NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

**AN OPTIMAL ALLOCATION OF
ARMY RECRUITING STATIONS WITH
ACTIVE AND RESERVE RECRUITERS**

by

Michael J. Teague

June, 1994

Thesis Advisor: Siriphong Lawphongpanich

Unclassified
Security Classification of this page

REPORT DOCUMENTATION PAGE

1a Report Security Classification: Unclassified		1b Restrictive Markings			
2a Security Classification Authority		3 Distribution/Availability of Report Approved for public release; distribution is unlimited.			
2b Declassification/Downgrading Schedule					
4 Performing Organization Report Number(s)		5 Monitoring Organization Report Number(s)			
6a Name of Performing Organization Naval Postgraduate School	6b Office Symbol *(if applicable)* *OR	7a Name of Monitoring Organization Naval Postgraduate School			
6c Address *(city, state, and ZIP code)* Monterey CA 93943-5000		7b Address *(city, state, and ZIP code)* Monterey CA 93943-5000			
8a Name of Funding/Sponsoring Organization	6b Office Symbol *(if applicable)*	9 Procurement Instrument Identification Number			
Address *(city, state, and ZIP code)*		10 Source of Funding Numbers			
		Program Element No	Project No	Task No	Work Unit Accession No

11 Title *(include security classification)* AN OPTIMAL ALLOCATION OF ARMY RECRUITING STATIONS WITH ACTIVE AND RESERVE RECRUITERS

12 Personal Author(s) Michael J. Teague

13a Type of Report Master's Thesis	13b Time Covered From To	14 Date of Report *(year, month, day)* 14 Jun 94	15 Page Count 81

16 Supplementary Notation The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 Cosati Codes			18 Subject Terms *(continue on reverse if necessary and identify by block number)*
Field	Group	Subgroup	Data Envelopment Analysis (DEA), Poisson Regression, United States Army Recruiting Command, USAREC, Nonlinear Integer Problem,

19 Abstract *(continue on reverse if necessary and identify by block number)*

This thesis addresses the problem of how to locate and staff recruiting stations with Active and Reserve recruiters in order to maximize the annual number of recruits. The problem is formulated as a nonlinear integer programming problem. The objective function for the problem, also referred to as the production function, describes the number of recruits obtainable from each zip code and can be estimated via Poisson regression. The resulting nonlinear integer programming problem is heuristically solved by decomposing decision variables into two sets: one to locate stations and the other to staff them with recruiters. Comparisons are made between problems with production functions derived from all zip codes and those derived from only zip codes belonging to efficient stations as defined in Data Envelopment Analysis.

20 Distribution/Availability of Abstract xx unclassified/unlimited ___ same as report ___ DTIC users	21 Abstract Security Classification Unclassified	
22a Name of Responsible Individual Siriphong Lawphongpanich	22b Telephone *(include Area Code)* (408) 656-2106	22c Office Symbol OR/Lp

DD FORM 1473,84 MAR 83 APR edition may be used until exhausted security classification of this page

All other editions are obsolete Unclassified

An Optimal Allocation of Army Recruiting Stations with Active and Reserve Recruiters

by

Michael J. Teague
Captain, United States Army
B.S., Norwich University, 1985

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
June, 1994

Author: ______________________
Michael J. Teague

Approved by: ______________________
Siriphong Lawphongpanich, Thesis Advisor

____
David Morton, Second Reader

____
Peter Purdue, Chairman
Department of Operations Research

ABSTRACT

This thesis addresses the problem of how to locate and staff recruiting stations with Active and Reserve recruiters in order to maximize the annual number of recruits. The problem is formulated as a nonlinear integer programming problem. The objective function for the problem, also referred to as the production function, describes the number of recruits obtainable from each zip code and can be estimated via Poisson regression. The resulting nonlinear integer programming problem is heuristically solved by decomposing decision variables into two sets: one to locate stations and the other to staff them with recruiters. Comparisons are made between problems with production functions derived from all zip codes and those derived from only zip codes belonging to efficient stations as defined in Data Envelopment Analysis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGEMENTS

This thesis has provided a tremendous opportunity to use the skills and education that I have gained over the past two years. However, it would not have been completed without a great deal of assistance and support from some very special people who should be recognized here.

First, my wife, Dawn, who has always supported me throughout this roller coaster of the past six months. You have put up with a lot and always been the positive. I love you.

Second, to my wonderful children who have given up their soccer and baseball time. I promise to make it up this summer.

Finally, to Professor Lawphongpanich. Sir this thesis would not have been completed on time without your dedication and experience. For all of the late nights and corrected drafts thank you.

EXECUTIVE SUMMARY

To support the ongoing drawdown by the Department of Defense, the US Army Recruiting Command (USAREC) is in the process of realigning its organizational structure for recruiting young men and women to join the Army. Of great concern is the question of which stations are to be closed and how to staff the remaining stations with recruiters for both the Active and Reserve components. To aid in this decision making, this thesis develops an optimization model that takes as its inputs the number of stations and the numbers of Active and Reserve recruiters available to a recruiting battalion. Its output consists of a list of stations to remain open and the corresponding number of Active and Reserve recruiters to staff each of them.

An integral part of the optimization model is the production function which describes the expected number of recruits obtainable from each zip code. This production function is not known with certainty and has to be estimated using a statistical technique called Poisson regression. To observe the difference in the annual production of recruits under the assumption that all recruiters operate in an efficient manner, two types of production functions, average and efficient, are considered. The average production function is based on data from all zip codes and the efficient one is based on data from zip codes belonging to efficient stations. The thesis uses Data Envelopment Analysis to determine which stations are efficient.

To illustrate its utilities, the model was used to locate and staff stations in the Albany Battalion with recruiters. It was also observed that a significant number of recruits can be obtained if all recruiters are efficient. Although it is optimistic to make such an assumption, results from the model with efficient production functions can serve as a goal all recruiters should strive to achieve, especially in the current budget environment.

I. INTRODUCTION

After forty years of Cold War, when the missions and challenges facing the US Armed Forces were clearly defined and easily understood we find ourselves in a period of unprecedented change. An increased demand for social and domestic improvement has replaced the dissipating threat of the Warsaw Pact. This change in focus brought about a corresponding shift of resources, with the Department of Defense being a major target for reductions. These reductions affect the number of personnel, the operational funds, and the development and acquisition of weapon systems. While the recent number of regional conflicts and humanitarian missions indicate that the world remains volatile, the reductions will continue.

The US Army is the most people intensive of all of the Armed Services and therefore implementing the personnel drawdown is a point of great concern. To prevent the development of a hollow force the drawdown has not been accomplished solely through reduced accessions, but rather by making reductions at every level, using a variety of incentive and control programs. The budget cuts have been felt through the entire force, compelling every unit and organization to become more efficient: being able to do more with less.

A. BACKGROUND

The drawdown affects the US Army Recruiting Command (USAREC) in several ways. USAREC's primary mission is to recruit young men and women, mainly between the ages of 17 and 21, to join the Army. The current downsizing has reduced the requirement for new Army recruits from 127,100 in FY92 to 75,000 in FY93. For the current fiscal year, as well as next year, USAREC is required to produce 70,000 enlistments for the Active Army and 46,000 for the Reserves. This reduction has been accompanied by smaller recruiting and advertising budgets as well as a smaller recruiting force, marked by the elimination of 1,100 recruiters in 1993 alone [Ref. 1]. Meanwhile, colleges and other civilian job training institutions have increased their recruiting efforts as the population of 17-21 year old individuals is projected to decline by six percent from 1990 to 1995[Ref. 2]. In addition, today's emerging weapon technologies demand high quality and more capable recruits. These two factors combine to shrink the pool of possible recruits for USAREC. Compounding this unfavorable situation is the downward shift in the attitude of youths toward a career in the military. During the past three years, there has been a 31% decrease in the propensity of young men and women to join the military [Ref. 1]. This decline can be attributed to the publicity surrounding the continued drawdown, the recent Gulf War and US military involvement in Somalia, and other social and economic factors. In order to maintain its competitive advantage over other services and civilian organizations in recruiting young men and women, USAREC must become as efficient as possible in every facet of its operations.

B. PROBLEM DEFINITION

In recruiting, one of the most important resources are recruiters, for they generate enlistment contracts for the Army. Therefore, it is important that USAREC provides sufficient support for recruiters to perform their duty in the most effective and efficient manner possible. In particular, USAREC views recruiting stations as an important resource for its recruiters and success in recruiting depends in part on the placement and staffing of these stations. A recruiting station provides space for conducting business and a homebase for recruiters. Moreover, the presence of a recruiting station also serves as an important patriotic reminder in the surrounding community and in some cases attracts youths to join the Army. Therefore, USAREC is interested in determining optimal locations and staffing levels for its stations.

C. APPROACH

This thesis addresses the problem of determining the locations and staffing levels of Army recruiting stations in a manner similar to Schwartz [Ref. 3]. The thesis formulates the problem as an optimization model with the objective of maximizing the total number of yearly enlistments which is statistically estimated from historical data. However, this thesis differs from Schwartz in three critical respects. First, Schwartz addressed the problem for the Navy Recruiting Command which only recruits for the Active component of the Navy. However, USAREC recruits for both the Active and Reserve components of the Army and the model in this thesis addresses both of them. The Reserve component presents additional complexity in that recruits joining the Army

Reserve must reside with a 50 mile radius of his/her assigned Reserve Center. In addition, recruiters for the Active and Reserve do not necessarily share the same recruiting territories. In fact, Reserve recruiters generally must cover more area since there are fewer of them to cover the continental United States. Second, Data Envelopment Analysis (DEA) [Ref. 4] is used to focus the estimation of the annual enlistments on efficient use of resources. Finally, this thesis also employs Poisson regression instead of least squares regression to predict the number of yearly enlistments.

D. THESIS OUTLINE

In order to allow a thorough understanding of the underlying rationale used in the selection of certain techniques and specific explanatory variables, a description of USAREC's organization and current operations is included in Chapter II. Chapter III describes and formulates the Army Location Allocation optimization problem. The objective function for the problem, also referred to as the production function, describes the number of recruits expected from a zip code. Since this function is not known with certainty, Chapter IV uses Poisson regression to estimate it. Using DEA to determine which recruiting stations are efficient, this chapter concludes with an analysis of two different production functions: one using all zip codes and the other using only those zip codes that belong to an efficient station. With these production functions, the optimization problem in Chapter III is a nonlinear integer program, a difficult class of problems to solve. As an alternative, Chapter V develops a decomposition approach to produce near optimal solutions. Chapter V also presents the implementation of the decomposition technique and analyzes the resulting realignment for the Albany Recruiting Battalion. Finally, Chapter VI summarizes the thesis and suggest possible areas for future research.

II. RECRUITING AT USAREC

This chapter consists of two sections that provide basic information about recruiting in the United States Army Recruiting Command (USAREC). The first section provides historical information, organization, and structure. The second section describes the recruiting operations as they pertain to the problem outlined in Chapter I.

A. ORGANIZATION AND STRUCTURE

USAREC is the proponent organization for recruiting young men and women into the Active and Reserve Components of the Army and, as such, it is responsible for one of the most critical missions of any organization in the Army. It is one of the very few organizations that executes its wartime mission on a daily basis. In addition to recruiting into the enlisted ranks of the Regular Army (RA) and US Army Reserve (USAR) units, USAREC is also responsible for recruiting candidates for other programs such as Officer Candidate School (OCS), Warrant Officer Flight Training (WOFT), and Army Nurse Corps (ANC).

In December 1963, a committee commissioned to study all aspects of recruiting for the Army found that the organizational structure for recruiting had major inconsistencies and was ineffective. As a result, the US Army Recruiting Service was established in 1964. The organization's mission also underwent a major revision in the early 1970s when the draft ended and an all volunteer force was implemented. This transition brought about significant changes in the focus of the entire recruiting process. In 1978, at the direction of the Vice Chief of Staff of the Army, USAREC also assumed the mission of recruiting for the Army Reserve and became the Total Army's recruiting organization. Currently, USAREC is a field operating agency under the Office of the Deputy Chief of Staff for Personnel. In 1993, the Headquarters moved from Fort Sheridan, Illinois to its current location at Fort Knox, Kentucky. The current organizational structure of USAREC is presented in Figure 1 [Ref. 5]. The different elements of the organization will be explained in the subsections below.

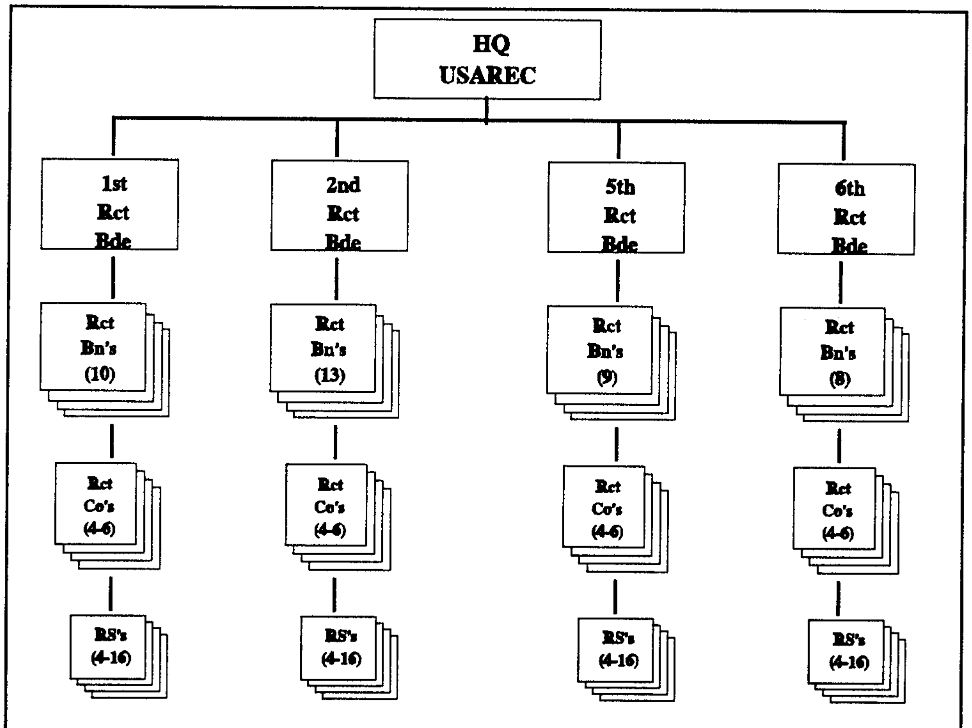

Figure 1. USAREC Organization

1. Headquarters, USAREC

Although the mission of USAREC is significantly different from any other Army organization, the headquarters and staff operate in much the same manner as any major unit. USAREC is commanded by a major general with a deputy commander who is a brigadier general and oversees the operations of the Recruiting Brigades. The staff is coordinated and led by the Deputy Commander/Chief of Staff and it consists of nine major directorates. The organization of the Headquarters is shown in Figure 2 [Ref. 6].

