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THE EFFECT OF GAPS ON PILE REACTIVITY

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ABSTRACT

The variation of the reactivity of a pile as a function of width of a transverse gap is obtained. The method involves first finding the boundary condition satisfied by the flux at the gap face. This, in principle, provides enough information for a complete solution of the pile equations. A method for calculating the reactivity change is presented. The calculated reactivity is compared with experiment and a brief discussion of the validity of the approximations is given.

THE EFFECT OF GAPS ON PILE REACTIVITY

S. Tamor

I. Introduction

The problem of a transverse gap in a cylindrical reactor has been treated in some detail by Goldberger, Goldberger, and Wilkins several years ago[1]. They develop a method for calculating the flux depression at the gap which is somewhat restricted in its validity because of an approximation which is invalid for small gaps. The method used here is exactly that of GGW, but the approximation to large gaps is not made. No serious complications is introduced by the generalization. Once the boundary condition at the gap is obtained, it is possible to replace the gap by an equivalent thin absorber; i.e., an absorber which produces the same flux depression. It is then a simple matter to calculate the reactivity change by means of perturbation theory (assuming, of course, that the adjoint flux for the reactor in question is known).

At the risk of repeating previous work, for the sake of completeness the principles of the GGW method will be presented in detail. We will restrict ourselves to reactors in the shape of a right (not necessarily circular) cylinder. As pointed out by GGW the general method is applicable to other geometries, the only requirement being that the flux be separable into parts normal and transverse to the gap.

II. The Method

Consider a bare cylindrical reactor of length ℓ, which has been cut along a transverse plane, and the two parts separated a distance h (fig. 1). For convenience, we let the gap lie in the mid-plane of the

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1) Goldberger, Goldberger, and Wilkins, CP 3443, hereafter referred to as GGW.

reactor although the generalization to an off-center gap is trivial. The flux in the one-velocity diffusion problem is separable into transverse and longitudinal parts, so that within the moderator the longitudinal part is a cosine distribution and the transverse part satisfies the two dimensional wave equation $\nabla^2\phi + \gamma^2\phi = 0$ with appropriate boundary conditions.

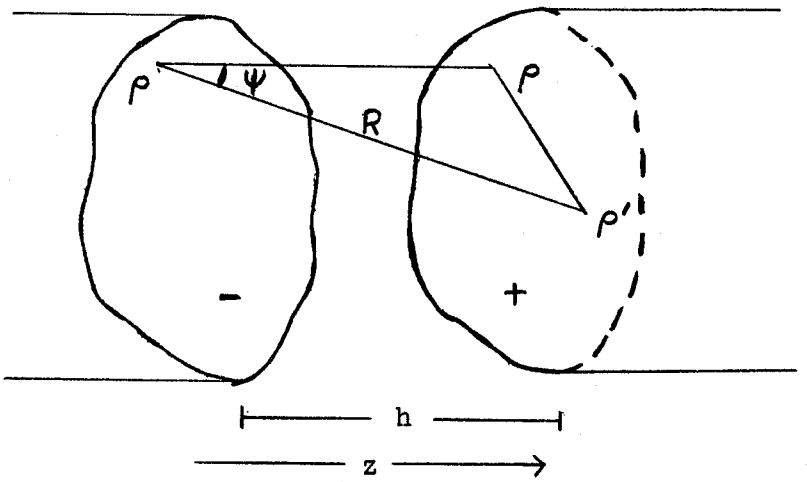

Figure 1

We assume the reactor to be sufficiently large that diffusion theory is applicable and gives correctly the flux emerging from the gap face. This restriction implies that for reactors with several gaps they must not be too close together. Following GGW, we now inquire as to how many neutrons leave an element of area at ρ' on the + face and arrive at ρ on the - face.

If $\vec{\Omega}$ is a unit vector directed from ρ' on the + face to ρ on the - face, the rate at which neutrons leave ρ' headed into $d\Omega$ is

$$\frac{1}{4\pi}(\phi_+(\vec{\rho}') + \lambda \nabla\phi_+(\vec{\rho}')\cdot\vec{\Omega})\cos 4\, d\Omega$$

so that the total number of neutrons arriving at ρ is given by

$$\frac{1}{4\pi}\int\left[\phi_+(\vec{\rho}') + \lambda\nabla\phi_+\cdot(\vec{\rho}')\cdot\vec{\Omega}\right]\cos\psi\, d\Omega$$

$$= \frac{1}{4\pi}\int\frac{\phi_+(\vec{\rho}') + \lambda\nabla\phi_+(\vec{\rho}')\vec{\Omega}}{R^2}\cos^2\psi\, dA' \qquad (1)$$

where **A'** is the element of area containing ρ'. This is to be equated to the total incoming current at ρ which is

$$\frac{\phi_-(\vec{\rho})}{4} + \frac{\lambda}{6}\frac{\partial\phi_-(\vec{\rho})}{\partial z} \qquad (2)$$

