

Spread Spectrum Modulation by Means of Time-Varying Linear Filtering

Howard L. Dyckman

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San Diego, CA 92152-5001





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NAVAL COMMAND, CONTROL AND OCEAN SURVEILLANCE CENTER RDT&E DIVISION San Diego, California 92152–5001

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MODULATION BY MEANS OF TIME-VARYING LINEAR FILTERING

Time-varying linear filtering is a very general technique that can generate a broad class of spread spectrum signals (as well as other forms of modulation*). As a spread spectrum technique, it admits a broader class of waveforms than are commonly used today, which may provide performance benefits in several areas of application.

Time-varying filter (TVF) spread spectrum modulators can spread in time as well as in frequency, thereby providing broad protection against errors. They can generate smooth signals taking on a continuum of values, which may facilitate hiding transmissions within background noise. They can generate strictly band-limited signals without impairing reception. These properties should provide benefits in antijam/low probability of intercept (AJ/LPI) and civilian code division multipleaccess (CDMA) mobile radio applications. Time-varying filters may also be used as modems in situations where bandwidth spreading is not needed.

From a theoretical point of view, time-varying linear filtering provides a coherent framework for understanding spread spectrum.

^{*} Amplitude modulation has been described as a form of time-varying filtering (Ericson, 1981).

DESCRIPTION OF TIME-VARYING FILTER SPREAD SPECTRUM

A time-varying filter (TVF) is a filter whose impulse response changes with time. A general linear time-varying filter in continuous time may be expressed in the form

$$y(t) = \int_{-\infty}^{\infty} h(t - t', t') x(t') dt' = \int_{-\infty}^{\infty} f(t, t') x(t') dt'$$
(1)

where x(t') is the input (data) signal and y(t) is the output (spread) waveform. Thus, TVF spread spectrum admits continuous-valued responses that can be overlapped in time.

To be suitable for spread spectrum modulation, the impulse response of a TVF must vary at a rate orders of magnitude greater than the input signal bandwidth. If the system operates in continuous time, the variation must be rapid in two respects:

- A. At each time instant t', the TVF has impulse response $h(\tau, t')$, which governs the effect of the input signal at that time instant on the output signal over all time. Each such instantaneous impulse response, $h(\tau, t')$ with t' held fixed, should be a rapidly fluctuating function of the difference time variable $\tau = t t'$, where t is the output time variable.
- B. The TVF's instantaneous impulse response, $h(\tau, t')$, should change rapidly when input time variable t' is varied with τ held fixed, for at least some values of the difference time variable τ .

Both forms of rapid variation are necessary (although not sufficient) when dealing with continuous-time inputs; without either one, the rapid variation introduced by the other will be smeared out, and little if any spreading will be effected.

More customarily, the input will be a discrete-time sequence, either binary values or Nyquist samples of a continuous-time waveform. We write $t' = m \tau$ and t = n T, where τ and T are sampling intervals for the input and output time domains respectively ($\tau = L T$, where L is the spreading ratio). The TVF becomes

$$y(n) = \sum_{m = -\infty}^{\infty} k(n, m) x(m)$$
(2)

Equations (1) and (2) represent time-varying filters in terms of the time-domain Green's functions f(t, t') and k(n, m), each a mapping from every point in time to every other point in time.

A TVF can also be represented in terms of a frequency-domain Green's function, a mapping from every frequency to every other frequency (Gardner, 1988). For a discrete-time system,

$$Y(f) = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} K(f, f') X(f') df'$$
(3)

(For a continuous-time system, the limits on the integral would be $-\infty$ to $+\infty$.)

When the restriction is made to time-invariant filters, these equations take on familiar forms: equations (1) and (2) become convolutions, and equation (3) becomes a multiplication.

$$y(t) = \int_{-\infty}^{\infty} h(t - t') x(t') dt', \qquad Y(f) = H(f) X(f)$$
(4a, b)

for continuous time, and

$$y(n) = \sum_{m = -\infty}^{\infty} h(n - m) x(m) , \qquad Y(f) = H(f) X(f)$$
 (5a, b)

for discrete time. The capability to shift energy among different frequencies enables a time-varying filter to produce spectral spreading, whereas a time-invariant filter cannot.