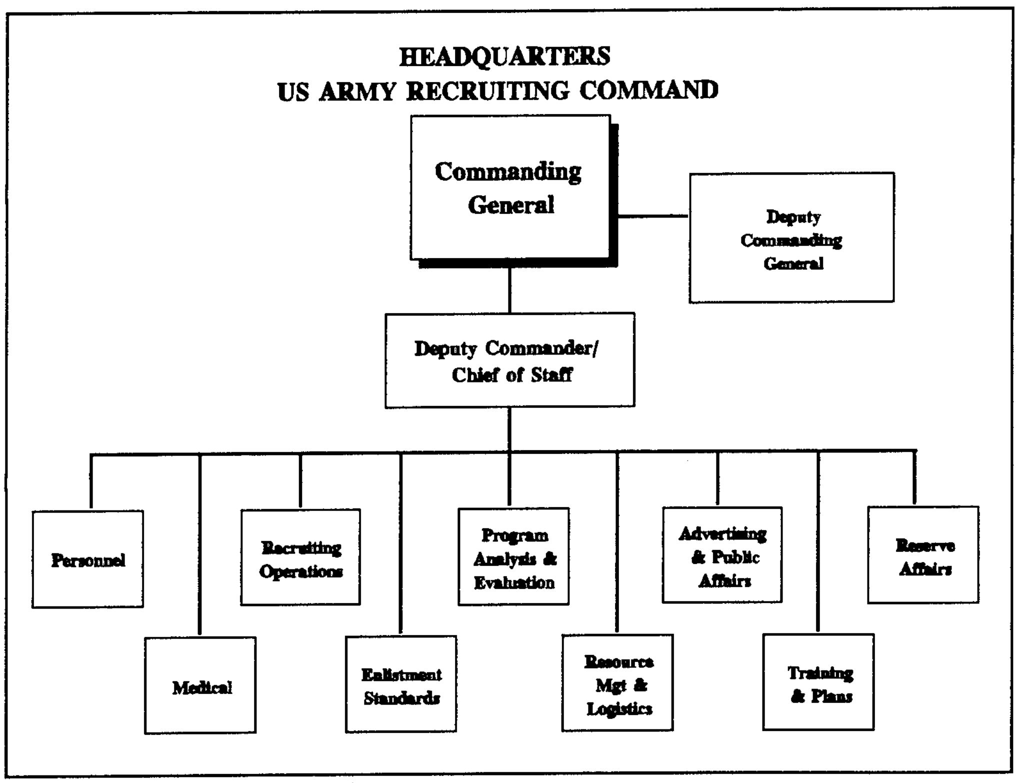

Figure 2. USAREC Headquarters

The missions of the directorates involve analyzing, resourcing, and executing the current annual recruiting mission. The staff is also involved in the long range planning of the entire organization. Of special note is the Program Analysis and Evaluation Directorate (PA&E); it is responsible, among many other tasks, for conducting analysis that will ensure that all recruiters have the market available to accomplish their assigned mission. PA&E provided much of the data used in this thesis and are also the intended end user of the methodology presented here.

2. Recruiting Brigades (Rct Bdes)

There are currently four Recruiting Brigades dispersed across the country. Their locations are shown in Table 1. Each of the brigades is commanded by a Colonel. Although the brigade staffs are not as large as the Headquarters', they still conduct a great deal of short term planning and analysis in order to accomplish their specific missions.

TABLE I. RECRUITING BRIGADE LOCATIONS

1st Recruiting Brigade (Northeast)	Ft. Meade, MD
2nd Recruiting Brigade (Southeast)	Ft. Gillem, GA
5th Recruiting Brigade (Southwest)	Ft. Sam Houston, TX
6th Recruiting Brigade (West)	Ft. Baker, CA

The brigade staff includes two very important branches that do not exist separately below the Rct Bde level: the Market Analysis Branch and the ANC Recruiting Branch. The Market Analysis Branch dispatches teams to conduct market studies (recruiter zone analyses or RZAs) that determine the boundaries of a particular recruiting station's territory. Some of the historical data used in this thesis are drawn from these studies.

The primary purpose of the Rct Bdes is to synchronize the plans and actions among the Recruiting Battalions under its control. Under the current alignment, a brigade is responsible for eight to thirteen battalions.

3. Recruiting Battalions (Rct Bns)

There are currently 40 Rct Bns located in the Continental United States (CONUS) and they are predominantly commanded by Lieutenant Colonels. The Rct Bn staffs are much smaller than those of the Rct Bdes, and are designed to deal with only near term planning and execution. The Rct Bns provide the lowest level dedicated planning organization within USAREC. Each Rct Bn controls between four and six companies.

4. Recruiting Companies (Rct Cos)

There are currently 216 Rct Cos commanded by Captains who have all had previous command experience. These command and control organizations are critical due to the dispersion of the recruiting stations. An average Rct Co covers an area of approximately 10,000 square miles. The Rct Cos represent the link between the policies and programs of USAREC and the recruiters at the stations. Their focus is on mission accomplishment and on recruiter training. Each Rct Co is assigned four to sixteen recruiting stations.

5. Recruiting Stations (RS)

There are currently 1,466 recruiting stations located throughout the United States and many of its territories. Typically located in high traffic commercial areas (shopping malls and office buildings), they are essentially the liaison between the Army and the civilian community. The number of recruiters assigned to a station varies between one and nine. A recruiter either recruits for the Active (RA) or Reserve (USAR) component, but not both. Some stations also have recruiters whose primary mission is to recruit Army nurses. Generally, there is at least one RA recruiter and at most three USAR recruiters at every recruiting stations. However, some stations have no USAR recruiters. This is because the Reserves have different requirements for its recruits and recruiters. First, each recruit must live within a 50 mile radius of his/her assigned Reserve Center, where reservists train one weekend of each month. This radius restricts the area in which USAR recruiters can recruit. In addition, USAR recruiters are sometimes required to recruit for a particular Reserve Center when it has vacancies needed to be filled immediately. Finally, RA recruiters mainly recruit individuals with no prior military service between 17 and 21 years old whereas USAR recruiters focus on a wider population of 17 to 29 year old.

B. RECRUITING OPERATIONS

Recruiters operate much like a saleperson selling an Army career to American youths. To avoid unnecessary competition and duplication of efforts, USAREC views the Continental United States (CONUS) as a collection of zip codes. For Regular Army recruiting, each zip code is assigned to one RA recruiter. A collection of zip codes belonging to the same RA recruiter is call a recruiter zone. The recruiting territory of a station consists of zones of recruiters who are assigned to the same station. The same method also applies to the Reserves. However, because of the previously mentioned special requirements, reserve recruiter zones are not generally aligned with the territories of the recruiting stations. For areas outside CONUS, the division of zones and territories depends on local geographical structure and overseas postal divisions. To simplify our presentation, this thesis focuses only on CONUS.

The Regular Army's target population of individuals between 17 and 21 years old with no prior military experience may be further divided into two major categories: GSA and Non-GSA. A GSA recruit is a high school graduate or senior with a category A classification that refers to those who score in the upper fifty percentile of the Armed Forces Qualification Test (AFQT) test. Last year, 95 percent of 77,600 recruits were high school graduates without prior experience and 70 percent scored in the upper 50 percentile on their AFQT. For the Reserve Army, the target market is larger and includes individuals between 17 and 29 years old without regard to prior military experience. However, recruits with prior military service are valuable to the Reserve Army, for they save training costs and are knowledgeable about current tactics, doctrine, and equipment modernizations. These factors are important for keeping Reserve units in synchronization with units in the Regular Army. In fact, soldiers separated from the Army are highly encouraged to join the Reserve and over 50 percent of recruits that joined the Reserve Army in FY93 have prior military service. [Ref. 7]

III. OPTIMAL ARMY LOCATION AND ALLOCATION PROBLEM

This chapter presents an optimization problem that determines both the locations for recruiting stations and the number of Active and Reserve recruiters for each station. In the first section, the problem and its assumptions are stated. The second section provides a discussion of prior research related to this type of problem. Finally, the formulation of the problem is presented in the last section.

A. Problem Description

A set of candidate locations for recruiting stations is assumed known. This is a reasonable assumption because downsizing is being considered and the set of candidate locations is taken to be the existing station locations. Next, it is assumed that there are two production functions, for RA and USAR recruiting, respectively. These functions describe the expected number of recruits that can be obtained annually from a given zipcode based on (i) demographic and economic factors, (ii) distance to its assigned station and (iii) amount of time recruiters (measured, e.g., in man-years) spent recruiting in the zipcode. (This recruiting time is also referred to as "recruiter share.") Given this information, the problem has four sets of decisions. The first set is to determine which candidate stations to open. The second is to assign zipcodes to open stations in order to establish the territory of each station. The third is to allocate Active and Reserve recruiters to the open stations. Finally, the last set is to decide the recruiter share for each zipcode in a station's territory. In the optimization problem, these four sets of decisions are made to maximize the annual number of Active and Reserve recruits.

B. Related Research

Extensive research has been conducted recently on realigning the structure of military recruiting organizations. In 1992, Celski [Ref. 8] developed a methodology to realign the Army Recruiting Battalions and Companies. In realigning the battalions, his model also takes into account state boundaries. When realigning companies within a battalion, he assumed that CONUS consists of a collection of counties and his model determines which counties belong to which company in an optimal manner. Doll [Ref. 9] and Schwartz [Ref. 3] addressed problems similar the one described above; Doll's work applied to the Marine Corps and Schwartz's to the Navy. One key difference between our model and those of Doll and Schwartz is the fact that theirs take into account only the active component of the respective services.

C. Problem formulation

Below is the formulation of the Army Location and Allocation (A-LOCAL) problem.

INDICES:

s = Candidate Recruiting Station

z = Zipcode

DATA:

WA	= Weight for Active production function
WR	= Weight for Reserve production function
NA	= Number of available Active recruiters
NR	= Number of available Reserve recruiters
NS	= Number of available recruiting stations
$f_Z(d,r)$	= Active component production function, where d is the distance from zipcode z to its assigned station and r is the recruiter share devoted to zipcode z
$g_Z(d,r)$	= Reserve component production function
$d_{Z\,s}$	= Distance from zipcode z to station s

VARIABLES:

Y_s	= indicates whether station s is open or closed
$AX_{Z\,s}$	= indicates whether zipcode z is assigned to station s for Active recruiting
$RX_{Z\,s}$	= indicates whether zipcode z is assigned to station s for Reserve recruiting
ASH_Z	= recruiter share devoted to zipcode z for Active recruiting
RSH_Z	= recruiter share devoted to zipcode z for Reserve recruiting
AR_s	= number of Active recruiters assigned to station s
RR_s	= number of Reserve recruiters assigned to station s

MAXIMIZE $$\sum_{z}\sum_{s} WA * f_{z}(d_{zs}AX_{zs}, ASH_{z}) + WR * g_{z}(d_{zs}RX_{zs}, RSH_{z})$$

SUBJECT TO:

$$\sum_{s} Y_{s} = NS \qquad (1)$$

$$AX_{zs} \leq Y_{s} \qquad \forall z, s \qquad (2)$$

$$\sum_{s} AX_{zs} \leq 1 \qquad \forall z \qquad (3)$$

$$\sum_{s} AR_{s} = NA \qquad (4)$$

$$\sum_{z} (ASH_{z} * AX_{zs}) \leq AR_{s} \qquad \forall s \qquad (5)$$

$$RX_{zs} \leq Y_{s} \qquad \forall z, s \qquad (6)$$

$$\sum_{s} RX_{zs} \leq 1 \qquad \forall z \qquad (7)$$

$$\sum_{s} RR_{s} = NR \qquad (8)$$

$$\sum_{z} (RSH_{z} * RX_{zs}) \leq RR_{s} \qquad \forall s \qquad (9)$$

$$Y_{s} \in \{0, 1\} \qquad \forall s \qquad (10)$$

$$AX_{zs} \in \{0, 1\}, \quad RX_{zs} \in \{0, 1\} \qquad \forall z, s \qquad (11)$$

$$AR_{s} \in \{0, 1, 2 ..., NA\}, \ RR_{s} \in \{0, 1, 2 ..., NR\} \qquad \forall s \qquad (12)$$

$$ASH_{z} \geq 0, \quad RSH_{z} \geq 0 \qquad \forall z \qquad (13)$$

The Active and Reserve objective function weights allow for several possibilities. When both *WA* and *WR* are one, then the objective is to maximize the expected number of Active and Reserve recruits. For other values of *WA* and *WR*, the objective function represents a weighted combination of the two type of recruits. Setting one of them to zero reduces the problem to either maximizing the expected number of Active or Reserve recruits. Constraint (1) ensures that *NS* stations are open. The next four sets of constraints pertain to the Active component. Constraints (2) allow zip codes to be assigned only to stations that are open. Constraints (3) guarantee that each zipcode is assigned to at most one station. Constraint (4) allocates *NA* recruiters to open stations. Constraints (5) apportion recruiter share to each zipcode. Contraints (6) to (9) are for the Reserve component and they are analogous to constraints (2) to (5). The remaining constraints define which variables are binary, integer and nonnegative.

