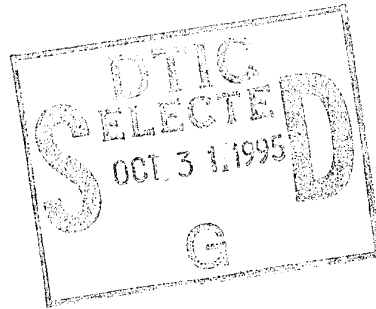


Final Report

Grant Number: N00014-92-J-1330
ONR Sponsored Research for 1992-1994

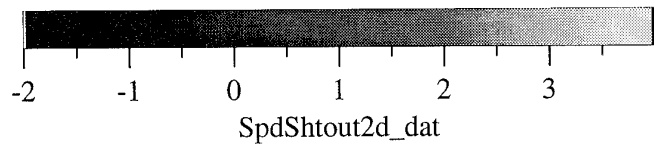
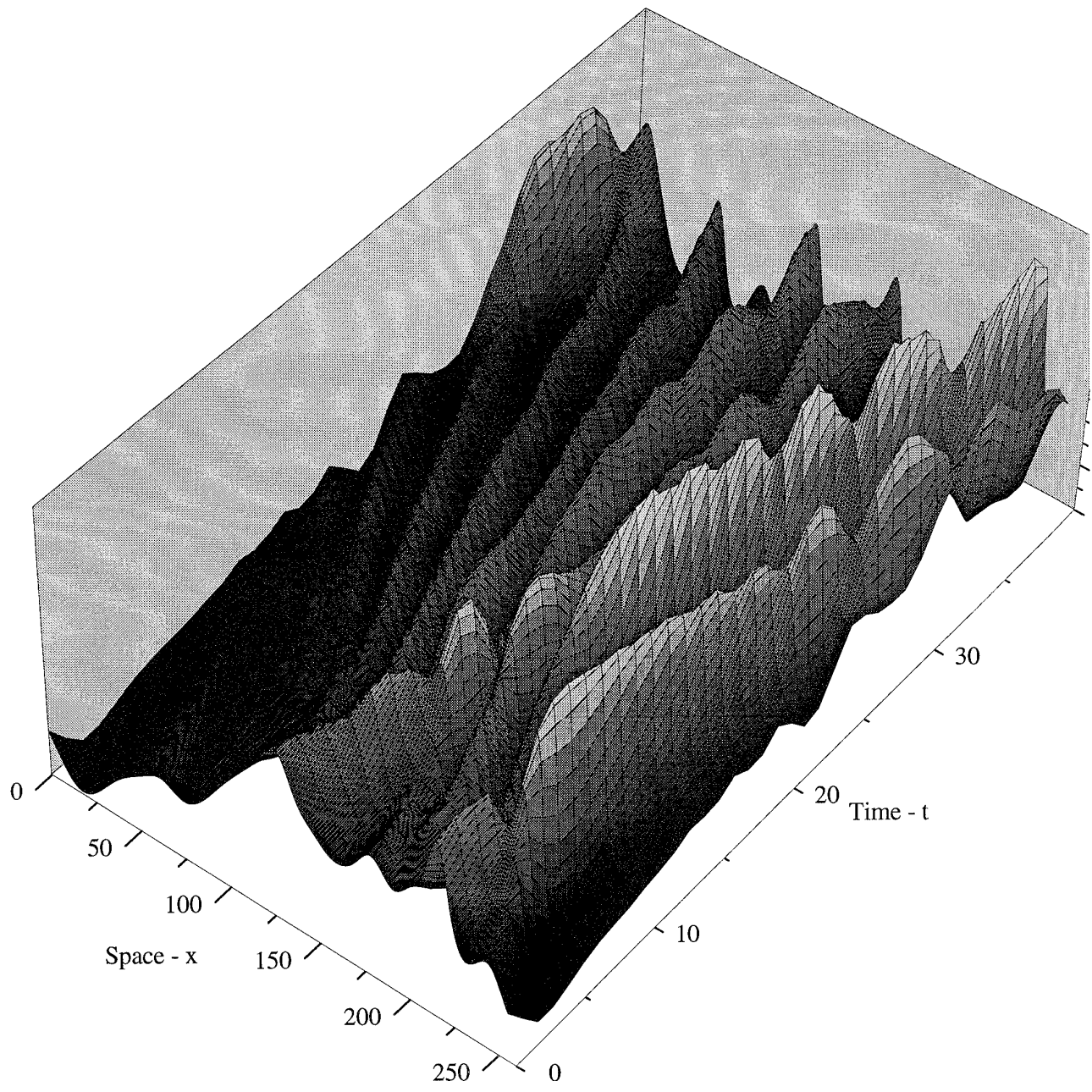
**Nonlinear Fourier Structure of Complex Surface Wave Trains:
A Search for Integrable and Chaotic Effects
in the Dynamics of Ocean Waves**

A. R. Osborne



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Cover Illustration: Shown is an example of the nonlinear evolution of a wave train in the *shallow water internal wave field*. The initial wave train at time $t = 0$ is assumed to be *linear and random*; its spectrum is governed by that of Garrett and Munk. Surprisingly, the random initial wave train *spontaneously emits a burst of internal solitons* for $t > 5$ [9, 15, 20] (see also Figs. 5(a), (b)). *Wave trains of this type are known to have a substantial and often devastating impact on many kinds of offshore operations, including drilling and production operations, ship and submarine traffic, and acoustic wave propagation.*

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Grant Number: N00014-92-J-1330

***Nonlinear Fourier Structure of
Complex Surface Wave Trains:
A Search for Integrable and Chaotic Effects
in the Dynamics of Ocean Waves***

Principal Investigator:

A. R. Osborne

*Istituto di Fisica Generale, Università di Torino
Via Pietro Giuria 1
10125 Torino, Italy*

INTERNET: osborne@to.infn.it
Telephone: (39) 11-670-7455 or (39) 11-822-4739
FAX: (39) 11-658444

Nonlinear Fourier Structure of Complex Surface Wave Trains: A Search for Integrable and Chaotic Effects in the Dynamics of Ocean Waves

A. R. Osborne

The main focus of the research reported on here is the development of *new nonlinear Fourier spectral analysis methods*, based upon the *inverse scattering transform (IST)*, for analyzing *measured time series* of laboratory and oceanic wave trains. A major emphasis has been the use of the technique for the analysis of substantial quantities of wave data in order to better understand the *nonlinear effects* which are important in oceanic surface and internal wave fields. A secondary objective of the research has been the analysis of time series of measured wave trains for the *effects of chaos* and to contrast these results with *nonlinear stochastic models*. *New data analysis procedures for the study of chaotic and stochastic dynamics in ocean waves* have also been developed.

An important underlying motivation for this work has been the potential impact of the methods on offshore operations of various types, e.g. how *nonlinear Fourier techniques* can be used to assess and contribute to the understanding of the behavior of surface and subsurface floating and tethered vehicles, on the propagation of acoustic waves in the internal wave field, etc.

Overview of the Scientific and Technological Objectives

1. New data analysis tools, which we refer to as the *numerical inverse scattering transform (NIST)*, are a *generalization of ordinary linear Fourier analysis to nonlinear wave motions*. One kind of nonlinear Fourier basis function is the so-called "hyperelliptic oscillation mode," which is generally not a sine wave, but has other, less-familiar shapes (see Fig. 1).

