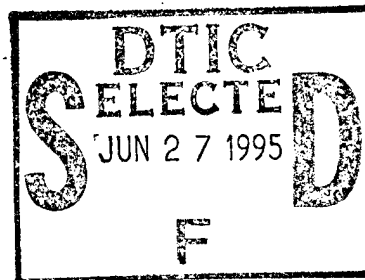WL-TR-95-4053

NUMERICAL SIMULATION OF ALUMINUM
EXTRUSION PROCESSES

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APRIL 1995

FINAL REPORT FOR 02/15/95--04/15/95

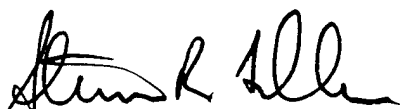APPROVED FOR PUBLIC RELEASE; DISTRIBUTION IS UNLIMITED.
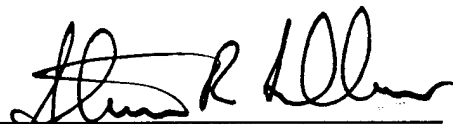
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This technical report has been reviewed and is approved for publication.

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NUMERICAL SIMULATION OF ALUMINUM EXTRUSION PROCESSES

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This presentation describes a research program directed towards the development of automated design procedures for aluminum extrusion technology. The objective is to eliminate costly trial and error by being able to simultaneously design the product, die, billet, and process (e.g., extrusion temperatures and speeds, uniformizing metal flow, etc.), within constraints of feasibility, and satisfying objectives including, but not limited to, optimizing shape, surface finish, and properties of the product, processing costs, time to market, and full utilization of capabilities. The approach is baed on the development of efficient and effective analysis of the whole processing system employing newly developed finite element solution technologies for complex, multiregion, multiphysical behavior. Generalizations of these methodologies to include Arbitrary Lagrangian-Eulerian (ALE) mesh descriptions for nonlinear, elasto-viscoplastic mechanical constitution equations will allow the faithful modeling of the metal flow within the die system and the accurate attainment of final shape upon exit. Automatic meshing and adaptive remeshing will insure efficient and accurate simulation of the entire forming process. New element technologies facilitating the use of general meshing procedures for difficult metal-forming processes involving a variety of kinematical constraints, such as incompressibility, contact, etc., are utilized. Feature based design methodologies, parametric modeling, and knowledge-based engineering techniques will constitute the fundamental methodologies for representing designs, managing the hierarchy of analysis models, performing model reduction and feature removal, and effectively utilizing design knowledge. Modern, three-dimensional, interactive visualization procedures are employed to animate simulations and design evolution. The software will be accessed through a modern and easy-to-use graphical user interface developed for extruders, die makers, and designers. The current status of our software and plans for executing the research and development program will be presented.

AUTOMATED DESIGN, FINITE ELEMENT SOLUTION TECHNOLOGIES, ARBITRARY LAGRANGIAN-EULERIAN (ALE), MESHING, FEATURE BASED DESIGN, PARAMETRIC MODELING, KNOWLEDGE-BASED ENGINEERING

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Numerical Simulation of Aluminum Extrusion Processes

Thomas J.R. Hughes and Arthur Muller

Centric Engineering Systems, Inc.

April, 1995

1. Summary

This is the final report for Centric's contract to Technical Management Concepts, Inc. under contract 95-TMC-01. This project was the first of several which are aimed at creating a Simulation Based Design System (SBD) for designing extrusion dies and processes. Ultimately, technology from Centric, Technosoft, and RPI will be combined to create the SBD system. However, the objective of this initial project was to demonstrate the applicability of the Spectrum Solver to the analysis of aluminum extrusion processes. The approach here is to simulate a two-dimensional cross-section of an extrusion process and to observe Spectrum's ability to model phenomena such as die-swelling, viscous heat dissipation, and the effects of die deformation on the shape of the final extruded part.

In our analysis the aluminum billet is treated as a Newtonian fluid whose balance laws are written in an arbitrary Lagrangian-Eulerian reference frame. Die-swelling is modeled as a free surface whose final position in a steady-state flow represents the billet's final shape.

Variations of viscosity with temperature and shear rate are not taken into account; instead, this preliminary study assumes a constant value of viscosity.

Spectrum's fluid-solid interaction capability is employed to study the effect of the die's deformation on the billet's final shape. Heat conduction from the billet into the die and then into the surrounding environment is also modeled with Spectrum's fluid-solid interaction capability.

2. Introduction

The extrusion process can be described by the insertion of the aluminum billet at the inflow surface. The objective of the die is to shape the final part to some pre-specified dimension. As the aluminum comes out from the bearing, there's a tendency for the material to swell to relieve the stresses generated during the billet's insertion. Depending on the

extruding conditions, this swelling can remain constant or it can oscillate. Oscillations will lead to a variation of the part's dimensions as a function of its length and, if excessive, the part must be discarded. Controlling die-swelling is one of the main goals of an extruding operation, and can be accomplished by modifying the die's shape, and the thermal and mechanical conditions on the die, as well as thermal conditions on the environment.

Figure 1 shows a two-dimensional extrusion process. The billet is the aluminum compound that will be pushed through the die in order to manufacture the final part. Extrusions are usually done at high temperatures, on the order of a few hundred degrees Celsius [Ref 1]. Controlling the excessive heat is a major component of the extrusion process.

The die consists of a very stiff structure, usually made out of steel, where the final manufactured shape is defined. The billet is pushed through the die in order to form the final part. Therefore, controlling the die swelling phenomenon is an important component of die design: the closer the extruded part is to the final shape, the better the die.

The inflow surface is where one considers the input of the billet into the die. At the interface between the aluminum and the die, the resulting forces are often of a complex behavior and a generalized friction force is often assumed. In this analysis we will only consider the case of a uniform plug flow, where a constant velocity profile through the inflow cross-section of the die is assumed. No-slip boundary conditions between the aluminum and the die are also assumed, and the plug-flow profile breaks down close to the walls.

Special attention must be payed to the portion of the die known as the bearing: the bearing shapes the aluminum compound to its final form. In the region around the bearing one may observe large pressures and temperatures, the latter due to the large shear rates and consequent viscous heating.

As the aluminum compound exits the die, it is cooled by its surroundings and solidifies. Because the aluminum is being treated as a Newtonian fluid, one must arbitrarily terminate the computational domain. The outflow surface is where one defines the end of the computational domain. At the outflow surface, the manufactured part has already achieved its final shape, so it is important that this surface be placed sufficiently far from the bearing region.

The analysis will be done in an arbitrary Lagrangian-Eulerian reference frame, where we allow the flow of mass from one element to the other, at the same time that we allow these elements to deform and adapt to the loads acting on them. We pay particular attention to the surface of the extruded part as it exits the die. As the material expands, one must ensure that this surface maintain a Lagrangian characteristic, i.e., special conditions must be added to the simulation that guarantee mass conservation through this surface. We refer to it as the "free surface".

The current investigation will not analyze elastic effects; the aluminum material will be treated solely as a Newtonian viscous fluid. In the arbitrary Lagrangian-Eulerian reference frame where the problem is being solved, elasticity is an important but complex component in this analysis and will be left for future investigations.

All analysis are done with the Spectrum Solver. Spectrum is a three-dimensional program. All runs were performed with one layer of three-dimensional elements with appropriate boundary conditions to simulate the two-dimensionality of the problem.

