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An investigation into the levels of internal tide energy in the Iceland-Faeroes ridge region has been carried out using both current meter observations and predictions from a three-dimensional ocean circulation model forced by tidal current predictions from a barotropic model. The work has proved successful in that the model runs are stable and in places make reasonable predictions. Two tidal constituents have been examined, M2 and O1. In general the predicted internal tide energy levels are too low, however, and further work is planned.

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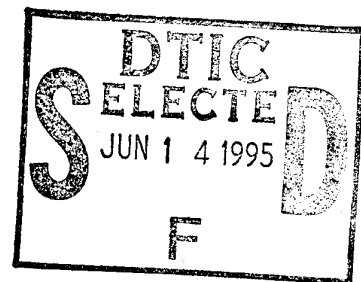| 17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
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TO MODEL TIDAL STRUCTURE
IN THE SHALLOW WATER
OF THE ICELAND-FAEROES RIDGE REGION

FINAL REPORT

by

T J SHERWIN

PROJECT NO: UCES 105

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ABSTRACT

An investigation into the levels of internal tide energy in the Iceland-Faeroes ridge region has been carried out using both current meter observations and predictions from a three-dimensional ocean circulation model forced by tidal current predictions from a barotropic model. The work has proved successful in that the model runs are stable and in places make reasonable predictions. Two tidal constituents have been examined, M2 and O1. In general the predicted internal tide energy levels are too low, however, and further work is planned.

REPORT STATUS

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1 INTRODUCTION

The aim of this work has been an investigation into the distribution of internal tide energy over the Iceland-Faeroes ridge using the extensive current meter data set collected by SACLANT and model predictions. An earlier progress report (see appendix) described the results of the current meter analysis; here results from a numerical model will be compared with the observations.

Internal tides are internal waves induced by the barotropic tide with energy peaks at periods of about 12.5 h and 25 h. Forcing at the semi-diurnal period has been successfully described by Baines (1982) and Craig (1987) in terms of the interaction of the flow of barotropic tidal currents with a shelf break bottom topography in the presence of oceanic stratification. These models consider generation in a two-dimensional vertical plain, but recent work by Perkins *et al* (1994) and Dale & Sherwin (1995) has suggested that in regions of complex topography a three-dimensional approach may be needed. Thus the motivation for the present work is to extend the understanding derived from two-dimensional forcing theory. In this report we look at the preliminary results of a three-dimensional primitive equation model of internal tidal waves forced over the Iceland-Faeroes ridge by the output from a barotropic tidal model.

2 THEORY

The investigations used the Bryan and Cox general ocean circulation model, which is a well established primitive equation model. Only a brief description of the model is given here since it is extensively discussed elsewhere, for example by Semtner (1986) and Cox (1984). The governing equations can be summarized in vector notation as:

$$\frac{\partial u}{\partial t} + u.\nabla u + w\frac{\partial u}{\partial z} + f(k \times u) = -\frac{1}{\rho}\nabla p + A_V\frac{\partial^2 u}{\partial z^2} + A_H\nabla^2 u \qquad (1)$$

$$\frac{\partial T}{\partial t} + u.\nabla T + w\frac{\partial T}{\partial z} = K_V\frac{\partial^2 T}{\partial z^2} + K_H\nabla^2 T \qquad (2)$$

$$\frac{\partial p}{\partial z} = -\rho g \qquad (3)$$

$$\nabla \cdot \mathbf{u} + \frac{\partial w}{\partial z} = 0 \qquad (4)$$

where **u** is the horizontal velocity vector; w is vertical velocity; p is pressure; A_V, A_H, K_V and K_H are the vertical and horizontal components of viscosity and diffusion; and T is temperature. Density, ρ, is a function of T and salinity, although for clarity the salinity equation equivalent to (2) is omitted in this discussion. The differential operator, ∇, applies only to the horizontal plane; z is positive upwards; **k** is the unit vertical vector; and t is time. Standard simplifications have been made in deriving these equations - the Boussinesq approximation in (1); the hydrostatic assumption in (3); incompressibility in (4); and Fickian parametrizations of viscosity and diffusion in (1) and (2). For computational purposes (1) to (4) are transformed onto spherical co-ordinates and solved on a finite difference grid.

An important feature of the model is that is uses a rigid lid to suppress the propagation of surface gravity waves. Barotropic flows are represented by a stream function, ψ, defined as

$$(\mathbf{k} \times \nabla)\psi = \overline{\mathbf{u}} h \qquad (5)$$

where

$$\overline{\mathbf{u}} = \frac{1}{h} \int_{-h}^{0} \mathbf{u} \, dz \qquad (6)$$

is the barotropic velocity vector in water of depth h.

The internal tide is forced in the present configuration by replacing $w\dfrac{\partial T}{\partial z}$ in (2) with $\left(w + w_0\right)\dfrac{\partial T}{\partial z}$ where w_0 is the oscillating vertical velocity due to the barotropic tide. This is equivalent to the linear forcing technique used by Baines (1982), although it is incomplete because it omits terms such as $(\mathbf{u}+\mathbf{u}_0)\cdot\nabla T$ where $\mathbf{u}_0$ is the horizontal current vector of the barotropic tide. However, except near a front, we expect $w_0\dfrac{\partial T}{\partial z}$ to be much greater than $\mathbf{u}_0\cdot\nabla T$.

