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OPTIMIZATION MODELING FOR AIRLIFT MOBILITY

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OPTIMIZATION MODELING FOR AIRLIFT MOBILITY

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Abstract: We describe a multi-period optimization model, implemented in GAMS, to help the U.S. Air Force improve logistical efficiency. It determines the maximum on-time throughput of cargo and passengers that can be transported with a given aircraft fleet over a given network, subject to appropriate physical and policy constraints. The model can be used to help answer questions about selecting airlift assets and about investing or divesting in airfield infrastructure.

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1. INTRODUCTION

In an Operation Desert Storm type scenario, massive amounts of equipment and large numbers of personnel must be transported over long distances in a short time. The magnitude of such a deployment imposes great strains on air, land and sea mobility systems.

The U.S. military services are well aware of this problem and various optimization and simulation models have been developed to help improve the effectiveness of limited lift assets and alleviate the problem. Congress commissioned the Mobility Requirement Study (MRS) in 1991, when post-operation analysis of Desert Storm revealed a shortfall in lift capability.

Two linear programming (LP) optimization models that were developed as part of MRS and subsequent studies form the primary background of this research. They are: (i) the Mobility Optimization Model (MOM) developed for MRS by the Joint Staff's Force Structure Resource, and Assessment Directorate (J8) [Wing *et al.*, 1991] and (ii) the THRUPUT Model developed by the USAF Studies and Analyses Agency (USAF/SAA) [Yost, 1994]. MOM considers both air and sea mobility, whereas THRUPUT and the model developed here cover only the air aspects of the problem. The model of this paper was first described in a Naval Postgraduate School master's thesis [Lim, 1994], which was sponsored by USAF/SAA.

In this research, the strategic airlift assets optimization problem is formulated as a multi-period, multi-commodity network-based linear programming model, with a large number of side constraints. It is implemented in the General Algebraic Modelling System (GAMS) [Brooke et al., 1992], and its purpose is to minimize late deliveries subject to physical and policy constraints, such as aircraft utilization limits and airfield handling capacities. For a given fleet and a given network, the model can help provide insight for answering many mobility questions, such as: 1) Are the aircraft and airfield assets adequate for the deployment scenario? 2) What are the impacts of shortfalls in airlift capability? 3) Where are the system bottlenecks and when will they become noticeable? This type of analysis can be used to help answer questions about selecting airlift assets and about investing or divesting in airfield infrastructure.

2. OVERVIEW OF MODEL

The analyses described above are accomplished through repeated runs of the model. Each run assumes a particular scenario as defined by a given set of time-phased movement requirements and a given set of available aircraft and airfield assets. It is then solved for optimal values for the number of missions flown, and the amounts of cargo and passengers carried, for each unit, by each aircraft type, via each route, in each time period.

2.1 Model Features

The model has been designed to handle many of the airlift system's particular features and modes of operation. For example, the payload an aircraft can carry depends on range (the shorter the range, the heavier the load), and aircraft with heavy loads may be required to make

one or more enroute stops. Also, there is a need to ensure cargo-to-carrier compatibility since some military hardware is too bulky to fit into certain aircraft. These features have been incorporated in the model to make it as realistic as possible. Others, such as the use of tanker aircraft for aerial refueling of airlift aircraft are recommended as follow-on work. (See CONOP by RAND [Killingsworth and Melody, 1994] for extensive treatment of aerial refueling in another GAMS-based optimization model.) The major features of the airlift system currently captured by the model include:

- Multiple origins and destinations: In contrast to MOM, the current model allows the airlift to use multiple origin, enroute and destination airfields.

- Flexible routing structure: The air route structure supported by the model includes delivery and recovery routes with a variable number of enroute stops (usually between zero and three). This provision allows for a mixture of short-range and long-range aircraft. The model can thus analyze trade-offs between higher-payload, shorter-range flights and lower-payload, longer-range flights. For further routing flexibility, the model also allows the same aircraft to fly different delivery and recovery routes on opposite ends of the same mission.

- Aircraft-to-route restrictions: The user may impose aircraft-to-route restrictions; e.g., only military aircraft may use military airfields for enroute stops. This particular provision arises because the USAF Air Mobility Command (AMC) may call upon civilian commercial airliners to augment USAF aircraft in a deployment, under the Civil Reserve Airfleet (CRAF) program. The model distinguishes between USAF and CRAF aircraft.

- Aircraft assets can be added over time. This adds realism to the model, because CRAF and other aircraft may take time to mobilize and are typically unavailable at the start of a deployment.

- Delivery time windows: In a deployment, a unit is ready to move on its *available-to-load date (ALD)* and has to arrive at the theater by its *required-delivery-date (RDD)*. This aspect of the problem has been incorporated in the model through user-specified time windows for each unit. The model treats the time windows as "elastic" in that cargo may be delivered late, subject to a penalty.

2.2 Conceptual Model Formulation

This section gives a verbal description of the key components of the airlift optimization model. The mathematical formulation is covered in detail in Section 3.

The primary decision variables are the number of missions flown, and the amounts of cargo and passengers carried, for each unit, by each aircraft type, via each available route, in each time period. Additional variables are defined for the recovery flights, for aircraft inventoried at airfields, and for the possibility (at high penalty cost) of not delivering required cargos or passengers.

2.2.1 Objective Function

The purpose of the optimization model is to maximize the effectiveness of the given airlift assets, subject to appropriate physical and policy constraints. The measure of effectiveness is the minimization of total weighted penalties incurred for late deliveries and non-deliveries. The penalties are weighted according to two factors: the priority of the unit whose movement requirement is not delivered on time, and the degree of lateness. The penalty increases with the amount of time late, and non-delivery has the most austere penalty.