These time-varying filter models describe the complex envelope of an RF spread spectrum waveform, i.e., in-phase and quadrature components at baseband. The complex envelope is y(t) in the case of continuous time, or a smoothed version of y(n) in the case of discrete time. This assumes upconversion to RF prior to transmission and downconversion upon reception.

RELATIONSHIP TO DIRECT SEQUENCE AND FREQUENCY-HOPPED SPREAD SPECTRUM

Direct Sequence (DS) spread spectrum is a special case of time-varying filtering. The DS timevarying filter consists of multiplying the data waveform by a binary (or multiphase) continuous-time waveform. Alternatively, it may be viewed as convolving an input sequence of discrete-time samples with a time-varying filter whose instantaneous impulse responses are short, as shown in Figure 1. This method is described by the formulas

$$y(t) = f(t) x(t)$$
 $Y(f) = F(f) * X(f)$ (6a, b)

which arise as a special case of equation (1) with

$$h(t - t', t') = f(t, t') = f(t) \,\delta(t - t') \tag{7}$$

where f(t) is the conventional Direct Sequence spreading code. There is no spreading in time; the responses to successive data samples cannot be overlapped.

Coherent frequency hopping is also a special case of time-varying filtering. Each hop in frequency by Δf appears in the TVF as a multiplication by a complex frequency shift of the form exp (j $2\pi \Delta f$ (t - t₀)).





TRANSVERSAL FILTER ARCHITECTURE

A time-varying filter can be modeled in discrete time for implementation with digital hardware using a transversal filter as shown in Figure 2, assuming the input data sequence is provided in the form of either binary values or analog-valued Nyquist samples of a continuous-time waveform. This data sequence is prepared for the TVF by interspersing the provided data samples with zeros to create a sequence whose rate is faster by the spreading ratio. This higher-rate sequence is presented to the transversal filter, whose analog weights may be time-varying or fixed. The transversal filter's output, a sequence of samples, would be smoothed to continuous-time form. An example of time-varying filtering of binary data is illustrated in Figure 3.

If we are given a continuous-time input signal, we may, alternatively, sample it at the sample rate of the final spread waveform; then we must carefully make sure that our time-varying filter satisfies condition (B),* so that the rapid variation inherent in condition (A) does not become smeared out by the slowly varying oversampled input. We envision sampling the input signal at the slower (Nyquist) rate, not only because it reduces the computational burden, but also because it may be viewed as pre-filtering the oversampled version with a time-varying filter that establishes condition (B) by setting L-1 out of every L samples to zero, where L is the bandwidth spreading ratio. Then, the transversal filter weights are easily chosen to satisfy condition (A). They would typically be taken from a pseudorandom number generator, and can be time-varying or even fixed, because condition (B) has already been established by the ersatz prefiltering.

Likewise, interspersing binary input data with zeros not only matches data rates but also establishes condition (B).



Figure 2. Shift register implementation of time-varying filter as modulator.

^{*} Refer to Description of Time-Varying Filter Spread Spectrum.



Figure 3. Example of time-varying filtering.

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FREQUENCY ANALYSIS/SYNTHESIS ARCHITECTURE

Equation (3) suggests a frequency domain architecture; however, the integral over all time is impractical. A practical analogue would involve breaking up the signal into blocks and performing three operations in sequence on each block: frequency analysis using an FFT; a linear transformation which may be different or the same from block to block; and finally, synthesis using an inverse FFT to form a time-domain signal (see Figure 4). These techniques are similar to those described in Crochiere and Rabiner (1983) for multirate digital signal processing. The initial FFT, which operates on only a small number of points, may be omitted, i.e., subsumed into the linear transformation.

In general, the linear transformation on the frequency components may be any Green's function transformation from every component in the input segment to every component in the output segment, and it may be as complicated as the transformation one might use in a time domain approach. The added work of the Fourier transforms would then seem to make the Fourier domain approach more costly. However, we may be able to perform a simple transformation in the frequency domain (for example, a permutation) and still achieve useful spreading (just as Direct Sequence is a simple transformation in the time domain). (Permutation of frequency components and other orthogonal or unitary transformations have been used for voice scrambling [Wyner, May 1979 and July 1979; Sakurai, Koga, & Muratani, 1984; and Ishii & Kakashita, 1990].)