The problem as stated above can be applied to the entire CONUS. However, such a problem would be too large for many computers. Our implementation in Chapter V restricts the problem to the territory of a single battalion. Finally, the A-LOCAL problem is a large nonlinear integer programming problem and, therefore, quite difficult to solve. A few commercially available software packages, e.g., GAMS/DICOPT [Ref. 10], are designed for small to medium size problems. However, none are available to handle a problem of this size. Thus, our implementation in Chapter V uses a heuristic approach to obtain a good solution to the A-LOCAL problem.

IV. FORECASTING RECRUITING PRODUCTION

One key component of the A-LOCAL problem described in the previous chapter is the production functions which estimate the number of recruits for RA and USAR for each zip code. In the past, many authors [Ref. 2, 3, and 8] have used standard least squares regression to estimate these functions. Least squares regression was the method of choice due its wide spread use and its intuitive appeal. Sometimes it provides reasonable estimates. Bohn and Schmitz reported that they obtained coefficients of determination, R^2, between .53 and .60 for their production models using least squares regression [Ref. 2]. This low R^2 can be in part explained by the fact that least squares regression asumes that residuals from the forecasted model are normally distributed; this may not be the case in recruiting. In fact, if each individual makes the decision to join the Army independently, then the number of recruits from a given zip code has a binomial distribution which, in certain limiting cases, can be approximated by either a Poisson or normal distribution. However, it is shown below that the Poisson approximation is more appropriate for Army data.

Previous studies have also estimated production functions using data from all zip codes. Such an approach does not distinguish efficient recruiters from the inefficient ones. This results in production functions that apply to average recruiters -- an "average" production function. However, when resources are limited, it is more appropriate to estimate the number of recruits that can be obtained by an efficient recruiter -- an "efficient" production function. In fact, an ongoing research project at the Naval Postgraduate School is trying to identify factors which will aid in the selection of efficient recruiters. Furthermore, data from efficient recruiters may also yield more significant relationships between dependent and independent variables. For example, one explanatory variable is the distance from a zip code to its assigned recruiting station. The hypothesis is that fewer recruits can be obtained from zip codes that are further away from the station. For recruiters who do not perform their duty efficiently, distance may not be a factor affecting their performance. However, for recruiters who habitually visit potential recruits, distance or travel time between the station and zip codes may be a significant factor.

The next section of this chapter describes how to determine efficiency in recruiting via Data Envelopement Analysis (DEA). The subsequent section estimates efficient production functions based on Poisson regression.

A. Efficient Recruiting

In Data Envelopment Analysis (DEA), Charnes, Cooper, and Rhodes [Ref. 4] define efficiency for a non-profit organizational unit as the ratio of a weighted sum of outputs produced by the unit over a weighted sum of inputs used to produce those outputs, i.e.,

$$Efficiency = \frac{weighted\ sum\ of\ outputs}{weighted\ sum\ of\ inputs} \quad .$$

The weights are scaled so that the maximum value for the ratio is one, representing the highest efficiency rating.

Given the available data, recruiting stations are considered as non-profit organizational units for Active recruiting. For our purpose, it would be more precise to treat individual recruiters as organizational units. However, there is no accurate data at that level of detail. The outputs and inputs for Active recruiting are listed below. The list for Reserve recruiting is similar.

Active Recruiting

Outputs:

- number of GSA recruits produced by the station
- number of Non-GSA recruits produced by the station

Inputs:

- number of RA recruiters at the station
- population of 17-21 year old individuals in the station's territory
- number of secondary schools in the station's territory
- inverse of the area, in square miles, of the station's territory
- inverse of the average distance from the assigned zip codes to the station
- average unemployment rate of the assigned zip codes
- average relative military pay in assigned zip codes; defined as the ratio of Army base pay to the per capita income

In the above lists of inputs, the number of recruiters at each station can be changed at the request or the discretion of the station commander. However, inputs such as population size and unemployment rate are not under the control of the station commander. These inputs are called non-discretionary inputs. [Ref. 11,12] To determine the efficiency of recruiting station k, the following optimization, or DEA, problem must be solved.

Data Envelopment Analysis (DEA) Problem

INDICES:

s = the Recruiting Station

i = input for the station

d = discretionary inputs for the station

nd = non-discretionary inputs for the station

o = output for the station

DATA:

$x_{s,i}$ = amount of input used at station s

$y_{s,o}$ = amount of output o produced by station s

VARIABLES:

u_o = the weight given to output o

v_i = the weight given to input i

MAXIMIZE

$$\frac{\sum_{o} u_o y_{k,o} - \sum_{nd} v_{nd} x_{k,nd}}{\sum_{d} v_d x_{k,d}}$$

SUBJECT TO:

$$\frac{\sum_{o} u_o y_{s,o} - \sum_{nd} v_{nd} x_{s,nd}}{\sum_{d} v_d x_{s,d}} \leq 1 \qquad \forall\, s$$

$$u_o > 0 \qquad \forall\, o$$

$$v_i > 0 \qquad \forall\, i$$

In both the objective function and the constraints, the ratio is slightly different from the traditional definition of efficiency used in Charnes et al. [Ref. 4]. In the numerator, the weighted sum of outputs is adjusted by the weighted sum of non-discretionary inputs. The basic idea in calculating efficiency in DEA is to maximize the efficiency ratio of station k subject to constraints that normalize the largest efficiency rating to one. The difficulty in the above formulation is in satisfying the requirements that the weights must be strictly positive. To handle this difficulty, the technique developed by Springer [Ref. 11] is used.

To illustrate the use of DEA, consider the stations of the 1st Recruiting Brigade. The majority of the data for these stations came from the Army Territory Assignment System (ATAS) database maintained by USAREC. Other demographic information such as the unemployment percentage, and the per capita income, are from CACI Marketing Systems [Ref. 13]. The problem was solved using the General Algebraic Modeling System (GAMS) and a nonlinear program solver called MINOS [Ref. 14]. The results are presented in Tables II and III. To obtain the data presented in the first row of

Table II, 32 DEA problems must be solved, i.e. one for each station in Battalion 1A, or the Albany Battalion. Data for the other rows in both tables are similarly obtained.

TABLE II. ACTIVE COMPONENT DEA RESULTS

Rct Bn	# Stations	# Efficient	Avg Efficiency
1A	32	12	0.76
1B	44	11	0.71
1D	45	13	0.73
1E	32	10	0.79
1G	38	9	0.68
1J	37	9	0.68
1K	36	8	0.65
1L	37	15	0.83
1M	38	5	0.40
1N	33	11	0.85

TABLE III. RESERVE COMPONENT DEA RESULTS

Rct Bn	# Zones	# Efficient	Avg Efficiency
1A	65	17	0.58
1B	48	15	0.64
1D	72	22	0.63
1E	49	19	0.69
1G	74	10	0.48
1J	25	9	0.70
1K	80	18	0.54
1L	63	17	0.64
1M	25	13	0.74
1N	49	20	0.73

In summary, the purpose of the DEA analysis is to preselect recruiting data that represents the work of efficient stations/recruiters. This data will be used in the next section to develop efficient production functions. For example, the efficient production function for Battalion 1A, the Albany Battalion, will be based on data from the zip codes belonging to the 12 efficient stations. On the other hand, the average production function will be based on zip codes belonging to all 32 stations in the battalion.

B. ESTIMATING THE PRODUCTION FUNCTION

As applied to recruting, production functions give the number of recruits that can be obtained from a given zip code. To estimate such functions, this thesis assumes that individuals independently decide to join the Army. Also the numbers of recruits from distinct zip codes are regarded as independent random variables. Within a single zip code, the accession of individuals can be represented as a sequence of independent Bernoulli trials. In this case the number of recruits from each zip code has a binomial distribution with parameters n and p. Here, n represents the population of 17 to 21 (or 17 to 29) year old individuals and p is the probability that a person will join the Army. To estimate p, one can assume that it is a function of some explanatory varibles and maximize the corresponding likelihood function to obtain the necessary coefficients as in logistic regression. Such an approach yields a likelihood function which is nonconcave and may produce multiple local optimal solutions. Moreover, it is not clear how to model p appropriately under the binomial assumption. An alternate approach is to use the fact that, in some cases, the poisson distribution provides a good approximation to the binomial distribution.

When the binomial parameters, n and p, satisfy the conditions: $n \geq 100$, $p \leq 0.01$ and $np \leq 20$, then the Poisson distribution provides a good estimate to the binomial distribution [Ref. 15]. The 1993 data from the 1st Brigade show that the number of 17 - 21 year olds, with no prior service, varies from zero to 10,801 with an average of 502 per zip code, and the number of 17 -29 varies from zero to 18,516 with an average of 1,040 per zip codes. Although the largest number of recruits from a single zip code is 30 and 27 for RA and USAR recruits, respectively, this number of recruits is only greater than 20 in ten of over 7,400 zip codes. Thus, data from this brigade seem to satisfy the above conditions. Under the Poisson distribution, $\lambda = np$ represents the expected number of recruits from a given zip code. To estimate λ for each zip code z, the following model based on the Cobb-Douglas production function [Ref. 3] is assumed for RA recruits:

where

$$\lambda_z = e^{\beta_0} S_Z^{\beta_S} U_Z^{\beta_U} ASH_Z^{\beta_{SH}} \left(\frac{1}{D_Z} \right)^{\beta_D}$$

- S_z = the number of public secondary schools in the zip code.
- U_z = the unemployment rate for the zip code.
- ASH_z = the recruiter's share devoted to zip code z.
- D_z = the distance from the zip code's centroid to its assigned station.

The above model is similar to those described in Bohn and Schmitz [Ref. 2]. Instead of least squares regression, the exponents, β_0, β_S, β_U, β_{SH}, and β_D are obtained as follows.

MAXIMIZE

$$LLF = \sum_z (-\lambda_z + n_z \ln(\lambda_z)) + R$$

SUBJECT TO:

$$0.01 \le \beta_D \le 1.0$$

$$0.01 \le \beta_{SH} \le 1.0$$

where

$$\lambda_z = e^{\beta_0} S_Z^{\beta_S} U_Z^{\beta_U} ASH_Z^{\beta_{SH}} \left(\frac{1}{D_Z} \right)^{\beta_D}$$

The objective function is simply the log-likelihood function of the Poisson distribution [Ref. 16], where R is the remainder term that is constant with respect to β's. The constraints ensure that the resulting model for λ_z is concave with respect to the recruiter share, ASH_Z, and the inverse distance to recruiting stations, $(1/D_Z)$.

Using the data from the Albany Battalion, Poisson regression is used to estimate the number of GSA, NPS and PS recruits. For Active recruiting, the focus is on recruiting GSA recruits since they constitute approximatly 75% of total Active recruits. The explanatory variables for GSA are as given above. In Reserve recruiting, NPS and PS recruits constitiute the total USAR recruits and they are separated here because each requires different recruiting tactics/strategies. The additional explanatory variables to account for NPS and PS are listed below:

- NRC_z = the number of Reserve Centers located within 50 miles of a zip code.
- RCD_z = the average distance to those Reserve Centers within 50 miles of the zip code's centroid.

As before, data for the explanatory variables for the 1,131 zip codes in the Albany Battalion are from ATAS and CACI Marketing Systems. For each type of recruit, two productions are estimated using Poisson regression. One is the average production function, estimated from all 1,131 zip codes and the other, the efficient production function, uses data from zip codes belonging to stations with an efficiency rating of one. The resulting exponents are in Table IV.

TABLE IV. COEFFICIENTS FOR THE ALBANY BATTALION

Var	β_0	β_S	β_U	β_{NRC}	β_{RCD}	β_D	β_{SH}
GSA	1.482	.062	.304	-	-	.01	.676
NPS	2.45	.152	.202	.488	-.344	.094	.601
PS	3.01	.058	.414	.208	-.582	.152	.739

To test how well the data fit the model, chi-square test statistics based on the Freeman-Tukey deviates [Ref. 17] were computed and are displayed in Table V along with the corresponding p-values. The Freeman-Tukey deviates and the Denominator Free goodness of fit test were used to account for the small number of recruits from each zip code. Note that the p-values generally lie in a reasonable range ($0.1 \leq p \leq 0.9$) indicating that the fit is acceptable for both models, i.e., with efficient and with all zip codes. Later, these two models will be compared to assess how they affect the decision to locate stations and allocate recruiters. Results for the other battalions of the 1st Recruiting Brigade are available in Appendix A.

