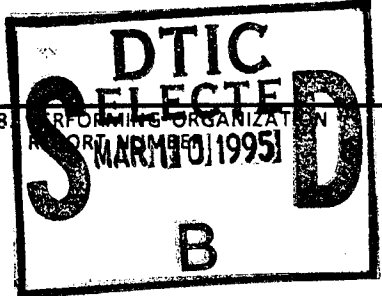



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STOCHASTIC CONTROL AND TOPICS IN APPLIED PROBABILITY  
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1 June 1991 - 30 November 1994

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FINAL REPORT  
15 January 1995

We outline our research findings, achieved during the last three years under the auspices of the above grant, on the topics of

- (i) Stochastic control under partial observations;
- (ii) Dynamic Allocation and Multi-armed Bandit Problems;
- (iii) Deterministic and anticipative aspects of stochastic optimization;
- (iv) Predictable representation properties for semimartingales.
- (v) The stochastic version of Pontryagin's maximum principle;
- (vi) Representation of additive functionals of Markov processes;

On the first topic, we focused on **Adaptive Control Problems of the Bayesian type** which can be formulated equivalently as stochastic control problems with partial observations. Roughly speaking, one tries to control the state of a system to a desired goal by optimizing a certain performance index in the presence of unobservable parameters in the plant equations and subject to random disturbances. These parameters are modelled by random variables with known "prior" distributions. A natural question arises then: can one obtain optimal control laws in this setup by simply plugging least-squares estimates of the unobserved parameters into the formulae for the optimal laws, which are valid for the system with all the parameters known? This "certainty-equivalence" (or separation) principle is well-known to hold for linear-quadratic-gaussian systems.

A significant thrust of our research has focused on exploring the validity of this very appealing principle for other, more complicated dynamics and cost structures, as well as for constrained control sets (cf. Benes et al. (1991), Karatzas & Ocone (1992), (1993)). The analytical problems one encounters are quite challenging, and have led us to the resolution of very interesting questions in the theory of random processes and in fully nonlinear partial differential equations of the second order.

The so-called **Multi-armed Bandit**, or **Dynamic Allocation**, problem, on the other hand, "is important as one of the simplest nontrivial situations on which one must face a conflict between taking actions which yield immediate reward, and taking actions (such as acquiring information, or preparing the ground) whose benefit will come only later" (P. Whittle (1980)). It has become a classic, not least through the pioneering work of J. Gittins and his collaborators who managed to crack open the infinite-horizon, discrete-time, Markovian version of this problem. In the papers El Karoui & Karatzas (1993a,b) we studied the general problem of optimal stopping in such detail as to permit a very simple probabilistic resolution of the general (non-Markovian) discrete-time dynamic allocation problem, in the spirit of Gittins-Whittle. We then extended these methodologies, in order to deal with the far more difficult continuous-time version of this problem, in El Karoui & Karatzas (1994), (1995). We obtained a very simple resolution of the problem in the special case of "decreasing rewards", and used refined tools from the stochastic analysis of multi-parameter processes, stopping times and martingales, to reduce the general case to this simple one, via the lower-envelopes of Gittins index processes. We also explored the implications of this approach in the study of diffusion processes with singular drift coefficients, such as "skewed Brownian motions".

In the paper Davis & Karatzas (1994) we offered a **deterministic approach to the problem of optimal stopping**, which reduces this difficult problem to pathwise maximization for a modified reward process. The modification is described in terms of a process  $L(\cdot)$  that plays the role of a Lagrange multiplier (enforcing the non-anticipativity constraint corresponding to the definition of stopping time) and which, at any given time, is given as  $L(t) = M(T) - M(t)$ , where  $T$  is the terminal time, and  $M(\cdot)$  is the martingale in the Doob-Meyer decomposition  $Z=M-A$  for the Snell envelope  $Z(\cdot)$  of the reward process  $Y(\cdot)$ . This idea works in both discrete- and continuous-time; in the former case, it yields a very simple proof for the famous "prophet inequality" of Krengel & Sucheston. The doctoral student I. Pikovsky is working on extending this idea to more general stochastic control problems, and on its connections with anticipative stochastic analysis and optimization (cf. Accardi & Pikovsky (1994) for a very general, unified approach to non-anticipative stochastic integration, based on ideas of Quantum Probability and on the Malliavin Calculus; and Pikovsky & Karatzas (1994), for additional aspects of stochastic control with side-information, or "anticipation").

Much of our research in optimal stopping and stochastic control has been the subject of considerable interest here at Columbia, on the part of our strong graduate students. Prompted by this, we have offered advanced doctoral courses on the Probabilistic Aspects of Optimal Stopping and Control and tried to compile as complete a collection of Lecture Notes as was possible (Karatzas (1993)); our hope is that they will form the nucleus of a monograph or textbook.

Along a different tack, we studied with the former doctoral student Abel Cadenillas the **Stochastic Maximum Principle** for control problems with linear dynamics, general random (adapted) coefficients, and convex but quite singular cost criteria; see Cadenillas (1992), Cadenillas & Karatzas (1995). Without imposing  $L_p$ -bounds on the controls, we solved explicitly for the associated adjoint equation and succeeded in obtaining integral and local versions of the stochastic maximum principle; these led, in turn, both to necessary and sufficient conditions for optimality of a control.

The version of the stochastic maximum principle in these references is the only one, to our knowledge, that covers the consumption/investment problem. When applied to problems like the Predicted-Miss or the Linear-Regulator, it gives results stronger than those possible with previous versions of the principle.

The work of the former graduate student and post-doctoral associate X. Xue (1994) on the martingale representation property for the filtration of two independent semimartingales, is an outgrowth of his doctoral dissertation here at Columbia. Finally, the paper Hoehnle & Sturm (1994) of the visiting research associate R. Hoehnle, extends, to general Markov processes, earlier work of X. Xue on the additive functionals of the Bessel process.

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