One way this implementation is lacking in generality is that it does not provide for overlapping among blocks; rather than propose modifications, we look at another architecture which is more flexible: one based on unitary transformations.





UNITARY TRANSFORMATION ARCHITECTURE

A time-varying filter can be built out of many 2 x 2 elementary linear transformations. The form of these elementary transformations is shown in the inset in Figure 5. Each transformation takes two complex inputs and maps them into two complex outputs by means of four complex multiplicative factors, i.e., an arbitrary linear mapping from each of its inputs to each of its outputs. (When elements are cascaded, the number of multiplications can be greatly reduced by rearrangement.) With on the order of N(N-1)/2 of these 2 x 2 transformations, an arbitrary mapping from N inputs to N outputs can be constructed. For spread spectrum applications, each of these building blocks could be controlled by a pseudorandom generator.

If we admit arbitrary elementary transformations as building blocks, we face the problem that individual elements of the data sequence may face degenerate or nearly degenerate transformations, so that some of the input data samples may be represented only weakly or not at all in the output. Simply stated, portions of the signal can get lost in the maze of connections between input and output.

If we restrict the elementary transformations to be unitary, we can implement a TVF in a distributed fashion without risk of losing the signal. The energy-preserving nature of the transformations ensures that every input sample is represented in the output with equal weight and therefore cannot get lost. A mathematical description of a 2×2 unitary transformation (a complex Givens rotation) is given in Appendix A.

Unitary transformations can also simplify the problem of demodulation because if they are suitably arranged (as described in the next section), optimal matched filter detection in additive white Gaussian noise may be performed merely by inverting the transformations.

Note that Direct Sequence modulation using a binary or pure phase waveform is a unitary transformation in continuous time.

One complication of discrete-time transformational architectures (elementary transformation architectures, as well as frequency analysis/synthesis techniques) is that they tend to produce critically sampled representations of their output signals; hence, accurately smoothing the output samples to a continuous-time signal may not be simple.



Figure 5. Time-varying filter composed of overlapped transformations (hard to invert).

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DEMODULATION

The optimal way to demodulate TVF spread spectrum is by matched filtering. Consider general TVF modulators modeled in discrete time. At the time of each input data sample, the modulating TVF has a different impulse response. To demodulate, one should matched filter to each of these instantaneous impulse responses, properly timed. Note that these responses are in general overlapping, and it may be necessary to remove intersymbol interference (ISI) by some other method (such as a decision technique for binary data).

This demodulation procedure is equivalent to a particular time-varying filter, which we call the matched time-varying filter. If the modulator is a unitary transformation, and the noise, plus other interference, is modeled as additive white Gaussian noise (AWGN), then the matched TVF is simply the inverse of the modulating TVF, and there is no ISI. This is an important benefit of unitary transformations. It is one of the reasons binary or pure-phase Direct Sequence, a unitary transformation whose inverse is just its complex conjugate, has been so successful.

We are thus led to consider under what conditions the modulating TVF is easy to invert. This brings up the issue of FIR vs. IIR (finite vs. infinite impulse response).

A TVF can be IIR and have an inverse which is IIR. Also, a FIR TVF can have an IIR inverse; an example, built from elementary transformations, is shown in Figure 5. Because of the way the elementary transformations in this TVF overlap (which may be described as convolutional), the inverse most likely would be difficult to implement. (Although we could easily design a structure with enough degrees of freedom to implement the inverse to a good approximation, computing what the coefficients should be would be difficult, since they would in general be time-varying and would not correspond one-to-one with the forward coefficients.)

In fact, a FIR time-varying filter can be constructed whose inverse is FIR (unlike the case for time-invariant filters, where a nontrivial FIR filter must have an IIR inverse). One way to construct such a FIR TVF is to build it out of elementary unitary transformations organized in a nonoverlapping manner at each level, as shown in Figure 6. Its inverse will be FIR and will have the same architecture, with the order of the transformations reversed; the coefficients will be in one-toone correspondence with the coefficients of the forward filter, greatly facilitating implementation.



Figure 6. Time-varying filter composed of elementary unitary transformations, non-overlapping at each level.