TABLE V. SUMMARY OF THE ALBANY BATTALION REGRESSION RESULTS

Dependent Variable	Average Production function			Efficient Production function		
	Dev	df	p	Dev	df	p
GSA	443.1	590	1.0	188.2	213	0.8890
NPS	313.8	297	0.2413	298.3	274	0.1497
PS	288.9	297	0.6215	248.9	274	0.8591

V. IMPLEMENTATION OF THE A-LOCAL MODEL

From the analysis of Chapter IV, the production functions used in A-LOCAL, f_Z and g_Z, have the following forms:

$$f_Z\,(d_{ZS}*AX_{ZS},\,ASH_Z) \;=\; p_Z\,(ASH_Z)^{\rho}\,\left(\frac{AX_{ZS}}{d_{ZS}}\right)^{\gamma}$$

$$g_Z\,(d_{ZS}*RX_{ZS},\,RSH_Z) \;=\; q_Z\,(RSH_Z)^{\delta}\,\left(\frac{RX_{ZS}}{d_{ZS}}\right)^{\theta}$$

where $\quad p_Z = e^{\beta_O}\,S_Z^{\beta_S}\,U_Z^{\beta_U}$

and $\quad q_Z = e^{R\beta_O}\,S_Z^{R\beta_S}\,U_Z^{R\beta_U}\,NRC_Z^{R\beta_{NRC}}\,RCD_Z^{R\beta_{RCD}}$

where all exponents are estimated via DEA and Poisson regression. With these production functions, A-LOCAL is a nonlinear, integer programming problem, a very difficult class of optimization problems. This chapter employs a heuristic technique to obtain a good solution.

The heuristic technique is based on the observation that A-LOCAL has two basic sets of decision variables: one set locates stations and their territories (i.e., Y_s, AX_{zs} and RX_{zs}) and the other allocates recruiters (i.e, AR_s, ASH_{zs}, RR_s, and RSH_{zs}). These two sets of variables are linked mainly by the above production functions. When optimal values of one set are known, optimal values for the other set can be determined by solving an independent and smaller subproblem. Thus, the first section below determines the optimal locations for stations by assuming that the values of variables in the set for allocating recruiters are given. Using these optimal locations for stations, the second section then optimally allocates recruiters to the stations. It should be noted that Schwartz also used a similar decomposition [Ref. 3]. However, he decomposed his problem into four subproblems instead of two. In essence, the technique described below is a streamlined and generalized version of Schwartz', for it uses only two subproblems and applies to both Active and Reserve recruiting. Finally, the last section in this chapter describes our implementation of the heuristic technique using GAMS.

A. LOCATING STATIONS

The heuristic approach for locating recruiting stations first assumes that recruiter shares, ASH_z and RSH_z, have been predetermined in some manner. Note that values of ASH_z and RSH_z implicitly determine the number of Active and Reserve recruiters, AR_s and RR_s, at each station since they must equal the sum of recruiter share assigned to zip codes in their territories. Therefore, variables ASH_z, RSH_z, AR_s and RR_s can be discarded from A-LOCAL, for they become constant and are of no consequence to the problem. This reduces A-LOCAL to the following:

<u>Problem A1:</u>

$$\text{MAX} \quad \sum_z \sum_s p_z \, (\overline{ASH_z})^{\rho} \left(\frac{AX_{zs}}{d_{zs}} \right)^{\gamma} + \sum_z \sum_s q_z \, (\overline{RSH_z})^{\delta} \left(\frac{RX_{zs}}{d_{zs}} \right)^{\theta}$$

SUBJECT TO:

Constraints (1), (2), (3), (6), (7), (10), and (11)

where $\overline{ASH_z}$ and $\overline{RSH_z}$ are constants representing the predetermined recruiter share. To further simplify problem A1, note that, since each zip code z can be assigned to at most one station, AX_{zs} can equal one for only a single station, s, and, when γ is non-zero,

$$\sum_{s} \left(\frac{AX_{zs}}{d_{zs}} \right)^{\gamma} = \sum_{s} \left(\frac{1}{d_{zs}} \right)^{\gamma} AX_{zs} \qquad \text{for each z.}$$

Applying similar analysis to RX_{zs}, the objective function of problem A1 can be written as

$$\text{MAX} \quad \sum_{z} p_z \left(\overline{ASH_z} \right)^{\rho} \left(\sum_{s} \left(\frac{1}{d_{zs}} \right)^{\gamma} AX_{zs} \right) + \sum_{z} q_z \left(\overline{RSH_z} \right)^{\delta} \left(\sum_{s} \left(\frac{1}{d_{zs}} \right)^{\theta} RX_{zs} \right)$$

Observe that, written in this form, the objective function for problem A1 is linear, thereby making it a linear integer program. Moreover, the problem also has a structure similar to the well-known uncapacitated plant location problem [Ref. 18]. The distinguishing feature of problem A1 is that it has two commodities, Active and Reserve. Special techniques can be developed to take advantage of this structure, however, these are beyond the scope of this thesis. Also, a commercially available solver, the X-System [Ref. 19], solves problem A1 in a reasonable amount of time (see Section C of this chapter).

B. ALOCATING RECRUITERS

Solving problem A1 yields optimal locations and territories for recruiting stations, i.e., the optimal values for Y_s, AX_{zs} and RX_{zs} are known. Setting these variables to their optimal values, i.e., Y_s*, AX_{zs}* and RX_{zs}*, reduces and decomposes A-LOCAL into two subproblems: one for the Active and the other for the Reserve.

Problem B1-a (Active):

MAXIMIZE

$$\sum_z \sum_s p_z (ASH_z)^\rho \left(\frac{AX^*_{zs}}{d_{zs}} \right)^\gamma$$

SUBJECT TO:

$$\sum_{s:Y^*_s=1} AR_s \le NA$$

$$\sum_z (ASH_z * AX^*_{zs}) \le AR_s \qquad \forall\, s:\ Y^*_s=1$$

$$AR_s \in \{0,1,2,\ldots NA\}$$

$$ASH_z \ge 0 \qquad \forall\, z$$

Problem B1-b (Reserve):

MAXIMIZE

$$\sum_z \sum_s q_z (RSH_z)^\delta \left(\frac{RX^*_{zs}}{d_{zs}} \right)^\theta$$

SUBJECT TO:

$$\sum_{s:Y^*_s=1} RR_s \le NR$$

$$\sum_z (RSH_z * RX^*_{zs}) \le RR_s \qquad \forall\, s:\ Y^*_s=1$$

$$RR_s \in \{0,1,2,\ldots NR\}$$

$$RSH_z \ge 0 \qquad \forall z$$

Consider problem B1-a. Note that it can be decomposed as follows:

Master Problem:

MAXIMIZE
$$\sum_{S:\ Y_S^*=1} H_S(AR_S)$$

SUBJECT TO:
$$\sum_{S:\ Y_S^*=1} AR_S \le NA$$

$$AR_S \in \{0,1,2,\ldots NA\} \qquad \forall\, S:\ Y^*_S = 1$$

Subproblem:

MAXIMIZE
$$H_S(AR_S) = \sum_{Z:\ AX_{ZS}^*=1} \hat{p}_Z\,(ASH_Z)^{\rho}$$

SUBJECT TO:
$$\sum_{Z:AX^*_{ZS}=1} ASH_Z \le AR_S$$

$$ASH_Z \ge 0 \qquad \forall Z:\ AX_{ZS}^*=1$$

where
$$\hat{p}_Z = p_Z\left(\frac{1}{d_{ZS}}\right)^{\gamma}$$

When ρ is between 0 and 1, the Karush-Kuhn-Tucker (KKT) conditions [Ref. 20], yield the following solution to the subproblem:

$$ASH^*_Z = AR_S \; [\, \hat{p}_Z^{\frac{1}{1-\rho}} \; / \sum_{Z:\; AX^*_{ZS}=1} \hat{p}_Z^{\frac{1}{1-\rho}} \,]$$

This solution yields the following objective function value for the Subproblem:

$$H_S(AR_S) \; = \sum_{Z:AX^*_{ZS}=1} \hat{p}_Z \; [\, AR_S\, (\hat{p}_Z)^{\frac{1}{1-\rho}} \; / \sum_{Z:AX^*_{ZS}=1} (\hat{p}_Z)^{\frac{1}{1-\rho}} \,]^{\rho}$$

$$= AR_S^{\rho} \; (\sum_{Z:\; AX^*_{ZS}=1} \hat{p}_Z^{\frac{1}{1-\rho}})^{(1-\rho)}$$

Substituting $H_S(AR_S)$ into the Master problem yields the following problem:

<u>Problem B2:</u>

MAXIMIZE

$$\sum_{S:\; Y^*_S=1} \Phi_S \, (AR_S)^{\rho}$$

SUBJECT TO:

$$\sum_{S:\, Y^*_S=1} AR_S \le NA$$

$$AR_S \in \{0, 1, 2, \ldots NA\} \qquad \forall S:\; Y^*_S = 1$$

where:

$$\Phi_S \; = \; (\sum_{Z:\; AX^*_{ZS}=1} \hat{p}_Z^{\frac{1}{1-\rho}})^{(1-\rho)}$$

This problem is a nonlinear (integer) allocation problem. However, from the statistical analysis of Chapter IV, ρ is always between zero and one. Therefore, the objective function is concave [Ref. 3] and the problem can be solved optimally by the Maximal Marginal Return algorithm [Ref. 21] which is stated below for completeness.

Maximal Marginal Return Algorithm

Step 1: Set $AR_S = 1$ for each s such that $Y^*_S = 1$, (Every open station must have at least one RA recruiter)

Step 2: Find the station t with the maximum marginal return of an additional recruiter.

$$t = \underset{S:\ Y_S^*=1}{\arg\max} \{\phi_S((AR_S+1)^\rho - (AR_S)^\rho)\}$$

Step 3: Set $AR_t = AR_t + 1$.

Step 4: If there are no more recruiters to allocate, stop. Otherwise, return to Step 2.

problem B1-Reserve can be solved in a similar manner. However, in Step 1, RR_S is set to zero since there is no requirement for every station to have a Reserve recruiter.

C. IMPLEMENTATION

In this section, the A-LOCAL problem is applied to the Albany Recruiting Battalion in order to illustrate the technique developed in this chapter. The Albany Battalion is the largest battalion in the 1st Recruiting Brigade, containing 1,131 zip codes. The results discussed below assume that there are 44 existing stations, of which 36 are to remain open.

From Chapter IV, the production function for the active component that estimates the number of GSA recruits is given below.

$$f_Z\,(d_{ZS}*AX_{ZS},\,ASH_Z) \;=\; e^{1.48}\,S_Z^{0.06}\,U_Z^{0.30}\,ASH_Z^{0.68}\,\left(\frac{AX_{ZS}}{d_{ZS}}\right)^{0.01}$$

For the Reserve, the production functions consists of two components: one is for recruits with prior service and the other is for those without any prior service.

$$PS_Z(d_{ZS}*RX_{ZS},RSH_Z) = e^{3.0}\,S_Z^{0.06}\,U_Z^{0.41}\,NRC_Z^{0.21}\,RCD_Z^{-0.58}\,RSH_Z^{0.74}\,\left(\frac{RX_{ZS}}{d_{ZS}}\right)^{0.15}$$

$$NPS_Z\,(d_{ZS}*RX_{ZS},\,RSH_Z) = e^{2.45}\,S_Z^{0.15}\,U_Z^{0.20}\,NRC_Z^{0.49}\,RCD_Z^{-0.34}\,RSH_Z^{0.60}\,\left(\frac{RX_{ZS}}{d_{ZS}}\right)^{0.09}$$

However, in order to apply the MMR algorithm, they are combined into one as follows.

$$g_Z\,(d_{ZS}*RX_{ZS},\,RSH_Z) \;=\; (\,e^{3.00}\,S_Z^{0.06}\,U_Z^{0.41}\,NRC_Z^{0.21}\,RCD_Z^{-0.58}\,RSH_Z^{0.74} + e^{2.45}\,S_Z^{0.15}\,U_Z^{0.20}\,NRC_Z^{0.49}\,RCD_Z^{-0.34}\,) * \left(\frac{RX_{ZS}}{d_{ZS}}\right)^{\delta}$$

$$where \quad \delta = \frac{0.15+0.09}{2} = 0.12$$

Note that δ is simply the average of the distance coefficients for PS and NPS.

Based on a sample of ten problems, this approach for the Reserves, produces answers within 10% of optimality. Considering the fact that the objective function is obtained through statistical estimation, solutions within 10% of optimality are judged as acceptable. Finally, the values for WA and WR are 0.4906 and 0.5094, respectively; they are the fractions of the Active and Reserve recruits for FY93.

To locate the 36 stations, problem A1 was solved using GAMS with the X-System as the integer program solver. For Albany, problem A1 contains 77,539 binary variables and 79,756 constraints. It took an average of 7.5 CPU minutes to solve the problem on an Amdahl 5995 computer at the Naval Postgraduate school. To allow for an easy interface between problem A1 and problem B1, the Maximal Marginal Return (MMR) Algorithm is also implemented in GAMS. It took GAMS another 33 CPU seconds to execute and print solution reports for the MMR algorithm. Recall that the MMR algorithm requires no solver. The output from this solution process is shown in Table VI below. Columns titled 'AUTH AR' and 'PROP AR' provide the current and 'optimal' allocations of Active recruiters. Columns titled 'AUTH USAR' and 'PROP USAR' have similar meaning. Note that stations with zero PROP AR, e.g., station 1A1H, are to be closed. The remaining columns give predicted number of recruits in each category: GSA, NPS and PS. It should be noted that Schwartz reported that his approach obtains solution within 10% of optimality. Since our approach is similar, it is expected that similar solution quality is obtained. One method for verifying such a claim involves solving the A-LOCAL problem as nonlinear programming problem while ignoring the integrality restriction. However, the resulting problems would require an excessive amount of computing time due to the large number of variables. In fact, Schwartz reported a CPU time of five hours for problems approximately half the size of A-LOCAL.