The anticipated use of the model is for situations when the given airlift resources are insufficient for making all the required deliveries on time. On the other hand, if there are enough resources for complete on-time delivery, then the model's secondary objective function is to choose a feasible solution that maximizes unused aircraft. The motivation of the secondary objective is that if the available aircraft are used as frugally as possible, while still meeting the known demands and observing the known constraints, then the mobility system will be as well prepared as it can be for unplanned breakdowns and unforeseen requirements, such as an additional nearly simultaneous regional contingency.

2.2.2 Constraints

The model's constraints can be grouped into the five categories: demand satisfaction, aircraft balance, aircraft capacity, aircraft utilization, and airfield handling capacity.

- Demand Satisfaction Constraints: The cargo demand constraints attempt to ensure for each unit that the correct amounts of cargo move to the required destination within the specified time window. The passenger demand constraints do the same for each unit's personnel. The demand constraints have elastic variables for late delivery and non-delivery. The optimization will seek to avoid lateness and non-deliveries if it is possible with the available assets, or to minimize them if not.

- Aircraft Balance Constraints: These constraints keep physical count of aircraft by type (e.g., C17, C5, C141, etc.) in each time period. They ensure that the aircraft assets are used only when they are available.

- Aircraft Capacity Constraints: There are three different kinds of constraints on the physical limitations of aircraft -- troop carriage capacity, maximum payload, and cabin floor space -- which must be observed at all times.

- Aircraft Utilization Constraints: These constraints ensure that the average flying hours consumed per aircraft per day are within AMC's established utilization rates for each aircraft type.

- Aircraft Handling Capacity at Airfields: These constraints ensure that the number of aircraft routed through each airfield each day is within the airfield's handling capacity.

2.3 Assumptions

Some major assumptions of the model are listed below. These are known to be sacrifices of realism, but such assumptions are needed in modeling most real-world problems due to the limitations of data availability or the need to avoid computational intractability.

- Airfield capacity is represented by Air Force planners by a measure called *Maximum-on-Ground (MOG)*. The literal translation of MOG as the maximum number of planes that can be simultaneously on the ground at an airfield is somewhat misleading, because the term MOG means more than just the number of parking spaces at an airfield. In actuality, airfield capacity depends on many dimensions in addition to parking, including material handling equipment, ground services capacity and fuel availability. Some Air Force planners use the terms *parking MOG* and *working MOG* to distinguish between parking space limits and servicing capability. Working MOG is always smaller than parking MOG, and is the only MOG for which we have data. Working MOG is an approximate measure because it attempts to aggregate the capacities of several kinds of services into a single, unidimensional figure. Disaggregation of airfield capacity into separate capacities for parking spaces and for each of the specific services available would yield a more accurate model. Unfortunately, data are not currently available to support this modeling enhancement.

- Inventoried aircraft at origin and destination airfields are considered not to affect the aircraft handling capacity of the airfield. This assumption is not strictly valid since an inventoried aircraft takes up parking space, but, as noted, working MOG dominates parking MOG.

- Deterministic ground time: Aircraft turnaround times for onloading and offloading cargo and enroute refueling are assumed to be known constants, although they are naturally stochastic. This ignores the fact that deviations from the given service time can cause congestion on the ground. To offset the optimism of this assumption, an efficiency factor is used in the formulation of aircraft handling capacity constraints to cushion the impact of randomness. Better handling of stochastic ground times is a subject of ongoing research.

Other approximations of reality employed in the model for computational tractability are aggregation of airfields, discretiztion of time, and continuous decision variables. A limitation on the scope of the model is that it considers only *inter*-theater, not *intra*-theater deliveries.

3. OPTIMIZATION MODEL

This section gives a mathematical formulation of the conceptual optimization model discussed previously in Section 2.2.

The airlift optimization problem is formulated as a multi-period, multi-commodity network-based linear program with a large number of side constraints. Two key concepts are

employed in the model. The first is the use of a time index to track the locations of aircraft for each time period. The modeling advantages of knowing when an aircraft will arrive at a particular airfield are that it enables us to model aircraft handling capacity at airfields and to determine unit closures (i.e., the time when all of a unit's deliveries are completed). This approach is in contrast to the THRUPUT model of Yost [1994], which takes a static-equilibrium or steady-state approach.

The second key concept is model reduction through data aggregation and the removal of unnecessary decision variables and constraints prior to optimization. This is necessary as the airlift problem is potentially very large. Without this model reduction step, the number of decision variables would run into the millions eeven for a nominal deployment. The unnecessary decision variables and constraints are removed by extensive checking of logical conditions, performed by GAMS during model generation. (See Lim [1994] for details.)

3.1. Indices

u	indexes units, e.g., 82nd Airborne
a	indexes aircraft types, e.g., C5, C141
t,t'	index time periods
b	indexes all airfields (origins, enroutes and destinations)
i	indexes origin airfields
k	indexes destination airfields
r	indexes routes

3.2 Index Sets

Airfield Index Sets

B	set of available airfields
$I \subseteq B$	origin airfields
$K \subseteq B$	destination airfields

Aircraft Index Sets

A	set of available aircraft types
$A_{bulk} \subseteq A$	aircraft capable of hauling bulk-sized cargo
$A_{over} \subseteq A_{bulk}$	aircraft capable of hauling over-sized cargo
$A_{out} \subseteq A_{over}$	aircraft capable of hauling out-sized cargo

Bulk cargo is palletized on 88 x 108 inch platforms and can fit on any milatary aircraft (as well as the cargo-configured 747). Over-sized cargo is non-palletized rolling stock: it is larger than bulk cargo and can fit on a C141, C5 or C17. Out-sized cargo is very large non-palletized cargo that can fit into a C5 or C17 but not a C141.

