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Finite element methods for penetration mechanics are developed. A pinball contact-impact algorithm which is easily vectorizable has been implemented on partitioned memory SIMD computers. The pinball algorithm is further extended to problems with friction and erosion; Lagrange and augmented Lagrange multiplier methods, and its convergence have also been studied. Multiple-quadrature elements with hourglass control and physical stabilization and multi-time step integration have also been studied. Numerical results of the multiple-quadrature elements with stabilization showed that simple stabilization forces can be obtained which are convergent and based on physical parameters. The implementation of this algorithm on massively parallel machines has also been investigated.

14. SUBJECT TERMS
Finite element methods, penetration mechanics, pinball contact-impact algorithm, partitioned memory SIMD computers, friction and erosion, Lagrange and augmented Lagrange multiplier methods Multiple-quadrature elements, hourglass control, stabilization

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FINITE ELEMENT TECHNOLOGY FOR PENETRATION PROBLEMS

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A. Statement of the Problem Studied

Finite element methods for penetration mechanics are developed. A pinball contact-impact algorithm which is easily vectorizable has been implemented on partitioned memory SIMD computers. Multiple-quadrature elements with hourglass control and stabilization and multi-time step integration have also been studied. The essential feature of the pinball contact-impact algorithm is to treat the impenetrability condition between bodies and the gap between bodies in contact in terms of the interpenetration of spheres embedded in the elements of the respective bodies. The pinball algorithm is further extended to problems with friction and erosion; Lagrange and augmented Lagrange multiplier methods, and its convergence have also been studied. Numerical results of the multiple-quadrature elements with stabilization showed that simple stabilization forces can be obtained which are convergent and based on physical parameters. The implementation of this algorithm on massively parallel machines has also been investigated.

B. Summary of the Most Important Results

New multiple-quadrature-point under-integrated finite elements with hourglass control are developed. The elements are selectively under-integrated to avoid volumetric and shear locking and save computational time. An approach for hourglass control is proposed such that the stabilization operators are obtained simply by taking the partial derivatives of the generalized strain rate vector with respect to the natural coordinates so that the elements require no stabilization parameter. To improve accuracy over the traditional one-point-quadrature elements, several quadrature points are used to integrate the internal forces, especially for tracing the plastic fronts in the mesh during loading and unloading in elastic-plastic analysis. Two-point and four-point quadrature elements are proposed for use in the two and three dimensional elements, respectively. Other multiple quadrature points can also be employed. Several numerical examples such as thin beam, plate and shell problems are presented to demonstrate the applicability of the proposed elements.

New element formulations based on extensions of assumed strain methods have been developed. It was found that by suppressing certain terms in the shear strains and modifying the extensional strains so that the higher order terms in the volumetric strains always vanish, the performance of the element in nearly incompressible cases, such as elastic-plastic materials, is enhanced dramatically. It was furthermore found that these assumed strain fields could be treated either with one-point quadrature and stabilization based on the higher order terms or by 4-point quadrature without stabilization. The latter is more expensive, but it compensates for this by increased accuracy whenever elastic-plastic fronts occur in the element.

New and more stable methods for subcycling have been developed for the time integration of the equations of motion. Subcycling is crucial for efficient explicit computer solutions of many problems, especially those involving h-adaptivity, because in explicit solutions the critical time step is set by the smallest element, so a few small elements can result in order of magnitude increases in running time. With subcycling, only the smaller elements are integrated at the smaller time step, and in fact in some implementations, all elements are integrated close to their stable time step. In previous versions of subcycling, a constant velocity interpolation was used on nodes connected to elements with different time steps. In this implementation, a constant acceleration interpolation is used. Numerical studies show that it is substantially more stable. A partial proof of stability has been developed for this formulation; however, certain key steps in the stability proof still need to be resolved.

The pinball contact-impact algorithm has been extended by incorporating an automatic splitting of pinballs when the elements are too irregular to permit accurate solutions on the basis of a single pinball. This formulation has been implemented for shell elements and it has been shown that highly accurate solutions can be obtained.

