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LAW FOR SPACE INTERCEPTION

by

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DESIGNING OF A NONLINEAR OPTIMAL TERMINAL GUIDANCE LAW FOR SPACE INTERCEPTION

Abstract: This article concerns a nonlinear kinematics model of space interception. Nonlinear optimal control is sought through use of inverse systematic methods. A theoretical nonlinear terminal guidance law of global state feedback is advanced. In addition, a simulation test is carried out and a comparison is made with proportional guidance. Results show that this guidance law performs satisfactorily.

Key words: nonlinear guidance law, inverse system methods, nonlinear optimal control, pseudolinear system.

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I. FORWARD

Conventional proportional guidance is simple and easy to carry out. However, precision is relatively low. Moreover, line of sight angular velocity divergence phenomena [4] exist. For this reason, there is a need to search out various types of optimal guidance laws. Due to actual interception kinematics models being nonlinear, as a result, one has only to solve optimal control problems associated with nonlinear systems and only then is it possible to obtain relatively high precision terminal guidance laws. With regard to general nonlinear optimal control problems, people still have not found a unified method of solution. However, we use inverse system methods, and it is possible to solve this concrete nonlinear optimal control problem encountered in terminal guidance.

II. NONLINEAR OPTIMAL CONTROL INVERSE SYSTEM METHODS

Assume the general form of nonlinear systems to be

(2-1-a)

$$\Sigma: \begin{cases} \dot{X} = F(X,U) \quad X(t_0) = X_0 & \text{(2-1-a)} \\ Y = G(X,U) & \text{(2-1-b)} \end{cases}$$

(2-1-b)

In this, U is an m dimension input vector. Y is an r dimension output vector. X is an n dimension state vector.

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The output equation (2-1-b) is also capable of being written as

$$\begin{cases} y_1 = g_1(X,U) \\ \vdots \\ y_r = g_r(X,U) \end{cases} \qquad (2\text{-}2)$$

First, carry out derivation and transformations of equation (2-2), that is, in sequence, solve for the relevant a1,a2,...ar order derivatives associated with time period t for y1,y2,...yr, constructing the new equation

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$$\begin{cases} y_1^{(a_1)} = h_1(X,U) \\ \vdots \\ y_r^{(a_r)} = h_r(X,U) \end{cases} \qquad (2\text{-}3)$$

In this, numerical values for ai(1≤i≤r) are defined in accordance with the forms below

$$\begin{cases} \frac{\partial}{\partial U}(F^k g_i) \equiv 0 \quad k = 0,1,2\cdots,a_i - 1 & (2\text{-}4\text{-}a) \\ \frac{\partial}{\partial U}(F^k g_i) \neq 0 \quad k = a_i & (2\text{-}4\text{-}b) \end{cases}$$

In this, $Fg_i \triangleq \left(\frac{\partial}{\partial X^T} g_i\right)F$, $F^k g_i \triangleq F(F^{k-1} g_i)$, and, when k = 0, one has $F^0 g_i \triangleq g_i$.

Assuming that, during the process of the derivatives described above, with regard to 1≤i≤r, one has ai < ∞, in the equation set (2-3), if U can act as a function of x and $y_1^{(a_1)}, \cdots, y_r^{(a_r)}$, then, it is possible to solve for the explicit representation as

(2-5) $$U = H^{-1}(X, Y^a)$$

In this, $Y^a \triangleq [y_1^{(a_1)}, \cdots, y_r^{(a_r)}]^T$. Then, it is conveniently possible, from equation (2-1), to construct a new system equation. The state equations can be obtained from substituting equation (2-5) into equation (2-1-a). The output equation is then form (2-5). As a result, system π:YU equations can be expressed as

(2-6-a)

$$\pi: \begin{cases} \dot{X} = F[X, H^{-1}(X, Y^a)], \ X(t_0) = X_0 \\ U = H^{-1}(X, Y^a) \end{cases}$$

(2-6-b)

It can be demonstrated [8] that the system expressed by the equations described above is the inverse systems to system (2-1).

With regard to the initially given system expressed by equation (2-1), take Ya in the inverse system (2-6) and replace it to be $\varphi = [\varphi_1, \varphi_2, \cdots \varphi_r]^T$. It is then conveniently possible to obtain the a order integral inverse system.

