random Number of Casualties among the Ships to Be Protected Behaves Probabilistically

How the random number of casualties among the ships to be protected behaves probabilistically in the multiple-target, multiple-weapon configured encounter between them and the attackers that successfully penetrate the defended area surrounding them is determined by the associated configural casualty probability density, the equation for which is given in Appendix A as equation (A-21.1). In this preliminary examination, each attacker that penetrates the defended area independently attempts to acquire one of those ships. It succeeds in acquiring some ship with a 0.9 probability. The ship a successful attacker acquires and attacks is uniformly randomly distributed among the ships to be protected (a restriction to be eliminated in subsequent research). Any ship that is attacked is damaged with a 0.5 probability. In this report, the damage events for a particular ship are postulated to be probabilistically independent, noncumulative, and have no degrees (restrictions also to be eliminated in subsequent research). A particular ship among the ships to be protected can be attacked by any number of the attackers that acquire some ship; and the targets acquired by attackers are postulated to be independent of previous successful attacks (additional restrictions to be eliminated in subsequent research). Any attacks on a particular ship after a first successful attack, of course, do not produce additional casualties.

A defender configuration is associated with each of the postulated air defense missile systems. It comprises the ships to be protected and a defended area that screens those ships from the attackers and contains the emplacements of the associated air defense missile system. The high-performance missile system is associated with the high-performance defender configuration, and the high-lethality missile system is associated with the high-lethality defender configuration. The defended areas of defender configurations are (essentially) identical, and each contains only the number of the associated multiple-round emplacements that is required for the specified configured encounter. Those emplacements are uniformly randomly and independently distributed within the corresponding defended area at the beginning of the attack and do not move appreciably during the time the attackers are within the defended area. Of course, the number of ships to be protected, which is ten, is the same for both the high-performance and the high-lethality defender configurations, and they are uniformly randomly and independently distributed in the protected area of the appropriate configuration.

How the behavior of the random number of casualties among the ships to be protected depends upon the weapon system deployed and its emplacement density can be illustrated by examining and comparing the configural casualty probability densities for the high-performance and the high-lethality defender configurations for combat situations in which the number of emplacements deployed in each defender configuration is the number necessary for the high-lethality system respectively to provide configural attrition rates that are low to moderate (say, 0.5), high (about 0.9), and very high (about 0.95) against attackers attempting to penetrate the corresponding defended areas. Because the emplacements of both systems are designed (in concept) to satisfy weight and cubage constraints and are essentially identical in each of those respects, the high-performance and high-lethality defender configurations being compared are constrained to have the same number of air defense emplacements in their respective defended areas. That number, of course, is taken to be the smallest number that provides the required configural attrition rate among the attackers in an attack that is as heavy as expected.

In combat situations in which one hundred or more attackers use an attack corridor five nautical miles wide through the defended area, the high-lethality system provides each of the specified configural attrition rates with a smaller number of emplacements than the high-performance system. Accordingly, each system is deployed in the number required by the high-lethality system for each specified configural attrition rate. As a result, in each comparison the average number of casualties among the attackers is slightly larger for a high-lethality defender configuration than for its high-performance counterpart.

For the resulting defender configurations, the configural probability densities of the random number of casualties among the ships to be protected can be similar to the binomial casualty probability densities of the corresponding free encounters in which the average casualty production is identical or they can be strikingly dissimilar. In defender configurations with low to moderate configural attrition rates against the attackers, those configural probability densities, even though they can have much larger probabilities of no survivors among the ships than the binomial casualty probability densities for the corresponding free encounters, are nonetheless similar to them. For at least high configural attrition rates, like the probability densities of the random number of casualties among the attackers previously discussed, the configural probability densities of casualties among the ships to be protected differ strikingly from the binomial casualty probability densities of the corresponding free encounters.

For high to very high configural attrition rates, the configural probabilities of no casualties among those ships are much greater than those of the binomial casualty probability densities for the corresponding free encounters. For instance, for the high-performance defender configuration with the nearly 0.95 configural attrition rate, the configural probability of no casualties among the ships is greater by a factor of four, as Section I notes. Each of those probability densities and comparisons is discussed in more detail in the following subsections.

a. Low to Moderate Configural Attrition Rates among the Attackers and the Associated Configural Probability Densities of Casualties among the Ships to Be Protected

This subsection examines how casualty production among the ships to be protected differs between the high-performance defender configuration and the high-lethality defender configuration in configured encounters for low to moderate configural attrition rates. For the number of air defense emplacements in each configuration that provides a 0.5 configural attrition rate among one hundred attackers attempting to penetrate the high-lethality defender configuration in an attack corridor five nautical miles wide, the associated configural casualty probability densities are compared in Exhibit III-2, following this page. As the exhibit notes, the average number of casualties among the ships to be protected that results from the specified attack on the high-performance defender configuration is 9.43, and from an identical attack on the high-lethality configuration, 8.62 . There is thus almost one additional casualty on the average among the ships to be protected in the high-performance defender configuration than among those in its equal-density, high-lethality counterpart. However, both air defense deployments are so inadequate, whether or not the ships have any point defense or close-in weapon systems, that the difference has little operational significance.

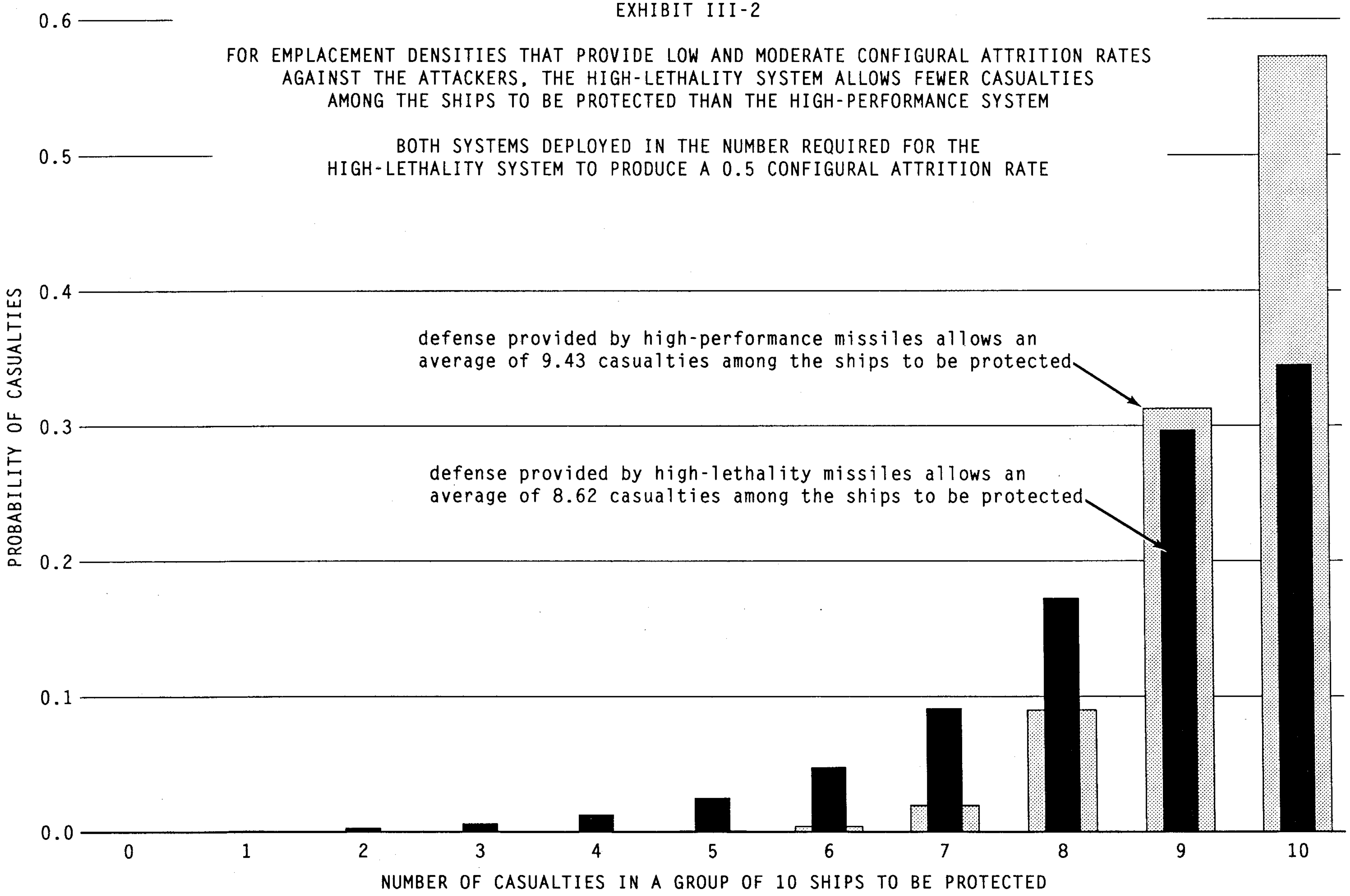
The configural probability density (dotted bars) of the random number of casualties among the ships to be protected in the high-performance defender configuration shows that the associated probability that no ships survive is high, 0.57 . It is substantially higher than the corresponding probability (0.34) of the configural probability density (black bars) for casualties among the ships to be protected in the high-lethality defender configuration. The high-performance system also has a higher probability of exactly nine casualties among the ships to be protected than the high-lethality system. For eight or fewer casualties, the probabilities associated with the high-lethality system are all larger than the corresponding probabilities associated with the high-performance system. Thus, neither air defense system provides adequate protection by itself, but the high-lethality system provides slightly better protection than the high-performance system.

For such emplacement densities in the defended areas, the configural probability densities for the random number of casualties among the ships to be protected in the respective configurations do not differ greatly from the binomial casualty probability densities of the corresponding

EXHIBIT III-2

FOR EMPLACEMENT DENSITIES THAT PROVIDE LOW AND MODERATE CONFIGURAL ATTRITION RATES AGAINST THE ATTACKERS, THE HIGH-LETHALITY SYSTEM ALLOWS FEWER CASUALTIES AMONG THE SHIPS TO BE PROTECTED THAN THE HIGH-PERFORMANCE SYSTEM

BOTH SYSTEMS DEPLOYED IN THE NUMBER REQUIRED FOR THE HIGH-LETHALITY SYSTEM TO PRODUCE A 0.5 CONFIGURAL ATTRITION RATE



defense provided by high-performance missiles allows an average of 9.43 casualties among the ships to be protected

defense provided by high-lethality missiles allows an average of 8.62 casualties among the ships to be protected

HORRIGAN ANALYTICS

free encounters (which are not displayed). In particular, for the high-performance defender configuration, there is only a slight difference between the configural casualty probability density and its binomial counterpart, which has a 0.943 success probability. The largest configural probability, that for ten casualties, which is 0.573, is only three percent larger than that (0.556) of its free-encounter, binomial counterpart. None of the other point-to-point differences are much greater. The differences between the configural casualty probability density for the high-lethality defender configuration and its free-encounter, binomial counterpart, which has a 0.862 success probability, are greater. In particular, the configural probability of no survivors among the ships to be protected is 0.345, which is slightly more than fifty percent greater than the corresponding probability (0.227) in its binomial, free-encounter counterpart. Nonetheless, the probability densities are similar.

b. A High Configural Attrition Rate among the Attackers and the Associated Configural Probability Densities of Casualties among the Ships to Be Protected

Of the combat situations so far examined, the configural casualty probability densities of the ships to be protected differ most for the combat situation in which the number of air defense emplacements in the respective defended areas of the high-performance defender configuration and the high-lethality defender configuration is that required for a high (0.9) configural attrition rate among the one hundred attackers of the high-lethality defender configuration. Those two configural probability densities also differ greatly from the binomial probability densities that are their free-encounter counterparts.

