Congrès International sur la Modélisation Mathématique des Ecoulements en Milieux Poreux

SAINT-ETIENNE
22 - 26 MAI 1995

TABLE DES MATIERES

LES ORGANISATEURS REMERCIENT :

- La société S.N.F.- Floerger
- La Municipalité de Saint-Etienne
- Le C.N.R.S.
- Le Ministère de l'Enseignement Supérieur
- L'European Research Office, (U S A R D S G-UK)
- La Région Rhônes Alpes
- Le Conseil Générale de la Loire
- L'Université de Saint-Etienne
- L'Ecole Nationale Supérieure des Mines de Saint-Etienne
- La Faculté des Sciences et Techniques de l'Université Jean Monnet

pour l'aide qu'ils ont bien voulu apporter au Congrès International sur la Modélisation Mathématique des Ecoulements en Milieux Poreux.

ORGANISATION DU CONGRÈS

Coordination générale :	Claude CARASSO
Secrétariat général :	Arlette BERNARD
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	Andro MIKELIC
Edition des actes :	Mario AHUES
Sessions de posters :	Abdelkader HIDANI
Contact avec la Presse :	Jean -Luc FOURY
Restauration :	Hélène CHABREUIL

PROGRAMME DU CONGRES

Dimanche 21 mai	18H30	Cocktail d'accueil des participants à l'Altea et inscriptions
Lundi 22 mai	8H30	Inscriptions sur le lieu du Congrès (Ecole des Mines 29 rue Ponchardier)
	9H45	Inauguration du congrès par le Président de l'Université et le Directeur des Mines
	10H	Information sur le déroulement du Congrès
	10H15 - 11H05	Conf. de 50' GLIMM
	11H05 - 11H20	Pause café
	11H20 - 11H50	Conf. de 30' ANTONTSEV
	11H55 - 12H25	Conf. de 30' CUSHMAN
	12H30 - 14H	Repas
	15H30 - 17H	Posters 1 à 13
	17H - 17H50	Conf. de 50' AVELLANEDA
	17H55 - 18H25	Conf. de 30' GREENKORN
	19H	Réception à la Mairie de St Etienne
Mardi 23 mai	9H - 9H50	Conf. de 50' KNABNER
	9H55 - 10H25	Conf. de 30' ALLAIRE
	10H25 - 11H	Pause café et photographie du congrès
	11H - 11H50	Conf. de 50' FASANO
	11H55 - 12H25	Conf. de 30' RUSSEL
	12H30 - 14H	Repas
	14H30	Conférence de presse
	15H30 - 17H	Posters 14 à 26
	17H - 17H50	Conf. de 50' DOUGLAS
	17H55 - 18H25	Conf. de 30' LENORMAND
	18H30 - 19H	Conf. de 30' GALLOUET
	19H	Coktail offert par l'Université
	20H30	Concert offert par la Mairie choeur Contrechant (salle Aristide Briand à la Mairie)
Mercredi 24 mai	9H - 9H50	Conf. de 50' EWING
	9H55 - 10H25	Conf. de 30' THOMAS
	10H25 - 11H	Pause café
	11H - 11H50	Conf. de 50' PANFILOV
	11H55 - 12H25	Conf. de 30' BADEA
	12H30 - 14H	Repas

Mercredi 24 mai		Aprés-midi libre :
		Prestations proposées aux participants (à leur charge) : - visite du Musée d'Art Moderne, - visite du Musée de la Mine, - séance au planetarium - golf,
	18h30	Cocktail et présentation de l'économie de la région stéphanoise à la Chambre de Commerce et d'Industrie
Jeudi 25 mai	9H - 9H50	Conf. de 50' WHEELER
	9H55 - 10H25	Conf. de 30' RISEBRO
	10H25 - 11H	Pause café
	11H - 11H50	Conf. de 50' MOLENAAR
	11H55 - 12H25	Conf. de 30' YORTSOS
	12H30 - 14H	Repas
	15H30 - 17H	Posters 27 à 40
	17H - 17H50	Conf. de 50' VAZQUEZ
	17H55 - 18H25	Conf. de 30' CHAVENT
	18H30 - 19H	Conf. de 30' GILBERT
	19H45	Départ des cars pour le Banquet devant l'Altea pour le Superflu à St Romain le Puy
Vendredi 26 mai	9H - 9H50	Conf. de 50' ZHIKOV
	9H55 - 10H25	Conf. de 30' HORNUNG
	10H25 - 11H	Pause café
	11H - 11H30	Conf. de 30' AGANOVIC
	11H35 - 12H05	Conf. de 30' QUINTARD
	12H10 - 12H40	Conf. de 30' GIPOULOUX
	12H45 - 14H15	Repas

PROGRAMME SCIENTIFIQUE

Lundi	22 mai	10H15 - 11H05	Conf. de 50'	GLIMM
		11H05 - 11H20	Pause café.	
		11H20 - 11H50	Conf. de 30'	ANTONTSEV
		11H55 - 12H25	Conf. de 30'	CUSHMAN
		12H30 - 14H00	Repas	
		15H30 - 1700H	Posters 1 à 13	
		17H00 - 17H50	Conf. de 50'	AVELLANEDA
		17H55 - 18H25	Conf. de 30'	GREENKORN
Mardi	23 mai	09H00 - 09H50	Conf. de 50'	KNABNER
		09H55 - 10H25	Conf. de 30'	ALLAIRE
		10H25 - 11H00	Pause café	
		11H00 - 11H50	Conf. de 50'	FASANO
		11H55 - 12H25	Conf. de 30'	RUSSEL
		12H30 - 14H00	Repas	
		15H30 - 17H00	Posters 14 à 26	
		17H00 - 17H50	Conf. de 50'	DOUGLAS
		17H55 - 18H25	Conf. de 30'	LENORMAND
		18H30 - 19H00	Conf. de 30'	GALLOUET
Mercredi	24 mai	09H00 - 09H50	Conf. de 50'	EWING
		09H55 - 10H25	Conf. de 30'	THOMAS
		10H25 - 11H00	Pause café	
		11H00 - 11H50	Conf. de 50'	PANFILOV
		11H55 - 12H25	Conf. de 30'	BADEA
		12H30 - 14H00	Repas	
Jeudi	25 mai	09H00 - 09H50	Conf. de 50'	WHEELER
		09H55 - 10H25	Conf. de 30'	RISEBRO
		10H25 - 11H00	Pause café	
		11H00 - 11H50	Conf. de 50'	MOLENAAR
		11H55 - 12H25	Conf. de 30'	YORTSOS
		12H30 - 14H00	Repas	
		15H30 - 17H00	Posters 27 à 40	
		17H00 - 17H50	Conf. de 50'	VAZQUEZ
		17H55 - 18H25	Conf. de 30'	CHAVENT
		18H30 - 19H00	Conf. de 30'	GILBERT
Vendredi	26 mai	09H00 - 09H50	Conf. de 50'	ZHIKOV
		09H55 - 10H25	Conf. de 30'	HORNUNG
		10H25 - 11H00	Pause café	
		11H00 - 11H30	Conf. de 30'	AGANOVIC
		11H35 - 12H05	Conf. de 30'	QUINTARD
		12H10 - 12H40	Conf. de 30'	GIPOULOUX
		12H45 - 14H15	Repas	

50' LECTURES

AVELLANEDA M. Courant Institute, NEW YORK (U.S.A.)
Exact relations between electrical measurements and permeability : a porescale analysis

DOUGLAS J. PURDUE University (U.S.A.)
Anomolous diffusion in immiscible displacement resulting from fractal permeabilities

EWING R.E Texas A & M University, COLLEGE STATION (U.S.A.)
Mathematical modelling and simulation for applications of fluid flow in porous media

FASANO A. University of FIRENZE (ITALY)
Satured flows through deformable porous media with mechanical and chemical interactions

GLIMM J. University of STONY BROOK (U.S.A.)
Scaling and scale up for flow in porous media

KNABNER P. University of ERLANGEN (GERMANY)
Finite element approximation of contaminant transport problems in porous media

MOLENAAR J. DELFT University of Technology (THE NETHERLANDS)
Density driven two-phase flow in heterogeneous porous media

PANFILOV M. University OIL & GAZ Research Institute, MOSCOU (RUSSIA)
Averaged models governing tranfer process through highly heterogeneous porous media

VAZQUEZ L. Universitad Autonoma de MADRID (SPAIN)
Mathematical model for two-phase non equilibrium flows in porous media

WHEELER M. University of HOUSTON (U.S.A.)
non parvenue

ZHIKOV V.V. Pedagogical University, VLADIMIR (RUSSIE)
Homogenization of monotone operations in perforated domains

30' LECTURES

AGANOVIC I. University of ZAGREB (CROATIE)
Homogeneization of micropolar flow through a porous medium

ALLAIRE G. C.E.A. and University of Paris 6, PARIS (FRANCE)
Multi-scale convergence and homogenization in porous media with an infinitnumber of length scales

ANTONTSEV S.N. University of OVIEDO (SPAIN)
Filtration of immiscible fluids : effects capillary locking and hysteresis self-propelling under wave impact

BADEA A. Université Jean Monnet, ST ETIENNE (FRANCE)
Homogenization of two-phase flow through random porous media

CHAVENT G. I.N.R.I.A., ROCQUENCOURT (FRANCE)
Methology for the numerical treatment of two phase flows through porous media with two rock-types

CUSHMAN J.H. PURDUE University (U.S.A.)
Comparaison of eulerian nonlocal models to lagrangian models of chemical transport in heterogeneous media

ENE H.I. Institut de Mathématiques, BUCAREST (ROMANIA)
On the homogenization of clays

GALLOUET T. Ecole Normale Supérieure, LYON (FRANCE)
Schémas de volumes finis pour les problèmes d'écoulement en milieu poreux

GILBERT R. University of Delaware, NEWARK (U.S.A.)
Boundary layer corrections in the poroelastic seabed lying under a shallow ocean

GIPOULOUX O. Université Jean Monnet, ST ETIENNE (FRANCE)
Theorical and numerical homogenization of polymer flows

GREENKORN R.A. PURDUE University (U.S.A.)
An examination of a stochastic-nonlocal theory for modeling dispersion in scale-dependent porous media

HORNUNG U. University B W M, MUNCHEN (GERMANY)
A model of multicomponent diffusion in reactive acid soil

LENORMAND R. Institut Français du Pétrole, RUEIL-MALMAISON (FRANCE)
Transport equations for fluid displacements in heterogeneous porous media : the MHD model

QUINTARD M. E.N.S.A.M., BORDEAUX (FRANCE)
Calculation of porous media effective properties : computational problems and required unit cell features

RUSSEL T.F. University of Colorado, DENVER (U.S.A.)
Upscalling of dispervisity in modelling of solute transport : mathematical theory and simulations of laboratory experiments

THOMAS J.M. Université de PAU (FRANCE)
Analysis of finite volume methods and application to reverse simulation

YORSTOS Y.C. University of Southern California, LOS ANGELES (U.S.A.)
Studies with asymptotic regimes of displacements in porous media

POSTERS

Monday 22th of May, 15 H 30

Tuesday 23th of May, 15 H 30

Thursday 25th of May, 15 H 30

CONFERENCES DE 50 MINUTES

EXACT RELATIONS BETWEEN ELECTRICAL MEASUREMENTS AND PERMEABILITY: A PORE-SCALE ANALYSIS

Marco Avellaneda
Courant Institute, New York University

Electrical measurements have been used for many years by petroleum engineers to characterize the formation and fluid conductivity of porous media. In this talk, relations between electrical measurements and permeability are discussed and analyzed through simple models and computer simulations of Stokes flow through capillaries. It is shown that certain estimators based on electric measurements proposed by Johnson *et al.* (Jr. Fluid Mech., 176, 379 (1986)) and Zhou and Sheng (Phys. Rev. Lett. 57, 2565. (1986)) are. generically, good predictors of the effective pore-radius and the Darcy constant. However, the accuracy of such estimators may deteriorate if the porous medium has a wide range of (active) pore sizes or due to the asperity of pore walls.

Anomolous Diffusion in Immiscible Displacement Resulting from Fractal Permeabilities

Jim Douglas, Jr. * Felipe Pereira* Frederico Furtado†

March 15, 1995

Abstract

The analysis, through high resolution numerical simulations, of mixing lengths in water-oil fronts in heterogeneous porous media can be accomplished through efficient computations on fast, multi-processor computers. This paper is devoted to the description of an algorithm for such a computation and to the presentation of numerical results that lead to the characterization of anomalous diffusion arising from fractal permeabilities. It is shown that the exponent γ describing the scaling behavior (for large time) of the mixing region lies in the range $\frac{1}{2} \leq \gamma \leq 1$. For physically typical heterogeneity strengths and mobility ratios occurring in waterflooding processes, $\gamma = 1$; thus, typical waterflooding displays non-Fickean behavior, while the behavior tends to Fickean as the mobility ratio tends to one.

The computational algorithm is based on a domain-decomposition technique applied to a mixed finite element approximation of the nonlinear equations for two-phase, immiscible, incompressible flow expressed in an Eulerian form and is implemented to be portable to several state-of-the-art parallel architectures.

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†Department of Mathematics, UNICAMP, Campinas, Brazil

MATHEMATICAL MODELING AND SIMULATION FOR APPLICATIONS OF FLUID FLOW IN POROUS MEDIA

Richard E. Ewing
Institute for Scientific Computation
Texas A&M University

ABSTRACT

Understanding the fate and transport of contaminants to determine water quality and to develop remediation strategies or optimizing the recovery of hydrocarbons in petroleum applications each require the ability to model multiphase flow in heterogeneous three-dimensional reservoirs. Model equations and corresponding parameters must be determined at the appropriate length scales to describe the scaled physics of flow. Effective simulators require accurate numerical methods on general geometries. Use of mixed finite element methods and local grid refinement will be discussed. Example calculations for field simulations in aquifers or reservoirs with complex boundaries will be presented. Parallelization of the codes will also be discussed.

Saturated flows through deformable porous media with mechanical and chemical interactions

Antonio FASANO

University of Firenze, ITALY

We will examine a variety of filtration processes in which there is a mutual interaction between the flow and the porous medium. Typical cases are the dissolution of substances from the porous matrix to the flow, the removal of small particles from the skeleton and their transport by the flow, the deformation of the medium induced by the flow with the consequent change of the porosity and of the hydraulic conductivity. In the above framework many different situations can arise. For instance the deformations can be reversible or not, they can be instantaneous or can obey some relaxation kinetics.

Also we can have a wetting front (i.e. a free boundary problem) or other types of free boundaries due to the generation of internal inhomogeneities because of the transport of solid components. The variety of the physical situations is obviously reflected in the diversity of the mathematical structure.

Scaling and Scale up for Flow in Porous Media

James GLIMM

University at Stony Brook, Stony Brook NY 11794-3600

Geological variation occurs across all length scales. Both fine and coarse scale variability produce important effects upon the coarse scale fluid flow, which is of direct engineering interest. The (scale up) problem of predicting the influence of fine scale features on coarse scale flow is further complicated by the fact that the fine scale variation is only specified stochastically, as a random field.

As a simple model of multi-length scale geological variation, we use fractal random fields. For the case of linear transport (flow of a passive scalar concentration), results of the author and colleagues will be presented, based on direct simulation and ordinary and renormalized perturbation theory. Comparison to results of others will be included. Results of the author and others concerning nonlinear stochastic transport (which is a considerably more difficult problem) will also be presented.

Finite element approximation of contaminant transport problems in porous media

Peter Knabner

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Martensstr. 3, 91058 Erlangen
Germany

Models for the transport of reactive solutes in porous media as the adsorption–diffusion–advection model or more involved multi–component models exhibit various aspects which distinguish them from smooth parabolic equations or systems:

- non–smooth nonlinearities, as e.q. the Freundlich isotherm,
- non–equilibrium,
- advection dominance.

I.e., emerging mathematical problems are nonlinear parabolic systems with various degenerations:

- parabolic–hyperbolic locally
  (as in the porous medium equation),
- parabolic–ordinary differential equation,
- parabolic close to hyperbolic globally
  (as singular limit).

In the first part of the lecture we will discuss order of convergence results for the conformal finite element method taking the first two phenomena into account. We allow for numerical quadrature and time discretization and

elucidate the role of regularization and kinetic relaxation of equilibrium reactions. If in addition the third aspect is prominent, the conformal finite element approach is no longer appropriate and we select the Lagrange–Gelerkin method (or modified method of characteristics). The main problem consists in a self–consistent definition of the chracteristics to insure an accurate tracking of fronts and shocks also for large time steps. The evolving "fully implicit scheme" is discussed and preliminary order of convergence estimates indicated.

– joint work with John Barrett (London) and Holger Kappmeier (Erlangen).

Density driven two-phase flow in heterogeneous porous media

J. Molenaar
TWI, Delft University of Technology,
p.o.box 5031, 2600 GA Delft, The Netherlands

The contamination of groundwater by Dense Non-Aqueous Phase Liquids is increasingly studied by hydrologists. A DNAPL exists as a separate phase in the subsurface, and does not mix with the water on time scales of practical interest. Therefore the spreading of a DNAPL in the soil can be described by an immiscible, incompressible two-phase flow model. In the absence of capillary forces the 1D flow through a homogeneous porous medium is modeled by the classical Buckley-Leverett equation.

In this presentation we consider the flow of a DNAPL in a heterogeneous porous medium taking capillarity into account. It is well known that at heterogeneity a certain capillary pressure must built up, before the DNAPL can enter a low permeability region. The minimum pressure needed, is called the displacement or threshold pressure. A nonzero displacement pressure in the capillary pressure model (e.g. the Brooks-Corey model) has interesting mathematical consequences. By a regularization technique we derive conditions to match the solution on both sides of an interface, where the soil properties are discontinuous. There are two conditions: a flux condition and an extended capillary pressure condition. To illustrate the implications of these interface conditions we consider the stationary 1D problem. Without gravity this problem has a unique steady-state solution, however with gravity there are multiple steady-state solutions. Moreover neither the DNAPL saturation, nor the capillary pressure need to be continuous at the heterogeneity. This means that the usual assumptions for standard numerical simulators are in general not valid.

We present a numerical algorithm that is developed specially to deal with this situation: at a heterogeneity we assume continuity of flux and the extended interface condition for the capillary pressure. To validate the numerical method we consider two problems with heterogeneities that can be treated analytically: the 1D steady state situation with gravity, and a 2D time-dependent problem without gravity. Finally we will show some computational results for cases which are of more practical relevance.

Averaged Models Governing Transfer Processes Through Highly Heterogeneous Porous Media

M. PANFILOV

Moscow, RUSSIA

Macroscopic models are proposed to describe convection-diffusion transfer through multiply heterogeneous media. The heterogeneity is given by three periodic fields of porosity, conductivity and dispersion parameter. The high heterogeneity corresponds to the case, when the oscillation amplitude of one of fields is large. Dual porosity medium presents one example of examined systems. The convection velocity is derived from the pressure distribution described by parabolic equation.

All media are shown to be nonuniform. They include 15 classes, with different averaged models and internal flow structures. Media when the convection predominates in blocks present essential interest. They can be decomposed into three sub-classes depending on convection flow type. Closed results were obtained for the through-type flow in blocks or source-type flow using homogenization method. Averaged models contain the exchange term that is no symmetric in bloc and in fracture. Homogenization results depend upon the direction of pressure variation in time. In case when the time derivative of pressure changes its sign, the averaged model consists of equations with delaying argument. Example of its solution is constructed. It corresponds to the special inverse method of oil well investigation by means of tracer injection.

Media with through-source flow type in blocs was examined by the way of simulations on the network model of porous media. The analysis of exchange process is shown. Some results for nonlinear case were presented also. This system corresponds to both two-phase flow or one-phase concentration transfer, when fluid viscosity depends on concentration. The case of two-phase flow was examined without a contribution of high heterogeneity. Generalization of Darcy's law including the capillary dynamics is derived. It is shown by averaging of numerical simulations, that new type of capillary pressure function has a form of non monotone curves.

TITLE: **Mathematical model for two-phase non-equilibrium flows in porous media**

Juan Luis Vazquez, Dpto. de Matemáticas, Univ. Autónoma de Madrid, 28049 Madrid, Spain

ABSTRACT

We study a model for the flow of two immiscible fluids in a homogeneous and isotropic porous medium, based on the Muskat-Leverett model, classical in the oil science, with the non-equilibrium effects introduced by Barenblatt in describing the process of water-oil displacement.

It leads to the following system for the saturation $s \in (0,1)$ and the effective saturation σ:

$$\epsilon \Delta \phi(\sigma) = \sigma - s,$$
$$\tau s_t + s = \sigma,$$

where ϵ and τ are positive constants and Φ is a monotone constitutive function.

We pose the problem for $x > 0$ and $t > 0$ with intial data $s(x,0) \geq 0$ and boundary data $\sigma(0,t) = 1$. Our main results concern the existence and properties of the **free boundary** or **leading front** of the set $\{s(x,t) > 0\}$ and the large-time behaviour of the solution and free boundary.

This is joint work with G.I. Barenblatt (Urbana, USA), and Jesús García Azorero and Arturo de Pablo, from Madrid. Previous analysis is due to G.I. Barenblatt and A. Gilman.

Homogenization of monotone operators in perforated domains

V.V.Zhikov

January 10,1995

Vladimir Pedagogical University
Department Mathematics
600024, Vladimir, Russia

Let Q be a connected periodic domain in $\mathbb{R}^N, Q_\varepsilon = \varepsilon Q, \square = (0,1)^N$ be a cell of periodicity. We consider the problem

$$\begin{cases} -\mathrm{div}(a(\varepsilon^{-1}x, \nabla u^\varepsilon) + b(\varepsilon^{-1}x, u^\varepsilon)) = f \text{ in } \Omega \cap Q_\varepsilon, \\ a(\varepsilon^{-1}x, \nabla u^\varepsilon) \cdot n|_{\Omega \cap \partial Q_\varepsilon} = 0 \text{ (Neumann's condition)}, \\ u^\varepsilon|_{\partial\Omega \cap Q_\varepsilon} = 0 \text{ (Dirichlet's condition)}. \end{cases}$$

Here Ω is a bounded domain in $\mathbb{R}^N$ and $a(y,\xi), b(x,s)$ verifies the following structure conditions

1) $a(\cdot,\xi), b(\cdot,s)$ are $\square$-periodic and Lebesgue measurable on Q;

2) for a.e. $x \in Q$ and every $\xi_1, \xi_2 \in \mathbb{R}^N$

$$\left.\begin{array}{c} (a(x,\xi_1) - a(x,\xi_2)) \cdot (\xi_1 - \xi_2) \geq \alpha|\xi_1 - \xi_2|^p, \alpha > 0, \\ |a(x,\xi_1) - a(x,\xi_2)| \leq \alpha^{-1}(1 + |\xi_1|^{p-2} + |\xi_2|)^{p-2})|\xi_1 - \xi_2| \end{array}\right\} \text{ in the case } p \geq 2$$

$$\left.\begin{array}{c} (a(x,\xi_1) - a(x,\xi_2)) \cdot (\xi_1 - \xi_2) \geq \alpha|\xi_1 - \xi_2|^2(|\xi_1| + |\xi_2|)^{p-2} \\ |a(x,\xi_1) - a(x,\xi_2)| \leq \alpha^{-1}|\xi_1 - \xi_2|^{p-1} \end{array}\right\} \text{ in the case } 1 < p < 2$$

3) $a(x,0) = b(x,0) = 0$

4) for a.e. $x \in Q$ and for every $s_1, s_2 \in \mathbb{R}^1$

$$\left.\begin{array}{c} (b(x,s_1) - b(x,s_2)) \cdot (s_1 - s_2) \geq \beta|s_1 - s_2|^q, \beta > 0, \\ |b(x,s_1) - b(x,s_2)| \leq \beta^{-1}(1 + |s_1|^{q-2} + |s_2|^{q-2})|s_1 - s_2| \end{array}\right\} \text{ in the case } q \geq 2$$

$$\left.\begin{array}{c} (b(x,s_1) - b(x,s_2)) \cdot (s_1 - s_2) \geq \beta|s_1 - s_2|^2(|s_1| + |s_2|)^{q-2} \\ |b(x,s_1) - b(x,s_2)| \leq \beta^{-1}|s_1 - s_2|^{q-1} \end{array}\right\} \text{ in the case } 1 < q < 2$$

Note that there is not connection between the exponents p and q.

The homogenized operator $a_0 : \mathbb{R}^N \to \mathbb{R}^N$ is defined for every $\xi \in \mathbb{R}^N$ by

$$A_0(\xi) = \int\limits_{\square \cap Q} a(y, \xi + \mathrm{D}w(y))dy,$$

where w is the unique solution to the problem

$$\begin{cases} \int\limits_{\square \cap Q} a(y, \xi + \mathrm{D}w) \cdot \mathrm{D}\phi dy = 0 \text{ for every } \phi \in W^{1,p}_{per}(\square) \\ w \in W^{1,p}_{per}(Q). \end{cases}$$

Let also set

$$\chi(x) = \begin{cases} 1 \text{ if } x \in Q \\ 0 \text{ if } x \in \mathbb{R}^N \setminus Q \end{cases}$$

$$b_0(s) = \int\limits_{\square} \chi(x) b(x, s)dx, \quad \theta = |\square \cap Q|$$

Theorem. *If $f \in L^{q'}(\Omega)$. $q' = \frac{q}{q-1}$, then we have*

$$\lim_{\varepsilon \to 0} \int\limits_{\Omega \cap Q_\varepsilon} |u^\varepsilon - u^0|^q dx = 0,$$

$$\chi(\varepsilon^{-1}x) a(\varepsilon^{-1}x, \nabla u^\varepsilon) \rightharpoonup a_0(\nabla u^0) \quad \textit{weakly in} L^p(\Omega)$$

where u^0 is the unique solution to homogenized equation

$$\begin{cases} -div(a_0(\nabla u^0) + b_0(u^0)) = \theta f \text{ in } \Omega, \\ u^0|_{\partial\Omega} = 0. \end{cases}$$

The case of $Q = \mathbb{R}^N$ see Tartar [1] and Fusco, Moscariello [2]. The case of variational problem in perforated domains see Zhikov [3].

References

[1] Tartar L., *Cours Peccot au Collegè de Franc.*, Paris, 1977

[2] Fusco N., Moscoriello G., *On the homogenization of quasilinear divergence structure operators.* Ann. Mat. Pura Appl. **146**, (1987), pp. 1-13

[3] Zhikov V.V. *On the homogenization of Nonlinear Variational Problems in perforated Domains.* Russian Journal of Math. Physics, v. 2, No 3, (1994), pp. 393-408.

CONFERENCES DE 30 MINUTES

Homogenization of Micropolar Flow through a Porous Medium

I. Aganović, Zagreb

Homogenization of an incompressible micropolar fluid flow through a periodic porous medium is discussed. The homogenized problem consists of the corrected Darcy law and divergence free condition for macroscopic velocity. Under appropriate scaling the corresponding convergence result is proved.

The presented results are a part of the joint research of I. Aganović and Z. Tutek.