Making use of the symmetry of the system about the mid-plane,

$$\phi_-(\vec{\rho}) = \phi_+(\vec{\rho}) \text{ and } \phi'(\vec{\rho}) \equiv \phi_+'(\vec{\rho}) \equiv \frac{\partial\phi_+(\vec{\rho})}{\partial z} = -\frac{\partial\phi(\vec{\rho})}{\partial z}$$

we have

$$q \equiv \frac{\phi}{4} - \frac{\lambda}{6}\phi' = \frac{1}{4\pi}\int\frac{\phi + \lambda\nabla\phi_+\cdot\vec{\Omega}}{R^2}\cos^2\psi\, dA \qquad (3)$$

At this point it is desirable to make one more approximation. Let us assume the gap to be small compared to the transverse dimensions of the reactor so that there is no appreciable communication between the outside and the middle of the reactor. This is the region of maximum importance, and the incoming flux will not be seriously falsified by continuing the integration in (3) over the infinite plane. We will see that this device leads to a very great simplification in that the spatial distribution of the entering flux reproduces that of the flux emerging.

To evaluate the intergrals it is convenient to view figure 1 as projected onto the gap face (fig. 2). The coordinate system is chosen arbitrarily and $\vec{u}$ is directed from $\vec{\rho}$ to $\vec{\rho}'$

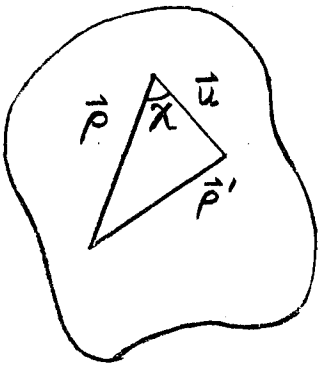

Figure 2

Then

$$q(\vec{\rho}) = \frac{\phi(\vec{\rho})}{4} - \frac{\lambda}{6}\phi'(\vec{\rho})$$

$$= \frac{1}{4\pi}\int \frac{\phi(\vec{\rho}') + \lambda\,\phi'(\vec{\rho}')\,\frac{k}{R} + \lambda\,\nabla\phi(\vec{\rho}')\cdot\frac{\vec{u}}{R}}{R^2}\,\cos^2\psi\,dA' \qquad (4)$$

Here the GGW calculation neglects the last term in the numerator of the integrand. It is easily seen that this is equivalent physically to ignoring the azimuthal asymmetry of the emerging flux. Since the flux is directed preferentially toward regions of low density, the azimuthal distribution favors those neutrons heading toward the edge of the reactor, thereby contributing a net streaming out of the gap. We will see that this gives most of the neutron loss for sufficiently small gaps.

Since

$$\nabla\phi(\vec{\rho}')\cdot\vec{u} = u\,\frac{\partial\phi(\vec{\rho}')}{\partial u}\,, \quad R^2 = u^2 + h^2$$

and $\cos\psi = h/R$, eq.(4) becomes

$$q(\vec{\rho}) = \frac{1}{4\pi}\int \frac{h^2}{(h^2+u^2)^2}\left\{ \phi(\vec{\rho}') + \frac{\lambda\, h\, \phi'(\vec{\rho}')}{\sqrt{u^2+h^2}} + \frac{\lambda u}{\sqrt{u^2+h^2}}\,\frac{\partial\phi(\vec{\rho})}{\partial u}\right\} dA' \qquad (5)$$

But the flux is separable so that $\phi'(\vec{\rho}')$ is equal to $\phi(\vec{\rho}')$ times a function of z which is constant over the gap face. Therefore the integral may be written in the form $\int K(u)\phi(\vec{\rho}')dA'$ where K(u) is explicitly a function of u alone.