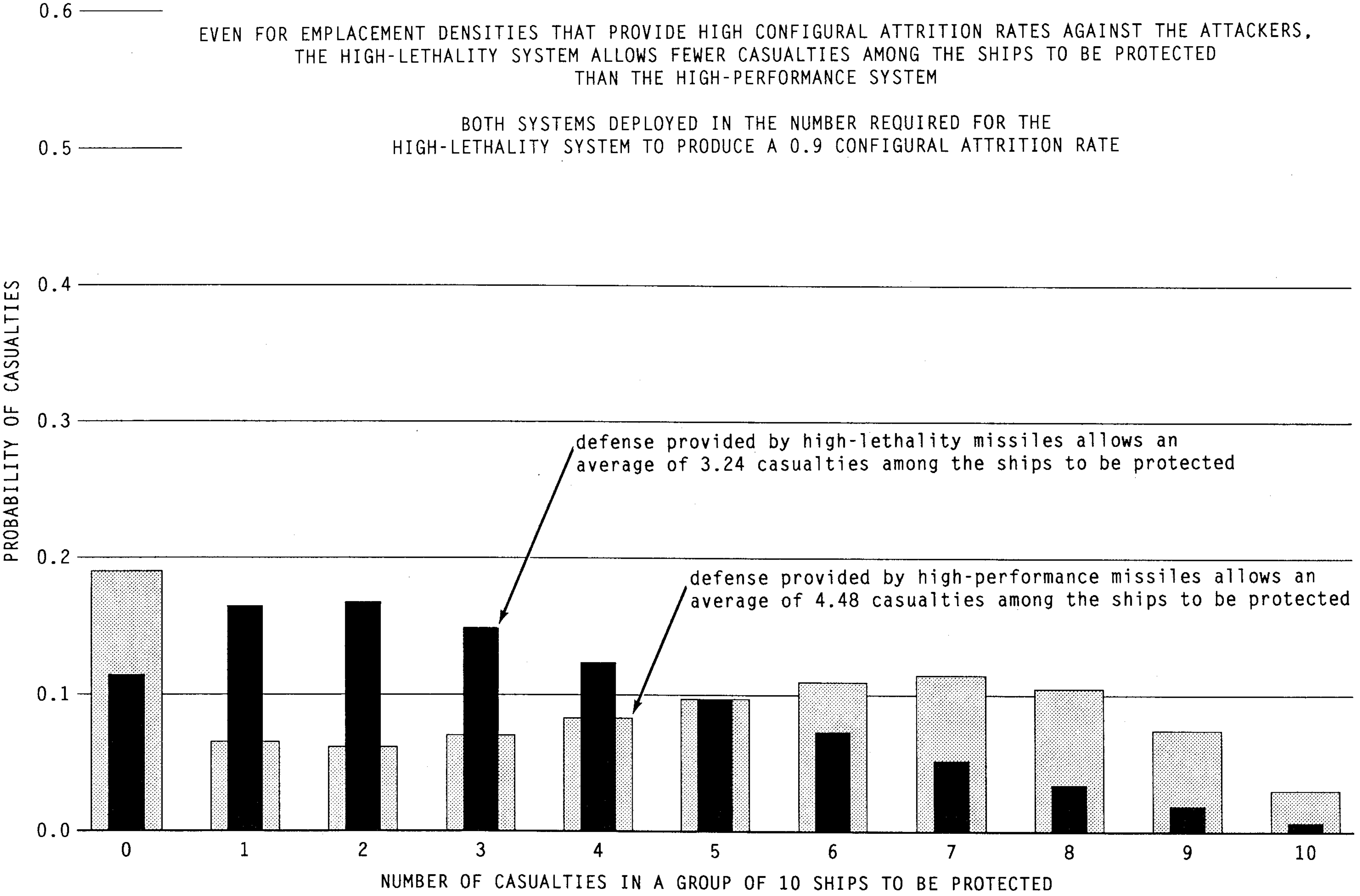
Inspection of the configural probability density (dotted bars) for the random number of casualties among the ships to be protected in the high-performance defender configuration with that number of emplacements, which is displayed in Exhibit III-3, following this page, shows that it has modes at zero casualties and at seven casualties. In contrast, the corresponding configural probability density (black bars) of casualties among the ships to be protected in the high-lethality defender configuration has only one mode, which is at two casualties.

Although the most probable number of casualties among the ships to be protected in the high-performance defender configuration is zero and the probability (0.47) of at most four casualties is substantial, the probability (0.43) of six or more casualties is almost the same. Small and large numbers of casualties are thus nearly equally probable. In contrast, the probability (0.72) of at most four casualties among the ships in the high-lethality configuration is almost four times the probability (0.19) of at least six casualties. Also, the probability of at most four casualties among the ships to be protected in the high-lethality configuration is fifty percent more than that for the high-performance defender configuration, and the probability of at least six casualties in the high-lethality configuration is less than fifty percent of that for the high-performance defender configuration. Those differences, of course, favor the high-lethality defender configuration and are reflected in the average numbers

EXHIBIT III-3

EVEN FOR EMPLACEMENT DENSITIES THAT PROVIDE HIGH CONFIGURAL ATTRITION RATES AGAINST THE ATTACKERS, THE HIGH-LETHALITY SYSTEM ALLOWS FEWER CASUALTIES AMONG THE SHIPS TO BE PROTECTED THAN THE HIGH-PERFORMANCE SYSTEM

BOTH SYSTEMS DEPLOYED IN THE NUMBER REQUIRED FOR THE HIGH-LETHALITY SYSTEM TO PRODUCE A 0.9 CONFIGURAL ATTRITION RATE



of casualties among the ships to be protected in the respective defender configurations. The high-performance defender configuration on the average allows thirty-eight percent more casualties among those ships, which amounts to 1.24 ships on the average, than the high-lethality defender configuration.

The binomial casualty probability density (which is not displayed) that is the free-encounter counterpart of the configural casualty probability density for the ships to be protected in the high-performance defender configuration has a success probability of 0.448 . The most probable number of casualties in the corresponding free encounter is four, and its probability is 0.24, which is almost three times that of the same number of casualties in the configural casualty probability density for the ships to be protected in the high-performance defender configuration. The probability (0.003) of zero casualties in the corresponding free encounter is very much smaller than that (0.19) for the configured encounter, the larger mode of the configural casualty probability density. The probability (0.07) of exactly seven casualties in the free encounter, which corresponds to the second mode in the configural casualty probability density, is also smaller than the corresponding probability (0.11) in the configured encounter.

The difference between the configural probability density for casualties among the ships to be protected in the high-lethality defender configuration and the binomial casualty probability density for the corresponding free encounter, which has a success probability of 0.324, is also large. The probability of zero casualties in the free encounter, for instance, is 0.02, but in the configured encounter, as Exhibit III-3 indicates, it is 0.11 . The free-encounter casualty probability density also understates substantially the probability of at most one casualty, 0.12 instead of 0.28, and understates substantially the probability of six or more casualties, 0.07 instead of 0.19 . The values in between are overstated, mainly substantially.

c. A Very High Configural Attrition Rate among the Attackers and the Associated Configural Probability Densities of Casualties among the Ships to Be Protected

In a combat situation in which the ten ships to be protected are to be defended either by high-lethality emplacements in the high-lethality configuration in sufficient quantity to produce a 0.95 configural attrition rate among the one hundred attackers in the five-mile-wide attack corridor or by an equal number of high-performance emplacements in the high-performance defender configuration, the high-performance configuration averages slightly fewer casualties among the ships to be protected than the high-lethality configuration, although it does not for the lower configural attrition rates discussed. That occurs even though the high-lethality defender configuration averages slightly *more* casualties among the attackers than the high-performance defender configuration.

Aside from that, the most conspicuous of the two principal differences between the configural probability densities for the random number of casualties among the ships to be protected by, respectively, the high-performance defender configuration and the high-lethality defender configuration, which are displayed in Exhibit III-4, following this page, is the difference between the probabilities that all the ships survive. For the high-performance defender configuration, the probability (dotted bar) of no casualties among the ships to be protected is 0.56 . The same number of high-lethality emplacements, however, produces a probability (black bar) of no casualties among the ships of only about 0.28 . The probability of no casualties among the ships protected by the high-performance system is twice that for the high-lethality system, even though the average numbers of casualties among the ships protected by the respective systems (1.79 for the high-performance system and 1.84 for the high-lethality system) differ by less than three percent.

The other principal difference between the two configural casualty probability densities is in their respective probabilities of a relatively large number of casualties. As the exhibit shows, both configural casualty probability densities have pronounced tails, but that for the high-performance defender configuration is the greater. The probability that the random number of casualties among the ships to be protected is at least six, for instance, is almost 0.13 for the high-performance defender configuration, despite its much higher probability of no casualties, but is only 0.05 for the high-lethality defender configuration.

The difference in the probabilities of large numbers of casualties is important. In an assessment based on the average number of casualties among the ships to be protected, the high-performance system is preferred because the average number of casualties among them favors it, although to an operationally negligible degree. Moreover, having no casualties among the ships to be protected with a 0.56 probability in the high-performance defender configuration is usually likely to be strongly preferred to the 0.28 probability associated with the corresponding high-lethality defender configuration. However, unless a high probability of no casualties among those ships is so important for a high probability of mission success that the associated substantial probability (0.13) of at least six casualties is acceptable, that preference may be a mistake.

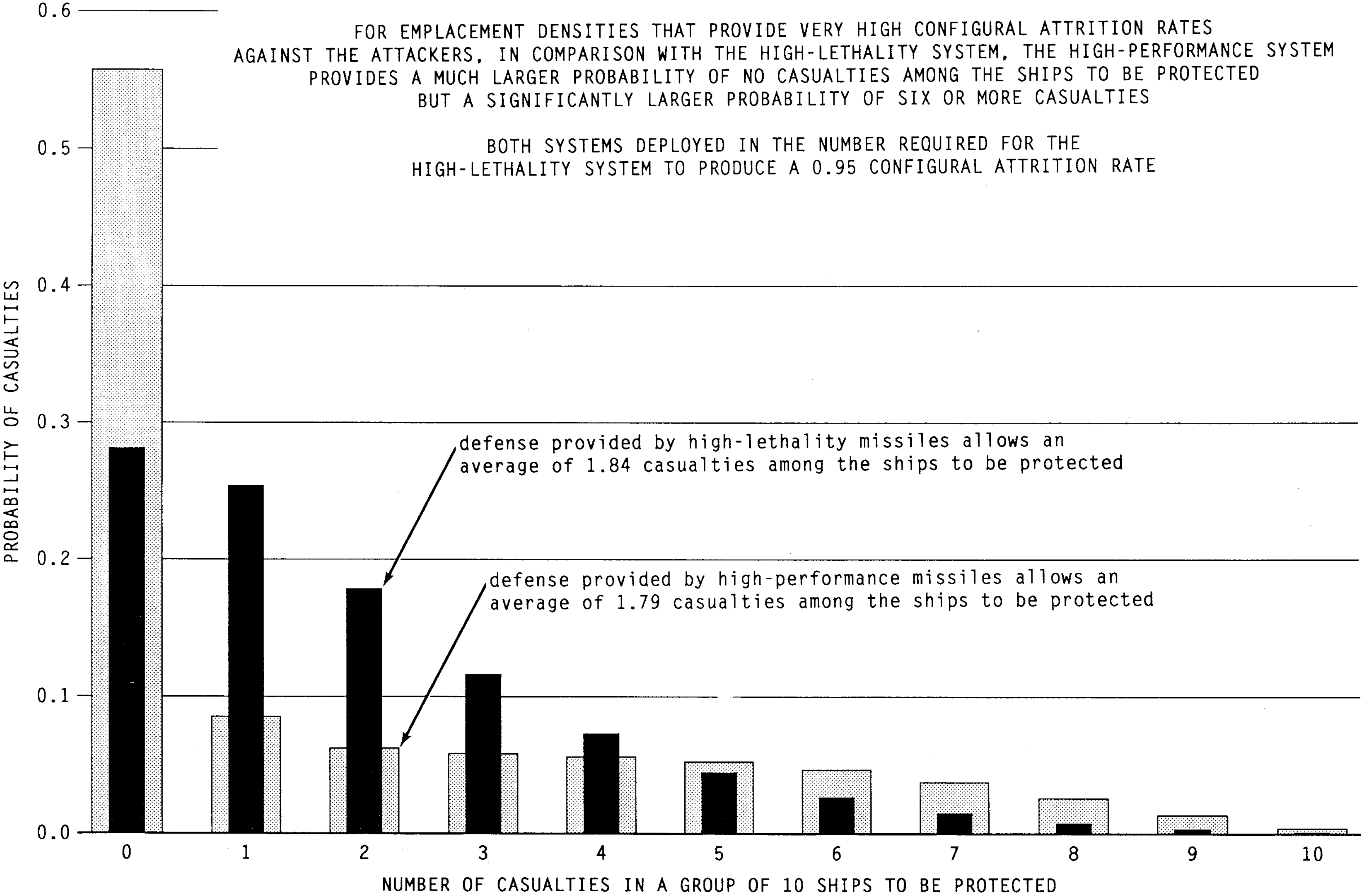
Specifically, with probability 0.44, at least one casualty occurs in a configured encounter with the high-performance defender configuration. Given that at least one casualty occurs, the average number of casualties becomes 4.04 and the probability of at least six casualties becomes 0.29 . However, the corresponding values for the high-lethality defender configuration — 2.56 and 0.07 respectively — are much smaller. Furthermore, to reduce the probability of a large number of casualties among the ships to be protected in the high-performance defender configuration to that for the high-lethality defender configuration with a 0.95 configural attrition rate, which apparently is what prudence would most often dictate, requires substantially more high-performance emplacements.