(2-7-a)

$$\begin{cases} \dot{X} = F[X, H^{-1}(X, \varphi)] & X(t_0) = X_0 \\ U = H^{-1}(X, \varphi) \end{cases}$$

(2-7-b)

After that, before taking the a order integral inverse system obtained and connecting it with the original system, combined systems which are further obtained are then corresponding pseudo linear systems. The transmission relationship can be expressed as

(2-8) $$diag(D^{a_1}, D^{a_2}, \cdots, D^{a_r}) \cdot Y = \varphi$$

In this, Dai is the ai order operator. i = 1,2,...,r.

Pseudo linear systems can also be expressed as state spacial forms

(2-9-a)

$$\begin{cases} \dot{Z} = AZ + B\varphi \\ U = CZ \end{cases}$$

(2-9-b)

Assuming that optimal performance indicators corresponding to system (2-1) are

$$J'(X,U) = \frac{1}{2}X^T(t_f)R'X(t_f) + \frac{1}{2}\int_{t_0}^{t_f} [X^T(t)Q'X(t) + U^T(t)S'U(t)]dt$$

(2-10)

Choosing pseudo linear system (2-9)'s optimal performance index to be

$$J(Z,\varphi) = \frac{1}{2}Z^T(t_f)RZ(t_f) + \frac{1}{2}\int_{t_0}^{t_f} [Z^T(t)QZ(t) + \varphi^T(t)S\varphi(t)]dt$$

(2-11)

If, at the same time that $J(Z,\phi)$ min, one also has $J'(X,U)$ min', then, it is possible to first, with regard to pseudo linear (2-9) and in accordance with solution methods for linear second order type optimal regulator devices, get pseudo linear system optimal control

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(2-12) $$\varphi^{*}=\varphi^{*}(Z^{*},t)$$

Assuming the relationship between state vectors X and Z

(2-13) $$Z=T(X)$$

comprehensive consideration of forms (2-7-b), (2-12), as well as (2-13), it is finally possible to obtain nonlinear system (2-1) optimum controls which are

(2-14) $$U^{*}(X^{*},t)=H^{-1}[X^{*},\varphi^{*}(T(X^{*}),t)]$$

III. THE SETTING UP OF INTERCEPTION DEVICE AND TARGET RELATIVE MOTION EQUATIONS

Assuming that targets move along elliptical orbits, in studying instants with targets at point M and interception devices at point D, the relative distance between the two is ρ. Introducing line of sight coordinate system $D\xi\eta\zeta$, , the relationship between it and the interception device three dimensional coordinate system Dx1y1z1 is determined by altitude angles θ and azimuth angles ψ as shown in Fig.1. The mathematical relationship between the two coordinate systems is

(3-1)
$$\begin{bmatrix} \vec{\xi} \\ \vec{\eta} \\ \vec{\zeta} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\theta & -\cos\theta\sin\psi \\ -\sin\theta\cos\psi & \cos\theta & \sin\theta\sin\psi \\ \sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} \vec{x}_1 \\ \vec{y}_1 \\ \vec{z}_1 \end{bmatrix}$$

Assuming that angular velocities of rotation of line of sight coordinate systems with respect to inertial coordinate systems are:

(3-2)
$$\vec{\omega} = \vec{\omega}_\xi + \vec{\omega}_\eta + \vec{\omega}_\zeta$$

In this, $\vec{\omega}_\xi, \vec{\omega}_\eta, \vec{\omega}_\zeta$: are projections of absolute angular velocities $\vec{\omega}$ the three line of sight coordinate system axes. When target orbit altitudes are between 300km – 2500km, considerations of terminal guidance laws can ignore differences in gravity. Then, in studying three axis stability plans, and, assuming conditions where attitude controls guarantee

(3-3) $$\vec{\omega}_\xi = \vec{o}$$

relative motion equations for targets and interception devices can be written as:

(3-4-a)

(3-4-b) $$\begin{cases} \ddot{\rho} - \rho(\omega_\eta^2 + \omega_\zeta^2) = a_\xi \\ 2\dot{\rho}\omega_\zeta + \rho\dot{\omega}_\zeta = a_\eta \\ -2\dot{\rho}\omega_\eta - \rho\dot{\omega}_\eta = a_\zeta \end{cases}$$