Route Index Sets

R	set of available routes
$R_a \subseteq R$	permissible routes for aircraft type a
$R_{ab} \subseteq R_a$	permissible routes for aircraft type a that use airfield b
$R_{aik} \subseteq R_a$	permissible routes for aircraft type a that have origin i and destination k
$DR_i \subseteq R$	delivery routes that originate from origin i
$RR_k \subseteq R$	recovery routes that originate from destination k

A *delivery route* is a route flown from a specific unit's origin to its destination for the purpose of delivering cargo and/or passengers. A *recovery route* is a route flown from a unit's destination to that unit's or some other unit's origin, for the purpose of making another delivery. Since recovery flights carry much less weight than deliveries, the recovery routes from k to i may have fewer enroute stops than the delivery routes from i to k.

Time Index Sets

T	set of time periods
$T_{uar} \subseteq T$	possible launch times of missions for unit u using aircraft type a and route r

The set T_{uar} covers the allowed time window for unit u, which starts on the unit's available-to-load date and ends on the unit's required delivery date, plus some extra time up to the maximum allowed lateness for the unit.

3.3 Given Data

Movement Requirements Data

$MovePAX_{uik}$	Troop movement requirement for unit u from origin i to destination k
$MoveUE_{uik}$	Equipment movement requirement in short tons (stons) for unit u from origin i to destination k
$ProBulk_u$	Proportion of unit u cargo that is bulk-sized
$ProOver_u$	Proportion of unit u cargo that is over-sized
$ProOut_u$	Proportion of unit u cargo that is out-sized

Penalty Data

$LatePenUE_u$	Lateness penalty (per ston per day) for unit u equipment

$LatePenPAX_u$ Lateness penalty (per soldier per day) for unit u troops

$NoGoPenUE_u$ Non-delivery penalty (per ston) for unit u equipment

$NoGoPenPAX_u$ Non-delivery penalty (per soldier) for unit u troops

$MaxLate$ Maximum allowed lateness (in days) for delivery

$Preserve_{at}$ Penalty (small artificial cost) for keeping aircraft type a in mobility system at time t

Cargo Data

$UESqFt_u$ Average cargo floor space (in sq. ft.) per ston of unit u equipment

$PAXWt_u$ Average weight of a unit u soldier inclusive of personal equipment

Aircraft Data

$Supply_{at}$ Number of aircraft of type a that become available at time t

$MaxPAX_a$ Maximum troop carriage capacity of aircraft type a

$PAXSqFt_{ua}$ Average cargo space (in sq. ft.) consumed by a unit u soldier for aircraft type a

$ACSqFt_a$ Cargo floor space (in sq. ft.) of aircraft type a

$LoadEff_a$ Cargo space loading efficiency (<1) for aircraft type a. This accounts for the fact that it is not possible in practice to fully utilize the cargo space.

$URate_a$ Established utilization rate (flying hours per aircraft per day) for aircraft type a

Airfield Data

$MOGCap_{bt}$ Aircraft capacity (in narrow-body equivalents) at airfield b in time t

$MOGReq_{ab}$ Conversion factor to narrow-body equivalents for one aircraft of type a at airfield b

$MOGEff_{bt}$ MOG efficiency factor (<1), to account for the fact that it is impossible to fully utilize available MOG capacity due to randomness of ground times

Aircraft Route Performance Data

$MaxLoad_{ar}$ Maximum payload (in stons) for aircraft type a flying route r.

$GTime_{abr}$ Aircraft ground time (due to onload or offload of cargo, refueling, maintenance, etc.) needed for aircraft type a at airfield b on route r

$DTime_{abr}$ Cumulative time (flight time plus ground time) taken by aircraft type a to reach airfield b along route r

$FltTime_{ar}$ Total flying hours consumed by aircraft type a on route r

$CTime_{ar}$ Cumulative time (flight time plus ground time) taken by aircraft type a on route r

$DaysLate_{uart}$ Number of days late unit u's requirement would be if delivered by aircraft type a via route r with mission start time t

3.4 Decision Variables

Mission Variables

X_{uart} Number of aircraft of type a that airlift unit u via route r with mission start time during period t

Y_{art} Number of aircraft of type a that recover from a destination airfield via route r with start time during period t

Aircraft Allocation and De-allocation Variables

$Allot_{ait}$ Number of aircraft of type a that are allocated to origin i at time t

$Release_{ait}$ Number of aircraft of type a that were allocated to origin i prior to time t but are not scheduled for any missions from time t on

Aircraft Inventory Variables

H_{ait} Number of aircraft of type a inventoried at origin i at time t

HP_{akt} Number of aircraft of type a inventoried at destination k at time t

$NPlanes_{at}$ Number of aircraft of type a in the air mobility system at time t

Airlift Quantity Variables

$TonsUE_{uart}$ Total stons of unit u equipment airlifted by aircraft of type a via route r with mission start time during period t

$TPAX_{uart}$ Total number of unit u troops airlifted by aircraft of type a via route r with mission start time during period t

Elastic (Nondelivery) Variables

$UENoGo_{uik}$ Total stons of unit u equipment with origin i and destination k that is not delivered in the prescribed time frame

$PAXNoGo_{uik}$ Number of unit u troops with origin i and destination k who are not delivered in the prescribed time frame

3.5 Formulation of the Objective Function

Minimize

$$\sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{t\in T_{uar}} LatePenUE_u * DaysLate_{uart} * TonsUE_{uart}$$

$$+ \sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{t\in T_{uar}} LatePenPAX_u * DaysLate_{uart} * TPAX_{uart}$$

$$+ \sum_{u}\sum_{i}\sum_{k} (NoGoPenUE_u * UENoGo_{uik} + NoGoPenPAX_u * PAXNoGo_{uik})$$

$$+ \sum_{a}\sum_{t} Preserve_{at} * NPlanes_{at}$$

The $DaysLate_{uart}$ penalty parameter has value zero if $t+CTime_{ar}$ is within the prescribed time window for unit u. Thus, the first two terms of the objective function take effect only when a delivery is late. The third term in the objective function corresponds to cargo and passengers that cannot be delivered even within the permitted lateness. Late delivery and non-delivery occur only when airlift assets are insufficient for on-time delivery.