Both configural casualty probability densities differ significantly from their free-encounter counterparts, that for the high-performance configuration greatly. First, as shown by the casualty probability densities

EXHIBIT III-4

FOR EMPLACEMENT DENSITIES THAT PROVIDE VERY HIGH CONFIGURAL ATTRITION RATES AGAINST THE ATTACKERS, IN COMPARISON WITH THE HIGH-LETHALITY SYSTEM, THE HIGH-PERFORMANCE SYSTEM PROVIDES A MUCH LARGER PROBABILITY OF NO CASUALTIES AMONG THE SHIPS TO BE PROTECTED BUT A SIGNIFICANTLY LARGER PROBABILITY OF SIX OR MORE CASUALTIES

BOTH SYSTEMS DEPLOYED IN THE NUMBER REQUIRED FOR THE HIGH-LETHALITY SYSTEM TO PRODUCE A 0.95 CONFIGURAL ATTRITION RATE



defense provided by high-lethality missiles allows an average of 1.84 casualties among the ships to be protected

defense provided by high-performance missiles allows an average of 1.79 casualties among the ships to be protected displayed in Exhibit I-2 and reproduced in Exhibit III-5 following this page, in the configured encounter (dotted bars) between the attackers and the high-performance defender configuration, the probability of zero casualties among the ships to be protected is 0.56, which is four times larger than that in the corresponding free encounter (white bars), which is only approximately 0.14 . Second, in the corresponding free encounter, the casualty probability density has essentially no tail. The free-encounter probability of at least six casualties, for instance, among the ships to be protected is less than 0.004 . Accordingly, the free-encounter assessment implies that six or more casualties are essentially impossible operationally. However, as noted, the probability of at least six casualties in the configured encounter with the high-performance defender configuration is almost 0.13, which is substantial.

The configural probability density of casualties among the ships to be protected in the high-lethality defender configuration differs from its free-encounter counterpart (which is not displayed) in the same two ways, but not to such a large degree. The free-encounter probability (0.13) of zero casualties among the ships is smaller than that (0.28) for the configured encounter, a difference that exceeds a factor of two, but is substantially less than the factor-of-four difference between the corresponding probabilities for the high-performance defender configuration. The free-encounter probability of at least six casualties among the ships to be protected is, for the high-lethality defender configuration, only slightly more than 0.004 . Again, the free-encounter assessment implies that six or more casualties among the ships to be protected are essentially impossible operationally. However, as noted, in the configured encounter, that probability is 0.05 . Six or more casualties are thus unlikely operationally in the configured encounter with the high-lethality defender configuration, but not essentially impossible, as the free-encounter probability density implies.

3. Configural Theory Shows That the Free-Encounter Conceptualization of Weapons Effectiveness and the Derivative Mathematical Representations Are Inappropriate for the Mathematical Modeling of Fleet Air Defense and Ship Self-Defense Systems

Numerical examination of the configural casualty probability densities (in particular, the probability densities of the random number of casualties that the air defense emplacements produce among the attackers and the probability densities of the random number of casualties that the attackers that successfully penetrate the defended area produce among the ships to be protected, the equations for which are established in Appendix A) shows that intuition and mathematical representations based on free encounters are inapplicable to encounters that include essential configural elements of combat situations in which fleet air defense and ship self-defense systems are intended to be employed. That is illustrated in the first subsection of this section for casualty production among the attackers and in the second subsection for casualty production among the ships to be protected.

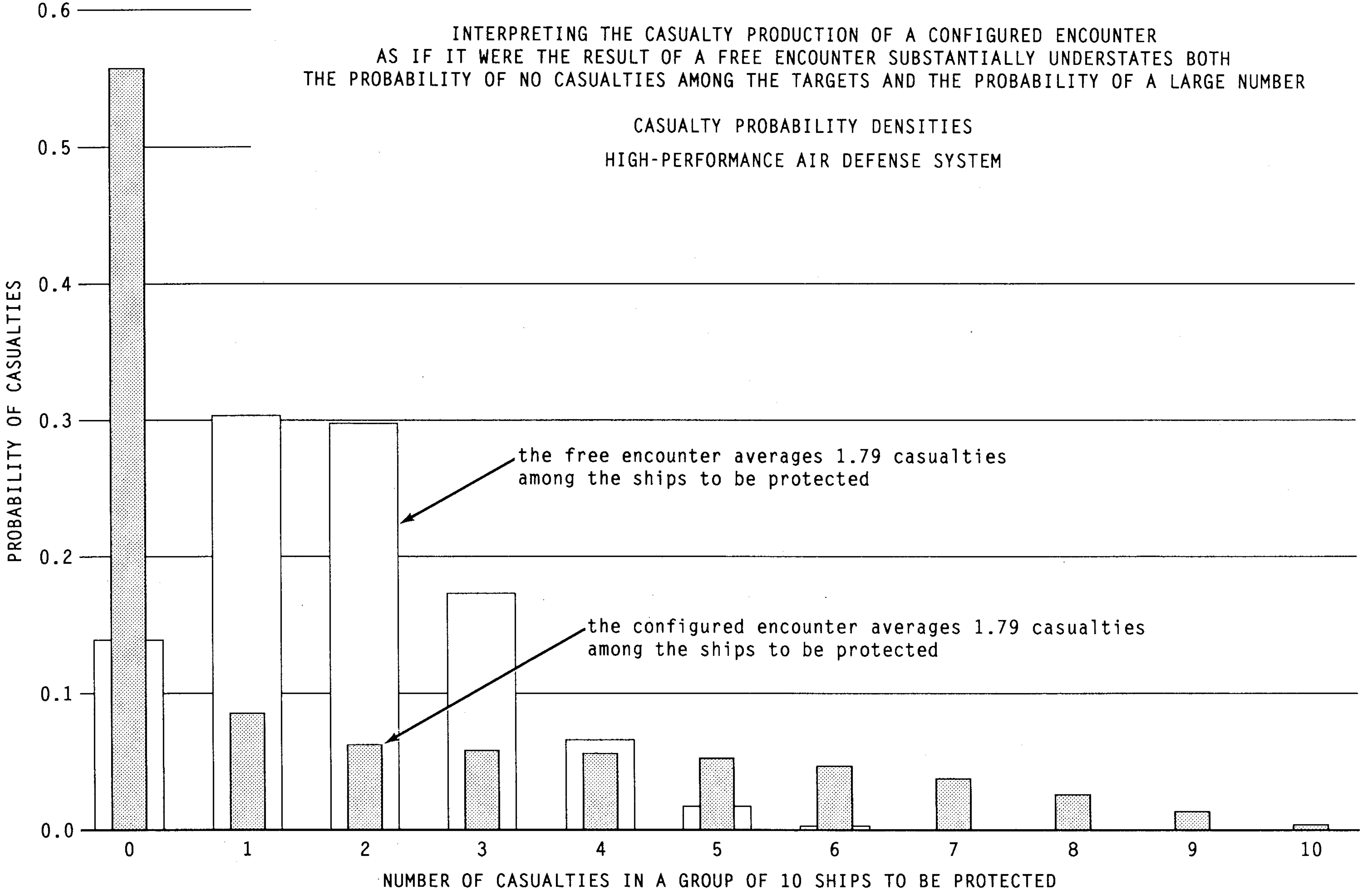
For emplacement densities that correspond to low configural attrition rates among moderate or smaller numbers of attackers in narrow attack

EXHIBIT III-5

INTERPRETING THE CASUALTY PRODUCTION OF A CONFIGURED ENCOUNTER AS IF IT WERE THE RESULT OF A FREE ENCOUNTER SUBSTANTIALLY UNDERSTATES BOTH THE PROBABILITY OF NO CASUALTIES AMONG THE TARGETS AND THE PROBABILITY OF A LARGE NUMBER

CASUALTY PROBABILITY DENSITIES

HIGH-PERFORMANCE AIR DEFENSE SYSTEM



the free encounter averages 1.79 casualties among the ships to be protected

the configured encounter averages 1.79 casualties among the ships to be protected

HORRIGAN ANALYTICS

corridors, as noted, the attacker casualty probability densities have large probabilities of zero casualties, that is, large probabilities that all attackers penetrate the defended area. Furthermore, both average casualties and the corresponding free-encounter casualty probability densities conceal those probabilities. For emplacement densities that correspond to moderate attrition rates among the attackers, the free-encounter casualty probability densities are concentrated about their means, whereas the configured-encounter probability densities are not. Small or large numbers of casualties are almost as probable as numbers close to the average numbers. For emplacement densities that correspond to high attrition rates, the configural probability that no attackers penetrate the defended area is large even for a large number of attackers, but the corresponding free-encounter probability is small. Moreover, the configural standard deviations of casualties among the attackers are much greater than the corresponding free-encounter standard deviations.

Such behavioral characteristics of casualty production among the attackers and their consequences for casualties among the ships to be protected cannot be discerned in models and simulations based on free encounters. Free encounters conceal important characteristics of the behavior of the air defense weapons that provide the area defense in both small-scale and large-scale encounters that include the key configural elements of important combat situations. In particular, the large probabilities of no casualties among the ships to be protected and the substantial probability of a large number are concealed. As a result, what could be contributed by an effective close-in weapon system that itself eliminates the substantial probability of a large number of casualties among the ships is also concealed. Thus, not only do the misstatements of the relative effectiveness of alternative air defense systems for area defense by factors of two to ten and more that are illustrated in Section II and the correlative, great understatements of numerical requirements for high-performance systems result from presuming free encounters, but also what point defenses, especially close-in weapon systems, can contribute to the survival of the ships to be protected in the defender configuration cannot be properly assessed in free encounters.

4. Configural Theory Can Provide the Conceptual and Mathematical Means to Enhance Weapons Effectiveness by Identifying the Design Trade-Offs That Contribute Most to Combat Effectiveness

In short, factors and relationships that are manifestly important for weapon design, numerical requirements estimates, effectiveness assessments, weapon systems mixes, and operational deployment in appropriate quantities, if even discerned, are misstated in assessments based on the customary conceptualization of weapons effectiveness and derivative mathematical models. As a consequence, free-encounter effectiveness assessments do not focus our technological resources on those operational characteristics of weapon systems that contribute most to combat effectiveness or that most need improvement. In contrast, as this report illustrates, configural theory can provide the means better to discern and to identify those characteristics that contribute most to combat effectiveness as well as those that most need improvement and to identify and to assess the design trade-offs that are most efficacious for overall mission success.