(3-4-c)

In these, a_ξ, a_η, a_ζ are thrust acceleration $\vec{a}$ projections on the three line of sight coordinate system axes. Due to the forms above implicitly containing target movements, this article, therefore, hypothesizes that targets are a type of special case. Assume that, on interception devices, there are installed four normal thrust engines along the periphery of the center of mass as shown in Fig.2. That is, $\vec{a}_{x_1} = \vec{o}$. From Fig.1, it can be known that:

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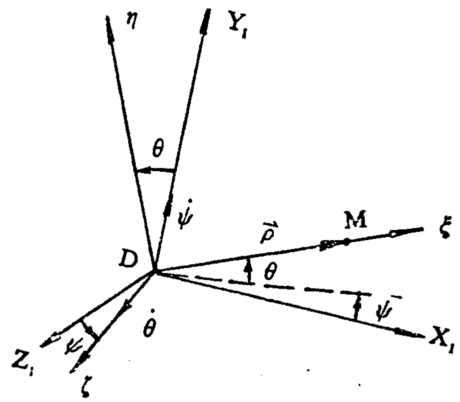

Fig.1 Line of Sight Coordinate System and Three Dimensional Coordinate System

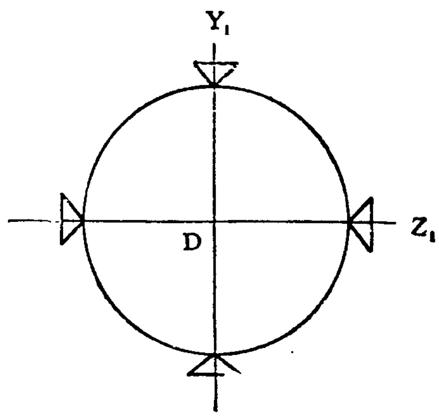

Fig.2 Interception Device Thrust Engine Installation Diagram

(3-5)

$$\begin{bmatrix} \vec{a}_\xi \\ \vec{a}_\eta \\ \vec{a}_\zeta \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\theta & -\cos\theta\sin\psi \\ -\sin\theta\cos\psi & \cos\theta & \sin\theta\sin\psi \\ \sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} \vec{o} \\ \vec{a}_{y_1} \\ \vec{a}_{z_1} \end{bmatrix},$$

Assume that attitude control systems guarantee $\theta \to 0$, $\psi \to 0$, Then, one has

(3-6-a)

$$\begin{cases} a_\xi = 0 \\ a_\eta = a_{y_1} \\ a_\zeta = a_{z_1} \end{cases}$$ (3-6-b)

(3-6-c)

Moreover, equation (3-4) changes to become

(3-7-a)

$$\begin{cases} \ddot{\rho} - \rho(\omega_\eta^2 + \omega_\zeta^2) = 0 \\ 2\dot{\rho}\omega_\zeta + \rho\dot{\omega}_\zeta = a_{y_1} \\ -2\dot{\rho}\omega_\eta - \rho\dot{\omega}_\eta = a_{z_1} \end{cases}$$ (3-7-b)

(3-7-c)

The equations above are nothing else than target and interception device relative motion equations in three dimensional space. Attitude control system engine fuel combustion will give rise to changes in interception device centers of mass. Moreover, once four normal thrust engines are installed, the locations are not easy to change. Therefore, in the process of interception, interception device centers of mass will deviate by small amounts the planes determined by the centers of mass of the four normal direction thrust motors. Assuming that the errors given rise to from this factor as well as the errors given rise to by coupling of attitude and center of mass movements can be ignored in engineering terms, then kinematics model (3-7) is reasonable. From this, it can be seen that model (3-7) is one type of approximation model. This type of approximation causes the model to be very greatly simplified. As a result, the carrying out of guidance law design is facilitated.

