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Hydrodynamic descriptions of one-dimensional granular systems have emphasized that the usual master variables, density, velocity and temperature are inadequate. A minimal description requires a fourth variable which is the third moment or "skewness" of the single particle velocity distribution function. These supplemented hydrodynamic systems have been used to study the linear stability of the spatially homogeneous state and good agreement with numerical simulation has been claimed. The linear stability analysis also shows that the hydrodynamic equations are ill-posed because the instability has no high wavenumber cut-off. That is, arbitrarily small scale disturbances are linearly unstable. This problem with the hydrodynamic description is physically justified because inelastic collapse is a particle scale mechanism through which hydrodynamics fails.

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Final technical report: AASERT component of: Development and Evolution of Nearshore Topography

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Scientific Results

This AASERT award supported a graduated student, Sean McNamara, for two years. McNamara graduated in the early fall of 1994 and is now a post-doctoral fellow in the condensed matter group at the Université de Rennes.

McNamara's thesis was directed at understanding a fundamental problem in the continuum theory of granular flows. This is the problem of "granular cooling". Imagine starting a collection of identical, inelastic, hard, spherical particles in a thermalized state and letting the ensemble evolve freely. The dissipation of energy that accompanies the collision of macroscopic particles is modelled using a "coefficient of restitution" denoted by r. The classical hard core gas is the special case $r = 1$. Collisions conserve momentum, but the relative velocity of collision is reduced by a factor $0 \leq r \leq 1$. This means that each collision results in a loss of energy $\delta E \sim (1 - r^2)(\delta U)^2$ where δU is the relative velocity of collision.

If there is no addition of energy to the system then the ensemble of particles is losing energy at each collision so that the medium is "cooling down". Simple kinetic arguments[4] suggest that the total energy of the medium should decay like t^{-2}. If the medium sinply slowed down then granular cooling would be boring. However, the problem is interesting because the particles spontaneously bunch up so that clusters and voids appear in the medium[2,3,5]. That is, the homogeneously cooling state is unstable. We illustrate this instability in figure 1 by showing two simulations of the one dimensional granular medium (point particles) with $r = 0.8$. The particles are confined by reflective boundaries (also with $r = 0.8$) at $x = 0$ and $x = 1$. In the top panel there are $N = 10$ particles in the domain and they remain dispersed. In this case the t^{-2} cooling law is satisfied. In the lower panel there are $N = 20$ particles and 16 of these particles are bunching up near the wall at $x = 1$. The formation of this cluster invalidates the assumption that ℓ is constant as the medium cools.

The dynamics of the particles within the cluster is amusing: this bunch of particles collides infinitely often in a finite time[1,5]. Thus the particle spacing becomes zero and all of the energy of the cluster is dissipated in a finite time. This singularity, called inelastic collapse, requires a certain minimum number of particles, $N_{\min}(r)$, so that in figure 1 (a), $10 < N_{\min}(0.8)$ and the singularity never occurs. The elastic limit is singular because $N_{\min}(r) \to \infty$ as $r \to 1$.

Cluster formation and inelastic collapse also occur in the two dimensional granular cooling problem[2,3,7]. In figure 2 shows a simulation of a two dimensional granular medium in a doubly periodic domain. The snapshot in figure 2 is after about 68 inelastic collisions per disk. The 14 black disks, which lie in a roughly linear string, participated in all of the two hundred collisions which occured immediately before the snapshot in figure 2. Further diagnosis of the simulation reveals that the time between collisions is approaching zero geometrically with the collision count. Thus the black disks in figure 2 are collapsing inelastically and the one dimensional phenomenology is recapitulated in two dimensions. Indeed the number of particles in the string (13 or 14 in this case) agrees with analytic estimates[1,5] which have been mode for the one dimensional problem.

The discovery of these particle strings provides an additional impetus to study one-dimensional granular hydrodynamics: the time scales for the evolution of the string are much shorter than those in the rest of the system so that one-dimensional theories have local utility. Hydrodynamic descriptions of one-dimensional granular systems have emphasized that the usual master variables, density, velocity and temperature are inadequate. A minimal description requires a fourth variable which is the third moment or "skewness" of the single particle velocity distribution function[6,8]. These supplemented hydrodynamic systems have been used to study the linear stability of the spatially homogeneous state and good agreement with numerical simulation has been claimed. The linear stability analysis also shows that the hydrodynamic equations are ill-posed because the instability has no high wavenumber cut-off. That is, arbitrarily small scale disturbances are linearly unstable. This problem with the hydrodynamic description is physically justified because inelastic collapse is a particle scale mechanism through which hydrodynamics fails.