MULTI-SCALE CONVERGENCE AND HOMOGENIZATION IN POROUS MEDIA WITH AN INFINITE NUMBER OF LENGTH SCALES

Grégoire ALLAIRE
Commissariat à l'Energie Atomique
DRN/DMT/SERMA, C.E. Saclay, F-91191 GIF sur YVETTE
&
Laboratoire d'Analyse Numérique, Université Paris 6

In collaboration with M. Briane, we generalized the notion of two-scale convergence to the case of multiple separated scales of periodic oscillations. We introduced a multi-scale convergence method for the so-called reiterated homogenization of partial differential equations with oscillating coefficients. This new method is applied to a diffusion problem in a porous media modeled by an infinite number of periodic scales of heterogeneities.

Filtration of Immiscible Fluids: Effects of Capillary Locking and Hysteresis Self-Propelling under Wave Impact

Antontsev S.N.
Lavrentyev Institute of Hydrodynamics, Novosibirsk, Russia
FICYT, University of Oviedo, Spain

Penkovskii V.I.
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There are studied qualitative properties of solutions of the classical model of filtration of two immiscible incompressible liquids

$$\vec{v}_i = -K_0(x)k_i(s_i)/\mu_i \nabla(p_i + \rho_i g h), \quad i = 1,2,$$

$$\frac{\partial (m s_i \rho_i)}{\partial t} + \operatorname{div}(\rho_i \vec{v}_i) = 0, \quad (i = 1,2, \quad s_1 + s_2 = 1)$$

$$p_2 - p_1 = p_c(s_1, x).$$

It is shown that the domain of filtration flow may contain "dead" cores of two types where either $\vec{v}_1 = 0$ or $\vec{v}_2 = 0$. The former happens in the presence of the limiting values of the saturation s_1^0, s_2^2 which make the relative phase permabilities zero, $k_1(s_i^0) = 0$. Besides, the saturation distribution in the dead core is constant (either $s = s_1^0$ or $s = s_2^0$).

The second type is caused by the action of the capillary locking effect. In the cores of this type the saturation needn't be constant but so is the presssure in the corresponding phase, ($p_1 = const.$). It is shown that among with the well-known effect of the "boundary locking" of the wetting phase an analogous phenomenon can occure inside the flow domain.

There are considered peculiarities of the filtration flow of two immiscible incompressible fluids in mixed-wetted porous media where the capillary pressure function $p_c(s_1)$ is of the hysteresis character. Besides, the effect of the wave change of the pressure at the production and injection wells is discussed.

There are presented some results of numerical and physical experiments.

where $\vec{v}_i$, ρ_i, p_i and s_i are the velocity , tensity, pressure, saturation of the phase i, $i = 1,2$, $m(x)$ is porosity of the medium, $K_0(x)$ is the tensor of absolute permeability, $k_i(s) = k_{0i}/\mu_i$, μ_i is the viscosity, $k_{0i}(s_i) \geq 0$ ($k_{0i}(0) = 0, k_{0i}(s_i)$ increasing in s_i) is the relative permeability, g is the acceleration due to gravity, h is the distance to a fixed horizontal reference plane, $q_i(x,t)$, ($q_1 + q_2 = 0$), is the injected mass of the phase i, p_c is the capillary pressure (increasing in s_1 ($\partial p_c 7 \partial s \leq 0$) and $p(x, s_1) = 0$).

HOMOGENIZATION OF TWO-PHASE FLOW THROUGH RANDOM POROUS MEDIA

by Anca BADEA† and Alain BOURGEAT†

We are interested to apply the theory of homogenization for random coefficients, developped mainly for linear operators, to the modelisation of incompressible two-phase flow in randomly heterogeneous porous media.

After giving the equations of two-phase flow in porous media, we define the problem of homogeneization (or scaling up) we are considering.

In a second part, we recall the theoretical results on this problem as obtained in [1] and in [2].

Due to the difficulty to use these theoretical results as a mean of computing the effective coefficients (permeability and porosity), in the ergodic case, we design a way of doing this scaling-up numerically by using the notion of Volume Averaging. We prove that this method gives back the theoretical effective coefficients.

In the last part we present numerical tests. The first test is for a one-phase flow problem and shows the convergence of the method. The second one is for two-phase flow and shows interaction between fingering, heterogeneities and scaling-up.

References

[1] A. BOURGEAT, S. KOZLOV, A. MIKELIC: Effective Equations of Two Phase Flow in Random Media , To appear in Calculus of Variations and P.D.E.

[2] A. BOURGEAT, A. MIKELIC. S. WRIGHT: Stochastic two-scale convergence in the mean and applications , J.reine angew.Math. **456** (1994), 19–51

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Mixed finite elements and cell-centered finite volumes for two-phase flow in porous media with two rock-types

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Finite Volumes methods are very popular,as they are very close to the physics and allow the implementation of complicated physical laws. However their design may become delicate in complex geometries. On the other hand, Finite Elements methods are less close to the physics, but they come with a good mathematical background, and their analysis is quite advanced for structured as well unstructured meshes.

We show how to combine the advantages of finite volumes and finite elements methods into a numerical procedure for the resolution of two-phase incompressible flows through a porous medium with two different rock-types, by a joint use of a mixed-hybrid finite elements method and of a Godunov method.

Comparison of Eulerian Nonlocal Models to Lagrangian Models of Chemical Transport in Heterogeneous Media

John H. Cushman, Bill X. Hu, and Fei-Wen Deng

Abstract

When developing transport theory for heterogeneous porous media there are basically two frame works one may adopt: Eulerian and Lagrangian. The standard starting point for both types of theories is a "Darcy-scale" CDE. In the particle tracking (Lagrangian) framework, the Darcy-scale dispersive term in the CDE is neglected so that the particles move along streamlines. In this approach one does not obtain the mean concentration, but rather various order mean spatial moments. Using the Eulerian perspective, there is no need to neglect Darcy-scale dispersion as particles are not tracked along streamlines, but rather they are viewed from a fixed position. Thus as currently employed the Eulerian framework which explicitly obtains the mean concentration is more general than the Lagrangian models. We show that if the local-scale dispersion is neglected in the Eulerian models and if these models are not localized for reactive transport, then the mean spatial moments agree with those obtained through the Lagrangian picture. However, if the local-scale dispersive flux is kept in the Eulerian model, then the Eulerian moments can be substantially different than the Lagrangian moments.

ON THE HOMOGENIZATION OF CLAYS

By Horia I. Ene and Bogdan Vernescu

In studying clays, an intensive research has been done in two directions: a microscopic phenomenological study, on one hand, and a macroscopic constitutive modeling, on the other. The present research intends, by using the homogenization techniques, to make a link between the two theories.

In modeling the microscopic behavior of clays it is significant to consider the presence of negative surface charges caused by imperfections of the crystal lattice of the clay particles. The unbalanced charges are compensated by the accumulation of cations from the solution surrounding the mineral particles. A diffuse double layer is created.

The problem is described at the microscopic level by the balance of momentum in both parts of the porous medium (the fluid part and the solid skeleton)

$$-\mathrm{div}\sigma^f = f, \quad \text{and} \quad -\mathrm{div}\sigma^s = f \tag{1}$$

where the constitutive equation for the fluid phase involves an electric stress part:

$$\sigma^f_{ij} = \sigma^0_{ij} - \frac{1}{2}\eta_{ij}E^2 + \eta_{ik}E_kE_j \tag{2}$$

with σ^0_{ij} being the stress tensor in the fluid in the absence of the electric field.

We have to adjoin the balance of mass:

$$\mathrm{div}v = 0 \tag{3}$$

the Maxwell's laws:

$$\mathrm{div}(\eta E) = q \quad \text{in the fluid}, \qquad \mathrm{div}(\eta E) = 0 \quad \text{in the solid} \tag{4}$$

with q the free charge density and η the electric permeability,

$$\mathrm{curl}E = 0 \tag{5}$$

and the conservation of free charge in the fluid:

$$\mathrm{div}J = 0 \tag{6}$$

with the free current density:

$$J = \sigma E + qv \tag{7}$$

σ being the conductivity.

The boundary conditions are:

$$[\sigma \cdot n] = 0, \quad [v \cdot n] = 0, \quad [v_\tau] = \beta(\sigma^f n)_\tau. \tag{8}$$

where $v_\tau = v - (v \cdot n)n$.

We have to adjoin to these, the appropriate boundary conditions for the electric field and free current:

$$[E_\tau] = 0, \quad [\eta E \cdot n] = q^s, \quad J \cdot n = 0 \tag{9}$$

with q^s the surface charge density.

It is important to note that the first jump condition in (8) reduces to the double layer condition, that is usualy used to describe the presence of the electric charges on the surface between the fluid and a rigid solid.

The third condition in (8) reduces, in the absence of the volume and surfaces charges, to the slip condition used in the case of composites with imperfect interfaces. For the case of small viscous stress compared to the Maxwell stress part $(\sigma^0 n)_\tau << ((\sigma^f - \sigma^o)n)_\tau$ the condition reduces to the ζ potential condition in the Debye-Hückel approximation.

The homogenized equations display a coupled non-linear behaviour.

Schémas de volumes finis pour les problèmes d'écoulements en milieu poreux.

Robert Eymard[1], Thierry Gallouët[2] et Raphaèle Herbin[3]

Les problèmes d'écoulements en milieu poreux possèdent de nombreux aspects physiques, induisant des comportements variés, de transports convectifs et diffusifs notamment. Pour résoudre numériquement ces problèmes, des schémas numériques de type "Volumes Finis" sont fréquemment employés. Nous présentons ici les principes généraux permettant de bâtir ces schémas, fondés, en particulier, sur un principe de conservativité. Nous donnons ensuite quelques résultats de convergence des solutions approchées données par ces schémas pour certains problèmes simples d'écoulements en milieu poreux.

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Boundary layer corrections in the poroelastic seabed Lying under a Shallow Ocean

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We consider the problem of acoustic wave propagation in a shallow ocean, with a reactive seabed. We shall model the seabed as a poroelastic structure using the method of homogenization. The effect of the ocean will be taken into account by computing boundary layer terms. It will be assumed that the ocean occupies the region

$$I\!R_{[0,h]} := \{(x_1, x_2, x_3) : \ 0 < x_3 < h, \ (x_1, x_2) \in I\!R^2\};$$

whereas the seabed occupies $I\!R_{[0,-\infty]} := \{(x_1, x_2, x_3) : -\infty < x_3 < 0, (x_1, x_2) \in I\!R^2\}$. The poroelastic structure in the seabed will be modeled by assuming a periodic microstructure dependent on a small parameter ϵ which is of the same order of magnitude as the pore size. In $I\!R_{[0,-\infty]}$, we shall refer to the region $\Omega_{s\epsilon}$ as that part occupied by the solid, elastic matrix and $\Omega_{f\epsilon}$ as that part occupied by the fluid. Moreover we assume that all the pores are filled so that $I\!R_{[0,-\infty]} = \Omega_{s\epsilon} \cup \Omega_{f\epsilon}$. We consider the solid to consist of an elastic porous material, which we model in the solid space-time region $\Omega_{s\epsilon} \times (0, T)$ by using the elastic equilibrium equation

$$\rho^{s\epsilon} \frac{\partial^2 u_i^\epsilon}{\partial t^2} - \frac{\partial \sigma_{ij}^{s\epsilon}}{\partial x_j} = f_i^{s\epsilon}, \tag{1}$$

and in the fluid space-time region $\Omega_{f\epsilon} \times (0, T)$ using the viscous fluid equilibrium equation

$$\rho^{f\epsilon} \frac{\partial v_i^\epsilon}{\partial t} - \frac{\partial \sigma_{ij}^{f\epsilon}}{\partial x_j} = f_i^{f\epsilon}. \tag{2}$$

Here $\vec{u} := (u_1, u_2, u_3)$ is a solid displacement; whereas $\vec{\sigma} := (v_1, v_2, v_3)$ is a flux (or velocity). The fluid stress-tensor is given by

$$\sigma_{ij}^{f\epsilon} := -p^\epsilon \delta_{ij} + 2\mu(\epsilon) e_{ij}(\vec{v}^\epsilon). \tag{3}$$

where p^ϵ is the pressure, $\mu(\epsilon)$ a viscosity which depends on ϵ, and $e_{ij}(v^\epsilon)$ a rate of strain tensor. In addition, we assume that the fluid is incompressible; hence

$$\nabla \cdot v^\epsilon = 0 \quad \text{in} \quad (\Omega_{f\epsilon} \cup I\!R_{[0,h]}) \times (0, T). \tag{4}$$

The elastic coefficients of the solid matrix are taken to be anisotropic. The boundary layer corrections in the seabed due to the ocean abutting upon the poroelastic structure are calculated following the scheme used in [1].

Reference

[1] Ene, H. I. and Vernescu, B. Viscosity dependent behavior of viscoelastic porous media, in Asymptotic Methods Shell and Plate Theory, Pitman, 1995.

THEORICAL AND NUMERICAL DERIVATION OF POLYMER FLOW THROUGH TWO MEDIA

O. GIPOULOUX, A. MIKELIČ

After a short review on classical isotherm viscosity laws (Power law, Carreau law and his variants), we propose the theorical derivation of the microscopic polymer flow in two particular cases to the different filtration laws depending on the injection velocity or the Reynolds number.

At first, we consider an incompressible bidimensionnal polymer flow through a thin layer, by classical homogenization, we determine three filtrations laws wich are function of the order of the injection velocity in regard of the thin layer's depth. When the injection velocity is small enough compared with the thin layer's depth, we obtain an explicit relation between the filtration velocity and the pressure gradient. when the injection velocity is great enough compared with the layer depth, we obtain an explicit filtration law connecting the filtration velocity and a certain power of the pressure gradient. Between this two case, we obtain an implicit transition filtration law. Some numerical experiments are proposed to illustrate this results.

In a second part, we consider the case of a polymer flow through a porous medium. After a review of the filtration laws obtained by theorical homogenization, we try to propose some numerical algorithms to solve the homogenized problems wich present a coupling between the microscopic scale and the macroscopic scale due to the two scale convergence.

AN EXAMINATION OF A STOCHASTIC-NONLOCAL THEORY FOR MODELING DISPERSION IN SCALE-DEPENDENT POROUS MEDIA

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A stochastic-nonlocal theory used to predict mean concentration in megascopic systems was investigated. The investigation involved numerically solving the stochastic-nonlocal theory and comparing the results to experimentally measured laboratory data. The theory uses the Darcy scale transport equation along with the log-hydraulic conductivity concept. The velocity, log-hydraulic conductivity and concentration are decomposed into their mean and fluctuation terms and the resulting mean and mean removed equations are solved by use of Fourier and Laplace transforms. A nonlocal constitutive flux is incorporated into the theory. A FORTRAN program was used to numerically solve the theory. The resulting concentration profiles and second spatial moments were compared to data from a one-dimensional tracer experiment characterized by nonlocal dispersion. The experiment involved a cylindrical column packed with glass beads in a heterogeneous fashion. It was found that the simulation of the stochastic-nonlocal theory did not adequately model the experimental results. The simulation concentration profiles did not match the experimental concentration profiles. Specifically, the simulation significantly overpredicted the size of the experimental mixed zone. Further, the simulation did not model the nonlocal behavior well since the results matched the local advection-dispersion equation better than the nonlocal experimental results. It was also found that, while the comparison of the concentration profiles was poor, a comparison of the second spatial moments of the concentration profiles was quite good. Two reasons for the poor performance of the simulation are hypothesized: 1) the simulation solves for the mean concentration while the experiment corresponds to a single realization of concentration; 2) the covariance function chosen for the log-fluctuating conductivity may not actually model this property very well.

A Model of Multicomponent Diffusion in a Reactive Acid Soil

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Saint Etienne, 22 au 26 mai 1995

In structured soils, a considerable amount of soil water can be immobile. As this immobile water is confined to the smallest pore classes, it may be nevertheless in contact with most of the reactive surface areas of the solid phase. A model is proposed for the following general case: i) The mobile soil water is characterized by a relatively low pH and high Al^{3+} concentrations due to internal or external inputs of acids; ii) the solid phase consists of silicate minerals that weather irreversibly and relatively independent of pH by releasing base cations, and iii) all soil water is exposed to a high partial pressure of CO_2.

In the simplest case we model this situation as a one-dimensional multicomponent diffusion problem in which the connection to the mobile phase and the solid phase are described by Dirichlet and Neumann boundary conditions, respectively. Precipitation of secondary minerals ($Al(OH)_3$) in the water column is linked to decreased diffusion coefficients of dissolved ions.

The resulting set of coupled non-linear partial differential equations has a nonstandard form, since a reaction-diffusion system is coupled to a condition of electric neutrality everywhere in space. Therefore, special care has to be taken when applying numerical techniques for parabolic systems.

Test calculations show that the model exhibits a new mechanism for SO_4^{2-} accumulation in acid soils that interferes in a novel way with variations in NO_3^- in the mobile water.

TRANSPORT EQUATIONS FOR FLUID DISPLACEMENTS IN HETEROGENEOUS POROUS MEDIA : THE M.H.D. MODEL

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CPU time for numerical simulations can be reduced by using improved transport equations in an equivalent "homogenised" medium instead of fine-scale simulations. Most of simulators use the Koval non-linear equation for large viscosity ratios and homogeneous media. We have derived the average equations to be used when the medium is highly heterogeneous, and especially layered. Our method is based on a "two-fluid" approach and the determination of a matrix of relative permeabilities that accounts for anisotropy and gravity effects. The originality of the method, called M.H.D., is to combine the numerical calculation of the streamtubes for unit viscosity ratio, in any of the three main directions, to the stochastic calculation of the displacement inside the streamtubes.

The form of the transport equations, and the values of the parameters that govern the equations are found to depend strongly on the flow regimes. For the limiting cases :

- dispersive regime: a standard dispersion equation is valid with a dispersion coefficient "D" proportional to the correlation length and permeability variance (macrodispersion).
- fingering regime (due to viscosity or gravity) : the transport equation reduces to the known Koval's equation with an effective viscosity ratio "M".
- for channeling, at unit mobility ratio, it is shown that the Koval non linear equation is still a good approximation, with a heterogeneity parameter "H" instead of M.

A general equation is then proposed for displacements in reservoirs with long-range correlations and viscosity effects. The equation combines the dispersive term (parameter D) and the non-linear convective term with the product HM.

The advantage of the method is that the streamtubes and the D and H factors are determined from a monophasic flow. It can also be used for calculating the pseudo-permeabilities for immiscible displacements with negligible capillary effects.

CALCULATION OF POROUS MEDIA EFFECTIVE PROPERTIES: COMPUTATIONAL PROBLEMS AND REQUIRED UNIT CELL FEATURES

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ABSTRACT

The calculation of effective transport properties starting with the description of the porous medium in terms of a periodic unit cell is particularly attractive. It provides bounds or even estimates that can be very accurate and useful for modeling purposes. Besides the theoretical aspects associated with the derivation of the local problems that are needed to compute the effective properties, two problems are of a particular importance. The first problem is the implementation of effective algorithms, which can be a very difficult task if convective effects are important, and if the geometry is very complicated. The second problem is associated with the degree of complexity required in the unit cell geometry in order to reasonably represent actual characteristics of "real" porous media. In this paper, several algorithms are presented to solve for local problems associated with dispersion-like processes. High accuracy schemes can be implemented by solving pseudo-transient problems with operator splitting. Details and recommendations are provided.

Resulting numerical models are used to test for the importance of the unit cell geometry and the type of process under consideration, i.e., diffusion or dispersion. It is shown that highly dispersive systems calls for very complex, disordered systems.

Upscaling of Dispersivity in Modeling of Solute Transport: Mathematical Theory and Simulations of Laboratory Experiments

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One need in the current theory of subsurface transport in porous media is an improved understanding of upscaled transport physics in highly heterogeneous subsurface environments. This understanding should take the form of mathematical models that are valid at multiple scales. This investigation first focuses on a Lagrangian approach, similar to a formulation of Dagan, for calculating the dispersion tensor coefficients of the transport equation. The tensor is defined in terms of the second spatial moment of a plume with respect to the plume's centroid. This form of the dispersion tensor is derivable from the transport equation. The Lagrangian approach characterizes the dispersion tensor as a stochastic process, which in turn leads to consideration of the transport equation as a stochastic partial differential equation. The lecture discusses this theory and numerical models using it to simulate flow and transport in an experimental tank packed with sands of varying hydraulic conductivities. Comparisons to experimental data are presented.

Analysis of Finite Volume Methods and Application to Reservoir Simulation

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A new Mixed Finite Volume Method for elliptic problems on quadrilateral meshes will be presented and analyzed. We obtain so a conforming and conservative convergent scheme, with a 9-point stencil in 2-D (27-point in 3-D). Generalization with a triangular unstructured mesh is then given. These method is well adapted for domain decomposition. Some numerical examples in reservoir simulation and developed by Elf-Aquitaine Society will be presented.

STUDIES IN THE ASYMPTOTIC REGIMES OF DISPLACEMENTS IN POROUS MEDIA

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ABSTRACT

We study the asymptotic behavior of miscible displacements in porous media of 2-D rectilinear geometry in the two limits where a permeability-modified aspect ratio R_L becomes large or small, respectively. In either case, the problem reduces to the solution of an integro-differential equation. We investigate the validity of the asymptotic description and derive appropriate matching conditions in regions where it is not valid. For the case of unstable displacement, the linear stability characteristics of the equation as well as the non-linear finger evolution are described. Comparison with the numerical solution of the full problem delineates the range of validity of the asymptotic regimes.

POSTERS

On a Model of Fractured Porous Media

by

Ioana-Andreea ENE* and Jeannine SAINT JEAN PAULIN*

In this paper we study a double porosity model in a double periodicity media. From a mechanical point of vue this model represents a fractured porous media. From a mathematical point of vue we study a Neumann problem with double periodicity. We prove existence and unicity for such a problem and using the three-scale convergence we obtain the homogenized equation and the homogenized coefficients. From a mechanical point of vue the result we obtain is a Darcy law at the macroscale.

At least in the steady case these result show us that both the double periodicity model, introduced by Th. Levy [3] and P. Donato and J. Saint Jean Paulin [2], and the double porosity model, introduced by T. Arbogast, J. Douglas and U. Hornung [1], are the same.

References

[1] T. Arbogast, J. Douglas, U. Hornung, "Derivation of the double porosity model of single phase flow via homogenization theory", SIAM J. Math. Anal., 21, p. 823-836, 1990

[2] P. Donato et J. Saint Jean Paulin, "Stokes flow in a porous medium with double periodicity", Progress in Partial Differential Equations: the Metz surveys, Eds. M. Chipot, J. Saint Jean Paulin and I. Shafrir, Pitman, Longman Press, p. 116-129, 1994

[3] T. Levy, "Filtration in a porous fissured rock: influence of the fissures connexity", European J. Mech., B/Fluids, 9,4, p. 309-327, 1990

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Non-linear Darcy's Law

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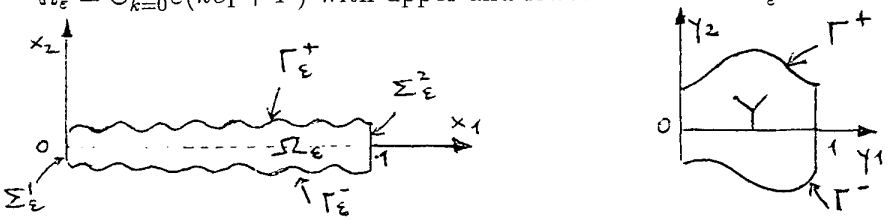
We study the stationary Navier-Stokes system in 1-dimensional porous medium. Let $Y \subset [0,1]^2$ be periodic in e_1 direction with upper and lower boundaries Γ^+ and Γ^- ($\text{dist}(\Gamma^+, \Gamma^-) > 0$). For $\varepsilon = 1/n$, $n \in \mathbf{N}$ we define the domain $\Omega_\varepsilon = \cup_{k=0}^{n-1} \varepsilon(ke_1 + Y)$ with upper and lower boundaries Γ_ε^{+-}



Let $\Sigma = \overline{Y} \cap \{x_1 = 0\} = \overline{Y} \cap \{x_1 = 1\}$. Let $g^i \in C_0^\infty(\Sigma)$, $i = 1, 2$ be such that $\int_\Sigma g^1 = \int_\Sigma g^2 = \theta$. For $g_\varepsilon^i(x) = g^i(x/\varepsilon)$ we consider the problem

$$\begin{cases} -\Delta u^\varepsilon + (u^\varepsilon \nabla) u^\varepsilon + \nabla p^\varepsilon = 0 \ , \ \text{div}\, u^\varepsilon = 0 \ \text{ in } \Omega_\varepsilon \\ u^\varepsilon = 0 \text{ on } \Gamma_\varepsilon^{+-} \ , \ u^\varepsilon = \frac{1}{\varepsilon} g_\varepsilon^i e_1 \text{ on } \Sigma_\varepsilon^i \end{cases} \tag{1}$$

for $\Sigma_\varepsilon^i = \varepsilon \Sigma^i$. Using the asymptotic expansions in the form

$$u^\varepsilon = \frac{1}{\varepsilon}\{u^0(x_1, y) + \cdots\} \ , \ p^\varepsilon = \frac{1}{\varepsilon^3}\{p^0(x_1) + \varepsilon p^1(x_1, y) + \cdots\} \ , \ y = \frac{x}{\varepsilon} \tag{2}$$

we obtain the two-scale homogenised problem ·

$$\begin{cases} -\Delta_y u^0 + (u^0 \nabla_y) u^0 + \nabla_y p^1 = -\frac{dp^0}{dx_1} e_1 \ , \ \text{div}_y\, u^0 = 0 \ \text{ in }]0,1[\times Y \\ u^0 = 0 \text{ on } \Gamma^{+-} \ , \ (u^o, p^1) \text{ is 1-periodic in } y_1 \ , \ \int_Y u^0 = \theta \ . \end{cases} \tag{3}$$

For $|\theta| < \sqrt{|Y|}/2\sqrt{2}$ we prove that the problem (3) has the unique solution $u^0 \in L^2(0,1; W(Y))$, $p^1 \in L^2(]0,1[\times Y)/\mathbf{R}$, $p^0 \in L^2(0,1)/\mathbf{R}$ where

$W(Y) = \{\phi \in H^1(Y)\ ;\ \mathrm{div}\phi = 0\ ,\ \phi|_{\Gamma^{+-}} = 0\ ,\ \phi \text{ is 1-periodic in } y_1\}$. Moreover $u^0 = u^0(y) \in W(Y)$, $p^1 = p^1(y) \in L^2(Y)$ and p^0 is the linear function $p^0(x_1) = -\alpha x_1 + \beta$.
By computing boundary layers u^i_{bl} on Σ^ε_i , $i = 1, 2$ and estimating the diference we obtain the following result:

Theorem 1 *Let* $C_p\{|u^0|_{L^\infty} + |u^1_{bl}|_{L^\infty} + |u^2_{bl}|_{L^\infty}\} < 1$. *Then*

$$\begin{array}{l} |\varepsilon u^\varepsilon - [u^0(\frac{x}{\varepsilon}) + u^1_{bl}(\frac{x}{\varepsilon}) + u^2_{bl}(\frac{x}{\varepsilon})]|_{H^1(\Omega_\epsilon)} \le Ce^{-1/\varepsilon} \\ |\varepsilon^3 p^\varepsilon - \{p^0 + \varepsilon[p^1(\frac{x}{\varepsilon}) + p^1_{bl}(\frac{x}{\varepsilon}) + p^2_{bl}(\frac{x}{\varepsilon})]\}|_{L^2(\Omega_\epsilon)} \le Ce^{-1/\varepsilon}\ . \end{array} \tag{4}$$

As the consequance we conclude that

$$\begin{array}{l} \int_{\Sigma^\epsilon(\frac{\cdot}{\varepsilon})} u^\varepsilon(\,\cdot\,, x_2)dx_2 \rightharpoonup \int_Y u^0 \ \text{ weakly in } \ L^2(0,1) \\ \varepsilon^2 \int_{\Sigma^\epsilon(\frac{\cdot}{\varepsilon})} p^\varepsilon(\,\cdot\,, x_2)dx_2 \to p^0 \ \text{ strongly in } \ L^2(0,1)\ , \end{array} \tag{5}$$

where $\Sigma^\varepsilon(y_1) = \varepsilon\Sigma(y_1)$ and $\Sigma(t) = \overline{Y} \cap \{y_1 = t\}$.
Finally we find the relation between filtration velocity $v = \int_Y u^0_1$ and the pressure gradient $\frac{dp^0}{dx_1}$ in the form of non-linear law

$$v = k(-\frac{dp^0}{dx_1}) \tag{6}$$

where $k(\alpha) = \int_Y u_1(\alpha)$ for $u(\alpha) \in W(Y)$ the solution of the problem

$$\left\{ \begin{array}{l} -\Delta u(\alpha) + (u(\alpha)\nabla)u(\alpha) + \nabla p(\alpha) = \alpha e_1\ ,\ \mathrm{div}\, u(\alpha) = 0 \ \text{ in } Y \\ u(\alpha) = 0 \text{ on } \Gamma^{+-}\ ,\ (u(\alpha), p(\alpha)) \text{ is 1-periodic in } y_1\ . \end{array} \right. \tag{7}$$

For the function k we find the Taylor's expansion in neighbourhood of 0 with $k^{(m)}(\alpha) = \int_Y u^{(m)}_1(\alpha)$ where $u^{(m)}(\alpha)$ are the solutions of the linear problems obtained by formal derivation of (7) with respect to α. We prove that $k(0) = k''(0) = 0$ so that

$$v = -k'(0)\frac{dp^0}{dx_1} - \frac{1}{6}k'''(0)(\frac{dp^0}{dx_1})^3 + \cdots\ . \tag{8}$$

moreover $k'(0)$ is the standard Darcy's permeability (as in the linear law) and $k'''(0) > 0$.