If one lets $x_1=\rho,\ x_2=\dot{\rho},\ x_3=\omega_\eta,\ x_4=\omega_\zeta,\ u_1=a_{z_1},\ u_2=a_{y_1},$ then, equation (3-7) can be written as the state equations

$$\begin{cases} \dot{x}_1 = x_2 & \text{(3-8-a)} \\ \dot{x}_2 = x_1(x_3^2 + x_4^2) & \text{(3-8-b)} \\ \dot{x}_3 = -\dfrac{2x_2x_3}{x_1} - \dfrac{u_1}{x_1} & \text{(3-8-c)} \\ \dot{x}_4 = -\dfrac{2x_2x_4}{x_1} + \dfrac{u_2}{x_1} & \text{(3-8-d)} \end{cases}$$

In these, x1 is the relative distance between interception devices and targets. x2 is their relative velocity. x3 is their longitudinal planar angular line of sight velocity. x4 is transverse planar angular line of sight velocity. u1 and u2 are, respectively, accelerations produced in association with interception device thrusts on axes Z1 and Y1.

IV. NONLINEAR OPTIMUM GUIDANCE LAW DESIGN

For the sake of convenience in the study of guidance laws, it is possible to take equation (3-8) and divide it into two dimensional models, that is, longitudinal planar relative motion equations

(4-1-a)

(4-1-b)

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1 x_3^2 \\ \dot{x}_3 = -\dfrac{2x_2x_3}{x_1} - \dfrac{u_1}{x_1} \end{cases}$$

(4-1-c)

as well as transverse planar relative motion equations

(4-2-a)

(4-2-b) $$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1 x_4^2 \\ \dot{x}_4 = -\frac{2x_2x_4}{x_1} + \frac{u_2}{x_1} \end{cases}$$

(4-2-c)

Now, taking the transverse planar case in order to design guidance laws, assume system (4-1)'s output equation is

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$$y_1 = x_3$$

(4-3)

With regard to equation (4-3), carry out derivation and transformation to obtain

(4-4) $$\dot{y}_1 = \dot{x}_3 = -\frac{2x_2x_3}{x_1} - \frac{u_1}{x_1}$$

After transformation, one has:

(4-5) $$u_1 = -x_1\dot{y}_1 - 2x_2x_3$$

Let $\varphi = \dot{y}_1$, , then, pseudo linear system numerical models are:

$$\dot{y}_1 = \varphi$$

(4-6)

Assume that pseudo linear system state variables are

(4-7) $$z = y_1$$

Then, pseudo linear system (4-6) can also be written as state controls forms:

(4-8-a)

$$\begin{cases} \dot{z} = \varphi \\ y_1 = z \end{cases}$$

(4-8-b)

If one takes equation (4-6) and substitutes into equation (4-5), then, one has:

(4-9) $$u_1 = -x_1\varphi - 2x_2x_3$$

After considering equations (4-3) and (4-7), equation (4,9) is changed to be:

(4-10) $$u_1 = -x_1\varphi - 2x_2z$$

From equation (4-10), it is possible to write pseudo linear system characteristic indices as

(4-11) $$J(z,\varphi) = \frac{1}{2}rz^2(t_b) + \frac{1}{2}\int_{t_0}^{t_b}[qz^2(t) + s\varphi^2(t)]dt$$

In the equation, r, q, and s are weighting coefficients. to it terminal guidance origination time. tb is time of entry into guidance blind area. From equation (4-10) and equation (4-11), it is possible to see that one has only to appropriately select weighting coefficients. Then, when equation (4-11) reaches a minimum, it is possible to make the absolute values of line of sight angular velocity x3 and control u1 diminish. That is, it is possible to make equation (2-11), which corresponds to a matrix associated with a certain set of weighting coefficients R',Q',S', reach minimums. As a result, making connections, it is possible to carry on in accordance with methods described in Section II.

