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## ELECTROMAGNETIC SCATTERING BY MAGNETIC SPHERES: THEORY AND ALGORITHMS

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RESEARCH AND TECHNOLOGY DIRECTORATE

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# ELECTROMAGNETIC SCATTERING BY MAGNETIC SPHERES: THEORY AND ALGORITHMS

## 1. INTRODUCTION

The scattering of electromagnetic radiation by spheres is a classic physics problem for which the theoretical solution has been known for well over fifty years. The development of the theory for the scattering of light by a sphere, which culminated in the general solution given by Gustav Mie,<sup>1</sup> dates to the late nineteenth and early twentieth centuries. Usually the Mie theory is formulated to describe the scattering for materials for which the magnetic permeability ( $\tilde{\mu}$ ) is one. Kerker<sup>2</sup> has explored a number of the unusual scattering effects for magnetic spheres. The approach taken here is to develop the theory and algorithms for scatterers with both a complex permittivity ( $\tilde{\epsilon}$ ) and a complex permeability; the usual Mie solution for a homogenous, nonmagnetic spherical scatterer will then appear as a special case of this more general theory.

The computational history of these scattering problems is much shorter than that of the formal solution since scattering computations flourished only with the development of electronic computers in latter half of this century. The number of scattering calculations carried out before the modern computer era was understandably small since the computational labor involved in evaluating scattering functions such as Ricatti-Bessel functions for complex arguments was extreme. Electronic computation brought problems of its own: early computer algorithms were plagued with numeric difficulties that stemmed principally from ill-conditioning produced by subtraction of nearly equal numbers and instability in recursion relations. Gradually several generally reliable scattering subroutines appeared;<sup>4-6</sup> these Mie scattering codes have been the mainstay of researchers in the atmospheric sciences over the past two decades.

Advances in computational technique have led to the development of a general purpose subroutine package<sup>7</sup> for computing Bessel functions of complex argument and nonnegative order. These subroutines fill a void in complex Bessel function software and implement features such as multiple computational schemes, exponential scaling, and internal error checking including underflow and overflow detection, which until now have not been incorporated in electromagnetic scattering codes. The purpose of this paper is to formulate the electromagnetic scattering problem and computational algorithms for magnetic spheres to take maximum advantage of the features of these general purpose Bessel function routines.

In what follows the theory for the scattering of electromagnetic radiation by magnetic spheres is developed by means of scaling functions. This theory leads in a natural way to the development of scattering algorithms which use exponential scaling to overcome computational overflow problems. The design and testing of the algorithm is described. Finally, a Fortran code which implements the algorithmic design is presented and examples of its usage are given.

## 2.

## ELECTROMAGNETIC SCATTERING THEORY

For spheres, the efficiencies (ratio of the optical cross section to the geometric cross section) for extinction,  $Q_e$ , scattering,  $Q_s$ , absorption  $Q_a$ , backscatter,  $Q_b$ , and the amplitude functions ( $S_1, S_2$ ) are:<sup>8</sup>

$$Q_e = \frac{2}{X^2} \sum_{n=1}^{\infty} (2n+1) \operatorname{Re}(a_n + b_n) \quad (1)$$

$$Q_s = \frac{2}{X^2} \sum_{n=1}^{\infty} (2n+1) [ |a_n|^2 + |b_n|^2 ] \quad (2)$$

$$Q_a = Q_e - Q_s \quad (3)$$

$$Q_b = \frac{1}{X^2} \left| \sum_{n=1}^{\infty} (-1)^n (2n+1) (a_n - b_n) \right| \quad (4)$$

$$S_1(\theta) = \sum_{n=1}^{\infty} \frac{(2n+1)}{n(n+1)} [a_n \pi_n(\mu_1) + b_n \tau_n(\mu_1)] \quad (5)$$

$$S_2(\theta) = \sum_{n=1}^{\infty} \frac{(2n+1)}{n(n+1)} [a_n \tau_n(\mu_1) + b_n \pi_n(\mu_1)] \quad (6)$$

The angular scattering functions ( $\pi_n, \tau_n$ ) corresponding to the scattering angle ( $\theta$ ) are defined in terms of the Legendre polynomials ( $P_n$ )<sup>9</sup> as:

$$\pi_n(\mu_1) = P'(\mu_1) \quad (7)$$

$$\tau_n(\mu_1) = \mu_1 \pi_n(\mu_1) - (1 - \mu_1^2) \pi_n'(\mu_1) \quad (8)$$

where  $\mu_1 = \cos(\theta)$ . All of which are given in terms of the scattering coefficients ( $a_n, b_n$ ):<sup>2</sup>

$$a_n = \frac{\tilde{\mu} U_n^s(m, X) - m V_n^s(m, X)}{\tilde{\mu} W_n^s(m, X) - m Z_n^s(m, X)} \quad (9)$$

$$b_n = \frac{m U_n^s(m, X) - \tilde{\mu} V_n^s(m, X)}{m W_n^s(m, X) - \tilde{\mu} Z_n^s(m, X)} \quad (10)$$

where

$$\begin{aligned}
 U_n^s(m, X) &\equiv \psi_n(X) \psi_n'(mX) \\
 V_n^s(m, X) &\equiv \psi_n(mX) \psi_n'(X) \\
 W_n^s(m, X) &\equiv \zeta_n(X) \psi_n'(mX) \\
 Z_n^s(m, X) &\equiv \psi_n(mX) \zeta_n'(X)
 \end{aligned}
 \tag{11}$$

$\psi_n, \zeta_n$  are the Riccati-Bessel functions and are defined in terms of the  $n$ th order spherical Bessel functions of the First and Second Kind ( $j_n, y_n$ ) as  $\psi_n(z) = zj_n(z)$  and  $\zeta_n(z) = z[j_n(z) - iy_n(z)]$ .<sup>9</sup>  $X = \pi D/\lambda$  is the size parameter where,  $D$ , is the diameter;  $\lambda$ , the wavelength; and  $m = \sqrt{\tilde{\mu}\tilde{\epsilon}}$ , the refractive index for which  $\tilde{\mu}$ , the permeability, and  $\tilde{\epsilon}$ , the permittivity, are complex quantities. Primes denote derivatives with respect to the argument.

It will now be shown that the scattering coefficients can be expressed directly in terms of the Bessel functions of half-integer order only. The Riccati-Bessel functions are related to the spherical Bessel functions as described above, and the spherical Bessel functions are related to the half-integer order Bessel functions by:<sup>9</sup>

$$\begin{aligned}
 j_n(z) &= \sqrt{\frac{\pi}{2z}} J_{n+1/2}(z) \\
 y_n(z) &= \sqrt{\frac{\pi}{2z}} Y_{n+1/2}(z)
 \end{aligned}
 \tag{12}$$

The above equations can be used with recurrence relations for derivatives<sup>9</sup> and (11) to find the scaled functions:

$$\begin{aligned}
 \tilde{U}_n^s(m, X) &= J_{n+1/2}(X) J_{n+1/2}'(mX) \\
 \tilde{V}_n^s(m, X) &= J_{n+1/2}(mX) J_{n+1/2}'(X) \\
 \tilde{W}_n^s(m, X) &= H_{n+1/2}^{(2)}(X) J_{n+1/2}'(mX) \\
 \tilde{Z}_n^s(m, X) &= J_{n+1/2}(mX) H_{n+1/2}^{(2)'}(X) .
 \end{aligned}
 \tag{13}$$

The set of all scaled functions ( $\tilde{g}_1$ ) is related to the set of all unscaled functions ( $g_i$ ) by:

$$\begin{aligned}\tilde{g}_1 &= \left\{ \tilde{U}_n^s(m, X), \tilde{V}_n^s(m, X), \tilde{W}_n^s(m, X), \tilde{Z}_n^s(m, X) \right\} \\ &= \left\{ \frac{2}{\pi X \sqrt{m}} g_s \right\}\end{aligned}\quad (14)$$

where

$$g_s \equiv \left\{ U_n^s(m, X), V_n^s(m, X), W_n^s(m, X), Z_n^s(m, X) \right\}.$$

Since the sphere scattering coefficients are invariant when the scaled functions,  $\tilde{g}_1$ , are used to compute them, the scattering coefficients may be expressed in the form:

$$a_n = \frac{\sqrt{\tilde{\mu}} J_{n+\frac{1}{2}}(X) J'_{n+\frac{1}{2}}(mX) - \sqrt{\tilde{\epsilon}} J_{n+\frac{1}{2}}(mX) J'_{n+\frac{1}{2}}(X)}{\sqrt{\tilde{\mu}} H_{n+\frac{1}{2}}^{(2)}(X) J'_{n+\frac{1}{2}}(mX) - \sqrt{\tilde{\epsilon}} J_{n+\frac{1}{2}}(mX) H_{n+\frac{1}{2}}^{(2)'}(X)} \quad (15)$$

$$b_n = \frac{\sqrt{\tilde{\epsilon}} J_{n+\frac{1}{2}}(X) J'_{n+\frac{1}{2}}(mX) - \sqrt{\tilde{\mu}} J_{n+\frac{1}{2}}(mX) J'_{n+\frac{1}{2}}(X)}{\sqrt{\tilde{\epsilon}} H_{n+\frac{1}{2}}^{(2)}(X) J'_{n+\frac{1}{2}}(mX) - \sqrt{\tilde{\mu}} J_{n+\frac{1}{2}}(mX) H_{n+\frac{1}{2}}^{(2)'}(X)} \quad (16)$$

This scaling of the scattering coefficients is referred to as scaling of the first kind; in developing the algorithms for computing the scattering by spheres a scaling of the second kind will be employed.

### 3. SCATTERING ALGORITHMS

Based on Equations 15 and 16 and a set of subroutines<sup>7</sup> that compute Bessel functions for complex arguments and nonnegative order, the development of algorithms for computing the scattering by spheres will be described.

One of the first decisions to be made when constructing a scattering algorithm is the number of terms to be included in the sums for cross sections and scattered amplitudes. Dave<sup>4</sup> adopted a dynamic procedure for terminating the Mie series summations, which as pointed out by Wiscombe<sup>10</sup> can fail in some circumstances. Wiscombe<sup>4,10</sup> replaced Dave's

procedure for estimating the number of terms in the Mie series summations with an improved *a priori* calculation of the number of terms required for convergence ( $N$ ):

$$N = X + c X^{\frac{1}{3}} + n_m, \quad (17)$$

where  $c$  and  $n_m$  are numeric constants, and  $N$  is the largest integer that does not exceed the value of  $N(X)$ . Bohren and Huffman<sup>5</sup> employed the same type of function for sphere calculations. This type of estimate was first suggested, without the " $n_m$ " term, by Khare.<sup>11</sup> The practical significance of  $n_m$  is to provide a lower bound on the number of summation terms.  $n_m$  becomes the total  $N$  when the integer part of  $X + cX^{1/3} < 1$ .