3 DERIVATION OF BAROTROPIC FORCING

The barotropic forcing term, w_0, has been derived from a barotropic model of the tides of the NE Atlantic due to Roger Flather at the Proudman Oceanographic Lab. The

model results specify the amplitude and phase of vertical elevation (which is ignored) and horizontal velocity for different tidal constituents. In the presence of variable topography the continuity equation of the barotropic tide can be written as

$$\frac{\partial w_0}{\partial z} + \nabla \cdot \left(\boldsymbol{Q} h^{-1} \right) = 0 \qquad (7)$$

where

$$\boldsymbol{Q} = h \left(u_\sigma \cos\left(\sigma t - G_{u\sigma} \right) \boldsymbol{i} + v_\sigma \cos\left(\sigma t - G_{v\sigma} \right) \boldsymbol{j} \right) \qquad (8)$$

is the volume flow. Here σ is the frequency of a specific tidal constituent; u_σ and v_σ are the amplitude of the barotropic tidal currents in the $\boldsymbol{i}$ and $\boldsymbol{j}$ directions; and G_u and G_v are the corresponding phase lags. On the assumption that $\nabla \cdot \boldsymbol{Q}$ is small the expression

$$\frac{\partial w_0}{\partial z} = -\boldsymbol{Q} \cdot \nabla h^{-1} \qquad (9)$$

has been used to derive w_0. In the Cox model the expression

$$w_0(z) = -\boldsymbol{Q} \cdot \nabla h^{-1} z \qquad (10)$$

has been used to give the vertical velocity at depth z with the rigid lid assumption. The computation of w_0, is complicated by the fact that the Flather model uses an Arakawa 'C' grid, whilst the Cox code uses an Arakawa 'B' grid.

4 THE GRIFFITHS MODEL OF THE ICELAND-FAEROES RIDGE

The Griffiths (1995) modification of the Cox code rigid lid General Ocean Circulation Model has been used to establish the background density field for the Iceland-Faeroes ridge region. The model parameters are given in Table 1. The topography was taken from the DBDB5 data set and the model initialized with a temperature and salinity field from the Levitus data set. The model computes baroclinic fields of density and momentum and a separate barotropic momentum (or stream function) field. Passive boundary conditions are applied to the baroclinic fields and to the north and south stream function boundaries. However, values of the stream function on the east and west boundaries are forced to equal values predicted by a larger scale model due to Stevens (1990). The model was run for typically five days during which time inertial

3

oscillations (period about 13.7 h), which are caused by the initialization process, were gradually damped out.

5 TIDAL FORCING AND ANALYSIS

Throughout a model run the internal tide was forced using (10) over the model domain. For the last wave period of a run (duration $T = 2\pi/\sigma$) an analysis of the tidal current in the i direction at each grid point was conducted using

$$U(x,y,z) = \frac{2}{N} \sum_{n=1}^{N} u(x,y,z,t) \exp(i\sigma n\Delta_t) \qquad (11)$$

where U is complex; u is the speed of the current at time t in the i direction and $(N\Delta_t = T)$. A similar analysis was conducted to derive V with v (the speed in the j direction) instead of u in (11). If

$$\overline{U}(x,y) = \frac{1}{h} \int_{-h}^{0} U(x,y,z) \, dz \qquad (12)$$

is the i component of the (complex) depth mean current, and $\overline{V}$ is the equivalent j component, then the baroclinic components of the tidal current are

$$u_b(x,y,z) = U(x,y,z) - \overline{U}(x,y) \qquad (13a)$$

and

$$v_b(x,y,z) = V(x,y,z) - \overline{V}(x,y) \qquad (13b)$$

A baroclinic tidal energy parameter was thus computed as

$$r^2 = \frac{1}{2h} \int_{-h}^{o} \left(u_b u_b * + v_b v_b * \right) dz \qquad (14)$$

where u_b*, v_b* are the complex conjugates of u_b and v_b.

6 RESULTS

The model was separately forced with two tidal constituents, M2 (period 12.42 h) and O1 (period 25.82 h) using Levitus data to prescribe a climatic spring time density field. The distributions of r for each constituent within a box bounded by 67°N 15°W, 62°N 3°W (within the Griffiths model domain) are compared in Figs 1 and 2 with

similar plots derived from current meter observations (described in Progress Report No 2, see Appendix).

The predictions with M2 are about a factor of 4 smaller than those observed, but within the comparison area encouragement can be drawn from the fact that the two energy distribution patterns seem comparable. For example, large values are both predicted and observed along the SE edge of the Iceland shelf, whilst they are much smaller along NE shelf edge. Overall we notice that the model predicts relatively high semi-diurnal internal tide levels along the crest of the ridge which is reasonable since strong barotropic tidal currents cross it.

These patterns are not reproduced with O1. Here the magnitude of the predictions and observations are very similar on the SE edge of the Iceland shelf, but the predictions are very much smaller than the observations on the north side of the ridge and on the NE Icelandic shelf edge.