The reason for including elastic variables that allow late delivery and non-delivery is to ensure that the model produces useful information even when the given assets are inadequate for the given movement requirements. The alternative of using an inelastic model (i.e., a model with hard constraints that insist upon complete on-time delivery) is inferior because it would report infeasibility without giving any insight about what *can* be done with the assets available.

A useful modeling excursion that is made possible by the elastic variables is to vary the number of time periods. As the horizon is shortened, it is interesting to observe the increase in lateness and non-delivery.

As noted, the model's anticipated use is in cases when the airlift assets are insufficient for full on-time delivery. In the opposite case, the model will be governed by the fourth term of the objective function, which rewards asset preservation for the reasons given in Section 2.2.1.

Some care must be taken in selecting the lateness and non-delivery penalties and the aircraft preservation rewards to ensure consistency. Late delivery should be preferred to non-delivery. The weights will be consistent with this preference provided the late penalty (per ston per day) is less than the corresponding non-delivery penalty (per ston) divided by the maximum allowed lateness (in days).

3.6 Formulation of the Constraints

As noted in the conceptual model, there are five categories of constraints. Their mathematical formulations are as follows.

3.6.1 Demand Satisfaction Constraints

There are four different kinds of demand constraints, corresponding to troops and the three classes of cargo (bulk, over-sized and out-sized). Separate constraints are required for the different cargo types to ensure cargo-carrier compatibility. For example, a carrier of over-sized cargo cannot be used to carry the larger out-sized cargo. On the other hand, it is possible to use a carrier of out-sized cargo to carry over-sized cargo. The model accounts for this asymmetry.

The demand constraints also account for the desired delivery time-windows by use of the index sets T_{uar} and the lateness parameters $DaysLate_{uart}$.

Demand Satisfaction Constraints for All Classes of Cargo:

$$\sum_{a \in A_{bulk}} \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TonsUE_{uart} + UENoGo_{uik} = MoveUE_{uik} \quad \forall\, u,i,k:\ MoveUE_{uik} > 0$$

Demand Satisfaction Constraints for Out-Sized Cargo:

$$\sum_{a \in A_{out}} \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TonsUE_{uart} + UENoGo_{uik} \geq ProOut_{u} * MoveUE_{uik}$$

$$\forall\, u,i,k:\ MoveUE_{uik} > 0$$

Demand Satisfaction Constraints for Over-Sized Cargo:

$$\sum_{a \in A_{ovr}} \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TonsUE_{uart} + UENoGo_{uik} \geq (ProOver_u + ProOut_u) * MoveUE_{uik}$$

$$\forall\ u,i,k:\ MoveUE_{uik} > 0$$

Demand Satisfaction Constraints for Troops:

$$\sum_{a} \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TPAX_{uart} + PAXNoGo_{uik} = MovePAX_{uik} \quad \forall\ u,i,k:\ MovePAX_{uik} > 0$$

3.6.2 Aircraft Balance Constraints

There are five kinds of aircraft balance constraints enforced for each aircraft type in each time period. At origin airfields, they ensure that the number of aircraft assigned for delivery missions plus those inventoried for later use plus those put in the released status equal the number inventoried from the previous period plus recoveries from earlier missions and the new supply of aircraft that is allocated to the origin.

The meaning of *releasing,* or *de-allocating,* an airplane in period t is that it is not flown on any missions from period t through the end of the horizon. In practice, the analyst can interpret a release in the model's solution in a variety of ways. It can mean, as in the case of the civilian CRAF aircraft, that the plane is literally sent back to its owner, but not necessarily. The aircraft can also be kept in the mobility system, available as a replacement in case of breakdowns or for unforeseen demands.

The second kind of aircraft balance constraints concerns destinations. They are similar to the first kind except releases are not allowed and the roles of delivery and recovery missions are reversed. The third kind of aircraft balance constraint ensures that if any new planes become available in period t, they are allotted appropriately among the origins. There is a potential gain in efficiency to allow the optimizer to make these allocation decisions, rather than relying on the user to preassign them to origin airfields. The fourth type of aircraft balance constraints is a set of accounting equations for defining the $NPlanes_{at}$ variables based on cumulative allocations and releases.

Aircraft Balance Constraints at Origin Airfields:

$$\sum_{u} \sum_{r \in DR_i} X_{uart} + H_{ait} + Release_{ait} = H_{ai,t-1} + Allot_{ait} + \sum_{r \in R_{ai}} \sum_{t' + [CTime_{ar}] = t} Y_{art'} \quad \forall\ a,i,t$$

where $[CTime_{ar}]$ is $CTime_{ar}$ rounded to the nearest integer.

Aircraft Balance Constraints at Destination Airfields:

$$\sum_{r \in RR_k} Y_{art} + HP_{akt} = HP_{ak,t-1} + \sum_{u} \sum_{r \in R_{ak}} \sum_{\substack{t' \in T_{uar} \\ t' + [CTime_{ar}] = t}} X_{uart'} \quad \forall\ a,k,t$$

Aircraft Balance Constraints for Allocations to Origins:

$$\sum_{t'=1}^{t} \sum_{i} Allot_{ait} \leq \sum_{t'=1}^{t} Supply_{at} \quad \forall\ a,t$$

This constraint is in the cumulative form, rather than in the simpler form $\Sigma_i Allot_{ait} \leq Supply_{at}$, to allow aircraft that become available in period t to be put into service at a later period.