In this investigation, typical billet dimensions are 3in; the final manufactured part has a cross-section dimensioned at around 1in., with some of its members as thin as 0.18in. Whenever available, we will employ material properties for 6063 Aluminum, since it is the most frequently extruded alloy. These dimensions are extracted from [Ref 2],where typical extrusion dimensions are presented.

The remainder of this report contains the following: Section 3 describes a representative two-dimensional flow; boundary conditions and material properties are presented for the isothermal flow problem. Results are presented for this isothermal flow. Section 4 adds thermal effects to the flow. Thermal boundary conditions and material properties are presented. Section 5 introduces the die, along with notions of an interface. Material properties for the steel die are also presented, along with the boundary conditions employed. Results are shown for two values of viscosity. Section 6 extends the notion of an interface to include thermal effects. Once again, results are shown for two different viscosities. Sections 7 and 8 present conclusions and directions for future research. References are found in Section 9.

3. The Isothermal Flow Problem

The isothermal flow problem can be described by Figure 2, where AB is set to 3.188, BC to 3.0, CD to 3.0, DE to 0.25 in, and EF is set to 3.0. All dimensions are in inches.

Boundary Conditions:

The inflow surface is identified by segment AB, where a horizontal velocity of 16 inches/ minute is applied. This corresponds to common ram speeds as described in [Ref 2]. Because of the no-slip boundary conditions applied at the walls, the plug flow profile disappears at the node by the wall, point B

At the walls, identified by the segments BC, CD, and the bearing DE, we apply no-slip boundary conditions of zero velocity. As pointed out earlier, the stick/slip condition at the wall can be significantly more complex than the no-slip approximation; however, we believe that it serves the purpose of demonstrating the applicability of the Spectrum Solver to extrusion problems.

Symmetry conditions are applied on the lower segment AG. These translate into two distinct boundary conditions: zero vertical particle velocity, and zero vertical mesh velocity. The particle velocity stands for the velocity of the fluid particle at a point as a function of time. At a particular geometric point in space, several material particles may flow through that point. In an arbitrary Lagrangian-Eulerian reference frame, the analysis domain may vary with time. The Spectrum Solver will automatically reposition nodes internal to the domain as one moves its boundary. Boundary motion can be imposed by constraining the displacements of the mesh nodes.

A free surface is specified in segment EF. On this surface, the following constraint is imposed:

$$v^p_n = v^m_n$$

where v^p_n is the component of the particle velocity normal to the surface, and v^m_n is the component of the mesh velocity normal to the surface. Enforcing that these two velocity components are equal to each other precludes mass from escaping through the free surface. On top of that, we also specify zero pressure on the free surface, corresponding to a traction free boundary.

The outflow condition is represented by the segment FG. In this segment, Spectrum is able to avoid imposing any boundary conditions by computing the tractions at the surface from the flow's internal degrees-of-freedom (pressure and velocity), as well as its material properties; these tractions are then applied on the outflow surface as a boundary condition Zero pressure is also specified at the outflow.

Two other boundary conditions are applied that simplify the analysis: the first of them prevents the horizontal motion of the mesh at all nodes. Note that the free surface is still allowed to move freely in the vertical direction while satisfying the constraint above. Constraining the horizontal motion of the mesh does not imply any physical simplifications on the problem.

The second boundary condition recognizes the fact that the flow within the die does not require any mesh motion. We then restrain the motion of these nodes in both the horizontal and vertical directions.

Material Properties:

The fluid aluminum material can be characterized by two material properties, density and viscosity. In SI units, the density was set to 2690. Kg/m3. The viscosity was set to 1.E+6 Ns/m2.

Although the density accurately represents aluminum characteristics, the viscosity is but a gross approximation: the true viscosity is a complex function of both temperature and shear rate, whose representation goes beyond the scope of this preliminary analysis. However, it is important to note that the Spectrum Solver contains several built-in functions for viscosity behavior, such as Bingham, Carreau, and Power Law. On top of that, Spectrum also allows users to define the viscosity in a separate subroutine to be linked with the Solver at run time.

We also point out that elastic effects were not taken into consideration.

Numerical Simulation:

The finite element mesh consisted of 570 nodes and 236 elements. Although relatively coarse, this mesh was sufficient to represent all the phenomena we wanted to observe, from die swelling, to free surfaces and, later on, thermal effects as well as the influence of the die on the flow.

The simulation was carried out with a time-increment of 0.01 seconds for a total of 2 seconds. At this point, the final steady-state condition had been achieved. Spectrum's multi-staggered approach was used, with two staggers defined: one for the fluid flow, and the other for the mesh update. A sparse direct solver was employed on both staggers, with the left-hand-side matrix reformed at every nonlinear iteration. The simulation was carried out in approximately 50 minutes on an IBM RS/6000 550. Memory requirement was 11Mb.

Results:

The result shown in Figure 3 reveals the effect of die-swelling on the extruded part. In this simulation, no oscillations were observed in the final extruded part. Figure 4 shows the velocity vectors, while Figure 5 shows the pressure contours.

The swelling in this analysis was of 0.036 in, which is about 19% of the extruded thickness of 0.188 in..

4. Thermally Coupled Flow

The thermally coupled flow problem is solved on the same geometry as the isothermal problem.

Additional Boundary Conditions:

An additional inflow condition must be added to those of the isothermal flow in order to simulate the balance of energy in the flow. In this analysis, we assume that the aluminum enters the die at a uniform ambient temperature of 25C. Simulating real aluminum flow requires knowledge of the proper inflow conditions, where the aluminum is heated beyond its melting point.

By assuming that the viscosity is independent of the temperature, the energy balance does not couple back into the linear momentum balance; energy is conducted and convected in the existing flow which is heated by viscous dissipation effects. The temperature rise is then independent of the inflow temperature, which is used only as a reference.

Proper thermal boundary conditions must also be applied to the exterior surfaces. These comprise the portions of the model identified as the walls, and the free surface of the extruded part. We assume that the exchange of heat with the surrounding environment takes place via natural convection. The heat flux is given as

$$q_n = h\,(T - T_\infty)$$

where q_n is the normal heat flux, h is a heat transfer coefficient, T is the surface temperature, and T_∞ is a reference environmental temperature. For natural convection, we assumed a heat transfer coefficient of 10W/m2 C and a reference temperature of 25C.

Just like for the mechanical problem, the energy balance equation also requires special treatment at the outflow surface. Neither the normal heat flux nor the surface temperature are known at the outflow surface; the normal heat flux is computed from the temperature degrees-of-freedom and the material properties and applied as a boundary condition. Boundary conditions such as these are stable so long as the flow is outward to the surface, which is indeed the case here [Ref 3].

Symmetry is enforced on the lower surface by assuming it adiabatic.

Additional Material Properties:

The material model is completely defined by specifying the specific heat at constant pressure and the thermal conductivity. Phase change in the aluminum implies a strong dependency of the specific heat on temperature. The thermal conductivity, in its most general form, is also strongly dependent on the temperature. For simplicity, these two properties were assumed constant for the purposes of this preliminary investigation, at values of 9000 J/KgC and 170 W/mC, respectively.

Numerical Simulation:

The simulation was performed with 3 staggers, an additional one being employed for the thermal analysis. A sparse direct linear solver was also employed in the thermal stagger, for a total memory usage of 13Mb.

CPU requirements were slightly higher in this problem due to the necessity of bringing the thermal problem to a steady-state condition. 1,000 time steps were computed for a total CPU time of 1 hour and 5 minutes.