APPENDIX A

CONFIGURAL CASUALTY PROBABILITY DENSITIES USED IN THE PRELIMINARY EXAMINATION

The principal objective of this appendix is to establish two configural probability densities used in this report:

- the configural probability density of the random number of casualties among the attackers that attempt to penetrate the defended area around the targets to be protected and
- the configural probability density of the random number of casualties among those targets that are produced by the attackers that successfully penetrate the defended area.

That is accomplished in three main steps which correspond to the three main sections of this appendix:

(1) define the attacker-defender configuration for the simplified, idealized combat situations considered in this report;

(2) identify and define the variables that quantify the configural interaction between an attacker and a battery of air defense emplacements and derive the associated probability density of the random number of attackers that survive a sequence of probabilistically independent, identical such batteries (see equation (A-18.1) on page A-18), which in turn determines the configural probability density of the random number of casualties among the attackers (equation (A-18.2) on page A-18); and

(3) identify and define the variables that quantify the configural interaction between an attacker that successfully penetrates the defended area and the targets to be protected and derive the associated probability density of the random number of casualties produced among those targets by the random number of such attackers (equation (A-21.1) on page A-21).

Only those elements of the attacker-defender configuration that are needed to establish the key equations are discussed in this appendix. A more complete discussion is in Section II of this report.

1. The Attacker-Defender Configuration and the Configured Encounter

As discussed in Section II, in concept the defender configuration consists of ships that provide area air defense with identical air defense weapons surrounding ships to be protected, which are the targets to be protected. In the research discussed in this report, which is a very limited first step, the region of the defender configuration in which the air defense weapons are distributed is represented as a rectangular region of unlimited length and finite width rather than an annulus. The targets are behind the defended area and in the immediate vicinity of the attack corridor. In effect, for situations such as those examined in this report in which the range of the air defense weapons is short, the part of the annular region in the vicinity of the attack corridor is approximated by a rectangle. The air defense weapons are postulated to be batteries that comprise identical numbers of identical multiple-tube emplacements. Relative to the range of their weapons, all the emplacements associated with a battery are postulated to be co-located. The batteries themselves are postulated to be deployed uniformly randomly and independently in the defended area.

Each battery has a common magazine from which the tubes or rails that constitute the launcher at each of its emplacements can be reloaded with missiles. The missiles are command-guided, and each emplacement can control at most one missile at a time. The battery performs detection and acquisition for its emplacements. In this initial research, the attackers in the attack corridor are postulated to be separated sufficiently to ensure that there is never more than one attacker at a time in the encounter region of a battery and that the individual emplacements can reload between assigned attackers.

The attacker configuration consists of antiship missiles, all of which maintain the same, constant, specified altitude, on linear trajectories. It attempts to penetrate the defender configuration through an attack corridor that is centered around a random attack axis that intersects the inner part of the defender configuration within which the targets are deployed. In concept, the linear trajectories of the attackers are parallel to the attack corridor and uniformly randomly and independently distributed across it. In the initial research this report discusses, the attack corridor is *semiconfigured*; that is, the random separations between the trajectory of an attacker and the batteries in the interaction corridor are postulated to be probabilistically independent.

Also in this initial research, the attackers do not attack the area air defense batteries but remain in the attack corridor until they reach the vicinity of the target subconfiguration, unless they are destroyed by fire from the air defense weapons. Destroyed attackers are instantly eliminated and draw no additional fire. Attackers that successfully penetrate the area air defense may or may not acquire a target. An attacker that acquires a target attacks it, but it may or may not produce a casualty. A target that is successfully attacked by at least one attacker is a casualty, but it does not sink in the course of the engagement. Consequently, in this initial research, a target may be attacked successively by any number of attackers.

a. The Characterizing Parameters

The hypothetical, highly simplified air defense weapon systems examined in the initial research are characterized by the following parameters:

β	battery availability
α	battery acquisition probability (the conditional probability that an attacker that encounters an available battery is detected and acquired)
a	emplacement availability
ρ	weapon reliability (in a multitube emplacement, the probability of a successful launch and proper flight)
Δ	weapon lethality (the conditional probability of kill given that the missile is successfully launched while the attacker is in the encounter region of the battery)
R	weapon range
L	number of tubes or rails in the launcher at each emplacement in the battery
G	number of multitube emplacements in the battery
M	number of rounds initially at the battery

The N targets to be protected are deployed in the inner part of the defender configuration.

The n attackers of the attacker configuration use an attack corridor of half-width W . They are characterized by the following parameters:

φ	the probability that an attacker that penetrates the defended area detects and acquires a target among the targets to be protected
γ	the conditional probability that an attacker that penetrates the defended area attacks and damages a target it detects and acquires

2. The Configural Probability Density of the Random Number of Casualties among the Attackers That Attempt to Penetrate the Area Air Defense in the Defender Configuration

The configural probability density that is used in the initial research for the random number of attackers that successfully penetrate the area defense of the defender configuration is established in four steps that correspond to the four subsections that follow:

(a) define the attacker-battery interaction in a configured encounter;

(b) derive the joint probability density of the random number of missiles used and casualties produced in a configured encounter between a single attacker and a single available battery in the interaction corridor;

(c) using the result in (b), develop the joint probability density for the random number of missiles used and casualties produced in a configured encounter between a sequence of n attackers and a single available battery in the interaction corridor; and

(d) using the probability density derived in (c), which determines the random number of attackers that survive a single available battery in the interaction corridor, establish the unconditional probability density for the random number of attackers that successfully penetrate the area defense of the defender configuration for a semiconfigured interaction corridor with a random number of available batteries.

The resulting equation (A-18.1), which is the first equation on page A-18, of course, also determines the probability density for the random number of casualties among the attackers, which is given by equation (A-18.2).

For the combat situations discussed in the body of the report, the random number of available batteries in the interaction corridor has a Poisson probability density. Equation (A-19.1) particularizes the probability density of the random number of attackers that successfully penetrate the defended area for such situations. Substituting that equation into equation (A-18.2) yields the probability density of the random number of casualties among the attackers in those situations.

a. How the Attacker-Battery Interaction Is Represented

Because the extent of the attacker configuration that is within the area air defense part of the defender configuration is that of the attack corridor itself, only defender air defense batteries that are in the attack corridor or are sufficiently close to it to have a positive probability of being encountered by an attacker within it can interact with an attacker. The smallest region containing an attack corridor and outside of which an air defense battery in the defender configuration has a zero probability of being encountered by an attacker in the attack corridor is the *interaction corridor*. An attack corridor, an encounter region for an air defense battery, and the associated interaction corridor are illustrated in Exhibit A-1, following this page. The cross-corridor position of a battery that is in the interaction corridor is specified by its location on the *battery reference line*, a line that is in the ground plane of the interaction corridor, passes through the center point of the battery, and is perpendicular to the attack axis. A configured encounter between an attacker in the attack corridor and an air defense battery in the interaction corridor is illustrated in Exhibit A-2, following Exhibit A-1.

As noted, detection and acquisition of attackers is performed by a battery for all its emplacements. Designate by Z the location on the battery reference line of a particular but unspecified battery in the interaction corridor. Designate by X the location on that reference line of the intersection with the projection of an attacker trajectory (at a specified altitude) in the attack corridor onto the ground plane. Define the function $A(\cdot,\cdot)$ to be the conditional probability that a battery given to be available detects and acquires an attacker given to be on a particular trajectory (at the specified altitude) as a function of the cross-corridor

EXHIBIT A-1

THE INTERACTION CORRIDOR OF AN ATTACK CORRIDOR IS THE LARGEST REGION THROUGHOUT WHICH A BATTERY OF A PARTICULAR AIR DEFENSE SYSTEM HAS A POSITIVE PROBABILITY OF BEING ENCOUNTERED BY AN ATTACKER IN THE ATTACK CORRIDOR

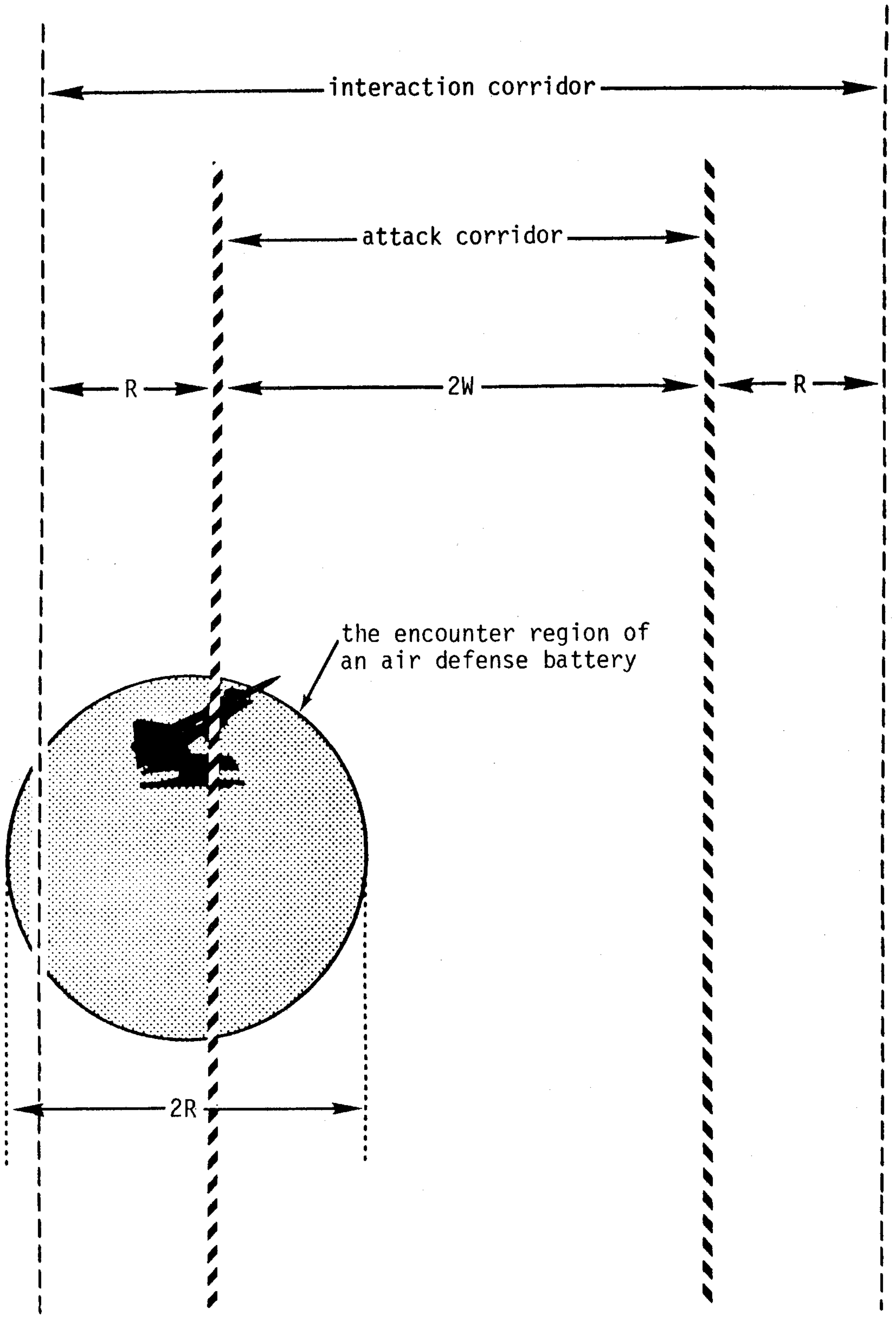
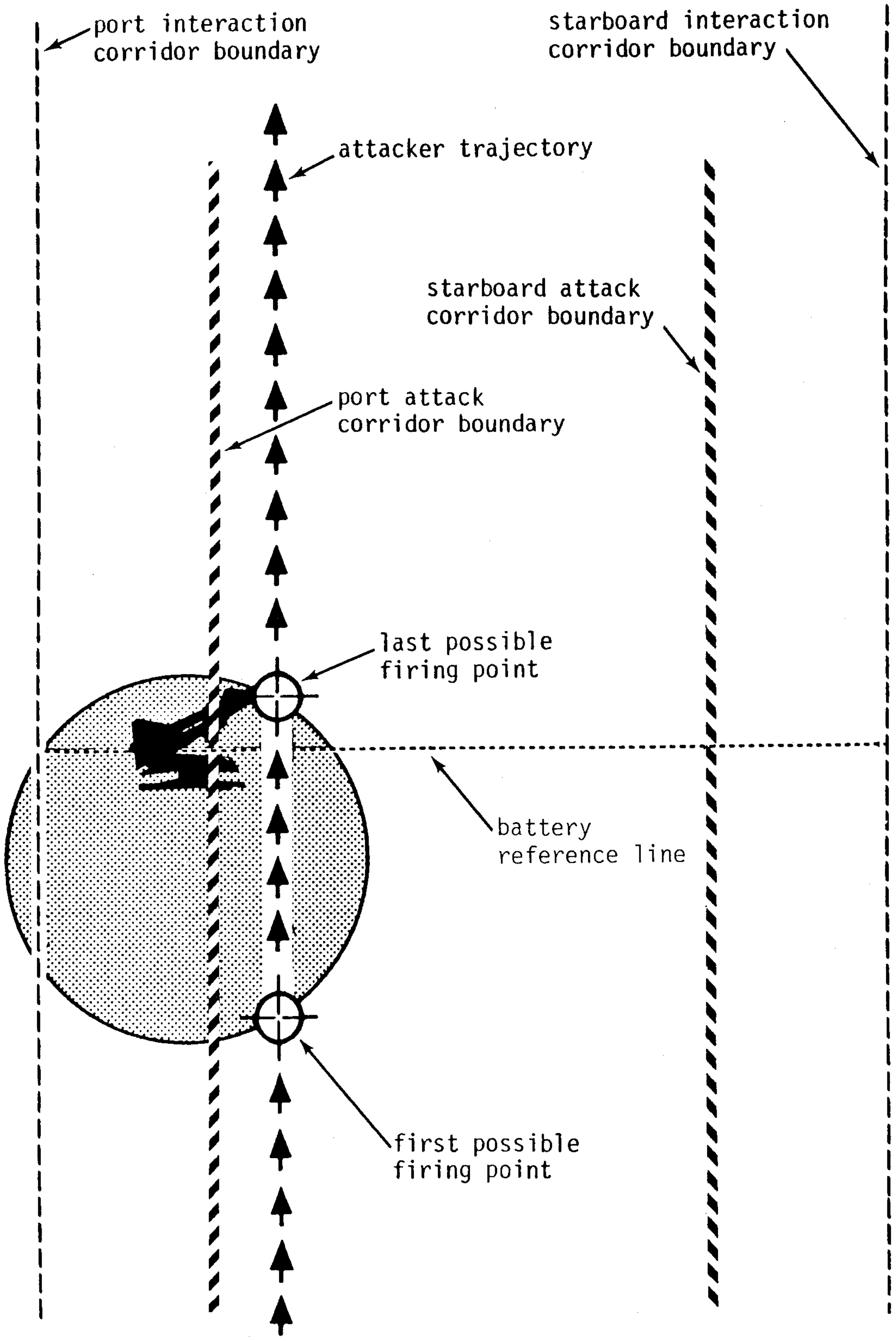