2. More recent work, completed only during the past few months, relates to the discovery of new methods, also based on the inverse scattering transform, which allow for a *decomposition of ocean surface waves into cnoidal waves* (see Fig. 2). The ordinary linear Fourier transform (among the most useful of all existing tools for the analysis of data and for the engineering design of floating vessels of all types, etc.) uses a linear superposition of sine waves; this technique does not easily include nonlinear effects in models of ocean waves. Nonlinear Fourier analysis, as discussed herein, consists of a *linear superposition of cnoidal waves plus their mutual nonlinear interactions*. The approach, sought by many investigators for the past 100 years, now has a complete and exact formulation thanks to recent research sponsored by ONR [9, 15, 20]. Progress has been made, not only in the one dimensional problem of Fig. 2, but also in the two dimensional problem illustrated in Figs. 3, 4 [21]; the latter results suggest that a full accounting of *nonlinear directional spreading of ocean surface waves* is now possible.

3. Further work relates to the search for numerical algorithms for NIST which are *fully automatic* [11]. These are the first steps toward the eventual development of numerical algorithms which could be used by individuals who are not experts in inverse scattering theory. Additional aspects of this work include extension of NIST algorithms to cases where the waves are directionally spread and propagate over uneven bathymetry.

4. Two aspects of *chaotic dynamics* are being pursued [12, 16, 17, 18]. The first consists of the *analysis of wave data* using several of the classical dynamical systems diagnostics. A second study is under way to extend NIST to higher order to provide for *new approaches to study chaotic surface wave motions*.

Approach:

Certain *nonlinear partial differential wave equations* are completely integrable (i.e. solvable) by the inverse scattering transform. *Inverse scattering theory is just a kind of nonlinear Fourier analysis for the equation of interest*. The particular nonlinear wave

equations under study by us are the Korteweg-deVries equation (one dimensional, shallow water waves), the nonlinear Schroedinger equation (one dimensional, shallow and deep water waves) and the Kadomtsev-Petviashvili equation (two dimensional, shallow water waves). In the context of these equations we have developed a number of numerical algorithms for the time series analysis of ocean surface wave data.

Tasks Completed or Technical Accomplishments:

We have made considerable theoretical progress with regard to IST and to the development of numerical codes [1-5, 7, 11, 19]. We have applied these approaches to the analysis of both laboratory [6, 10] and oceanic field data [8, 13, 14, 20]. A separate study of chaotic motions in wave data has also been conducted [1,12, 16, 17, 18].

Results:

1. A basis for computing *stochastic solutions* of shallow water and internal wave dynamics has been developed using IST [9,15, 19, 20]. This is a first step toward an understanding of *nonlinear random processes* and *nonlinear power spectral analysis* techniques. An example of the application of this new technology to one aspect of ocean engineering is given on the cover of this report and in Figs. 5(a), (b).

2. One of the most important results of recent research is the discovery of the *cnoidal wave spectral decomposition* of measured wave data. Cnoidal waves are a well-known nonlinear wave form discovered 100 years ago by Korteweg and deVries; these waves are nonlinear generalizations of the sine wave and are close relatives of the Stokes wave. An important open question in mathematical physics for the last century has been: Can one do Fourier analysis with cnoidal waves instead of with the ordinary sine wave? Thanks to recent research [9,15, 20, 21] the answer is "yes." An example is shown in Fig. 2 in which one sees that the *spectral representation for shallow water waves consists of a linear superposition of cnoidal waves plus their mutual nonlinear interactions*. In the small-

amplitude linear limit the interaction term tends to zero, the cnoidal waves tend to sine waves, and in this way the ordinary linear Fourier transform is recovered. It is in this sense that nonlinear Fourier analysis is a generalization of linear Fourier analysis.

3. Progress has also been made with regard to nonlinear Fourier analysis for the Kadomtsev-Petviashvili equation. This work addresses the understanding of nonlinear spectral properties in waves which are *directionally spread* [21]. Numerical examples are given in Figs. 3, 4. *These are the first fully nonlinear, spectrally based, two-dimensional wave trains ever generated.*

4. An automatic algorithm has been developed for computing the NIST of time series and for *nonlinear filtering of data* [11]. We have made an extensive *analyses of shallow water wave data from the Adriatic Sea and a wave tank in Florence* [6, 10, 14, 13, 20]. See Fig. 6 for an example of the analysis of the Adriatic Sea data.

5. A study of chaotic motions in both the laboratory and in the Adriatic Sea has been conducted during the past two years [1, 12, 16, 17, 18]. The primary data analysis tools have been the Grassberger and Procaccia correlation dimension and Lyapunov exponents. In spite of the fact that both of these methods often suggest the possibility of chaotic behavior in our data, the correct interpretation is that *the data are essentially stochastic*, and that the finite correlation dimensions and positive Lyapunov exponents result from the anomalous statistical behavior of certain near-Gaussian random processes. Our results bring into question the ability of certain diagnostics to perceive the presence of chaos in "real world" or "large degree of freedom" data. The search for more reliable indicators of deterministic chaos in complex experimental systems will be a major task for future studies. See Fig. 7 for an example of the analysis of the Adriatic Sea and Florence wave tank data.

6. *Coherent structures* in laboratory data and in the mathematical structure of the IST solutions of the KdV equation have been found [6, 10]. This work establishes the import

role of these structures in the *spectrum* of nonlinear Fourier analysis. Thus for the first time coherent structures (this likely also includes vortices, eddies, etc. in other types of fluid motions) can be analyzed in data as *single nonlinear spectral components* in the inverse scattering transform.

Impact for Science or Systems Applications

Applications will come from use of nonlinear Fourier analysis techniques in a wide range of scientific fields. Systems applications in which nonlinear Fourier methods are important include not only *surface, but also internal wave motions, acoustic wave propagation and the design of both fixed and floating offshore structures.*

Relationship to Other Programs or Projects

An intimate relationship between our results and other projects exists because the sea surface provides a major forcing input to many kinds of offshore activities, including the *dynamics of floating and drilling vessels, barges, risers, and tethered and submerged vehicles.*

PUBLICATIONS FROM ONR SPONSORED WORK

Grant Number: N00014-92-J-1330

1 January 1992 - 31 December 1994

A. R. Osborne

1. Osborne, A. R. and A. Pastorello, Simultaneous Occurrence of Low-dimensional Chaos and Colored Random Noise in Nonlinear Physical Systems, *Phys. Lett. A*, **181** (1993) 159-171.
2. Osborne, A. R., Construction of Nonlinear Wave Train Solutions of the Periodic, Defocusing Nonlinear Schroedinger Equation, *J. Comp. Phys.*, **109**(1) (1993) 93-107.
3. Osborne, A. R., The Numerical Inverse Scattering Transform for the Periodic, Defocusing Nonlinear Schroedinger Equation, *Phys. Lett. A*, **176** (1993) 75.
4. Osborne, A. R., The Numerical Inverse Scattering Transform: Nonlinear Fourier Analysis for Laboratory and Oceanic Wave Data, to appear in *Future Directions of Nonlinear Dynamics in Physical and Biological Systems*, edited by P. L. Christiansen, C. Eilbeck and R. Parmentier, Elsevier, Amsterdam (1993).
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6. Osborne, A. R. and M. Petti, The Numerical Inverse Scattering Transform Analysis for Laboratory-Generated Surface Wave Trains, *Phy. Rev. E*, **47**(2) (1993) 1035.
7. Osborne, A. R., Numerical Construction of Complex, Nonlinear Wave Train Solutions of the Periodic Korteweg-deVries Equation, *Phy. Rev. E*, **48**(1) (1993) 296.
8. Osborne, A. R., The numerical inverse scattering transform: Nonlinear Fourier analysis and nonlinear filtering of oceanic surface waves. Proceedings of the AHA HULIKO'A HAWAIIAN WINTER WORKSHOP, Edited by Peter Müller and Diane Henderson, University of Hawaii Press (1993).
9. Osborne, A. R., The behavior of solitons in random-function solutions of the periodic Korteweg-deVries equation, *Phys. Rev. Lett.*, **71**(19), 3115-3118 (1993).