Results:

The thermal profile is shown in Figure 6. It is important to note the effect of the viscous dissipation on the temperature rise. The maximum temperature observed was 108.8C, an increase of 83.8C over the inflow boundary condition value. This is seen mostly in the area close to the bearing, where one sees strong velocity gradients.

It is important to note that this value is extremely low, given that aluminum extrusions usually occur at temperatures around hundreds of degrees. This reflects the simplifications employed in this analysis, namely: Newtonian flow at constant viscosity, and room temperature inflow boundary conditions.

5. Fluid-Solid Interaction: Mechanical Coupling

In order to analyze the effects of the die's flexibility on the flow, we employ Spectrum's fluid-solid interaction capability. This functionality is based on the notion of a master and a slave surface [Ref 4]: the fluid mechanics problem is solved on a deformable mesh that follows the deformation of the solid die. At the interface, the velocity of the fluid particle is matched to that of the solid particle. To the linear momentum stagger, we add the solution of the degrees-of-freedom associated with the die and solve simultaneously for the solid displacements, as well as the fluid's velocity and pressure degrees-of-freedom.

The presence of the solid die obviates the need to impose wall (no-slip) boundary conditions on the fluid nodes; instead, these conditions will be simulated via the interface, where these velocities are matched to those of the solid. As the system comes to a steady-state condition, the die comes to a rest and the interface condition automatically sets the fluid velocity to zero.

Die Geometry:

The die is shown in Figure 7. The added dimension BH was set to 0.5 in.

Solid Material Model:

The solid is modeled as steel, with Young's modulus $E = 200MPa$ and Poisson's ratio $V = 0.3$. Since only a steady-state solution is sought, the solid's density is not required.

Solid Boundary Conditions:

Two sets of boundary conditions are specified on the solid die. The first of them, along edge BH, holds the die in place as a cantilever beam by setting all displacement components to zero. Note that the die's restraints are important for an accurate computation of its deformations; the conditions imposed here provide us with a minimum set of restraints that allow us to solve the physical problem.

Because of the simplistic model utilized, we felt the need to add another boundary condition to further restrain the deformation of the die. Due to the dimensions and material properties selected for both the fluid and the solid, the cantilever condition by itself led to an opening of the die in the area of the bearing that significantly increased the exit area.

We arbitrarily chose the nodes on point E to constrain the displacement in the x-direction.

Numerical Simulation:

We utilized the same discretization for the fluid region as the analysis above. The solid mesh consisted of 250 nodes and 96 elements.

The analysis was performed over 1,000 time steps in order to allow a steady-state condition to settle in. Memory usage was 15Mb and the total CPU time was 1 hour 44 minutes.

Results:

Several sets of results are presented:

- Figure 8 shows the final deformed configuration.
- Figure 9 presents the pressure drop through the flow region.
- Figure 10 shows the velocity field in the flow region.
- In Figure 11 we present the Von Mises stress in the die.

We note that the excessive deformations shown in figures 8-11 are not representative of a real aluminum extrusion process. However, it does point to the strength of the Spectrum Solver, which was able to solve the coupled fluid-solid interaction problem even at deformations of this magnitude.

6. Thermo-Mechanical Fluid-Solid Interaction

We take the fluid-solid coupled problem described in the previous section a step further by solving the energy balance equation on both the fluid aluminum compound and the solid die. A coefficient of thermal expansion on the solid die provides a very weak coupling back to the flow by deforming the die; however, with the dimensions and properties utilized in the analysis, this effect is negligible as compared to the pressure exerted on the die by the flowing aluminum.

The fluid-solid interface in the Spectrum Solver obviates the need to impose thermal boundary conditions on the surfaces of the fluid in contact with the die. Energy flows naturally from one region to the other via a constraint that enforces equal temperatures on both sides of the interface. The convective heat flux conditions are transferred from the surface of the fluid to the exterior surface of the solid die. We also assume that the cantilevered section of the die is thermally insulated.

Additional steel properties:

To the material properties already defined, one must add thermal properties for the die. In general, these properties vary with temperature, but were assumed constant for simplicity.

The isotropic thermal conductivity and the coefficient of thermal expansion were set to 45 W/m2C and 1.E-5, respectively.

Results:

Figure 12 shows the final temperature distribution. It is important to observe that the temperature increase is significantly smaller than the fluid-only problem. This is due to the wide opening of the die, especially in the area around the bearing. This opening leads to a much smaller shear rate and, consequently, to a smaller viscous heat generation.

7. Conclusions and Future Research

Several conclusions can be drawn from the present analysis. The first is that the Spectrum Solver is a suitable tool for simulating extrusion processes. Spectrum is able to combine the die's deformation and its effects on the flow field in a single analysis. Computing the thermal problem on this deformed geometry is also a critical step that was successfully achieved with this demonstration. Once the variation of the viscosity with both temperature and shear rate are incorporated into the model, full two-way coupling can be achieved.

The effect of the die's deformation on the resulting temperature field is worth noting. By altering the geometry to account for the die's flexibility, the shear rates and the resulting viscous heating was cut in almost half.

Finally, we observe that die-swelling can be successfully modeled with the use of the free-surface functionality available in the Spectrum Solver.

Several directions can be taken in future research, of which we suggest a few.

As a starting point, it is important that this work be extended to a full three-dimensional analysis, where realistic dimensions for both the billet and the die are employed. This will increase the complexity of the modeling process substantially. To keep the overall time required to complete a three-dimensional simulation within the practical limits will require a close cooperation between Centric, Technosoft, and RPI. In as much as fifty percent of the time spent completing a simulating is typically consumed by model creation; integration of the three technologies should become a priority of this program.

Another important point is the fact that only no-slip boundary conditions were employed in this analysis. It is important that one also studies the effects of a slip boundary condition on the extrusion process. We expect that a more realistic scenario should include some regions where the aluminum billet slips through the die or, in a more general case, where friction is present.

The present analysis was done with a Newtonian flow with a constant viscosity. The material model employed for the aluminum can be perfected in two steps: the first step would be towards replacing the constant viscosity by a more realistic function of both the shear rate and the temperature. The second step would be to extend the analysis towards a visco-elastic fluid. It is our impression that elastic effects are an important aspect of the die-swelling phenomenon that should be included in the model.

Material properties for the die were assumed temperature-independent, and should be replaced by a material model that includes temperature dependence.

Finally, thermal and mechanical boundary conditions should match what is observed in the field. The conditions applied here, namely natural convection to the surrounding environment, aluminum inflow at ambient temperature, and the structural restraints serve the purpose of demonstrating Spectrum's ability to simulate the extrusion process; however, their effects on the results are of sufficient size that they should be replaced with more accurate and realistic conditions.

We are confident that the technology is working and that we can make significant progress in the future by combining these efforts with those of RPI and Technosoft.

8. Recommendations for Follow-on Work

We recommend that two projects be started in parallel.

The first will be to integrate software from Centric, Technosoft, and RPI to create a prototype SBD system. The prototype will comprehend modelling, meshing, and visualization functions and will require the efforts of all three companies. Centric will readily coordinate the efforts to generate a definitive proposal for this project.

The second project will be to complete the work necessary to carry out an incrementally more realistic simulation. It will include the following tasks:

1. Complete a fully three-dimensional simulation.
2. Develop more realistic material models
   - Allow viscosity to vary as a function of shear rate and temperature.
   - Add a viscoelastic fluid model.
3. Add a temperature-dependent material model for the die.
4. Define an approach that will lead to accurate modeling of friction.
5. Define more accurate and realistic thermal and mechanical boundary conditions.