7 CONCLUSIONS

This work has demonstrated that it is worthwhile trying to map internal tide energy levels in regions of complex topography. The method of coupling the baroclinic mode of a rigid lid general ocean circulation model to forcing from a barotropic tidal model is stable and permits runs to take place in a realistic time (the results shown here took about 2 h on a Sun Sparc 10). However, comparison with observations shows that further work is required before the results can be treated with confidence. For example it will be necessary to look at the sensitivity of the predictions to the prescribed density field. It is likely that the stratification used to the north of the Iceland-Faeroes ridge is much weaker than actually exists and may explain, in particular, why the predicted O1 magnitudes are too small in that region. It may also be worthwhile to examine the sensitivity to vertical resolution in the surface layers.

It should be evident that this contract has stimulated some innovative work which is not yet complete. This work will be continued with funding from another agency and will eventually be reported in the scientific press with due acknowledgement to the support of NRL.

ACKNOWLEDGEMENTS

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Table 1

The Model Parameters

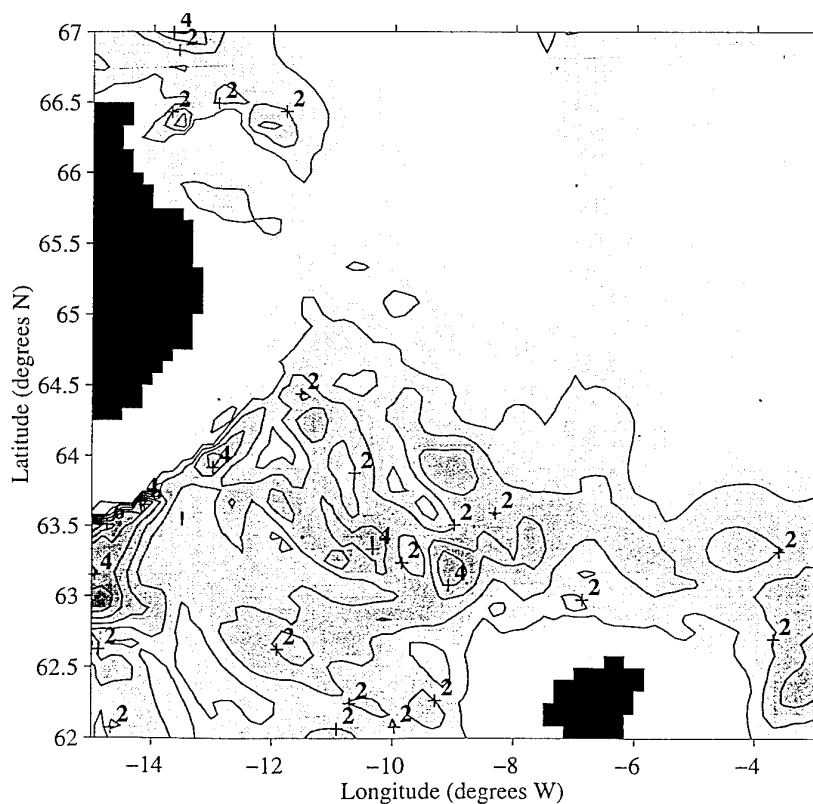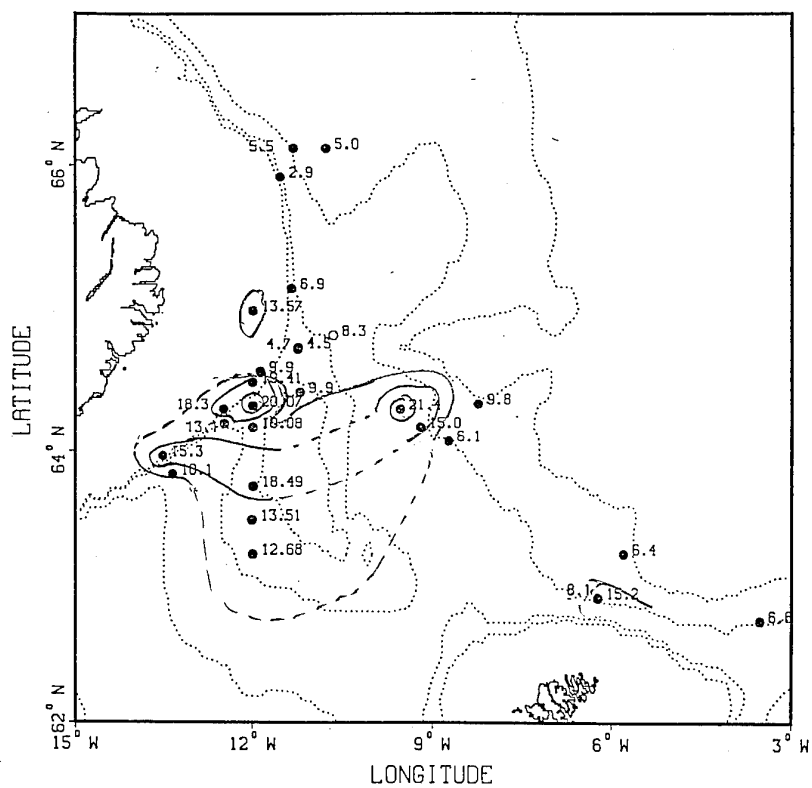|	
NW corner	67°N 20°W
SE corner	59°N 0°W
Number of grid points (x×y×z)	121×97×11
Δ_X (longitudinal grid spacing)	5'
Δ_Y (latitudinal grid spacing)	10'
Δ_Z (vertical grid spacing, from surface)	50, 50, 50, 100, 100, 100, 100, 100, 300, 300, 1500 m
Δ_t (time interval)	900 s
A_H	5×10^6 cm^2 s^{-1}
A_V	1 cm^2 s^{-1}
K_H	1×10^6 cm^2 s^{-1}
K_V	1 cm^2 s^{-1}