Based on numerical tests of sphere scattering codes for size parameters in the range  $0.001 \leq X \leq 5000$  the effective minimum number of Bessel function orders is set at three, and  $c = 4$ . This corresponds to  $X_{\min} \approx 1.5 \times 10^{-2}$ . In practice this was accomplished by setting the number of Bessel function orders (starting with order zero) by using a function as specified by Equation 15 for  $c = n_m = 4$ . The zero order functions are not used directly in the Mie sums, but are used to find the Bessel function derivatives.

As pointed out by Dave,<sup>3</sup> computation of the scattering coefficients for complex values of the refractive index leads to potential overflow conditions since functions that increase exponentially with  $kX$  are required. The standard method for overcoming this computational problem is to employ logarithmic derivatives of the Riccati-Bessel functions ( $\psi'_n/\psi_n, \zeta'_n/\zeta_n$ ), a technique that dates back to Infield<sup>12</sup> (1947). Here another approach has been taken to this problem: the use of exponential scaling.

Scaled values of the Bessel functions appearing in the unified expressions for the scattering coefficients (Equations 13 and 14) are available as an option in the Amos<sup>7</sup> subroutine package; these functions are defined as:

$$\begin{aligned} J_{n+1/2}^\sigma &= e^{-|kX|} J_{n+1/2} \\ Y_{n+1/2}^\sigma &= e^{-|kX|} Y_{n+1/2} \end{aligned} \quad (18)$$

This gives rise to a scaling of the second kind for which:

$$\begin{aligned} \bar{g}_2 &= \left\{ \bar{U}_n(m, X), \bar{V}_n(m, X), \bar{W}_n(m, X), \bar{Z}_n(m, X) \right\} \\ &= \left\{ e^{-|kX|} g \right\} \end{aligned} \quad (19)$$

where

$$g \equiv \left\{ U_n(m, X), V_n(m, X), W_n(m, X), Z_n(m, X) \right\}.$$

$U_n$ ,  $V_n$ ,  $W_n$ , and  $Z_n$  are the previously defined functions.

The scattering coefficients are invariant to any combination of these scalings so that no algorithmic compensation for scaling is required. Exponential overflow and underflow limits for the Bessel function package are machine dependent quantities that are defined as:

$$\begin{aligned} e^{\pm\delta} &= B^{\pm K} / 10^{\pm 3} \\ e^{\pm\epsilon} &= (e^{\delta} \cdot \Delta)^{\pm 1} \end{aligned} \quad (20)$$

where  $\delta$  = overflow or underflow limits; epsilon = near - overflow or near - underflow limit; B = number base; K = maximum exponent; and  $\Delta$  = unit round - off. Overflow quantities are produced by the upper signs; underflow quantities are produced by the lower signs. An offset of  $10^3$  is included to allow for some impreciseness in tests. The near-overflow condition is a convenient criterion for shifting from unscaled to scaled Bessel functions; for  $|kX| > \epsilon$  scaled functions are used.

Computation of the angular scattering presents no particular difficulties. For the scattered amplitudes the procedure advocated by Wiscombe<sup>10</sup> has been adopted. In this procedure scattered amplitudes are cumulated in a loop over the summation index, n, which is nested within a loop over the scattering angles. The arithmetic is more efficient if the quantities:

$$\begin{aligned} S^+(\mu_1) &= S_1(\mu_1) + S_2(\mu_1) \\ S^-(\mu_1) &= S_1(\mu_1) - S_2(\mu_1) \end{aligned} \quad (21)$$

are computed.  $S_1$  and  $S_2$  are then computed outside of the n-loop from:

$$\begin{aligned} S_1(\mu_1) &= 1/2 [S^+(\mu_1) + S^-(\mu_1)] \\ S_2(\mu_1) &= 1/2 [S^+(\mu_1) - S^-(\mu_1)]. \end{aligned} \quad (22)$$

The angular functions are computed from the recursion relations:<sup>10</sup>

$$\pi_{n+1}(\mu_1) = \mu_1 \pi_n(\mu_1) + \left(\frac{n+1}{n}\right) [\mu_1 \pi_n(\mu_1) - \pi_{n-1}(\mu_1)] \quad (23)$$

$$\tau_n(\mu_1) = n [\mu_1 \pi_n(\mu_1) - \pi_{n-1}] - \pi_{n-1}(\mu_1) \quad (24)$$

The usual procedure is to initialize these functions with  $\pi_0 = 0$  and  $\pi_1 = 1$ . However, it has been found that by changing the initialization for  $\pi_1$  to  $\pi_1 = 1/2$  the post n-loop multiplications in Equation 22 can be avoided.

Algorithmic testing was carried out by comparison with special cases, limiting cases, and published results. The code was checked for internal consistency according to several *a priori*<sup>5</sup> criteria: no calculation ever yielded extinction and scattering efficiencies that were negative, the extinction efficiency was always greater than the scattering efficiency except for nonabsorbing particles for which these efficiencies were found to be equal, and predictions of the asymptotic limits of the efficiencies for both very small and very large size parameters were verified. Published results were used to confirm the calculations for both efficiencies and angular scattering quantities. The tabulated values of Wiscombe,<sup>10</sup> Dave,<sup>3</sup> and Bohren and Huffman<sup>5</sup> were used to check the code.

The maximum size parameter for which a valid scattering calculation can be done is determined by the ability to compute  $J_\nu(mX)$  (Equations 15 and 16). Scaled values of these Bessel functions are computed by means of a uniform asymptotic expansion for  $X$ ,  $\nu \gg 1$  which is valid for

$$\nu \cdot \text{Re} \left\{ \ln \left[ \frac{1 + \sqrt{1+z^2}}{z} \right] - \sqrt{1+z^2} + z \right\} > \delta, \quad (25)$$

where  $z^2 = - \left( \frac{mX}{\nu} \right)^2$ ,  $|mX| \leq \sqrt{\Gamma}$ , and  $2 \cdot \Gamma =$  largest positive machine integer. The

first two terms on the left in Equation 25 arise from the theory for the uniform asymptotic expansion of Bessel functions,<sup>9</sup> and the last term comes about because scaled functions are being computed. For large size parameters it is known<sup>4</sup> that  $\nu \approx X$  which leads to the following approximations for estimating the largest size parameters for which valid scattering calculations can be performed:

$$X_{\max} = \frac{\delta}{\text{Re} \left\{ \ln \left[ \frac{1 + \sqrt{1+m^2}}{im} \right] - \sqrt{1+m^2} + im \right\}}$$

(26)

or

$$X_{\max} = \frac{\sqrt{\Gamma}}{|m|}$$

The Figure shows a plot of results computed from Equation 26 on an AT&T 3B2-600G computer, which had the machine constants:  $\delta \approx 702$ ,  $2 \cdot \Gamma = 2^{31} - 1$ . The Table displays a comparison of directly computed size parameters with the approximate values at the "three corners" shown in the Figure, and the maximum and minimum size parameters.

Table. Size Parameter Comparisons\*

n	k	$X_{\max}$	$X_{\min}$	$X_{\text{est}}$
1.05	0.95	2697	2700	2841
1.05	5.00	6401	6421	6414
5.00	5.00	4614	4635	4634
5.00	0.10	6468	6899	6552
1.40	0.10	23098	23118	23346

\*n and k are the real and imaginary parts respectively of the complex refractive index.  $X_{\max}$  is the largest size parameter for which a calculation was completed.  $X_{\min}$  is the smallest size parameter for which the calculation failed.  $X_{\text{est}}$  is the estimate of  $X_{\max}$  from Equation 29. Size parameters for finding  $X_{\max}$  and  $X_{\min}$  were equally spaced logarithmically with 50 values per decade.

In general the approximation overestimates the size parameter for which valid calculations can be performed; the maximum overestimation based on the data shown is on the order of five percent.

#### 4. RESULTS AND DISCUSSION

The listings of a main (driver) program (msmain), the magnetic sphere scattering program (magsph) and associated programs are given in Appendix A. For each



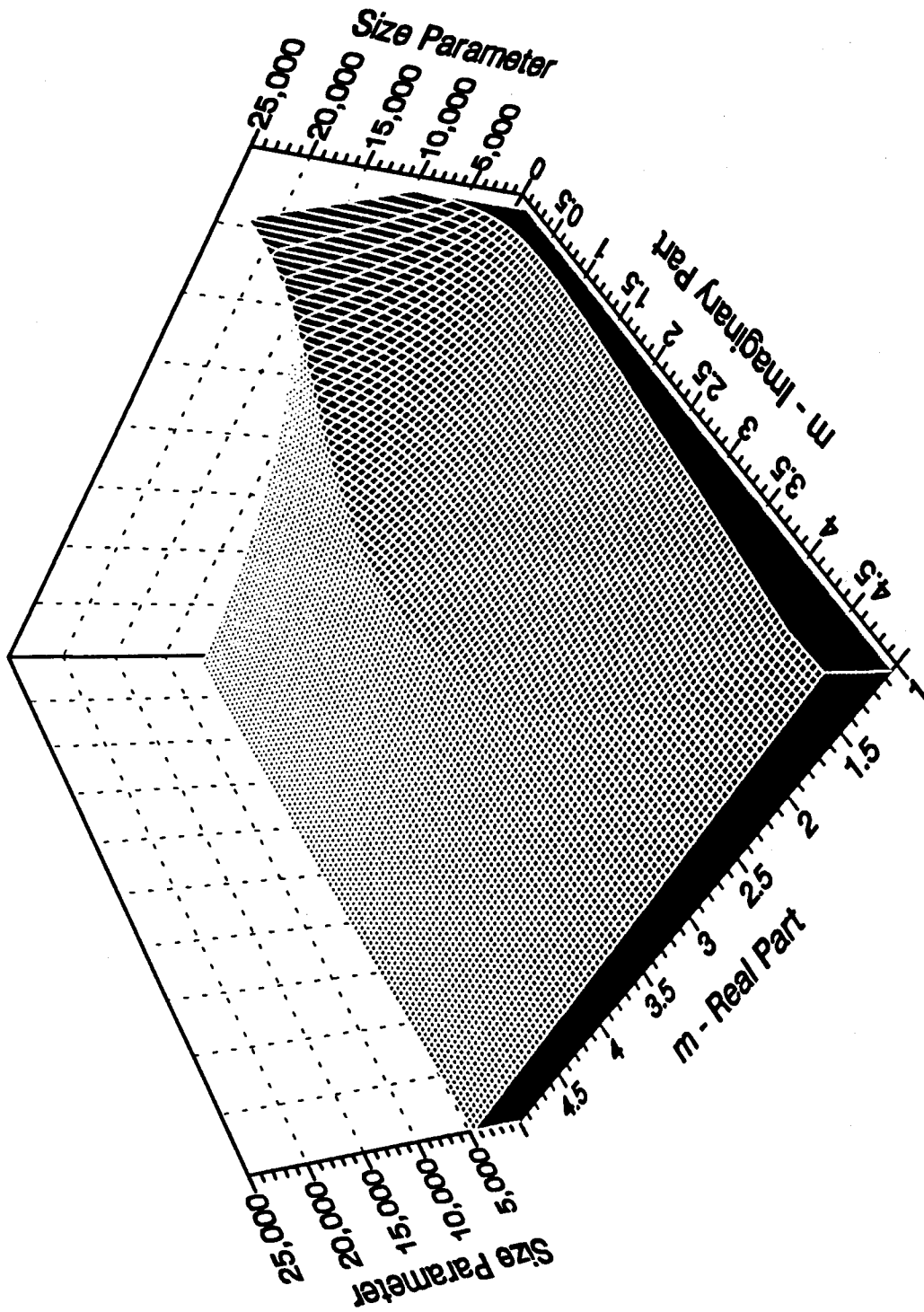


Figure. Maximum Size Parameter Estimates for Valid Scattering Calculations as a Function of the Real and Imaginary Parts of the Refractive Index