INSTABILITY THRESHOLDS in MISCIBLE FLUID FLOWS

by

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ABSTRACT

When a fluid of lower viscosity displaces a fluid of higher viscosity, the interface between the two fluids is unstable. The resulting interface pattern is refereed to as viscous fingering [1]. When a denser fluid sits on the top of a lighter one, the situation is also unstable leading to gravity fingering [2]. When gravity and viscosity act on simultaneously, they can contribute either in the same way to the interface stability (both stabilizing or destabilizing) or in opposite. In the latter case, two situations have to be described. A lighter and less viscous fluid is injected from top to bottom against a denser and more viscous fluid (flow 1): at low flow rate, q , the gravity stabilizing effect overcomes the viscous destabilizing one leading to an overall stability. Above a large enough flow rate q_1 viscous effects prevail leading to an overall instability. In the reverse situation (flow 2) the denser and more viscous is injected from the top against the other one in which case large flow rates lead to an overall stability due to viscous effects. Below a small enough flow rate q_2, gravity effects prevail resulting to instability. In the pioneering approach of Hill [3] the linear stability analysis (long waves,

LW) predicts a similar threshold of instability for the two reverse flow $q_h = q_1 = q_2$, in reasonable agreement with the experiment with small gravity and viscosity contrasts.

$$q_h = kg\frac{\rho_+ - \rho_-}{\mu_+ - \mu_-} \tag{1}$$

If the *LW* aproach is relevant for immiscible fluids interfaces where the interface thickness (b) can always be considered as small compared to the wave length (λ) of the the perturbation involved in the stability analysis, this no longer true for miscible fluids because of longitudinal dispersion which spreads the interface as times goes on ($b \sim \sqrt{t}$). Therefore the details of the front concentration profile can be felt by short waves ($\lambda \ll b$) leading to new predictions [4-5]. For miscible fluids a non-monotonic viscosity or a gravity-viscosity profiles can leads to SW instabilty whereas LW theory predicts stability. These new instability branches in the dispersion relation leads to $q_1 \# q_2$.

$$q_2 = kg\frac{\partial \rho}{\partial \mu}\lfloor_{Max} \qquad q_1 = kg\frac{\partial \rho}{\partial \mu}\lfloor_{Min} \tag{2}$$

In the poster, we discuss the state of the art, present a derivation of this new thresholds of instabilty and compute accurately the dispersion relation, i.e. growth rate of the disturbance versus wave-vector. For the numerical computation, we address the case of real fluids for the concentration viscosity and density relationships to get results suitable for comparison with experiments [6].

REFERENCES

1. G.M. Homsy, *Annu.Rev.Fluid Mech.* **19**, 271 (1987).

2. R.A. Wooding, *J. Fluid Mech.* **39**, 477 (1969).

3. S. Hill, *Chem. Eng.Sci.* **1**, 247 (1952).

4. F.J Hickernell and Y. C. Yortsos, *Stud. Appl. Math.* **74**, 93 (1986).

5. O. Manickam and G. M. Homsy, *Phys. Fluids.* **A5**, 1356 (1993).

6. D. Loggia, N. Rakotomalala, D. Salin and Y. Yortsos *submitted.*

HOMOGENEISATION DES EQUATIONS DE STOKES ET NAVIER-STOKES DANS UN MILIEU POREUX SANS PROLONGEMENT DE LA PRESSION ET ESTIMATION DES CORRECTEURS

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On considère un domaine fluide Ω_ε obtenu en retirant d'un ouvert Ω des parties solides εY_s par périodicité. Pour ε fixé, on considère le système de Stokes

$$\left\{\begin{array}{ll} \text{Trouver} \quad (u^\varepsilon, p^\varepsilon) \in H^1(\Omega_\varepsilon)^n \times L^2(\Omega_\varepsilon) \ / & \\ -\Delta u^\varepsilon + \nabla p^\varepsilon = f & \text{dans } \Omega_\varepsilon \\ \operatorname{div}(u^\varepsilon) = 0 & \text{dans } \Omega_\varepsilon \\ u^\varepsilon = 0 & \text{sur } \partial\Omega_\varepsilon \end{array}\right. \tag{1}$$

et de Navier-Stokes

$$\left\{\begin{array}{ll} \text{Trouver} \quad (v^\varepsilon, q^\varepsilon) \in H^1(\Omega_\varepsilon)^n \times L^2(\Omega_\varepsilon) \ / & \\ -\varepsilon^\gamma \Delta v^\varepsilon + (v^\varepsilon.\nabla)v^\varepsilon + \nabla q^\varepsilon = f & \text{dans } \Omega_\varepsilon \\ \operatorname{div}(v^\varepsilon) = 0 & \text{dans } \Omega_\varepsilon \\ v^\varepsilon = 0 & \text{sur } \partial\Omega_\varepsilon \end{array}\right. \tag{2}$$

où f est une fonction appartenant à $H^1(\Omega)^n$ et $0 \leq \gamma < 3/2$. On définit ensuite $(w^{j,\varepsilon}, \pi^{j,\varepsilon})$ et $\gamma^{j,i,\varepsilon}$ par

$$\left\{\begin{array}{ll} -\varepsilon^2 \Delta w^{j,\varepsilon} + \varepsilon \nabla \pi^{j,\varepsilon} = e^j & \text{dans } \Omega_\varepsilon \\ \operatorname{div}(w^{j,\varepsilon}) = 0 & \text{dans } \Omega_\varepsilon \\ w^{j,\varepsilon} = 0 & \text{sur } \partial\Omega_\varepsilon \setminus \Omega \end{array}\right. \qquad \left\{\begin{array}{ll} \operatorname{div}(\gamma^{j,i,\varepsilon}) = \frac{1}{|Y|}(w^{j,\varepsilon})_i - \frac{1}{|Y_f|} K_{i,j} & \text{dans } \Omega_\varepsilon \\ \gamma^{j,i,\varepsilon} = 0 & \text{sur } \partial\Omega_\varepsilon \setminus \Omega \end{array}\right.$$

avec K, matrice de perméabilité de composante $K_{i,j} = \int_{Y_f} (w^j)_i dx$. On considère le problème suivant, qui va permettre de définir la vitesse de Darcy ou vitesse homogénéisée dans Ω_ε.

$$\left\{\begin{array}{ll} \text{Trouver } p^0 \in H^1(\Omega) \ / & \\ \operatorname{div}\left(K(f - \nabla p^0)\right) = 0 & \text{dans } \Omega \\ K(f - \nabla p^0).\nu_{ext} = 0 & \text{sur } \partial\Omega \end{array}\right.$$

On définit ensuite la vitesse de Darcy par $v = K(f - \nabla p^0)$. Nous rappelons que si A est un ouvert de R^n, alors on note χ_A, la fonction caractéristique de l'ensemble A. On a alors

Théorème 0.1 *Soit $(u^\varepsilon, p^\varepsilon)$ la solution du problème de Stokes (1). On note $B^\varepsilon = \{x \in \Omega \ / \ dist(x, \partial\Omega) > \sqrt{\varepsilon}\}$ et $\tilde{u}^\varepsilon$ le prolongement par 0 sur Ω de u^ε. On suppose que $f \in H^1(\Omega)^n$. On a alors les résultats de convergence suivants:*

$$\frac{\tilde{u}^\varepsilon}{\varepsilon^2} - \sum_j w^{j,\varepsilon} (K^{-1} v)_j + \sum_{i,j} \gamma^{j,i,\varepsilon} \frac{\partial}{\partial x_i} ((K^{-1} v)_j) \underset{\varepsilon \to 0}{\longrightarrow} 0 \qquad \text{dans } L^2(\Omega)^n$$

et

$$\left\{\begin{array}{l} \chi_{B^\varepsilon \cap \Omega_\varepsilon} \cdot \left\{ p^\varepsilon - p^0 - \varepsilon \sum_j \pi^{j,\varepsilon} (K^{-1} v)_j \right\} \underset{\varepsilon \to 0}{\longrightarrow} 0 \qquad \text{dans } L^2(\Omega) \\ \chi_{B^\varepsilon \cap \Omega_\varepsilon} \cdot \left\{ p^\varepsilon - p^0 \right\} \underset{\varepsilon \to 0}{\longrightarrow} 0 \qquad \text{dans } L^2(\Omega) \end{array}\right.$$

où p^ε est fixé par la relation $\int_{B^\varepsilon \cap \Omega_\varepsilon} (p^\varepsilon - p_0) = 0$

Nous avons également un résultat similaire pour le problème de Navier-Stokes. Cependant, la convergence de la pression n'est acquise qu'avec une restriction supplémentaire sur γ. Ainsi, pour des valeurs de γ proches de $3/2$ nous allons être obligé de faire appel à la construction d'un opérateur de relèvement pour la pression.

Théorème 0.2 *Soit $(v^\varepsilon, q^\varepsilon)$ la solution du problème de Navier-Stokes (2). On suppose que $f \in H^1(\Omega)^n$, et que γ vérifie $0 \leq \gamma < \frac{5}{4}$ si $n = 3$ et $0 \leq \gamma < \frac{3}{2}$ si $n = 2$. En notant $\tilde{v}_\varepsilon$ le prolongement de v^ε par 0 sur Ω, on a les résultats de convergence suivants:*

$$\frac{\tilde{v}^\varepsilon}{\varepsilon^{2-\gamma}} - \sum_j w^{j,\varepsilon}(K^{-1}v)_j + \sum_{i,j} \gamma^{j,i,\varepsilon} \frac{\partial}{\partial x_i}((K^{-1}v)_j) \underset{\varepsilon\to 0}{\longrightarrow} 0 \qquad \text{dans } L^2(\Omega)^n$$

Si γ est tel que $0 \leq \gamma < \frac{3}{4}$ si $n = 3$ et $0 \leq \gamma < 1$ si $n = 2$, alors on a

$$\begin{cases} \chi_{B^\varepsilon \cap \Omega_\varepsilon} \cdot \left\{ q^\varepsilon - p^0 - \varepsilon \sum_j \pi^{j,\varepsilon}(K^{-1}v)_j \right\} \underset{\varepsilon\to 0}{\longrightarrow} 0 & \text{dans } L^2(\Omega) \\ \chi_{B^\varepsilon \cap \Omega_\varepsilon} \cdot \{q^\varepsilon - p^0\} \underset{\varepsilon\to 0}{\longrightarrow} 0 & \text{dans } L^2(\Omega) \end{cases}$$

où q^ε est fixé par la relation $\int_{B^\varepsilon \cap \Omega_\varepsilon} (q^\varepsilon - p_0) = 0$.
Si γ est tel que $\frac{3}{4} \leq \gamma < \frac{5}{4}$ si $n = 3$ et $1 \leq \gamma < \frac{3}{2}$ si $n = 2$, alors il existe $r \in]1, 2[$ et un relèvement $\tilde{q}^\varepsilon$ de q^ε appartenant à $L^r_0(\Omega)$ tel que

$$\begin{cases} \tilde{q}^\varepsilon - p^0 - \varepsilon \sum_j \pi^{j,\varepsilon}(K^{-1}v)_j \underset{\varepsilon\to 0}{\longrightarrow} 0 & \text{dans } L^r_0(\Omega) \\ \tilde{q}^\varepsilon - p^0 \underset{\varepsilon\to 0}{\longrightarrow} 0 & \text{dans } L^r_0(\Omega) \end{cases}$$

MIXED-HYBRID FINITE ELEMENT APPROXIMATION OF THE CHEMICAL SPECIES' TRANSPORT PROBLEM

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The uranium mining activities and remediation of the deposits in the chalk formation of Stráž of the Northern Bohemia is the main subject of the contribution proposal. Nowadays, the mining activity is being suppressed and the problems concerning the remediation of the deposits in Stráž appear to be very urgent. One of the most important aspect which is faced is the protection of the detached roof stratum – the reservoir of the drinking-water in this region.

It is necessary, most of all, to consider much larger region than it is in the current models and to compute the transport of contaminants applying three-dimensional models. With respect to the large size of the problems, necessity of non-standard use of the FEM and particularities of uranium mining in the considered region, it is possible to exploit the already known experience only partially. Thus the whole process of creating the new models is still open. The solution of this problem is divided into two parts: In the first part - the solution of stationary porous media flow of the technological solution. It is assumed that the chemical non-homogenity of the liquid does not affect its porous media flow. In the second part, the transport of chemical substances in the technological solution and the chemical changes in the rock are studied. Both particular problems are solved by several computational methods.

The solution of underground water flow problem in the real conditions must reflect complex geological structure of sedimented minerals. Layers of the stratified rocks with substantially different physical properties must be modelled using the appropriate discretization of the geological region. These geological characteristics can be correspondingly described by the mixed-hybrid finite element method using trilateral prismatic elements with vertical faces and generally nonparallel bases.

Let Ω be a bounded domain in R^3 with a Lipschitz continuous boundary $\partial\Omega$. The potential fluid flow in a saturated porous media can be modelled by the velocity $\mathbf{u}$ given by Darcy's law $\mathbf{u} = -\mathbf{A}^{-1}\nabla p$, where p is the piezometric potential (fluid pressure) and $\mathbf{A}^{-1}$ is symmetric and uniformly positive definite second rank tensor of hydraulic permeability of the porous medium. Consider also the continuity equation for the incompressible flow $\nabla \cdot \mathbf{u} = q$, where q represents the density of potential sources in the medium. The boundary conditions are given by $p = p_D$ on $\partial\Omega_D$, $\mathbf{u}\cdot\mathbf{n} = u_N$ on $\partial\Omega_N$, where $\partial\Omega = \overline{\partial\Omega_D} \cup \overline{\partial\Omega_N}$ are such that $\partial\Omega_D \cap \partial\Omega_N = \emptyset$ and $\mathbf{n}$ is the outward normal vector defined (almost everywhere) on the boundary $\partial\Omega$.

Assume from now that the domain Ω is polyhedron and is subdivided in a collection of subdomains $\mathcal{E}_h$, such that every subdomain is a trilateral prism. We denote the collection of

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faces of subdomains $e \in \mathcal{E}_h$ which are not adjacent to the boundary $\partial\Omega_D$ by $\Gamma_h = \cup_{e\in\mathcal{E}_h}\partial e - \partial\Omega_D$, where h is the discretization parameter.

The velocity function $\mathbf{u}$ will be approximated with the vector functions linear on every element $e \in \mathcal{E}_h$ and the pressure function p will be approximated with the element-wise constant function and the trace of the pressure on the structure of faces Γ_h will be approximated by the face-wise constant function.

Hybrid-mixed formulation of the porous media flow problem leads to the system of linear equations:

$$\begin{pmatrix} A & B & C \\ B^T & & \\ C^T & & \end{pmatrix} \begin{pmatrix} u \\ p \\ \lambda \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}.$$

In the second part, the transport of chemical substances in the technological solution and the chemical change in the rock are studied. The hybrid-mixed model of the transport of chemical substances dissolved in technological solution is described by Fick's law ${}^l\mathbf{i} = -\mathbf{D}\nabla {}^lc$, where ${}^l\mathbf{i}$ is the flow vector of l-th substance and ${}^lc$ denotes its concentration, $\mathbf{D}$ is the diffusivity-dispersivity tensor calculated by the formula $[\mathbf{D}]_{ij} = D_m\delta_{ij} + \alpha_T.|\mathbf{u}|.\delta_{ij} + (\alpha_L - \alpha_T)\frac{u_iu_j}{|\mathbf{u}|}$, where D_m denotes coeficient of molecular diffusion and α_T respectively α_L denotes coefficient of transversal dispersivity respectively coefficient of longitudinal dispersivity. The equation of the materials balance for l-th substance is $\frac{\partial {}^lc}{\partial t} + \mathbf{u}\cdot\nabla {}^lc + \nabla\cdot {}^l\mathbf{i} + {}^lcQ^- - {}^lr(c,t) = {}^lc^*Q^+$, where first member defines storage of l-th substance, second member defines convection and third expresses the influence diffusion and dispersion. The member ${}^lcQ^-$ determines the influence exhausting of l-th substance and the member ${}^lc^*Q^+$ determines the influence indentation of l-th substance about concentration ${}^lc^*$. Last member on the left side equation defines the complicated chemical reaction of components of the technological solution and component of the rock by that l-th substance either rises or extinguishes.

This member is analysed using reaction kinetics and decomposed on the part ${}^lc.{}^lR^-(t)$, where coefficient ${}^lR^-(t)$ determines a measure of a capacity of the environment bring into reaction with l-th substance and on the part ${}^lR^+(t)$, which determines a measure of a capacity of the environment generated l-th substance.

Balance equations for all substances are discretized in the time with the step $\Delta t = \frac{T}{N}$ and space on the equal decomposition $\mathcal{E}_h$ of the domain Ω as in the porous media flow problem. The finite-dimensional approximation of the transport chemical substances problem using the function spaces described above leads to the following system of equations:

$$\begin{pmatrix} D & B & C \\ G^T & {}^lH_n & \\ C^T & & \end{pmatrix} \begin{pmatrix} {}^li_n \\ {}^lc_n \\ {}^l\theta_n \end{pmatrix} = \begin{pmatrix} {}^ls_1 \\ {}^ls_{2,n} \\ {}^ls_3 \end{pmatrix}, \quad ({}^lH_n)({}^lc_n) = ({}^ls_{2,n}),$$

where superscripts $l = 1, ..., L_1$ denote substances in the solution and superscripts $l = L_1 + 1, ..., L$ denote substances in the rock.

We perform the numerical experiments with the problem of transport of chemical substances is testing at present time using a set of basis problems.

DISPERSION EN MILIEUX POREUX PÉRIODIQUES : DE LA MODÉLISATION NUMÉRIQUE A L'EXPÉRIENCE

Dispersion in spatially periodic porous media : from the numerical modelling to the experiments

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La prise de moyenne a rendu possible l'obtention de l'équation de transport macroscopique en milieux poreux. Dans le cas du problème de dispersion, l'équation macroscopique est similaire à l'équation microscopique si les variables locales sont remplacées par les variables moyennes et si le tenseur effectif de dispersion est introduit à la place du coefficient de diffusion moléculaire. Le tenseur peut être calculé si on connaît le champ des vitesses à l'intérieur de la structure poreuse et si on est en mesure de résoudre le problème de fermeture associé à la prise de moyenne. Le problème de fermeture, est un problème classique de convection-diffusion. Cependant, la géométrie du milieu est une grande inconnue et le choix que l'on fait pour aborder numériquement la résolution du problème de dispersion en milieux poreux peut être déterminant pour la réussite de la méthode. La question qui se pose est de savoir si l'on peut trouver des arrangements géométriques simples qui puissent représenter raisonnablement un milieu réel. Le choix c'est porté ici sur des arrangements périodiques bidimensionnels de cylindres carrés. Pour certains auteurs, si la périodicité spatiale autorise une résolution numérique, voire analytique, aisée du problème de dispersion, elle engendre nécessairement des comportements singuliers du tenseur. Une étude numérique complète de l'évolution, en fonction du nombre de Péclet des composantes longitudinale et transversales du tenseur de dispersion est proposée dans des arrangements d'abord simples (en ligne, en quinconce) puis plus complexes (disposition aléatoire de la phase solide dans la cellule unité). On s'intéressera plus particulièrement à l'influence de la direction θ de l'écoulement moyen par rapport aux axes de la cellule unité.
Pour des milieux dits "ordonnés", le coefficient de dispersion longitudinal présente, pour $\theta=0°$ et 45°, une variation proche de Pe_p^2, assez différente de celle obtenue pour d'autres valeurs de θ. Pour le coefficient de dispersion transversal, les résultats sont fortement dépendants de l'arrangement considéré.

Pour les 20 milieux "désordonnées" étudiés, les résultats sont quasi indépendants de l'angle θ et les variations moyennes sont en $Pe_p^{1,6}$ et $Pe_p^{0,4}$ pour le coefficient respectivement longitudinal et transversal.

Les résultats pour les milieux "ordonnés" sont confrontés à ceux issus de la théorie approchée de Koch et Brady dont on peut dire qu'elle est uniquement valable lorsque la porosité du milieu tend vers l'unité. Cette conclusion est confortée quand on aborde la comparaison des résultats obtenus ici avec ceux issus des études de Salles et al. puis Edwards et Eidsath et al. pour des arrangements de même type.

Dans le but d'élargir la comparaison à des résultats expérimentaux, peu présents dans la littérature, un dispositif expérimental est mis au point. L'ensemble est composé de deux dispositifs : le premier a pour objet la fabrication de milieux poreux modèles par photo polymérisation laser. Le principe de fabrication repose sur une technique appelée stéréophotolithographie laser développée par le GRAPP (ENSIC NANCY) permettant de réaliser des objets de très bonne définition géométrique. Les milieux ainsi obtenus sont formés d'un arrangement périodique de cylindres de section circulaire de 5 mm de hauteur et de 0,5 mm de diamètre disposés entre deux plaques de verre. Un milieu comporte en moyenne 4000 obstacles répartis sur une largeur égale à 25 mm pour un milieu de longueur égale à 130 mm. Les expériences de dispersion reposent sur la mesure de l'évolution de la concentration d'un traceur coloré introduit dans l'écoulement au sein du milieu. Cette mesure est réalisée par un ensemble vidéo, caméra et analyseur d'images. Les profils temporels de concentration, prise en valeur moyenne sur une période spatiale du milieu, analysés par une méthode d'ajustement sur le modèle théorique, permettent d'estimer la variation du coefficient de dispersion longitudinal en fonction du nombre de Péclet. L'intérêt majeur de l'ensemble expérimental est que les résultats obtenus peuvent être comparés de façon effective aux résultats numériques donnés pour des géométries identiques. Les premiers résultats concernent un arrangement en ligne étudié pour deux directions de l'écoulement moyen : 0° et 26°. L'exposant m de la loi limite pour le coefficient de dispersion longitudinal $\frac{Dxx}{\mathcal{D}} = A\ Pe_p^m$ présente bien une forte sensibilité à la direction de l'écoulement, conformément aux prédictions numériques. Le décalage sur les valeurs effectives du coefficient (inférieures à 0°, supérieures à 26°, comparées aux valeurs obtenues numériquement) semble devoir être attribué à la forme de l'obstacle. Un milieu désordonné est étudié pour une direction du vecteur vitesse moyenne. La cellule unité du milieu étudié expérimentalement est identique à l'une des 20 géométries aléatoires étudiées numériquement. Dans ce cas l'accord sur les valeurs effectives du coefficient de dispersion longitudinal est excellent cependant que l'on obtient m=1,2, valeur généralement admise pour les structures poreuses "réelles", sensiblement plus faible que la valeur prédite numériquement (1,38). Les trois expériences réalisées démontrent l'aptitude du dispositif pour étudier qualitativement et quantitativement le processus de dispersion. L'outil mis au point, même s'il reste encore perfectible, couplé à un code de calcul fiable doit permettre une meilleure compréhension des différents mécanismes d'un processus particulièrement complexe.

CORRECTION NON LINÉAIRE DE LA LOI DE DARCY : ÉTUDE NUMÉRIQUE DE L'ÉCOULEMENT BIDIMENSIONNEL POUR QUELQUES ARRANGEMENTS PÉRIODIQUES.

Non-linear correction to Darcy 's law : numerical study of hydrodynamics through some two-dimensional spatially periodic arrays

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Ce travail a pour but de déterminer l'ordre de variation des termes non linéaires de la loi de Darcy, c'est-à-dire comment varie la perte de charge en fonction du vecteur vitesse moyenne. La plage de nombre de Reynolds particulaire considérée est choisie de façon à ce que l'écoulement se trouve dans le régime laminaire inertiel [1]. Différents types d'arrangements périodiques de cylindres carrés ont été retenus : arrangements disposés en ligne, en quinconce, en chicane et de façon aléatoire pour une porosité de 0,6 environ. L'étude s'attache à faire varier le nombre de Reynolds particulaire et la direction du vecteur vitesse moyenne de l'écoulement par rapport aux axes du maillage.

La méthode des volumes finis est adoptée pour résoudre l'équation de Navier-Stokes en régime permanent avec des conditions aux limites périodiques sur la frontière du domaine de résolution, c'est-à-dire une cellule-unité du milieu poreux périodique. Deux types de schémas numériques sont utilisés pour la discrétisation des termes convectifs de l'équation : un schéma à trois points et un schéma quadratique à quatre points afin de traiter la diffusion numérique. Les termes diffusifs sont discrétisés a l'aide d'un schéma des différences centrées. La résolution utilise l'algorithme SIMPLE (Semi-Implicite Method for Pressure-Linked Equations) développé par Patankar et Spalding [2].