Figure 1. Estimates of r for the internal tidal constituent, M2, in cms⁻¹.

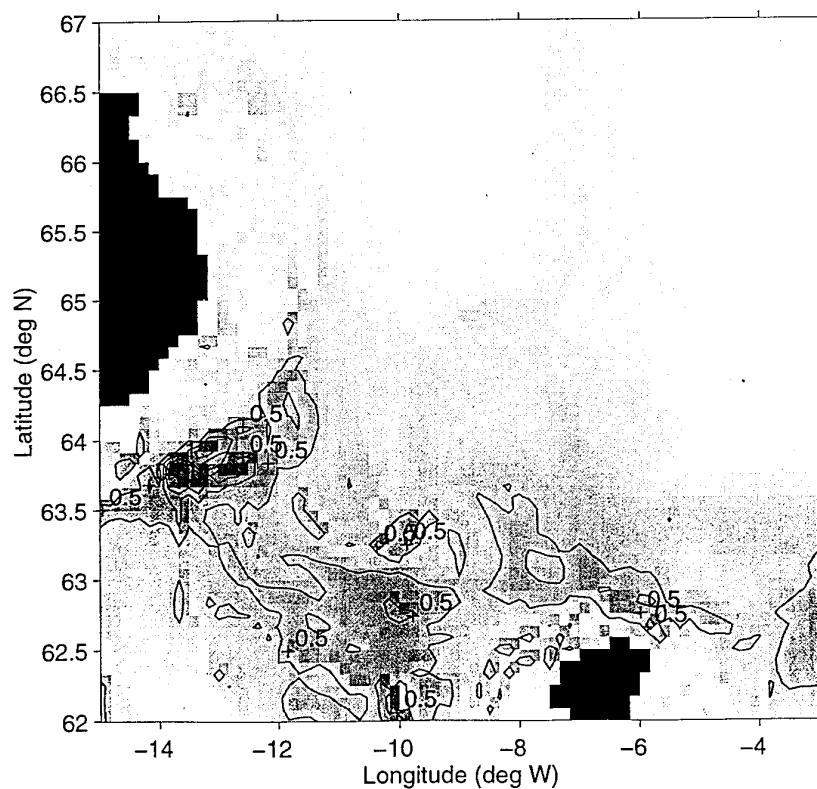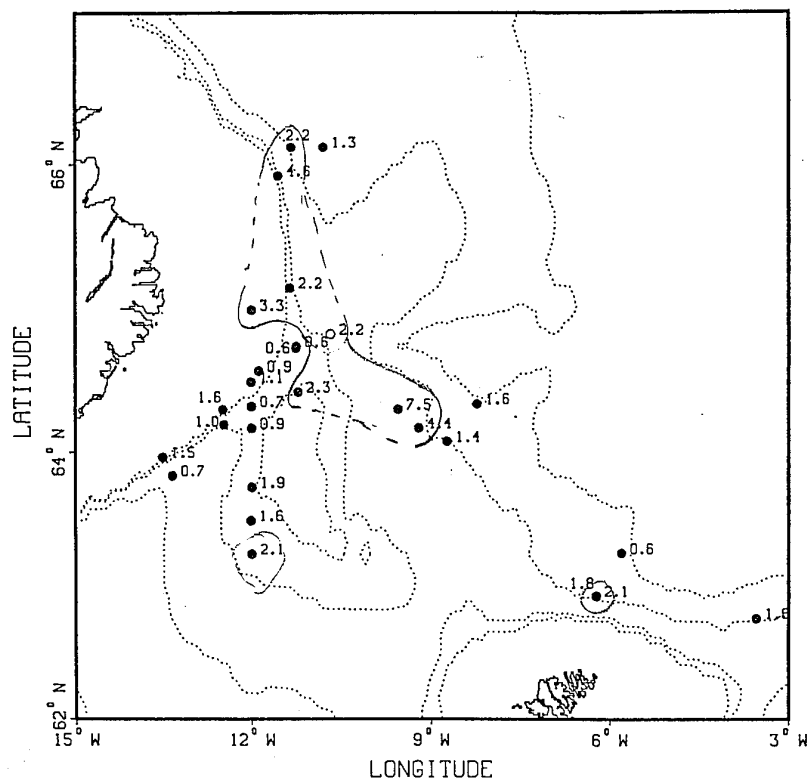a) from observations; b) from model predictions.