scattering angle the program computes the complex scattered field amplitudes ( $S_1$ ,  $S_2$ ), the scattered intensity

$$I = 1/2 [ |S_1|^2 + |S_2|^2 ], \quad (27)$$

the degree of polarization,

$$P = \frac{[ |S_2|^2 - |S_1|^2 ]}{[ |S_2|^2 + |S_1|^2 ]}, \quad (28)$$

the Mueller matrix elements ( $S_{11}$ ,  $S_{12}$ ,  $S_{33}$ ,  $S_{34}$ ) which are nonzero, independent quadratic combinations of  $S_1$  and  $S_2$  and elements of a 4 x 4 matrix relating the incident Stokes vector,<sup>5</sup>

$$\begin{aligned} S_{11} &= 1/2 ( |S_2|^2 + |S_1|^2 ) \\ S_{12} &= 1/2 ( |S_2|^2 - |S_1|^2 ) \\ S_{33} &= 1/2 ( S_2^* S_1 + S_2 S_1^* ) \\ S_{34} &= 1/2 ( S_2^* S_1 - S_2 S_1^* ) \end{aligned} \quad (29)$$

and the polarization

$$p = -S_{12}/S_{11}. \quad (30)$$

Efficiency factors (extinction, scattering, absorption, and backscatter) and the asymmetry factor,<sup>5</sup>  $g$ , ( $\langle \cos\theta \rangle$ ) given by,

$$g = \frac{4}{X^2 Q_s} \sum_{n=1}^{\infty} \frac{n(n+2)}{n+1} \text{Re} [ a_n a_n^* + b_n b_{n+1}^* ] + \frac{2n+1}{n(n+1)} \text{Re} [ a_n b_n^* ] \quad (31)$$

are also computed. If the sphere is small, a comparison is made between the rigorous magsph calculation and an approximate calculation valid for small spheres (msphsx).

Appendix B lists input data to msmain and then displays the resulting output for two sample calculations. The first sample problem computes the scattering from spheres with a refractive index of (1.5, -.1), a permeability of (1.0, 0.), and size parameters of 0.01

and 10.0. The results from the latter size parameter may be compared with the published results of Wiscombe.<sup>10</sup> The second sample problem computes the scattering from a magnetic sphere with  $\tilde{\mu} = \tilde{\epsilon} = (2.24, -0.3)$  and the same size parameters as above. As noted by Kerker<sup>2</sup> the backscatter from a sphere with the same permeability and permittivity vanishes; the computed results illustrate this.

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APPENDIX A  
PROGRAM LISTINGS

program msmain

```
C*****
C
C Program msmain computes the electromagnetic scattering for magnetic
C spheres. If the sphere is small a comparison is made between the
C results of a rigorous, complete calculation and an approximate
C calculation valid for small spheres.
C
C Merrill Milham >>> version 1.0 <<< JANUARY 1994
C
C inputs:
C ALL INPUT IS IN LIST DIRECTED FORMAT
C
C line #1
C flag = 'r' for refractive index data or
C 'p' for permittivity data (character*1)
C mior = number of complex indexes or
C complex permittivities to be read (integer)
C mp = complex index or permittivity values
C to be read, up to 10 values (complex*16)
C
C line #2
C muu = complex permeability values to be read
C mior values are required (complex*16)
C
C line #3
C nx = number of size parameters to be read (integer)
C x = size parameter values to be read (real*8)
C line #4 anginc = angular increment in degrees which is
C added to zero to produce the angles at
C which the scattering is to be produced.
C Values of anginc are restricted to those
C for which mod(90,anginc).eq.zero. (real*8)
C
C output:
C For each angle the following quantities are given:
C The complex scattered field amplitudes, the
C scattered intensity, the degree of polarization,
C the Mueller matrix elements, and the polarization.
C
C Efficiency factors for extinction, scattering, absorption, and
C backscatter, and the asymmetry factor.
C
C subroutines used:
C
C magsph - returns computed scattering quantities for a
C homogeneous magnetic sphere or homogeneous
C nonmagnetic sphere if the permeability = (1,0)
C
C amuelr - returns elements of the Mueller matrix for a
C spherical scatterer
C
C msphsx - returns approximate scattering quantities for a
C small homogeneous, magnetic sphere or a small,
```

```

c                                     homogeneous, nonmagnetic sphere if the
c                                     permeability = (1,0)
c
c*****
c
c      implicit none
*
c      integer nangl
c      real*8 one,zero
c      parameter (nangl=255,one=1.d0,zero=0.d0)
*
c      complex*16  s1(nangl), s2(nangl),sx1(nangl),sx2(nangl)
c      complex*16  eps,mu,muu(10),mp(10)
*
c      real*8  i1,i2,inten,theta(nangl),xx,x(10),anginc
c      real*8  isx1,isx2,intsx,dgplsx,angle,degpol
c      real*8  s11(nangl),s12(nangl),s33(nangl)
c      real*8  s34(nangl),s11nor,pol(nangl)
c      real*8  sx11(nangl),sx12(nangl),sx33(nangl)
c      real*8  sx34(nangl),sxpole(nangl)
c      real*8  qext,qscq,qbac,g,qabs
c      real*8  qesx,qssx,qasx,qbsx
*
c      integer sign,mior,i,nx,nmang1,nmang2,nior,nxx
*
c      logical xflag
*
c      character*1 flag
c
c      write(*,*) 'read(*,*) flag,mior,(mp(i),i=1,mior)'
c      read(*,*) flag,mior,(mp(i),i=1,mior)
c      write(*,*) 'read(*,*) (muu(i),i=1,mior)'
c      read(*,*) (muu(i),i=1,mior)
c      write(*,*) 'read(*,*) nx,(x(i),i=1,nx)'
c      read(*,*) nx,(x(i),i=1,nx)
c      write(*,*) 'read(*,*) anginc'
c      read(*,*) anginc
c      write(*,*)
c
c      if(dmod(90.d0,anginc).eq.zero) then
c          continue
c      else
c          stop 'angular increment error'
c      end if
*
c      nmang1=(180.d0/anginc)+1
c      nmang2=(90.d0/anginc)+1
*
c      do 1 i = 1,nmang1
c      1 theta(i) = dble(i-1)*anginc
c
c      do 100 nior = 1,mior

```



```

if(flag.eq.'p') then
  eps=mp(nior)
  else if(flag.eq.'r') then
    eps=(mp(nior))**2
  else
    stop 'material properties input error'
end if

mu=muu(nior)

*
sign=1
c
do 100 nxx = 1,nx
  xx=x(nxx)
  xflag=xx*zabs(zsqrt(mu*eps)).lt.one
*
write(*,1008) xx,eps,mu
if(flag.eq.'r') write(*,1000) xx,mp(nior)
c
call magsph(xx,eps,mu,nmang2,theta,qext,qzca,qbac,g,s1,s2)
qabs = qext-qzca
*
s1lnor=0.d0
call amuelr(s1,s2,theta,nmang1,sign,s1lnor,s11,s12,s33,s34,pol)
c
if(xflag) then
  call msphsx(xx,mu,eps,nmang2,theta,qesx,qssx,qasx,qbsx,sx1,sx2)
  s1lnor=0.d0
  call amuelr(sx1,sx2,theta,nmang1,sign,s1lnor,sx11,sx12,sx33,sx34
&                                     ,sxpole)
  else
  continue
end if
c
do 10 i = 1,nmang1
  angle = theta(i)
*
  i1 = (dble(s1(i)))**2+(dimag(s1(i)))**2
  i2 = (dble(s2(i)))**2+(dimag(s2(i)))**2
  inten = 0.5*(i1+i2)
  if(.not.(i1.eq.zero.and.i2.eq.zero)) then
    degpol = (i2-i1)/(i2+i1)
  else
  write(*,*)
  write(*,*) 'degree of polarization undefined for ',angle,' deg'
  write(*,*)
  degpol=zero
end if
*
if(xflag) then
  isx1 = (dble(sx1(i)))**2+(dimag(sx1(i)))**2
  isx2 = (dble(sx2(i)))**2+(dimag(sx2(i)))**2

```

```

intsx = 0.5*(isx1+isx2)
if(.not.(isx1.eq.zero.and.isx2.eq.zero)) then
dgplsx = (isx2-isx1)/(isx2+isx1)
                                else
write(*,*)
write(*,*)'degree of polarization undefined for',angle,' deg.(sx)'
write(*,*)
dgplsx=zero
end if
                                else
continue
end if

*
write(*,1004)
write(*,1001) angle,s1(i),s2(i)
if(xflag) write(*,1001) angle,sx1(i),sx2(i)
write(*,*)
write(*,1005)
write(*,1006) inten,degpol
if(xflag) write(*,1006) intsx,dgplsx
write(*,*)
write(*,1007)
write(*,1003) s11(i),s12(i),s33(i),s34(i),pol(i)
if(xflag) write(*,1003) sx11(i),sx12(i),sx33(i),sx34(i),sxp1(i)
10 write(*,*)
write(*,1002) qext,qscat,qabs,g,qbac
if(xflag) write(*,1002) qesx,qssx,qasx,g,qbsx
write(*,*)
100 continue
c
stop
c
1000 format(1x,'mie size parameter =',f10.5,5x,'refractive index ='
+ ,f7.3,e12.3//)
1001 format(f7.2,4e14.6)
1002 format( /29x,'extinction scattering absorption'/
+ 7x,'efficiency factors',3e14.6/ 7x,'asymmetry factor =',f9.6,
& 7x,'backscatter =',e14.6)
1003 format(5e14.6)
1004 format (' angle',11x,'s-sub-1',21x,'s-sub-2')
1005 format (11x,'intensity',4x,'deg of polzn')
1006 format(7x,2e14.6)
1007 format(7x,'s11',11x,'s12',11x,'s33',11x,'s34',11x,'pol')
1008 format(1x,'mie size parameter =',f10.5,5x,'permittivity ='
+ ,2e12.3,/36x,'permeability =',2e12.3//)
c
end