EXHIBIT A-2

A CONFIGURED ENCOUNTER BETWEEN AN ATTACKER ON A LINEAR TRAJECTORY IN THE ATTACK CORRIDOR AND AN AIR DEFENSE BATTERY IN THE INTERACTION CORRIDOR



location of the attacker relative to the battery and of the attacker bearing relative to the battery at or before which the detection and acquisition is to occur. Accordingly, the conditional probability that an available battery at the cross-corridor location Z detects and acquires an attacker on a trajectory (at the specified altitude), the projection of which intersects the battery reference line at X, before the attacker bearing exceeds θ is $A(X-Z,\theta)$ for appropriate values of those variables. For the simplified situations postulated for the initial research, $A(\cdot,\cdot)$ is a cookie cutter that has the values α for an attacker that enters the battery encounter region and 0 for an attacker that does not.

Given that a battery in the interaction corridor detects and acquires an attacker on a trajectory (at the specified altitude) in the attack corridor and has an available emplacement with at least one round, the attacker is assigned to such an available emplacement, and that emplacement begins to fire upon the attacker. Given a successful launch, the function $D(\cdot,\cdot)$ is the conditional probability that the round launched by the emplacement destroys the attacker as a function of the cross-corridor location of the attacker relative to the battery and its bearing relative to the battery at the instant of firing. In particular, the conditional probability that an available battery at cross-corridor location Z with an available emplacement with at least one round destroys an attacker on a trajectory (at the specified altitude) with bearing θ located at X with a single missile given successfully to be launched at the attacker is $D(X-Z,\theta)$. For the simplified situations postulated for the initial research, like $A(\cdot,\cdot)$, $D(\cdot,\cdot)$ is a cookie cutter that has the values Δ for an attacker that enters the battery encounter region and 0 for an attacker that does not.

Configurally, an engagement between an attacker and a battery takes place only while the attacker is so located that the battery has a positive probability of destroying it with a round fired at the corresponding instant. The targeting information available to defender gunners and their command and control is perfect: No more than one emplacement at a time engages an attacker. As noted, each available emplacement in a battery can launch and control at most one missile at a time, but it can do so simultaneously with the other available emplacements of the battery.

The number of rounds that can be fired at a particular attacker by a particular battery the attacker encounters cannot exceed the number of rounds at that battery and, for a specified fire discipline (which is shoot-look-shoot in the initial research), cannot exceed what the speed, range, and dynamics of the interceptor missile allow for the given trajectory of the attacker and its given bearing relative to the battery at the instant of detection and acquisition. The maximum number allowed by the dynamics of the missile and the other particulars of the encounter is a nonnegative, integer-valued function $I(\cdot,\cdot)$ of the cross-corridor location of the attacker (at a specified altitude) relative to the battery and of the bearing of the attacker relative to the battery at the instant of detection and acquisition. For a battery given to be at cross-corridor location Z and an attacker given to be on a trajectory (at the specified altitude) with the cross-corridor location X that is detected and acquired at the bearing θ, the maximum number of rounds that can be fired is $I(X-Z,\theta)$.

Given a successful firing from a particular multitube emplacement, the emplacement, by definition, engages no other attackers until its assigned attacker either is destroyed or leaves its encounter region. Determining whether a missile that reaches the vicinity of the attacker destroys the attacker is postulated to take a fixed time. If a missile does not destroy the attacker after reaching the vicinity of the attacker and the attacker is still in the encounter region at the instant that determination is made, the emplacement is postulated instantaneously to fire the missile in the next tube in the firing sequence at that emplacement. Missiles that are faulty and fail at launch or immediately thereafter, termed *launch aborts*, are distinguished from missiles that are faulty but reach the vicinity of the target before failing. Launch aborts are postulated instantaneously to result in firing the missile in the next tube in the firing sequence at that emplacement until a successful launch occurs or launches have been attempted from all loaded tubes at that emplacement. Should launches be attempted from all tubes before a successful launch occurs, an unengaged emplacement in the same battery, if available, is postulated instantaneously to commence firing. That process continues until one of the following happens:

- a successful launch occurs;
- all missiles that had been loaded in available emplacements at the start of the encounter are used;
- the attacker leaves the encounter region; or
- the attacker is destroyed.

As noted, destroyed attackers are instantly eliminated and draw no additional fire.

If a missile that is successfully launched is the last missile in the launcher at that emplacement or the last missile aborts at launch, the emplacement immediately initiates a reload cycle. In a reload cycle, any nonfires are postulated to be extracted from their tubes and missiles from the magazine are postulated to be loaded until all tubes are reloaded or the magazine becomes empty. A reload cycle is postulated to take a fixed time, and during a reload cycle an emplacement cannot fire upon attackers. In the initial research, the duration between encounters of a battery by successive attackers is postulated to be sufficient for a reload cycle.

b. Casualty Production in a Configured Encounter between a Single Attacker and a Battery of Multitube Emplacements

Consider an available battery in the interaction corridor with a random number G^* of available emplacements that is given to be g, with a random number M^* of rounds that is given to be m, and at a uniformly random cross-corridor location Z^* that is given to be Z in a configured encounter with a particular but unspecified attacker (at a specified altitude) on a trajectory in the attack corridor that crosses the battery reference line at a uniformly random cross-corridor location X^* that is given to be X . Such a configured encounter is described by two functions of the number of available emplacements in the battery, the number of missiles at the battery, the cross-corridor location of the battery, and the cross-corridor location of the trajectory of the attacker:

- $S_i(g,m,Z,X)$, the probability that the attacker survives and the battery uses $i = 0, 1, 2, \ldots, m$ rounds against it; and
- $D_i(g,m,Z,X)$, the probability that the attacker is destroyed and the battery uses i rounds against it.

Two related functions that are unconditionalized on the random cross-corridor location X of the trajectory of the attacker are the probabilities $S_i(g,m,Z)$ and $D_i(g,m,Z)$ that i rounds are used against a particular but unspecified attacker in the sequence of attackers in the attack corridor and the attacker respectively survives or is destroyed. The relationship between the two pairs of functions for an attack corridor of half-width W is, of course,

$$S_i(g,m,Z) = \frac{1}{2W}\int_{-W}^{+W} S_i(g,m,Z,X)\,dX \qquad \text{(A-7.1)}$$

$$D_i(g,m,Z) = \frac{1}{2W}\int_{-W}^{+W} D_i(g,m,Z,X)\,dX \qquad \text{(A-7.2)}$$

for $0 \le i \le m$ and appropriate values of the other variables. Those two functions determine the configural probability density for the random number of attackers that survive the specified battery.

What happens in a configured encounter between a single attacker and the battery is defined by the following:

- the random number R^* of rounds that are used against the attacker by the battery, which takes values between 0 and the given value m of the random number M^* of rounds at the battery, and

- the random number $D^{\star}$ of attackers destroyed, which, of course, takes only the values 0 and 1.

Accordingly, the outcome of the encounter is defined by the joint conditional probability density of $R^{\star}$ and $D^{\star}$,

$$\Pr\{R^{\star}=i, D^{\star}=k \mid G^{\star}=g, M^{\star}=m, Z^{\star}=Z, X^{\star}=X\} \qquad \text{(A-8.1)}$$

for $0 \le i \le m$ and $0 \le k \le 1$ and all appropriate values of the other variables. The probabilities $S_i(\cdot,\cdot,\cdot,\cdot)$ and $D_i(\cdot,\cdot,\cdot,\cdot)$ that the attacker respectively survives or is destroyed and the battery uses i rounds against it are defined by that joint conditional probability density:

$$S_i(g,m,Z,X) = \Pr\{R^{\star}=i, D^{\star}=0 \mid G^{\star}=g, M^{\star}=m, Z^{\star}=Z, X^{\star}=X\}$$

$$D_i(g,m,Z,X) = \Pr\{R^{\star}=i, D^{\star}=1 \mid G^{\star}=g, M^{\star}=m, Z^{\star}=Z, X^{\star}=X\} \ .$$

For notational convenience, the joint conditional probability density of $R^{\star}$ and $D^{\star}$ is determined by establishing expressions for those functions.