9. References

[Ref 1] Nagpal, V., Billhardt, C.F., and Altan, T., "Lubricated Extrusion of "T" Sections from Aluminum, Titanium and Steel Using Computer-Aided Techniques", *Journal of Engineering for Industry*, August 1979, Vol. 101, pp. 309-325.

[Ref 2] R. Ray and J. Malas, private communication.

[Ref 3] T.J.R. Hughes, private communication.

[Ref 4] 1994, Centric Engineering Systems, Spectrum Theory Manual.

The Extrusion Process
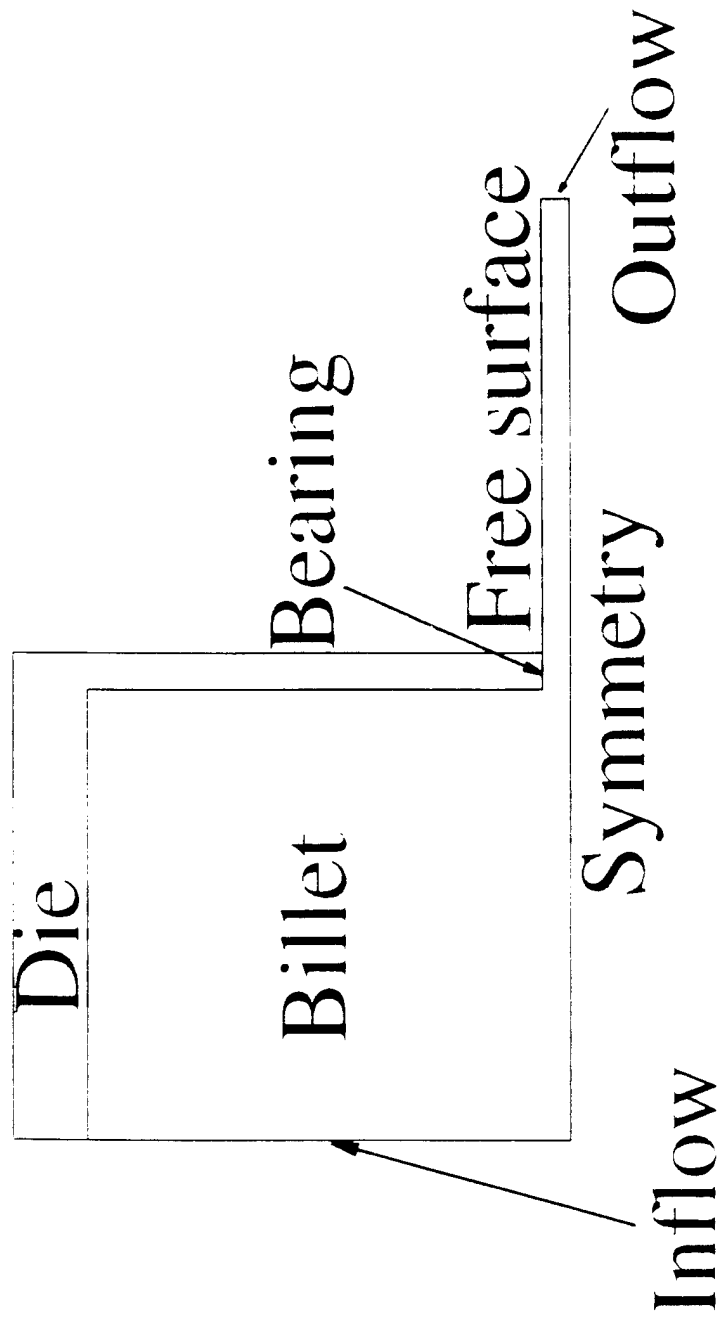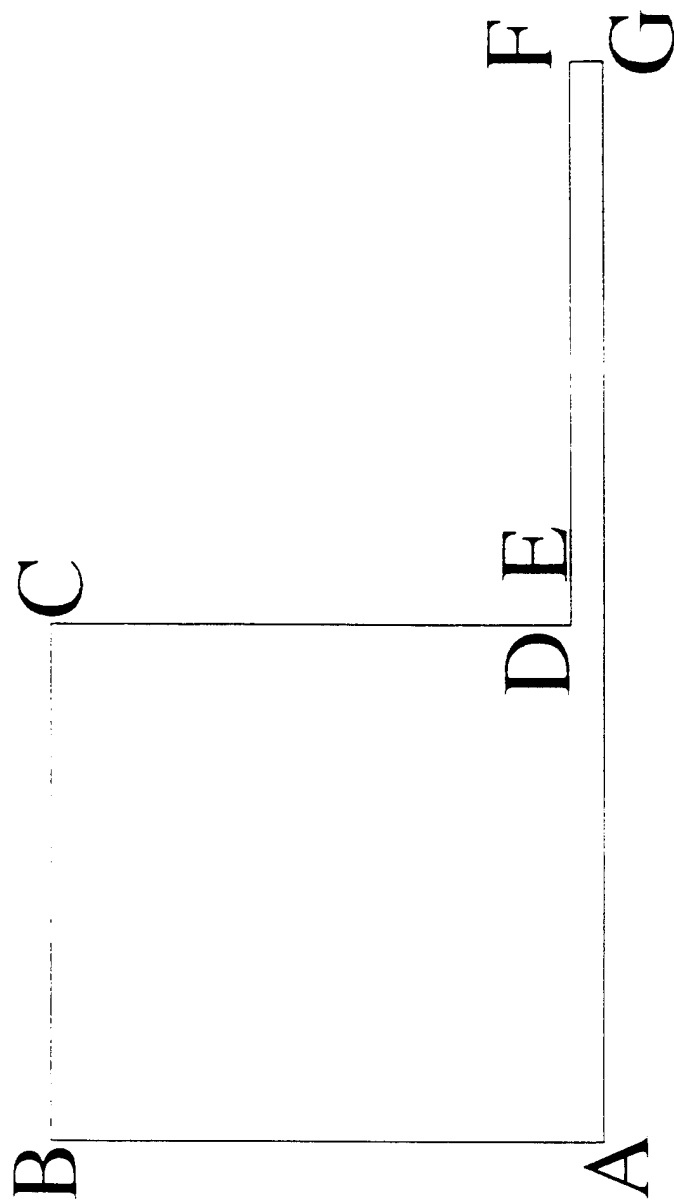