```

```
subroutine magsph(x,eps,mu,numang,theta,
& qext,qsca,qbac,g,s1,s2)
```

```
*****
```

```
Subroutine magsph computes the scattering cross sections and angular
scattering from a magnetic sphere. If the number of scattering angles
is set to zero, only the cross sections (efficiencies) are returned.
```

```
Merrill Milham
```

```
>>> version 2.0 <<<
```

```
SEPT 1993
```

```
Inputs:
```

```
  x = size parameter of the sphere (real*8)
  eps = complex permittivity: epsr -i*epsi (complex*16)
  mu = complex permeability: mur - i*mui (complex*16)
  numang = number of scattering angles (integer)
           between 0 & 90 deg.
  theta = scattering angles in degrees (real*8)
           theta(i) are entered between 0 & 90 deg.
           theta must increase monotonically. Results for
           supplementary angles (180 deg. - theta(i)) are
           also returned.
```

```
Outputs:
```

```
  qext = extinction efficiency (real*8)
  qsca = scattering efficiency (real*8)
  qbac = backscatter efficiency (real*8)
  g = asymmetry factor (real*8)
  s1 = scattered amplitude (complex*16)
  s2 = scattered amplitude (complex*16)
```

```
Subroutines used:
```

```
  zbjy - returns one-half integer order J & Y Bessel functions
```

```
References:
```

```
M. Kerker, D.-S. Wang, and C. L. Giles, "Electromagnetic scattering
by magnetic spheres," J. Opt. Soc. Am., 73, 765-767 (1983).
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D. E. Amos, "Algorithm 644:A portable package for Bessel functions of
a complex argument and nonnegative order," ACM Trans. on Math.
Software, 12, 265-273 (1986).
```

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M. Abramowitz and I. A. Stegun, "Handbook of Mathematical Functions,"
```

```
NBS Applied Math. Series 55, US Dept. of Commerce, Washington, DC
(1955).
```

```
W. J. Wiscombe, "Mie Scattering Calculations: Advances in Technique
```

and

c Fast, Vector-Speed Computer Codes," NCAR Tech. Note, NCAR/TN-140+STR  
c (1979)

c

c

\*\*\*\*\*

c

implicit none

c

real\*8 x  
complex\*16 eps,mu  
integer numang  
real\*8 theta(1)

\*

real\*8 gext,gsca,qbac,g  
complex\*16 s1(1),s2(1)

c

integer al,nangl,nangl2  
real\*8 third  
parameter (third=1.d0/3.d0,al=5100,nangl=255,nangl2=(nangl+1)/2)  
complex\*16 sp(nangl2),sm(nangl2),sps(nangl2),sms(nangl2)

\*

complex\*16 m,mcl,mxi,s,t,u,v,an,bn,xp  
real\*8 xi,dn,dnn,rn,tnp1,thetan  
real\*8 xmu(nangl),pi(nangl),pil(nangl),tau(nangl)

\*

real\*8 bjr(al),byr(al)  
real\*8 cjr(al),cji(al),cyr(al),cyi(al)  
real\*8 bjn,byn,bjl  
real\*8 sc,ca,t1,t2,t3,t4  
complex\*16 cjn,cyn,cjl,cyl,h2n,h2l  
complex\*16 anl,bnl,bs,anp,bnp,abp,abm,anpm,bnpm  
complex\*16 zt1,zt2

\*

integer kstop,k,mm,n,j,j2,j3

\*

real\*8 cpi,zero,one,two,rad,half,fnu  
complex\*16 cdblei,cdb1e1,cdb1e0  
parameter (cpi=3.1415926535897932384d0,zero=0.d0,one=1.d0)  
parameter (two=2.d0,rad=cpi/180.d0,half=0.5d0,fnu=half)  
parameter (cdb1e1=(0.d0,1.d0),cdb1e0=(1.d0,0.d0))  
parameter (cdb1e0=(0.d0,0.d0))

c

kstop=idint(x+4.d0\*x\*\*third+4.d0)

\*

if(kstop.le.al) then  
 continue  
 else  
 print\*,'magsph arrays too small: kstop =',kstop,' al =',al  
 stop  
 end if

c

```

if(numang.eq.0) then
  s1(1)=cdbl0
  s2(1)=cdbl0
  else
*
  if(numang.le.nangl2) then
    continue
  else
    print*,numang,'scattering angles input: only',nangl2,' allowed'
    stop
  endif
*
do 100 n=1,numang
  thetan=dabs(theta(n))
  theta(n)=thetan
*
  if(thetan.le.90.d0) then
    continue
  else
    print*, 'theta(',n,')=',thetan,'scattering angles must be < 90 deg'
    stop
  end if
*
  thetan=rad*thetan
  xmu(n)=dcos(thetan)
*
  sp(n)=cdbl0
  sm(n)=cdbl0
  sps(n)=cdbl0
  sms(n)=cdbl0
  pi(n)=half
  pil(n)=zero
*
100 continue
*
end if
c
m=zsqrt(mu*eps)
mcl=m/mu
*
xi=one/x
mxi=xi/m
c
call zbjy(x,m,kstop,fnu,bjr,byr,cjr,cji)
c
bjl=bjr(1)
cjl=dcmplx(cjr(1),cji(1))
cyl=dcmplx(cyr(1),cyi(1))
h2l=dcmplx(bjr(1),-byr(1))
*
qext=zero
qsc=zero

```

```

bs=cdouble0
g=zero
*
anl=cdouble0
bnl=cdouble0
dn=one
rn=one
tnpl=one
mm=1
c
do 300 k=2,kstop
*
tnpl=tnpl+two
t1=dn-rn
ca=one+rn
sc=rn
*
dnn=dn+one
rn=one/dnn
*
sc=sc+rn
*
bjn=bjr(k)
byn=byr(k)
cjn=dcmplx(cjr(k),cji(k))
cyn=dcmplx(cyr(k),cyi(k))
h2n=dcmplx(bjn,-byn)
*
xp=dn*mxi
s=cj1-xp*cjn
u=s*h2n
s=s*bjn
*
xp=dn*dcmplx(xi,zero)
t=cjn*(bj1-xp*bjn)
v=cjn*(h21-xp*h2n)
*
an=(s-mc1*t)/(u-mc1*v)
bn=(mc1*s-t)/(mc1*u-v)
abp=an+bn
abm=an-bn
*
zt1=dconjg(an)
zt2=dconjg(bn)
qext=qext+tnpl*dble(abp)
qsca=qsca+tnpl*(an*zt1+bn*zt2)
bs=bs-(dn+half)*mm*abm
g=g+t1*dble(anl*zt1+bnl*zt2)+sc*dble(an*zt2)
*
if(numang.eq.0) then
    continue
    else

```

```

anp=sc*abp
bnp=sc*abm
anpm=mm*anp
bnpm=mm*bnp
*
do 375 j=1,numang
t1=xmu(j)*pi(j)
t4=t1-pil(j)
tau(j)=dn*t4-pil(j)
t2=pi(j)+tau(j)
t3=pi(j)-tau(j)
*
sp(j)=sp(j)+anp*t2
sms(j)=sms(j)+bnpm*t2
sm(j)=sm(j)+bnp*t3
sps(j)=sps(j)+anpm*t3
*
pil(j)=pi(j)
pi(j)=t1+ca*t4
*
375 continue
end if
*
dn=dnn
mm=-mm
anl=an
bnl=bn
*
bjl=bjn
cjl=cjn
cyl=cyn
h2l=h2n
*
300 continue
c
if(numang.eq.0) then
continue
else
j2=2*numang
do 500 j=1,numang
j3=j2-j
s1(j)=sp(j)+sm(j)
s2(j)=sp(j)-sm(j)
s1(j3)=sps(j)+sms(j)
s2(j3)=sps(j)-sms(j)
500 continue
*
end if
c
xi=two*xi*xi
qext=xi*qext
qsca=xi*qsca

```

```
xi=two*xi
qbac=xi*bs*dconjg(bs)
g=xi/qsca*g
c
return
c
end
```



```

      subroutine zbjy(x,m,nstop,fnu,
&                    bjr,byr,cjr,cji)
c
c
c*****
c
c Subroutine zbjy gets J & Y Bessel functions for use in
c sphere (fnu=0.50) or cylinder (fnu=0.0d0) scattering calculations.
c Scaled or nonscaled functions for argument z = m*x are returned
depending
c on the magnitude of the product of the size parameter and the complex
c refractive index (zabs(z)). Nonscaled functions are returned if the
c imaginary part of the refractive index is zero. Nonscaled functions are
c returned for argument x.
c
c          Merrill Milham          >>> version: 2.0 <<<          JANUARY 1994
c
c inputs:
c          x = the size parameter of the cylinder.(real*8)
c          m = the complex refractive index, n - ik.(complex*16)
c          nstop = the highest order of the Bessel functions.(integer)
c          fnu = 0.5d0 for sphere calculations or
c              0.0d0 for cylinder calculations. (real*8)
c
c outputs:
c          bjr = real part of J(x) Bessel functions.(array: real*8)
c          byr = real part of Y(x) Bessel functions.(array: real*8)
c          cjr = real part of J(m*x) Bessel functions.(array: real*8)
c          cji = imag. part of J(m*x) Bessel functions.(array: real*8)
c
c subroutines used:
c
c          zbesj - returns J Bessel functions
c          zbesy - returns Y Bessel functions
c
c Reference: D. E. Amos, "Algorithim 644: A Portable Package for Bessel
c Functions of a Complex Argument and Nonnegative Order,"
c          ACM Transcations on Mathematical Software, 12,265-273(1986).
c*****
c**
c
c          implicit none
c          automatic cwrkr,cwrki,bji,byi
c
c          integer al
c          real*8 zero,xll,two
c          parameter (al=5100,zero=0.0d0,xll=1.d-1,two=2.d0)
c
c          real*8 bjr(1),byr(1)

```

```

real*8 x,fnu,cjr(1),cji(1)
real*8 zr,zi,dlmach
real*8 cwrkr(al),cwrki(al)
real*8 rlm5,elim,aa,alim
complex*16 m,z
integer kode,ierr,nz,nstop,n
integer k1,ilmach,k2,k
logical zflag,eflg1,eflg2

```

c