As noted, how many rounds out of the m rounds at a battery that the battery successfully launches against the attacker cannot exceed the maximum number of interceptions that are permitted by the following:

- the fire discipline (shoot-look-shoot in the initial research);
- the dynamics of the interceptor missile;
- the speed, altitude, and bearing (relative to the battery) of the attacker at the instant of detection and acquisition; and
- the cross-corridor location of the attacker.

For the given situation, that maximum number of interceptions is $I(X-Z,\theta)$. The number of rounds the battery successfully launches is also limited by the maximum number of tubes that can be successively fired in an encounter with a single attacker, which is the product Lg of the number L of tubes at each emplacement and the number g of available emplacements (the tubes at unavailable emplacements are postulated to be unloaded), and the number m of rounds at the battery. Consequently, the maximum number of missiles that can be successfully launched at the attacker, as given, by the battery, as given, is

$$\min(I(X-Z,\theta),Lg,m) \ .$$

Designate the maximum number $\min(Lg,m)$ of loaded tubes that can be successively fired at the attacker by U . Thus, U is the maximum number of missiles that can be used against the attacker. Accordingly, the maximum number of successful launches that can be made against the attacker is

$$\min(I(X-Z,\theta),U) \ .$$

A situation in which $I(X-Z,\theta)$ is smaller than U differs significantly from a situation in which it is not, for a missile that aborts at launch can be replaced in the former situation without necessarily decreasing the number of successful launches that can be made; otherwise, a launch abort reduces the number of successful launches that can be made. There are thus two basic situations to be considered: the situation in which there are at most as many loaded tubes as the maximum permitted number of interceptions,

$$U \le I(X-Z,\theta) \ ;$$

and the situation in which there are more loaded tubes than the maximum permitted number of interceptions,

$$U > I(X-Z,\theta) \ .$$

In the following discussion, the dependency of $I(\cdot,\cdot)$ on X, Z, and θ is suppressed for notational convenience; in particular, the value of $I(X-Z,\theta)$ is designated I . Also, for the simplified air defense systems postulated, the acquisition and damage functions are cookie cutters and do not depend on the attacker bearing relative to the battery at the instant of detection and the instant of firing, respectively; consequently, in $A(\cdot,\cdot)$ and $D(\cdot,\cdot)$, the second argument, attacker bearing, is suppressed.

The simplest situation is that in which the number U of loaded tubes at most equals the maximum permitted number I of interceptions, that is, $U \le I$, for a launch abort in that situation is just one fewer round fired at the attacker. In particular, g or m and consequently U may be zero. For $U = 0$,

$$S_i(g,m,Z,X) = \begin{cases} 1 , & i=0 \\ 0 , & 0<i\leq M \end{cases} \tag{A-10.1}$$

$$D_i(g,m,Z,X) = 0 , \quad 0\leq i\leq M \tag{A-10.2}$$

for the stated values of i and all appropriate values of the other variables.

For the situation in which U is positive but at most I, the possibilities are readily enumerated and appropriate expressions established by inspection. To determine the values of $S_i(g,m,Z,X)$ for $0\leq i\leq M$, it is sufficient to note that, provided that the attacker is detected and acquired, all U rounds are always used because $U\leq I$ and the attacker survives. Thus, for $0<U\leq I$,

$$S_i(g,m,Z,X) = \begin{cases} 1 - A(X-Z) , & i=0 \\ 0 , & 0<i<U \\ A(X-Z)[1-\rho D(X-Z)]^U , & i = U \\ 0 , & U<i\leq M \end{cases} \tag{A-10.3}$$

To determine the values of $D_i(g,m,Z,X)$ for $0\leq i\leq M$ in the same situation, it is sufficient to note that the random number of rounds required to destroy the attacker in such an encounter has a geometric probability density. Consequently, for $0<U\leq I$,

$$D_i(g,m,Z,X) = \begin{cases} 0 , & i=0 \\ \rho A(X-Z)D(X-Z)[1-\rho D(X-Z)]^{i-1} , & 1\leq i\leq U \\ 0 , & U<i\leq M \end{cases} \tag{A-10.4}$$

The probability of detection and acquisition appears as a multiplier of the geometric term, for a detection and acquisition is necessary for any missiles to be launched.

In situations in which the maximum number U of missiles that can be used against the attacker exceeds the maximum number of interceptions permitted, that is, $U > I$, instantaneous compensatory launches by the battery may offset some launch aborts and thereby the emplacement may make the maximum permitted number of interceptions despite the launch aborts. How many missiles are used to produce I successful launches is, of course, a random variable. The maximum number of interceptions permitted may be obtained, despite a number of launch aborts, with a probability that increases with additional loaded tubes. Specifically, define $J^*(I)$ to be the random number of missiles used in order to launch I missiles successfully. Then, provided the launch-abort events are probabilistically independent, $J^*(I)$ has the Pascal probability density

$$\Pr\{J^*(I)=j\} = \binom{j-1}{I-1} \rho^I (1-\rho)^{j-I} ,$$

in which ρ is the probability of a successful launch.

To establish the values of $S_i(g,m,Z,X)$ in such situations, five sets of values for the number i of rounds used against the attackers require consideration. First, consider the event that no missiles are used. For that event, there is only one value of i, and it is zero ($i = 0$). That event occurs only if the attacker is not detected and acquired, for U is positive. Hence, the probability of that event is the probability of not detecting and acquiring the attacker,

$$1 - A(X-Z) .$$

Second, consider the events in which at least one and fewer than I missiles are used; that is, $1 \le i < I$. Such events have probability zero because fewer than I missiles are used, the attacker is not destroyed, and more loaded tubes remain.

Third, consider the events in which at least I missiles and fewer than U missiles are used; that is, $I \le i < U$. As at least I missiles are used, launches stop while loaded tubes remain, and the attacker is not destroyed, exactly I successful launches occur. The probability of that event is the probability that the last of I interceptions occurs with the i-th missile used, and none of the I interceptions destroy the attacker; that is,

$$A(X-Z)[1-D(X-Z)]^I \Pr\{J^*(I)=i\} .$$

Replacing $\Pr\{J^*(I)=i\}$ in that expression by its explicit formula yields

$$A(X-Z)\binom{i-1}{I-1}[\rho\overline{1-D(X-Z)}]^I(1-\rho)^{i-I} \ ,$$

for $i \geq I$ and zero otherwise.

Fourth, consider the event that all U missiles are used ($i = U$). As all U missiles are used, fewer than I successful launches occur among the first U-1 launches, none of which destroy the attacker, and the last missile used also does not destroy the attacker. The probability of that event is

$$A(X-Z)\left\{\sum_{0}^{I-1}{}_{j}\binom{U-1}{j}[\rho\overline{1-D(X-Z)}]^j(1-\rho)^{U-1-j}\right\}[1-\rho D(X-Z)] \ .$$

The first factor in that expression is the probability that the attacker is detected and acquired. The second is the probability that fewer than I successful launches occur among the first U-1 missiles used and the attacker is not destroyed by any that do occur. The third is the probability that the last missile also does not destroy the attacker.

The probability of the fifth event, that more rounds are used than are loaded, is, of course, zero.

Therefore, because for $U > I$ those five events partition the range of i, $S_i(g,m,Z,X)$ is defined by the following:

$$S_i(g,m,Z,X) = \begin{cases} 1 - A(X-Z) \ , & i = 0 \\ 0 \ , & 1 \leq i < I \\ A(X-Z)\binom{i-1}{I-1}[\rho\overline{1-D(X-Z)}]^I(1-\rho)^{i-I} \ , & I \leq i < U \\ A(X-Z)\left\{\sum_{0}^{I-1}{}_{j}\binom{U-1}{j}[\rho\overline{1-D(X-Z)}]^j(1-\rho)^{U-1-j}\right\}[1-\rho D(X-Z)] \ , & i = U \\ 0 \ , & U < i \leq M \end{cases}$$

(A-12.1)

for situations in which the number U of loaded tubes exceeds the maximum permitted number I of interceptions.

To determine $D_i(g,m,Z,X)$ for those situations, it is necessary to note that, in the events that correspond to each value of i, the i-th missile destroys the attacker and fewer than I successful launches occur among the first i-1 missiles used (and, of course, none that do occur destroy the attacker). Consequently,

$$D_i(g,m,Z,X) = \begin{cases} 0\ , & i = 0 \\ \rho A(X-Z)D(X-Z) \sum\limits_{0}^{I-1}{}_j \binom{i-1}{j} [\rho\overline{1-D(X-Z)}]^j (1-\rho)^{i-1-j}\ , & 1 \le i \le U \\ 0\ , & U < i \le M \end{cases} \qquad \text{(A-13.1)}$$

for situations in which the number of loaded tubes exceeds the maximum permitted number of interceptions.

The expressions thus established for $S_i(g,m,Z,X)$ in equations (A-10.1), (A-10.3), and (A-12.1) and for $D_i(g,m,Z,X)$ in equations (A-10.2), (A-10.4), and (A-13.1) for all values of i and U define them and, accordingly, the corresponding joint conditional probability density for $R^\star$ and $D^\star$ as given in equation (A-8.1). Similarly, the functions $S_i(g,m,Z)$ and $D_i(g,m,Z)$ as given in equations (A-7.1) and (A-7.2) for the attacker with a random location on the battery reference line and the corresponding joint conditional probability density for $R^\star$ and $D^\star$ are defined.

c. Casualty Production in a Configured Encounter between a Sequence of Attackers and a Multitube Battery

It is straightforward to determine the joint conditional probability $\Pr\{R_n^\star=i, D_n^\star=k \mid G^\star=g, Z^\star=Z\}$ that, in a configured encounter between a particular available but unspecified battery in the interaction corridor, which initially has M missiles, and a sequence of n attackers in the attack corridor, the random number $R_n^\star$ of rounds used equals i and the random number $D_n^\star$ of attackers destroyed as a result equals k, given that the battery has g available emplacements and is located at Z in the interaction corridor.

A simple recursion determines the joint conditional probability density of R^*_{n+1} and D^*_{n+1} from that of R^*_n and D^*_n and from the joint conditional probability density of R^* and D^* previously established.

Define $B_{i,k,n}(g,Z)$ to be the joint conditional probability density for R^*_n and D^*_n given that the random number G^* of available emplacements at the battery equals g and the random cross-corridor location Z^* of the battery equals Z; that is,

$$B_{i,k,n}(g,Z) = Pr\{R^*_n=i, D^*_n=k | G^*=g, Z^*=Z\}$$

for $0 \le i \le M$, $0 \le k \le n$, and the appropriate values of the other variables. For the number n of attackers equal to 0, no missiles are fired, no casualties result, and that probability is zero unless $i=k=0$, in which case it is one. For the number n of attackers equal to 1, the right member of that equation is simply the conditional probability that the random number of rounds used against a particular but unspecified attacker by a particular available but unspecified battery with M missiles equals i and, for k equal to 0 and 1, the attacker respectively survives or is destroyed.