ANALOGY TO ERROR-CORRECTING CODING AND INTERLEAVING

TVF spread spectrum encompasses spreading in time as well as in frequency. Spreading in time introduces a form of protection from errors similar to error-correcting coding and interleaving. This benefit can be realized if we try to recover a transmitted message when a short interval of the transmission has been corrupted by interference. For Direct Sequence transmissions, the interference affects all data sent during that interval, and that data will generally be lost if the interference is strong. For TVF transmissions (even if the interference is strong), if the interval is short relative to the extent of spreading in time, one may ignore the received signal during that interval (i.e., treat it as an erasure) and have an excellent chance of reconstructing the signal. (A simple alternative to zeroing out reception during corrupted intervals is to apply amplitude limiting at all times.) If the signal is binary or discrete-valued, then squaring up the detected signal to the nearest admissible amplitude—a procedure analogous to finding the nearest codeword—may be sufficient to reproduce the message with no errors. If the signal is analog, the output of the demodulator may be a good approximation.

SYNCHRONIZATION

As for any spread spectrum receiver, synchronization is necessary; this requirement is as much a property of time-varying filters as it is of spread spectrum. Both Time ("Code") and Frequency ("Carrier") synchronization must be acquired.

An important consideration in devising a method for acquiring synchronization is to avoid having to perform a two-dimensional search over a large number of time and frequency offsets. Synchronization can be achieved more quickly if the search space is one-dimensional.

For Direct Sequence spread spectrum reception, a one-dimensional search just in the time variable is sufficient because the despreading TVF commutes with a frequency shift. Regardless of whether any frequency shift has occurred after modulation in the transmitter, code synchronization can be acquired first by attempting to despread with the DS code at various candidate timings and looking for a narrowing of the spectrum. Carrier sync (shifting that spectrum to take out any frequency shift) can be acquired afterwards.

For a general TVF spread spectrum system, the TVF modulator and demodulator do not commute with a frequency shift. In general, this would imply the need for a two-dimensional search to acquire sync since if one of the variables is off target, there may be no indication of closeness to sync. Whether the search in the frequency dimension needs to be extensive or minimal would depend on the amount of phase uncertainty over the time extent of the TVF's instantaneous impulse response. (One may truncate the impulse response of the despreading TVF to reduce that uncertainty.)

If the spreading TVF is a transversal filter with fixed weights, acquiring synchronization should be simpler because the time uncertainty is as short as the separation between data samples. (The frequency uncertainty would be no greater or less than otherwise.) This form of TVF may be particularly suitable for civilian CDMA applications because it provides some of the advantages of more general TVFs (in terms of potential for control of bandwidth utilization, and in terms of robustness to errors due to spreading in time) while keeping the hardware requirements and synchronization burden to a minimum.

Analogously, any of the three TVF models can be made easier to synchronize by restricting the coefficients to periodic rather than pseudorandom variation.

IMPLEMENTATION

One way to implement a general TVF would be with discrete-time digital methods on parallel microprocessor hardware. Because of the difficulty of performing the calculations in real time, operation at useful spread spectrum bandwidths may have to wait for a new, faster generation of microprocessor hardware.

The discrete-time nature of the samples leads to a problem of fractional time shifts: If the sampling instants at the receiver do not exactly match the sampling instants at the transmitter, then some method of shifting is necessary. It may be done either electronically (prior to discrete-time sampling and digitization) or mathematically (by interpolating between samples). Note that under a fractional time shift, a FIR filter would become IIR because an optimal interpolation would utilize the sinc function. Integration of the interpolation with the TVF would be desirable; however, computation of the composite filter may be quite complicated.

APPLICATIONS

When transmitting in a jamming or noisy environment, the capability of a TVF modulator to spread the effect of each data sample in time may diminish the need for additional coding as a supplement to spread spectrum. (Spreading the effect of each data sample in this way provides error protection similarly to error-correcting coding and interleaving.)

Time-varying filters may be useful for generating near-Gaussian noise-like spread spectrum signals which are hard to intercept. Two factors contribute to this: the potential of the filters' impulse responses to assume values from a continuum; and the capability of the filters to have very long impulse responses, so that the effects of many data bits overlap at each point in time.