We have performed an analysis of the consistency and accuracy of SPH methods and shown that consistency is not satisfied for an irregular finite SPH grid. Its accuracy is also poor in the vicinity of boundaries, as boundary conditions are not enforced effectively in current SPH methods. Since the accurate treatment of boundaries is crucial to the successful solution of problems of fracture mechanics or contact-impact, the success of SPH methods in penetration problems has been quite limited. Two gridless methods, moving least mean square interpolants and kernel functions, similar to smooth particle hydrodynamics (SPH) with correction functions, have been explored. A key feature of these methods is the introduction of a correction function on the kernel which is active primarily in the vicinities of the boundaries. We have learned that this correction function can dramatically improve the accuracy and stability of the kernel.

Introduction

Simulation of penetration and other nonlinear behavior can be of great benefit to the Army because it can improve the design of new weapons and weapon countermeasures. However, at the present time, these simulations often require as much as 100 hours of supercomputer time and still lack sufficient resolution of the important physical phenomena, which impairs their usefulness. This research is aimed at improving finite elements for penetration mechanics both from the viewpoints of speed and accuracy, the development of improved contact-impact algorithm and the implementation of these procedures on massively parallel computers.

Technical Objectives and Approach

The purpose of this project is to improve finite element methods for highly nonlinear problems such as penetration problems with particular emphasis on element stability and accuracy, contact-impact algorithms and parallelization of these algorithms. In addition, work has been started on gridless methods since they appear very promising in dealing with problems of penetration and failure.

Two new formulations of the 8-node hexahedral element for nonlinear analysis have been developed. Both forms share the characteristics that they are more stable than existing elements and do not require user-input of stabilization parameters. As a consequence, they permit efficient and accurate computation of three-dimensional penetration problems. These computations, when performed with tetrahedral elements, are often limited in resolution because they require as much as a 100 hours of supercomputer time. Compared to tetrahedral, these new elements are significantly faster and they retain the robustness of fully integrated elements.

Work is progressing on the implementation of these methods on the massively-parallel computer CM5. We have now developed a MIMD version of the exchange algorithm originally developed for the CM2. This algorithm halves the number of communications required during a time step, so it significantly speeds up the algorithm. Although our initial implementations were quite slow because they used message-passing, our latest implementation uses the gather operation in the CM5 to perform the exchange. The resulting algorithm is significantly faster than algorithms requiring both a gather and a scatter.

Our recent work has focused on gridless or meshless methods. A key breakthrough in this work has been our discovery that gridless methods such as smooth particle methods can be viewed as collocation methods based on moving least square interpolants. These constructs then enable us to overcome the stability and accuracy problems which have plagued SPH methods. At the present time, these stabilizing procedures are still quite expensive, but several promising avenues to improving their efficiency are under investigation.

C. List of All Publications and Technical Reports

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INVITED PAPER: Liu, W., K., "Multiple Scale Reproducing Kernel Methods" U. S. Association for Computational Mechanics August 16-18, 1993.

KEYNOTE: Liu, W., K., "Multiple Scale Reproducing Kernel and Wavelet Methods," *3rd World Congress for Computational Mechanics*, Chiba, Japan, August 1–5, 1994.

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INVITED PAPER: T. Belytschko, "Adaptive Methods for Nonlinear Structural Dynamics and Crashworthiness Analysis", NASA Workshop on Computational Methods for Crashworthiness, Sept.2-3, NASA Langley.

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KEYNOTE: T. Belytschko, "Computational Methods for contact-impact" Impact IV, August 23-24, Berlin, Germany.

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INVITED PAPER. "New Trends in High Resolution Algorithms", USA/ France Workshop on Numerical Simulation of Metal Forming Processes, Sophia Antipolis, France, September 27-29, 1993.

INVITED PAPER. "Parallel Computations of Shear Band Formation by Finite Elements" Symposium on Parallel Finite Element Computations, Supercomputer Institute, University of Minnesota, October 24-27, 1993.

INVITED PAPER. "Adaptive and Element Free Methods", International Symposium on Highly Advanced Computing, Sendai, Japan, November 9-11, 1993.

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KEYNOTE. "Element Free Galerkin Methods for Static and Dynamic Crack Propagation", 3rd World Congress for Computational Mechanics, Tokyo, Japan, August 1-5, 1994.

D. List of All Participating Scientific Personnel Showing any Advanced Degrees Earned by Them While Employed on the Project

Lee P. Bindeman, Ph. D., June 1992
I.S. Yeh, Ph. D., September1992
H.Y. Chiang, Ph.D., June 1993
Dan Organ

Report of Inventions

None.