Figure 1

Isothermal Problem Description

Figure 2

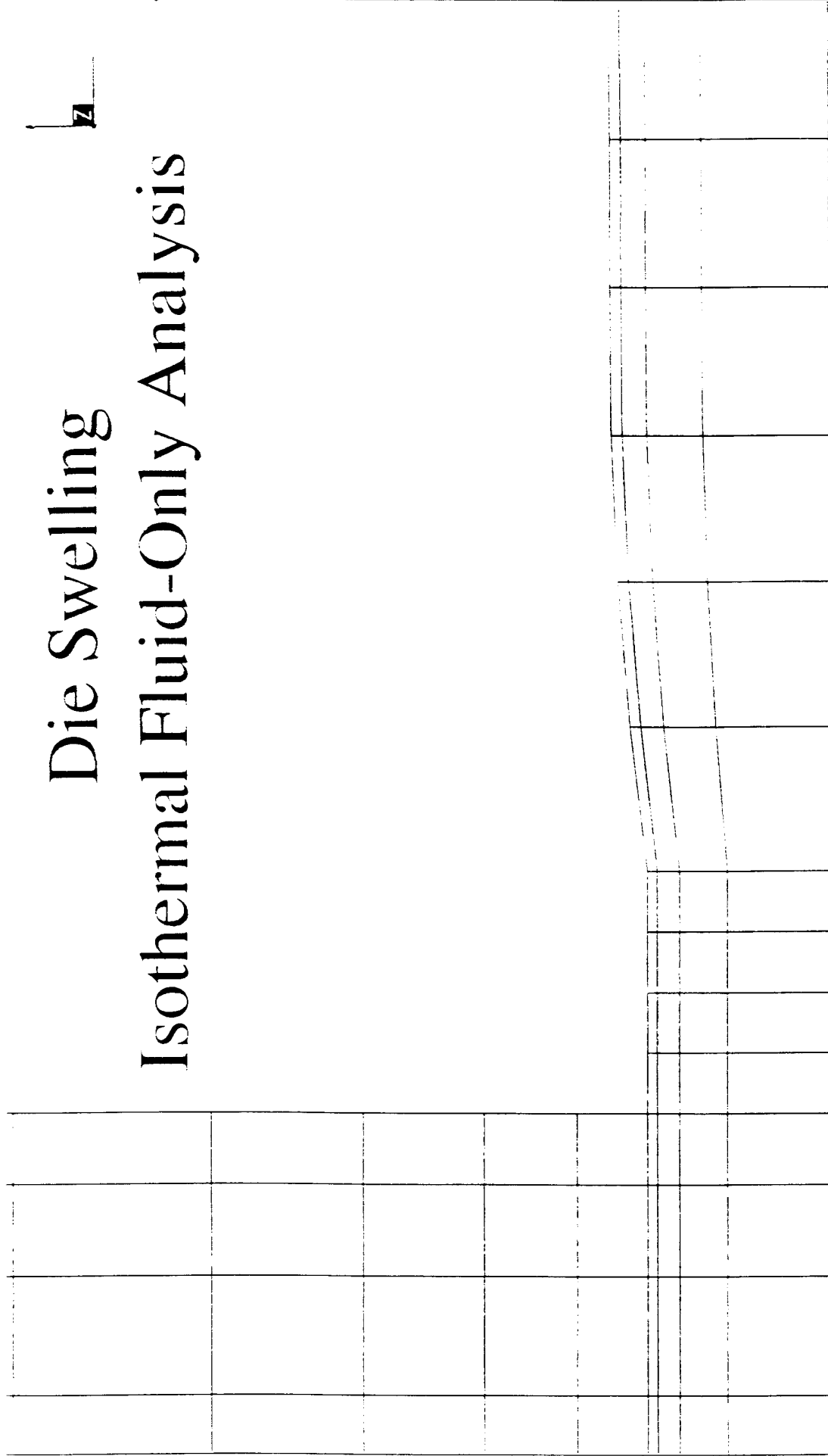# Die Swelling
Isothermal Fluid-Only Analysis