10. Osborne, A. R. and M. Petti, Laboratory-generated, shallow-water surface waves: Analysis using the periodic, inverse scattering transform, *Phys. Fluids*, **6**(5) (1994) 1727-1744.
11. Osborne, A. R., Automatic Algorithm for the Numerical Inverse Scattering Transform of the Korteweg-deVries Equation, *Mathematics and Computers in Simulation*, **37** (1994) 431-450.
12. Osborne, A. R., M. Serio, L. Bergamasco and L. Cavaleri, Are Ocean Surface Waves Chaotic?, Second Experimental Chaos Meeting, edited by M. Spano, et al (1994).
13. Osborne, A. R., M. Serio, L. Bergamasco and L. Cavaleri, Nonlinear Fourier analysis and nonlinear filtering of Adriatic Sea surface waves near Venice, to appear *Physica D* (1994).
14. Osborne, A. R., Nonlinear Fourier Structure of Oceanic Surface Waves, *Solitons, Chaos and Fractals*, December (1995).
15. Osborne, A. R., Soliton Physics and the Periodic Inverse Scattering Transform, to appear in *Physica D* (1994).
16. Bergamasco, L., M. Serio and A. R. Osborne, Lyapunov exponents and correlation dimension for a Gaussian random process, submitted for publication (1994).
17. Bergamasco, L., M. Serio, A. R. Osborne and L. Cavaleri, Finite correlation dimension and positive Lyapunov exponents for surface wave data obtained in the Adriatic Sea near Venice, *Fractals*, March (1995).
18. Serio, M., Bergamasco, L. and A. R. Osborne, Fractal properties of surface water wave data, Nonlinear Dynamics Conference, Pavullo, Italy (1994).
19. Osborne, A. R., The solitons of Zabusky and Kruskal revisited II: A hyperelliptic function representation, submitted for publication (1994).
20. Osborne, A. R., Solitons in the periodic Korteweg-deVries equation, the θ -function representation and the analysis of nonlinear, stochastic wave trains, to appear in *Physical Review E* (1995).
21. Osborne, A. R., Shallow water cnoidal wave interactions, *Nonlinear Processes in Geophysics*, **1** (1994) 241-251.

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PUBLICATIONS/PATENTS/PRESENTATIONS/HONORS REPORT

1 January 92 - 31 December 94

R&T Number: **412s007---01**

Contract/Grant Number: **N00014-92-J-1330**

Contract/Grant Title: **Nonlinear Fourier Structure of Complex Surface Wave**

**Trains: A Search for Integrable and Chaotic Effects in the Dynamics of
Ocean Waves**

Principal Investigator: **A. R. Osborne**

Mailing Address:

Istituto di Fisica Generale

Universita' di Torino

Via Pietro Giuria 1

10125 Torino

Italy

Telephone: (39) 11-670-7455 or (39) 11-822-4739

INTERNET: osborne@astrto.ph.unito.it

FAX: (39) 11-658444

- a. Number of Papers Submitted to Refereed Journals but not yet published: 2.
- b. Number of Papers Published (or accepted by) Refereed Journals: 15.
- c. Number of Books or Chapters Submitted but not yet Published: 7 chapters.
- d. Number of Books or Chapters Published: 0.
- e. Number of Printed Technical Reports & Non-Refereed Papers (presented at Conferences): 4
- f. Number of Patents Filed: 0.
- g. Number of Patents Granted: 0.

h. Number of Invited Presentations at Workshop or Prof. Society Meetings: 9.

i. Number of Presentations at Workshops or Prof. Society Meetings: 11.

j. Honors/Awards/ Prizes for Contract/Grant Employees: 0.

k. Total number of Graduate Students and Post-Docs Supported at least 25% this year on this grant: 9. Grad Students 7 and Post-Docs 2 including Grad Students Female 3 and Post-Docs Female 1. No minorities.

FIGURE CAPTIONS

Figure 1. A measured time series (a) can be spectrally decomposed into the *hyperelliptic function oscillation modes* of the inverse scattering transform as shown in (b). These latter modes generally have nonlinear shapes but, nevertheless, obey a linear superposition law, i.e. their linear sum returns the input time series or allows the input wave train to be nonlinearly filtered. Nonlinear filtering is an important aspect of the work presented herein (see Fig. 6).

Figure 2. Four degree-of-freedom solution to the KdV equation in the so-called θ -function formulation (for which *cnoidal wave modes* are employed), an alternative theoretical approach to the hyperelliptic modes shown in Fig. 1. In (a)-(d) are the four *cnoidal waves* in the inverse scattering transform spectrum. The *modulus m of each cnoidal wave component* ($0 \leq m \leq 1$) is shown in its respective panel. In (e) is the contribution due to *nonlinear interactions*. The four degree-of-freedom solution to KdV is given in (f) and corresponds to the linear superposition of the four cnoidal waves (a)-(d) plus nonlinear interactions (e). The decomposition of measured wave trains into cnoidal wave plus interactions is a major emphasis in the present program.

Figure 3. An example of *nonlinearly, directionally spread waves* is presented in this figure. Four directionally spread cnoidal wave solutions of the KP equation are given. The cnoidal wave moduli are: (a) $m = 0.98$, (b) $m = 0.88$, (c) $m = 0.70$ and (d) $m = 0.37$. These cnoidal waves are identical to those given in the example of Fig. 2 for a solution of the KdV equation. The wave directions, however, have been distributed in the horizontal plane of the motion, as shown in panels (a)-(d).

Figure 4. Example solution to the KP equation based upon the cnoidal waves in Fig. 3. In panel (a) is the *linear superposition of these cnoidal waves*. The *nonlinear interactions*

are shown in panel (b). The solutions to KP is the sum of (a) and (b) and is given in (c). This is a concrete example of a *fully two dimensional, shallow water sea state*, evidently the first ever generated.

Figure 5(a). Stochastic solution to the KdV equation from the cover of this report (see also the figure caption on the second page). The initial conditions correspond to an internal wave field which is governed by the Garrett-Munk spectrum. Shown in this figure are the solutions to the equation at $t = 0$ and at $t = 20$. No solitons are visible in the initial conditions which are taken to be *random* (dotted line), while at the later time the solitons are easily visible (solid line). This result illustrates how purely stochastic solutions to the KdV equation can have space/time dynamics consisting of solitons in a random sea of radiation.