Aircraft Balance Constraints Accounting for Allocations and Releases:

$$NPlanes_{at} = \sum_{t'=1}^{t} \sum_{i} Allot_{ait'} - \sum_{t'=1}^{t} \sum_{i} Release_{ait'} \quad \forall\ a,t$$

The fifth and final set of aircraft balance constraints helps to correct the discretization error that can result from rounding $CTime_{ar}$ to $[CTime_{ar}]$, the nearest integer, in the other balance constraints. For example, suppose $CTime_{ar}$ is less than half a day for some aircraft a and route r. When this time is rounded to zero in the balance constraints of the route's origin and destination, these constraints unrealistically permit an unlimited number of missions per day on that route. Solving the model with this deficiency would yield overly optimistic results.

One way to fix this problem would be to insist that $CTime_{ar}$ be rounded up to a higher integer. Then the model would be overly pessimistic, because it would rule out the possibility of an aircraft flying two or more missions in a day even when this is possible. This sort of problem is common in mathematical modeling whenever time is discretized. The approach taken here is to enforce the following additional constraints, based on the cumulative plane-days available.

Cumulative Aircraft Balance Constraints:

$$\sum_{r\in R_a}\sum_{t'=1}^{t}\sum_{u} K_{artt'}\, X_{uart'} + \sum_{r\in R_a}\sum_{t'=1}^{t} K_{artt'}\, Y_{art'} + \sum_{i}\sum_{t'=1}^{t} H_{ait'}$$

$$+ \sum_{k}\sum_{t'=1}^{t} HP_{akt'} \le \sum_{t'=1}^{t} NPlanes_{at'} \quad \forall\, a,t$$

where

$$K_{artt'} = \begin{cases} t-t'+1 & \text{if } t' \le t < t'+CTime_{ar}-1 \\ CTime_{ar} & \text{if } t \ge t'+CTime_{ar}-1 \end{cases}$$

The right-hand-side indicates the cumulative number of plane-days available for type *a* aircraft up to day *t*. The left-hand-side accounts for all possible plane activities up to day *t*, whether flying or inventoried. The inventory terms are straightforward. The delivery and recovery terms work as follows: if a delivery initiated on day *t'* is completed by the end of day *t*, then the entire time $CTime_{ar}$ (which may be integer or fractional) is included in the left-hand-side of the cumulative balance constraint for day *t*. On the other hand, if a delivery initiated on day *t'* is not completed by the end of day *t*, then only the time expended so far, *t-t'+1*, is counted in the day *t* constraint.

An experiment attesting to the value of the cumulative aircraft balance constraints is described in Section 5.4. If the $CTime_{ar}$'s were all integer, these constraints would be redundant and could be omitted.

3.6.3 Aircraft Capacity Constraints

Troop Carriage Capacity Constraints:

$$TPAX_{uart} \le MaxPAX_a * X_{uart} \quad \forall\, u,a,r,t:\ t\in T_{uar}$$

Maximum Payload Constraints:

$$TonsUE_{uart} + PAXWt * TPAX_{uart} \le MaxLoad_{ar} * X_{uart} \quad \forall\, u,a,r,t:\ t\in T_{uar}$$

Cargo Floor Space Constraints:

$$PAXSqFt_a * TPAX_{uart} + UESqFt_u * TonsUE_{uart} \leq ACSqFt_a * LoadEff_a * X_{uart}$$

$$\forall\ u,a,r,t:\ t \in T_{uar}$$

3.6.4 Aircraft Utilization Constraints

The aircraft utilization constraints ensure that the total flying hours consumed by the fleets of each aircraft type over the planning horizon are within AMC's established utilization rates [Wilson, 1985; Gearing *et al.*, 1988]. These rates are meant to capture spares availability, aircraft reliability, crew availability, and other factors. The utilization constraints are formulated by comparing the flying hours consumed by an aircraft fleet in delivery and recovery flights to the maximum achievable flying hours for the fleet according to the utilization rate.

$$\sum_u \sum_{r \in R_a} \sum_{t \in T_{uar}} FltTime_{ar} * X_{uart} + \sum_{r \in R_a} \sum_t FltTime_{ar} * Y_{art}$$

$$\leq \sum_t URate_a * NPlanes_{at} \quad \forall a$$

As an illustration of the above equation, consider a fleet of 5 aircraft of the same type made available from day 11. If the utilization rate for this aircraft type is 10 flying hours per aircraft per day and the horizon is 30 days, then the maximum achievable flying is 1000 hours *(10 hours/plane-day x 20 days x 5 planes)*. This total may not be exceeded for the whole fleet over the entire planning horizon, however, it is not unusual for a subset of aircraft to exceed utilization rates over a subset of the horizon, particularly during the early (surge) stage of a deployment.

3.6.5 Aircraft Handling Capacity of Airfields (MOG Constraint)

The aircraft handling constraints at airfields, commonly called MOG constraints, are perhaps the most difficult to model. This is because of two complicating factors that necessitate approximations. First, there is no airfield capacity data available that provides separate accounting of parking spaces and all the various services (refueling, maintenance, etc.). The MOG data provided by the Air Force is an approximation, attempting to aggregate all these services. Thus, the units of $MOGCap_{bt}$ are an idealized notion of airfield parking spaces (normalized to narrow-body sized aircraft), not a precisely defined physical quantity.