Les résultats montrent que pour les maillages et critère de convergence retenus, les valeurs calculées sont sensiblement équivalentes pour les différents schémas, ceci pour des nombres de Reynolds particulaires inférieurs à 400 environ.

Les résultats cherchent plus particulièrement à décrire au-delà du régime de Stokes les déviations obtenues par rapport à la loi de Darcy à savoir la correction à apporter à la perte de charge linéaire ainsi que la perte d'isotropie observée pour les arrangements périodiques réguliers possédant deux axes de symétrie géométriques perpendiculaires. Les variations de la perte de charge nous amènent à conclure à l'existence de deux types de correction non linéaire à la loi de Darcy. Pour une plage intermédiaire de nombre de Reynolds particulaire, la correction non linéaire est d'un ordre supérieur à une variation quadratique relativement à la vitesse moyenne $\langle u \rangle$, proportionnelle à $\langle u \rangle^n$ où n est supérieur a 2. Une valeur de l'exposant n=3 semble correcte sur une plage de nombre de Reynolds particulaires modérés pour des milieux isotropes lorsque les termes d'inertie sont importants mais ne sont pas prépondérants devant les termes visqueux. Cette constatation est en accord avec les résultats théoriques prévus par Wodie et Levy [3], Mei et Auriault [4] et Moura-Neto et al [5].

RÉFÉRENCES

[1] - Dybbs, A. Edwards, R.V., A New Look at Porous Media Fluid Mechanics - Darcy to Turbulent, Fundamentals of Transport Phenomena in Porous Media, Martinus Nijhoff Publishers, pp. 199-256, Dordrecht, The Netherlands, 1984.

[2] - Patankar, S.V. et Spalding, D.B., A Calculation Procedure for Heat, Mass and Momentum Transfer in Three-Dimensional Parabolic Flows, Int. J. Heat Mass Transfer, vol. 15, pp. 1787-1806, 1972.

[3] - Wodie, J.C. et Levy, T., Correction Non Lineaire de la Loi de Darcy, C. R. Acad. Sci. Paris, t.312, serie II, pp. 157-161, 1991.

[4] - Mei, C.C. et Auriault J.L., The Effect of Weak Inertie on the Flow through a Porous Medium, J. Fluid Mechanics, vol. 222, pp. 157-161, 1991.

[5] - Moura-Neto, F., Amaral Souto, H.P., Paes Leme, P.J. et Vargas, A. S., Inertial and 2nd Grade Effects on Macroscopic Equations for the Flow of a Fluid in a Porous Media, en preparation.

Models for water percolation during the preparation of espresso coffee

Gianni Baldini(*)

I describe the experiments I performed in the laboratory of Illycaffè S.p.A., Trieste, (Italy), in order to explain what happens during the water percolation. I made experiments with water temperature of 4°C, since I want to minimize the effects of extraction process. As qualitative results, I have that the flow rate q, corresponding to a fixed Δp, is decreasing with time and that the asymptotic value of the flow is not necessarily an increasing function of Δp [1]. I suppose there are two main processes related with this behaviour: first of all the finest component of the solid particles is removed from the coarse skeleton and transported by the water flow, so that it tends to form a compact layer whose hydraulic conductivity is considerably smaller; on the other hand I have a (partially irreversible) deformation of the skeleton of the porous medium induced by the flow. The first one is investigated in [2], where the proof of the well-posedness of the problem, of the unicity of the solution and of the continuous dependence upon the data and the coefficients is given; moreover, it is shown in [2] that there are sufficient conditions for a finite time extinction of the transport phenomenon and that its occurrence is indeed necessary in order that the asymptotic discharge can exhibit a nonmonotone dependence upon the external pressure. Concerning the model of the deformation of the skeleton, if $\delta(t)$ denotes the thickness reduction of the coffee bed at time t, I can relate it to the water flow q; for instance by assuming that $\dot{\delta} = \eta q(\beta q - \delta)_+$,

Mathematical models for freezing in porous media

FEDERICO TALAMUCCI

If we observe a column of porous soil saturated with water and subjected to a freezing temperature at the top (the lower side, on the contrary, is maintained at a positive temperature), we will notice fundamentally two facts: a *freezing front* moving downward (confining temperature permitting) and a *water migration* from the bottom towards higher parts of the soil. The mass transfer is due to capillarity phenomena, or rather to *cryosuction*.

We locate essentially two parts of the soil: an unfrozen region, with a porous matrix and liquid water, and a frozen part, where ice fills the porous space.

Nevertheless, what can happen in proximity to the separation front is the formation of pure ice layers, called *lences*, that are fed by the migration flux of water towards the freezed part. If we consider that the lower base of the soil is at rest, we will observe a lifting of the upper surface, due to the formation of lences: such a process is known as *frost heave*.

On the other hand, if the freezing process is too fast, or an *overburden pressure* (a load that the column must support) prevents the soil from separating somewhere, we will simply observe a downward movement of the freezing front (*frost penetration*). A *frost heave* can be observed even in this case, due to the increase of the mass of water (the soil is maintained saturated): instead of a macroscopic lens, a change in porosity can occur, after the passage of the frost front.

If we assume that the separation region between the two main parts of the soil is a surface (*freezing front*), we are dealing with a class of models which are more treatable from a mathematical point of view. In this case, the lifting of the soil is known as *primary frost heave*.

Nevertheless, the experimental observation of the process makes us remark the existence of an intermediate region, situated between the frozen and the unfrozen part and called *frozen fringe*, where ice and water coexist in the porous space. Models including this transition zone describe the *secondary frost heave*.

In any case, the starting point for writing the equations of any model is the conservation of heat and mass. Obviously, we need further information about the evolution of the process.

Three questions are of crucial importance:

— which is the composition of the *frozen fringe*, that is, which are the most relevant interfacial effects in the mixture ice-water-soil?

— which is the law regulating the water flux in the *frozen fringe*?

— are temperature and pressure independent in the same region?

Models proposed in literature differ one from the other just on the ground of the kind of answer given to the previous questions: the thermodynamical and transfer process in the frozen fringe are still something obscure and there is not a convergence of opinions among modelers about the governing laws.

The analysis that we are doing deals with a quasi-steady approach to the problem, assuming that the temperature is linear with respect to the spatial coordinate (we are considering one dimensional models).

As regards *primary frost heave*, we found out conditions on the (given) confining temperatures such that we are able to distinguish between *lens formation* and *frost penetration*. Furthermore, a proof of an existence theorem has been concluded in the general case (not linear temperature).

The investigation of *secondary frost heave* is started through the analysis of a particular model proposed recently by Y. Nakano: he assumes that temperature and water pressure are independent in the frozen fringe and the hydraulic flux is regulated by a combination of the thermal and pressure gradients.

Moreover, the volumetric water content in the *frozen fringe* is assumed to be a known function of the temperature. Ice is at rest with respect to the porous matrix (in opposition to that models where ice moves as one rigid body) and always at the atmospheric pressure (instead of a time and space dependence of the ice pressure, responsible in that case for the soil separation as soon as lens starts to form).

In particular, a criterion which allows us to discriminate between the two possibilities will be analyzed and the existence of the solution in a general case (without any simplification) will be investigated.

Diffusion and dissolution in reactive porous medium: Mathematical modeling and numerical simulations

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1 Introduction

This work originates from an attempt to model from mathematical and numerical points of view, the leaching of ions from cement matrices used for waste solidification and waste storage. The aim of this model is to study the leaching of ions on a large scale of time which can't be done by experimental measurements. Here, we present a simple model for leaching of several ionic chemical species accounting for phase change, diffusion and chemical reactions.

Some numerical results are discussed in the case of one chemical species (lime) and two chemical species (lime and lead). We present a modification of the original model to fit with precipitation phenomena.

2 Mathematical model for diffusion and dissolution

We consider the process of diffusion and dissolution on the macroscopic scale. Interstitial solution of cement is assumed to be diluted and at rest. The quantities studied are C_k the molar concentration of ionic chemical species k in liquid phase and S_p the molar concentration of chemical species p in solid phase. We denote $l(p)$ the set of indexes of species k yielding from dissolution of species p in solid phase. The porosity θ, $0 < \theta < 1$, is assumed to be constant.

The domain of the study is a cylinder Ω, filled with hydrated cement, during a period of time $[0, T]$.

$$\Omega = \left\{(x, y, z) \in \mathbf{R}^3,\ x^2 + y^2 < 1, \quad 0 < z < 1\right\} \tag{2-1}$$

Using the conservation principle, we consider the variation of the relative number of moles of species k filling an arbitrary subdomain V of Ω. This variation is given by the sum of the diffusive flux through the boundary and the internal production/destruction rate.

According to Fick's law, the outgoing diffusive flux of ionic species k, q_k through the surface ∂V (the boundary of subdomain V) expresses as:

$$q_k = D_k \nabla C_k \cdot \vec{n} \qquad 1 \leq k \leq N, \tag{2-2}$$

where the constant D_k stands for the diffusive coefficient and $\vec{n}$ for the outgoing normal to the volume V.

We denote by $R_k(C_1,..,C_N)$ the molar rate of production of species k due to chemical reactions in liquid phase, and for $1 \leq k \leq L$ we denote by $F_k(C_1,..,C_N,S_p)$ the molar production rate of species k yielding from dissolution of species p in solid phase.

Hence we have:

$$\begin{array}{rcl} \frac{d}{dt}\int_V \theta C_k\, dxdydz & = & \int_{\partial V} \theta D_k \nabla C_k \cdot \vec{n}\, d\sigma + \\ & & \int_V \theta R_k(C_1,..,C_N)\, dxdydz + \\ & & \int_V (1-\theta) F_k(C_1,..,C_N,S_p)\, dxdydz, \qquad 1 \leq k \leq N. \end{array} \tag{2-3}$$

By using Green's formula, and since equations 2-3 are valid in every smooth subdomain V, we have:

$$\theta \partial_t C_k - div(\theta D_k \nabla C_k) = \theta R_k(C_1,..,C_N) + (1-\theta) F_k(C_1,..,C_N,S_p) \quad 1 \leq k \leq N. \tag{2-4}$$

It remains to specify the dissolution kinetic equations which, because of mass balance between solid and liquid phases read:

$$\partial_t S_p = -\sum_{k \in l(p)} F_k(C_1,..,C_N,S_p) \qquad 1 \leq p \leq M. \tag{2-5}$$

One of the major assumptions is that chemical reactions in liquid phase are at equilibrium, thus $R_k \equiv 0$ for $1 \leq k \leq N$; Moreover, the model is supposed to be one-dimensional.

For $0 < T$ be given, the equations 2-4 and 2-5 become:

$$\theta \partial_t C_k - \partial_z(\theta D_k \partial_z C_k) = (1-\theta) F_k(C_1,..,C_N,S_p) 1 \leq k \leq N, \text{ in } (0,1)\times(0,T) \tag{2-6}$$

$$\partial_t S_p = -\sum_{k \in l(p)} F_k(C_1,..,C_N,S_p) 1 \leq p \leq M, \text{ in } (0,1)\times(0,T) \tag{2-7}$$

3 Phase change rates

The main phenomenon involved is dissolution of solid phase in open system (see [3]). There is no general expression in literature to express the dissolution rate in such a case. In this model we consider it is proportional to the difference between the concentration of the species in solution C_k and a reference concentration C_k^*, provided that solid phase is still present:

$$\begin{cases} F_k \equiv 0 \text{ for } L+1 \leq k \leq N; \\ F_k(C_1,..,C_N,S_p) = \alpha sgn^+(S_p)[C_k^* - C_k]^+ \text{ for } 1 \leq k \leq L \text{ with } 0 \leq \alpha. \end{cases} \tag{3-8}$$

C_k^* stands for thermodynamical equilibrium concentrations of ionic species k. It is determined by a system of equations including equilibrium relationships for species in solution, the electrical equilibrium relation and the equilibrium relationships for dissolution. The last ones are replaced by an equality between C_k and C_k^* when there is no more of the related solid phase.

Existence and uniqueness of solutions to the mathematical problem for a single species in solid phase have been proved in a previous article (see [1]). A numerical method based on a marching technique to approximate numerically the mathematical problem for diffusion and dissolution was also proposed (see [2]). This method was first tried in the case of a single species and extended to the case of two species.

In the case of two species, computations were done with lead and lime. The results showed that we reached values of pH where usely a phenomenon of precipitation appears.

This is due to the fact that we took into account two dissolved species for lead. The precipitation rate is not written our model.

To consider the possibility of precipitation, the model is changed as follows:

$$F_k(C_1,..,C_N,S_p) = \alpha_1 sgn^+(S_p)[C_k^* - C_k]^+ - \alpha_2[C_k^* - C_k]^-, \quad 1 \leq k \leq L. \tag{3-9}$$

Numerical simulations with a given C_k^* shows that C_k "follows" the equilibrium concentration and that precipitations can occur with this model. This expression is interesting because it represents both dissolution and precipitation and because it may be use for environemental flows too (see [4]). Results got with this model about the example lime and lead will be discussed.

References

[1] P. Moszkowicz, J. Pousin and F. Sanchez. *Diffusion and dissolution in reactive porous medium: Mathematical modeling.* In: Computational Methods For Transport in porous Media, Proceedings of Journées numériques de Besançon, Edited by J.M. Crolet, September 27-28, 1994

[2] P. Moszkowicz, J. Pousin and F. Sanchez. *Diffusion and dissolution in reactive porous medium: Mathematical modeling and numerical simulation.* International Congress on Computational and Applied Mathematics. Leuven University, Belgium, 1994.

[3] M. Buil, E. Revertegat and J. Olivier. *A model of attack of pure water or undersaturated lime solutions on cement.*, Stabilization and Solidification of Hazardous Radioactive, and Mixed Wastes 2nd Vol., STP 1123. Edited by T.M. Gilliam and C.C. Wiles, American Society for Testing and Materials, Philadelphia, 1992 pages 227–241

[4] C.I. Steefel and A.C. Lasaga, *A coupled model for transport of multiple chemical species and kinetic precipitation/dissolution reactions with application to reactive flow in single phase hydrothermal systems.* American Journal of Science, Vol. 294, May 1994, P529-592.

LATTICE GAS SIMULATION OF VISCOUS FINGERING FOR MISCIBLE FLUIDS

by

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ABSTRACT

Lattice gas has been proved useful to simulate flow as an alternative to Navier-Stokes finite difference method. We will demonstrate the relevance of such a simulation tool in the case of flow of miscible fluids in a Hele-Shaw cell. This is a particular and simple way to approach the properties of viscous flow inside porous media (Darcy's law holds). Using a recent and efficient lattice gas model we simulate the mixing at the interface of two viscous miscible fluids in such a flow, and adress the question of the resulting instabilities.

To simulate the flow we use the lattice gas BGK model proposed by Qian et al. [1] that describes the fluid dynamics in 2D. This model is an alternative to conventional lattice Boltzmann models ([2], [3]) which themselves evolved from the lattice gas automaton [4]. The basic variable in the model are $N_i(\vec{r},t)$ which denotes the mean occupation number of particles and $\Delta_i(\vec{r},t)$ which represent the relative concentration of the two fluids in the direction i at the node $\vec{r}$ at time t. These number are updated according to the Boltzmann equation

$$N_i(\vec{r}+\vec{c}_i,t+1) = N_i(\vec{r},t) + \lambda_\nu(N_i(\vec{r},t) - N_i^{eq}(\vec{r},t)) \tag{1}$$

and a convection-diffusion equation

$$\Delta_i(\vec{r}+\vec{c}_i,t+1) = \Delta_i(\vec{r},t) + \lambda_D(\Delta_i(\vec{r},t) - \Delta_i^{eq}(\vec{r},t)) \tag{2}$$

where λ_ν is the relaxation parameter that determines the kinematic viscosity ν and λ_D the one who determines the molecular diffusivity D_m and $\vec{c}_i$ denotes the velocity vector

connecting neighboring nodes on the lattice. In 2D we take a square lattice with eight directions of propagation (including the diagonals) plus one at rest (model $D2Q9$). In 3D a cubic lattice is used with fifteen or nineteen directions ($D3Q15$ or $D3Q19$ models).

The algorithm involves three steps: given an initial uniform distribution of N_i and the chosen initial amount of color Δ_i, the equilibrium distributions $N_i^{eq}(\vec{r},t)$ and $\Delta_i^{eq}(\vec{r},t)$ are calculated; then a collision step (right hand side of equation 1, 2) is performed; finally, the occupation numbers are propagated to the neighboring sites. In case the last site belong to a solid boundary (top or bottom walls), they are propagated back in the opposite direction to enforce a no-slip boundary condition at the walls. Along the flow direction x, periodic boundary conditions are applied for the N_i as for the Δ_i. Finally, to simulate an imposed flow, a fixed small amount ϵ of the mean occupation number on each node of the first column of lattice is transferred from the upstream to the downstream direction at each time step. The simulations are performed on a 256×32 lattice for the 2D studies and on a $256 \times 32 \times 8$ lattice for the 3D studies.

On 2D we perform simulation with a real 2D flow, i.e. in a channel, using both the original Navier-Stokes and convection-diffusion equations (1, 2). On the other hand we test the Rayleigh-Taylor instability, where the flow is induced by gravity. Finally we build a 3D Hele-Shaw cell with our lattice. Our results are compared to classical numerical simulations [5] and to an Hele-Shaw experiment performed in our laboratory. The effects of the viscosity ratio are presented, for the stable case and the instable one. The application to more complex systems, as real porous media, will be now considered.

References

[1] Y. H. Qian, D. d'Humieres and P. Lallemand, *Europhys. Lett.*, **1** (7), 479 (1992).

[2] G. R. McNamara and G. Zanetti, *Phys. Rev. Lett.*, **61**, 2332 (1988).

[3] F. J. Higuera and J. Jimenez, *Europhys. Lett.*, **9** (7), 663 (1989).

[4] U. Frisch, B. Hasslacher and Y. Pomeau, *Phys. Rev. Lett*, **56**, 1505 (1986).

[5] Homsy, *Annu. Rev. Fluid. Mech* **19**, 271 (1987).

Nonlinear diffusion in an inhomogeneous aquifer

by
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We consider the Cauchy problem

$$(P)\begin{cases}\rho(x)u_t = (u(1-u)u_x)_x & \text{in } \boldsymbol{R}\times\boldsymbol{R}^+\\ u(x,0)=u_0(x) & x\in\boldsymbol{R}.\end{cases}$$

We suppose that the functions ρ and u_0 satisfy the hypotheses

$$\rho\in C^1(\boldsymbol{R})\cap L^\infty(\boldsymbol{R})\cap L^1(\boldsymbol{R}),\quad \rho>0$$

and

$$u_0\in W^{1,\infty}(\boldsymbol{R}),\ 0\le u_0\le 1, u_0(1-u_0) \text{ has compact support.}$$

The related equation $\rho(x)u_t = \Delta u^m$ has been studied by Kamin and Rosenau and Kamin and Kersner. In particular they prove that in the case that the space dimension $n \geq 3$ the support of u may become unbounded in finite time if ρ decreases fast enough as $|x|$ tends to infinity. Our main purpose is to prove a similar property for Problem (P).

After having shown how Problem (P) modelizes the interface between fresh and salt groundwater in an inhomogeneous aquifer, we sketch the proofs of existence and uniqueness of a weak solution u of Problem (P).

We then give a result about the limiting behavior of u as t tends to infinity.

Theorem A (Large-time behaviour) *Let u be the solution of Problem (P). Then*

$$u(t)\to\bar{u}\overset{def}{=}\frac{\int_R\rho(x)u_0(x)\,dx}{\int_R\rho(x)\,dx}\quad as\quad t\to\infty,$$

uniformly on compact subsets of $\boldsymbol{R}$.

A solution u is said to exhibit finite time blow-up (FTB) if there exists a time T such that $\operatorname{supp} u(t)(1-u(t))$ is unbounded from above for all times $t > T$. For the formulation of the next result we also need an auxiliary density function σ defined by

$$\sigma(x) = \min\{\rho(\xi) : 0 \leq \xi \leq x\},$$

the reason being that the function σ is monotonic while ρ need not be.

Our main result is the following.

Theorem B *Let u be the solution of Problem (P). Then the following implications hold:*

1. *If $\int_0^\infty x\rho(x)\,dx < \infty$, then finite time blow-up occurs;*
2. *If $\int_0^\infty x\sigma(x)\,dx = \infty$, then there is no finite time blow-up.*

W. Anker, D. Graillot, J. Bourgois et M. Zelfani :

Spatialisations quantitative et qualitative en milieu poreux Application à l'estimation des risques dans la dépollution des sols

La spatialisation dans les méthodes quantitatives

Les méthodes quantitatives sont des méthodes largement utilisées pour décrire les phénomènes de transfert en milieu poreux souterrain. La résolution du problème de transfert fait appel :
(i) : à une équation aux dérivées partielles et à la connaissance des conditions limites et initiales,
(ii) : à la dimension spatiale du domaine pour une discrétisation de l'espace à considérer.

Si ces méthodes ne posent pas ou peu de problème pour des milieux poreux homogènes, il en va autrement pour les milieux poreux hétérogènes. Pour ces derniers cas, un maillage adapté de façon optimale et résultant d'une méthode de discrétisation aux différences finies avec mailles variables ou aux éléments finis est nécessaire de telle sorte que chaque maille corresponde à une partie du domaine relativement homogène (nous exclurons les milieux hétérogènes fissurés : milieu «poreux» à grande échelle). L'optimisation du maillage discrétisant le milieu considéré soulève cependant un problème particulier : il s'agit de déterminer le nombre et la forme des éléments du maillage en fonction des connaissances du milieu (mesures disponibles, localisation des hétérogénéités).

Deux approches différentes peuvent résoudre ce type de problème :
- les modèles déterministes qui prennent bien en compte les hétérogénéités mais qui demandent des données souvent inconnues dans la pratique.
- les modèles stochastiques qui fournissent des résultats en termes de probabilité puisque les données fournies par la géostatistique le sont également.

La spatialisation dans les méthodes qualitatives

Les méthodes qualitatives de résolution ne font pas appel à des données uniquement numériques, mais aussi à des données descriptives qualitatives ou semi-quantitatives (éloigné, rapproché, immédiat par exemple) et fournissent également des résultats descriptifs. Parmi ces méthodes, nous avons distingué les systèmes à bases de connaissances et les systèmes de simulation qualitative.

La simulation qualitative s'intéresse à la simulation des phénomènes physiques par l'intermédiaire de raisonnements sur leur comportement et sur leur changement caractéristique. Ces modèles s'appuient sur des règles logiques qui relient différents paramètres : «modèles de paramètres». Ces outils servent souvent à décrire des systèmes qui se formalisent difficilement par les méthodes conventionnelles (ébullition d'un liquide par exemple). De nombreux phénomènes physiques font appel à une dimension spatiale qui a été introduite par certains auteurs dans la simulation qualitative, mais une telle description se limite pour l'instant à des systèmes mécaniques très simples. Si la simulation qualitative est appliquée au déplacement d'une particule dans un domaine discrétisé, le nombre d'états qualitatifs croît de manière exponentielle très rapidement [86 états qualitatifs pour 4 mailles !]. Pour simuler un écoulement, il faudrait de plus tenir compte de l'hétérogénéité du domaine, de la notion des phénomènes continus, des niveaux piézomètriques, des phénomènes de transfert et de la dispersion dans le cas de pollution. Ce type de méthodes qualitatives, bien que prenant en compte la dimension spatiale pour des cas sim-

ples, ne sont pas encore en mesure de simuler des phénomènes aussi complexes que l'écoulement et le transfert de polluants dans un domaine poreux.

Dans les systèmes à bases de connaissances, la dimension spatiale a été introduite principalement pour la construction de topologies et la localisation d'objets dans une topologie. Les données et les mécanismes d'inférence sur les propriétés spatiales peuvent être introduits dans les systèmes à bases de connaissances pour construire des règles de logique (par exemple) qui servent de support au raisonnement spatial.

Application à l'estimation des risques des pollutions dans la décontamination des sols

Les bases de connaissances spatiales peuvent être utilisés comme bases de connaissances géographiques. En réalité, la complexité des phénomènes de pollution met en évidence la complementarité et la nécessité d'associer les méthodes de spatialisation quantitatives avec les méthodes qualitatives. Les méthodes quantitatives sont davantage destinées à simuler la propagation des phénomènes physico-chimiques, les méthodes qualitatives sont davantage destinées à raisonner sur les résultats quantitatifs pour une aide à la décision (mise en place d'un plan d'intervention). La contribution de chacune de ces deux catégories de méthodes est plus ou moins importante selon le cas. Elle est en particulier guidée par l'existence, la nature et la distribution des informations spatiales (localisation des polluants de différente nature, repérage des zones vulnérables).

La méthodologie proposée consiste à mettre en place un système d'analyse spatiale permettant d'évaluer la situation de la pollution en terme quantitatif (scénarios). A partir des résultats quantitatifs obtenus, une base de connaissances permet une évaluation en terme de risques ou d'impacts de la pollution (aide à la décision).

Water retention characteristics for fractal soil

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For soil structure modelled as a fractal with the pore space as it's complement, it is possible to derive the cumulative pore volume as a function of pore scale. From this powerlaw water retention models may be constructed. However, pore connectivity effects neglected in the above approach are likely to make the relation between pore volume and water retention characteristics more complex, and the feasibility of inferring fractal structure and fractal dimensions from experimental water retention data must then be questioned.

Pore connectivity effects relate to the accessibility of pore water to the air-water interface within the soil (resulting in delayed drainage), and the accessibility of the pore water to a drainage route through the soil (resulting in entrapped water).

In a conventional non-fractal problem, the air-interface accessibility issue diminishes as drainage progresses beyond air breakthrough. In contrast for a fractal structure, the accessibility to an air interface remains a problem at all scales and thus at all stages of the drainage process.

As desaturation of a fractal structure progresses, the probability that the remaining water-filled porosity percolates falls. If the connectivity of the water filled porosity is disrupted at an early stage of the drainage process, then a substantial quantity of entrapped water may result, and further drainage will occur for only a subset of the remaining water filled porosity ,hydraulically connected to the water sink. This ignores the hydraulic connections made by thin films, and the water filled porosity existing below the fractal regime, but in practical terms insufficient time may be allowed for these to operate.

These two effects combined may result in a nontrivial dependence of water retention data upon the pore volume distribution. Hence the interpretation of powerlaw exponents derived from such data in terms of fractal dimensions should be made with caution.

Contribution of Image Analysis to Accurate Numerical Flow Modelling by the Local Change of Scale Technique

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This work presents a progress report towards building physically and structurally realistic models of flow through porous media, based on the *local change of scale method.* The local change of scale technique leads, from first principles, to a spatially smoothed equation (e.g., Darcy's law) and to a closure problem, set at the microscopic scale, over a local Representative Elementary Volume (R.E.V.) of Darcy's type ($V_\beta(r_0)$).

$V_\beta(r_0)$ must be larger than the micro-geometrical R.E.V. ($V_\beta(\ell_0)$) -- (minimal volume of length-scale ℓ_0 describing the geometrical variability over some length-scale L ; $\ell_0 << L$).