Figure 3

Velocity Vectors
Isothermal Fluid-Only Analysis

z

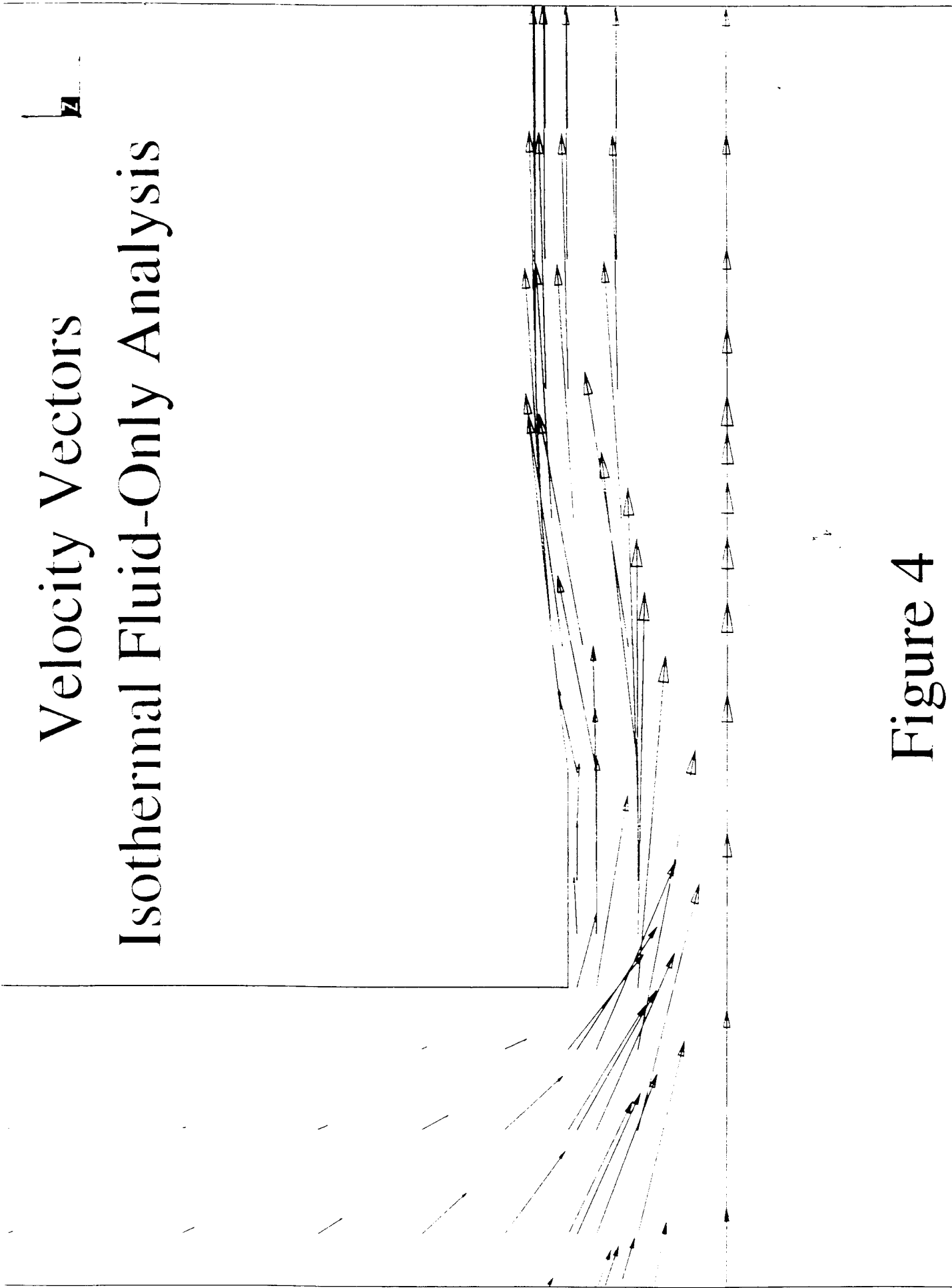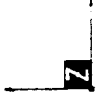# Figure 4
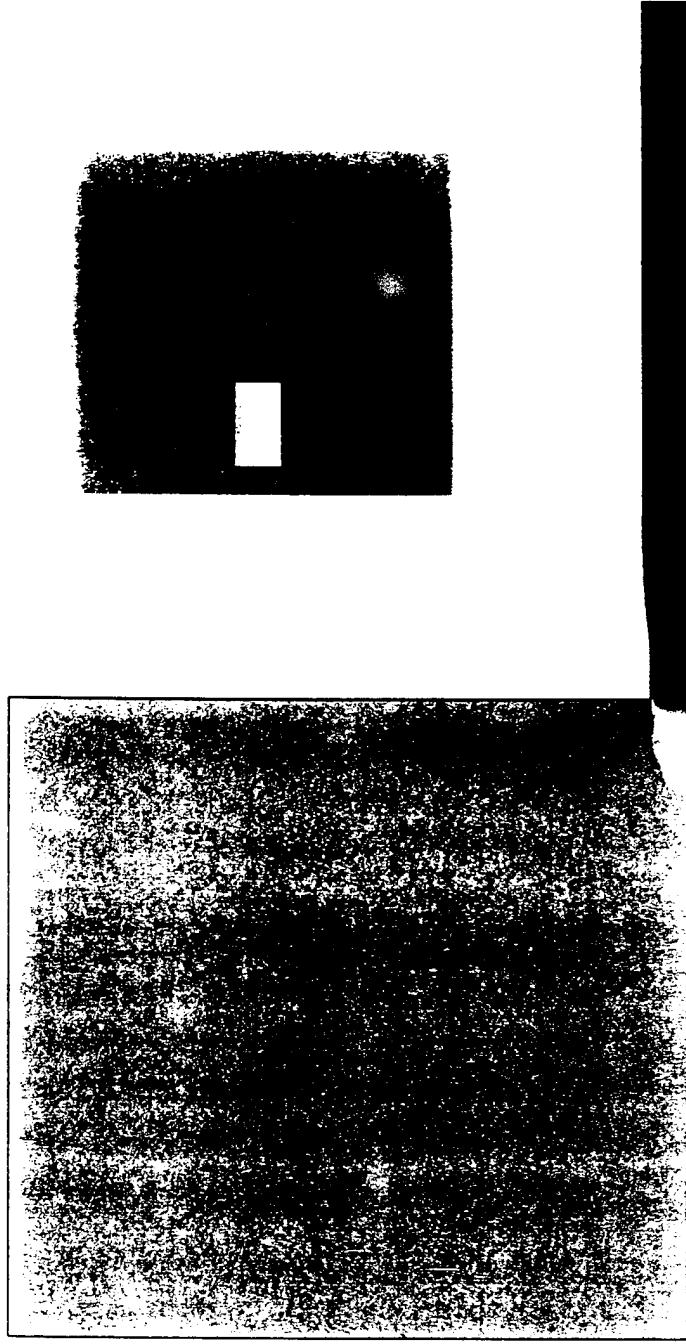
Pressure Contours
Isothermal Fluid-Only Analysis
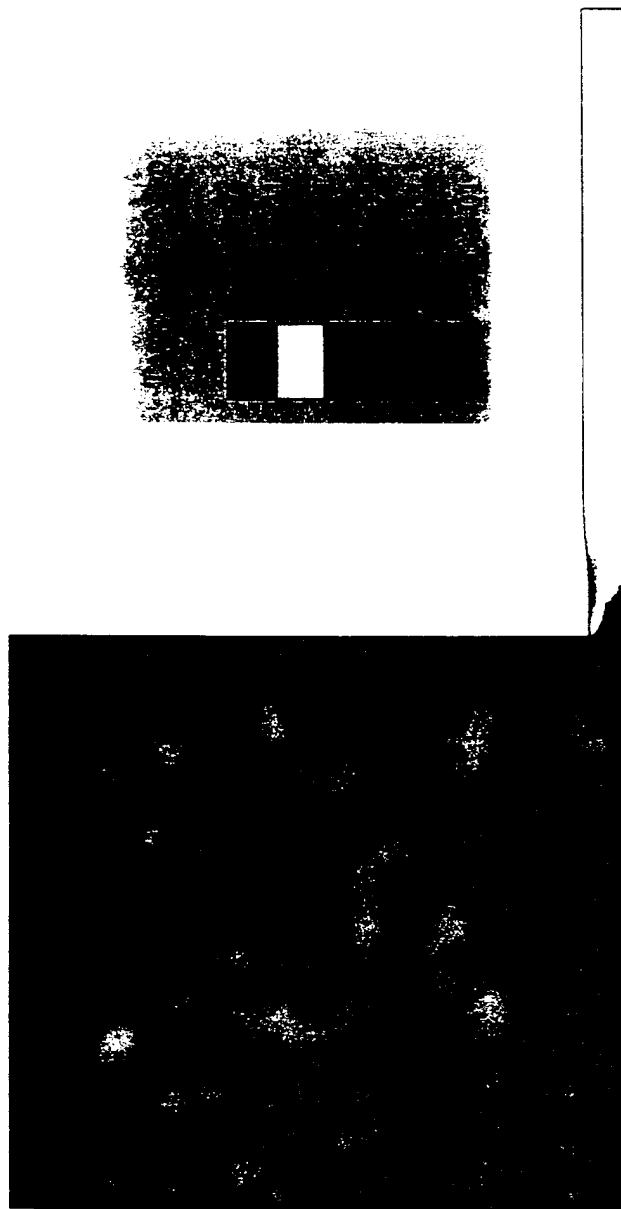

Figure 5

Temperature Contours
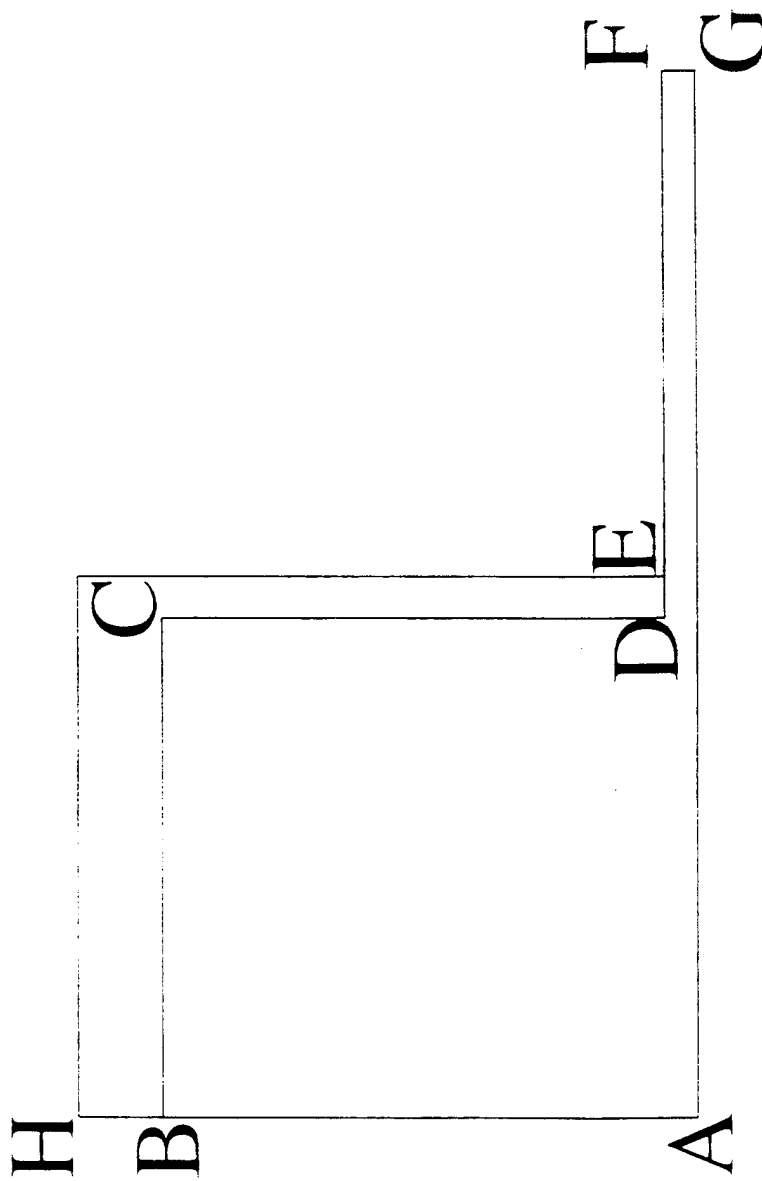Thermal Flow Analysis