Choosing the point $\vec{\rho}$ as a new origin, the Fourier Bessel transform of $\phi(\vec{\rho}')$ may be written

$$\phi(\vec{\rho}') = \sum_t \int ds\; a_{st}(\vec{\rho})\; J_t(su)\cos(t\chi + \eta_t).$$

Substituting into $\nabla^2\phi + \gamma^2\phi = 0$ and making use of the orthogonality property, we find

$$(s - \gamma^2)\, a_{st} = 0$$

so that

$$\phi(\vec{\rho}') = \sum_t a_{\gamma t}(\vec{\rho}) J_t(\gamma u)\cos(t\chi + \eta_t) \qquad (6)$$

Now $$\int K(u)\,\phi(\vec{\rho}')dA' = \int K(u)u\,du \int \phi(\vec{\rho}')d\chi$$

$$= 2\pi a_{\gamma 0}(\vec{\rho})\cos\eta_t \int K(u)J_0(\gamma u)u\,du \qquad (7)$$

But we now observe that if $\vec{\rho} = \vec{\rho}'$ (6) becomes

$$\phi(\vec{\rho}) = a_{\gamma 0}(\vec{\rho})\cos\eta_t \text{ so that}$$

$$\int K(u)\phi(\vec{\rho}')dA' = 2\pi\phi(\vec{\rho})\int K(u)J_0(\gamma u)u\,du \qquad (8)$$

This proves our assertion that the incoming flux is proportional to the emerging flux, and further that the constant of proportionality depends upon h, γ, λ, but is <u>independent of the geometry of the system</u>.

Returning to the original integral, (5) becomes

$$q(\vec{\rho}) = \frac{\phi(\vec{\rho})}{2}\left\{h^2\int\frac{J_0(\gamma u)}{(u^2+h^2)^2}\,u\,du - h^2\gamma\lambda\int\frac{J_1(\gamma u)}{(u^2+h^2)^{5/2}}\,u^2du\right\}$$

$$+ \frac{\phi'(\vec{\rho})}{2}\lambda h^3\int\frac{J_0(\gamma u)}{(u^2+h^2)^{5/2}}\,u\,du.$$

These integrals are all Hankel transforms and are[2)]

$$\int_0^\infty \frac{J_0(\gamma u)}{(u^2+h^2)^2}\,u\,du = \frac{\gamma K_1(\gamma h)}{2h}$$

$$\int_0^\infty \frac{J_0(\gamma u)}{(u^2+h^2)^{5/2}}\,u\,du = \frac{\gamma^{3/2}K_{3/2}(\gamma h)}{2^{3/2}h^{3/2}\Gamma(5/2)} = \left(1+\frac{1}{\gamma h}\right)\frac{\gamma e^{-\gamma h}}{3h^2}$$

$$\int_0^\infty \frac{J_1(\gamma u)}{(u^2+h^2)^{5/2}}\,u^2du = \frac{\gamma^{3/2}K_{1/2}(\gamma h)}{2^{2/3}h^{1/2}\Gamma(5/2)} = \frac{\gamma e^{-\gamma h}}{3h}$$

2) See Watson, Bessel Functions p.434.

where the K functions are the usual Bessel functions of the second kind with imaginary argument. We now have

$$q(\vec{\rho}) = \frac{\phi(\vec{\rho})}{4} - \frac{\lambda\phi'(\vec{\rho})}{6} = \phi(\vec{\rho})\left\{\frac{\gamma h}{4} K_1(\gamma h) - \frac{(\gamma\lambda)(\gamma h)e^{-\gamma h}}{6}\right\}$$

$$+ \phi'(\rho)\frac{\gamma(1+\gamma h)}{6} e^{-\gamma h}$$

or

$$\frac{\phi'}{\phi} = \frac{3}{2\lambda} \frac{1 - \gamma h K_1(\gamma h) + \frac{2}{3} (\gamma\lambda)(\gamma h)e^{-\gamma h}}{1 + (1+\gamma h)e^{-\gamma h}} \tag{9}$$

Note that as $h \to 0$ $\phi \to 0$ and as $h \to \infty$ $\frac{\phi'}{\phi} \to \frac{3}{2\lambda}$, the usual diffusion theory boundary condition.

The result of GGW is identical with (9) but for the omission of the last term in the numerator. For small gaps, the last term is linear in h, while $1 - zK_1(z)$ starts off as $z^2 \log z$. Thus for very small h the last term dominates. For reasonable dimensions of the reactor, the two parts become comparable for $h \approx \lambda/3$, which for graphite is about 1 cm. For gaps much larger than this the last term may be neglected.

