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## FINAL TECHNICAL REPORT AFOSR Grant 94-1-0091 Rarefied Gas Dynamics and Monte Carlo Methods

## Russel E. Caflisch, PI Mathematics Department, UCLA

## May 19, 1995

The research supported by this grant was primarily on quasi-random sequences, which are a deterministic alternative to random or pseudo-random sequences, and their use in Monte Carlo methods. These methods are vastly superior to standard Monte Carlo, at least in principle, because the errors are of size  $O(N^{-1})$  rather than the usual  $O(N^{-1/2})$ . On the other hand, this gain in accuracy can be lost if the domain of integration is of high dimension or the function to be integrated is not smooth. In joint work with Bradley Moskowitz, we succeeded in overcoming these limitations for a number of Monte Carlo methods, including the acceptance-rejection method, Feynman-Kac integrals and diffusion Monte Carlo.

In particular we developed the following new method that employ quasirandom sequences:

(1) A smoothed acceptance-rejection method.

(2) A rearrangement of the Feynman-Kac integral using the Brownian bridge.

(3) A hybrid method for diffusion Monte Carlo, using pseudo-random to get a good approximation to the solution, then quasi-Monte Carlo to reduce the statistical error.

Funds from this grant were used to support Bradley Moskowitz as a postdoctoral researcher and a visitor, Giovanni Russo. The duration of this grant was one year.

## Publications Supported by This Grant

1. B. Moskowitz, "Improved Stochastic Simulation Using Quasi-Monte Carlo: A Computational Study" UCLA CAM Report 94-40

2. B. Moskowitz, "Quasirandom Diffusion Monte Carlo" UCLA CAM Report 94-39

3. B. Moskowitz and R. Caflisch, "Smoothness and Dimension Reduction in Quasi-Monte Carlo Methods" UCLA CAM Report 94-17, to appear in J. Math. Comp. Modeling

Publications Supported by Previous AFOSR Grants that Appeared During this Period

1. W. Morokoff and R. Caflisch "A Quasi-Monte Carlo Approach to Particle Simulation of the Heat Equation" SIAM J. Num. Anal. 30 (1993) 1558-1573.

2. W. Morokoff and R. Caflisch "Quasi-Random Sequences and Their Discrepancies" SIAM J. Sci. Stat. Comp. 15 (1994) 1251-1279.