TABLE VI. AN OPTIMAL ALIGNMENT OF THE ALBANY BN

STATION	AUTH RA	PROP RA	PRED GSA	AUTH USAR	PROP USAR	PRED NPS	PRED PS
1A1D	3	3	44.76	3	1	32.95	15.54
1A1H	2	0	0.00	1	0	0.00	0.00
1A1M	2	4	61.15	1	1	31.84	13.99
1A1O	2	6	86.99	0	1	26.46	12.13
1A1R	4	2	28.70	2	2	54.81	22.58
1A1S	1	0	0.00	1	0	0.00	0.00
1A1W	3	2	29.85	2	1	30.85	15.41
1A3C	3	0	0.00	1	0	0.00	0.00
1A1G	2	4	60.09	1	1	24.25	12.36
1A3J	1	0	0.00	0	0	0.00	0.00
1A1N	3	5	71.92	2	2	52.79	26.31
1A3P	2	5	73.13	2	1	28.47	14.80
1A1Q	3	2	29.97	2	1	29.79	16.67
1A3Q	1	0	0.00	0	0	0.00	0.00
1A3B	2	7	103.62	1	1	26.49	12.97
1A3D	4	5	72.00	0	2	49.64	32.56
1A3N	1	0	0.00	0	0	0.00	0.00
1A3T	1	0	0.00	1	0	0.00	0.00
1A3V	2	7	101.05	1	1	30.15	14.20
1A4C	2	2	26.84	2	3	88.36	37.68
1A4F	3	2	29.99	3	3	85.16	36.35
1A4T	2	2	26.90	1	2	64.56	24.28
1A4V	3	4	57.20	3	3	128.03	47.44
1A5E	2	1	14.84	2	1	33.73	14.30
1A5F	2	2	28.98	2	3	81.42	41.57
1A5H	3	1	16.11	2	2	56.81	30.30
1A5J	3	1	15.39	2	2	56.83	27.10
1A5L	1	2	26.79	1	2	54.51	24.56
1A6J	1	1	9.47	1	1	24.16	11.39
1A6K	2	0	0.00	1	0	0.00	0.00
1A6L	1	1	15.17	1	1	33.76	12.65
1A6N	3	2	27.17	2	2	57.83	30.85
1A6P	3	2	29.33	1	2	55.05	22.68
1A6B	1	1	14.86	1	2	54.28	24.24
1A6D	4	3	44.78	2	3	118.50	46.35
1A6R	2	2	27.22	2	3	92.63	31.78
1A5T	1	2	29.15	1	1	34.95	12.76
1A5W	2	2	28.73	2	2	60.14	28.43
1A4D	2	3	45.21	1	1	31.30	12.95
1A4H	2	1	15.58	2	2	49.20	25.69
1A4M	1	1	15.56	1	1	29.06	12.89
1A4R	2	2	31.43	2	1	30.86	12.42
1A4W	3	2	29.88	2	2	59.91	29.52
1A6S	3	2	28.03	2	3	106.81	34.94
TOTAL	96	96	1397.83	63	63	1906.31	842.65

D. APPLICATIONS AND RESULTS

This section demonstrates two possible uses of the A-LOCAL problem. The first subsection studies the difference between using efficient and average production functions. The other subsection shows how results from solving the A-LOCAL problem with varying number of stations and recruiters can be used to determine the appropriate number of stations and recruiters.

1. Comparison of Efficient and Average Production Functions

Using efficient and average production functions, the A-LOCAL problem for the Albany Battalion was solved using 96 Active and 63 Reserve recruiters and stations varying from 18 to 44. Figures 3 to 5 graphically display the results. Figure 3 shows that, in terms of combined Active and Reserve recruits, USAREC can obtain an additional 1,426 recruits per year from the Albany Battalion if the all recruiters are assumed to be efficient. Figures 4 and 5 display the number of recruits for Active and Reserve, separately. From these figures, efficient recruiters would produce 500 and 926 more recruits for Active and Reserve, respectively. Although the assumption that all recruiters are efficient is unrealistic, the results obtained using the efficient production functions provide USAREC planners with goals against which they can measure their recruiting productivity.

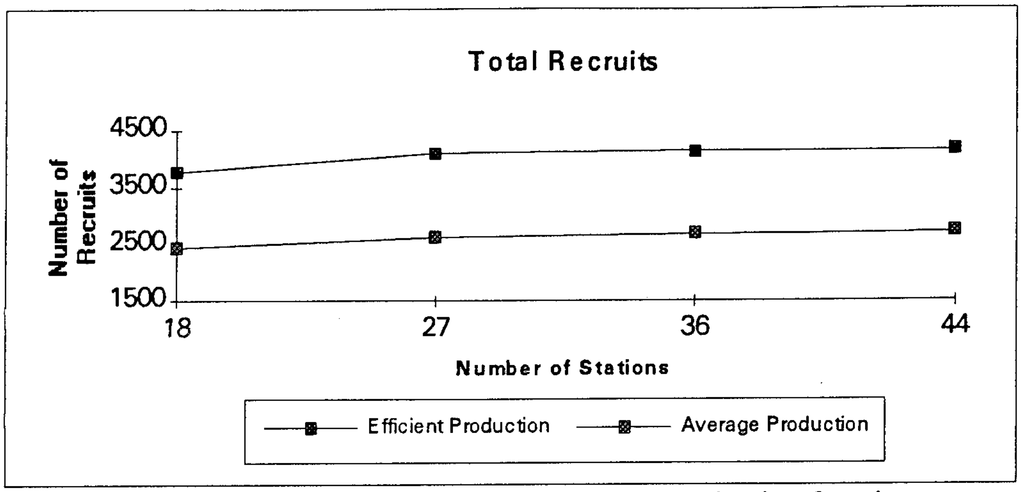

Figure 3. Total recruits using efficient and average production functions

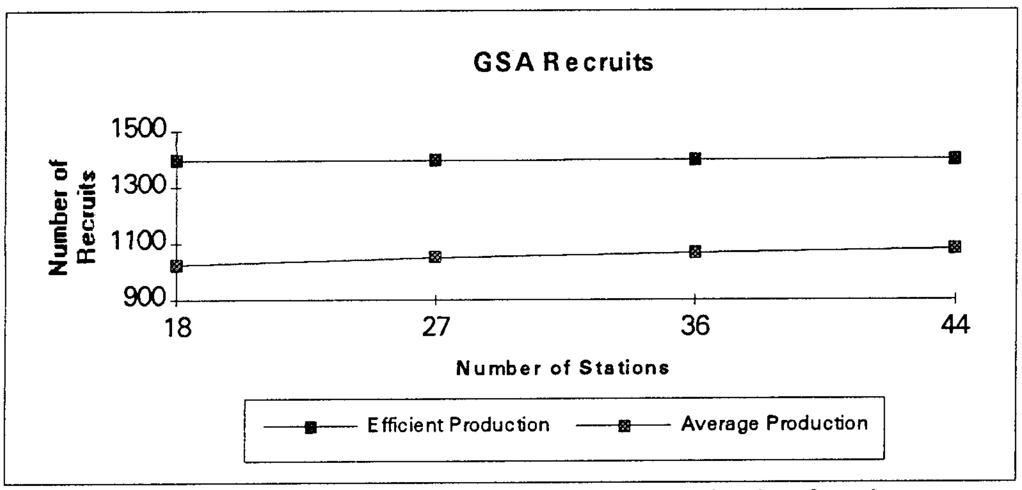

Figure 4. GSA recruits using efficient and average production functions

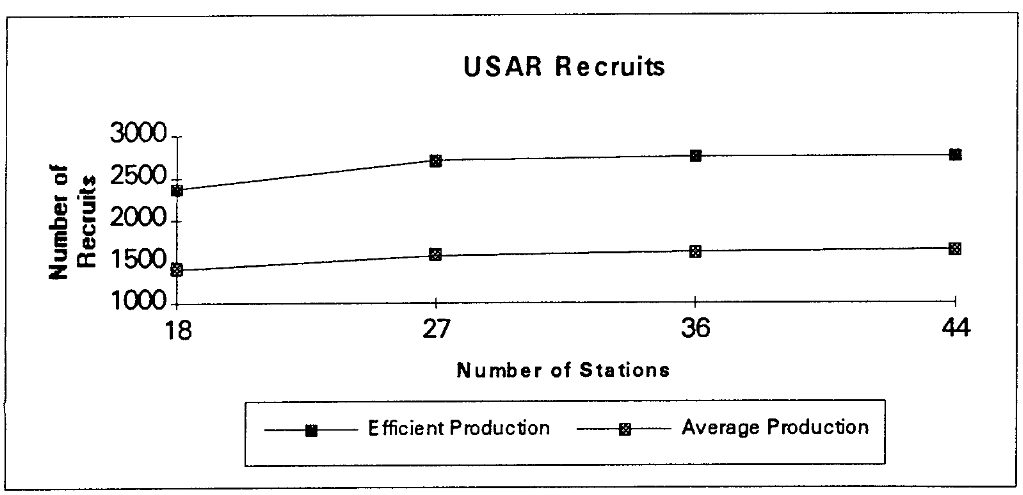

Figure 5. USAR recruits using efficient and average production functions

2. Determining the Number of Stations and Recruiters

Running the A-LOCAL problem with various numbers of stations and recruiters produces a series of results that can be used to analyze a wide variety of issues involving a single battalion. Presented here are the results of A-LOCAL for five different numbers of stations and three different combinations of Active and Reserve recruiters. To continue with the idea of efficient recruiting, the results in Table VII are based on efficient production functions.

TABLE VII. RESULTS OF A-LOCAL

Number of RA	Number of USAR	Number of Stations	Total Recruits
96	63	9	2825
96	63	18	3770
96	63	27	4105
96	63	36	4145
96	63	44	4150
75	50	9	2611
75	50	18	3307
75	50	27	3378
75	50	36	3395
75	50	44	3398
50	30	9	2328
50	30	18	2551
50	30	27	2580
50	30	36	2575
50	30	44	2573

The curves in Figure 6 show that the number of recruits stops increasing after 36 stations. This indicates that the Albany Battalion should contain no more than 36 stations. On the other hand, the differences in the three graphs in Figure 6 also indicate that more recruiters mean more recruits. This seems intuitive since it is the recruiters that generates recruits not the stations. However, it is also expected that the marginal increase due to additional recruiters will level off when the number of recruiters is sufficiently large. This is because our choice of production function also models the saturation of the market when an excessive number of recruiters is present.

The curves in Figure 6 also provide information for the appropriate number of stations and recruiters. For example, the top most curve indicates that 17 stations, 96 Active and 63 Reserve recruiters would produce approximately 3,700 total recruits in the Albany Battalion. By interpolating between the two top most graphs (see Figure 7), the same number of recruits can alternately be obtained with 25 stations and $86 \approx (96+75)/2$ Active and $56 \approx (63+50)/2$ Reserve recruiters.