The second complicating factor in modeling airfield capacity is the congestion caused by the uncertainty of arrival times and ground times. A deterministic, time-discretized optimization model cannot accurately treat events occurring within a time period. For example, suppose the time period of the model is one day and an airfield has 20 landings per day. How much

congestion occurs depends on when the landings occur during the day, a phenomenon not captured in the daily model. It is possible to attack these concerns with stochastic modeling techniques, however, the existing simulation and optimization models for air mobility have made very limited progress to date in this area [Morton and Rosenthal, 1994]. The MOG efficiency factor *MOGEff* is introduced to cushion the effect of not explicitly modeling uncertainty. The MOG constraints are formulated for each airfield and time period as follows:

$$\sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{\substack{t'\in T_{uar}\\ t'+[DTime_{abr}]=t}} (MOGReq_{ab} * GTime_{abr} / 24) * X_{uart'}$$

$$+ \sum_{a}\sum_{r\in R_a}\sum_{t'+[DTime_{abr}]=t} (MOGReq_{ab} * GTime_{abr} / 24) * Y_{art'}$$

$$\leq MOGEff_{bt} * MOGCap_{bt} \quad \forall\, b,t$$

Dimensional analysis is useful for understanding these constraints. The right-hand-side is in the units of narrow-body parking spaces, because $MOGCap_{bt}$ is in those units and $MOGEff_{bt}$ is dimensionless. The first term on the left-hand-side accounts for airfield capacity consumed by all delivery missions that pass through airfield *b* during period *t*. The second term on the left does the same thing for recovery missions. The dimension of $MOGReq_{ab}$ is narrow-body parking spaces per plane, the dimension of $GTIME_{abr}/24$ is days, and the dimensions of $X_{uart'}$ and $Y_{uart'}$ are planes per day; thus, the MOG constraints are dimensionally balanced.

Aircraft inventoried at origin or destination airfields do not consume any MOG capacity in the above formulation. This is not a mathematical limitation, but rather a modeling choice taken because inventoried planes do not consume ground services. It can be easily modified if data for "parking space MOG" and various "ground service MOG's" become available.

4. Performance

The performance of the optimization model is relatively fast. On an IBM RS6000 model 590 workstation with GAMS/OSL, it takes about 100 seconds to generate and an additional 100 seconds to solve a sample problem with 20 units, 7 aircraft types, 17 airfields and 30 time periods. A 486/66 laptop computer running the same software on the same problem takes about 28 minutes. After extensive variable and constraint reduction, the sample problem has 11,516 decision variables, 6,970 constraints and 189,351 nonzero coefficients. The data entry time for the sample problem is about one and a half hours. Excursions from a base model run take considerably less time to prepare. In short, turnaround time for the optimization model is

significantly faster than simulation models commonly used in the Air Force [Morton and Rosenthal, 1994].[1]

5. Analytic Insights

We now describe some examples of modeling excursions and the resulting analytic insights. The base case scenario, developed by the U.S. Air Force Studies and Analyses Agency, notionally resembles a Desert Storm scenario. This is the same problem instance whose dimensions (after model reductions) are given in the Performance section.

5.1 Diversion of Ramstein-Riyadh Demand to Dhahran

In the base case scenario, there are twenty origin-to-destination demand pairs, but they are dominated by the demand for airlifting two Army mechanized units from Ramstein, Germany to Riyadh, Saudi Arabia. These two units combined account for 66,400 short tons (stons), or 48%, of all unit equipment to be moved. When the base case is optimized, the given fleet delivers only 67% of the total unit equipment. The shortfall is due entirely to 45,000 undelivered stons of Ramstein-Riyadh demand, and a critical constraint appears to be MOG limitations at Riyadh's airfield.

In one modeling excursion, we examine the effects on the airlift system of changing the destination for one of the Ramstein-based mechanized units to Dhahran, Saudi Arabia, which is 250 miles northeast of Riyadh and closer to Kuwait and Iraq. Re-optimizing with this one change, the same fleet delivers 85% of all unit equipment, a dramatic improvement from 67%. However, the shortfall of 20,000 stons of unit equipment from Ramstein may still be a serious impediment to the Army's effectiveness, necessitating a re-evaluation of the scenario's war plans or augmentation of the mobility system.

The graphs in Figure 1 show a summary of this modeling excursion over time. The unit equipment demand profile has jumps in it at the required delivery dates (RDD's). Cumulative delivery profiles are shown for the base case and the excursion. When the demand curve is higher than the delivery profile, shortfalls occur. All passenger demands, though not shown in the figure, are delivered on time in both cases.

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[1]*Note added in final revision:* More recent runs of the model have been with a larger data set corresponding to a two-MRC scenario. This instance of the model contains 151 units, 7 aircraft types, 155 routes, and 47 time periods. The linear program has 95,111 constraints, 106,707 variables and 1,758,221 nonzeroes. It took 24 minutes to generate and 148 minutes to solve with GAMS/OSL on the RS6000/590.

5.2 Required-Delivery-Date Sensitivity

As a second excursion, after shifting some of the Ramstein demand to Dhahran, we investigated the effect of changes in the required delivery date for the unit whose equipment could not be delivered. With the given RDD, the total unit equipment delivered is 85%, as noted. If extra days are allowed, delivery increases as follows:

Extra Days Allowed	*Percent Unit Equipment Delivered*	*Objective Function Value*
0	85%	12.45
2	88%	11.35
4	93%	10.13
6	99%	8.56

The maximum allowed lateness is four days in all these runs. However, around 99% of all the deliveries made are on time.

5.3 Identifying Critical Resources

The overall performance of the air mobility system in our optimization runs can be characterized as having three phases. During the first third of the thirty days modeled, the system is airframe constrained. During the middle third (plus or minus a few days depending on location), the system is airfield-capacity constrained. During the final third, the system is in a sustainment phase with diminished demands. Neither airframes nor airfield capacities are critical resources, and it is too late to deliver cargos that were undelivered earlier.