```
kode=1
```

c

```
call zbesj(x,zero,fnu,kode,nstop,bjr,cji,nz,ierr)
```

\*

```

eflg1=ierr.eq.0
eflg2=nz.eq.0
if (eflg1.and.eflg2) then
  continue
  else if (.not.eflg2.and.eflg1) then
    nstop=nstop-nz
    print*,'zbesj error: ierr =',ierr,'nz =',nz
    else
    print*,'inputs =',x,zero,fnu,kode,nstop
    print*,'zbesj called from subroutine zbjy'
end if

```

c

```
call zbesy(x,zero,fnu,kode,nstop,byr,cji,nz,cwrkr,cwrki,ierr)
```

\*

```

eflg1=ierr.eq.0
eflg2=nz.eq.0
if (eflg1.and.eflg2) then
  continue
  else if (.not.eflg2.and.eflg1) then
    nstop=nstop-nz
    print*,'zbesy error: ierr =',ierr,'nz =',nz
  continue
  else
    print*,'zbesy error: ierr =',ierr,'nz =',nz
    print*,'inputs =',x,zero,fnu,kode,nstop
    print*,'zbesy called from subroutine zbjy'
end if

```

c

```

z=m*x
zr=dbl(z)
zi=dimag(z)
if(zi.ne.zero) then

```

\*

```

k1 = ilmach(15)
k2 = ilmach(16)
rlm5 = dlmach(5)
k = min0(iabs(k1),iabs(k2))
elim = 2.303d0*(dbl(float(k))*rlm5-3.0d0)
k1 = ilmach(14) - 1

```

```

aa = rlm5*dble(float(k1))
aa = aa*2.303d0
alim = elim + dmax1(-aa,-41.45d0)
*
    if(dabs(zi).gt.alim) kode=2
                                else
        continue
    end if
c
call zbesj(zr,zi,fnu,kode,nstop,cjr,cji,nz,ierr)
*
eflg1=ierr.eq.0
eflg2=nz.eq.0
if (eflg1.and.eflg2) then
    continue
    else if (.not.eflg2.and.eflg1) then
        nstop=nstop-nz
        print*,'zbesj error: ierr =',ierr,'nz =',nz
    continue
        else
        print*,'zbesj error: ierr =',ierr,'nz =',nz
        print*,'inputs =',zr,zi,fnu,kode,nstop
        print*,'zbesj called from subroutine zbjy'
end if
c
zflag=dabs(zi).lt.two*dlmach(1).and.x.lt.xll
if (.not.zflag) then
    continue
    else
        do 100 n=1,nstop
            cji(n)=zero
100    continue
end if
*
zflag=dabs(zr).lt.two*dlmach(1).and.x.lt.xll
if (.not.zflag) then
    continue
    else
        cji(1)=zero
        do 200 n=2,nstop
            if(mod(n,2).eq.0) then
                cjr(n)=zero
            else
                cji(n)=zero
            end if
200    continue
end if
c
return
c
end

```



```

parameter (cdbl0=(0.d0,0.d0),cdbl1=(0.d0,1.d0))
*
integer nangl,nangl2
parameter (nangl=255,nangl2=(nangl+1)/2)
real*8 xmu(nangl)
integer l,j,j2,j3
C
if(ntheta.eq.0) then
    s1(1)=cdbl0
    s2(1)=cdbl0
    else
*
        if(ntheta.le.nangl2) then
            continue
            else
                print*,ntheta,'scattering angles input: only',nangl2,' allowed'
                stop
            endif
*
do 100 l=1,ntheta
    thetal=dabs(theta(l))
    theta(l)=thetal
*
        if(thetal.le.90.d0) then
            continue
            else
                print*,'theta(',l,')=',thetal,'scattering angles must be <=90 deg'
                stop
            end if
*
    thetal=rad*thetal
    xmu(l)=dcos(thetal)
100 continue
*
end if
C
n=dbl0(eps)
k=dabs(dimag(eps))
epsi=dcmplx(n,-k)
*
n=dbl0(mu)
k=dabs(dimag(mu))
mui=dcmplx(n,-k)
C
ef=(epsi-one)/(epsi+two)
mf=(mui-one)/(mui+two)
*
xp=x*x*x
cx=dcmplx(zero,xp)
C
if(ntheta.eq.0) then
    continue

```

```

                else
j2=2*ntheta
do 500 j=1,ntheta
j3=j2-j
s1(j)=cx*(ef+mf*xmu(j))
s2(j)=cx*(ef*xmu(j)+mf)
s1(j3)=cx*(ef-mf*xmu(j))
s2(j3)=cx*(mf-ef*xmu(j))
500 continue
end if
c
xp=xp*x
qext=-four*x*dimag(ef+mf)
qsca=c0*xp*(zabs(ef)**2+zabs(mf)**2)
if(qext.le.zero) qext=qsca
qabs=ddim(qext,qsca)
qbac=four*xp*(zabs(ef-mf))**2
c
return
c
end

```

```

subroutine amuelr (s1,s2,theta,numang,sign,
& s1lnor,s11,s12,s33,s34,pol)
c
c *****
c subroutine amuelr computes Mueller matrix elements for either
c spheres or infinite cylinders
c
c Merrill Milham >>> version: 2.0 <<< JANUARY 1994
c
c inputs:
c s1 = amplitude scattering matrix element array.(complex*16)
c s2 = amplitude scattering matrix element array.(complex*16)
c theta = scattering angle array (real*8)
c numang = the number of scattering angles, i.e., number of
c elements in s1, s2. (integer)
c sign = arbitrary sign (+1 or -1) used to adjust the sign of
c s34 according to the users convention. (integer)
c
c outputs:
c
c s1lnor = s11 for a scattering angle of zero degrees, which
c is used to normalize s11(theta) (real*8)
c s11 = Mueller matrix element 1,1 (real*8)
c s12 = Mueller matrix element 1,2 (real*8)
c s33 = Mueller matrix element 3,3 (real*8)
c s34 = Mueller matrix element 3,4 (real*8)
c pol = polarization = pol=-s12/s11 (real*8)
c
c s12,s33,s34 are normalized by s11(theta)
c
c subroutines used: none
c
c *****
c
c implicit none
c
c *
c complex*16 s1(1),s2(1)
c real*8 theta(1),s1lnor
c integer numang,sign
c
c *
c real*8 s11(1),s12(1),s33(1),s34(1)
c real*8 pol(1)
c
c *
c real*8 rs1,is1,rs2,is2,s1li
c real*8 ts11,ts12,half,zero,one
c parameter (half=0.5d0,zero=0.d0,one=1.d0)
c complex*16 ts
c integer i
c
c do 100 i=1,numang
c rs1=dbl(s1(i))
c is1=dimag(s1(i))
c rs2=dbl(s2(i))

```

```

is2=dimag(s2(i))
*
ts11=rs1*rs1+is1*is1
ts12=rs2*rs2+is2*is2
s11(i)=half*(ts11+ts12)
s12(i)=half*(ts12-ts11)
*
if(s11(i).ne.zero) then
    s11i=s11(i)
    else
        s11i=one
        write(*,*)
        write(*,*) 'Unnormalized Mueller matrix elements for ',
&
        write(*,*)
        write(*,*) 'theta(i),' deg.'
end if
*
pol(i)=-s12(i)/s11i
*
ts=s2(i)*dconjg(s1(i))
s33(i)=dble(ts)
s33(i)=s33(i)/s11i
s34(i)=sign*dimag(ts)
s34(i)=s34(i)/s11i
*
if(i.eq.1) s11nor=s11i
*
s11(i)=s11i/s11nor
100 continue
c
return
c
end

```



APPENDIX B  
SAMPLE CALCULATIONS

INPUT DATA

```
'r',1,(1.5,-.1)
(1,0)
2,0.01,10.
10
```

OUTPUT

```
read(*,*) flag,mior,(mp(i),i=1,mior)
read(*,*) (muu(i),i=1,mior)
read(*,*) nx,(x(i),i=1,nx)
read(*,*) anginc
```

```
mie size parameter = 0.01000      permittivity = 0.224E+01 -0.300E+00
                                permeability = 0.100E+01 0.000E+00
```