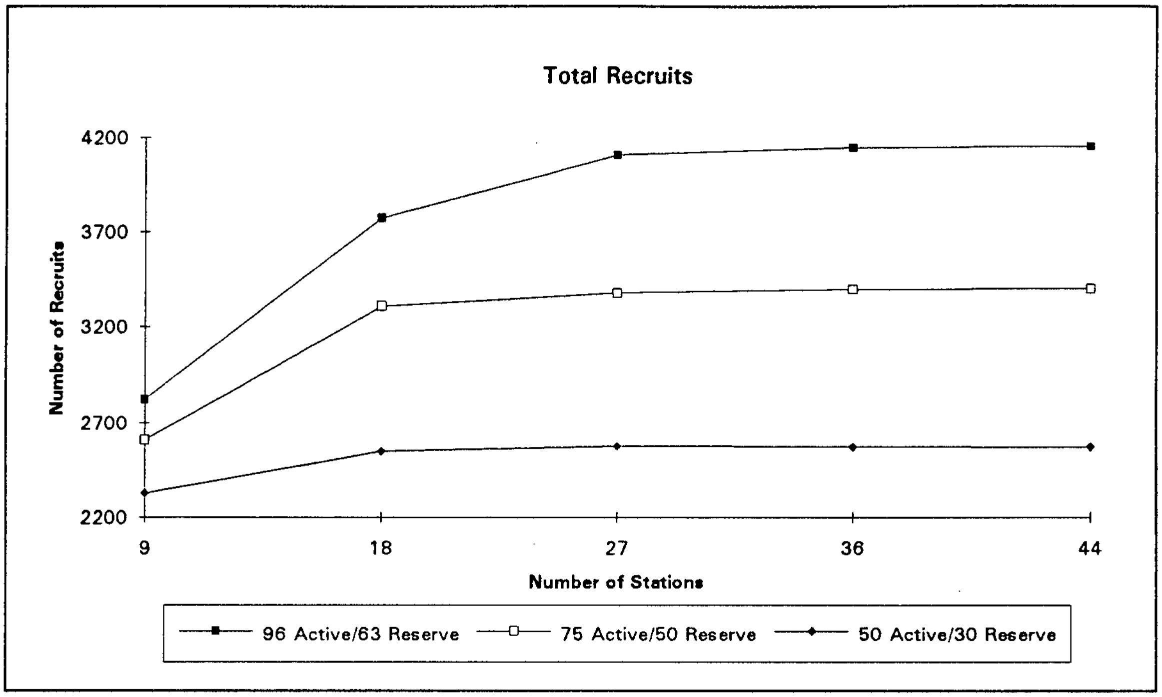

Figure 6. Results for the Albany Battalion

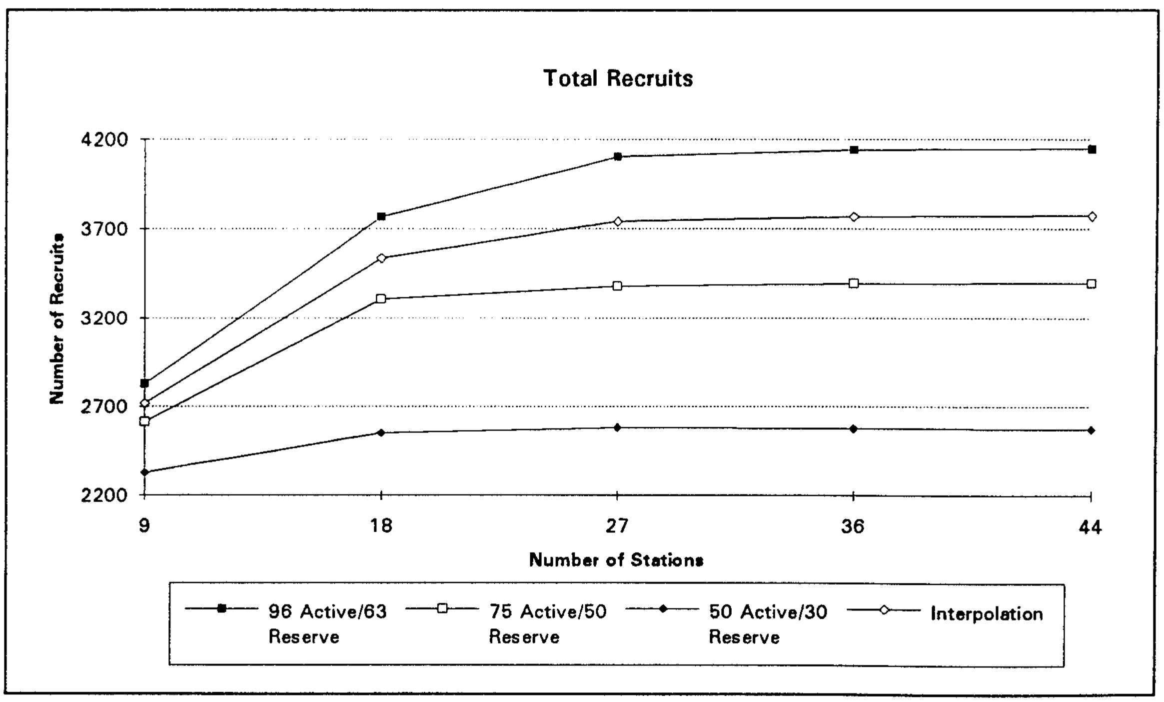

Figure 7. Selecting the Number of Stations and Recruiters by Interpolation

VI. CONCLUSIONS

This thesis addresses the problem of how to improve recruiting by better locating stations and staffing them with Active and Reserve recruiters. The problem, which is called A-LOCAL, is formulated as a nonlinear integer program with the objective of maximizing the total number of recruits. Since the number of recruits is not known with certainty, it is modelled with the Cobb-Douglas production function and statistically estimated using Poisson regression. To study the effect of efficiency in recruiting, Data Envelopment Analysis is used to determine stations which are efficient at recruiting. Then, two types of production functions, average and efficient, are estimated. The efficient production function is based on data from zip codes belonging to efficient stations and the average is based on all zip codes from a battalion.

Under both types of production functions, the resulting A-LOCAL problem is difficult to solve optimally. So, a heuristic procedure is developed to obtain a near optimal solution instead. This procedure is based on decomposing the problem into two: one is a linear integer program and the other is another nonlinear integer program with special structure. The linear integer program deals with locating stations and resembles an uncapacitated plant location problem, a problem well-known in the operations research literature. The other, the nonlinear integer program, allocates recruiters to open stations. This problem has a special structure that allows it to be further decomposed into a master and subproblems. The subproblems have closed form solutions, thereby permitting the master problem to be solved optimally using the Maximal Marginal Return Algorithm.

The results from A-LOCAL show that, using the efficient production functions, the Albany Battalion can obtain an additional 1,426 recruits or approximately 50 percent more than those that can be obtained by using the average functions. In addition, it is also demonstrated that 36 stations are sufficient for the Albany Battalion.

This thesis also identifies the following topics for future research.

1. As mentioned in Chapter III, A-LOCAL can be applied to CONUS instead of a single battalion. However, such an approach would produce an optimization problem too large for many existing computers. When applied to CONUS, techniques for decomposing A-LOCAL into smaller and more manageable subproblems need to be developed.

2. As formulated, A-LOCAL assumes that stations do not have capacity limitations in order to allow for possible expansion of existing stations. Although, it is possible to add station capacities to A-LOCAL, it is not clear how the resulting problem can be solved in practice. Therefore, exact or approximate solution techniques for handling station capacities need to be developed.

APPENDIX A POISSON REGRESSION

A. POISSON REGRESSION IMPLEMENTATION

```
$TITLE     ACTIVE ARMY REGRESSION MODEL          CPT Michael J. Teague
$STITLE    USING POISSON REGRESSION
*
*-----------GAMS AND DOLLAR CONTROL OPTIONS--------------------------
*             (See Appendice B & C)

$OFFUPPER OFFSYMLIST OFFSYMXREF INLINECOM{ } MAXCOL 150
$offlisting

OPTIONS
   LIMCOL =  0  , LIMROW  =  0   , SOLPRINT = OFF , DECIMALS = 4
   RESLIM =  100, ITERLIM = 10000, OPTCR    = 0.1 , SEED     = 78915;
*--------------------------------------------------------------
SETS
  A     attributes for a zipcode /
      DIST      dist to station
      AREA      area of zip code
      POP       population of 17 to 21 years old in zipcode
      SCHOOLS   number of secondary schools in zipcode
      RELPAY    relative military pay in zipcode in 1990
      UNEMP     percent unemployment in zipcode in 1990
      GSA       gsa contracts in 1993
      NGSA      non gsa contracts in 1993
      REG       total ra contracts in 1993
      RECSHR    share for regular army rec
      EFF       efficiency from DEA model
      /
```

```
   I(A) independent var / DIST, RECSHR, SCHOOLS, UNEMP/

   D(A) dependent var /GSA, REG/
   ;


SETS
   ZC    zip codes /
$INCLUDE %1.ZID
     / ;

ALIAS (Z,ZC);


TABLE
  INZIP(ZC,A)  information about zipcode
       DIST         AREA        POP     SCHOOLS   RELPAY  UNEMP  GSA
NGSA    REG    RECSHR   EFF
$INCLUDE %1.DAT
;



* BLOCK 1 is without DEA and BLOCK 2 is with DEA
* ----USE ONLY ONE AT A TIME----



SET ZIP(Z), NZIP(Z);
* BLOCK 1 divide data into two random groups for cross validation
    ZIP(Z) = YES$(UNIFORM(0,1) GE 1/2);
   NZIP(Z) = NOT ZIP(Z);

* BLOCK 2 fit the efficient zipcodes and check against the same number
*    ZIP(Z) = YES$( (INZIP(Z,'EFF') LE 1) AND (UNIFORM(0,1) GE 1/2) );
```

```
*    NZIP(Z) = (NOT ZIP(Z)) * YES$(INZIP(Z,'EFF') GE 1);


*  cannot have an independent variable with value of 0 for ln transform
INZIP(Z,I)$(INZIP(Z,I) EQ 0) = 0.01;


*  take ln of independent variables for cobb douglas transformation
PARAMETER LINF(Z,I) natural log of data;
  LINF(Z,I)$INZIP(Z,I) = LOG(INZIP(Z,I));




*----------------------MODEL--------------------------------------
PARAMETER DA(Z)  dependent variable;

VARIABLE
  B0      constant term or intercept
  B(I)    independent variable coefficients
  LLF     log likelihood function
  ;

POSITIVE VARIABLE
  B0
  ;


  B.UP('DIST') = -0.01;
  B.UP('RECSHR') = 1.0;
  B.LO('RECSHR') = 0.01;
  B.LO('UNEMP') = 0.0;
  B.LO('SCHOOLS') = 0.0;
```

```
EQUATION
   FUN      Log likelihood function for POISSON  ;



FUN.. LLF =E= SUM(ZIP,
              -EXP(B0)*PROD(I, INZIP(ZIP,I)**B(I))
              +DA(ZIP)*(B0+SUM(I,B(I)*LINF(ZIP,I)) )
             );



MODEL POISSON /FUN/;

PARAMETER EXPECT(*,*);

*  set up scalers for chi square goodness of fit test
SCALAR TESTSTAT     test statistic for goodness of fit
       DFTESTSTAT  denominator free test stat
       CHIPROB     'prob the chi-sq > TESTSTA'
       DFCHIPROB  'prob the chi-sq > DFTESTSTAT'
       CHI10       'chi-squared stat at .10 with d.f. > 40'
       CHI05       'chi-squared stat at .05 with d.f. > 40'
       CHI01       'chi-squared stat at .01 with d.f. > 40'
       DF          degree of freedom of the model
       NORMTEST    DRR test percent outside 1.96;


LOOP(D,

DA(Z) = INZIP(Z,D);
SOLVE POISSON USING NLP MAXIMIZING LLF;
```

```
EXPECT(ZIP,'ACTUAL') = DA(ZIP);
EXPECT(ZIP,'EST') = EXP(B0.L)*PROD(I, INZIP(ZIP,I)**B.L(I) );
EXPECT(ZIP,'DRR') = (SQRT(2+4*DA(ZIP)) -
SQRT(1+4*EXPECT(ZIP,'EST')))$DA(ZIP)
            + (1-SQRT(1+4*EXPECT(ZIP,'EST')))$(DA(ZIP) EQ 0);

EXPECT('TOTAL','COUNT') = CARD(ZIP);
NORMTEST = SUM(ZIP$(EXPECT(ZIP,'DRR') LT -1.96 OR
            EXPECT(ZIP,'DRR') GT 1.96), 1)/CARD(ZIP);
EXPECT('TOTAL','ACTUAL') = SUM(ZIP, EXPECT(ZIP,'ACTUAL'));
EXPECT('TOTAL','EST') = SUM(ZIP, EXPECT(ZIP,'EST'));

*DISPLAY EXPECT;
DISPLAY NORMTEST;

*  Do a CHI Square goodness of fit test
* DENOM FREE TEST
DFTESTSTAT = SUM(ZIP,
          SQR( SQRT(DA(ZIP)) + SQRT(1+DA(ZIP))
            - SQRT(1+4*EXPECT(ZIP,'EST')) )
          );


* CHI SQUARE
TESTSTAT = SUM(ZIP, SQR( EXPECT(ZIP,'ACTUAL') - EXPECT(ZIP,'EST'))/
              EXPECT(ZIP,'EST'));




* Compare test statistic with chi square
DF = CARD(ZIP) - (CARD(I)+1);

* Calculate CHI Square values using approximation from DEVORE
CHI10 = DF*POWER( 1 - 2/(9*DF) + 1.28*SQRT(2/(9*DF)),3);
```

```
CHI05 = DF*POWER( 1 - 2/(9*DF) + 1.64*SQRT(2/(9*DF)),3);
CHI01 = DF*POWER( 1 - 2/(9*DF) + 2.33*SQRT(2/(9*DF)),3);
CHIPROB = 1 - ERRORF((2/(9*DF) - 1 + (TESTSTAT/DF)**(1/3))/SQRT(2/(9*DF)));
DFCHIPROB = 1 - ERRORF((2/(9*DF) - 1 +
(DFTESTSTAT/DF)**(1/3))/SQRT(2/(9*DF)));

DISPLAY LLF.L, B0.L, B.L, TESTSTAT,CHIPROB,DFTESTSTAT,DFCHIPROB,
                    DF,CHI10,CHI05,CHI01;


*  Do a CHI Square goodness of fit test on cross validation data set
* DENOM FREE TEST
DFTESTSTAT = SUM(NZIP,
            SQR( SQRT(DA(NZIP)) + SQRT(1+DA(NZIP))
              - SQRT(1+4*EXP(B0.L)*PROD(I, INZIP(NZIP,I)**B.L(I)) ) )
          );

* CHI SQUARE
TESTSTAT = SUM(NZIP, SQR( DA(NZIP) - EXP(B0.L)*PROD(I,
INZIP(NZIP,I)**B.L(I)) )/
                  (EXP(B0.L)*PROD(I, INZIP(NZIP,I)**B.L(I)))
                  );

DF = CARD(NZIP) ;
CHI10 = DF*POWER( 1 - 2/(9*DF) + 1.28*SQRT(2/(9*DF)),3);
CHI05 = DF*POWER( 1 - 2/(9*DF) + 1.64*SQRT(2/(9*DF)),3);
CHI01 = DF*POWER( 1 - 2/(9*DF) + 2.33*SQRT(2/(9*DF)),3);
CHIPROB = 1 - ERRORF((2/(9*DF) - 1 + (TESTSTAT/DF)**(1/3))/SQRT(2/(9*DF)));
DFCHIPROB = 1 - ERRORF((2/(9*DF) - 1 +
(DFTESTSTAT/DF)**(1/3))/SQRT(2/(9*DF)));

DISPLAY TESTSTAT,CHIPROB,DFTESTSTAT,DFCHIPROB, DF,CHI10,CHI05,CHI01;

*

);
```

1. Poisson Results of Active Battalions with DEA

Bn	Var	ß0	Schools	Unemp	Dist	RecShr	Chi^2	p
1A	GSA	1.4824	.0624	.3039	.01	.6758	188.2	.8890
1B	GSA	2.2768	.3409	.0801	.0782	.3128	169.3	.1216
1D	GSA	1.7593	.0431	.2151	.0838	.6752	174.2	.6272
1E	GSA	2.0730	.0346	0	.0454	.7351	62.3	.8888
1G	GSA	1.4549	0	.3553	.0160	.8105	102.4	.3863
1J	GSA	1.8789	0	.4261	.0898	.5951	123.5	0
1K	GSA	2.3687	0	.1320	.01	.8574	37.4	.6715
1L	GSA	1.5420	.0192	.2551	.01	.7797	94.5	1.0
1M	GSA	3.4989	.0400	0	.03484	.6249	62.8	.0260
1N	GSA	2.0457	.0488	0	.0610	.5498	113.6	.7381