(4-12) $$\varphi^*(t) = -\frac{1}{s}p(t)z^*(t)$$

In this, p(t) satisfies Riccati scalar quantity integral equations [7]:

(4-13-a)

$$\begin{cases} \dot{p}(t) = \frac{1}{s}p^2(t) - q \\ p(t_b) = r \end{cases}$$

(4-13-b)

Through solving Riccati equations, it is finally possible to obtain pseudo linear system optimum control as:

$$\varphi^*(t) = -\sqrt{\frac{q}{s}} \cdot \frac{(r+\sqrt{sq}) + (r-\sqrt{sq})\exp\left[2\sqrt{\frac{q}{s}}(t-t_b)\right]}{(r+\sqrt{sq}) - (r-\sqrt{sq})\exp\left[2\sqrt{\frac{q}{s}}(t-t_b)\right]} x_3^*(t).$$

(4-14)

Take equation (4-14) and substitute into equation (4-9). One then obtains the longitudinal planar nonlinear optimum guidance law:

$$u_1^* = \sqrt{\frac{q}{s}} \cdot \frac{(r+\sqrt{sq})+(r-\sqrt{sq})\exp\left[2\sqrt{\frac{q}{s}}(t-t_b)\right]}{(r+\sqrt{sq})-(r-\sqrt{sq})\exp\left[2\sqrt{\frac{q}{s}}(t-t_b)\right]} x_1^* x_3^* - 2x_2^* x_3^*. \quad (4\text{-}15)$$

By the same reasoning, it is possible to obtain the transverse planar nonlinear optimal guidance law: /23

$$u_2^* = -\sqrt{\frac{q}{s}} \cdot \frac{(r+\sqrt{sq})+(r-\sqrt{sq})\exp\left[2\sqrt{\frac{q}{s}}(t-t_b)\right]}{(r+\sqrt{sq})-(r-\sqrt{sq})\exp\left[2\sqrt{\frac{q}{s}}(t-t_b)\right]} x_1^* x_4^* + 2x_2^* x_4^*. \quad (4\text{-}16)$$

Looking at the two equations above, this type of nonlinear optimal guidance law requires the carrying out of global state feedback on systems. This then brings with it certain difficulties in realizing guidance laws. In particular, it requires the measurement of distances. However, we are capable of using reconstructed state methods in order to solve this problem. In summary, although the guidance laws we put forward are a bit complicated, guidance performance will, however, be very greatly increased.

V. NUMERICAL VALUE SIMULATION

Using longitudinal planar nonlinear guidance laws as an example, simulation tests were carried out. When

$$x_1(t_0)=100\text{km}, x_2(t_0)=-10\text{km/sec}, \qquad x_3(t_0)=5\times 10^{-4}\text{rad/sec},$$

simulation results for guidance law (4-15) are as shown in Fig. 3, 4, and 5 (among these, selections are made for r=10, q=60, s=110, to = 0 sec, and tb = 10 sec). Moreover, proportional guidance law simulation results are seen in Fig.6 and 7. In accordance with calculation formulae for the amounts by which targets are missed [9],

$$\rho_{\min} = \frac{x_{1b}^2\sqrt{x_{3b}^2+x_{4b}^2}}{\sqrt{x_{2b}^2+x_{1b}^2(x_{3b}^2+x_{4b}^2)}} \tag{5-1}$$

it is possible to solve for the amounts by which targets are missed as associated with nonlinear guidance law (4-15) and conventional proportional guidance laws to be, respectively, 5.4x10-7 m and 10.32 m., assuming the aiming blind area range is 2000m. In nonlinear guidance laws, the values of weighting coefficients r, q, and s influence guidance characteristics. Through simulations, it is possible to know that, the larger the value of q/s is, the better guidance performance is. Moreover, the influence of the value of r on guidance characteristics is very tiny. Simulation results also tell us that the absolute values of control signals associated with nonlinear guidance law (4-15) are monotonic reduction functions of time. Moreover, they will not exceed maximum thrust ranges associated with interception device motors. Besides this, conventional proportional guidance indeed has line of sight angular velocity divergence phenomena. Moreover, as far as the nonlinear guidance laws discussed above are concerned, the line of sight angular velocities drop monotonically and converge to zero. In summary, the performance of the nonlinear guidance laws given in this article are indeed very good.