Figure 5(b). Space/time contours of the internal wave evolution shown on the cover. To the left of this diagram, for times less than $t = 5$, one see a rather featureless band (over all space) which corresponds to the random initial conditions. Thereafter, for times $t > 5$ one can see the *rays associated with the burst of solitons which are emitted by the initial conditions* (which is after all a kind of fission process). This *spontaneous emission of solitons can*, because of the large energetics involved, *have a substantial impact on floating vessels of all types, including submarines*.

Figure 6. θ -function inverse scattering transform analysis of surface wave data measured in the Adriatic Sea. The measured wave train is shown in (a). In (b) is the inverse scattering transform of the data given in (a); the solid jagged curve in (b) represents the radiation spectrum and the vertical arrows denote the solitons. Each spectral amplitude (solid, jagged line) corresponds to the amplitude of its associated cnoidal wave. The values of the spectral modulus, m ($0 \leq m \leq 1$), for each cnoidal wave component, are also shown. Values of $m \sim 1$ indicate the presence of *individual solitons* (to the left, at low frequency) or *soliton wave trains* (to the right, at

intermediate frequency). *Nonlinear filtering techniques* have been used to extract the low-frequency solitons from the data; these latter are shown in panel (a).

Figure 7. The search for chaotic motions in both the laboratory and the Adriatic Sea is summarized in panels (a)-(c). In (a) are the Grassberger and Procaccia correlation integrals (as a function of scaling length) for both sets of data and for numerical simulations of a Gaussian stochastic process based upon the well-known Pierson-Moskowitz power spectrum. The positive Lyapunov exponents of the data and Gaussian simulations are given in (b) as a function of the parameter $\phi \equiv 1 - f_p / f_N$ for f_p the peak spectral frequency and f_N the Nyquist frequency. Finally, the scaling exponent of Mandelbrodt is given in panel (c) as a function of ϕ . The results, for *particular* values of ϕ , suggest the presence of chaotic dynamics, *when in reality the dynamics are purely stochastic*. These results indicate the need for more sophisticated approaches for studying chaotic motions in "large degree of freedom" systems like ocean surface waves.

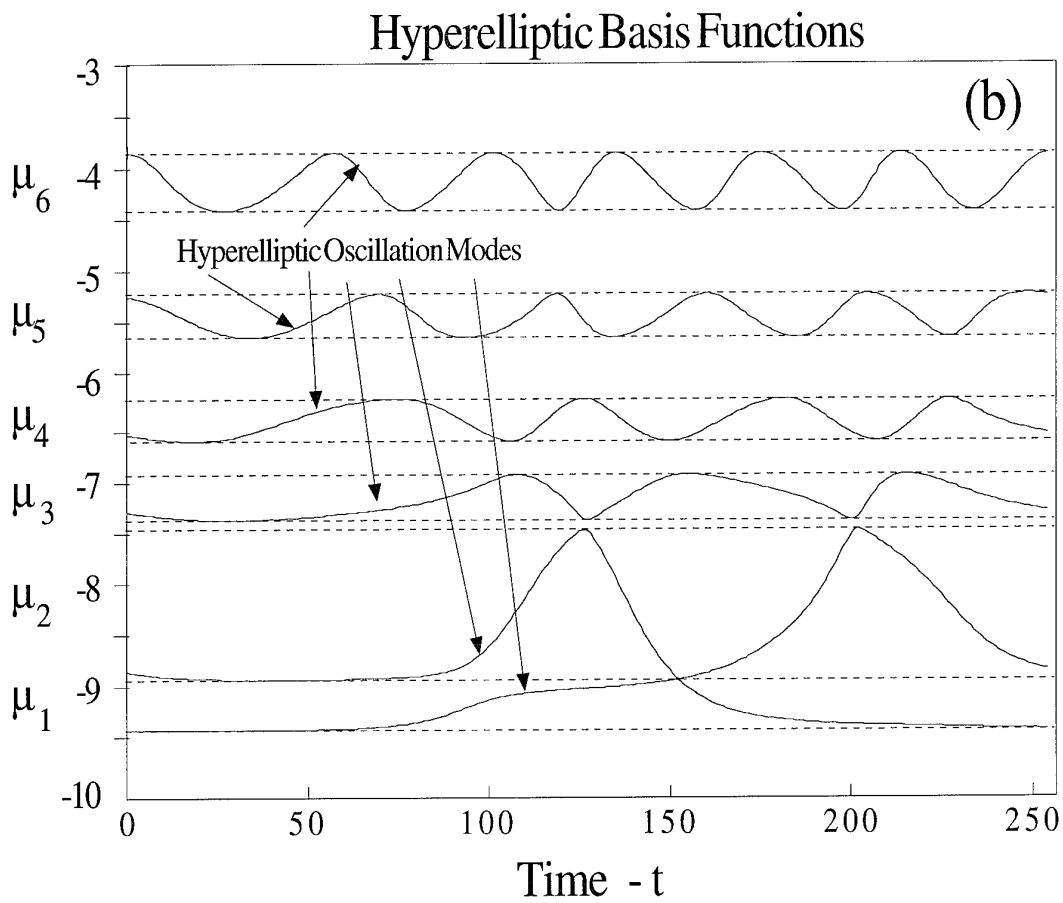
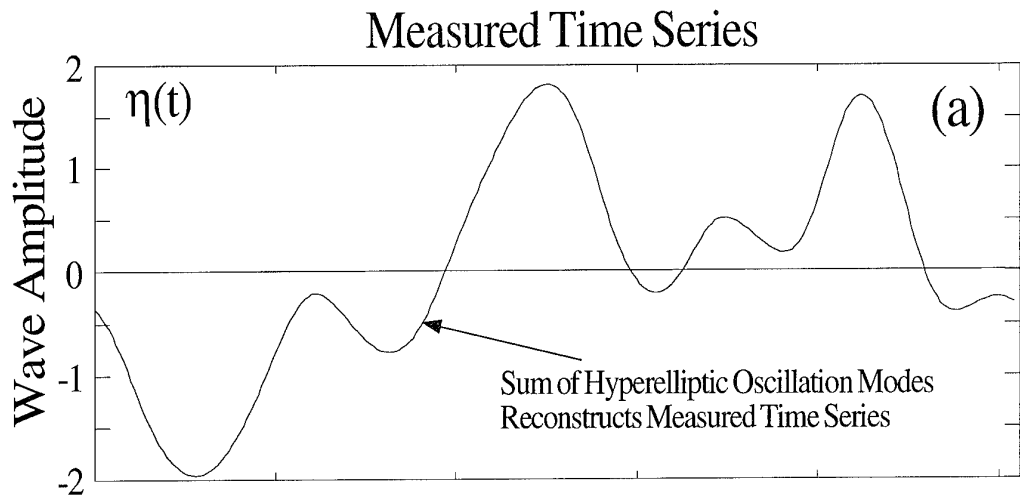