2. Poisson Results for Active Battalions without DEA

Bn	Var	ß0	Schools	Unemp	Dist	RecShr	Chi^2	p
1A	GSA	1.7320	.0389	.1832	.01	.7049	443.1	1.0
1B	GSA	2.1860	.0975	.0775	.2272	.3787	492.3	.0005
1D	GSA	1.7593	.0431	.2151	.0838	.6752	174.2	.6272
1E	GSA	2.1496	.0597	0	.0552	.7246	200.1	1.0
1G	GSA	1.4549	0	.3553	.0160	.8105	102.4	.3863
1J	GSA	2.1072	.0870	.2080	.1117	.4943	399.5	0
1K	GSA	1.2848	.0493	.3298	.0299	.6090	291.6	.3488
1L	GSA	1.9674	.0573	.0131	.01	.7298	251.5	1.0
1M	GSA	2.1923	.0769	.1407	.17664	.4573	410.0	.0031
1N	GSA	1.8762	.0338	.1540	.0202	.7129	259.7	.9422

3. Poisson Results for Reserve Battalions with DEA

Bn	Var	ß0	Schls	Unem	NRC	RCD	Dist	RShr	Chi^2	p
1A	NPS	2.45	.152	.202	.488	-.344	.094	.601	298	.150
	PS	3.01	.058	.414	.208	-.582	.152	.739	248	.859
1B	NPS	2.85	.125	.797	.784	-1.0	.028	.397	235	.001
	PS	2.76	.039	.787	.710	-1.0	.167	.411	174	.358
1D	NPS	2.69	.055	0	.518	-.281	.217	.743	250	.633
	PS	1.34	.011	.018	.399	0	.01	.921	278	.191
1E	NPS	1.99	.057	.311	.536	-.248	.01	.630	193	.014
	PS	1.99	.003	.187	.431	-.242	.176	.890	156	.134
1G	NPS	3.99	.180	0	.454	-.541	.01	.739	42	.843
	PS	3.41	.095	.226	0	-.445	.01	.745	65	.406
1J	NPS	2.24	0	.488	0	-.245	.115	.526	109	.052
	PS	0.00	0	.578	.157	-.012	.01	.351	82	.204
1K	NPS	2.26	.035	0	.302	0	.01	.999	85	.117
	PS	3.47	.004	.418	.155	-.664	.01	.999	71	.455
1L	NPS	1.15	.066	.448	.394	-.225	.01	.561	106	.868
	PS	4.75	.076	.553	.505	-1.34	.013	.844	131	.308
1M	NPS	4.34	.146	.123	0	-.756	.036	.380	221	0
	PS	5.62	.181	0	0	-1.22	.283	.289	126	.441
1N	NPS	1.67	.085	.392	.681	-.356	.072	.653	128	.879
	PS	1.87	.105	.341	.369	-.227	.225	.532	131	.838

4. Poisson Results for Reserve Battalions without DEA

Bn	Var	ß0	Schls	Unem	NRC	RCD	Dist	RShr	Chi^2	p
1A	NPS	2.24	.043	.355	.499	-.362	.077	.677	313	.241
	PS	3.67	0	.420	.059	-.661	.138	.829	288	.622
1B	NPS	4.12	.092	.535	.613	-1.28	.024	.354	548	0
	PS	4.07	.068	.539	.301	-1.16	.155	.344	437	.051
1D	NPS	2.38	.076	.260	.528	-.310	.186	.793	251	.643
	PS	1.16	.014	.160	.364	0	.01	.929	251	.153
1E	NPS	2.15	.025	.208	.015	0	.060	.729	316	.349
	PS	2.93	.074	.802	.643	-1.12	.01	.717	295	.937
1G	NPS	0.86	0	.565	..131	-.104	.093	.131	564	0
	PS	0.00	.017	.712	.307	-.325	.132	.392	340	.014
1J	NPS	3.31	.067	.205	.132	-.517	.01	.625	268	.589
	PS	1.19	.038	.509	.067	-.137	.046	.601	302	.117
1K	NPS	1.54	.002	.553	0	0	.01	.809	446	0
	PS	.035	0	.894	.234	-.157	.01	.758	279	.472
1L	NPS	1.15	.066	.448	.394	-.225	.01	.561	106	.868
	PS	4.75	.076	.553	.505	-1.34	.013	.844	131	.308
1M	NPS	2.52	.047	.415	.479	-.452	.356	.358	300	.086
	PS	2.00	.079	0	.858	-.560	.028	.545	243	.988
1N	NPS	2.03	.331	0	.263	-.296	.019	.649	308	.281
	PS	2.91	.046	.362	.024	-.360	.061	.816	267	.872