Development of a valid geometrical R.E.V., $V_\beta(\ell_0)$, coupled with its relationship to the local Darcy's R.E.V. $V_\beta(r_0)$ will allow explicit calculation of the permeability **K** as an implicit function of the micro-geometry.

In accordance with the existence of a local closure form, the three-dimensional micro-structure defines **K**. Nonetheless, realistically characterizing the three-dimensional micro-geometry is complex, current observations being of porosity exposed on slices intersecting the medium.

Development of a valid geometrical R.E.V., $V_\beta(\ell_0)$, and, a-fortiori, usefulness of the local change of scale method appears to be dependent on one's ability to remove several theoretical problems *upstream* from the local change of scale model in itself:

- *complete* characterization of the micro-geometry in terms of N two-dimensional measurable parameters $\{P^i\}_{i=1,N}$ -- Micro-geometry = $f(\{P^i\}_{i=1,N})$,
- reduction of the set $\{P^i\}_{i=1,N}$ to a *minimal sub-set* $\{P^i\}_{i=1,N'\ N'\leq N}$ relevant to the involved physical property, namely **K**,
- development of an "ad-hoc" procedure solely constrained by the *sub-set* $\{P^i\}_{i=1,N'}$, allowing generation of *synthetic* three-dimensional porous media acting as an interface between the *image-analysis-based* bi-dimensional knowledge one has of the micro-structure and the three-dimensional numerical codes based on the local change of scale technique.

Herein, one intends, in *two-dimensions*, to come up with partial answers to the aforementioned upstream problems, capitalizing on a more *empirical* approach due to Ehrlich et al. [Ehrlich et al., 1991a ; McCreesh et al., 1991]. The relevance of the plan to the volume has been empirically based by Ehrlich et al. who suggest that a finite set of parameters $\{P^i\}_{i=1,N}$, namely, *pore types and their relative proportions* (obtained by deconvolving the two-dimensional micro-geometry) are adequate to characterize the three-dimensional micro-structure:

$$\text{Micro-geometry} = f\left(\sum_{i=1}^{N} (\text{Pore Type}^i,\ \text{Relative Proportion}^i)\right)$$

Each pore-type is arranged in continuous and independent circuits at the origin of strong correlations between *pore-types* and *throat sizes*. Such relation between pore-types and throat-sizes has been quantified by combining two-dimensional image-analysis-based data, *pore types*, and three-dimensional petrophysical data, *mercury drainage*. In sandstones, the tendency of pore types to fill in mutually exclusive ranges of pressure intervals would be the consequence of the proclivity of pores of like type to be mutually adjacent, connected by similarly-sized throats, hence a *heterogeneous fabric that results in a series of quasi-parallel flow circuits.*

Using an "*interpretative*" implicit model of *percolation* type [Ehrlich et al., 1991b], the physically relevant geometrical information is reduced to a simpler sub-set, $\{P^i\}_{i=1,N}=\{d_i,NP_i\}_{i=1,N}$

to address the relation *k - Micro-geometry* under the form $k = g\{d_i,NP_i\}_{i=1,N}$, where Np_i is the number of pores of the i^{th} pore type per μm^2 and d_i is the diameter of throat of the i^{th} pore type per μm.

Herein, one means to develop a similar, but *devoid of any constitutive assumption* methodology. The two-dimensional micro-structure is fully-characterized by a measure of type $\{\mathcal{P}^i\}_{i=1,3}=\{F(\nu_x,\nu_y),\varphi(\nu_x,\nu_y),\varepsilon_\beta\}$ where $F(\nu_x,\nu_y)$ and $\varphi(\nu_x,\nu_y)$ are the Fourier modulus and phase of the micro-geometry and ε_β the associated probability density function. The *minimum subset* $\{\mathcal{P}^i\}_{i=N'}$ included in $\{F(\nu_x,\nu_y),\varphi(\nu_x,\nu_y),\varepsilon_\beta\}$ and held to describe accurately the coupling **K** - Micro-geometry by means of the change of scale method is $\{\mathcal{P}^i\}_{i=1,2} = \{|F(\nu)|^2 ,\varepsilon_\beta\}$. $|F(\nu)|^2$ allows quantitative assessment of the nature and length-scales of the structural components of the micro-structure i.e., of the minimum size of $V_\beta(\ell_0)$ wherein the micro-geometry is stationary and ergodic (locally homogeneous).

A set of synthetic media are generated from a single set $\{|F(\nu)|^2;\varepsilon_\beta\}$ derived from any medium by convolution of these parameters with a gaussian field. Adequacy of a "measure" of type $\{|F(\nu)|^2 ,\varepsilon_\beta\}$ as a measure of the two-dimensional micro-geometry is tested from an *empirical* (but practically validated) angle by linking *qualitatively* the structural parameters $\{P^i\}_{i=1,N} = \{\text{Pore type}^i , \text{Relative proportion}^i\}_{i=1,N}$ of Ehrlich et al. to characteristics of $|F(\nu)|^2$ [Anguy et al., in press ; Prince et al., in press]. This is done by showing that filtered-and-inverse-Fourier-transformed images might reveal a hierarchy of micro-structural components preferentially containing distinct pore types - throat sizes relationships. The spatial arrangement of pores of like type, as predicted by Ehrlich et al. would be the same as that one displayed on the filtered images. The amount and nature of the geometrical information carried by $\{|F(\nu)|^2 ;\varepsilon_\beta\}$ are *quantitatively* assessed by comparing the Pore-Types of *real* and *synthetic* media.

The local change of scale method is not absolutely restricted to periodic media. However, development of a "numerically-tractable" closure form requires a weak periodic condition imposed on the boundaries of $V_\beta(r_0)$. That is, **K** is not calculated using the local geometrical R.E.V. $V_\beta(\ell_0)$, but a local periodic Darcy's R.E.V., $V_\beta(r_0)$, derived from $V_\beta(\ell_0)$ [Aguy, 1993]. It is commonly assumed that **K** derived this way is a close approximation of the permeability tensor of natural porous media, provided that $V_\beta(r_0)$ is large enough. However, the random character of natural porous media requires assessment of the ratio "n" between the size ℓ_0 of $V_\beta(\ell_0)$ and the size $r_0=n\times\ell_0$ of $V_\beta(r_0)$. Because natural porous media are not totally periodic, $V_\beta(r_0)$ must be larger than $V_\beta(\ell_0)$ in order for random effects to be correctly accounted for. On account of practical constraints, this problem has been so far investigated in two dimensions. A 1st rough estimate of "n" is derived using synthetic isotropic media by calculating **K** for unit cells of increasing sizes and common centre. Results achieved this day verify that the Darcy's R.E.V., $V_\beta(r_0)$, is larger than the geometrical R.E.V., $V_\beta(\ell_0)$: $r_0\approx 30\times\ell_0$ [Anguy, 1993]. These results are also encouraging in that they suggest that the realizations of synthetic media constrained by the same set $\{|F(\nu)|^2;\varepsilon_\beta\}$ converge at about the same values of **K**. In this respect, these results can be taken as a *necessary* condition in view of basing that **K** is solely constrained by $\{|F(\nu)|^2;\varepsilon_\beta\}$.

References

- Anguy Y., thèse Univ. Bordeaux I, 1993.
- Anguy Y., Ehrlich R., Prince C.M., Riggert V. and Bernard D., *A.A.P.G. Special Publ.: Stochastic Modeling and Geostatistics, Practical Case and Case Histories, eds. Yarus J.M. and Chambers R.L.*, in press.
- Anguy Y., Bernard D. and Ehrlich R., *Advances in Water Resour.*, accepted.
- Ehrlich R., Crabtree S.J. Horkowitz K.O. and Horkowitz J.P., *A.A.P.G. Bulletin* 75, n°. 10, pp. 1547-1562, 1991a.
- Ehrlich R., E.L. Etris, D. Brumfield, L.P. Yuan and S.J. Crabtree, *A.A.P.G. Bulletin* 75, n°. 10, pp. 1579-1592, 1991b.
- McCreesh C.A., Ehrlich R. and Crabtree S.J., *A.A.P.G. Bulletin* 75, n°. 10, pp. 1563-1578, 1991.
- Prince C.M., Ehrlich R. and Anguy Y., *J. of Sed. Pet.*, in press.

A three dimensional model for flow and transport in saturated porous media
Application to salt intrusion into coastal aquifer

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A three dimensional mathematical model, for flow and solute or energy transport in saturated porous media is presented. This model allows to take into account the study of flow in a confined aquifer or in an unconfined aquifer; density and viscosity of the fluid might be considered as linearly dependant on transport (VOSS [5]); during mass transport, the miscible polluant can be chemically degraded or absorbed; finally this modeling can be realized in an steady or unsteady way.

This model is based on physical equations which one are described with accuracy in Bear [1],[2]. In this study, they are written in order to consider particular conditions like pumping, infiltration as boundary conditions of flux (Jacob [3]). This method allows to take into account, for example some specifications like a pumping in upper layer of the aquifer. A such problem induces to solve a system with two coupled partial differential equations. Each of them is solved by a finite element method. For reasons described by MOSE [4], a two degree interpolation is required to get a good approximation of the velocity, thus the finite element used is a hexahedron with 20 nodes, it permits to have the previous conditions on degree interpolation. The finite element method requires to solve at each time step some linear systems. In order to get a fast and accurate computation, a preconditioned conjugate gradient method is used; the same process is used to solve the non symmetric system produced by solving transport equation.

When the density or the viscosity depends on concentration or temperature, a coupled system of two equations has to be solved. For this, an iterative method on the approximation of the pressure P is developed.

A description of the model including the physical equations and the numerical schemes is presented. An example of salt intrusion into coastal aquifer is analysed, according to a study made with the Australian National University of Canberra.

REFERENCES

1. **BEAR.J** : Hydraulics of ground water. Mc-graw Hill,New-york, 1979.

2. **BEAR.J** : Dynamics of fluid in porous media.American Elsevier, New-york 1972.

3. **JACOB.F** : Modélisation tridimensionnelle par la méthode des éléments finis de l'écoulement et du transport en milieu poreux saturé. Thèse de l'Université de Franche-Comté, Mai 1993.

4. **MOSE.R** : Application de la méthode des éléments finis mixtes hybrides et de la "marche au hasard" à la modélisation de l'écoulement et au transport de masse en milieu poreux. Thèse de l'Université de Strabourg 1990.

5. **VOSS.C.I** : SUTRA a finite element simulation model, Reston Virginia 1984.

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Title: Use of the Fourier-Laplace transformation and of diagrammatical methods to interpret pumping tests in heterogeneous reservoirs.

Abstract

Progresses in the power of computers and in reservoir characterization allow the simulation of pressure-transients in complex reservoirs generated with stochastic tools. Classically, the interpretation of these transients gives useful informations about the reservoir structure : a major goal is to interpret these transients in a stochastic context. To adress this problem, an important question is to relate the pressure variations at time t to the local permeability map. Using the Fourier tranformation for spatial coordinates, as well as a Laplace transformation for time allows to tackle this task. This formalism is used in conjuction with a perturbation series expansion in powers of the permeability fluctuations to get an explicit solution. The N-th order term of this series involves the hydrodynamic interaction between N permeability heterogeneities.

The goal of the work is to ensemble average the pressure over all the random permeability field realizations to derive an equation obeied by the mean pressure. To adress this problem, Feynman graphs are introduced which allows a graphical interpretation of the perturbation series. It is shown that the sum of the so called one-particle irreducible graphs gives the kernel of a linear integro-differential equation obeyed by the ensemble averaged pressure. All the information about the heterogeneities is thus contained in this renormalized kernel. We recover thus directly results assumed by Indelman and we get a systematic and direct scheme to evaluate successive approximations of the kernel.

This equation itself is the starting point of useful asymptotic results and approximations . In particular it is shown that the interpretation of pumping tests yields the steady-state equivalent permeability after a sufficiently long time for an infinite reservoir, a rather intuitive result. This means that the homogeneization process performed by pumping tests is well understood.

It is also shown that the kernel of the mean equation is closely related to the time variations of some spatial moments of the pressure field. These moments are important as they quantify the size of the investigated volume at any time and play a key role for simplified pumping tests simulation methods. This equation is also a good starting point to study fractal-like media having correlations at all lengthscale. In that case, we can expect time and scale dependant behaviour of the apparent parameters known under the name of anomalous diffusion. Understanding of these phenomena will be of great theoretical and practical interest.

Compressible Flow in Porous Media. Checking of a macroscopic model by means of pore-level simulations

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Abstract : This paper is devoted to the modelling of compressible flow through porous media. In contrast with incompressible flows, compressible flows through porous media have been little studied in recent years. However, in addition to more traditional domain, new applications of porous media, such as the external insulation of spacecraft [1], require a proper modelling of compressible flows in porous media. Naturally, it exists a large variety of compressible flow depending on the Mach number and the Reynolds number, the coupling effect with thermal effects, etc. Here, we limit our scope to isothermal low Reynolds and Mach numbers transient flows. Traditionally, this type of flow at the macroscopic level is described by the classical Darcy's law combined with a mass balance that includes the transient term. This model is called the "classic model" in the following.

The aim of this paper is to check numerically the relevance of this classic model. To this end, we consider the flow of an ideal gas within a two dimensional model porous medium consisting of an arrangement of square cylinders. The flow is due to the imposed pressure decrease at the outlet of the fluid domain. At the microscopic level, the flow is computed by solving the full Navier-Stokes equations in two dimensions. We use the PISO (Pressure-Implicit with Splitting Operators) method, [2]. This method is well adapted to solving variable density low Mach flows. Special attention is given to the outlet boundary conditions by using the NSCBC's method (Navier Stokes Characteristic Boundary Conditions, [3]). The results are spatially averaged to obtain macroscopic data. At the macroscopic level, the classic model is solved by a Galerkin finite element method. The comparison between the macroscopic data, obtained on the one hand by spatially averaging the microscopic results and on the other hand by solving the problem directly at the macroscopic level is performed for various depressurization conditions. We exhibit situations for which a good agreement is found between the two series of data and situations for which discrepancies are observed. These various behaviours are discussed in terms of the various time scales controlling the flow.

1 A.Bouhouch, M.Prat & S.Bories. Transient compressible flow and thermal transfer within a heterogeneous porous system. Journal of Thermophysics and Heat transfer in press.

2 R..I. Issa, 1986, Solution of the implictly discretised fluid flow equations by operator-splitting. Journal of Comp. Phys., Vol.62, No.1, pp.40-65

3 T.J.Poinsot & S.K.Lele, 1992, Boundary conditions for direct solutions of compressible viscous flows, Journal of Comp. Phys., Vol.101, pp.104-129

Mathematical modelling of diagenetic processes in sedimentary basins

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In the field of geology diagenesis refers to the sum total of processes that bring about changes in a sediment or sedimentary rock subsequent to deposition in water. These processes may be physical, chemical and/or biological, see [1].

In this paper we present the mathematical model and numerical methods used to simulate a subset of the chemical diagentic processes within a sedimentary basin. The processes are the dissolution and precipitation of minerals and decay of organic carbon present in the porous sediments in contact with a moving fluid phase in the pores. The mineral reactions and the organic carbon decay processes are time dependent. In addition there are instantaneous equilibrium reactions in the fluid phase.

The results of the mineral dissolution/precipitation reactions and the decay of organic carbon are changes in the porosity and the permeability in the sediments and hence the fluid flow properties within a sedimentary basin. The main results we therefore want from our model are information about the behaviour of the mineral and organic carbon reactions combined with information about how these reactions affects the fluid flow properties within a sedimentary basin through geological time.

In our model we assume the sedimentary basin is two-dimensional and represent it in space by quadrilateral elements. Each element is assigned a temperature, a mineral composition and a fluid phase composition. Between the elements there are fluid flow and diffusion of ions and complexes dissolved in the fluid phase.

The diffusion depends on temperature and concentration gradients. Electrostatic potential is not considered. Parameters that influence on the fluid flow are viscosity, permeability and pressure. The chemical reactions taking place in the basin are temperature dependent. We assume Neumann conditions for the unknowns at the basin boundaries. Typical simulation periods are 1 - 10 million years. The temperature and the fluid flow history may be the result from a basin simulator.

In our model we are studying the time dependent development of the concentration of 12 minerals in the solid phase and 18 species in the fluid phase per basin element. The minerals are modelled by ordinary differential equations. The species in the fluid phase are modelled by 10 mass balances which are algebraic equations and 8 mass action laws. To reduce the number of equations to integrate, the 8 mass actions laws are eliminated. A typical simulation is 10 - 100 elements, i.e. 220 - 2200 equations to integrate and 300 - 3000 concentrations to compute.

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The full mathematical model is thus a Differential Algebraic Equation System (DAE-system):

$$y' = f(t, y, z), \tag{1a}$$
$$0 = g(t, y, z), \tag{1b}$$

with $t \in \mathbb{R}$, $y \in \mathbb{R}^{n_M}$, $z \in \mathbb{R}^{n_{aq}}$, $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^{n_M}$, $g : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^{n_{aq}}$ and $n = n_M + n_{aq}$. Here n is the total number of unknowns, n_M is the number of mineral unknowns and n_{aq} is the number of fluid phase unknowns, y is the mineral unknowns and z is the fluid phase unknowns.

This system of equations has the following properties:

- The equations are very non-linear.
- The unknowns are tightly coupled and can vary many orders of magnitude during a simulation period both individually and compared with each other.
- The mineral reactions switch between the state of dissolution, equilibrium and precipitation. This switching is extremely sensitive for changes in the fluid composition.
- The equations are very stiff.
- It is not known if there exists a unique solution for the set of parameters we are considering.

To solve this complicated DAE-system we have constructed a solver based on the RADAU5 code, see [2]. Our main modifications in this code are non-negativity handling and introduction of sparse Jacobian and iterative linear solver. Because of the complexity of the equations we have to use an analytical Jacobian.

We will present numerical results from the model, in both single element and multi-element basin configurations to show the validity of the model. The input data to the model are real data from sedimentary basins in the North Sea. The results will be presented as the time-dependent development of the mineral composition of the sediments and the pore fluid composition.

References

[1] Robert A. Berner (1980): *Early Diagenesis, A Theoretical Approach*,Princeton University Press

[2] E. Hairer, G. Wanner (1991): *Solving Ordinary Differential Equations II, Stiff and Differential-Algebraic Problems*, Springer Verlag

Medial Axis Analysis of Three Dimensional Tomographic Images of Drill Core Samples

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We introduce the medial axis transform as a tool in the analysis of geometric structure of void space in porous media. The medial axis of any volume filling object is intuitively the "spine" of the object running along its geometric middle. It defines the inherent shape of the object, for example, the medial axis of a cylinder is the axis of rotational symmetry, for a sphere it is the center point. The medial axis for any n-dimensional object can be found by an algorithm equivalent to the following process. Imagine burning the object by igniting a fire simultaneously everywhere on its entire surface. Let this fire burn into the object at a constant rate, reducing it layer-by-layer. The set of all points within the object where the fire extinguishes itself is the media axis. The fire is said to extinguish itself at a point if two components of the fire, traveling in opposite directions, simultaneously arrive at the point.

Volume information can also be associated with each elementary segment of the medial axis. As the rate of burn is constant, the time at which the fire burns any point in the object also gives the radius of the largest L_2–norm ball, centered on the point, which just fits completely inside the object. Thus each point in the object is assigned a unique radius value, which we refer to as the burn radius; in particular, each medial axis point is assigned a burn radius, which we refer to as the medial axis radius at the given point.

The medial axis of an object gives basic information concerning its topology and geometry. The attraction of the medial axis is that it has lower dimensionality than the object itself; for a two-dimensional object, the medial axis will consist of a union of lines and points, for a three-dimensional object: surfaces, lines and points.

We apply this analysis to the void spaces in high resolution (5 micron), three dimensional, tomographic images obtained using the National Synchrotron Light Source facility at Brookhaven National Laboratory. Three samples are analyzed, consisting of two drill core samples, Berea sandstone and Danish chalk, and a sample of packed, 100 micron diameter glass beads. A Berea sandstone sample was chosen as it is a reference standard rock for petrology. The Danish chalk core sample is representative of a low porosity rock that is difficult to work with using invasive laboratory techniques, such as microsanding/microslicing. The packed glass bead sample provides a realization of a non-overlapping sphere model, a model which has been extensively studied both theoretically and experimentally.

In this poster, we describe a discrete version of the medial axis "burn" algorithm applicable to digitized tomographic images consisting of a cubic array of voxels. Example medial axes obtained for the void spaces in the data samples are shown.

We present four analyses of the medial axes constructed for these three data sets. The first analysis we present is for the distribution, $n(r)$, of burn radii, which is a discrete analog of the pore-size distribution. We show that for the two drill core samples, the burn radius distribution is very well fit by exponential, $n(r) \sim \exp(r/r_0)$. This conclusion is also true for the distribution of the subset of medial axis radii. This contrasts with the burn radius distribution of the packed glass bead sample which is very well fit by a normal distribution, $n(r) \sim \exp((r/r_0)^2)$.

The second analysis concerns the size distribution of disconnected medial axis segments, which correspond to disconnected void volumes. All samples indicate a composite distribution consisting of a power-law distribution of the smallest medial axis fragments, followed by an essentially constant distribution of medial axis fragments of intermediate sizes. Finally, both the Berea and glass bead data sets contain a single, very large interconnected medial axis segment, occupying approximately 40% of the pore volume. Such a large segment is absent in the lower porosity chalk sample.

The third analysis concerns the tortuosity distribution of the medial axis between parallel planes in the sample. Assuming that the medial axis between two parallel planes consists of N voxels, there are on the order of $N!$ connecting paths to be considered. For the Berea and Bead samples considered here, the value of N can exceed 100,000. To simplify this CPU intensive computation, only shortest path connections along the medial axis between the planes were considered. For the Berea and chalk samples, the tortuosity distribution appears to be best fit by an incomplete Gamma distribution, which has two free parameters. The distribution for the Bead sample does not appear to be fit well by the same model.

Finally, considering the medial axis radius as a continuous function along the medial axis, we classify segments of the medial axis as throats (local radius minima) and nodal pores (local radius maxima). We present throat and nodal pore size distributions and the connectivity between them.

Etude numérique de la méthode de récupération secondaire du pétrole

Mazen SAAD[1]

On s'intéresse à l'étude numérique de la méthode de récupération secondaire du pétrole dans un gisement constitué d'un seul type de roche caractérisé par la porosité, la perméabilité intrinsèque, les pressions capillaires et les perméabilités relatives. Cette méthode consiste à injecter un fluide mouillant l'eau dans les puits d'injection pour déplacer le pétrole vers les puits de production.

On considère que la phase eau (w) et la phase huile (o) sont incompressibles tandis que la phase gaz (g) est compressible. La modélisation de ce type d'écoulement est décrite à l'échelle macroscopique par

$$(1)\qquad \phi\,\partial_t s_w + \boldsymbol{V}_T.\nabla v_w(s) + div(G_w(s,p_g)\boldsymbol{g}) + v_w\, div\boldsymbol{V}_T + div(\boldsymbol{D}_w(\boldsymbol{s})\nabla \boldsymbol{s}) = 0$$

$$(2)\qquad \phi\,\partial_t s_o + \boldsymbol{V}_T.\nabla v_o + div(G_o(s,p_g)\boldsymbol{g}) + v_o\, div\boldsymbol{V}_T + div(\boldsymbol{D}_o(s)\nabla \boldsymbol{s}) = 0$$

$$(3)\qquad \phi\,\partial_t(\rho_g s_g) + \boldsymbol{V}_T.\nabla(\rho_g v_g) + div(\rho_g G_g(s,p_g)\boldsymbol{g}) + \rho_g v_g\, div\boldsymbol{V}_T + div(\rho_g \boldsymbol{D}_g(\boldsymbol{s})\nabla \boldsymbol{s}) = 0$$

La vitesse totale V_T est donnée par

$$(4)\qquad \boldsymbol{V}_T = -\boldsymbol{K}\,M\,\nabla p_g + G(s,p_g)\boldsymbol{g} + \boldsymbol{D}_T(\boldsymbol{s})\nabla \boldsymbol{s}$$

On a de plus

$$(5)\qquad s_w + s_o + s_g = 1$$

Dans ce système, s_η représente la saturation de la phase η, $\boldsymbol{s} = (s_w, s_o)$, p_g la pression du gaz, $v_\eta = v_\eta(\boldsymbol{s})$ la fraction des flux, $\rho_g(p_g)$ la masse volumique du gaz, $G_\eta(\boldsymbol{s}, p_g)$ les termes gravitationnels, $\boldsymbol{D}_\gamma(\boldsymbol{s})$ les termes dus aux pressions capillaires ($\gamma = w, o, g, T$), $M(\boldsymbol{s})$ la mobilité totale.

Dans le domaine $\Omega =]0, L[\times]0, l[$, les conditions aux limites sont

– sur la paroi d'injection Γ_e

$$s_w(t,x) = 1,\ p_g(t,x) = p_e \ ou\ \boldsymbol{V}_T.\boldsymbol{n} = -Q \qquad sur\ \Gamma_e$$

– sur la paroi de production Γ_s

$$p_g(t,x) = p_s < p_e;\ (\boldsymbol{D}_\eta(\boldsymbol{s})\nabla \boldsymbol{s}).\boldsymbol{n} = 0\ sur\ \Gamma_s$$

– sur la paroi imperméable Γ_i

$$\boldsymbol{V}_T.\boldsymbol{n} = 0;\ (\boldsymbol{D}_\eta(\boldsymbol{s})\nabla \boldsymbol{s}).\boldsymbol{n} = 0\ sur\ \Gamma_i$$

Le système (1)-(3) est un ststème parabolique dégénéré ; la dégénérescence est due au fait que la fraction de flux v_η s'annule avec s_η . Le système (1)-(3) est formé de deux parties modélisant deux phénomènes physiques différents. Le premier correspond au transport sous l'action de la seule force extérieure (l'injection de l'eau à une vitesse ou

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une pression donnée) et sous l'action des termes gravitationnels. Le deuxième correspond à la diffusion sous l'effet des termes capillaires. Afin de prendre en compte, aussi précisément que possible, ces deux phénomènes une méthode de pas fractionnaires est utilisée.

Dans la formulation du système (1)-(3), l'opérateur est scindé en deux parties ; ainsi la méthode des pas fractionnaires employée ici est une méthode à deux pas. Nous décrivons schématiquement cette méthode.

Au premier pas, nous donnons une caractérisation pour que la première partie (correspondant au transport) soit hyperbolique ; ceci donne une façon naturelle de classer les modèles et permet de mettre en place des schémas quasi d'ordre deux.

Plus précisement, on se limitera aux modèles de perméabilités relatives donnant lieu à un système hyperbolique (l'étude de l'hyperbolicité de ces écoulements a été essentiellement faite dans [3], [5]). Pour résoudre ce problème, nous mettons en place un solveur approché de type Roe. Pour ce, nous explicitons une construction de la matrice de Roe pour une certaine classe de perméabilités relatives.