Figure 6

Fluid–Solid Interaction Geometry

Figure 7

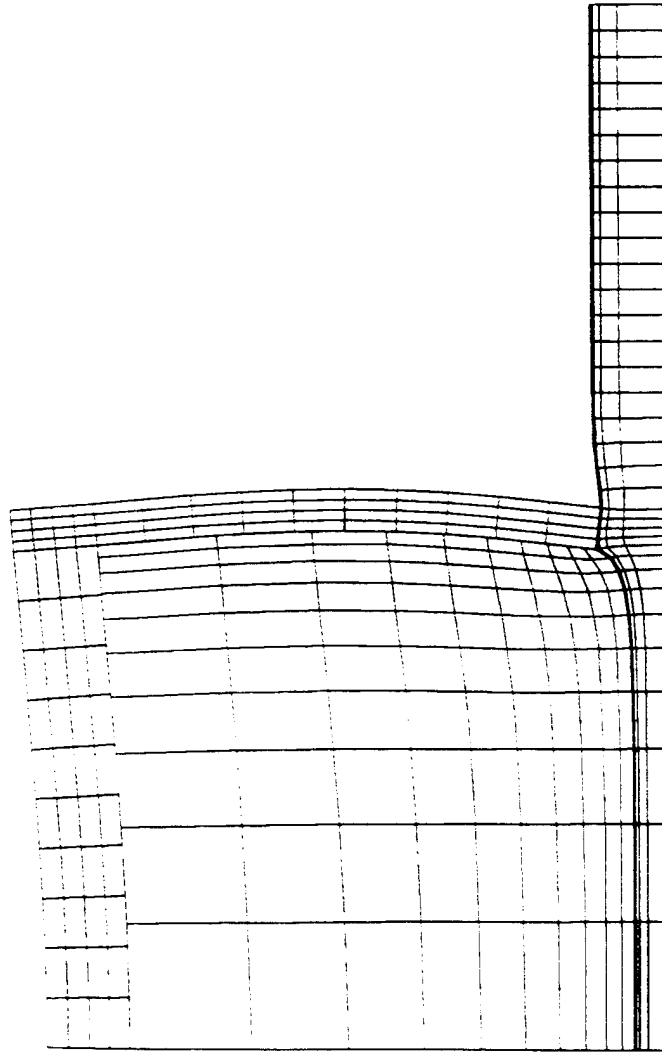# Fluid-Solid Interaction Geometry
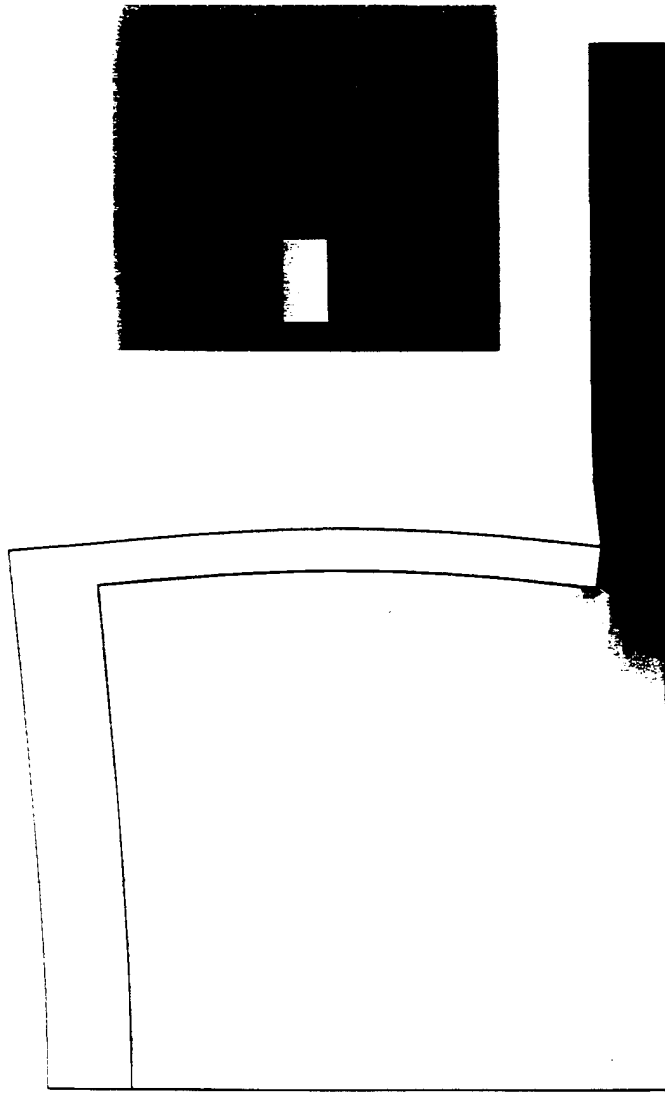## Final Configuration