APPENDIX B A-LOCAL GAMS PROGRAM

```
$TITLE          ARMY LOCATION ALLOCATION        CPT Michael J. Teague
$STITLE   Realign Recruiting Stations, Active and USAR Recruiters for a Bn
*
*-----------GAMS AND DOLLAR CONTROL OPTIONS------------------------------
*              (SEE APPENDICE B & C)

$OFFUPPER OFFSYMLIST OFFSYMXREF OFFUELLIST INLINECOM{ } MAXCOL
130
$OFFLISTING


OPTIONS
   LIMCOL = 0    ,    LIMROW = 0     , SOLPRINT = OFF , DECIMALS = 4
   RESLIM = 90000,    ITERLIM=9999999, OPTCR = 0.05   , SEED = 3141;

*-----------DATA AND SETS---------------------------------------------

SETS
  A   attributes  /
     ZIPX        x-coordinate of zipcode centroid
     ZIPY        y-coordinate of zipcode centroid
     POP1721     active target population of 17 to 21 year olds
     POP1729     reserve target population of 17 to 29 year olds
     SCHOOLS     number of secondary schools within the zipcode
     UNEMP       unemployment percentage of zipcode in 1990
     NRC50       number of reserve centers within 50 miles of zipcode
     RCDIST      average distance to reserve centers within 50 miles
     ASHR        share of active recruiters assigned to the zipcode
     RSHR        share of reserve recruiters assigned to the zipcode
     DIST        distance from zipcode to assigned stations
     GSA         number of active GSA contracts in 1993
     NPS         number of reserve NonPrior Service contracts in 1993
     /

  L   location  / RSID, XCOORD, YCOORD, OPRA, OPAGR /

  I(A) independent variables
```

```
       / SCHOOLS, UNEMP, ASHR, RSHR, DIST, NRC50, RCDIST, POP1721,
POP1729 /
       ;

SETS
  ZC    zipcodes /
$INCLUDE ZIPBN1A ZID
    /
  RC    reserve centers /
$INCLUDE RESCTR STA
    /
  S     station rsids /
$INCLUDE RSIDBN1A STA
    / ;


ALIAS (Z,ZC);


TABLE
  INZIP(ZC,A)  information about zipcode
          ZIPX              ZIPY            POP1721      POP1729    SCHOOLS
UNEMP    GSA      NPS
$INCLUDE ZIPBN1A DAT
  ;

TABLE
  INSTA(S,L)  information about recruiting stations
          XCOORD            YCOORD      OPRA   OPAGR
$INCLUDE  RSIDBN1A DAT
  ;

TABLE
  LOCRC(RC,L)  location of reserve centers
          XCOORD              YCOORD
$INCLUDE RESCTR DAT
  ;



SCALAR   B0      intercept for active production function
        RB0     intercept for reserve production function
        PB0
```

```
        NOPRA   total number of active recruiters available to BN
        NOPAGR  total number of reserve recruiters available to BN
        NS      total number of recruiting stations available to BN
        W1      weight for active production from ATAS 93
        W2      weight for reserve production
        RAD     maximum distance from zip to RS  helps solvability;

PARAMETER
        B(I)    coefficients of active production function
        PB(I)
        RB(I)   coefficients of reserve production function     ;

*------------------------Assign Startup Values to Scalars----------------
*Scalars
B0 =          1.4824 ;
RB0 =         2.4508 ;
PB0 =         3.0048 ;
NOPRA =       %1       ;
NOPAGR =      %2       ;
NS =          %3       ;
W1 =          0.4906 ;
W2 =     (1 - W1)     ;
RAD =         150      ;

*Parameters
B('SCHOOLS') =  0.0624 ;
B('UNEMP') =  0.3039   ;
B('ASHR') =  0.6758    ;
B('DIST') =  -0.01     ;
RB('SCHOOLS') = 0.1521 ;
RB('UNEMP') =  0.2022  ;
RB('RSHR') =  0.6006   ;
RB('DIST') =  -0.0938  ;
RB('NRC50') =  0.4883  ;
RB('RCDIST') = -0.3442 ;
PB('SCHOOLS') = 0.0581 ;
PB('UNEMP') =  0.4136  ;
PB('RSHR') =  0.7390   ;
PB('DIST') =  -0.1517  ;
PB('NRC50') =  0.2081  ;
PB('RCDIST') = -0.5825 ;
```

```
PARAMETER
  DR(ZC,RC)  distance from zip to reserve center;

  DR(Z,RC) =  69.171*SQRT(POWER(COS(3.14159*
              (INZIP(Z,'ZIPY')+LOCRC(RC,'YCOORD')) /360)*
              ((INZIP(Z,'ZIPX')-LOCRC(RC,'XCOORD'))  ), 2)
           + POWER((INZIP(Z,'ZIPY')-LOCRC(RC,'YCOORD')), 2));

  DR(Z,RC)$(DR(Z,RC) LT .5) = 0.5;

SET Z5(ZC)  set of zip codes which are within 50 mi of a reserve center;
  Z5(Z) = YES$(SUM(RC$(DR(Z,RC) LE 50), 1) gt 0);

INZIP(ZC,'NRC50') = SUM(RC$(DR(ZC,RC) LE 50), 1);

INZIP(ZC,'RCDIST')$INZIP(ZC,'NRC50') =
                          SUM(RC$(DR(ZC,RC) LE 50), DR(ZC,RC))
                          /INZIP(ZC,'NRC50');

PARAMETER
  D(ZC,S)  distance from zip to recruiting station;

  D(Z,S) =  69.171*SQRT(POWER(COS(3.14159*
              (INZIP(Z,'ZIPY')+INSTA(S,'YCOORD')) /360)*
              ((INZIP(Z,'ZIPX')-INSTA(S,'XCOORD'))  ), 2)
           + POWER((INZIP(Z,'ZIPY')-INSTA(S,'YCOORD')), 2));

  D(Z,S)$(D(Z,S) LT .5) = 0.5;


*  cannot have an independent variable with value of 0
INZIP(Z,I)$(INZIP(Z,I) EQ 0) = 0.01;


PARAMETER
  C(Z)     constant terms for active production function for each zip
  K(Z)     constant terms for reserve production function for each zip
  P(Z)
  CHAT(Z)  changed C(Z) based on first approximation
  KHAT(Z)  changed K(Z) based on first approximation
  PHAT(Z)  CHANGED P(Z) BASED ON FIRST APPROXIMATION
  YCOV(Z)  '=1 if zip is within RAD';
```

```
C(Z) = EXP(B0)*(INZIP(Z,'SCHOOLS')**B('SCHOOLS'))
         *(INZIP(Z,'UNEMP')**B('UNEMP')) ;

K(Z) = EXP(RB0)*(INZIP(Z,'SCHOOLS')**RB('SCHOOLS'))
         *(INZIP(Z,'UNEMP')**RB('UNEMP'))
         *(INZIP(Z,'NRC50')**RB('NRC50'))
         *(INZIP(Z,'RCDIST')**RB('RCDIST'));

P(Z) = EXP(PB0)*(INZIP(Z,'SCHOOLS')**PB('SCHOOLS'))
         *(INZIP(Z,'UNEMP')**PB('UNEMP'))
         *(INZIP(Z,'NRC50')**PB('NRC50'))
         *(INZIP(Z,'RCDIST')**PB('RCDIST'));

CHAT(Z) = C(Z)*( (NOPRA*INZIP(Z,'POP1721')/
              SUM(ZC, INZIP(ZC,'POP1721')))**B('ASHR')) ;

KHAT(Z) = K(Z)*( (NOPAGR*INZIP(Z,'POP1729')/
              SUM(ZC, INZIP(ZC,'POP1729')))**RB('RSHR')) ;

PHAT(Z) = P(Z)*( (NOPAGR*INZIP(Z,'POP1729')/
              SUM(ZC, INZIP(ZC,'POP1729')))**PB('RSHR')) ;

YCOV(Z) = 1$(SUM(S$(D(Z,S) LE RAD), 1) GT 0);

*------------------RECRUITING STATION ASSIGNMENT MODEL-------------------
VARIABLE
    CONTR    total number of contracts from this BN
    ;

*POSITIVE VARIABLE

BINARY VARIABLE
    Y(S)    open or close station s
    X(Z,S)  assign zip to station for active recruiting
    RX(Z,S) assign zip to station for reserve recruiting
    ;

EQUATIONS
    APPROX     obj function for linear approx to total contracts
    TOTSTA     limit the number of open stations
    AZIP(Z,S)  only assign a zip to an open station for active
    RZIP(Z,S)  only assign a zip to an open station for reserve
    ACEACH(Z)  assign a zip to one and only one station for active
```

```
      RESEACH(Z)  assign a zip to one and only one station for reserve
      ;

APPROX..  CONTR =E= W1*SUM(Z$YCOV(Z),
                CHAT(Z)*SUM(S$(D(Z,S) LE RAD),
                        (X(Z,S)*(D(Z,S)**B('DIST')) ) )) +
            W2*SUM(Z5$YCOV(Z5),
                KHAT(Z5)*SUM(S$(D(Z5,S) LE RAD),
                        (RX(Z5,S)*(D(Z5,S)**RB('DIST')) ) ))+
            W2*SUM(Z5$YCOV(Z5),
                PHAT(Z5)*SUM(S$(D(Z5,S) LE RAD),
                        (RX(Z5,S)*(D(Z5,S)**PB('DIST')) ) ));

TOTSTA..     SUM(S, Y(S)) =E= NS;

AZIP(Z,S)$(D(Z,S) LE RAD)..  X(Z,S) =L= Y(S);

RZIP(Z5,S)$(D(Z5,S) LE RAD)..  RX(Z5,S) =L= Y(S);

ACEACH(Z)$YCOV(Z)..  SUM(S$(D(Z,S) LE RAD), X(Z,S)) =L= 1;

RESEACH(Z5)$YCOV(Z5)..  SUM(S$(D(Z5,S) LE RAD), RX(Z5,S)) =L= 1;


MODEL STATIONS /ALL/;

$BATINCLUDE 'XSOPTION INC A' STATIONS GEN 400 200 200 1 * *
SOLVE STATIONS USING MIP MAXIMIZING CONTR;
SCALAR OBJ1, OBJ2, OBJ3;
OBJ1 = SUM(Z$YCOV(Z),
            CHAT(Z)*SUM(S$(D(Z,S) LE RAD),
                    (X.L(Z,S)*(D(Z,S)**B('DIST')) ) )) ;
OBJ2 = SUM(Z5$YCOV(Z5),
            KHAT(Z5)*SUM(S$(D(Z5,S) LE RAD),
                    (RX.L(Z5,S)*(D(Z5,S)**RB('DIST')) ) ));
OBJ3 = SUM(Z5$YCOV(Z5),
            PHAT(Z5)*SUM(S$(D(Z5,S) LE RAD),
                    (RX.L(Z5,S)*(D(Z5,S)**PB('DIST')) ) ));
DISPLAY OBJ1, OBJ2, OBJ3;
*--------------------RECRUITER ASSIGNMENT MODEL-------------------------
*  Uses the Max Marginal Return heuristic

SET   RCNT loop indice for number of recruiters assigned
```

```
   /1*500/
   ;

SCALAR
   NRCT     counter for number of recruiters assigned
   MXBFIT   the max marginal benefit for a single iteration
   MXFOUND  counter to id when the sta with MXBFIT is reached
   BETA     coefficient for recruiter share
   ;


PARAMETER
   MRBFIT(S)     marginal benefit of assigning an add recruiter to sta
   STAREP(*,*)  report for results
   ;





*------------------------ACTIVE RECRUITERS-------------------------------
PARAMETER
   DELTA(S)  approx active contracts for sta without ASHR
   RA(S)     number of active recruiters to assign to station
   ;

BETA = B('ASHR');
DELTA(S)$(Y.L(S) EQ 1) = ( SUM(Z$(X.L(Z,S) EQ 1),
               (C(Z)*(D(Z,S)**B('DIST')))**(1/(1-BETA)) )**(1-BETA)
                 );

* Initialize values based on at least one recruiter to each open station
RA(S)  = 1$Y.L(S);
MRBFIT(S) = DELTA(S)*(2**BETA - 1**BETA)$Y.L(S);
NRCT   = SUM(S, RA(S));

LOOP ( RCNT$(NRCT LT NOPRA),

    MXBFIT = SMAX(S$Y.L(S), MRBFIT(S));
    MXFOUND = 0;

    LOOP ( S$( Y.L(S) EQ 1 AND MXFOUND EQ 0),
        IF ( MRBFIT(S) EQ MXBFIT,
```

```
            RA(S) = RA(S) + 1;
            NRCT = NRCT + 1;
            MRBFIT(S) = DELTA(S)*( (RA(S)+1)**BETA -
                              RA(S)**BETA   );
            MXFOUND = 1;
         ); {endif}
     ); {end loop}
); {end loop}


**** REPORT RESULTS ****
STAREP(S,'OLD OPRA') = INSTA(S,'OPRA');
STAREP(S,'NEW OPRA') = RA(S);
STAREP(S,'GSA') = (DELTA(S)*(RA(S)**BETA))$Y.L(S);
STAREP(S,'NZIPS') = SUM(Z, X.L(Z,S) );
STAREP('TOTAL','OLD OPRA') = SUM(S, INSTA(S,'OPRA'));
STAREP('TOTAL','NEW OPRA') = SUM(S, RA(S));
STAREP('TOTAL','GSA') = SUM(S, STAREP(S,'GSA'));


PARAMETER
  RDELTA(S)  APPROX RESERVE CONTRACTS FOR STA WITHOUT RSHR
  RDEL1(S)  APPROX RESERVE CONTRACTS FOR STA WITHOUT RSHR
  RDEL2(S)  APPROX RESERVE CONTRACTS FOR STA WITHOUT RSHR
  USAR(S)  NUMBER OF RESERVE RECRUITERS TO ASSIGN TO STATION
  ;


*------------------------RESERVE RECRUITERS------------------------------
*  Uses the Max Marginal Return heuristic

BETA = RB('RSHR');

RDEL1(S)$(Y.L(S) EQ 1) = SUM(Z5$(RX.L(Z5,S) EQ 1),
            (K(Z5)*(D(Z5,S)**RB('DIST')))**(1/(1-BETA)))**(1-BETA);

BETA = PB('RSHR');
RDEL2(S)$(Y.L(S) EQ 1) = SUM(Z5$(RX.L(Z5,S) EQ 1),
            (P(Z5)*(D(Z5,S)**PB('DIST')))**(1/(1-BETA)))**(1-BETA);


BETA = (RB('RSHR') + PB('RSHR'))/2;
RDELTA(S)$(Y.L(S) EQ 1) = SUM(Z5$(RX.L(Z5,S) EQ 1),
```

```
    (K(Z5)*(D(Z5,S)**RB('DIST'))+P(Z5)*(D(Z5,S)**PB('DIST'))
    )**(1/(1-BETA)))**(1-BETA)
                        ;

DISPLAY RDELTA, RDEL1, RDEL2;
*  Initialize values based on no reqd min recruiters at each open station
   USAR(S)  = 0;
   MRBFIT(S) = RDELTA(S);
   NRCT   = 0;

LOOP ( RCNT$(NRCT LT NOPAGR),

     MXBFIT = SMAX(S$(Y.L(S) EQ 1 AND USAR(S) LT 3), MRBFIT(S));
     MXFOUND = 0;

     LOOP ( S$( Y.L(S) EQ 1 AND MXFOUND EQ 0 AND USAR(S) LT 3),
          IF (MRBFIT(S) EQ MXBFIT,
             USAR(S) = USAR(S) + 1;
             NRCT = NRCT + 1;
             MRBFIT(S) = RDELTA(S)*( (USAR(S)+1)**BETA -
                                USAR(S)**BETA   );
             MXFOUND = 1;
          );  {endif}
     );  {end loop}
);  {end loop}


**** REPORT RESULTS  ****

STAREP(S,'OLD OPAGR') = INSTA(S,'OPAGR');
STAREP(S,'NEW OPAGR') = USAR(S);
STAREP(S,'NPS') = (RDEL1(S)*(USAR(S)**RB('RSHR')))$Y.L(S);
STAREP(S,'PS') = (RDEL2(S)*(USAR(S)**PB('RSHR')))$Y.L(S);
STAREP(S,'NRZIPS') = SUM(Z, RX.L(Z,S) );
STAREP('TOTAL','OLD OPAGR') = SUM(S, INSTA(S,'OPAGR'));
STAREP('TOTAL','NEW OPAGR') = SUM(S, USAR(S));
STAREP('TOTAL','NPS') = SUM(S, STAREP(S,'NPS'));
STAREP('TOTAL','PS') = SUM(S, STAREP(S,'PS'));



FILE SUMMARY /'%4 REPORT'/;
PUT SUMMARY
```

```
PUT'                    * * * BATTALION 1A * * *     '/;
PUT'            '/;
PUT'          AUTH     PROP    PRED    AUTH    PROP    PRED   PRED'/;
PUT'STATION   OPRA     OPRA    GSA     OPAGR   OPAGR   NPS    PS'/;
LOOP(S,
   PUT S.TL:10,
   STAREP(S,'OLD OPRA'):4:0, STAREP(S,'NEW OPRA'):10:0,
   STAREP(S,'GSA'):10:2,
   STAREP(S,'OLD OPAGR'): 8:0, STAREP(S,'NEW OPAGR'):8:0,
   STAREP(S,'NPS'):10:2, STAREP(S,'PS'):9:2 /;
*  PUT STAREP(S,'NZIPS'):15:0, STAREP(S,'NRZIPS'):10:0/;
  );
PUT 'TOTAL':10,
    STAREP('TOTAL','OLD OPRA'):4:0, STAREP('TOTAL','NEW OPRA'):10:0,
    STAREP('TOTAL','GSA'):10:2,
    STAREP('TOTAL','OLD OPAGR'): 8:0,STAREP('TOTAL','NEW OPAGR'):8:0,
    STAREP('TOTAL','NPS'):10:2,
    STAREP('TOTAL','PS'):9:2;
PUT // "STATION CONSTRAINT : " NS;
PUT /  "OPRA CONSTRAINT    : " NOPRA;
PUT /  "OPAGR CONSTRAINT   : " NOPAGR;
PUTCLOSE SUMMARY;
```

LIST OF REFERENCES

1. Department of the Army, Headquarters, *ARMY FOCUS 1993*, pp. 17-18, US Army Publication and Printing Command, Washington, DC, 1993.

2. Bohn, D. and Schmitz, E., "Station Location Analysis," Research Report, Navy Recruiting Command, Arlington, VA, 1992.

3. Schwartz, G.S., *Realigning the U.S. Navy Recruiting Command*, Master's Thesis, Naval Postgraduate School, Monterey, CA, March 1993.

4. Charnes, A., Cooper, W.W., and Rhodes, E., "Measuring the Efficiency of Decision Making Units", *European Journal of Operational Research*, v. 2, pp. 429-444, 1978.

5. U.S. Army Recruiting Command, Headquarters, *USAREC Manual 100-5*, USAREC, Fort Sheridan, IL, September 1989.

6. U.S. Army Recruiting Command, Headquarters, *USAREC Regulation 10-1*, USAREC, Fort Sheridan, IL, 1991.

7. Department of the Army, Headquarters, *ARMY POSTURE STATEMENT FY95*, pp. 31, US Army Publication and Printing Command, Washington, DC, 1994.

8. Celski, R.J., *Realignment of the U.S. Army Recruiting Command*, Master's Thesis, Naval Postgraduate School, Monterey, CA, September 1992.

9. Doll, J.M., *An Optimal Allocation of Recruiters and Facilities in the 12th Marine Corps District*, Master's Thesis, Naval Postgraduate School, Monterey, CA, 1992.

10. GAMS Development Corp., *GAMS/DICOPT: A Solver for Mixed Integer Nonlinear Programs*, Washington, DC, July 1993.

11. Springer, J.M., *An Interservice Comparison of Recruiting Efficiency Using Data Envelopment Analysis*, Master's Thesis, Naval Postgraduate School, Monterey, CA, March 1992.

12. Banker, R.D., and Morey, R., "Efficiency Analysis for Exogenously Fixed Inputs and Outputs", *Operations Research*, v. 34, no. 4, pp. 513-521, 1986.

13. CACI Marketing Systems, *Sourcebook of Zipcode Demographics*, 8th ed. 1993.

14. Brook, A., Kendrick, D., Meeraus, A., *GAMS, A User's Guide*, Release 2.25, The Scientific Press, 1992.

15. Devore, J.L., *Probability and Statistics for Engineering and the Sciences*, 3d ed., p. 120, Brooks/Cole Publishing Co., 1991.

16. Jacobs, P., "Poisson Regression", Naval Postgraduate School OA4301 Course Notes, Monterey, CA, Winter 1994.

17. Velleman, P.F., and Hoaglin, D.C., *Applications, Basics, and Computing of Exploratory Data Analysis*, pp. 281, Duxbury Press, 1981.

18. Schrage, L., *Linear, Integer, and Quadratic Programming with LINDO*, 3d ed. p. 195, The Scientific Press, 1986.

19. Brown, G.G., and Olsen, M.P., "Elastic Modeling with the X-System and GAMS", Internal Instruction, Operations Research Department, Naval Postgraduate School, Monterey, CA, 1993.

20. Bazaraa, M. S., and Shetty, C.M., *Nonlinear Programming: Theory and Algorithms*, John Wiley and Sons, 1979.

21. Ibaraki, T., and Katoh, N., *Resource Allocation Problems: Algorithmic Approaches*, The MIT Press, 1988.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria VA 22304-6145	2
2.	Library, Code 052 Naval Postgraduate School Monterey CA 93943-5002	2
3.	Dr. Siriphong Lawphongpanich (OR/LP) Department of Operations Research Monterey CA 93942-5004	3
4.	Dr. David Morton (OR/MO) Department of Operations Research Monterey CA 93942-5004	1
5.	CDR, U.S. Army Recruiting Command ATTN: USARC-PAE-MKT (LTC Beatty) Ft. Knox KY 40121	1
6.	CDR, U.S. Army Recruiting Command ATTN: USARC-PAE-MM-RM (MAJ Broadwater) Ft. Knox KY 40121	1
7.	CPT Michael J. Teague 1 Miller Lane West Stockbridge MA 01266	3