After looking at Figure 1, one might disagree with the assertion that the mobility system is airframe-bound in the first phase, because there are no significant shortfalls until Day 16. This would be a mistake, however. In fact, all available aircraft are used to the maximum from the earliest available-to-load date (Day 1) through Day 11 (when a large portion of the military aircraft first become available), and the dual multipliers indicate that additional airframe assets in the first phase would have high marginal value. This is because if more aircraft were available earlier, then the optimization model would have made more early deliveries to prevent the shortfalls that it foresees but cannot avoid later in the middle phase.

The middle phase of the airlift has more overall flights than the first phase, because there are more aircraft in the system and demand is sufficient to keep them flying. The middle phase also has a higher percentage of the shorter Germany-to-Saudi flights, as compared to the longer CONUS-to-Saudi flights which predominate in the first phase. With more flights and with shorter flights (which consume MOG at a faster rate per plane), the mobility system becomes airfield-capacity constrained.

One might be tempted to conclude that adding more planes to the system during the middle phase would be unproductive. This would also be a mistake: the dual multipliers on aircraft consumption indicate that additional C17's and C5's would have high marginal value in the middle phase. Why does the optimization say that adding more planes would help the mobility system when airfield capacities are already hitting their limits?

The answer is that the optimization advocates adding more *efficient* and *versatile* planes. The meaning of *efficiency* for planes in a MOG-constrained environment is a high ratio of cargo-delivered-per-plane to MOG-hours-consumed-per-plane. The more efficient a plane is in this sense, the more cargo it can deliver per day to a MOG-limited destination. According to the data furnished by USAF/SAA and the evaluation of MOG-hours consumed per plane at the most congested airfields in the model, the C17 is the most efficient airframe for a MOG-constrained environment. The meaning of *versatility* in the present context is having the ability to carry all three types of cargo (bulk, over-size and out-size), as only the C17 and C5 can. The optimization determines that the mobility system would perform better on the entire airlift if some more efficient and versatile airframes were made available during the middle phase.

5.4 Sensitivity to Time Discretization

The cumulative aircraft balance constraints were added to lessen the effects of time discretization, as discussed in Section 3.6.2. The kinds of problems they are intended to remedy arise, for example, if the cycle time of a route is less than half the length of a time period. Without these constraints, such a cycle time would be rounded to zero and cause unrealistic results.

To test the effectiveness of the cumulative aircraft balance constraints, the model was run with time period lengths of 12, 24 and 48 hours. The resulting delivery profiles are displayed in Figure 2. The idea of the test is that in the absence of discretization error abatement measures, the error would increase as the time-step of the model gets larger. Figure 2, however, shows close agreement among the delivery profiles, regardless of time period length.

6. CONCLUSIONS AND FUTURE EXTENSIONS

The preceding analytic insights are typical of what that can be obtained through optimization, but not from simulation. They represent but a small sample of the kinds of questions that can be addressed with the optimization model. The model can give relatively rapid response to questions relating to major mobility issues such as: 1) Are the given aircraft and airfield assets adequate for the deployment scenario? 2) What are the impacts of shortfalls in airlift capability? 3) Where are the system bottlenecks and when will they become noticeable? This type of analysis can be used to help answer questions about selecting airlift assets and about investing or divesting in airfield infrastructure.

The optimization model has some limiting assumptions which must be taken into account when evaluating its results. As noted, they are: the approximation of airfield capacity by a uni-

dimensional MOG factor, deterministic ground times, the absence of aerial refueling, and the rounding problems that are inevitably caused by the discretization of time. The cumulative aircraft balance constraints help address the last difficulty, by preventing overly optimistic or pessimistic results. Nevertheless, the one-day time scale of the model that typically has been used to date cannot accurately represent what happens at airfields during smaller time intervals.

In the Air Force analysis community, simulation has more acceptance than optimization. The advantage of simulation over optimization is that it can more readily accommodate uncertainty and it can handle a higher level of detail, such as tracking individual airplanes by tail number. The disadvantage is that it can only answer *what-if* questions, not *what's-best* questions. Simulations also usually take longer to run. Air mobility simulations used by the Air Force have had such long run times that the stochastic elements are sometimes left out in order to make them run faster.

Ideally, optimization and simulation should be used in concert, with the optimization being used to suggest mobility system configurations and modes of operation that are then analyzed in detail by the simulation. Simulation runs, in turn, would suggest new scenarios to be investigated by the optimization.

The optimization model described here is capable of being used in concert with other Air Force planning models, or it can stand alone to provide rapid and realistic responses in emerging conflict situations. Ongoing research is attempting to enhance the model in the following ways:

- Currently the routes made available to the optimization model are entered manually, based on USAF/SAA analysts' judgement. An auxiliary model is under development for generating routes [Turker, 1995]. Turker's research is also addressing the issue of decreasing the effects of airfield aggregation (and associated unit aggregation).

- Stochastic programming methods are under investigation for incorporating random ground times [Goggins, 1995].

- The Air Force is currently studying the formation and transportation of *global reach laydown packages*. The idea is to bring these packages to remote airfields to quickly create or augment airfield capacity. A related optimization model is addressing the optimal deployment of these mobile assets [Chapates, 1995].