Si les flux sont des polynômes de degré inférieur ou égal à deux, Vila [6] construit explicitement une matrice de Roe. Nous avons généralisé ce résultat lorsque les flux s'écrivent comme quotient de deux polynômes.

Le schéma de Roe est d'ordre un et à trois points. L'inconvénient d'un tel schéma est sa diffusion numérique ; de nombreuses méthodes peuvent être utilisées pour essayer de la réduire. Par exemple, les limiteurs de pentes, les limiteurs de flux, ou des schémas numériques d'ordre supérieur (Harten [2]). La construction de cette nouvelle famille de matrice de Roe a permis, entre autres, la mise en place du schéma de Harten. Nous présentons tout d'abord, l'avantage numérique d'un tel schéma pour un écoulement monodimensionnel, ensuite son extension aux écoulements bidimensionnels.

Au deuxième pas, nous traitons les opérateurs d'ordre deux en espace par un schéma implicite en pression et semi-explicite en saturations. Dans cette étape, on établit une équation discréte en pression, non dégénérée, qui préserve la consistance du schéma avec l'opérateur induit.

Les résultats obtenus sont satisfaisants, on retrouve le comportement attendu : étalement des fronts dû à la diffusion capillaire, freinage des fluides dû à l'augmentation de la mobilité massique du gaz en compressible. On vérifie également la conservation de la masse de chaque phase.

Références

[1] G. Chavent et J. Jaffré: *Studies in Mathematics and its Applications*, 17, *North–Holland* (1986)

[2] A. Harten: *J. Comp. Phys.* 49 *p.p.* 357–393 (1983)

[3] L. Holden: *SIAM, J. Appli. Math, vol.* 50, 3, *p.p.* 667–682(1990)

[4] P. L. Roe: *J. Comp. Phys.* 4" *p.p.* 357–372 (1981)

[5] M. Shearer and J.A. Trangenstein: *Transport in porous media*, 4, *p.p.* 499–525 (1989)

[6] J. P. Vila: Third internationnal conference on hyperbolic problems, Vol I, II, p.p. 913-922 (1990).

Scaling-up Permeability in the Near Well Regions

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One of the major applications of modelling of flow in porous media can be found in the petroleum reservoir simulation. However, the reservoir heterogeneities, such as absolute permeability, generated on fine grids by geological/geostatistical models, can not be directly used for flow simulation, due to computational cost and memory storage capacity. Therefore, efficient techniques are needed to scale the fine grid petrophysical parameters up to the coarse grid[1-3].

It is well known that the equivalent gridblock permeability obtained by the scaling-up procedure from fine gridblocks depends on the boundary conditions[1-3]. In the literature, most boundary conditions considered represent a "linear" flow pattern. Using directly these scaling-up results in reservoir simulation might be erroneous, because the near well behavior can not be approximated by the "linear" flow.

On a reservoir field scale, the flow region can be divided into two types: a "radial" flow region with a high pressure gradient and a "linear" flow region with a low pressure gradient[4]. The "radial" flow region is usually more important for the prediction of production forecast, because it is directly related to the well. A specific scaling-up procedure is needed in the near well region by imposing a "radial" flow condition. The transmissibility will be scaled-up in stead of the permeability because it is more accurate for flow simulation[5]. In the well vicinity, the scaled-up parameters should not only consist of the equivalent transmissibility (permeability), but also numerical productivity index (PI). More precisely, the equivalent transmissibility (permeability) and numerical PI is calculated as follow[4]:

1. According to Darcy's law, the equivalent transmissibility T_{eq} (K_{eq}) between two coarse grids is defined by the quotient of equivalent flow F_{eq}, which is the sum of flows at the fine grid interfaces composing the coarse grid interface, and the difference in coarse grid pressures P_{CO}, which is defined as the storage-weighted mean of the pressures of all the fine grids composing the coarse grid:

$$T_{eq} = F_{eq} / \Delta P_{CO}$$

2. By the definition of numerical productivity index, the PI on a coarse grid is calculated by the following formula:

$$PI_{CO} = PI_{fine} (P_{fine} - P_W) / (P_{CO} - P_W) = Q / (P_{CO} - P_W)$$

where PI_{fine} is the numerical PI on fine grid, P_W is the wellbore pressure, P_{CO} and P_{fine} are the coarse grid and fine grid wellblock pressures, and Q is the well flow rate.

The goal of the scaling-up procedure is to reconstruct the fine grid simulation by a coarse grid simulation, especially for the well production forecast. We give an example to show the need of the specific scaling-up procedure in the near well region.

Example:

A heterogeneous configuration is generated on 99x99 fine gridblocks with a short correlation length (Figure 1a). Fourteen wells are distributed in this field with five injection wells (I1-I5) and nine production wells (P1-P9) (Figure 1b). Pressure conditions are imposed at all the wellbores to investigate flow rate calculations. A fine grid simulation is done with a standard finite-difference reservoir simulator, where the numerical productivity indices are calculated by a conventional formula[6]. The results of this simulation are considered as the reference solution. A 33x33 coarse grid is constructed based on the fine grid, and each coarse grid includes nine fine ones. Two scaling-up procedures are used and the results of flow simulation on coarse grids are compared to the reference solution.

The first scaling-up procedure is the standard one, i.e., no flow boundary conditions on the edges and constant pressure at the inlet and outlet faces[3]. The second one is new: it includes the standard procedure for "linear" flow pattern and the specific procedure in the well vicinity for "radial" flow pattern.

Single-phase incompressible flow is first tested. Figure 2 presents the flow rate for each well given by three simulations: a fine grid simulation and two coarse grid simulations with the scaling-up procedures presented above. It can be seen that the errors caused by the standard scaling-up procedure attain about 30% en the average, while those caused by the new procedure, which includes the scaling-up for "radial" flow pattern, are only about 1-2%. This test shows the need to use the new procedure when scaling-up in the vicinity of wells. With the new procedure, the error for well performance can be significantly reduced.

Two-phase flow simulation is also investigated. Water is injected in an oil reservoir. Figure 3 shows the water cut for three production wells P1, P2 and P3. Again, the simulation with new scaling-up procedure approaches the fine grid results much better. The breakthrough time can be calculated accurately by using the new scaling-up procedure.

It is clear that using a scaling-up procedure without considering the impact of the "radial" flow pattern in the near well regions might greatly bias the flow simulation results on a reservoir scale. The simulation accuracy can be significantly improved by using the new scaling-up procedure. This procedure is very easy to use in reservoir simulation.

References:

1. Durlofsky, L.J. : "Numerical Calculation of Equivalent Grid Block Permeability Tensors for Heterogeneous Porous Media" Water Resour. Res., 27(5), 1991 pp699-708

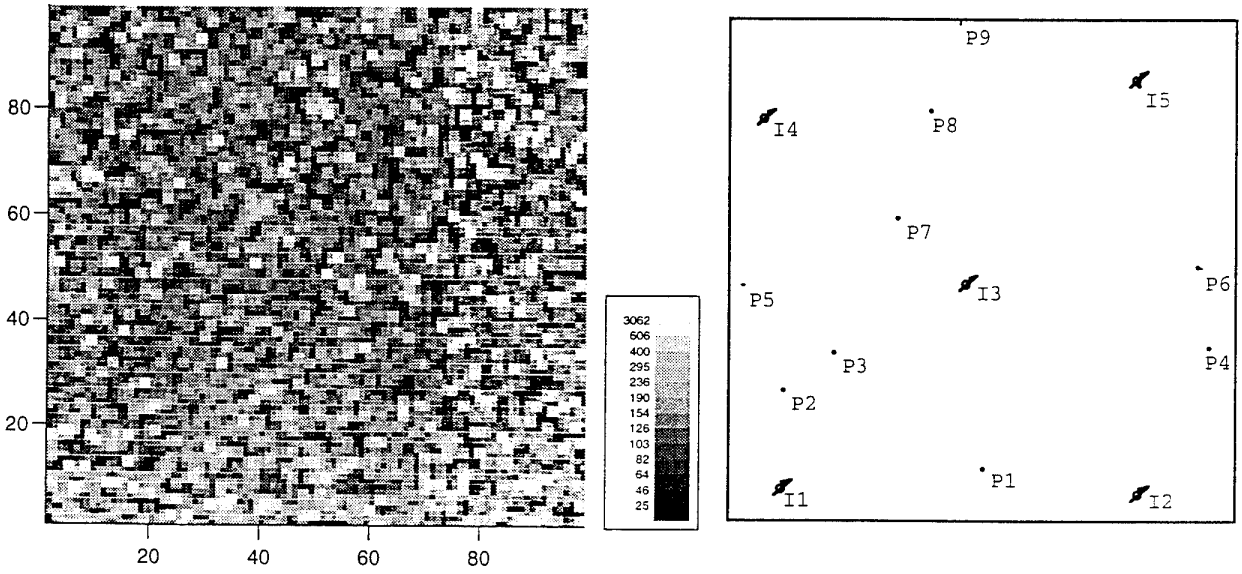
2. Gallouët, T. and Guérillot, D. : "An Optimal Method for Averaging Absolute Permeability", Third International Reservoir Characterisation Technical Conference, Tulsa, Nov. 3-5. 1991

3. King, P.R. and Williams, J.K. : "Upscaling Permeability: Mathematics of Renormalization" 4th Euro. Conf. on Math. of Oil Recovery, Norway, June 7-10 1994

4. Ding. Y. : "Scaling-up in the Vicinity of Wells in Heterogeneous Field" Paper SPE 29137 presented at the 13th SPE Symposium on Reservoir Simulation, San Antonio, Feb. 12-15, 1995

5. Romeu, R.K. and Noetinger, B. : "Calculation of Internodal Transmissibilities in Finite-Difference Models of Flow in Heterogeneous Media" to be published in Water Resour. Res.

6. Peaceman, D.W. : "Interpretation of Well-Block Pressures in Numerical Reservoir Simulation" SPEJ Jun. 1978 pp183-194



1a). Permeability field 1b). Well positions

Fig. 1--Heterogeneity and well configuration

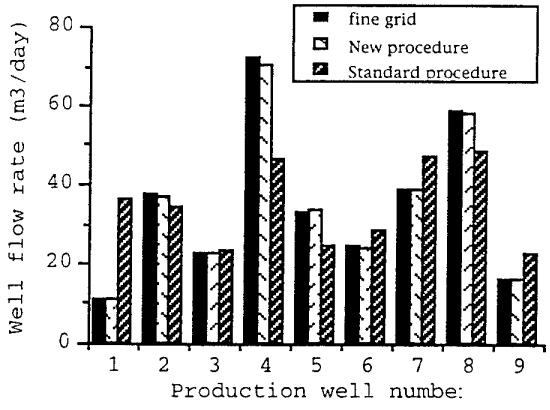


Fig. 2--Single-phase flow simulation

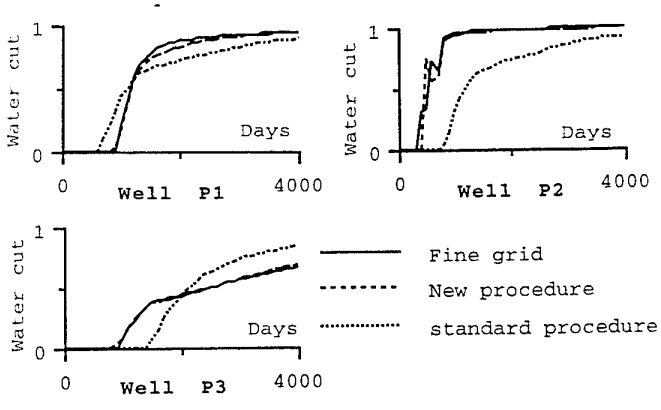


Fig. 3--Two-phase flow simulation

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On the Dam Problem with Leaky Boundary Conditions and Linear Darcy's Law

We would like to present various results on the Dam problem with leaky boundary conditions. The problem is to find a pair $(p, \chi) \in H^1(\Omega) \times L^\infty(\Omega)$ such that

$$(P) \qquad \begin{cases} p \geq 0,\ 0 \leq \chi \leq 1,\ p(1-\chi) = 0 \ \text{ a.e in } \Omega, \\ p = 0 \quad \text{on} \quad S_2, \\ \displaystyle\int_\Omega a(x)(\nabla p + \chi e).\nabla \xi dx - \int_{S_3} \beta(x, \varphi - p).\xi d\sigma(x) \leq 0 \\ \qquad\qquad\qquad\qquad \forall \xi \in H^1(\Omega),\ \forall \xi \geq 0 \text{ on } S_2, \end{cases}$$

where β is a nondecreasing Carathéodory function and $a \in L^\infty(\Omega, \mathcal{M}_n(\mathbb{R}))$.

References

H.W. Alt : A Free boundary Problem Associated with the Flow of Ground water. Arch. Rat. Mech. Anal.64, (1977), 111-126.

H. Brezis, D. Kinderlehrer, and G. Stampacchia : Sur une nouvelle formulation du problème de l'écoulement à travers une digue. C. R. Acd. Sci Paris Serie A 287, (1978), 711 - 714.

J. Carrillo, M. Chipot : On the Dam Problem. J. Differentiel Equations 45, (1982), 234 - 271.

J. Carrillo, M. Chipot : The Dam Problem with Leaky Boundary conditions. Applied Mathematics Optimisation 28, (1993), 57 - 85.

A. Lyaghfouri : Sur quelques problèmes d'écoulements dans les milieux poreux. Thèse de l'université de Metz. Juin 1994.

Numerical Modelling of Clay Drying

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Abstract
Drying is an important unit operation, which is used in many production processes. An example of a branch of industry, where drying plays an essential role in the production process, is the clay brick and tile manufacturing industry.
Shrinkage occurs during drying of many materials. Moisture concentration gradients in the material and corresponding gradients in the amount of shrinkage will lead to drying stresses. Controlling these stresses is important since they can lead to deformation or cracking of the product. A correct description of the evolution of moisture concentration profiles in the material is complicated by the influence of shrinkage on mass transfer.
Drying of clay can be modelled by equations expressing mass conservation of moisture and solid, together with a flux equation for moisture. This flux equation is essentially Darcy's law, where pressure and permeability depend on moisture content, i.e. the quotient of moisture and solid concentration.
These equations are supplemented with equations expressing the deformation of clay due to gradients in moisture content. It is assumed that clay behaves as an elastic material, i.e. Hooke's law applies, where Young's modulus and Poisson's ratio depend on the moisture content.
A careful analysis leads to a nonlinear parabolic equation for the moisture content and a nonlinear elliptic equation for the displacements, that depends on the moisture content.
These equations are discretised in space by finite elements and in time by Euler's method with frozen coefficients. The resulting systems of sparse linear equations are solved by preconditioned conjugate gradients.
Results of three-dimensional simulations are presented.

The Plane Potential Flow Through The Inhomogeneous Porous Line

Ivan Keglević*

November 25, 1994

We consider the plane stationary potential flow, i.e. a pair of functions $(\boldsymbol{v}, p)$, $\boldsymbol{v} : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $p : \mathbb{R}^2 \longrightarrow \mathbb{R}$, which satisfy

$$\begin{aligned} \operatorname{div} \boldsymbol{v}(x,y) &= 0\,, & (1)\\ \operatorname{rot} \boldsymbol{v}(x,y) &= 0\,, & (2)\\ (\boldsymbol{v}(x,y)\cdot\nabla)\boldsymbol{v}(x,y) + \frac{1}{\rho}\nabla p(x,y) &= 0\,, & (3) \end{aligned}$$

where ρ is a positive real constant (the density of an incompressible fluid). We assume that the flow is ideal (i.e. the formulae (1) to (3) are valid) in the whole $\mathbb{R}^2$ except on the straight line $y = 0$. There, we have the continuity condition

$$v_{y+}(x,0) = v_{y-}(x,0)\,, \tag{4}$$

where $(x,0)$ is in porous region and v_{y+} and v_{y-} are the limits of the v_y component of the velocity field form the $+$ $(y > 0)$ and $-$ $(y < 0)$ part of the plane. Let the porosity law be given by

$$\gamma_1(x)v_y(x,0) = \gamma_2(x)(p_+(x,0) - p_-(x,0))\,, \tag{5}$$

where γ_1, $\gamma_2 : \mathbb{R} \longrightarrow \mathbb{R}$ are two given nonnegative functions which do not vanish simultaneously and p_+ and p_- are the limits of pressure field from the $+$ resp. $-$ part of the plane. We also assume

$$\boldsymbol{v}_+(x,0) = \boldsymbol{v}_-(x,0)\,, \quad x \in \mathbb{R}\setminus \operatorname{supp}\gamma_1\,. \tag{6}$$

If the velocity in infinity

$$\lim_{|(x,y)|\to\infty} \boldsymbol{v}(x,y) = \boldsymbol{v}_\infty = (v_{\infty x}, v_{\infty y}) \tag{7}$$

and the functions

$$\gamma_1, \gamma_2 : \mathbb{R} \longrightarrow [0,\infty) \tag{8}$$

are given, we look for velocity and pressure fields $\boldsymbol{v}$ and p.

The form (5) of the porosity law is derived by SANCHEZ-PALENCIA [6], MURAT [5] and DAMLAMIAN[1] and already used in WOLFERSDORF [7] and in MIKELIĆ-SUHADOLC-VESELIĆ [4].

If the function γ_2 is zero at some points, the hindrance is completely impervious. If γ_1 is zero, the hindrance is completely open. We assume that γ_1 and γ_2 are piecewise continuous and have the limit in the infinity.

For $v_{\infty x} \neq 0$ the solution of this problem can be explicitely given (through the formula) if there is the limit

$$\lim_{x\to\pm\infty} \frac{\gamma_1(x)}{\gamma_2(x)} = d < \infty, \tag{9}$$

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in case supp $\gamma_1 = \mathbb{R}$ and if

$$\lim_{x \to \pm\infty} \frac{\gamma_1(x)}{\gamma_2(x)} = 0\,. \tag{10}$$

hold in case supp $\gamma_1 \neq \mathbb{R}$. The second possibility also includes the cases when only one or more than one bounded line segments are porous or imprevious. The uniqueness of this solution in the set of all functions v with their boundary value on Γ in $L^2_{loc}(\mathbb{R})$ can also be proved.

In the case $v_{\infty x} = 0$ we do not have any solutions in the space of all functions v with boundary values on Γ in $L^2_{loc}(\mathbb{R})$. If supp γ_1 consist of finitely many bounded line segments, we do get some solutions (with boundary values not in $L^2_{loc}(\mathbb{R})$) but there is always more than one solution, and all solutions show the unexpected property that the flow do not get through hindrance then round it. In this cases the model do not describes properly the expected physical situation.

References

[1] DAMLAMIAN, A. Le probleme de la passoire de Neumann. *Rend. Sem. Mat. Univers. Politech. Torino*, 43:427–450, 1985.

[2] KEGLEVIĆ, I. On the plane potential flow through the inhomogeneous porous line. To appear in *Z. angew. Math. Mech. (ZAMM)*.

[3] KEGLEVIĆ, I. *Über die ebene wirbelfreie Strömung durch eine poröse Oberfläche.* PhD thesis, Fernuniversität Hagen, 1994.

[4] MIKELIĆ, A., SUHADOLC, A., and VESELIĆ, K. On the potential flow of an ideal incompressible fluid through a porous boundary. *IMA Journal of Applied Mathematics*, 47:109–125, 1991.

[5] MURAT, F. The Neumann sieve. In *Nonlinear Variational Problems.* Research Notes in Mathematics 127, pages 24–33. London: Pitman, 1985.

[6] SANCHEZ-PALENCIA, E. Un probleme d'écoulement lent d'un fluide incrompressible au travers d'une paroi finement preforée. In *Les méthodes de l'homogenization: théorie et applications en physique.* Collection de la direction des études et recherches d'Électricité de France 57, pages 371–400. Paris: Eyrolles, 1985.

[7] WOLFERSDORF, L.V. Potential flow past a circular cylinder with permeable surface. *Z. angew. Math. Mech. (ZAMM)*, 68(1):11–19, 1988.

Implementing a Non-local Theory for Flow and Transport Through Porous Media

John F. Peters[1] and Stacy E. Howington[2]

Abstract

At many sites with subsurface contamination, potential health risks make remediation necessary. As a rule, remediation of subsurface contamination is expensive and the consequences of failure are considerable. Therefore, a thorough assessment of remediation candidates in the context of the particular site is warranted before a selection is made. At the heart of many such assessments is the prediction of fluid flow and contaminant transport through media with multiple scales of heterogeneity. In the past decade, this need has prompted a reevaluation of the existing theory for flow and transport in porous media. The central difficulty in devising new models from existing theory is that model parameters often depend on the scales at which their measurements are made. For example, an apparent scale dependency arises as a result of the macroscopic Fickian description for dispersion, which is evidently a non-Fickian process. Many non-Fickian dispersion models currently proposed are non-local, wherein the rate of change of concentration in an elemental volume depends not only on a concentration gradient in the element, as for a local continuum law, but on the history of the gradient in a finite region around the element. Accordingly, the non-local form of the equation is represented in terms of convolution integrals. The non-local form contains a length scale that permits the introduction of scale dependence.

A theoretical approach is presented based on representing a discrete porous medium in terms of continuous variables. A non-local governing equation, similar in form to those presented by others [1, 2, 3] results. Regardless of the origin of the non-local equation, the convolution integral may be approximated discretely as a network. This network is distinguishable from traditional network models in three ways: (1) the flow paths represent the larger-scale primary advection paths in the porous material, not individual pores, (2) variations in advection paths capture dispersion whereas spatial averaging reproduces Darcy-scale properties, and (3) because the network is a discrete form of the non-local model, each location in the domain may be connected to all other locations, resulting in unusually high coordination numbers and a very dense pattern of flow paths. This approximation is valid at any scale from the pore scale to the field scale. A non-local advection equation will necessarily display dispersion not found in its local continuum counterpart. The observed scale effect in apparent dispersivity is reproduced by including sufficient spatial range and structure in the kernel function of the convolution integral.

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A discrete network model was constructed to numerically simulate non-local advection. Flow paths are laid out such that spatial averages honor the Darcy-scale flow parameters. Variations in small-scale flow direction and resistance endow the network with the desired dispersion characteristics. Flow paths are discretized along their length to capture small-scale details and reduce numerical dispersion. Several notable results are observed: (1) the dispersion characteristics of statistically homogeneous media depend on the range of the kernel function, (2) by incorporating random variability in the throat parameters, early scaling behavior is matched, (3) the dilemma of coupling is made clear by the network model; in the physical system, processes are coupled at the finest scale, a feature lost by continuum models but only approximated by coarse network models, (4) model parameters in these equations do not depend on scale.

The network represents an alternative discrete system to the real porous medium. Different network systems yield the same Darcy-level averages and, therefore, produce spatially averaged behavior indistinguishable from one another. In this sense, the network is simply a coarsened discrete system that yields the same dispersive behavior as the real medium, provided both are viewed through the same broad band filter.

Research to this point shows that, in principle, a network may be constructed that gives the best representation of the real medium *for a given level of resolution*. The key problem for practical applications is determining procedures to calibrate these networks, especially when non-linear processes are involved. Efforts are underway to calibrate the network model to specific laboratory and field data and to extend the approach to other processes such as nonconservative solute transport and immiscible flow.

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[1] Koch, D. L. and J. F. Brady. 1987. "A Non-Local Description of Advection-Diffusion with Application to Dispersion in Porous Media," *Journal of Fluid Mechanics*, **180**, 387-403.

[2] Cushman, J. H. and T. R. Ginn. 1993. "Nonlocal Dispersion in Media with Continuously Evolving Heterogeneity," *Transport in Porous Media*, **13**, 123-138.

[3] Neuman, S. P. and S. Orr. 1993. "Prediction of Steady State Flow in Nonuniform Geologic Media by Conditional Moments: Exact Nonlocal Formalism, Effective Conductivities, and Weak Approximation," *Water Resources Research*, **29(2)**, 341-364.

Effective Permeability of Strongly Heterogeneous Porous Media

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Finding effective parameters to describe a flow in large scale heterogeneous porous media is a problem of major concern to the fields of hydrogeology, petroleum engineering and the chemical process industry. In such systems, the velocity field $\underline{u}$ of the flow is related to the gradient of pressure ∇p by Darcy's law

$$\underline{u} = -\kappa(\underline{r})\ \nabla p \tag{1}$$

where $\kappa(\underline{r})$ is the spatially varying permeability which varies on length scales much longer than the characteristic pore scale. In practice, it is desired to describe the heterogeneous medium in terms of an effective homogeneous medium, i.e.:

$$< \underline{u} >= -\mathrm{K}_{eff} \cdot < \nabla p > \tag{2}$$

where K_{eff}^{jm} is the constant effective permeability tensor.

The problem of computing the effective permeability of a heterogenous medium has been the subject of numerous analytical and numerical efforts [1]. Most of these works have focused on systems for which the logarithm of the permeability has a normal probability distribution function. In our work [2], we derive the effective permeability tensor $\mathbf{K}_{eff}$ of strongly heterogeneous media for arbitrary distributions of the log-permeability $ln[\kappa(\underline{r})]$. We perform a perturbation expansion of Darcy's law in the variance σ^2 of $ln[\kappa(\underline{r})]$ and show how to solve the problem at higher orders using spectral methods. The only assumption is that the spatially varying permeability $\kappa(\underline{r})$ is a stationary random function of position. This approach expresses the effective permeability in terms of the moments of the distribution of $ln[\kappa(\underline{r})]$, i.e. $\mathbf{K}_{eff}$ can formally be computed for any given distribution of the fluctuations

of the log-permeability. We then analyze the computed effective permeability tensor for different particular situations. As a special case of the theory, we examine $\mathbf{K}_{eff}$ for a normal distribution function of $ln[\kappa(\underline{r})]$ for both isotropic and anisotropic media. In this case, all odd moments of the distribution function are zero. In the case of isotropic systems, a conjecture has been made in the past [3] according to which, for three-dimensional porous media, the scalar effective permeability $\kappa_{eff} = K_G \exp[\sigma^2/6]$ where K_G is the geometric mean of the log-permeability. It is shown here that this conjecture is incorrect as the σ^6-order term of κ_{eff} contains additional terms than those corresponding to the development of the above formula. Moreover, these additional terms depend on the shape of the two-point correlation function of $ln[\kappa]$. The resulting κ_{eff} lies below the exponential formula. For anisotropic systems, $O(\sigma^4)$ corrections to the effective permeability tensor are given for an arbitrary orientation of stratification.

[1] L.W. Gelhar and C.L. Axness, "Three-dimensional stochastic analysis of macrodispersion in aquifers", Water Resour. Res., **19**, 161-180 (1983).

B. Dykaar and P. Kitanidis, "Determination of the effective hydraulic conductivity for heterogeneous porous media using a numerical spectral approach. 2, Results.", Water Resour. Res., **28**, 1167-1178 (1992).

G. Dagan, "Higher-order correction of effective permeability of heterogeneous isotropic formations of log-normal conductivity distribution", Transp. Porous Media, **12**, 279-290 (1993).

P. Indelman and B. Abramovich, "A higher-order approximation to effective conductivity in media of anisotropic structure", Water Resour. Res., **30**, 1857-1864 (1994).