Figure 1

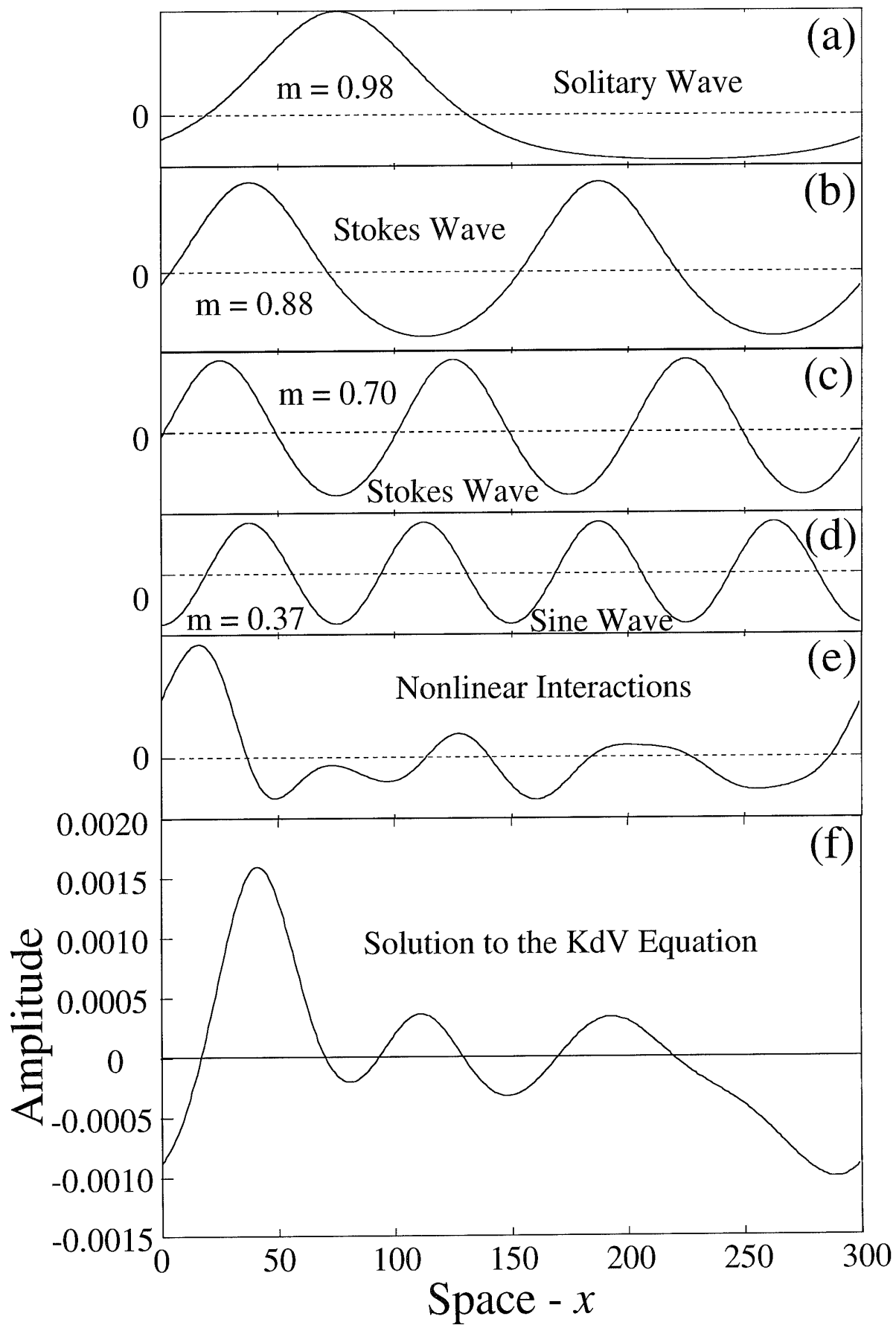


Figure 2

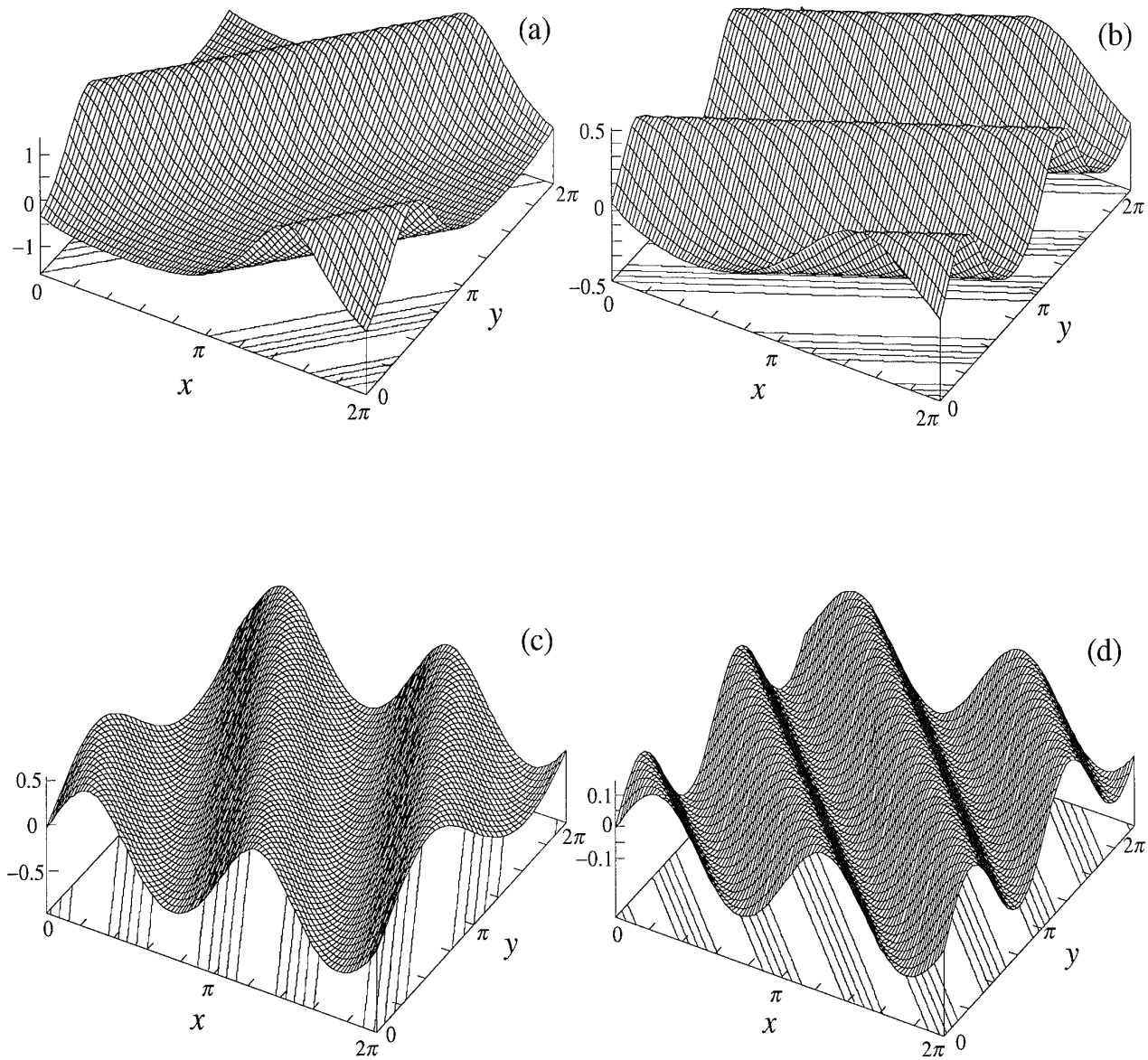


Figure 3

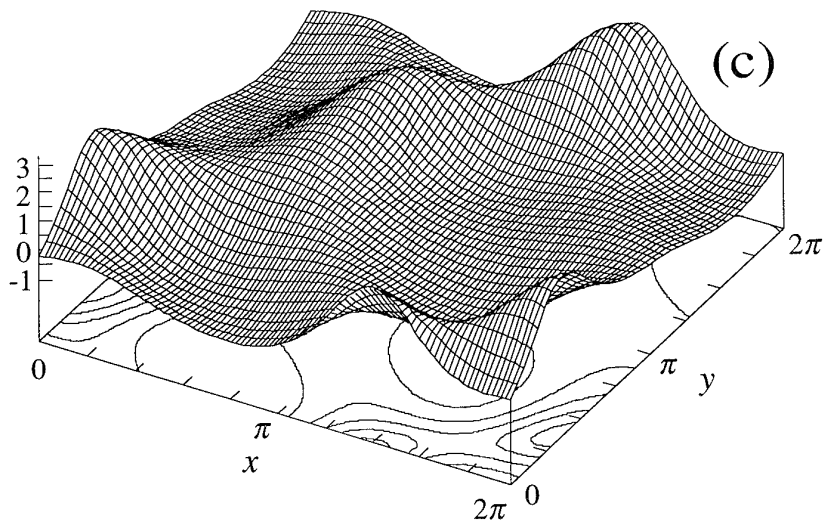