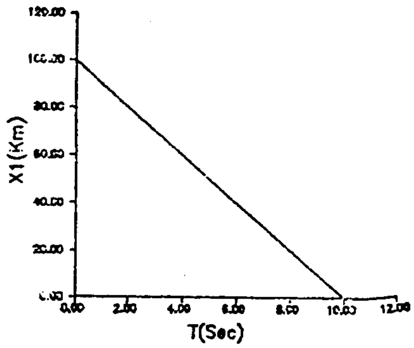

Fig.3 Changes in Relative Distances Under the Control of Nonlinear Guidance Laws

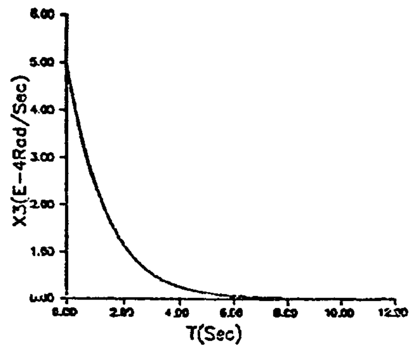

Fig.4 Changes in Line of Sight Angular Velocities Under Nonlinear Guidance Law Control

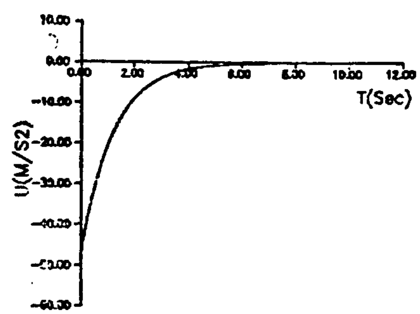

Fig.5 Change States Associated with Control Signals in Nonlinear Guidance Laws

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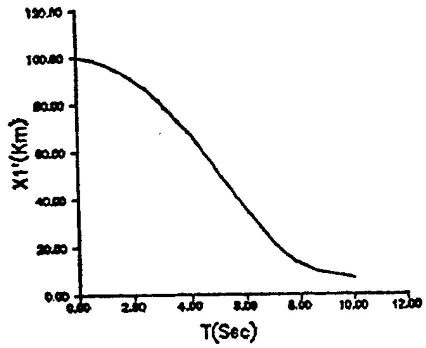

Fig.6 Changes in Relative Distance Under Proportional Guidance Law Control

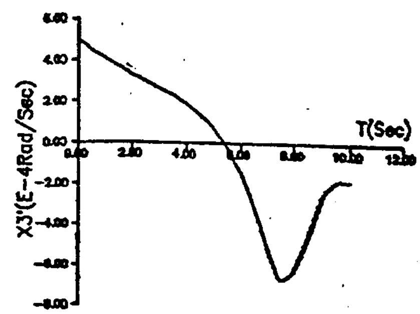

Fig.7 Changes in Line of Sight Linear Velocities Under Proportional Guidance Law Control

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VI. CONCLUSIONS

This article makes use of inverse system methods and studies nonlinear optimal control problems. In conjunction with that, it puts forward nonlinear optimal guidance laws. The guidance laws in question furnish a theoretical foundation for increasing space intercept terminal guidance performance. Of course, during actual realization, there still exist a number of problems--for instance, problems with obtaining relative distance signals x1(t) in it. Please see reference [6]. This article will not provide superfluous details.

REFERENCES

1. Tempelman W. Linear Guidance Laws for Space Missions. Journal of Guidance and Control. 1986, 9(4): 495—502.
2. Guelman M. and Shinar, J., Optimal Guidance Law in the Plane. Journal of Guidance and Control, 1984, 7(4): 471—475.
3. Webber R F and Bonfanti G. A Nonlinear Guidance Scheme for Intercept Mission, AIAA Paper No 71—916.
4. Murtaugh S A and Criel H E. Foundamentals of Proportional Navigation. IEEE Spectrum, December. 1966, 75—85.
5. Nazaroff G G. An optimal terminal guidance law. IEEE Transactions on Automatic Control. June 1976. AC-21, (3): 407—408.
6. Xiao-Hua Xia and Wei-Bin Gao. Nonlinear Observer Design by Observer Canonical Forms. INT. J. Control, 1988, 47 (4): 1081—1100.
7. 解学书。最优控制理论与应用。北京：清华大学出版社，1986年7月，第1版。
8. 李春文，冯元琨。多变量非线性控制的逆系统方法。北京：清华大学出版社，1991 年8月，第1版。
9. 姚郁，王子才。一种空间拦截导引规律的设计。制导与引信，1990. 2.
10. 雷特曼G. 著，陈介山译。最佳控制引论，国防工业出版社，1982年2月，第1版。

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