Figure 2. Estimates of r for the internal tidal constituent, O1, in cms^{-1}.

a) from observations; b) from model predictions.

9

APPENDIX

To Model Tidal Structure in the Shallow Water of the Iceland-Faeroes Ridge Region

Progress Rpt No 2

Project UCES 105

ONR Grant N00014-93-1-6011

Dr T.J. Sherwin, UCES, Marine Science Labs, Menai Bridge, Gwynedd, UK

1. Introduction

The main objective of this work is to investigate the distribution of internal tide energy over the Iceland-Faeroes ridge, using (i) the extensive existing current meter data set available and (ii) predictions from the UCES two-dimensional linear slice model of internal tide generation. To date we are close to finishing the first part of the work and this report gives a summary of the observed distribution of two semi-diurnal constituents (M2 and S2) and two diurnal constituents (K1 and O1). We are about to start the second stage of the work which involves explaining the distribution using the linear model. The final phase of the work will be the preparation of a report, the main part of which will be published in a scientific journal.

2. Data Acquisition

The observations described below all come from the SACLANT archive. We have trawled the relevant data banks (principally the British Oceanographic Data Centre) for archived current meter data of the region. In the event only a small number of useful current meter sites have been located - in the archives of the Bundesant für Seeschiffahrt und Hydrographie, Hamburg. These observations were made in 1973 using Richardson current meters and are of variable quality. So far they have not been included in the analysis although they cover the south eastern half of the ridge for which there is no other data.

3. Data Evaluation

The SACLANT current meter data archive have been analysed by our partners, under Dr H Perkins, and his results are discussed below. The method of analysis was as follows: depth integrated (barotropic) currents were computed at each site and compared with the predictions from the tidal model due to R.A. Flather. From this we derived sufficient confidence in the Flather model to use it as a reliable estimate of barotropic tidal currents over the whole domain (Fig. 1). This was necessary because on some rigs there existed too few current meters to justify the computation of a depth mean. The Flather tidal currents were then subtracted from the individual current meter time series and a tidal analysis was conducted on these residual time series to produce individual estimates of the internal tide for each of the constituents, M2, S2, O1 and K1.

From these constituents estimates of the internal tide activity, r, have been calculated as

$$r^2 = \frac{1}{2N} \sum_{1}^{N} \left(u^2 + v^2 \right)$$

where N is the number of current meters on a rig, and u and v are the amplitude of the internal tidal current in the east and north directions respectively. Values of r are shown in Figs 2 to 5.

There is a clear difference in the spatial distribution of activity between the super-inertial, semi-diurnal constituents (M2 and S2) and the sub-inertial, diurnal ones (K1 and O1). The peaks in activity of M2 and S2 are found in the region where the ridge meets the Icelandic shelf and to some extent along both flanks of the ridge on its shallower northern end (Figs 2 and 3). By contrast K1 has relatively high values around the Icelandic shelf, with some outlying regions of higher activity on both flanks of the ridge (Fig. 3); and the O1 internal tide is mainly located on the eastern side of Iceland and the northern flank of the ridge (Fig. 4).

There are probably two explanations for this variability which will be investigated in the next part of this work. One reason is that the spatial distribution of the barotropic tide (which is the forcing agent) appears to vary in a way which is compatible with the internal tide observations (see Fig. 1). It is particularly noticeable that the internal tide distributions of K1 and O1 match their barotropic counterparts quite well. The other explanation concerns the fact that on a rotating Earth super-inertial waves can propagate into the ocean away from their source whereas sub-inertial ones remain trapped to the topography. Perkins *et al.* (J.P.O., 1994) have shown that the internal tide generated at the edge of the Iceland shelf has the potential to propagate a significant way across the ridge.