[2] A. De Wit, "Effective Permeability of Strongly Heterogeneous Porous Media", submitted to Phys. Fluids.

[3] G. Matheron, *Eléments pour une théorie des milieux poreux* (Masson, Paris, 1967).

Different Pressure and Saturation Grids in Heterogeneous Porous Media

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Background and Motivation

More and more geological models are available to describe the internal structure of oil and gas reservoirs. These models are the results of geoscientific work aiming to integrate the data and knowledge about the field. Generally, these models are represented on a very high resolution grid (HR Grid). It is not unusual to obtain a grid with millions of cells. However, it is not possible to solve over this HR Grid while fluid flow simulating, because of the hugeness of the linear system to be solved. So, one must upscale petrophysical parameters.

The conventional method aims to obtain a lower resolution grid (LR Grid) by coarsening the mesh. The motivation of this coarsening is generally to perform fluid flow simulations at a reasonable cost. The results are averaged phase pressures and saturations, and, for compositional modelling, components of the oil or gas. These classical methods prevent the dynamical upscaling to the extent that one homogeneizes the petrophysical parameters before doing the fluid flow simulation. Some attempts have been made to overcome this difficulty [1].

To overcome this drawback, an original method based on two grids is proposed. The principle of this one consists in solving the parabolical pressure equation over a LR Grid, and the hyperbolical transport equations over a HR Grid. So, a specific discretisation in space for each unknown allows to keep the information on the distribution of the saturation while computing accurate averaged parameters for the pressure equation. This method can be seen as a way to dynamically update the pseudo-functions for the relative and capillary pressure curves.

The Model Problem

Let us consider the following dead-oil problem with the hypothesis of incompressibility of the fluids and the rock. The diffusion and capillary pressure are neglected.

The problem is: find the pressure p and the saturation s such that:

$$(1) \qquad -\mathrm{div}\left(\bar{\bar{k}}m(s)\mathrm{grad}p\right) = f_w + f_o \qquad \text{Pressure Equation}$$

$$(2) \qquad \phi\frac{\partial S}{\partial t} - \mathrm{div}\left(\frac{\bar{\bar{k}}k_{rw}}{\mu_w}\mathrm{grad}p\right) = f_w \qquad \text{Saturation Equation}$$

with initial conditions in saturation, and with boundary conditions in pressure and flow rate, such that the problem is well posed.

In the proposed method, each unknown will have its own discretisation. An IMPES scheme is used. So, we apply an implicit scheme for the pressure equationon on the LR Grid, and an explicit scheme for the saturation equation on the HR Grid.

The Dual Mesh Algorithm

Let us describe an extension of the Dual Mesh Algorithm presented for the pressure equation [2].

We consider the update from p_H^n, s_h^n to p_H^{n+1}, s_h^{n+1}

H is relative to the LR Grid and h to the HR Grid (cf fig 1)

<u>Step 1:</u> Homogeneization of the petrophysical parameters $\overline{\bar{\bar{k}}m(s_h^n)}$ over the LR Grid

<u>Step 2:</u> Resolution of the pressure equation; calculation of p_H^{n+1}

$$-\mathrm{div}\left(\overline{\bar{\bar{k}}m(s_h^n)}\mathrm{grad}(p_H^{n+1})\right) = f_w + f_o$$

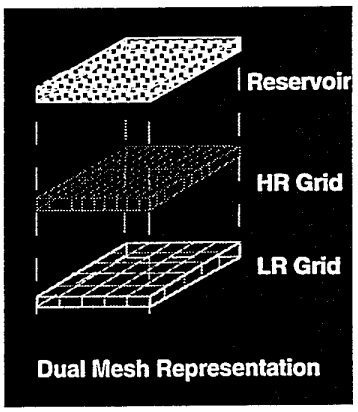

fig 1

Step 3: Reconstruction of the pressure $\tilde{p}_h^{n+1}$ and the flow rate $\tilde{\bar{q}}_h^{n+1}$ thanks to p_H^{n+1} and $\bar{q}_H^{n+1}$

Step 4 : Calculation of the saturation s_h^{n+1} by resolution of the transport equation over the HR Grid:

$$s_h^{n+1} = s_h^n + \phi \Delta t^n \mathrm{div}\left(\bar{\bar{k}} \frac{k_{rw}}{\mu_w} \mathrm{grad} p_h^{n+1} \right) + f_w$$

Step 3 - Reconstruction of the flow rate and the pressure

Let us describe into details the step 3. This is one of the most important in this algorithm.

In order to take into account the heterogeneities of the porous media, we propose an original method by solving for each cell (called M_i - cf fig 2) of the LR Grid a local problem using the same interpolator as the one used to model the global system.

There are several options to define

- the region $r(M_i)$ around each cell M_i.
- the boundary conditions on $r(M_i)$

We choose first $r(M_i) = M_i$

The problem of the boundary conditions is probably the most difficult point. After taking boundary conditions in pressure, such that the flow rate Q is kept on the boundary of M_i, we decide to take Neuman conditions on M_i with one Dirichlet condition.

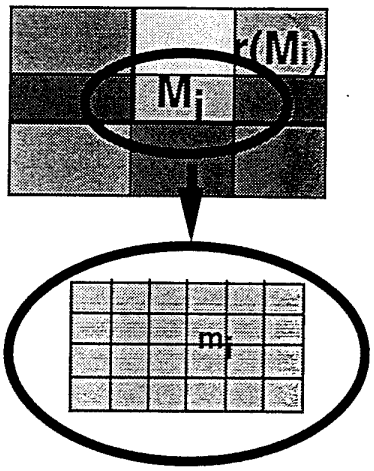

fig 2

Numerical Results

This Dual Mesh Method was successfully implemented on synthetic cases. In our poster, one could notice how the interpolator follows the variation of the pressure due to the discontinuity permeability field. At last, the Dual Mesh Method is completely validated in homogeneous case, with a good estimation of the saturation.

Conclusion

The paper describes an algorithm which allows to solve the transport equation over the HR Grid after solving the pressure equation on a LR Grid, and obtaining the flow rate at a smaller scale by solving local problems. This algorithm is implemented for a dead-oil problem. These results are encouraging and could be extended to more general multiphase flow.

Nomenclature

Symbols pertaining to the LR grid are written in capital letters, those for the HR grid are written in small letters.

f_w, f_o:	source terms vector	$\tilde{p}$:	interpolated pressure vector (over the HR Grid)
$\bar{\bar{k}}$:	permeability tensor	p:	pressure vector (over the HR grid)
k_{rw}:	relative permeability of the water	s:	water saturation
m_i:	current cell	μ_w:	viscosity
m:	mobility	ϕ :	porosity

Acknowledgement

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References

[1] Rame, M. , Killough, J. , "A new Approach to the Simulation of Flows in Highly Heterogeneous Porous Media", paper presented at the 11th SPE Symposium on Reservoir Simulation, Anaheim, February 1991

[2] Guerillot, D. , Verdiere, S. , "Different Pressure Grids for Reservoir Simulation in Heterogeneous Porous Media." paper presented at the 13th Symposium on Reservoir Simulation, San Antonio, February 1995

Contribution à l'étude du comportement de la solution d'équations non linéaires de diffusion-convection

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La modélisation mathématique des phénomènes naturels rencontrés en mécanique des milieux continus fait appel à la traduction de lois de conservation (de masse, de l'énergie, ...). On est alors amené à considérer des équations non linéaires de type divergentiel, dites équations de continuité. Elles sont génériquement formulées, au sens des distributions dans $Q =]0,T[\times \Omega$ (Ω domaine borné régulier de $\mathbf{R}^n$),

I) soit sous la forme d'un problème de Cauchy de diffusion-convection

$$(I) \quad \frac{\partial \beta(U)}{\partial t} - \Delta\phi(U) + \mathrm{Div}\left[\Psi(U)\vec{G}\right] = f,$$

$$\beta(U)(0,.) = \beta(U_0),$$

auquel on adjoint des conditions aux limites appropriées,
(β, ϕ et Ψ sont au moins des fonctions continues, ϕ étant croissante et β strictement croissante)

II) soit sous la forme d'une équation hyperbolique non linéaire du premier ordre :

$$(II) \quad \frac{\partial U}{\partial t} + \mathrm{Div}\left[\Psi(U)\vec{G}\right] = f,$$

$$U(0,.) = U_0,$$

lorsque l'effet de transport est prépondérant par rapport à l'effet de diffusion-dispersion. Ceci est par exemple le cas dans une phase fluide hétérogène, lorsque l'on désire suivre l'évolution de la fraction massique d'un constituant. En effet, la diffusion-dispersion moléculaire de ce dernier (due à l'existence de gradients de concentration au sein de la phase) peut être "négligeable" au regard de la vitesse de filtration du fluide.
(la fonction Ψ est généralement supposée de classe C^2, U_0 bornée et à variation bornée)

Lorsque l'on cherche à montrer l'unicité de la solution de tels problèmes par des méthodes de troncature dans L^1, il est nécessaire de prendre en compte les éventuelles ondes de chocs d'une solution.

I. i) Ainsi, lorsque la fonction ϕ est injective (problème de type *parabolique dégénéré*), s'il existe une solution U de l'équation (I) telle que $\frac{\partial \beta(U)}{\partial t} \in L^1(\Omega)$, alors cette solution est unique. Bien sûr, on n'est jamais assuré de l'existence d'une telle solution "forte". Cependant, moyennant certaines hypothèses de régularité sur la donnée initiale et sur les non-linéarités β et ϕ, G.Vallet a su donner dans [12] un résultat d'existence d'une solution "forte" pour l'équation intégro-différentielle

$$\frac{\partial \beta(U)}{\partial t} - \Delta\phi(U) = J(U)\mathrm{Exp}\left(-\int_0^t J(U(s,.))ds\right) \text{ dans } Q,$$

vérifiant des conditions de bord de type Fourier. Sa démarche s'appuie sur un résultat développé dans [2] et montre même que cette solution est en fait élément de $W^{1,\infty}(0,T;L^1(\Omega))$.

I. ii) Lorsque l' on n'est pas en mesure d'établir l'existence d'une solution "forte" de (I), il est néanmoins possible de montrer l'unicité d'une solution U vérifiant : $\phi(U) \in H^1(Q)$ et $\beta(U) \in BV(0,T;L^1(\Omega)) \cap L^\infty(Q)$. La démonstration repose sur la validation de la formule de dérivation de la composition d'applications dans l'ensemble BV des fonctions à variation bornée. Pour

cela, on montre que l'onde de chocs (au sens de la limite approximative) est négligeable par rapport à la mesure de Radon $\frac{\partial \beta(U)}{\partial t}$ (cf. par exemple [6], [3] ou encore [10]).

Enfin, l'adaptation des résultats exposés dans [4], a permis à G.Gagneux & M.Madaune-Tort [7] de montrer qu'une solution faible de l'équation (I) posée dans $H_0^1(\Omega)$ vérifie implicitement une formulation de Kruzkov. Sans hypothèses supplémentaires sur les non-linéarités, son unicité est alors prouvée en reprenant l'idée de la méthode généralement dévellopée dans le cas d'une équation hyperbolique non linéaires du premier ordre. (voir aussi [5] pour des problèmes de barrages, [11] lors que le tenseur de diffusivité dépend de l'évolution du problème ou encore [12] dans le cadre d'équations de la thermodynamique)

II. La solution de l'équation (II) est obtenue par l'étude du comportement, lorsque ε tend vers 0^+, de la suite des solutions des problèmes (P_ε) de *"diffusion lente"* :

$$(I_\varepsilon) \quad \frac{\partial U_\varepsilon}{\partial t} - \varepsilon \Delta \phi(U_\varepsilon) + \mathrm{Div}\left[\Psi(U_\varepsilon)\vec{\mathbf{G}}\right] = f,$$

$$U_\varepsilon(0,.) = U_0.$$

associés à des conditions de bord Dirichlet sur tout ou une partie de la frontière.

Reprenant l'idée de la méthode de viscosité artificielle, on est amené à estimer la solution U_ε du problème (P_ε) dans l'espace $W^{1,1}(Q) \cap L^\infty(Q)$, dans les cas de solutions "fortes", voire dans l'espace $BV(Q) \cap L^\infty(Q)$, dans le cas de solutions faibles. Ainsi, en raisonnant par compacité, on établit l'exitence d'une *solution faible entropique* U *à variation bornée*. (cette technique de perturbations singulières, introduite par [1], a notamment été reprise par M.J.Jasor [8] pour des problèmes à effet de puits et par L.Lévi dans le cas de problèmes unilatéraux [9]).

Remarque : Puisque l'onde de chocs (au sens de la limite approximative) n'est pas négligeable par rapport aux mesures de Radon $\frac{\partial U}{\partial t}$ et $\frac{\partial U}{\partial x_i}$, i=1..n, on comprend mieux la nécessité d'une formulation entropique, qui permet de "contrôler" les sauts de U au niveau d'une onde de chocs (relation de Rankine-Hugoniot) et d'assurer ainsi son unicité.

Références

[1] C.Bardos, A.Y.LeRoux, J.C.Nédélec : First Order Quasilinear Equations with Boundary Conditions, Comm. in Partial Differential Equations, 4 (9), 1017-1034, (1979).

[2] Ph.Bénilan, R.Gariepy : Strong solutions in L1 of Degenerate Parabolic Equations, J. Diff. Equa., à paraître.

[3] J.B.Betbeder, G.Vallet : La question de l'unicité pour une classe de problèmes d'évolution non linéaires intégro-différentiels, C.R.Acad.Sci.Paris, T.317, Série 1, 1993, pp.319-322.

[4] J.Carrillo : 1. Unicité des solutions de type Kruskov pour des problèmes elliptiques avec des termes de transport non linéaires, C.R.Acad. Sci. Paris, T.303, Série 1, n°5, 1986, pp.189-192.

[5]J.Carrillo : On the Uniqueness of the Solution of the Evolution Dam Problem, Nonlinear Analysis, Vol. 22, n°5, 1994, pp.573-607.

[6] G.Gagneux, M.Madaune-Tort : Sur la question de l'unicité pour des inéquations des milieux poreux, C.R.Acad.Sci. Paris, T.314, Série 1, 1992, pp.605-608.

[7] G.Gagneux, M.Madaune-Tort : Unicité des solutions faibles d'équations de diffusion-convection, C.R.Acad.Sci.Paris, T.318, Série 1, 1994, pp.919-924.

[8] M.J.Jasor : Perturbations singulières d'équations non linéaires de diffusion-convection modélisant des écoulements diphasiques incompressibles en milieu poreux. Thèse de doctorat d'université, Université de Pau (1992).

[9] L.Lévi : Equations quasi linéaires du premier ordre avec contrainte unilatérale, C.R.Acad.Sci.Paris, T.317, Série 1, 1993, pp 1133-1136

[10] L.Lévi : Perturbations singulières d'une inéquation parabolique dégénérée, publi. interne 93/15 URA 1204-CNRS, Univ. de Pau.

[11]A.Plouvier : Sur une classe de problèmes d'évolution quasi linéaires dégénérés, Revista Matematica de la universidad Complutense de Madrid, à paraître.

[12]G.Vallet : Strong Solution of a Nonlinear Integro-differential Parabolic Equation, Ann. Fac. Sc. Toulouse, à paraître.

Sur la correction non-lineaire de la loi de Darcy pour les écoulements en milieux poreux à faible nombre de Reynolds

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1. INTRODUCTION. – On démontre une conjecture annoncée dans Wodié–Levy [5] et Mei–Auriault [2]: dans les milieux poreux périodiques dont les inclusions sont de taille equivalente à la période, la correction non linéaire à la loi de Darcy est quadratique en fonction du nombre de Reynolds $R_\epsilon = \rho \nabla P_0 l^3/\mu^2$, où ∇P_0 est l'échelle de gradient de pression macroscopique, l l'échelle spatiale microscopique, ρ la masse volumique du fluide et μ sa viscosité dynamique. On montre que l'hypothèse d'isotropie faite dans [5] et [2] n'est pas nécessaire, mais peut être remplacée par une hypothèse d'invariance par rapport au sens du flux. Des tests numériques illustrent ce résultat.

2. POSITION DU PROBLÈME. – On suppose le milieu poreux périodique. On se donne pour cellule périodique Ω un parallépipède de $\mathbb{R}^n$ engendré par les vecteurs $(t_1, \ldots, t_n)$ qu'on suppose linéairement indépendants, c'est à dire $\det_e(t_1, \ldots, t_n) = \mathrm{mes}(\Omega) \neq 0$, où e est la base canonique de $\mathbb{R}^n$ (*ie.* en se donnant e, on fixe l'échelle microscopique). La base $(t_1, \ldots, t_n)$ n'étant pas forcément orthonormée, on définit $(t^1, \ldots, t^n)$ la base covariante. Pour $i = 1, \ldots n$ on note $\partial\Omega_i^-$ la face de Ω passant par l'origine de $\mathbb{R}^n$ (qu'on munit de sa structure affine) et telle que $\partial\Omega_i^- . t^i = 0$; on pose aussi $\partial\Omega_i^+ = \partial\Omega_i^- + t_i$. Soit Ω_0 un ouvert connexe de Ω, de frontière régulière (disons Lipschitz et d'un seul côté de sa frontière). Pour $i = 1, \ldots, n$ on pose $\partial\Omega_{i,p}^\pm = \partial\Omega_i^\pm \cap \partial\Omega_0$ et on suppose que Ω_0 est compatible avec la périodicité, c'est à dire $\partial\Omega_{i,p}^+ = \partial\Omega_{i,p}^- + t_i$. On définit enfin, $\partial\Omega_{0,s} = \partial\Omega_0 \backslash \cup_{i=1}^n \partial\Omega_{i,p}^\pm$. On suppose que $\mathrm{mes}(\partial\Omega_{0,s}) \neq 0$ et $\mathrm{mes}(\partial\Omega_{i,p}^-) \neq 0$ pour $i = 1, \ldots, n$. On introduit maintenant le cadre fonctionnel habituel: soient $\mathrm{H}^1_{0,p}(\Omega_0) = \{v \in \mathrm{H}^1(\Omega_0),\ v_{|\partial\Omega_{0,s}} = 0,\ v_{|\partial\Omega_{i,p}^-} = v_{|\partial\Omega_{i,p}^+}\}$ et $V = \{v \in \mathrm{H}^1_{0,p}(\Omega_0),\ \mathrm{div} v = 0\}$.

Dans un premier temps on considère le problème suivant: pour $\lambda \in S_n(0,1)$ (la sphère unité de $\mathbb{R}^n$, *ie.* on fixe le gradient de pression macroscopique de référence), trouver $u_0(\lambda) \in V$ tel que

$$\forall v \in V, \qquad (\nabla u_0(\lambda), \nabla v) = (\lambda, v). \tag{2.1}$$

On introduit maintenant le tenseur de perméabilité $K_0 \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ tel que $K_0\lambda = \int_{\Omega_0} u_0(\lambda)$. Ce tenseur est symétrique défini positif. Dans le cadre de la théorie de l'homogénéisation on interprète la relation $K_0\lambda = \int_{\Omega_0} u_0(\lambda)$ comme la loi de Darcy (*cf.* Bensoussan–Lions–Papanicolaou [1], Sanchez-Palencia [3] et Tartar [4]). L'objectif de ce travail est d'étudier la correction non linéaire de cette relation dans le cadre des équations de Navier–Stokes.

On se place maintenant en dimension $n = 2$ ou 3. On introduit la forme trilinéaire $b(u, v, w) = (u.\nabla v, w)$; b est continue sur $\mathrm{H}^1_{0,p}(\Omega_0)^3$, pour $n = 2$ ou 3, on note c_b la norme de b sur $\mathrm{H}^1_{0,p}(\Omega_0)^3$. On s'intéresse au problème de Navier–Stokes: pour $\epsilon \geq 0$ et $\lambda \in S_n(0,1)$ trouver $u_\epsilon(\lambda) \in V$ tel que

$$\forall v \in V, \qquad (\nabla u_\epsilon(\lambda), \nabla v) + \epsilon b(u_\epsilon(\lambda), u_\epsilon(\lambda), v) = (\lambda, v). \tag{2.2}$$

Si $\epsilon < c_0(\Omega_0)^2/2c_b\mathrm{mes}(\Omega_0)$, où $c_0(\Omega_0)$ est la constante de Poincaré du domaine Ω_0, il existe une solution unique au problème ci-dessus. Par la suite, on caractérise la dépendence de $\int_{\Omega_0} u_\epsilon(\lambda)$ par rapport à λ pour ϵ petit.

3. CORRECTION NON LINÉAIRE DE LA LOI DE DARCY – On introduit maintenant l'hypothèse centrale de ce travail:

(H) On suppose que pour tout $\lambda \in S_n(0,1)$ et tout ϵ suffisamment petit, $u_\epsilon(\lambda)$ satisfait:

$$\int_{\Omega_0} u_\epsilon(\lambda) = -\int_{\Omega_0} u_\epsilon(-\lambda) + \mathcal{O}(\epsilon^2)$$

Remarque 1 : Cette hypothèse signifie qu'un renversement du gradient de pression macroscopique implique un renversement de la vitesse de filtration sans altération notable de son module, et ce indépendamment du nombre de Reynolds (supposé petit), le résultat de l'expérience pouvant dépendre de la direction (non orientée) du gradient de pression en question.
Remarque 2 :Si Ω_0 est invariant par symétrie centrale, (H) est vérifiée et la correction $\mathcal{O}(\epsilon^2)$ est exactement nulle indépendamment de ϵ.
Remarque 3 : Si Ω_0 est invariant par rapport aux symétries S_i, $i = 1, \ldots, n$, telles que $S_i(t_i) = t_i$ et $S_i(t_j) = -t_j$ si $j \neq i$, alors Ω_0 est invariant par symétrie centrale.

Le résultat central de ce travail est le suivant

Théorème 3.1 *Si l'hypothèse* (H) *est satisfaite, on a*

$$\forall \lambda \in S_n(0,1), \qquad \int_{\Omega_0} u_\epsilon(\lambda) = K_0\lambda + \mathcal{O}(\epsilon^2). \tag{3.1}$$

Ce résultat signifie que pour ϵ petit, la vitesse de filtration vérifie quasiment la loi de Darcy avec le tenseur de perméabilité K_0, la norme de l'erreur étant bornée par $c\epsilon^2$, où la constante c ne dépend que de la géométrie du domaine.

On peut s'affranchir de l'hypothèse (H) si l'écoulement de filtration est monodimensionnel. Plus précisement, on se place dans le cas $\ker(K_0)^\perp = \mathrm{Im}(K_0) =< t_1 >$, c'est à dire K_0 se réduit à la multiplication par une constante; on montre alors

Théorème 3.2 *Dans le cas monodimensionnel, on a*

$$\forall \lambda \in \{t_1/|t_1|, -t_1/|t_1|\}, \qquad \int_{\Omega_0} u_\epsilon(\lambda) = K_0\lambda + \mathcal{O}(\epsilon^2). \tag{3.2}$$

4. ILLUSTRATIONS NUMÉRIQUES – Les résultats annoncés ici font en fait suite à une série d'expérimentations numériques en dimension 2 sur des réseaux périodiques. Nous rapportons ci-dessous quelques unes de ces expériences afin d'illustrer les théorèmes 3.1 et 3.2.

Les figures 1, 2, 3 et 4 résument des résultats numériques significatifs pour quatres configurations différentes. A gauche on représente l'évolution de $|\int_{\Omega_0} u_\epsilon(\lambda) - K_0\lambda|$ normalisé par son maximum en fonction du nombre de Reynolds relatif $R_e/R_{\max}$ pour différentes valeurs du Reynolds maximum $R_{\max}$. A droite des figures on a representé la configuration périodique correspondante. Dans tous les cas on a choisi $|t_1| = 1$.

La figure 1 concerne un réseau rectangulaire de rapport $|t_1|/|t_2| = 2$. Le tenseur K_0 n'est pas sphérique, le milieu homogénéisé n'est donc pas "isotrope" pour les équations de Stokes (il n'entre pas dans le cadre de [2], [5]). Ce réseau est à symétrie centrale; il vérifie donc (H). On vérifie sur la figure que pour $R_e \leq 20$, la correction non linéaire est parfaitement quadratique. Le cas présenté correspond à $\lambda = e^{i\pi/4}$.

La figure 2 concerne un réseau rectangulaire qui n'est pas à symétrie centrale mais qui satisfait (H) expérimentalement. On vérifie encore ici que la correction non linéaire est quadratique (on a pris $\lambda = e^{i\pi/2}$).

La figure 3 concerne un cas monodimensionnel. Il n'y a pas de réversibilité (*ie.* (H) n'est pas vérifiée), mais le théorème 3.2 s'applique. Indépendamment du sens de l'écoulement et pour $R_e \leq 10$ on vérifie que la correction non linéaire est quadratique.

La figure 4 concerne un cas bidimensionnel fabriqué de telle sorte qu'il ne satisfait pas (H). Ce cas a été assez difficile à exhiber. Pour $\lambda = e^{i5\pi/6}$ on vérifie que la correction non linéaire est linéaire en fonction du nombre de Reynolds.

Références

[1] A. BENSOUSSAN, J.-L. LIONS, G. PAPANICOLAOU, *Asymptotic analysis for periodic structures*, North Holland, 1978.

[2] C.C. MEI, J.-L. AURIAULT The effect of weak inertia on flow through a porious medium, *J. Fluid Mech.*, **222**, 1991, 647–663.

[3] E. SANCHEZ-PALENCIA, *Non homogeneous media and vibration theory*, Lecture Notes in Physics, Springer–Verlag, 1980.

[4] L. TARTAR, Convergence of the homogenization process, Appendice de [3], 1980.

[5] J.-C. WODIE, T. LEVY, Correction non linéaire de la loi de Darcy, *C. R. Acad. Sci. Paris*, Série II, **312**, 1991, 157–161.