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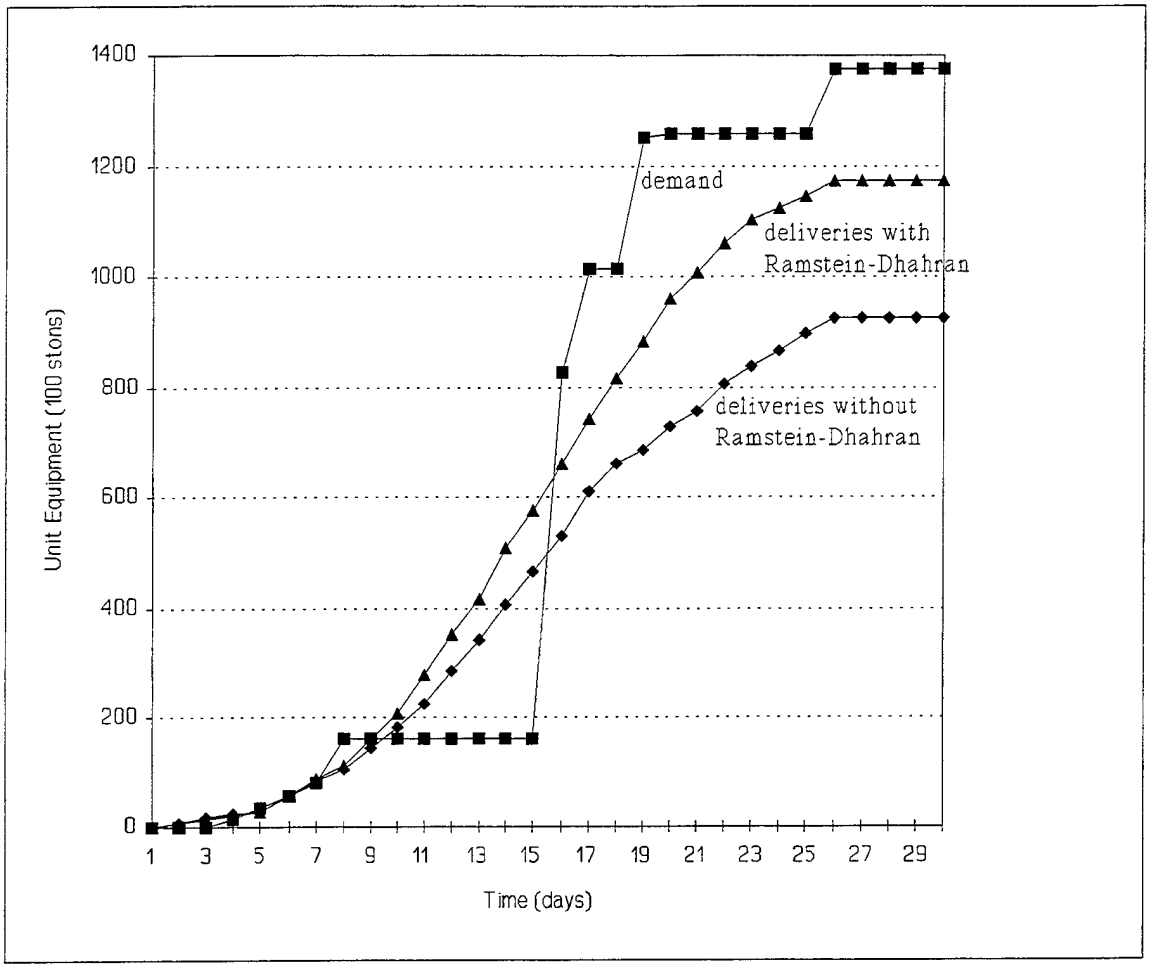

Figure 1. A modeling excursion: after changing a Ramstein-based unit's destination from Riyadh to Dhahran, the amount of undelivered cargo decreases from 45,000 stons to 20,000 stons.

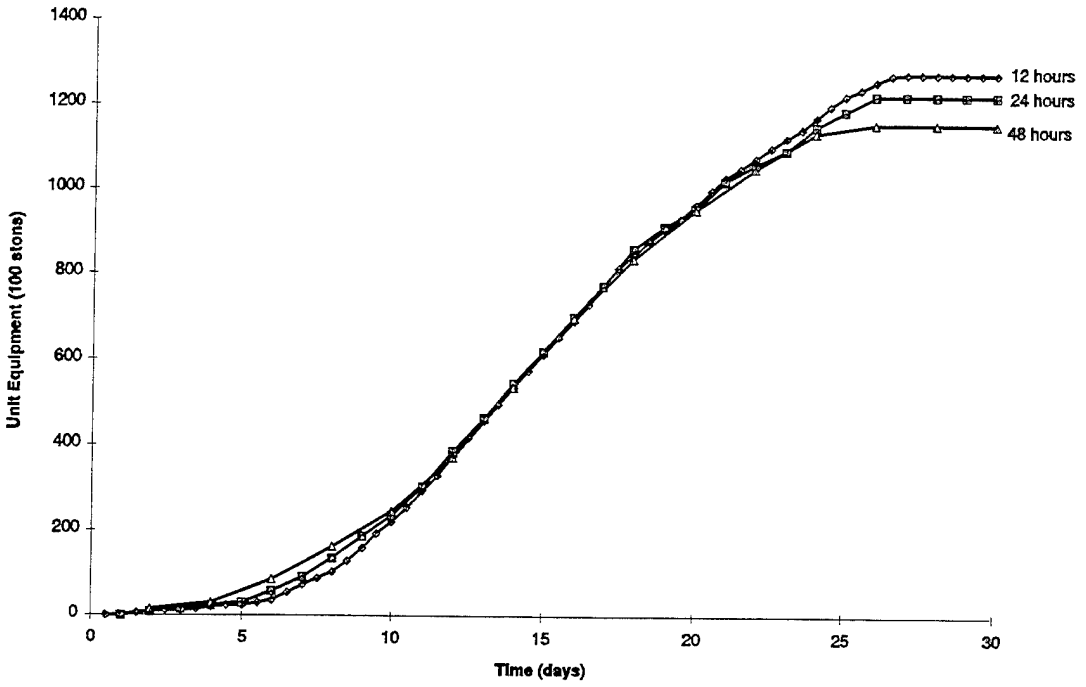

Figure 2. Agreement among delivery profiles when time periods have length 12, 24 or 48 hours. Larger time-steps in linear programming yield smaller, easier-to-solve models, but usually cause greater discretization errors. In this model, however, the cumulative aircraft balance constraints effectively reduce discretization error.

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