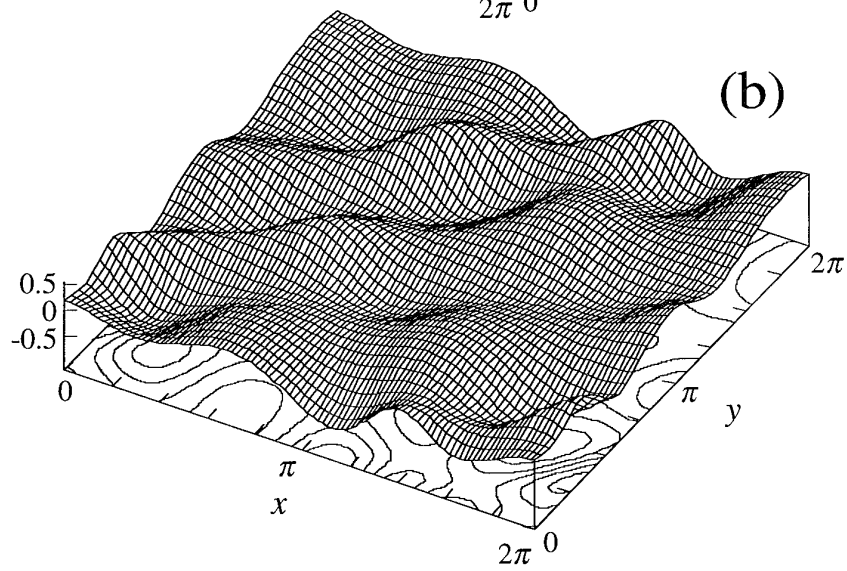
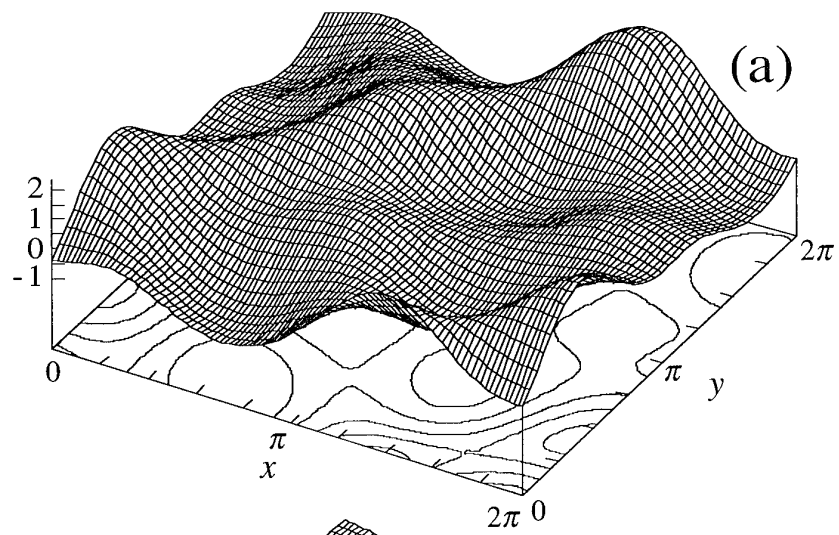


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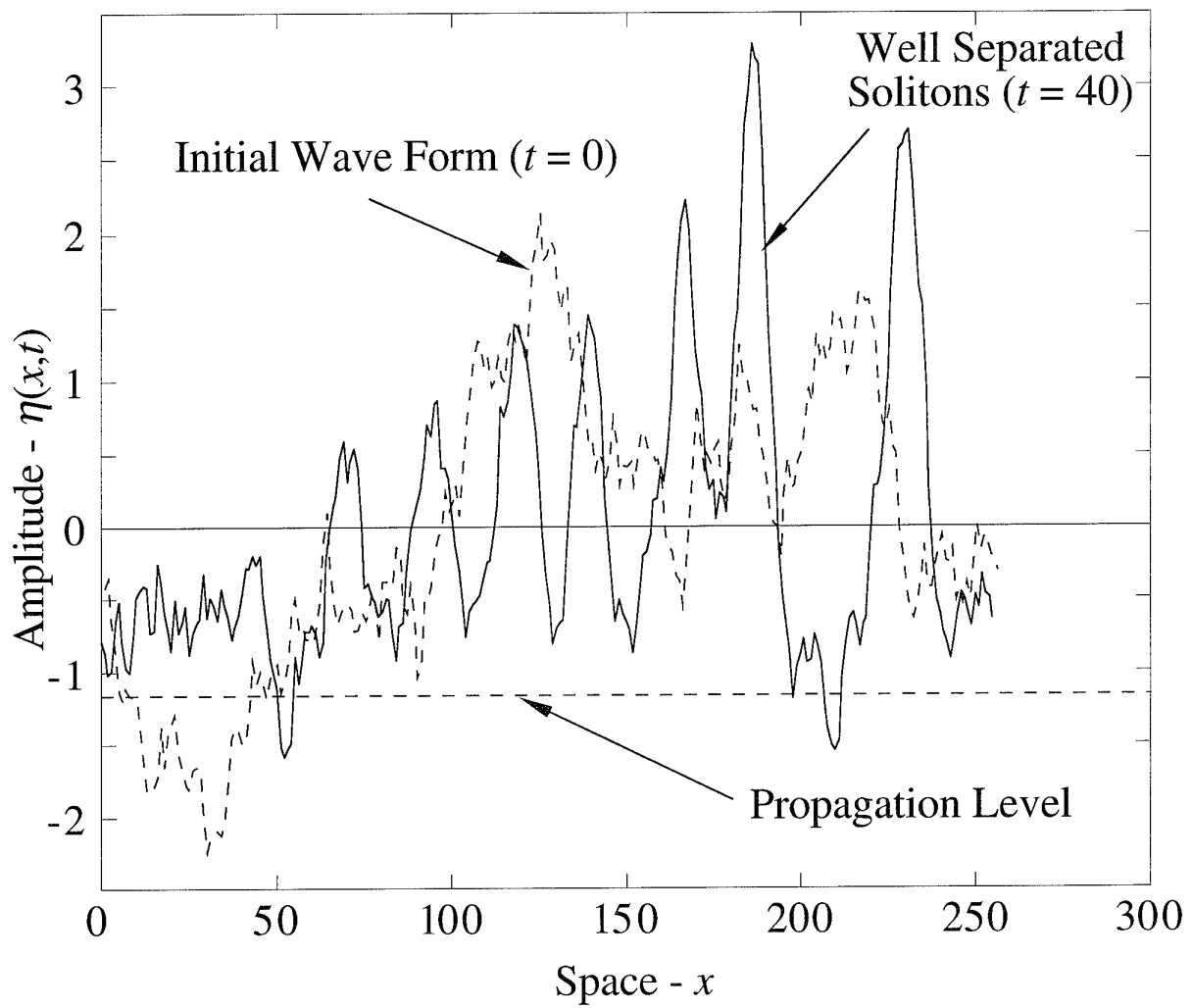