Bibliography *

[1] B. Bernu and R. Mazighi, "One-dimensional bounce of inelastically colliding marbles on a wall," *J. Phys. A: Math. Gen.* **23**, 5745 (1990).

[2] I. Goldhirsch and G. Zanetti, "Clustering instability in dissipative gases," *Phys. Rev. Lett.*, **70**, 1610 (1993).

[3] I. Goldhirsch, M.L. Tan, and G. Zanetti, "A molecular dynamical study of granular fluids I: the unforced granular gas in two dimensions," *J. Sci. Comp.*, **8**, (1993).

[4] P.K. Haff, "Grain-flow as a fluid mechanical phenomenon," *J. Fluid Mech.*, **134**, 401 (1983).

[5] S. McNamara and W.R. Young, "Inelastic collapse and clumping in a one-dimensional granular medium," *Phys. Fluids A* **4**, 496 (1992).

[6*] S. McNamara and W.R. Young, "Kinetics of a one-dimensional granular medium in the quasielastic limit," *Phys. Fluids A* **5**, 34 (1993).

[7*] S. McNamara and W.R. Young, "Inelastic collapse in two dimensions," *Phys. Rev. E* **50**, R28 (1994).

[8] N. Sela and I. Goldhirsch, "Hydrodynamics of a one-dimensional granular medium," *Phys. Fluids,* **7**, 507 (1995).

[9*] S. McNamara, "Hydrodynamic modes of a uniform granular medium," *Phys. Fluids A* **5**, 3056 (1993).

[10*] S. McNamara and W.R. Young, "On the dynamics of a freely evolving two dimensional granular medium," submitted to *Phys. Rev. E* (1995).

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Figure 1: From reference 5. (a) World lines of $N = 10$ particles with $r = 0.8$. The particles are confined in the interval $0 < x < 1$ by inelastic walls (also with $r = 0.8$). In this example the particles remain dispersed. (b) The same as part (a) except that there are now $N = 20$ particles. In this case a cluster forms near $x = 1$. Subsequently this cluster collapses: sixteen particles come into mutual contact via an infinite number of collisions in a finite time.

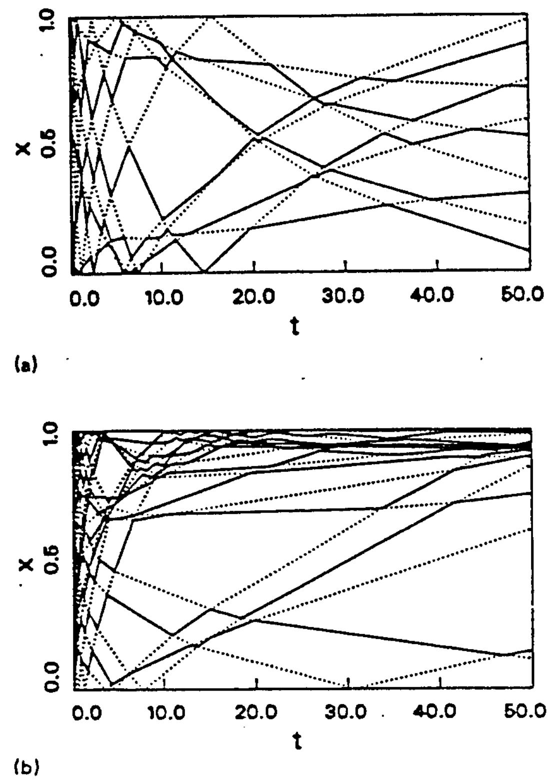

Figure 2: From reference 7. A snapshot of a two dimensional simulation with $N = 1024$ disks and coefficient of restitution $r = 0.6$. The disks occupy $1/4$ of the area and the doubly periodic computation domain is the dashed square. The system started in a kinetic state which was established by running with $r = 1$ for several hundred collisions per disk. Then r was reduced to 0.6 and the snapshot at right was taken after about 68 inelastic collisions per disk. At this time a system spanning cluster has formed. All of the disks which participated in the two hundred collisions that immediately preceeded this snapshot are shaded in black.