4.   Future Work

It is planned to compliment the analysis of the observation with a series of two-dimensional internal tide model runs along sections that will make it possible to map the distribution over most of the region shown in the figures. If these mapped predictions agree with the distribution seen in the observations then we will have confidence in the estimates thus produced and this may provide the stimulus to conduct a similar analysis in other shelf edge and shelf regions.
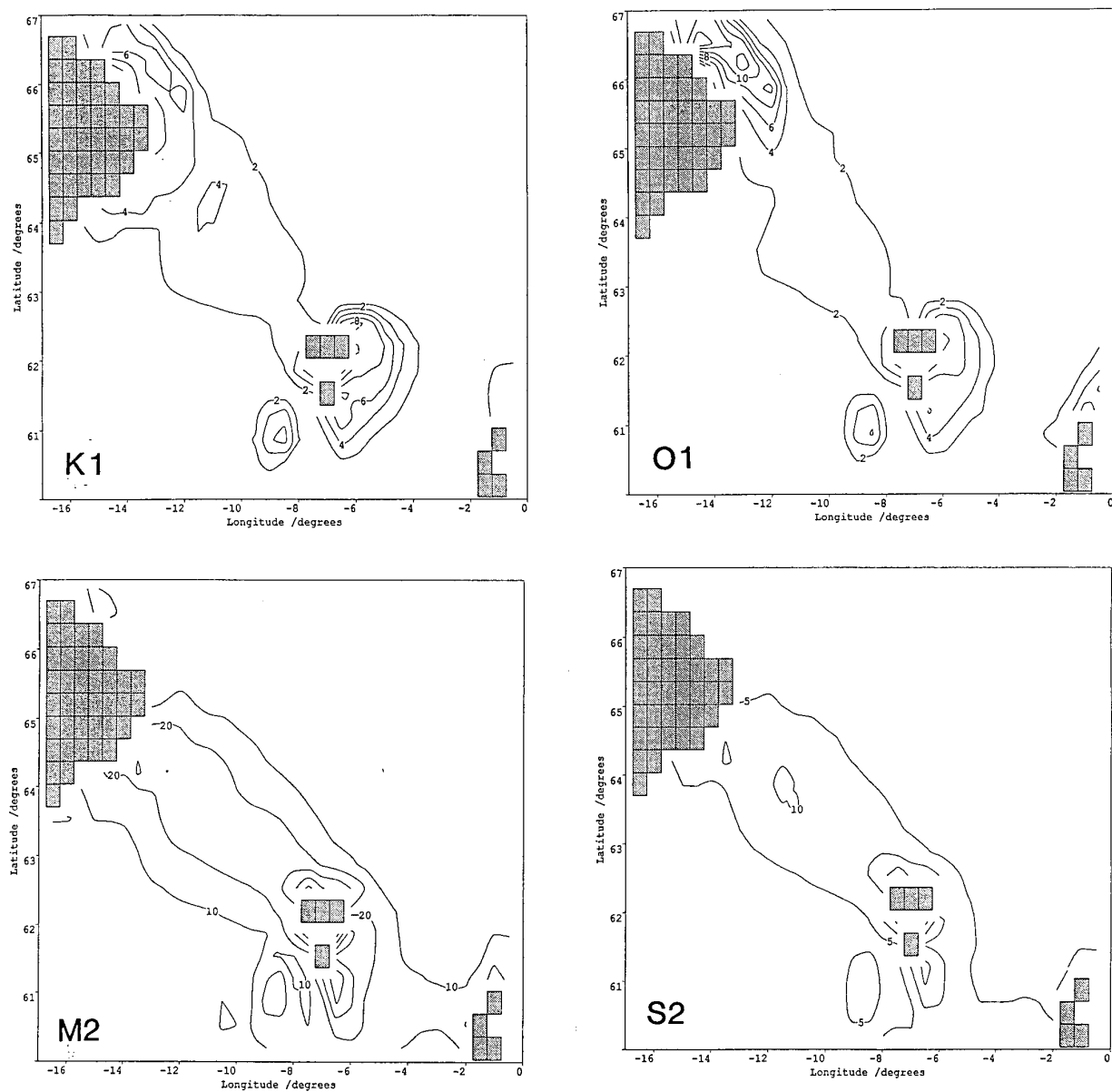