For small h eq. (9) can be checked by an elementary method. Let h be small enough that the flux and its angular distribution are essentially the same in the gap as in the neighboring moderator. The total flux leakage out of the gap is then

$$-\frac{2}{3}\lambda h \int \frac{d\phi}{dn} ds \tag{10}$$

where ds is an element of arc along the gap edge, and $\frac{d\phi}{dn}$ is the outward gradient of ϕ at the edge.

The rate at which neutrons are fed into the gap is

$$-2 \cdot \frac{2}{3}\lambda \int \nabla \phi . d\vec{A} \tag{11}$$

where $d\vec{A}$ is an element of area on the + face. For $\gamma h \ll 1$

$\frac{\phi'}{\phi} = \frac{\gamma^2 h}{2}$ so that (11) becomes

$$\frac{2}{3}\gamma^2 \lambda h \int \phi dA.$$

But ϕ satisfies $\nabla^2\phi + \gamma^2\phi = 0$ giving

$$-\frac{2}{3}\lambda h \int \nabla^2 \phi dA.$$

Integrating by parts, if we let $\vec{s}_1$ be a unit vector along ds, we have $-\frac{2}{3}\lambda h \int (\nabla \times \vec{s}_1)\phi . d\vec{s}$. Since ϕ is constant along the gap edge, this is simply $-\frac{2}{3}\lambda h \int \frac{d\phi}{dn} ds$, which is precisely the result (10).

III. Calculation of the Reactivity Change

So far we have considered the effect of the gap on neutrons diffusing with constant velocity. While this is an unrealistic picture of a reactor, our result does provide enough information to obtain the δk of a real system. For a truly thermal reactor the following somewhat crude method may be used.

We start with the pile equation[3)]

$$k_{eff} = \frac{k}{p} \frac{\bar{P}_{\infty}(B^2)}{1+L^2B^2}$$

where k is the infinite medium multiplication constant, p is the resonance escape probability, $\bar{P}_{\infty}$ is the Fourier transform of the infinite medium slowing-down kernel, L is the diffusion length, and B the geometrical buckling.

For h = 0 the boundary condition at the gap in $\phi' = 0$. Knowledge of $\frac{\phi'}{\phi}$ thus determines the increase in buckling of the system. If B is small, we have

$$\frac{\delta k_{eff}}{k_{eff}} = -\frac{\frac{\bar{r}^2}{6} + L^2}{1+L^2B^2} \delta(B^2)$$

when $\bar{r}^2$ is the mean square distance traveled in slowing down to thermal.

Fitting a cosine distribution to the new boundary condition, we find

$$\delta(B^2) = \frac{4}{\ell} \frac{\phi'}{\phi}$$

so that $$\frac{\delta k}{k} = -\frac{4}{\ell} \frac{L^2 + \frac{\bar{r}^2}{6}}{1+L^2B^2} \frac{\phi'}{\phi} \qquad (12)$$

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3) See A. M. Weinberg and L. C. Noderer, Theory of Neutron Chain Reactions, Vol. II, Part I, Chapter V, (CF 51-5-98).

This method assumes that all the fissions are caused by thermal neutrons and breaks down if an appreciable fraction of the fissions are fast.

It is, however, possible to obtain the δk for more general reactors provided that the flux and its adjoint for the unperturbed system are known. This can be done by noting that the gap may be replaced by an equivalent thin absorber[4]. The presence of the gap causes a net flow of neutrons into the gap from the two faces of $\frac{2}{3}\lambda\phi'$. These neutrons are effectively "absorbed" by the gap. If $\frac{\phi'}{\phi}$ is small, we replace the gap by an absorber whose cross-section is $\Sigma(z) = \theta\ \delta(z-z_o)$ where z_o defines the plane at which the reactor is opened. This will absorb the correct number of neutrons if

$$\frac{2}{3}\lambda\phi' = \int \phi\Sigma(z)dz = \int \phi\theta\delta(z-z_o)dz = \phi(z_o)\theta$$

or

$$\theta = \frac{2}{3}\lambda\frac{\phi'}{\phi}\ . \tag{13}$$

The effective absorption cross-section is seen to be a function of energy through its dependence on λ.

In the spirit of the multigroup method we consider the slowing-down process as a succession of one-velocity diffusion proccesses each providing a source for the next lower one in energy. At each energy the result (9) is applicable. If we have solved the slowing-down problem for the unperturbed reactor, $\frac{\delta k}{k}$ may be obtained from perturbation theory. We have

$$\frac{\delta k}{k} = \int q(\vec{r},u)\ \frac{\Sigma(z)}{\xi\Sigma_t}\ q^+(\vec{r},u)d\vec{r}\ du \tag{14}$$

4) This method was suggested to the author by D. K. Holmes.

where $q(\vec{r},u)$ is the slowing-down density at lethargy u and coordinate $\vec{r}$, $\Sigma(z)$ is defined by (13), and the other symbols have their usual meaning.