Figure 5(a)

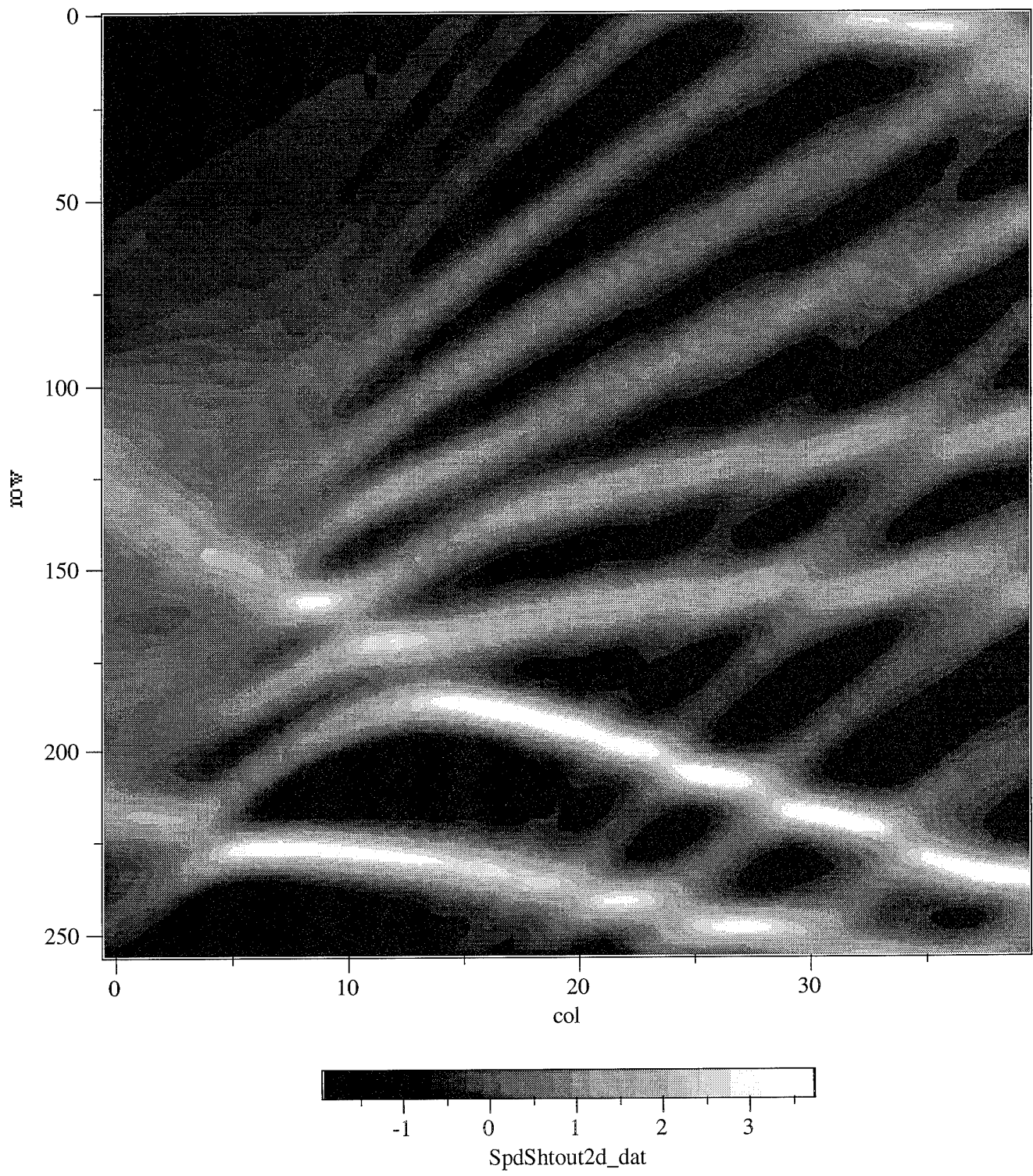


Figure 5(b)