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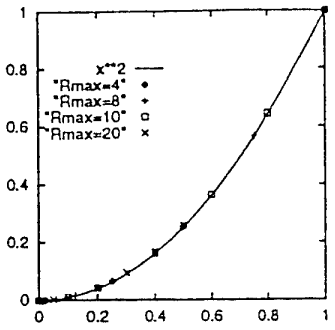


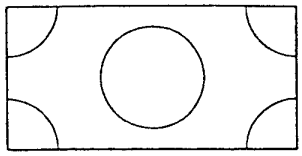
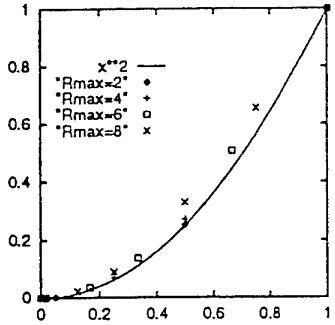


Figure 1: Cas anisotrope à symétrie centrale

Figure 1: *Anisotropic case with central symmetry*



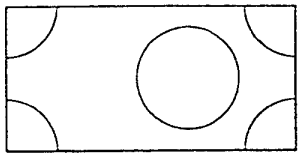
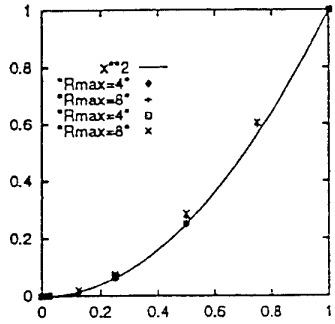


Figure 2: Cas anisotrope avec (H) verifiée expérimentalement

Figure 2: *Anisotropic case for which* (H) *is satisfied experimentally*



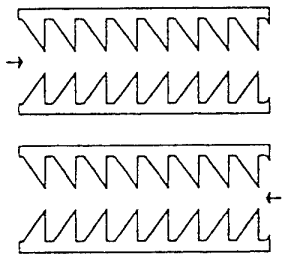
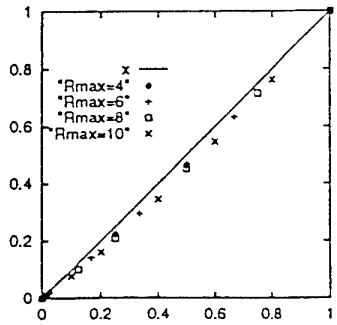


Figure 3: Cas monodimensionnel sans reversibilité du flux

Figure 3: *Monodimensional case without reversibility of the flux*



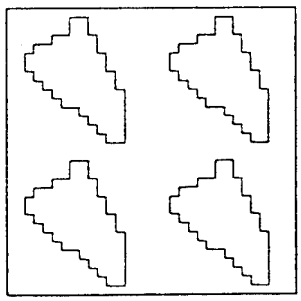


Figure 4: Cas anisotrope avec (H) non verifiée

Figure 4: *Anisotropic case for which* (H) *is not satisfied*

The Cauchy Problem for Hyperbolic Conservation Laws with Three Equations

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Abstract

This paper considers the Cauchy problem for the following nonlinear system

$$\begin{aligned} &v_t - u_x = 0 \\ &u_t - \sigma(v,s)_x + \alpha u = 0 \\ &s_t + \frac{\beta \cdot\{s-f(v)\}}{\tau} = 0 \end{aligned} \tag{1.1}$$

with bounded L^2 measurable initial data

$$(v,u,s)_{/t=0} = (v_0(x),u_0(x),s_0(x)) \,, \tag{1.2}$$

where α, β, τ are nonnegative constants. When $\beta = 0$, system (1.1) can be used to model the adaiabatic gas flow through porous media, where v is specific volume, u denotes velocity, s stands for entropy and σ denotes pressure. Its form in Euler coordinates is also a model of isothermal unsteady two phase flow in pipelines. In this paper we study the global generalized solution for the case $\beta = 0$.

When $\beta \neq 0$, which, when written in Euler coordinates, can be used to model the chemically reacting flow, where again v is specific volume, u denotes velocity but s is the mass fraction of one mode of the two-mode gas and f(v) is a given equilibrium distribution in v. In this case, τ denotes a reaction time. We show that the solution of the equilibrium system

$$\begin{aligned} &v_t - u_x = 0 \\ &u_t - \sigma(v,f(v))_x + \alpha u = 0 \end{aligned} \tag{1,3}$$

is given by the limit of the solutions of the viscous approximation

$$\begin{aligned} &v_t - u_x = \varepsilon v_{xx} \\ &u_t - \sigma(v,s)_x + \alpha u = \varepsilon\, u_{xx} \\ &s_t + \frac{\beta \cdot\{s-f(v)\}}{\tau} = \varepsilon\, s_{xx} \end{aligned} \tag{1.4}$$

as the dissipation and the reaction time τ go to zero.

MODELLING OF A HORIZONTAL AND A VERTICAL-FRACTURED WELL

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A horizontal-well and vertical-fractured-well production rate-time analysis needs a different approach and treatment. Reservoir simulation data and a well-production data may be compared to the modelling data. The difference to the wellbore rate-time data will be analysed and discussed. The rate-time and pressure-time comparison data analysis should provide more realistic future rate prediction. Comparison analysis will be based on developed type-curves data, reservoir simulation rate-time data and well production data. The rate-time data will be compared and obtained results discussed. The approach will comprise modelling, simulation analysis of both a horizontal-well and a vertically fractured well in a single porosity reservoir that may be extend to the dual porosity case. The comparison analysis will start with a simplified horizontal-well case and in a future the proposed simplified model can be extended to a model that will include heterogeneities (vertical to horizontal permeability differences, faults and multilayers or composite structures).

Résolution numérique d'un problème de restauration biologique en milieu poreux

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On s'intéresse à la résolution numérique d'un problème modélisant la restauration biologique par des bactéries B d'un milieu poreux pollué par un substrat miscible S en présence d'oxygène O [4]. Le modèle étudié est celui de Borden et Bedient [1] où l'adsorption des substrats en solution est supposée linéaire et instantanée, et les termes de réactions qui régissent le développement des bactéries et la dégradation du polluant sont issus de la cinétique de Monod [1] :

$$\Phi R \frac{\partial S}{\partial t} - \nabla((\lambda(S)+D(V)).\nabla S) + V.\nabla S = -\Phi R_S(O,S,B) \tag{1}$$

$$\Phi \frac{\partial O}{\partial t} - \nabla((\lambda(O)+D(V)).\nabla O) + V.\nabla O = -\Phi R_O(O,S,B) \tag{2}$$

$$\frac{dB}{dt} = R_B(O,S,B) \tag{3}$$

$$\mu(S)\,V = -K.(\nabla p - \beta(S).e_z) \tag{4}$$

$$\nabla.V = 0 \tag{5}$$

La vitesse de l'écoulement est notée $\boldsymbol{V}$, la porosité du milieu Φ, la perméabilité du milieu K, R est le facteur de retard du substrat S, μ et β la viscosité et la densité du fluide, λ le tenseur de diffusion moléculaire et $\boldsymbol{D}$ le tenseur de dispersion

$$D(V) = \alpha_l \begin{bmatrix} u^2 & uv \\ uv & v^2 \end{bmatrix} + \alpha_t \begin{bmatrix} v^2 & -uv \\ -uv & u^2 \end{bmatrix}$$

où α_l et α_t sont les coefficients de dispersion longitudinale et transversale.

La vitesse de filtration $\boldsymbol{V}$ étant donnée par la résolution de l'équation de Darcy (4) et de l'équation de continuité (5), les équations (1), (2), et (3) sont découplées par une méthode de pas fractionnaires [3] : le système d'équations différentielles issu des termes de réaction est alors résolu par une méthode de Runge Kutta explicite d'ordre quatre. La principale difficulté réside dans le traitement des équations d'advection diffusion. En effet lorsque le transport domine la diffusion, les méthodes usuelles génèrent soit de la diffusion numérique soit de la dispersion numérique source d' étalements des fronts et d'instabilités.

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A partir du schéma de Tackacs [6], qui est du type Lax-Wendroff auquel un terme de troisième ordre en espace est ajouté minimisant ainsi les erreurs de dispersion et de dissipation, on construit par la technique des limiteurs de flux de Roe ou Davis [5], un schéma explicite quasi d'ordre deux à variation totale décroissante pour l'approximation des termes de transport [4]. Les termes de diffusion sont discrétisés de façon implicite par un schéma centré. Les résultats obtenus sont trés satisfaisants.

L'extension bidimensionnelle du schéma de Tackacs est basée sur la discrétisation par la méthode des volumes finis sur un maillage cartésien de l'équation

$$\frac{\partial C}{\partial t} - \frac{\Delta t}{2} \nabla(\boldsymbol{D}^*(\boldsymbol{V})\nabla C) + \boldsymbol{V}.\nabla C = 0 \quad ; \quad \boldsymbol{D}^*(\boldsymbol{V}) = \begin{bmatrix} u^2 & uv \\ uv & v^2 \end{bmatrix} \tag{6}$$

issue du développement de Taylor à l'ordre deux en temps de la solution C de l'équation de transport

$$\frac{\partial C}{\partial t} + \boldsymbol{V}.\nabla C = 0. \tag{7}$$

Le flux $\boldsymbol{D}^*(\boldsymbol{V})\nabla C$ est discrétisé et limité de façon à avoir un schéma L^∞ stable.

Des essais numériques ont été effectués pour le problème modèle (7) avec des champs de vitesse à divergence nulle pour lesquels on connait une solution analytique. La comparaison de la solution calculée avec la solution exacte montre que le schéma est faiblement diffusif et minimise les effets dispersifs dus au maillage tout en conservant la masse. C'est ce schéma qui est proposé pour resoudre la partie transport du système (1) - (5).

Références

[1] R. C. Borden, P. B. Bedient - Transport of dissolved hydrocarbons influenced by oxygen-limited biodegradation 1. Theorical development ; *Water Resour. Res.*, 22, pp. 1973-1982, 1986

[2] Ch.H. Bruneau, P. Fabrie, P. Rasetarinera - Accurate schemes to solve advection dominated-diffusion equations, *International Journal for Numerical Methods in Fluid*, soumis

[3] C. H. Chiang, C. N. Dawnson, M. F. Wheeler - Modeling of in-situ biorestoration of organic compounds in groundwater, *Transport in porous media*, 6, pp. 667-702, 1991

[4] P. Fabrie, P. Rasetarinera - Analyse mathématique d'un système de transport-diffusion-réaction modélisant la restauration biologique d'un milieu poreux, *Revista Matemàtica de la Universidad Complutense de Madrid*, accépté pour publication

[5] E. Godlewski, P. A. Raviart - Hyperbolic systems of conservation laws, *édition de la S.M.A.I*, 1991

[6] L. L. Takacs - A two-step scheme for the advection equation with minimised dissipation and dispersion errors, *Monthly Weather Rewiev*, 113, pp. 1050-1065, 1985

ASYMPTOTIC BEHAVIOUR OF SOLUTIONS OF NEUMANN PROBLEMS FOR NONLINEAR NONVARIATIONAL ELLIPTIC EQUATIONS IN DOMAINS WITH COMPOSITE STRUCTURE

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The asymptotic behaviour of $W^{1,m}$-generalized solutions of Neumann problems for elliptic equations

$$-\sum_{i=1}^{n} \frac{d}{dx_i} a_i(x, \nabla u) + a(x, u) = f\,, \quad x \in \Omega_s\,, \quad s = 1, 2, \ldots,$$

is studied. The following qualitatively different kinds of domains $\Omega_s \subset \Omega$ are considered:

1) strongly connected perforated domains;

2) weakly connected domains of framework type with thin channels;

3) perforated domains with so called accumulators.

The first kind of domains Ω_s is characterized with existence of a sequence of linear continuous extension operators $p_s : W^{1,m}(\Omega_s) \to W^{1,m}(\Omega)$ such that $\sup_s \|p_s\| < \infty$. In this case solutions of the problems under consideration converge to solution of Neumann problem for an equation of the form

$$-\sum_{i=1}^{n} \frac{d}{dx_i} \hat{a}_i(x, \nabla u) + \alpha a(x, u) = \alpha f\,, \quad x \in \Omega\,, \quad \alpha > 0\,.$$

In the second case domains Ω_s have representation $\Omega_s = \Omega_s^{(1)} \cup H_s \cup \Omega_s^{(2)}$, $\operatorname{mes} H_s \to 0$ and there exist sequences of linear continuous extension operators $p_s^{(i)} : W^{1,m}(\Omega_s^{(i)}) \to$ $\to W^{1,m}(\Omega)$ such that $\sup_s \|p_s^{(i)}\| < \infty$ $(i = 1, 2)$. In this situation solutions of the initial Neumann problems converge to solution of an equation with operator defined on $(W^{1,m}(\Omega))^2$.

In the third case domains Ω_s have representation $\Omega_s = \Omega_s^{(1)} \cup H_s \cup E_s$, $\operatorname{mes} H_s \to 0$, there exists a sequence of operators $p_s^{(1)} : W^{1,m}(\Omega_s^{(1)}) \to W^{1,m}(\Omega)$ such as in the second case, open sets E_s are unions of nonintersected domains E_s^j $(j \in J_s)$, which are called as accumulators. In this case solutions of the problems under consideration converge in a certain sense to solution $(u, \psi) \in W^{1,m}(\Omega) \times L^m(\Omega)$ of a problem for functional–differential system of the form

$$-\alpha_1 b(x, u - \psi) + \alpha_2 [a(x, \psi) - f] = 0\,, \quad x \in \Omega\,,$$

$$-\sum_{i=1}^{n} \frac{d}{dx_i} \hat{a}_i(x, \nabla u) + \alpha_0 a(x, u) + \alpha_2 a(x, \psi) = (\alpha_0 + \alpha_2) f\,, \quad x \in \Omega$$

$$(\alpha_i > 0,\ i = 0, 1, 2).$$

Stockage de gaz réactif dans un milieu poreux Modélisation mathématique et numérique

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Le stockage rapide de grandes quantités de tritium, isotope radioactif et toxique de l'hydrogène, est un problème important pour les installations utilisées dans la fusion thermonucléaire contrôlée. Une des méthodes préconisée par les ingénieurs du CEA consiste à faire absorber ce gaz par une poudre métallique, cette réaction étant réversible. L'absorption du gaz se fait par une réaction d'hydruration qui est exothermique. Cette réaction se produit sous certaines conditions de température et de pression. Durant l'hydruration la poudre gonfle, modifiant la porosité du milieu. Le stockage maximum, c'est à dire l'hydruration maximale, demande de contrôler, par un système de refroidissement inclus dans les parois, la température du four de stockage; en particulier pour empêcher la formation en entrée de four d'un bouchon d'hydrure bloquant la progression du gaz. Le processus de destockage est symétrique, à un léger hystérésis près . La réaction de deshydruration est activée par chauffage de la poudre. Le phénomène est donc caractérisé par des couplages non-linéaires raides entre dynamique et chimie et entre chimie et thermique.

La modélisation mathématique et numérique du problème a été entreprise dans le but d'optimiser la forme du four et le contrôle de la réaction. Elle est basée sur l'équation de la cinétique chimique de la réaction d'hydruration/deshydruration, sur le bilan de masse et sur le bilan d'énergie. Si le problème est relativement standard dans le cas du transport d'un seul gaz, il est moins classique quand on prend en compte un mélange tritium - hélium. L'hélium qui peut apparaître à des concentrations non négligeables dans le système est inerte dans la poudre. Nous proposons dans ce cas une modélisation de la dynamique du mélange en définissant des vitesses propres pour chacun des gaz ; ces vitesses sont calculées elles-mêmes à partir d'une unique loi de Darcy fondée sur une viscosité dynamique moyenne.

La modélisation numérique est faite en volumes finis avec des schémas conservatifs, semi-implicites en temps et décentrés amont en espace. Des résultats d'existence et de stabilité ont pu être établis ainsi qu'un principe du maximun discret analogue du principe du maximum obtenu sur le modèle mathématique continu.

Le code de calcul, prenant en compte la géométrie détaillée du prototype industriel (poudre, masses métalliques du four et volumes d'entrée et de sortie) donne des résultats en accord avec les expériences.

MACROSCOPIC PERMEABILITY OF THE SYSTEM OF THIN FISSURES FILLED BY MATERIAL WITH RANDOM PERMEABILITY TENSOR

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Consider the simliest model of lattice-structures: two-dimensional rectangular lattice.

Definition 1. *The union*

$$B_{\varepsilon,\mu} = U_{k=-\infty}^{+\infty} (\{(x_1,x_2) \in R^2 \mid\mid x_2 - k\varepsilon \mid < \varepsilon\mu/2 \}$$

$$U \{(x_1,x_2) \in R^2 \mid\mid x_1 - k\varepsilon \mid < \varepsilon\mu/2 \})$$

is called the two-dimensional rectangular lattice.

Thus the rectangular lattice is a union of thin strips of the width $\varepsilon\mu$ stretched in each coordinate direction and forming the $\varepsilon-$ periodic system in each dimension. We also denote

$$B_{\varepsilon,\mu}^j = U_{k=-\infty}^{+\infty} \{(x_1,x_2) \in R^2 \mid\mid x_{3-j} - k\varepsilon \mid < \varepsilon\mu/2 \}$$

the unions of horisontal ($j=1$) and vertical ($j=2$) strips, so

$$B_{\varepsilon,\mu} = B_{\varepsilon,\mu}^1 \, U \, B_{\varepsilon,\mu}^2.$$

Let G be a domain with the bondary $\partial G \in C^\infty$ which is independent of ε and μ.

Definition 2. *Let $u_{\varepsilon,\mu}(x)$ is a sequence of functions from $L_2(B_{\varepsilon,\mu} \cap G)$, $u_0(x) \in L_2(G)$. One sais that $u_{\varepsilon,\mu}$ L - converges to $u_0(x)$ at $B_{\varepsilon,\mu} \cap G$ if and only if*

$$\frac{\|u_{\varepsilon,\mu} - u_0\|_{L_2(B_{\varepsilon,\mu} \cap G)}}{\sqrt{meas(B_{\varepsilon,\mu} \cap G)}} \to 0, \quad (\varepsilon,\mu \to 0).$$

The normalisation factor $1/\sqrt{meas(B_{\varepsilon,\mu} \cap G)}$ is necessary because $\|1\|_{L_2(B_{\varepsilon,\mu} \cap G)} = \sqrt{meas(B_{\varepsilon,\mu} \cap G)}$. Notice that L-convergence is not a convergence in common sense because the domain depends on small parameters.

Consider a lattice $B_{\varepsilon,\mu}$ and for each strip $B_k^j = \{(x_1,x_2) \in R^2 \mid\mid x_{3-j} - k\varepsilon \mid < \varepsilon\mu/2 \}, j = 1,2$ we associate a random-valued constant $(2\times 2)-$matrix $A_{B_k^j}$, independent of ε and μ. All $A_{B_k^j}$ are independent in aggregate and have the same discrete distribution:

$$P\{A_{B_k^j} = A^{(s)}\} = p_s, \quad s = 1,...,r,$$

where

$$\sum_{s=1}^{r} p_s = 1, \quad A^{(s)} = (A^{(s)})^T > 0.$$

Let Π be an intersection of the strips:

$$\Pi = \cup_{k,j=-\infty}^{+\infty}(B_k^1 \cap B_j^2),$$

and $A^{(0)}$ be a fixed constant matrix such that $A^{(0)} = (A^{(0)})^T > 0$. We pose $A = A^{(0)}$ on the set Π and $A = A_{B_k^j}$ in each strip $B_k^j \backslash \Pi$ without Π.

Let G be a domain with the bondary $\partial G \in C^\infty$ which is independent of ε and μ, $f \in C^1(G)$. Consider the problem with random coefficients

$$-div\ (\ A\ grad\ u_{\varepsilon,\mu}) = f(x)\ , \tag{1}$$

$$(\ A\ grad\ u_{\varepsilon,\mu}\ ,\ n\ ) = 0,\ for\ x \in \partial B_{\varepsilon,\mu} \cap G, \tag{2}$$

$$u_{\varepsilon,\mu} = 0,\ for\ x \in \bar{B}_{\varepsilon,\mu} \cap \partial G. \tag{3}$$

Problems (1)-(3) simulate a problem of permeability of a fissured rock filled with porous substance , with A being the permeability tensor of the substance in the fissures , $u_{\varepsilon,\mu}$ is a microscopic pressure, and u_0 is a macroscopic pressure. Numerical solution of problems (1)-(3), with $\varepsilon << 1$, $\mu << 1$, is very difficult since the step size of the grid must have an order much less than ε. The realization of the standard homogenization procedure is also impeded, since the problem on a cell depends on the small parameter μ, and in order to solve it numerically, we must select the step size of the grid to be much less than μ. Hence, an asymptotic investigation of the problem is needed.

Introduce the notation $\bar{A} = (\bar{a}_{ij})$,

$$\bar{a}_{11} = \sum_{s=1}^{r} p_s 0.5(a_{11} - a_{12}a_{22}^{-1}a_{21}),\ \bar{a}_{12} = 0,$$

$$\bar{a}_{22} = \sum_{s=1}^{r} p_s 0.5(a_{22} - a_{21}a_{11}^{-1}a_{12}),\ \bar{a}_{21} = 0.$$

Let u_0 be a solution of the homogenized averaged problem

$$-div\ (\ \bar{A}\ grad\ u_0) = f(x)\ ,\ x \in G,\quad u_0|_{\partial G} = 0, \tag{4}$$

Theorem *For each $\delta \in (0,1)$ the estimate takes place*

$$P\{\frac{\|u_{\varepsilon,\mu} - u_0\|_{L_2(B_{\varepsilon,\mu} \cap G)}}{\sqrt{meas(B_{\varepsilon,\mu} \cap G)}} > (\sqrt{\varepsilon} + \sqrt{\mu})^{1-\delta}\} \leq c\varepsilon^\delta.$$

Application of convective transfers on geology

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Convective heat transfer in porous media is of fundamental importance to a number of geological applications. such as oil recovery, water supply management in hydrogeology, geothermal exploitation, ground heat storage, radioactive waste management, ground water flow modelling and is also of interest in environmental sciences and geophysics. But problems encountered in that kind of application are different from those encountered in laboratories for several reasons: i) large spatial extension, ii) 3D complex geometry, iii) scarce, irregular or partial data, iv) geometry of objects not known. v) large amount of data. In such situation, there is no uniqueness of solution and consequently, robust benchmarks are necessary to select the most appropriated model compatible with the available data.

The purpose of this work is to present, firstly, a global process for 3D modelling of transfer problems encountered in geology, and secondly, indirect procedure based on direct measurements for validating the model. In a first step, the geometry is modelled using surfaces defining the limits of the studied object (layers, volume,...) and the discontinuities (fractures. faults). Then, properties of the objects (permeability, thermal conductivity,..) are interpolated into the space from a given set of "control points". This step is done by the GOCAD software, developed at the Ecole de Gologie de Nancy, a powerful tool designed for interactive 3D modelling of surfaces and volumes, of complex geological structures.

In a second step, a special scheme allowing modelling of coupled heat and mass transfer in 2D and axisymmetric. anisotropic and heterogeneous porous media has been developed. The media is saturated with a single fluid phase. Darcy flow characteristics are assumed for the liquid phase. Solid characteristics depend on the temperature. A dimensionless formulation is used to simplify the heat and Darcy equations. For two dimensional and axisymmetric problems, the resulting equations are strictly similar and can be solved numerically using the same procedure. Numerical studies are in agreement with results obtained by theoretical approach for homogeneous isotropic medium.

This approach has been applied to the site of Soultz-sous-Forts, selected by the European Hot Dry (HDR) geothermal project as a pilot zone for exploiting low enthalpy energy. It is situated east of the Merkwiller-Pechelbronn oil field , along the western limit of the upper Rhine Graben in northern Alsace (France). Geophysical and geochemical studies carried out on Rhine Graben show evidences of water circulation, within the Triassic and altered granitic rocks. Firstly, the

sedimentary cover is asymmetric with a depth of 1400m along the west border of the graben compared to 3500m on the eastern part, causing a general circulation pattern from east to west, associated with a higher surface heat flow on the west (up to 150mW m^{-2} compared to 70-80mW m^{-2} on the east). Secondly, the vertical heat flow decreases vertically from a mean constant value of 150 mW m^{-2} in the sedimentary cover to a lower value of 70 and 30 mW m^{-2} at 1700-2000m depth, in the bedrock in boreholes GPK1 and EPS1, respectively. Finally, fluid circulation is presently observed in the granitic fractured bedrock. Regional geochemical studies suggest that these fluids result from a mixing of low salinity surface water from the west with higher salinity deep formation water from the east. The decrease of the geothermal heat flow at depth is likely attributed to fluid circulation through the sediment cover. Physical parameters and boundary conditions are sometime difficult to define. Thermal conductivity has been measured directly on a conductivimeter from rock samples whilst permeability is estimated from empirical approximations and injection tests in boreholes. Boundary conditions are defined from hydrological and geophysical information.

To obtain a "good model", several tries have been necessary, implying the modification of the geometry, the physical parameters and the boundary conditions. The final model shows three hydrothermal circulation systems. The first one is expressed at the surface next to Baden-Baden. The second has a complex path, consisting of six convective cells extending from the Rhine axis to Pechelbronn. The third and smallest one consists of two cells, which originate in the western part of the graben and continue to Hochwald. The maximum filtration velocity observed is about 1m.yr^{-1}. The temperature obtained from the numerical model are in good agreement with those from direct borehole observations and with temperature estimated from fluid inclusions, suggesting that the temperature field would be stable from the formation of the fluid inclusions to the present time. The geochemical study of water shows that fluid exchange has been taken place between the Triassic and the fractured granite layers, resulting from a mixing of high salinity formation fluids and surface water. These two last arguments are in favour of convective circulation and show that water exchange between the Triassic and the fractured granite layers has been taken place. The C^{14} fluid dating provides ages ranging from 5000 to 20 000 yr., compatible with the velocity calculated by model.

The above methodology shows that fluid transfer in natural porous media implies a feedback procedure between the modelling and the available data. The solution is not unique, but indirect methods (fluid dating, etc...) can be used to constrain the final model. Such procedures are used to provide temperature estimation at depth for geothermal purpose. Together with geometrical reconstitution throughout times, they give also interesting information for oil formation and migration.

The Explicit Solution of a Free Boundary Problem for a Nonlinear Absorption Model of Mixed Saturated-Unsaturated Flow

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ABSTRACT

In wetted soils, zones of saturation develop naturally in the vicinity of impermeable strata, surface ponds and subterranean cavities. Hidrology must be concerned with transient flow trough coexisting unsaturated and saturated zones. Models of advancing saturated zones necessarily involve a nonlinear free boundary problem.

A closed–form analytic solution is presented for a nonlinear diffusion model under conditions of ponding at the surface. The soil water diffusivity is restrict to the special funcional form $D(\theta)=\frac{a}{(b-\theta)^2}$, where θ is the water content field to be determined and, a and b are positive constants. The explicit solution depends of a parameter C (determined by the data of the problem), according to two cases : $1 < C < C_1$ or $C \geq C_1$, where C_1 is a constant which is obtained as the unique solution of an equation. This result complements the study given in P. Broadbridge, "Solution of a Nonlinear Absorption Model of Mixed Satured–Unsatured Flow", Water Resources Research, 26 (1990), 2435–2443.

On some new models of nonhomogeneous fluid filtration in porous media

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In this report I propose some new models of nonhomogeneous fluid filtration in porous media. The models based on Navier-Stokes and boundary layer equations. Existence theorems are proved as well as numerical results are given.

Multiscale models : A tool to describe the porosity of cement-based materials and to predict their transport properties

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The porosity of cement-based materials covers several orders of magnitude, generally ranging from nanometers to a few micrometers. Therefore, it is difficult to relate porosity and transport properties for such materials. In this paper, a method based on the representation of the pore structure by networks of capillary tubes is proposed. The basic idea is to sort the pore-size-distribution into several classes and to distribute each class on networks of various sizes. The different networks are then superposed by following rules based on renormalization theory. This model has been applied to correct the pore-size distribution (PSD) obtained from experimental mercury intrusion porosimetry curves. Using this "corrected" PSD, transport coefficients, such as saturated (water, air) permeability and mixed (water vapour+water) transfer coefficients, can be estimated using numerical methods. The estimated values are compared with experimental results.