For a bare reactor $q(\vec{r},u)$ is separable into $q(u)\cdot Q(\vec{r})$ where $Q(\vec{r})$ is self-adjoint and (14) becomes

$$\frac{\delta k}{k} = \int Q^2(\vec{r})\ \delta(z-z_o)d\vec{r} \int q(u)\ \frac{\theta(u)}{\xi \Sigma_t}\ q^+(u)\ du\ ,$$

and if Q is properly normalized

$$\frac{\delta k}{k} = \cos^2(\epsilon z_o)\ \frac{2}{\ell}\int q(u)\ \frac{\theta(u)}{\xi \Sigma_t}\ q^+(u)\ du \qquad (15)$$

where ϵ is the longitudinal buckling.

It is easily shown that (15) is valid also for the case of the off-center gap. The result implies that the δk produced by a gap of given width is proportional to the importance of the plane along which it is cut.

Substituting into (15) from (9) and (13) we have

$$\frac{\delta k}{k} = \cos^2(\epsilon z_o)\ \frac{2}{\ell}\ \left[\alpha I_1 + \beta I_2\right] \qquad (17)$$

where

$$\alpha = \frac{1 - (\gamma h)K_1(\gamma h)}{1 + (1+\gamma h)e^{-\gamma h}} \qquad \beta = \frac{2}{3}\frac{\gamma^2 h\ e^{-\gamma h}}{1 + (1+\gamma h)e^{-\gamma h}}$$

$$I_1 = \int q(u)\ \frac{1}{\xi \Sigma_t}\ q^+(u) \qquad I_2 = \int q(u)\ \frac{\lambda(u)}{\xi \Sigma_t}\ q^+(u)\ du.$$

The GGW result is obtained by setting $\beta = 0$.

By this method δk is calculable in a straightforward way involving only two integrations. In many cases, λ is independent of u over a wide range, and may be considered constant so that actually only one integral need by evaluated.

IV. Validity of Approximations

The approximations made in obtaining the boundary conditions at the gap are a) use of diffusion theory for the outgoing flux at the gap face, and b) the continuation of the surface integrals to infinity.

A qualitative justification of the diffusion approximation is simply this. The diffusion approximation gets poorer as the angular distribution of the neutron flux deviates more and more from isotropy. The worst possible case from this point of view is when h becomes infinite. However this leads to the elementary diffusion boundary condition which, for large systems, is an excellent approximation to the correct one. The error arising from this source is then probably no worse than that from using the diffusion approximation in the pile or multigroup equations.

To justify the continuation to infinity, one observes that the integrals involved go something like $h^2/(u^2 + h^2)^2$ times a relatively slowly varying function of u. The first factor is very sharply peaked in the neighborhood of u = h and falls off as a high power of u. If the distance from the edge of the reactor is large compared to h, the controbution to the integral from large u is small. For points close to the edge the approximation breaks down, but these regions have low importance and can be ignored. The error was estimated by numerical integration at the center of a circular cylinder of radius = 10h (a relatively large gap). In this case the leakage is overestimated by about 1.5 %. For smaller gaps it is easy to see that the accuracy improves very rapidly.

The replacement of the gap by an equivalent absorber and the use of perturbation theory require that the flux depression at the gap be small.

V. Comparison With Experiment

The methods developed above were used to calculate the reactivity change as a function of gap width for the CA-4 graphite moderated critical assembly[5]. This reactor was a rectangular parallelepiped 130 x 112 x 112 cm with a gap perpendicular to the long dimension and slightly off center. Measurements of $\frac{\delta k}{k}$ for gap widths up to about 1 cm were made, the maximum opening determined by the available control. It was unfortunately extremely difficult to obtain accurate measurements of the table separations for very small gaps, hence the large experimental uncertainty.

Figure 3 compares the experimental points with theory, curve 1 representing the GGW result obtained from eq. (17) but with $\beta = 0$, while curve 2 is calculated from the complete expression. Eq. (12) does not apply to this system since a large fraction of the fissions are caused by fast neutrons. For this case eq. (12) gives about one-third of the total effect.