Because R^*_1 and D^*_1 apply only to an encounter with a single attacker — in this case, the first attacker in the sequence — they are an instance of R^* and D^* for appropriate values for their defining parameters. As a result, after they are expressed as R^* and D^* as established in the preceding subsection, it follows that

$$B_{i,0,1}(g,Z) = Pr\{R^*=i, D^*=0 | G^*=g, M^*=M, Z^*=Z\} = S_i(g,M,Z)$$

$$B_{i,1,1}(g,Z) = Pr\{R^*=i, D^*=1 | G^*=g, M^*=M, Z^*=Z\} = D_i(g,M,Z)$$

for $0 \le i \le M$ and appropriate values of the other variables. The probability $B_{i,0,n}(g,Z)$ of no casualties among the first n attackers follows directly by recursion. By definition,

$$B_{i,0,n+1}(g,Z) = Pr\{R^*_{n+1}=i, D^*_{n+1}=0 | G^*=g, Z^*=Z\}$$

for $0 \le i \le M$ and $0 \le n$. Because R^*_{n+1} equals i if and only if R^*_n equals j for $0 \le j \le i \le M$ and i-j rounds are used against the next attacker, the possible values of R^*_n define a partition of the event $\{R^*_{n+1}=i, D^*_{n+1}=0\}$ into the sequence of disjoint events $\{R^*_{n+1}=i-j, D^*_{n+1}=0, R^*_n=j, D^*_n=0\}$, which is termwise equal to the sequence $\{R^*=i-j, D^*=0, R_n^*=j, D_n^*=0\}$ for $0 \le j \le i$. Consequently, after suppressing the dependency on $\{G^*=g, Z^*=Z\}$ for notational convenience, the right member of the previous equation can be expressed as the sum over the disjoint events that constitute the partition:

$$B_{i,0,n+1}(g,Z) = \Pr\{R_{n+1}^*=i, D_{n+1}^*=0\}$$

$$= \sum_{0}^{i}{}_{j} \Pr\{R^*=i-j, D^*=0, R_n^*=j, D_n^*=0\}$$

$$= \sum_{0}^{i}{}_{j} \Pr\{R^*=i-j, D^*=0 \mid R_n^*=j, D_n^*=0\} \Pr\{R_n^*=j, D_n^*=0\} .$$

Therefore, because the pair of random variables R^* and D^* is conditionally independent of the pair R_n^* and D_n^*, by substitution it follows from the definitions of $B_{i,k,n}(\cdot,\cdot)$ and $S_i(\cdot,\cdot,\cdot)$ that

$$B_{i,0,n+1}(g,Z) = \sum_{0}^{i}{}_{j} S_{i-j}(g, M-j, Z) B_{j,0,n}(g,Z)$$

for $0 \le n$ and $0 \le i \le M$. Thus, a recursion that determines $B_{i,0,n}(g,Z)$ from $S_i(\cdot,\cdot,\cdot)$ for $0 \le i \le M$ and $1 \le n$ is established.

Similarly, because D^* takes only the values 0 and 1, the general term in the conditional probability density $B_{i,k,n}(\cdot,\cdot)$ is given by

$$B_{i,k+1,n+1}(g,Z) = \sum_{0}^{i}{}_{j} [\Pr\{R^*=i-j, D^*=0, R_n^*=j, D_n^*=k+1\} + \Pr\{R^*=i-j, D^*=1, R_n^*=j, D_n^*=k\}] .$$

Proceeding as before and using the definition of $D_i(\cdot,\cdot,\cdot)$ yields

$$B_{i,k+1,n+1}(g,Z) = \sum_{0}^{i}{}_{j} [S_{i-j}(g, M-j, Z) B_{j,k+1,n}(g,Z) + D_{i-j}(g, M-j, Z) B_{j,k,n}(g,Z)]$$

for $0 \le i \le M$, $0 \le k \le n$, and appropriate values of the other variables. Thus, the joint conditional probability density $B_{i,k,n}(\cdot,\cdot)$ is determined from the functions $S_i(\cdot,\cdot,\cdot)$ and $D_i(\cdot,\cdot,\cdot)$ by a simple recursion.

The conditional probability density $\Pr\{D_n^\star=k|G^\star=g,Z^\star=Z\}$ for the random number $D_n^\star$ of attackers destroyed in a sequence of n attackers by an available battery in the interaction corridor that is given to have g available emplacements and the cross-corridor location Z consequently is

$$\Pr\{D_n^\star=k|G^\star=g,Z^\star=Z\} = \sum_{0\,i}^{M} B_{i,k,n}(g,Z)$$

for $0 \le k \le n$ and appropriate values of the other variables. In this initial research, the cross-corridor location of the battery is postulated to be uniformly random. Specifically, $Z^\star$ is uniformly random in the interval $[-R-W,R+W]$ in which R is the weapon range and W is the half-width of the attack corridor. Accordingly, because the random number $G^\star$ of available emplacements is postulated to have a binomial probability density with success probability a, unconditionalizing both members of the above equation with respect to $G^\star$ and $Z^\star$ yields

$$\Pr\{D_n^\star=k\} = \frac{1}{2(R+W)} \int_{-R-W}^{R+W} \left\{ \sum_{0\,g}^{G} \binom{G}{g} \left[\sum_{0\,i}^{M} B_{i,k,n}(g,Z) \right] a^g(1-a)^{G-g} \right\} dZ \ , \qquad \text{(A-16.1)}$$

the probability density for the random number $D_n^\star$ of attackers in the sequence that are destroyed in a configured encounter with an available battery in the interaction corridor.

d. The Configural Probability Density of the Random Number of Casualties among the Attackers That Attempt to Penetrate the Defender Area Air Defense in a Semiconfigured Attack Corridor That Interacts with a Random Number of Multitube Batteries

Because the interaction corridor is postulated to be semiconfigured, the probability density of the random number $S_n^\star(b)$ of attackers that successfully penetrate the area defense in a combat situation in which b available batteries are given to be in the interaction corridor can be determined from the probability density of the random number of attackers destroyed among a sequence of n attackers that encounters a particular available but unspecified battery in the interaction corridor. Define $D_{k,n}$ to be that probability density; specifically, for $0 \le k \le n$,

$$D_{k,n} = \Pr\{D_n^\star=k\} \ .$$

Because the probability $Pr\{S_n^*(1)=j\}$ that the random number $S_n^*(1)$ of attackers that survive the first battery equals j is simply the probability that the first battery destroys n-j of the initial n attackers,

$$Pr\{S_n^*(1)=j\} = D_{n-j,n}$$

for $0 \le j \le n$. For $b>1$, the attackers in the sequence that approaches the last battery are the attackers that survive the first b-1 batteries. As a result, the random number $S_n^*(b)$ of attackers that survive all b batteries equals i if and only if the random number $S_n^*(b-1)$ of attackers that survive the previous b-1 batteries equals j and j-i attackers are destroyed by the last battery for some j for $i \le j \le n$. Hence, the event $\{S_n^*(b)=i\}$ is partitioned into the disjoint sequence $\{S_n^*(b)=i, S_n^*(b-1)=j\}$ for $i \le j \le n$ by the possible numbers of survivors of the previously encountered batteries. Consequently, its probability can be expressed as the sum

$$Pr\{S_n^*(b)=i\} = \sum_{i\,j}^{n} Pr\{S_n^*(b)=i, S_n^*(b-1)=j\}$$

$$= \sum_{i\,j}^{n} Pr\{D_j^*=j-i, S_n^*(b-1)=j\}$$

$$= \sum_{i\,j}^{n} D_{j-i,j} Pr\{S_n^*(b-1)=j\}$$

for $0 \le i \le n$. The probability densities of the random numbers of survivors of a succession of available batteries in the interaction corridor are thus determined from the values of $D_{k,n}$ by a simple recursion.

Defining $S_n^*(0)$ to equal n with probability 1 and unconditionalizing $Pr\{S_n^*(b)=i\}$ with respect to the random number b^* of available batteries in the interaction corridor yields the probability density of the random number $S_n^*(b^*)$ of the n attackers that successfully penetrate the defended area. Thus,

$$\Pr\{S_n^\star(b^\star)=i\} = \sum_{0}^{\infty}{}_{b} \Pr\{S_n^\star(b)=i\}\Pr\{b^\star=b\} \ , \qquad \text{(A-18.1)}$$

the probability that exactly i attackers survive the $b^\star$ available batteries that are in the interaction corridor, is the probability that exactly i of the n attackers successfully penetrate the defended area and can attack the targets to be protected.

Correlatively, the random number $D_n^\star(b^\star)$ of attackers destroyed by the defender configuration is $n-S_n^\star(b^\star)$; and the probability that exactly k casualties are produced among the attackers is, of course,

$$\Pr\{D_n^\star(b^\star)=k\} = \Pr\{S_n^\star(b^\star)=n-k\} \ . \qquad \text{(A-18.2)}$$

That is the configural probability density of the random number of attackers destroyed by the defender area air defense.

In the situations discussed in the body of this report, as previously noted, the batteries are postulated to be uniformly randomly and independently distributed within the defended area at the start of an attack. As a result, the random number $b^\star$ of available batteries in the interaction corridor, of course, is binomially distributed. For computations made during the initial research, however, that binomial probability density is replaced by its Poisson approximation to facilitate comparisons between configured encounters and free encounters. In fact, to be correct for even a single attacker, free-encounter assessments require that the random number of batteries a single attacker can encounter in attempting to penetrate a defended area in which multiple encounters are possible has a Poisson probability density. Consequently, to eliminate any differences between configured-encounter and free-encounter assessments that might arise from a probability density of the number and locations of weapons in the defended area that is incompatible with that implicit in the free-encounter concept, the Poisson approximation to the binomial is used in *both* assessments, not as an approximation but as the actual probability density. Using the Poisson probability distribution also simplifies determining numerically the number of batteries that are needed, for instance, to provide a specified configural attrition rate among the attackers.

The parameter of that Poisson distribution, the expected number of available batteries in the interaction corridor, is defined in terms of the density δ of batteries per nautical mile of the (unlimited) width of the defended area, their availability β, the weapon range R, and the half-width W of the attack corridor. Specifically, because the width of the interaction corridor is 2(R+W), the expected number of batteries in it is $2(R+W)\delta$ and the expected number of available batteries in it is $2(R+W)\beta\delta$. As a result,

$$\Pr\{S_n^*(b^*)=i\} = e^{-2(R+W)\beta\delta} \sum_{0}^{\infty}{}_{b} \frac{[2(R+W)\beta\delta]^b}{\lfloor\underline{b}} \Pr\{S_n^*(b)=i\} \qquad \text{(A-19.1)}$$

is the configural probability density of the random number of attackers that successfully penetrate the area air defense in those situations.

The configural probability density of the random number of attackers that are destroyed in those situations, of course, follows from substitution in the right member of equation (A-18.2). It is the configural probability density of the random number of attackers destroyed by the defender air defense for the situations discussed in the body of this report.

3. The Configural Probability Density of the Random Number of Casualties among the Targets to Be Protected

How the attackers that successfully penetrate the defender area air defense detect, acquire, and attack their targets, which are the ships to be protected, is greatly simplified and highly idealized in the initial research. In particular, the attackers that detect and acquire a ship are postulated to detect and acquire any ship with the same probability.

In actuality, configuration imposes greater limitations on the attacker: Not all ships in the inner part of the defender configuration need even be simultaneously visible to a low-altitude antiship missile. Depending on the deployment and the point at which the attack axis enters the inner part of the defender configuration, some target ships may be masking others, some may be beyond the horizon of the attacker. Moreover, the probabilities that any particular ships are detected and acquired can differ greatly, and the associated random events need not be probabilistically independent from attacker to attacker. As a consequence, in a contemporary combat situation, any particular number of antiship missiles would detect and acquire, and therefore attack, a smaller fraction of the ships to be protected, and some of the ships attacked would be attacked by disproportionately large numbers of attackers. In actuality, casualty production among the ships to be protected thus would be less than in the postulated situations examined in this report and the associated configural casualty probability densities would differ from those examined as well. However, configural theory suggests ways of forcing more even distributions of the attackers among more of the targets without introducing additional attack axes. In such situations, casualty production among the ships to be protected could exceed that in the postulated situations considered in this report.

In those postulated situations, in concept, each attacker that penetrates the defended area scans the portion of the defender configuration in which the N targets are deployed. Each such attacker, independently of the other attackers, finds the target subconfiguration with probability φ and finds nothing and permanently leaves the vicinity of the targets and the defender configuration with probability $1-\varphi$. Consequently, of a given random number $S_n^*(b^*) = i$ of attackers that penetrate the defended area, the random number H_i^* of those attackers that find the target subconfiguration has a binomial probability density with success probability φ. An attacker that finds the target subconfiguration acquires any particular ship with probability 1/N and attacks and damages it with probability γ. Thus, an attacker that finds the target subconfiguration attacks and damages a particular ship among the N to be protected with probability γ/N.