```
mie size parameter = 0.01000      refractive index = 1.500 -0.100E+00
```

angle	s-sub-1		s-sub-2	
0.00	0.498158E-07	0.295985E-06	0.498158E-07	0.295985E-06
0.00	0.498129E-07	0.295977E-06	0.498129E-07	0.295977E-06

intensity	deg of polzn
0.900888E-13	0.000000E+00
0.900839E-13	0.000000E+00

s11	s12	s33	s34	pol
0.100000E+01	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00
0.100000E+01	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
10.00	0.498158E-07	0.295985E-06	0.490590E-07	0.291488E-06
10.00	0.498129E-07	0.295977E-06	0.490562E-07	0.291481E-06

intensity	deg of polzn
0.887305E-13	-0.153075E-01
0.887257E-13	-0.153076E-01

s11	s12	s33	s34	pol
0.984923E+00	-0.135824E-14	0.999883E+00	-0.333333E-07	0.153075E-01
0.984923E+00	-0.135818E-14	0.999883E+00	0.355640E-16	0.153076E-01

angle	s-sub-1		s-sub-2	
20.00	0.498157E-07	0.295985E-06	0.468115E-07	0.278135E-06
20.00	0.498129E-07	0.295977E-06	0.468088E-07	0.278128E-06

intensity deg of polzn  
0.848194E-13 -0.621218E-01  
0.848150E-13 -0.621224E-01

s11 s12 s33 s34 pol  
0.941509E+00 -0.526913E-14 0.998069E+00 -0.135275E-06 0.621218E-01  
0.941511E+00 -0.526891E-14 0.998069E+00 -0.186019E-16 0.621224E-01

angle s-sub-1 s-sub-2  
30.00 0.498156E-07 0.295984E-06 0.431417E-07 0.256330E-06  
30.00 0.498129E-07 0.295977E-06 0.431393E-07 0.256324E-06

intensity deg of polzn  
0.788273E-13 -0.142856E+00  
0.788234E-13 -0.142857E+00

s11 s12 s33 s34 pol  
0.874995E+00 -0.112609E-13 0.989744E+00 -0.311081E-06 0.142856E+00  
0.875000E+00 -0.112605E-13 0.989743E+00 -0.400318E-16 0.142857E+00

angle s-sub-1 s-sub-2  
40.00 0.498155E-07 0.295984E-06 0.381611E-07 0.226737E-06  
40.00 0.498129E-07 0.295977E-06 0.381589E-07 0.226732E-06

intensity deg of polzn  
0.714769E-13 -0.260377E+00  
0.714737E-13 -0.260379E+00

s11 s12 s33 s34 pol  
0.793405E+00 -0.186109E-13 0.965507E+00 -0.566993E-06 0.260377E+00  
0.793412E+00 -0.186103E-13 0.965506E+00 0.220742E-16 0.260379E+00

angle s-sub-1 s-sub-2  
50.00 0.498153E-07 0.295983E-06 0.320210E-07 0.190255E-06  
50.00 0.498129E-07 0.295977E-06 0.320191E-07 0.190251E-06

intensity deg of polzn  
0.636548E-13 -0.415248E+00  
0.636522E-13 -0.415252E+00

s11 s12 s33 s34 pol  
0.706578E+00 -0.264325E-13 0.909708E+00 -0.904240E-06 0.415248E+00  
0.706588E+00 -0.264317E-13 0.909706E+00 0.000000E+00 0.415252E+00

angle s-sub-1 s-sub-2  
60.00 0.498151E-07 0.295982E-06 0.249080E-07 0.147992E-06  
60.00 0.498129E-07 0.295977E-06 0.249065E-07 0.147989E-06

intensity deg of polzn  
0.563043E-13 -0.599995E+00  
0.563024E-13 -0.600000E+00

s11	s12	s33	s34	pol
0.624987E+00	-0.337823E-13	0.800003E+00	-0.130654E-05	0.599995E+00
0.625000E+00	-0.337815E-13	0.800000E+00	-0.280223E-16	0.600000E+00

angle	s-sub-1		s-sub-2	
70.00	0.498148E-07	0.295981E-06	0.170382E-07	0.101232E-06
70.00	0.498129E-07	0.295977E-06	0.170370E-07	0.101230E-06

intensity	deg of polzn
0.503122E-13	-0.790541E+00
0.503109E-13	-0.790546E+00

s11	s12	s33	s34	pol
0.558473E+00	-0.397738E-13	0.612409E+00	-0.172148E-05	0.790541E+00
0.558489E+00	-0.397730E-13	0.612403E+00	0.000000E+00	0.790546E+00

angle	s-sub-1		s-sub-2	
80.00	0.498146E-07	0.295979E-06	0.865075E-08	0.513976E-07
80.00	0.498129E-07	0.295977E-06	0.864992E-08	0.513959E-07

intensity	deg of polzn
0.464009E-13	-0.941455E+00
0.464001E-13	-0.941458E+00

s11	s12	s33	s34	pol
0.515058E+00	-0.436844E-13	0.337139E+00	-0.205011E-05	0.941455E+00
0.515077E+00	-0.436838E-13	0.337131E+00	0.000000E+00	0.941458E+00

angle	s-sub-1		s-sub-2	
90.00	0.498143E-07	0.295978E-06	0.556044E-12	0.133481E-11
90.00	0.498129E-07	0.295977E-06	0.000000E+00	0.000000E+00

intensity	deg of polzn
0.450423E-13	-0.100000E+01
0.450420E-13	-0.100000E+01

s11	s12	s33	s34	pol
0.499976E+00	-0.450423E-13	0.938616E-05	-0.217760E-05	0.100000E+01
0.500000E+00	-0.450420E-13	0.000000E+00	0.000000E+00	0.100000E+01

angle	s-sub-1		s-sub-2	
100.00	0.498141E-07	0.295977E-06	-0.864959E-08	-0.513946E-07
100.00	0.498129E-07	0.295977E-06	-0.864992E-08	-0.513959E-07

intensity	deg of polzn
0.464000E-13	-0.941461E+00
0.464001E-13	-0.941458E+00

s11	s12	s33	s34	pol
0.515048E+00	-0.436838E-13	-0.337122E+00	-0.205013E-05	0.941461E+00
0.515077E+00	-0.436838E-13	-0.337131E+00	0.000000E+00	0.941458E+00

angle	s-sub-1		s-sub-2	
110.00	0.498139E-07	0.295976E-06	-0.170368E-07	-0.101229E-06
110.00	0.498129E-07	0.295977E-06	-0.170370E-07	-0.101230E-06

intensity	deg of polzn
0.503103E-13	-0.790550E+00
0.503109E-13	-0.790546E+00

s11	s12	s33	s34	pol
0.558452E+00	-0.397728E-13	-0.612397E+00	-0.172151E-05	0.790550E+00
0.558489E+00	-0.397730E-13	-0.612403E+00	0.000000E+00	0.790546E+00

angle	s-sub-1		s-sub-2	
120.00	0.498136E-07	0.295975E-06	-0.249064E-07	-0.147986E-06
120.00	0.498129E-07	0.295977E-06	-0.249065E-07	-0.147989E-06

intensity	deg of polzn
0.563014E-13	-0.600005E+00
0.563024E-13	-0.600000E+00

s11	s12	s33	s34	pol
0.624954E+00	-0.337811E-13	-0.799997E+00	-0.130658E-05	0.600005E+00
0.625000E+00	-0.337815E-13	-0.800000E+00	0.280223E-16	0.600000E+00

angle	s-sub-1		s-sub-2	
130.00	0.498134E-07	0.295974E-06	-0.320191E-07	-0.190247E-06
130.00	0.498129E-07	0.295977E-06	-0.320191E-07	-0.190251E-06

intensity	deg of polzn
0.636506E-13	-0.415256E+00
0.636522E-13	-0.415252E+00

s11	s12	s33	s34	pol
0.706531E+00	-0.264313E-13	-0.909705E+00	-0.904265E-06	0.415256E+00
0.706588E+00	-0.264317E-13	-0.909706E+00	0.000000E+00	0.415252E+00

angle	s-sub-1		s-sub-2	
140.00	0.498132E-07	0.295973E-06	-0.381589E-07	-0.226728E-06
140.00	0.498129E-07	0.295977E-06	-0.381589E-07	-0.226732E-06

intensity	deg of polzn
0.714715E-13	-0.260382E+00
0.714737E-13	-0.260379E+00

s11	s12	s33	s34	pol
0.793344E+00	-0.186098E-13	-0.965506E+00	-0.567010E-06	0.260382E+00
0.793412E+00	-0.186103E-13	-0.965506E+00	-0.220742E-16	0.260379E+00

angle	s-sub-1		s-sub-2	
150.00	0.498131E-07	0.295972E-06	-0.431393E-07	-0.256319E-06
150.00	0.498129E-07	0.295977E-06	-0.431393E-07	-0.256324E-06

intensity deg of polzn  
 0.788207E-13 -0.142858E+00  
 0.788234E-13 -0.142857E+00

s11	s12	s33	s34	pol
0.874922E+00	-0.112602E-13	-0.989743E+00	-0.311091E-06	0.142858E+00
0.875000E+00	-0.112605E-13	-0.989743E+00	0.400318E-16	0.142857E+00

angle	s-sub-1		s-sub-2	
160.00	0.498130E-07	0.295972E-06	-0.468088E-07	-0.278122E-06
160.00	0.498129E-07	0.295977E-06	-0.468088E-07	-0.278128E-06

intensity deg of polzn  
 0.848118E-13 -0.621229E-01  
 0.848150E-13 -0.621224E-01

s11	s12	s33	s34	pol
0.941424E+00	-0.526876E-14	-0.998069E+00	-0.135280E-06	0.621229E-01
0.941511E+00	-0.526891E-14	-0.998069E+00	0.186019E-16	0.621224E-01

angle	s-sub-1		s-sub-2	
170.00	0.498129E-07	0.295971E-06	-0.490561E-07	-0.291475E-06
170.00	0.498129E-07	0.295977E-06	-0.490562E-07	-0.291481E-06

intensity deg of polzn  
 0.887222E-13 -0.153078E-01  
 0.887257E-13 -0.153076E-01

s11	s12	s33	s34	pol
0.984831E+00	-0.135814E-14	-0.999883E+00	-0.333345E-07	0.153078E-01
0.984923E+00	-0.135818E-14	-0.999883E+00	-0.355640E-16	0.153076E-01

angle	s-sub-1		s-sub-2	
180.00	0.498129E-07	0.295971E-06	-0.498129E-07	-0.295971E-06
180.00	0.498129E-07	0.295977E-06	-0.498129E-07	-0.295977E-06

intensity deg of polzn  
 0.900803E-13 0.000000E+00  
 0.900839E-13 0.000000E+00

s11	s12	s33	s34	pol
0.999906E+00	0.000000E+00	-0.100000E+01	0.000000E+00	-0.000000E+00
0.100000E+01	0.000000E+00	-0.100000E+01	0.000000E+00	-0.000000E+00

	extinction	scattering	absorption
efficiency factors	0.199263E-02	0.240226E-08	0.199263E-02
asymmetry factor =	0.000020	backscatter =	0.360321E-08

	extinction	scattering	absorption
efficiency factors	0.199252E-02	0.240224E-08	0.199251E-02
asymmetry factor =	0.000020	backscatter =	0.360336E-08

mie size parameter = 10.00000      permittivity = 0.224E+01 -0.300E+00  
permeability = 0.100E+01 0.000E+00

mie size parameter = 10.00000      refractive index = 1.500 -0.100E+00

angle                    s-sub-1                    s-sub-2  
0.00 0.614948E+02 -0.317799E+01 0.614948E+02 -0.317799E+01

intensity      deg of polzn  
0.379171E+04 0.000000E+00

s11                    s12                    s33                    s34                    pol  
0.100000E+01 0.000000E+00 0.100000E+01 0.000000E+00 -0.000000E+00

angle                    s-sub-1                    s-sub-2  
10.00 0.377490E+02 0.936674E+00 0.374475E+02 -0.207314E+01

intensity      deg of polzn  
0.141624E+04 -0.679779E-02

s11                    s12                    s33                    s34                    pol  
0.373510E+00 -0.962731E+01 0.996770E+00 -0.800255E-01 0.679779E-02

angle                    s-sub-1                    s-sub-2  
20.00 0.181377E+01 0.410068E+01 0.215939E+01 0.280384E+00