2nd June 1994

Figure 1. The amplitude of the eastward component of current velocity for the four major tidal constituents over the Iceland-Faeroes ridge (K1, O1, M2 and S2) from the model due to R.A. Flather. Units are in cms s⁻¹, but note that the contour intervals vary between the figures. The northward currents show a similar distribution.
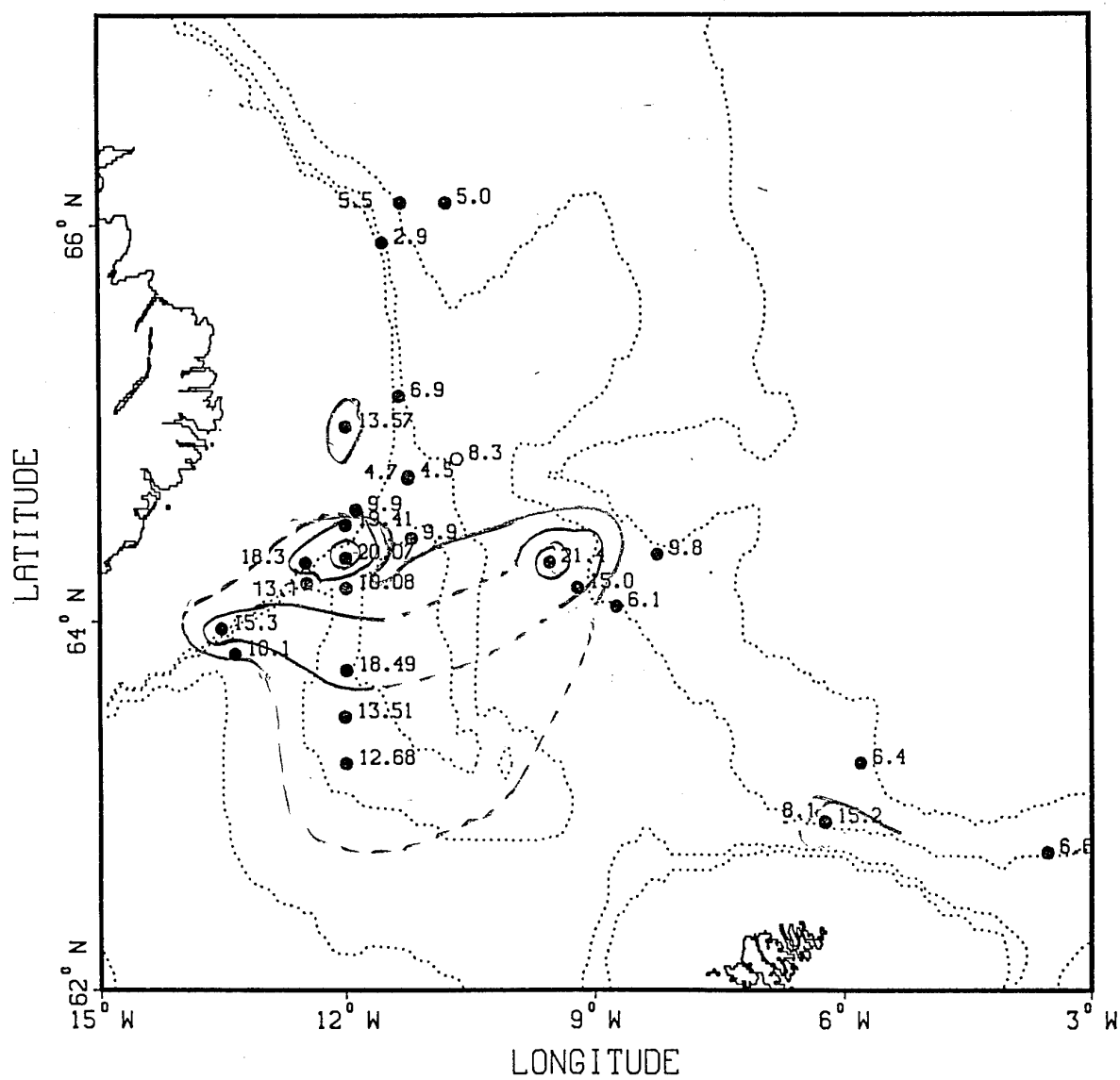
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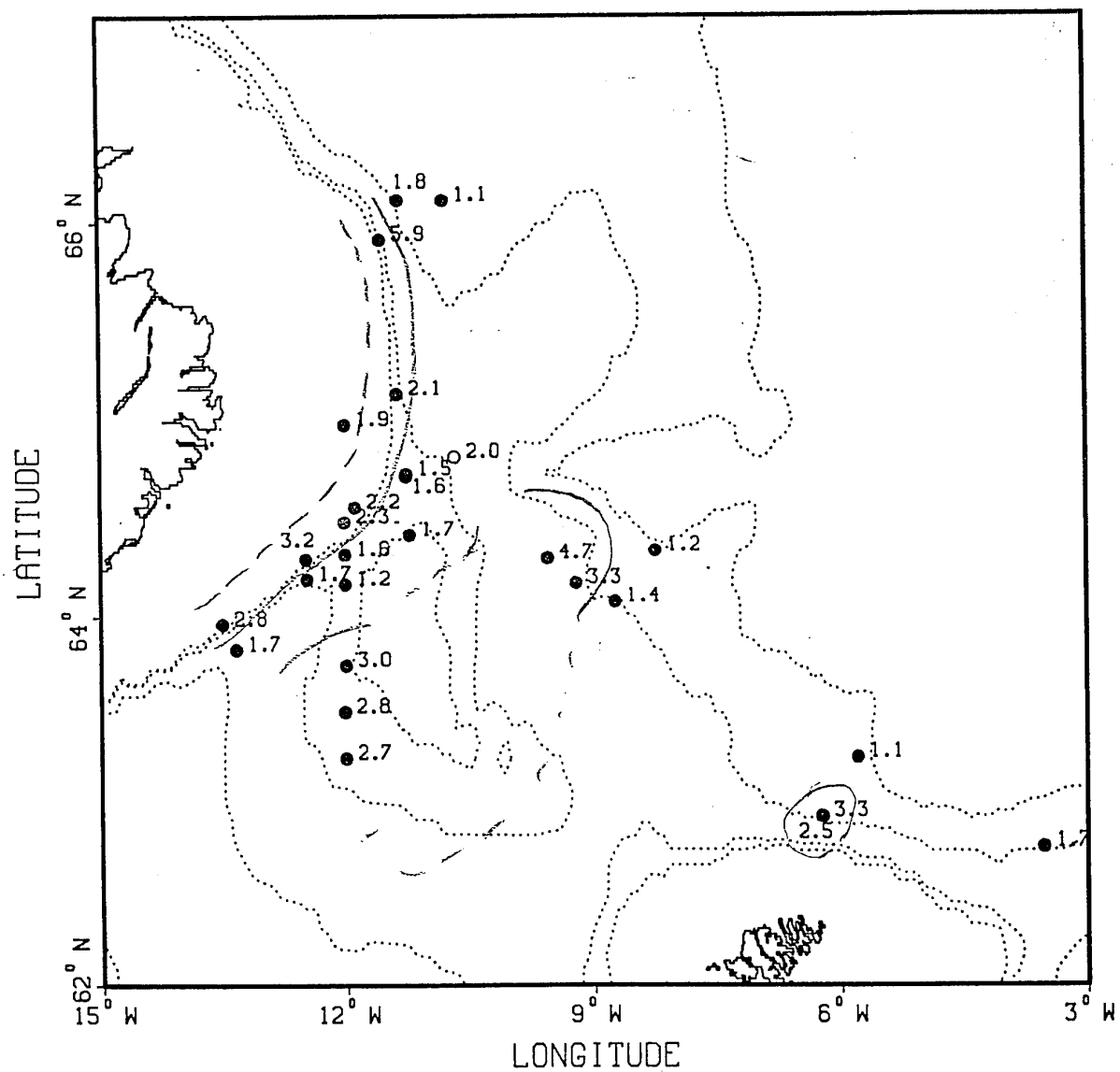**Figure 2.** Estimates of r for the internal tidal constituent, M2. Units are in cms s^{-1}.