TVF spread spectrum may also be useful in civilian code division multiple-access applications, particularly if bandwidth constraints are important, because it provides more control than DS to assure that transmissions fit into bandwidth constraints. (Smoothing samples generated in discrete time constrains the output to the Nyquist bandwidth without impairing reception.) To simplify hardware and synchronization requirements, the form of TVF modulator may be used that consists of a transversal filter with fixed weights.

Conversely, TVF techniques may be used to optimally demodulate DS signals that were filtered to satisfy bandwidth constraints.

Choosing good multiple-access codes is likely to be easier for TVF spread spectrum than for DS because for a given bandwidth spreading ratio, overlapping the responses allows the codes to be longer. The longer codes are likely to cause less statistical fluctuation in the near-orthogonality conditions governing separation between channels. (Interference between channels can never be eliminated entirely, since exact orthogonality of all channels for all time shifts is theoretically impossible.)

A TVF with a bandwidth spreading ratio too small for spread spectrum may nevertheless be useful as a modern if it provides a degree of protection from errors through spreading in time.

REFERENCES

Crochiere, R. E., and L. R. Rabiner. 1983. Multirate Digital Signal Processing. Prentice-Hall.

Ericson, T. H. 1981. "Modulation by Means of Linear Periodic Filtering," *IEEE Transactions on Information Theory*, vol. IT-27, no. 3. May 1981.

Gardner, W. A. 1988. Statistical Spectral Analysis: A Nonprobabilistic Theory. Prentice-Hall.

- Ishii, R., and M. Kakishita. 1990. "A Design Method for a Periodically Time-Varying Digital Filter for Spectrum Scrambling," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 7, pp. 1219–1222. July 1990.
- Sakurai, K., K. Koga, and T. Muratani. 1984. "A Speech Scrambler Using the Fast Fourier Transform Technique," *IEEE Journal on Selected Areas in Communications*, vol. SAC-2, no. 3. May 1984.
- Wyner, A. D. 1979. "An Analog Scrambling Scheme which Does Not Expand Bandwidth, Part I: Discrete Time," *IEEE Transactions on Information Theory*, vol. IT-25, no. 3, pp. 261–274. May 1979.
- Wyner, A. D. 1979. "An Analog Scrambling Scheme which Does Not Expand Bandwidth, Part II: Continuous Time," *IEEE Transactions on Information Theory*, vol. IT-25, no. 4, pp. 415–425. July 1979.

APPENDIX A

2 X 2 UNITARY TRANSFORMATION (MATHEMATICAL DESCRIPTION)

Any complex 2 x 2 unitary transformation can be expressed by a matrix Q where Q $Q^{\dagger} = 1$ ([†] denotes complex conjugate transpose). This condition implies that the matrix may be expressed in the form

$$Q = \begin{bmatrix} e^{j\alpha}\cos\theta & -e^{j\beta}\sin\theta \\ e^{j\gamma}\sin\theta & e^{j\delta}\cos\theta \end{bmatrix}$$

where $\alpha - \beta - \gamma + \delta = 0$ modulo 2π .

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Time-varying linear filtering can be used to generate a broad class of spread spectrum signals which includes Direct Sequence spread spectrum as a special case. The technique is based on a filter of arbitrary length whose impulse response changes rapidly. It can spread the data signal in time as well as in frequency, thereby introducing a form of protection from errors similar to error-correcting coding and interleaving. Time-varying filter (TVF) systems can produce a broad range of noise-like waveforms and can be designed to minimize spillover into adjacent frequency regions. These properties should provide benefits in defense as well as in civilian radio communication systems. Three architectures for time-varying filters were identified: time-domain transversal filter, frequency-domain filter, or a combination of unitary transformations. All three architectures have similar complexity, dependent on the extent of spreading in both frequency and time. The unitary transformation approach, because of its energy-preserving nature, allows a TVF to be implemented in a distributed fashion without risk of losing the signal. Also, if the unitary transformations are suitably arranged, optimal matched-filter detection in additive white Gaussian noise may be performed merely by inverting the transformations. Synchronizing the receiver may be challenging because a TVF is part of a broader class than Direct Sequence and, in general, does not commute with a frequency shift.					
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