Figure 8

Fluid–Solid Interaction Geometry
Pressure Contours

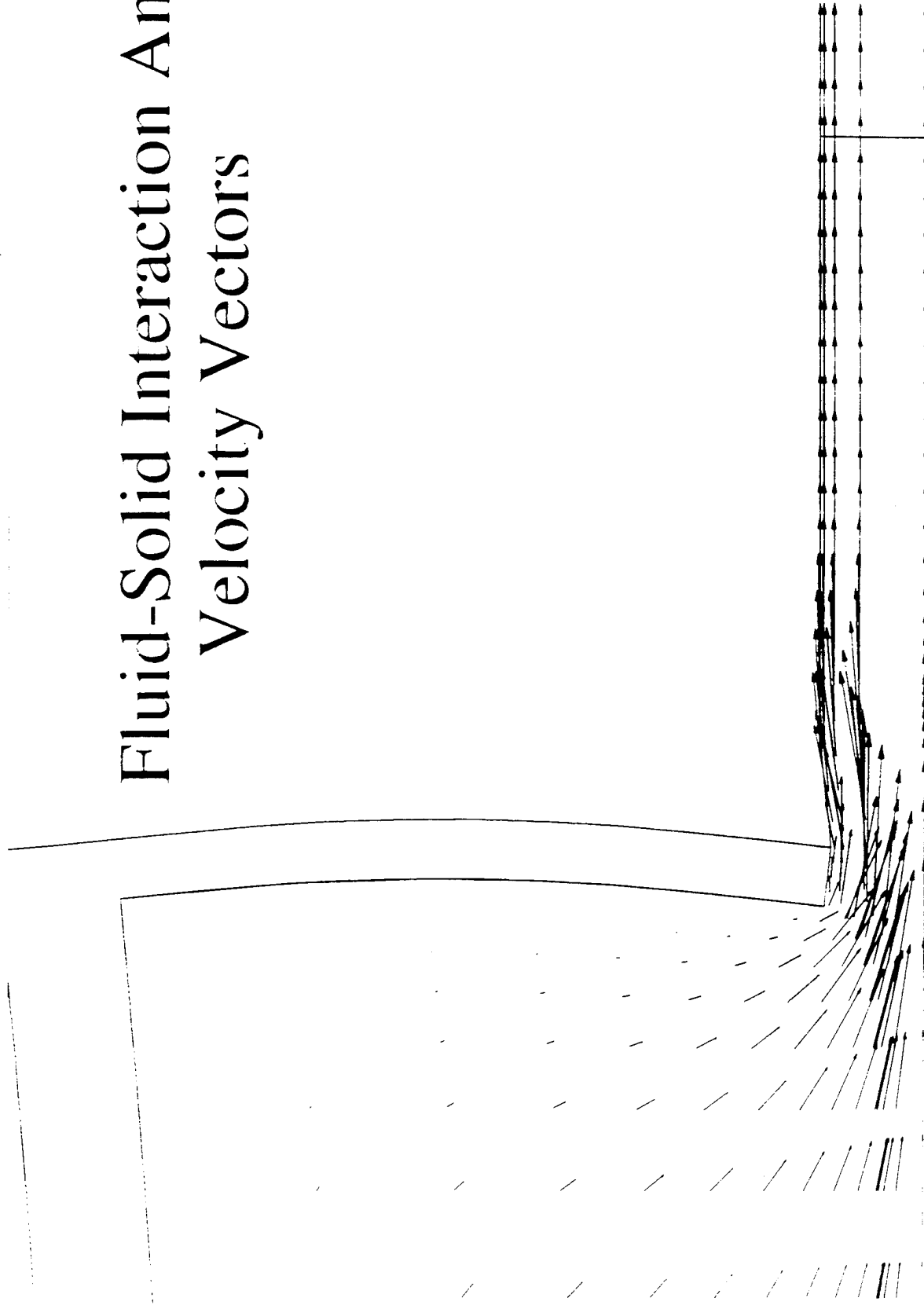Figure 9

Fluid-Solid Interaction Analysis
Velocity Vectors

Figure 10
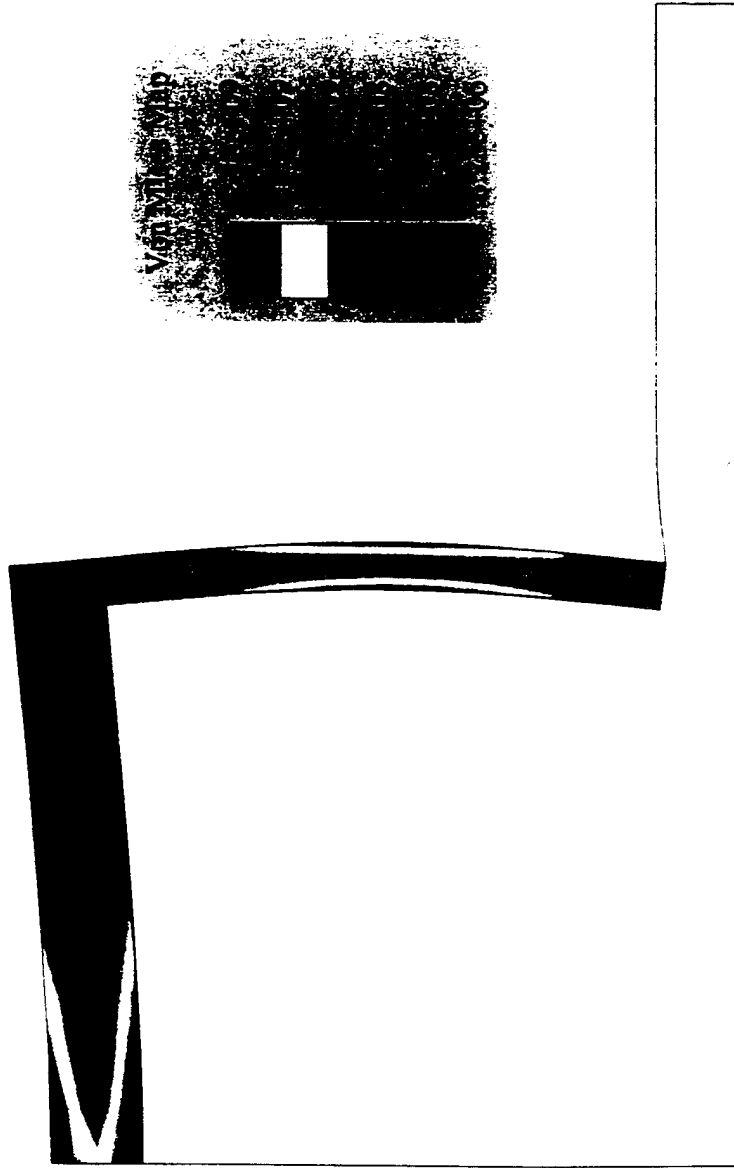
Fluid-Solid Interaction Analysis
Von Mises Stress

Figure 11

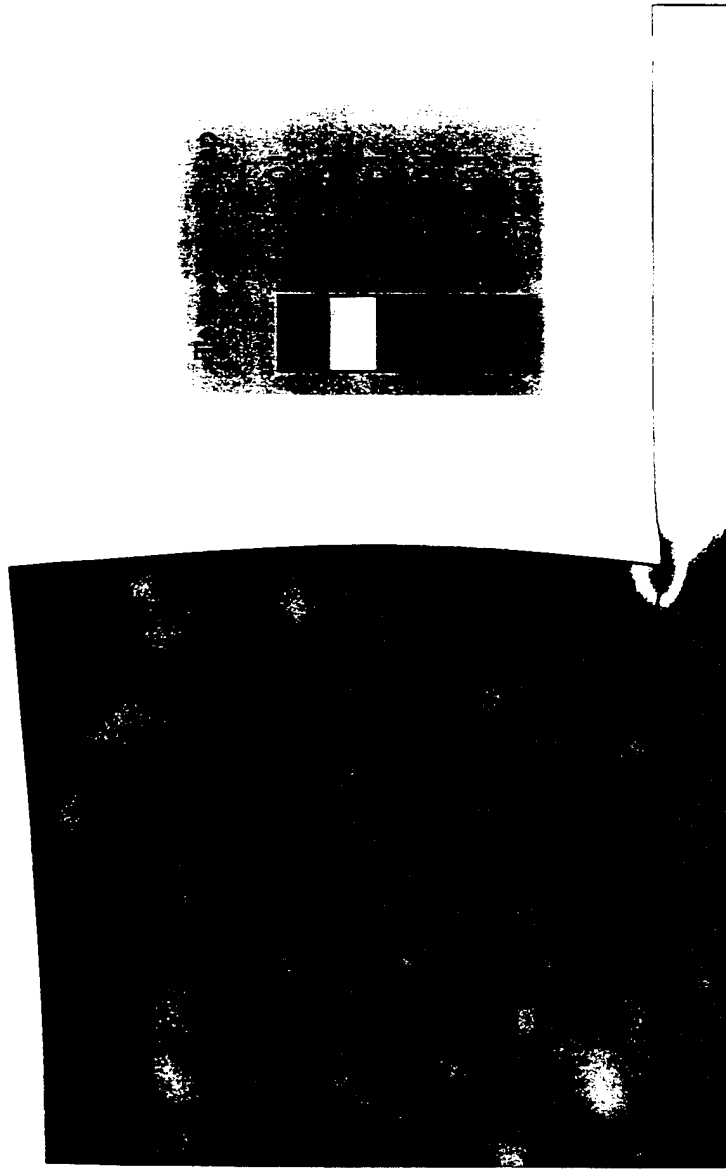# Thermal Fluid Solid Interaction
Temperature Contours

Figure 12