Figure 3. Estimates of r for the internal tidal constituent, S2. Units are in cms s^{-1}.

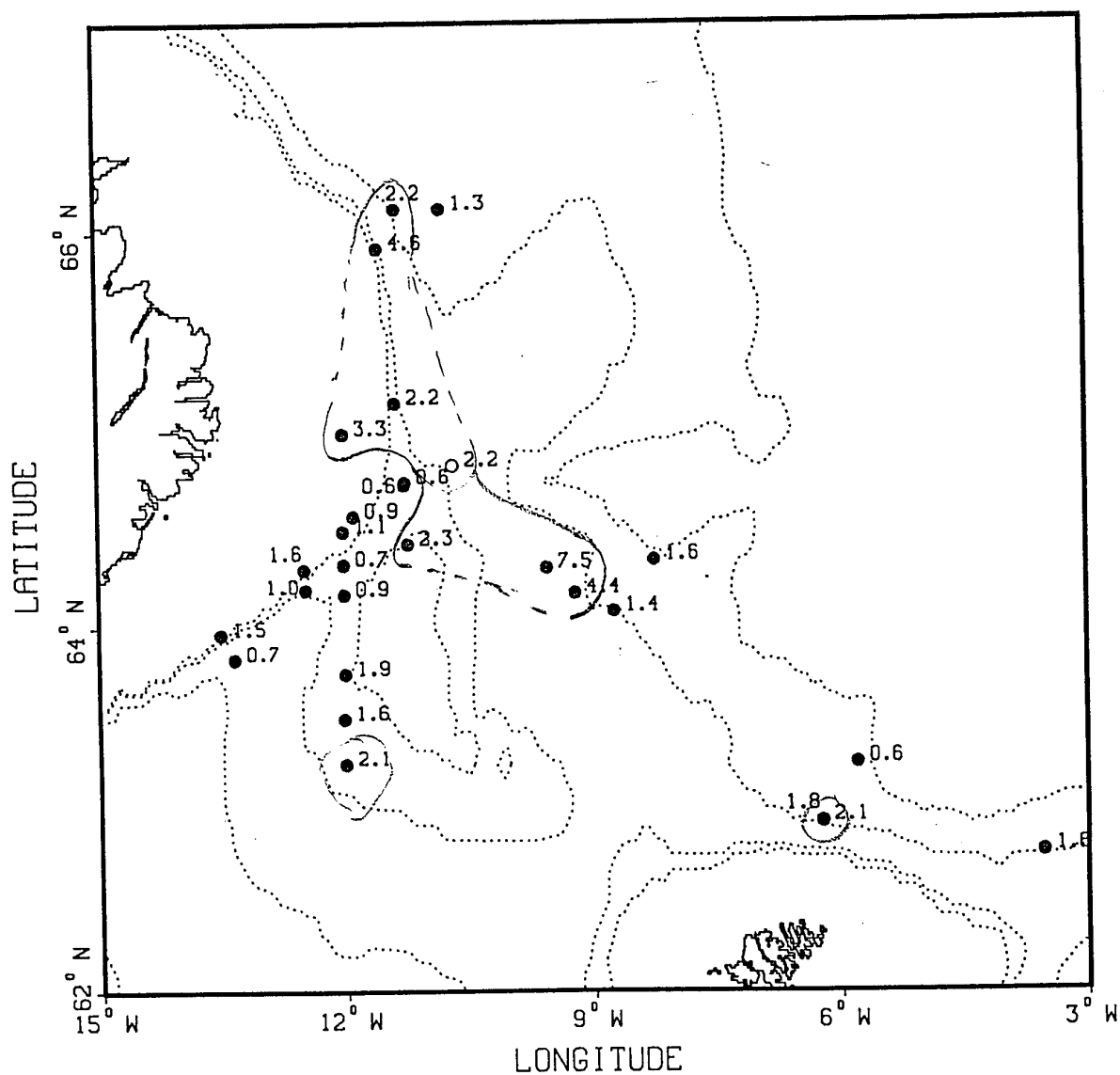**Figure 4.** Estimates of r for the internal tidal constituent, K1. Units are in cms s^{-1}.

Figure 5. Estimates of r for the internal tidal constituent, O1. Units are in cms s⁻¹.