intensity      deg of polzn  
0.124235E+02 -0.618335E+00

s11                    s12                    s33                    s34                    pol  
0.327648E-02 -0.768186E+01 0.407810E+00 -0.671828E+00 0.618335E+00

angle                    s-sub-1                    s-sub-2  
30.00 -0.579008E+01 -0.121935E+01 -0.442757E+01 0.132155E+00

intensity      deg of polzn  
0.273164E+02 -0.281719E+00

s11                    s12                    s33                    s34                    pol  
0.720424E-02 -0.769552E+01 0.932586E+00 -0.225651E+00 0.281719E+00

angle                    s-sub-1                    s-sub-2  
40.00 0.115404E+01 -0.407492E+01 0.740364E+00 -0.184894E+01

intensity      deg of polzn  
0.109517E+02 -0.637801E+00

s11                    s12                    s33                    s34                    pol  
0.288834E-02 -0.698503E+01 0.765967E+00 0.806435E-01 0.637801E+00

angle s-sub-1 s-sub-2  
50.00 0.166733E+01 0.121208E+01 -0.309906E+00 -0.269918E+00

intensity deg of polzn  
0.220902E+01 -0.923542E+00

s11 s12 s33 s34 pol  
0.582592E-03 -0.204012E+01 -0.382016E+00 -0.336851E-01 0.923542E+00

angle s-sub-1 s-sub-2  
60.00 -0.693755E+00 0.314952E+01 -0.281778E-01 0.159291E+01

intensity deg of polzn  
0.646946E+01 -0.607670E+00

s11 s12 s33 s34 pol  
0.170621E-02 -0.393130E+01 0.778496E+00 -0.157099E+00 0.607670E+00

angle s-sub-1 s-sub-2  
70.00 -0.113609E+01 -0.117949E+01 0.125557E+01 -0.241202E+00

intensity deg of polzn  
0.215826E+01 -0.242616E+00

s11 s12 s33 s34 pol  
0.569207E-03 -0.523630E+00 -0.529101E+00 0.813136E+00 0.242616E+00

angle s-sub-1 s-sub-2  
80.00 -0.273185E+00 -0.219728E+01 -0.830355E+00 -0.629697E+00

intensity deg of polzn  
0.299433E+01 -0.637312E+00

s11 s12 s33 s34 pol  
0.789705E-03 -0.190832E+01 0.537836E+00 -0.551875E+00 0.637312E+00

angle s-sub-1 s-sub-2  
90.00 0.135105E+01 0.417250E+00 -0.102255E+01 0.791253E+00

intensity deg of polzn  
0.183556E+01 -0.892756E-01

s11 s12 s33 s34 pol  
0.484100E-03 -0.163871E+00 -0.572777E+00 0.814835E+00 0.892756E-01

angle s-sub-1 s-sub-2  
100.00 0.117126E+01 0.118164E+01 0.658108E+00 -0.351174E+00

intensity deg of polzn  
0.166228E+01 -0.665261E+00

s11 s12 s33 s34 pol

0.438398E-03 -0.110585E+01 0.214077E+00 -0.715262E+00 0.665261E+00

angle s-sub-1 s-sub-2  
110.00 -0.857020E+00 0.821659E+00 0.178123E+00 -0.127272E+01

intensity deg of polzn  
0.153058E+01 0.790379E-01

s11 s12 s33 s34 pol  
0.403666E-03 0.120974E+00 -0.782971E+00 0.617016E+00 -0.790379E-01

angle s-sub-1 s-sub-2  
120.00 -0.145257E+01 0.316204E+00 0.255067E+00 0.235420E+00

intensity deg of polzn  
0.116521E+01 -0.896600E+00

s11 s12 s33 s34 pol  
0.307304E-03 -0.104472E+01 -0.254085E+00 -0.362697E+00 0.896600E+00

angle s-sub-1 s-sub-2  
130.00 -0.630302E+00 -0.824923E+00 0.128067E+01 0.877085E+00

intensity deg of polzn  
0.174359E+01 0.381861E+00

s11 s12 s33 s34 pol  
0.459842E-03 0.665806E+00 -0.877924E+00 0.288845E+00 -0.381861E+00

angle s-sub-1 s-sub-2  
140.00 -0.154838E+00 -0.112897E+01 0.566968E-01 0.542568E+00

intensity deg of polzn  
0.798073E+00 -0.627109E+00

s11 s12 s33 s34 pol  
0.210479E-03 -0.500479E+00 -0.778529E+00 -0.250616E-01 0.627109E+00

angle s-sub-1 s-sub-2  
150.00 0.205857E+00 -0.889334E+00 -0.919354E+00 0.994698E+00

intensity deg of polzn  
0.133396E+01 0.375326E+00

s11 s12 s33 s34 pol  
0.351811E-03 0.500672E+00 -0.805025E+00 -0.459418E+00 -0.375326E+00

angle s-sub-1 s-sub-2  
160.00 0.812356E+00 -0.110065E+01 0.709831E-01 0.104108E+01

intensity deg of polzn  
0.148012E+01 -0.264325E+00



s11 s12 s33 s34 pol  
0.390358E-03 -0.391233E+00 -0.735212E+00 0.624176E+00 0.264325E+00

angle s-sub-1 s-sub-2  
170.00 0.131138E+01 -0.469165E+00 -0.399742E+00 0.187150E+00

intensity deg of polzn  
0.106733E+01 -0.817470E+00

s11 s12 s33 s34 pol  
0.281490E-03 -0.872507E+00 -0.573413E+00 0.542291E-01 0.817470E+00

angle s-sub-1 s-sub-2  
180.00 0.149343E+01 0.296366E+00 -0.149343E+01 -0.296366E+00

intensity deg of polzn  
0.231818E+01 0.000000E+00

s11 s12 s33 s34 pol  
0.611381E-03 0.000000E+00 -0.100000E+01 0.000000E+00 -0.000000E+00

extinction scattering absorption  
efficiency factors 0.245979E+01 0.123514E+01 0.122465E+01  
asymmetry factor = 0.922350 backscatter = 0.927271E-01

INPUT DATA

'p',1,(2.24,-.3)  
 (2.24,-.3)  
 2,0.01,10.  
 10

OUTPUT

read(\*,\*) flag,mior,(mp(i),i=1,mior)  
 read(\*,\*) (muu(i),i=1,mior)  
 read(\*,\*) nx,(x(i),i=1,nx)  
 read(\*,\*) anginc

mie size parameter = 0.01000      permittivity = 0.224E+01 -0.300E+00  
 permeability = 0.224E+01 -0.300E+00

Unnormalized Mueller matrix elements for 180.0000000000000 deg.

angle	s-sub-1		s-sub-2	
0.00	0.996372E-07	0.591982E-06	0.996372E-07	0.591982E-06
0.00	0.996258E-07	0.591955E-06	0.996258E-07	0.591955E-06

intensity	deg of polzn
0.360371E-12	0.000000E+00
0.360336E-12	0.000000E+00

s11	s12	s33	s34	pol
0.100000E+01	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00
0.100000E+01	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
10.00	0.988804E-07	0.587485E-06	0.988804E-07	0.587485E-06
10.00	0.988691E-07	0.587458E-06	0.988691E-07	0.587458E-06

intensity	deg of polzn
0.354916E-12	0.000000E+00
0.354882E-12	0.000000E+00

s11	s12	s33	s34	pol
0.984865E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00
0.984865E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
20.00	0.966327E-07	0.574131E-06	0.966327E-07	0.574131E-06
20.00	0.966218E-07	0.574105E-06	0.966218E-07	0.574105E-06

intensity	deg of polzn
0.338964E-12	0.000000E+00

0.338932E-12 0.000000E+00

s11	s12	s33	s34	pol
0.940600E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00
0.940602E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
30.00	0.929626E-07	0.552326E-06	0.929626E-07	0.552326E-06
30.00	0.929522E-07	0.552301E-06	0.929522E-07	0.552301E-06

intensity	deg of polzn
0.313706E-12	0.000000E+00
0.313677E-12	0.000000E+00

s11	s12	s33	s34	pol
0.870508E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00
0.870513E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
40.00	0.879815E-07	0.522731E-06	0.879815E-07	0.522731E-06
40.00	0.879718E-07	0.522709E-06	0.879718E-07	0.522709E-06

intensity	deg of polzn
0.280989E-12	0.000000E+00
0.280964E-12	0.000000E+00

s11	s12	s33	s34	pol
0.779721E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00
0.779728E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
50.00	0.818409E-07	0.486247E-06	0.818409E-07	0.486247E-06
50.00	0.818321E-07	0.486228E-06	0.818321E-07	0.486228E-06

intensity	deg of polzn
0.243134E-12	0.000000E+00
0.243114E-12	0.000000E+00

s11	s12	s33	s34	pol
0.674679E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00
0.674688E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
60.00	0.747273E-07	0.443982E-06	0.747273E-07	0.443982E-06
60.00	0.747194E-07	0.443966E-06	0.747194E-07	0.443966E-06

intensity	deg of polzn
0.202705E-12	-0.186801E-15
0.202689E-12	0.000000E+00

s11	s12	s33	s34	pol
0.562489E+00	-0.378653E-28	0.100000E+01	-0.311334E-16	0.186801E-15

0.562500E+00 0.000000E+00 0.100000E+01 0.000000E+00 -0.000000E+00

angle s-sub-1 s-sub-2  
70.00 0.668568E-07 0.397221E-06 0.668568E-07 0.397221E-06  
70.00 0.668499E-07 0.397208E-06 0.668499E-07 0.397208E-06

intensity deg of polzn  
0.162254E-12 -0.311160E-15  
0.162243E-12 0.000000E+00

s11 s12 s33 s34 pol  
0.450243E+00 -0.504871E-28 0.100000E+01 -0.388950E-16 0.311160E-15  
0.450255E+00 0.000000E+00 0.100000E+01 0.000000E+00 -0.000000E+00

angle s-sub-1 s-sub-2  
80.00 0.584687E-07 0.347384E-06 0.584687E-07 0.347384E-06  
80.00 0.584628E-07 0.347373E-06 0.584628E-07 0.347373E-06

intensity deg of polzn  
0.124094E-12 -0.305134E-15  
0.124086E-12 0.000000E+00

s11 s12 s33 s34 pol  
0.344352E+00 -0.378653E-28 0.100000E+01 -0.762834E-16 0.305134E-15  
0.344363E+00 0.000000E+00 0.100000E+01 0.000000E+00 -0.000000E+00

angle s-sub-1 s-sub-2  
90.00 0.498177E-07 0.295986E-06 0.498177E-07 0.295986E-06  
90.00 0.498129E-07 0.295977E-06 0.498129E-07 0.295977E-06

intensity deg of polzn  
0.900892E-13 -0.700515E-15  
0.900839E-13 0.000000E+00

s11 s12 s33 s34 pol  
0.249991E+00 -0.631089E-28 0.100000E+01 -0.210155E-15 0.700515E-15  
0.250000E+00 0.000000E+00 0.100000E+01 0.000000E+00 -0.000000E+00

angle s-sub-1 s-sub-2  
100.00 0.411668E-07 0.244587E-06 0.411668E-07 0.244587E-06  
100.00 0.411630E-07 0.244581E-06 0.411630E-07 0.244581E-06

intensity deg of polzn  
0.615177E-13 -0.923279E-15  
0.615145E-13 0.000000E+00

s11 s12 s33 s34 pol  
0.170707E+00 -0.567980E-28 0.100000E+01 -0.230820E-15 0.923279E-15  
0.170714E+00 0.000000E+00 0.100000E+01 0.000000E+00 -0.000000E+00