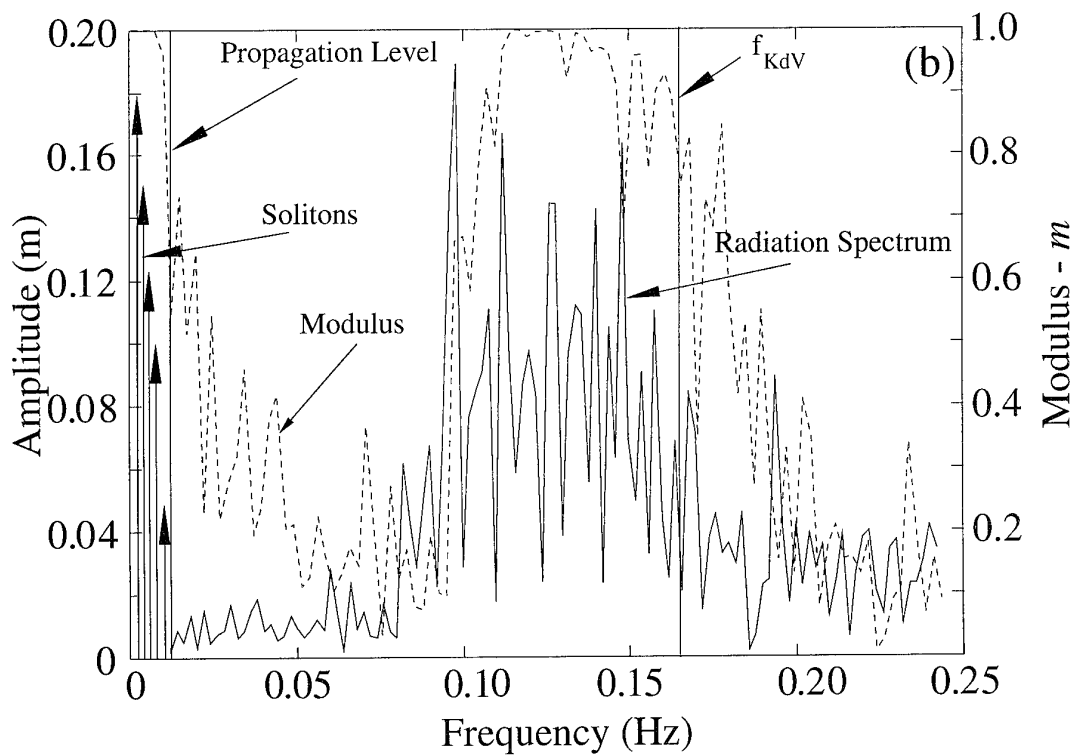
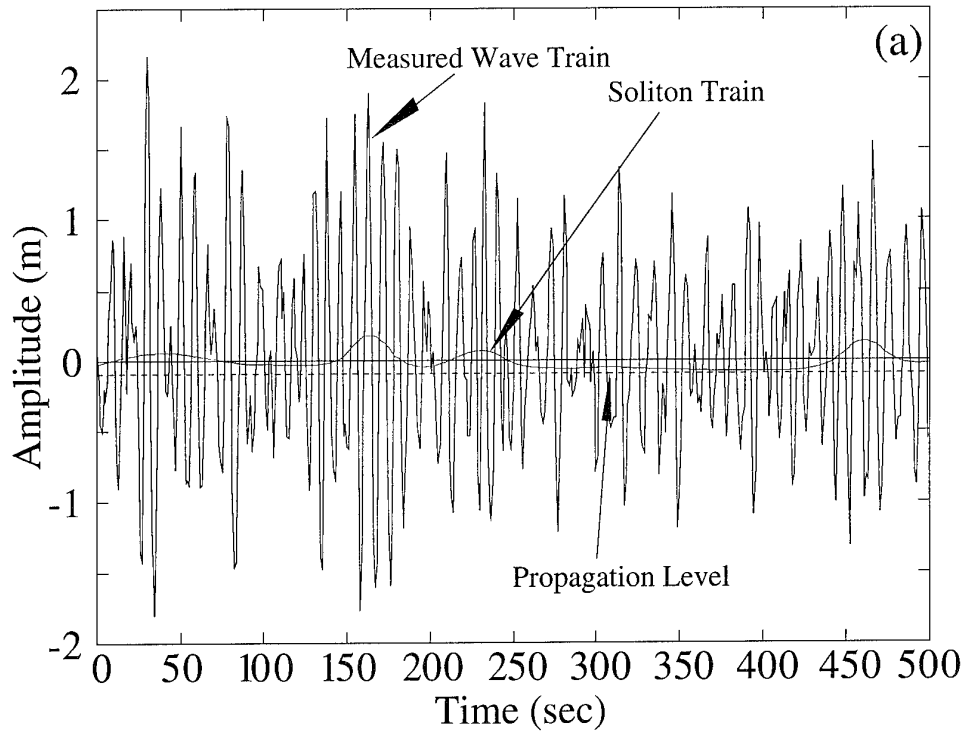


Figure 6

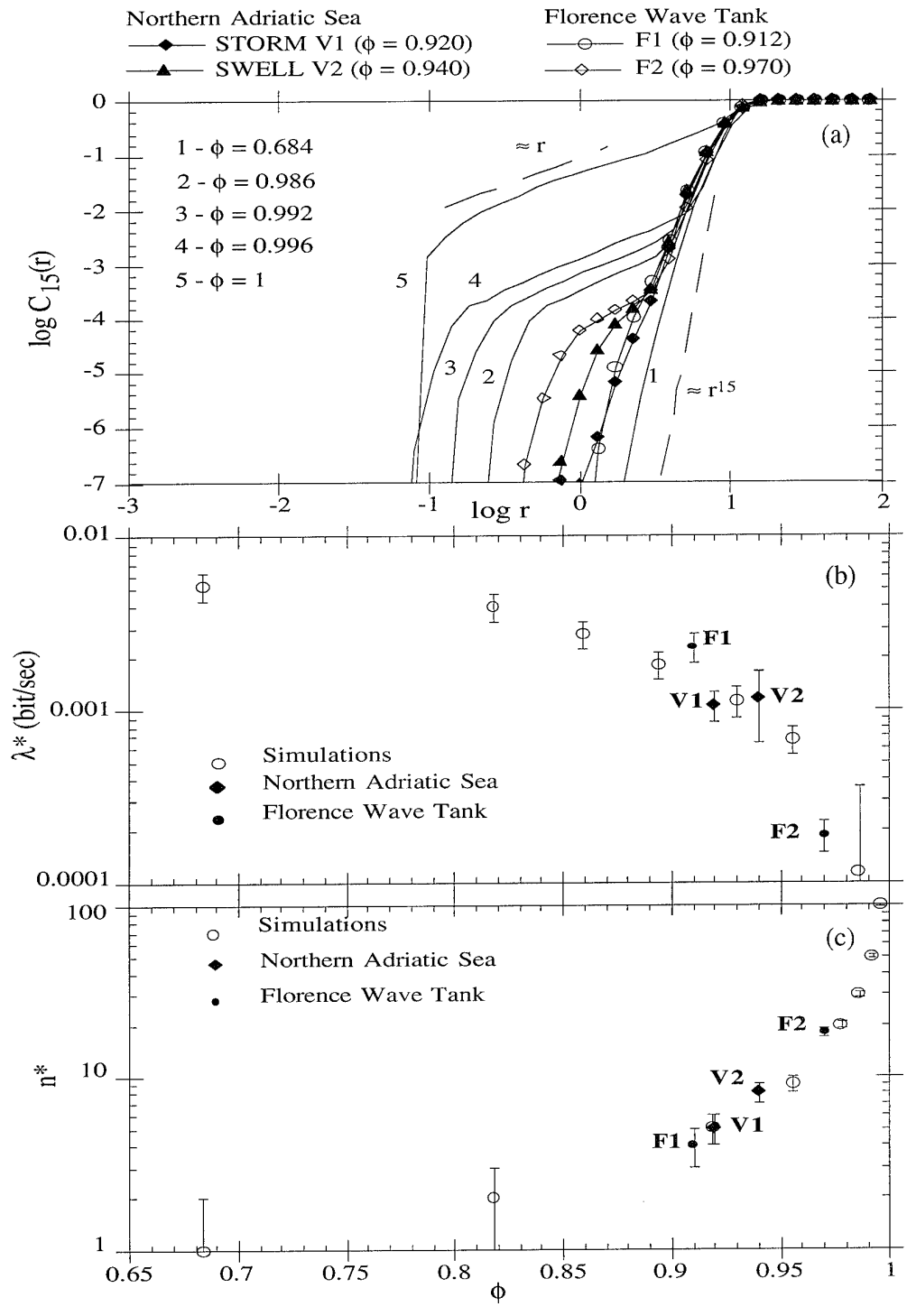


Figure 7



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N00014-92-J-1330
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