Given that i attackers penetrate the defensive screen and h of them find the target subconfiguration, the random number C^* of the N ships to be protected that are damaged by at least one attacker has the generalized occupancy probability density

$$\Pr\{C^*=k|S_n^*(b^*)=i,H_i^*=h\} = \binom{N}{k}\sum_{j=0}^{k}\binom{k}{j}(-1)^j[1-(N-k+j)\gamma/N]^h$$

for $0 \le k \le N$. That probability density is conditioned on exactly $H_i^* = h$ of the $S_n^*(b^*) = i$ attackers that penetrate the defensive screen finding the target subconfiguration. Unconditionalizing for the random number of attackers that find the target subconfiguration yields

$$\Pr\{C^*=k|S_n^*(b^*)=i\} = \binom{N}{k}\sum_{j=0}^{k}\binom{k}{j}(-1)^j[1-(N-k+j)\gamma\varphi/N]^i ,$$

another generalized occupancy probability density. The unconditional probability density for the random number of casualties among the ships to be protected is, therefore,

$$\Pr\{C^*=k\} = \binom{N}{k}\sum_{j=0}^{k}\binom{k}{j}(-1)^j\sum_{i=0}^{n}[1-(N-k+j)\gamma\varphi/N]^i\Pr\{S_n^*(b^*)=i\} \tag{A-21.1}$$

for $0 \le k \le N$ and $0 \le n$. The configural probability density for the random number of casualties among the targets to be protected that is used for the situations discussed in the body of this report is the result of replacing $\Pr\{S_n^*(b^*)=i\}$ in the right member of that equation with the right member of equation (A-19.1).

APPENDIX B

CALCULATING RELATIVE EFFECTIVENESS

Relative effectiveness for weapon systems of essentially equal weight and cubage is defined in the body of this report to be the ratio of the number of units of the system with the larger numerical requirement for the defended area for specified casualty production to the number of units of the system with the smaller numerical requirement for equivalent casualty production. In other words, for a particular casualty production with a particular weapon system and the corresponding numerical requirement N,

$$\text{relative effectiveness} = N_{\text{larger}}/N_{\text{smaller}} \cdot$$

Of course, the numerical requirement can be expressed as a density δ, which is the number of units of the weapon system per unit width of defended area front. Thus,

$$\text{relative effectiveness} = \delta_{\text{larger}}/\delta_{\text{smaller}} \cdot$$

The system with the smaller numerical requirement in such comparisons is, by definition, the more effective or the superior system.

For the systems considered in the body of this report, the air defense missile battery is the unit, and relative effectiveness is determined for numerical requirements only for casualty production among the attackers.

1. Relative Effectiveness in Configured Encounters

In the configural assessments of relative effectiveness that are used in the body of this report, any particular battery density δ' produces a configural attrition rate (average fractional casualty production) r among the n attackers in the attacker configuration. That attrition rate satisfies the equation

$$E[D_n^\star(b^\star)] = nr ,$$

in which $D_n^\star(b^\star)$ is the random number of casualties produced among the attackers by the $b^\star$ available batteries in the interaction corridor in a configured encounter in which the batteries of the specified system are deployed in the defender configuration with the density δ'. Appendix A establishes the probability distribution of $D_n^*(b^*)$ in terms of the probability distribution of $S_n^*(b^*)$, the random number of attackers that successfully penetrate the defended area. Using the relationship $D_n^*(b^*) = n-S_n^*(b^*)$ in the previous equation for $E[D_n^*(b^*)]$ and simplifying results in

$$E[S_n^*(b^*)] = (1-r)n \ .$$

As Appendix A discusses, for the situations discussed in the main body of this report, the random number of available defender batteries in the interaction corridor is postulated to have a Poisson probability density. Consequently, as a result of equation (A-19.1), the density δ' of batteries with availability β and weapon range R that produces the configural attrition rate r for n attackers using an attack corridor of half-width W must satisfy the equation

$$e^{-2(R+W)\beta\delta'}\sum_{0}^{\infty}{}_{b}\frac{[2(R+W)\beta\delta']^{b}}{\lfloor b} E[S_n^*(b)] = (1-r)n \ , \qquad \text{(B-2.1)}$$

which implicitly defines δ' as a continuous function of r . The number δ' of batteries per nautical mile of defended area front of the weapon system with specified values of its characteristics that is needed to produce exactly any specified configural attrition rate r in an attack corridor of specified width is obtained by numerically solving that equation.

2. Relative Effectiveness in Free Encounters

This section establishes an expression for relative effectiveness as it is customarily determined in free encounters for the kinds of weapon systems considered in the body of this report. The hypothetical, simplified air defense systems considered are characterized by the following parameters:

β	battery availability
α	battery detection and acquisition probability
a	emplacement availability
ρ	missile reliability
Δ	missile lethality
R	weapon range (encounter distance)

I	maximum permitted number of interceptions
L	number of tubes in the launcher at each available emplacement
G	number of emplacements in the battery
M	number of rounds at the battery

The (average) number of identical batteries, each with sufficient rounds to load all tubes at all emplacements ($LG \leq M$), that an attacker must freely encounter to be destroyed is customarily determined for such systems in terms of the probability that a single attacker is destroyed in a free encounter with a single battery. Designate that probability by C . Because the encounters are free and the batteries are identical, the random number necessary to destroy the attacker has a geometric probability distribution with success probability C . Hence, the average number of batteries that the attacker must freely encounter to be destroyed is $1/C$. Because those batteries must be within a distance R of the trajectory of the attacker, the average battery density is $1/2RC$.

Whether an attack corridor is specified does not matter in the free encounter, as is easily shown by the kind of expected-value argument often used in defense analysis. For an attack corridor of width $2W$, the corresponding interaction corridor has width $2(R+W)$. Consequently, the probability that a single attacker encounters a particular battery given to be in the interaction corridor is $R/(R+W)$. As a result, the probability that the attacker is destroyed by a particular battery in the interaction corridor is $CR/(R+W)$. Again, the random number of batteries necessary to destroy the attacker has a geometric probability distribution with $CR/(R+W)$ as its success probability, and the average number of batteries that must be in the interaction corridor to destroy a single attacker is $(R+W)/CR$, the reciprocal of that probability. For a defender configuration with B batteries and a defended area front of width F, the probability that a particular battery is in the interaction corridor (which, by definition, is entirely within the defended area) is $2(R+W)/F$. Consequently, the average number of batteries that are in the interaction corridor is $2B(R+W)/F$, and it must equal the average number of batteries $(R+W)/CR$ that must be in the interaction corridor to destroy a single attacker. Hence,

$$\frac{2B(R+W)}{F} = \frac{(R+W)}{CR}$$

and, as a result, the density of defender batteries is given by

$$\frac{B}{F} = \frac{1}{2CR} \text{ ,}$$

which is independent of the width of the attack corridor.

In a free encounter with a battery that has all L tubes loaded at each available emplacement, the conditional probability S(g) that a single attacker survives given that the battery and g of its G emplacements are available and the attacker is detected and acquired is

$$S(g) = \sum_{0\,j}^{I-1} \binom{Lg}{j} [\rho(1-\Delta)]^j (1-\rho)^{Lg-j} + (1-\Delta)^I \sum_{I\,j}^{Lg} \binom{Lg}{j} \rho^j (1-\rho)^{Lg-j} \text{ .}$$

In that expression, the first extended summation is the probability that fewer than I successful launches of the Lg missiles loaded at the battery occur and the attacker survives, and the term that includes the second extended summation is the product of two factors: the probability that the attacker survives I successful launches and the probability that at least I of the Lg missiles loaded at the battery can successfully launch. Consequently, the unconditional probability C that an attacker given to encounter a battery of multitube emplacements with all tubes loaded at each of its G^* available emplacements, is destroyed thereby is

$$C = \alpha\beta\left[1 - \sum_{0\,g}^{G} \binom{G}{g} S(g) a^g (1-a)^{G-g}\right] \text{ .} \qquad \text{(B-4.1)}$$

For the single-emplacement batteries (G=1) used in the body of this report,

$$C = \alpha\beta[1 - S(0)(1-a) - S(1)a] = \alpha\beta a[1 - S(1)] \text{ ,} \qquad \text{(B-4.2)}$$

in which the product βa in the right member is, in effect, the availability of the single-emplacement battery.[1]

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[1] The value of the product βa for each of the hypothetical weapon systems considered in the body of this report is identified as launcher availability in Table II-2 on page 17.

A particular case of that equation is used to determine the free-encounter relative effectiveness of alternative, individual missiles discussed in the body of this report. In that case, the battery consists of a single emplacement with only a single round and a single-tube launcher, and the equation for the probability the attacker is destroyed reduces to

$$C = \beta a \rho \alpha \Delta \ , \qquad \text{(B-5.1)}$$

in which the product $\beta a \rho$ is, in effect, the reliability of the single missile considered as an emplacement or, in short, the (single-round) availability.

In general, the maximum permitted number I of interceptions in such an encounter is a function $I(\cdot,\cdot)$ of the cross-corridor location of the trajectory of the attacker relative to the location of the battery and the bearing of the attacker relative to the battery at the instant of detection and acquisition. The corresponding general expression for C is much more complicated. However, for the situations examined in the body of this report, I is a constant and the expression established for C in equation (B-4.1) is appropriate.

Thus, to compare two weapon systems, system 1 and system 2, in free encounters, designate by N_1 and N_2 the average numbers of units of the respective systems that are required to destroy a single attacker in a free encounter. Furthermore, for $i = 1, 2$, designate the operational characteristics of system i as previously defined with the particular subscript i . Designate by C_i the probability given by equation (B-4.1) that a particular but unspecified battery of system i destroys an attacker in a free encounter. Again using the previously illustrated expected-value argument which is often used in defense analysis, the average number of batteries of system i that an attacker must freely encounter to be destroyed is

$$N_i = \frac{1}{C_i} \ .$$

Because those batteries must be uniformly randomly and independently distributed in the interaction corridor for system i, the density with which the system must be deployed in the defended area, as previously shown, is the reciprocal of its "lethal width" $2R_iC_i$, where R_i is the range of system i; that is,

$$\delta_i = \frac{1}{2R_iC_i} .$$

Consequently, designating the system with the smaller numerical requirement as system 1, the relative effectiveness of the two systems in free encounters is

$$\frac{\frac{1}{2R_2C_2}}{\frac{1}{2R_1C_1}} = \frac{R_1C_1}{R_2C_2} . \qquad \text{(B-6.1)}$$

That ratio, which is the ratio of their "lethal ranges", is the expression on which the free-encounter relative-effectiveness assessments made in the body of this report are based. Because a free encounter is the same as a configured encounter in which there is only one attacker and the number of available batteries in the interaction corridor has a Poisson distribution, that ratio is equal to the relative effectiveness calculated from the configural probability density of the random number of casualties among the attackers that attempt to penetrate the defended area of the defender configuration, which is defined by equation (A-18.2), as particularized for a Poisson distribution of available batteries of the respective systems in the interaction corridors and for a single attacker or, equivalently, from equation (B-2.1).

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The reported research is a product of a very limited, initial effort to develop configural theory for application to the design and assessment of ship self-defense and fleet air defense weapon systems and to illustrate its importance to analysis, modeling, and simulation in that context. Configural theory is a mathematical theory for quantifying the relationships between the behavior of weapons in use in combat and their individual characteristics. Its name is derived from its central concept, configuration, which is the mathematical expression of the fact that the disposition in space and time of the targets and weapons of the attacker and the defender is inseparable from the outcome of the engagement and the combat effectiveness of those weapons. Customary mathematical representations usually exclude much that is essential and thus are nonconfigural. Configural and nonconfigural assessments of the relative effectiveness of weapon systems compared in the report for situations that minimize the effects of configuration differ by factors from two to ten and more, and systems that are less effective in combat appear in nonconfigural assessments to be substantially superior to more effective systems. Configural theory can help produce more effective weapons and tactics for their use with no increase in required resources.

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