angle s-sub-1 s-sub-2  
110.00 0.327789E-07 0.194751E-06 0.327789E-07 0.194751E-06

110.00 0.327759E-07 0.194747E-06 0.327759E-07 0.194747E-06

intensity deg of polzn  
0.390025E-13 -0.970843E-15  
0.390007E-13 0.000000E+00

s11	s12	s33	s34	pol
0.108229E+00	-0.378653E-28	0.100000E+01	-0.303389E-15	0.970843E-15
0.108234E+00	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
120.00	0.249086E-07	0.147991E-06	0.249086E-07	0.147991E-06
120.00	0.249065E-07	0.147989E-06	0.249065E-07	0.147989E-06

intensity deg of polzn  
0.225219E-13 -0.133100E-14  
0.225210E-13 0.000000E+00

s11	s12	s33	s34	pol
0.624965E-01	-0.299767E-28	0.100000E+01	-0.385291E-15	0.133100E-14
0.625000E-01	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
130.00	0.177953E-07	0.105728E-06	0.177953E-07	0.105728E-06
130.00	0.177938E-07	0.105727E-06	0.177938E-07	0.105727E-06

intensity deg of polzn  
0.114952E-13 -0.240189E-14  
0.114948E-13 0.000000E+00

s11	s12	s33	s34	pol
0.318982E-01	-0.276101E-28	0.100000E+01	-0.720567E-15	0.240189E-14
0.319002E-01	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
140.00	0.116550E-07	0.692465E-07	0.116550E-07	0.692465E-07
140.00	0.116540E-07	0.692455E-07	0.116540E-07	0.692455E-07

intensity deg of polzn  
0.493091E-14 -0.399957E-14  
0.493076E-14 0.000000E+00

s11	s12	s33	s34	pol
0.136829E-01	-0.197215E-28	0.100000E+01	-0.119987E-14	0.399957E-14
0.136838E-01	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
150.00	0.667421E-08	0.396539E-07	0.667421E-08	0.396539E-07
150.00	0.667367E-08	0.396534E-07	0.667367E-08	0.396534E-07

intensity deg of polzn  
0.161698E-14 -0.701301E-14

0.161693E-14 0.000000E+00

s11	s12	s33	s34	pol
0.448698E-02	-0.113399E-28	0.100000E+01	-0.210390E-14	0.701301E-14
0.448730E-02	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
160.00	0.300433E-08	0.178498E-07	0.300433E-08	0.178498E-07
160.00	0.300409E-08	0.178496E-07	0.300409E-08	0.178496E-07

intensity	deg of polzn
0.327641E-15	-0.166282E-13
0.327633E-15	0.000000E+00

s11	s12	s33	s34	pol
0.909178E-03	-0.544807E-29	0.100000E+01	-0.498469E-14	0.166282E-13
0.909245E-03	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

angle	s-sub-1		s-sub-2	
170.00	0.756830E-09	0.449660E-08	0.756830E-09	0.449660E-08
170.00	0.756770E-09	0.449656E-08	0.756770E-09	0.449656E-08

intensity	deg of polzn
0.207922E-16	-0.669882E-13
0.207918E-16	0.000000E+00

s11	s12	s33	s34	pol
0.576968E-04	-0.139283E-29	0.100000E+01	-0.200446E-13	0.669882E-13
0.577011E-04	0.000000E+00	0.100000E+01	0.000000E+00	-0.000000E+00

degree of polarization undefined for 180.00000000000000 deg.(sx)

angle	s-sub-1		s-sub-2	
180.00	-0.198521E-22	0.158818E-21	0.198521E-22	-0.158818E-21
180.00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

intensity	deg of polzn
0.256172E-43	0.000000E+00
0.000000E+00	0.000000E+00

s11	s12	s33	s34	pol
0.710856E-31	0.000000E+00	-0.100000E+01	0.000000E+00	-0.000000E+00
0.277519E+13	0.000000E+00	0.000000E+00	0.000000E+00	-0.000000E+00

efficiency factors	extinction	scattering	absorption
	0.398549E-02	0.480485E-08	0.398549E-02
asymmetry factor =	0.500006	backscatter =	0.102469E-38

efficiency factors	extinction	scattering	absorption
	0.398503E-02	0.480448E-08	0.398503E-02

asymmetry factor = 0.500006

backscatter = 0.000000E+00

mie size parameter = 10.00000

permittivity = 0.224E+01 -0.300E+00

permeability = 0.224E+01 -0.300E+00

angle                    s-sub-1                    s-sub-2  
0.00   0.604902E+02 -0.800379E+01   0.604902E+02 -0.800379E+01

intensity      deg of polzn  
0.372313E+04   0.000000E+00

s11                    s12                    s33                    s34                    pol  
0.100000E+01   0.000000E+00   0.100000E+01   0.000000E+00   -0.000000E+00

angle                    s-sub-1                    s-sub-2  
10.00   0.364065E+02 -0.285146E+01   0.364065E+02 -0.285146E+01

intensity      deg of polzn  
0.133357E+04   0.000000E+00

s11                    s12                    s33                    s34                    pol  
0.358185E+00   0.000000E+00   0.100000E+01   -0.213125E-16   -0.000000E+00

angle                    s-sub-1                    s-sub-2  
20.00   0.255793E+00   0.305124E+01   0.255793E+00   0.305124E+01

intensity      deg of polzn  
0.937548E+01   -0.189468E-15

s11                    s12                    s33                    s34                    pol  
0.251817E-02   -0.177636E-14   0.100000E+01   -0.710507E-15   0.189468E-15

angle                    s-sub-1                    s-sub-2  
30.00   -0.650130E+01   0.601006E+00   -0.650130E+01   0.601006E+00

intensity      deg of polzn  
0.426280E+02   0.000000E+00

s11                    s12                    s33                    s34                    pol  
0.114495E-01   0.000000E+00   0.100000E+01   -0.135431E-15   -0.000000E+00

angle                    s-sub-1                    s-sub-2  
40.00   0.170607E+01   -0.232783E+01   0.170607E+01   -0.232783E+01

intensity      deg of polzn  
0.832946E+01   -0.213262E-15

s11                    s12                    s33                    s34                    pol  
0.223722E-02   -0.177636E-14   0.100000E+01   0.159946E-15   0.213262E-15

angle                    s-sub-1                    s-sub-2

50.00 0.223135E+01 0.134911E+00 0.223135E+01 0.134911E+00

intensity deg of polzn  
0.499713E+01 0.444344E-15

s11 s12 s33 s34 pol  
0.134219E-02 0.222045E-14 0.100000E+01 -0.466561E-15 -0.444344E-15

angle s-sub-1 s-sub-2  
60.00 -0.976135E+00 0.172588E+01 -0.976135E+00 0.172588E+01

intensity deg of polzn  
0.393152E+01 -0.564781E-15

s11 s12 s33 s34 pol  
0.105597E-02 -0.222045E-14 0.100000E+01 -0.564781E-16 0.564781E-15

angle s-sub-1 s-sub-2  
70.00 -0.105808E+01 -0.219556E+00 -0.105808E+01 -0.219556E+00

intensity deg of polzn  
0.116774E+01 -0.570447E-15

s11 s12 s33 s34 pol  
0.313645E-03 -0.666134E-15 0.100000E+01 -0.570447E-15 0.570447E-15

angle s-sub-1 s-sub-2  
80.00 0.166837E+00 -0.116371E+01 0.166837E+00 -0.116371E+01

intensity deg of polzn  
0.138205E+01 -0.722984E-15

s11 s12 s33 s34 pol  
0.371207E-03 -0.999201E-15 0.100000E+01 0.140580E-15 0.722984E-15

angle s-sub-1 s-sub-2  
90.00 0.647575E+00 -0.683249E-01 0.647575E+00 -0.683249E-01

intensity deg of polzn  
0.424022E+00 -0.202919E-14

s11 s12 s33 s34 pol  
0.113889E-03 -0.860423E-15 0.100000E+01 -0.474570E-15 0.202919E-14

angle s-sub-1 s-sub-2  
100.00 0.342863E+00 0.567668E+00 0.342863E+00 0.567668E+00

intensity deg of polzn  
0.439801E+00 -0.258748E-14

s11 s12 s33 s34 pol  
0.118127E-03 -0.113798E-14 0.100000E+01 0.100975E-14 0.258748E-14



angle s-sub-1 s-sub-2  
110.00 -0.204251E+00 0.304099E+00 -0.204251E+00 0.304099E+00

intensity deg of polzn  
0.134195E+00 -0.320587E-14

s11 s12 s33 s34 pol  
0.360436E-04 -0.430211E-15 0.100000E+01 -0.149952E-14 0.320587E-14

angle s-sub-1 s-sub-2  
120.00 -0.348717E+00 0.223846E-01 -0.348717E+00 0.223846E-01

intensity deg of polzn  
0.122104E+00 -0.681930E-14

s11 s12 s33 s34 pol  
0.327962E-04 -0.832667E-15 0.100000E+01 -0.161248E-14 0.681930E-14

angle s-sub-1 s-sub-2  
130.00 -0.134797E+00 -0.107422E+00 -0.134797E+00 -0.107422E+00

intensity deg of polzn  
0.297099E-01 -0.648115E-14

s11 s12 s33 s34 pol  
0.797983E-05 -0.192554E-15 0.100000E+01 -0.992608E-15 0.648115E-14

angle s-sub-1 s-sub-2  
140.00 -0.461748E-02 -0.147281E+00 -0.461748E-02 -0.147281E+00

intensity deg of polzn  
0.217131E-01 -0.511316E-14

s11 s12 s33 s34 pol  
0.583194E-05 -0.111022E-15 0.100000E+01 -0.320072E-14 0.511316E-14

angle s-sub-1 s-sub-2  
150.00 0.560250E-02 -0.764339E-01 0.560250E-02 -0.764339E-01

intensity deg of polzn  
0.587352E-02 -0.826970E-14

s11 s12 s33 s34 pol  
0.157758E-05 -0.485723E-16 0.100000E+01 0.117216E-14 0.826970E-14

angle s-sub-1 s-sub-2  
160.00 0.258230E-01 -0.216056E-02 0.258230E-01 -0.216056E-02

intensity deg of polzn  
0.671494E-03 0.161461E-14

s11 s12 s33 s34 pol

0.180358E-06 0.108420E-17 0.100000E+01 0.219385E-13 -0.161461E-14

angle s-sub-1 s-sub-2  
170.00 0.203439E-01 0.744313E-02 0.203439E-01 0.744313E-02

intensity deg of polzn  
0.469274E-03 -0.247211E-13

s11 s12 s33 s34 pol  
0.126043E-06 -0.116010E-16 0.100000E+01 0.488645E-13 0.247211E-13

angle s-sub-1 s-sub-2  
180.00 0.726314E-15 -0.526075E-15 -0.726314E-15 0.526075E-15

intensity deg of polzn  
0.804287E-30 0.000000E+00

s11 s12 s33 s34 pol  
0.216025E-33 0.000000E+00 -0.100000E+01 0.000000E+00 -0.000000E+00

extinction scattering absorption  
efficiency factors 0.241961E+01 0.116202E+01 0.125759E+01  
asymmetry factor = 0.958194 backscatter = 0.321715E-31