In view of the rather large uncertainty in the data the check between theory and experiment should be taken as only qualitative. At this point it cannot be said whether the apparent divergence between theory and experiment for large h is real.

The comparison is, however, sufficiently encouraging that for reactor in which diffusion theory is applicable one may expect to predict the effect of a small gap with some degree of confidence.

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5) For further details of the experiment see Report Y-881.

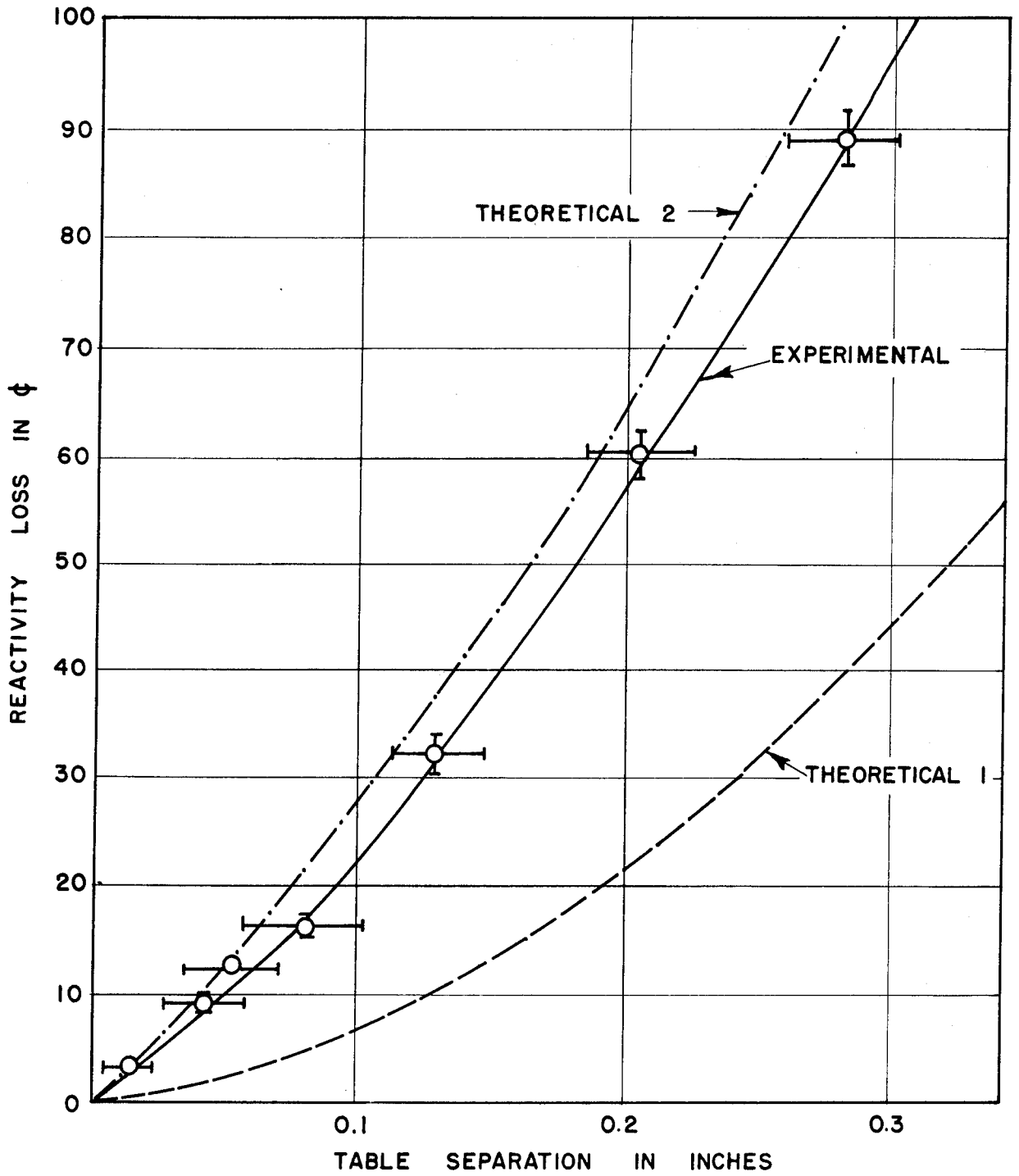

FIGURE 3. REACTIVITY LOSS vs TABLE SEPARATION

$1\,¢ = \frac{\delta k}{k}$ OF 7.3×10^{-4}

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