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November 1994

**US Army Corps
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Waterways Experiment
Station

*Computer-Aided Structural
Engineering (CASE) Project*

User's Guide: Computer Program for Analysis of Beam-Column Structures with Nonlinear Supports (CBEAMC)

by *William P. Dawkins, Oklahoma State University*

WES

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Prepared for Headquarters, U.S. Army Corps of Engineers

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Final report

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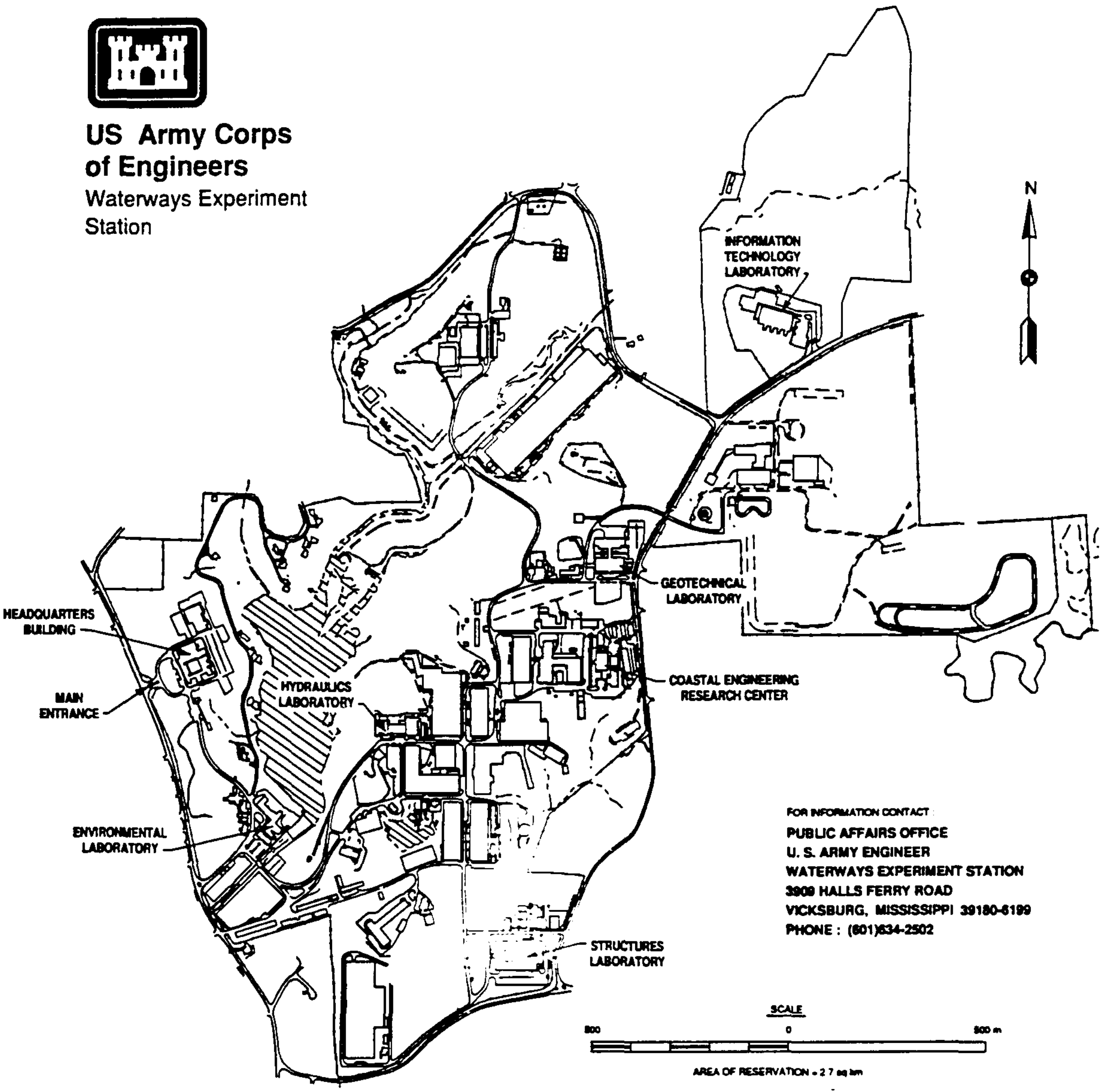

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Contents

Preface

This user's guide describes the computer program CBEAMC which can be used for analysis of general beam-column structures supported and/or loaded by components which interact with the displacements of the beam-column. The work in writing the computer program and user's guide was accomplished with funds provided to the U.S. Army Engineer Waterways Experiment Station (WES), Vicksburg, MS, by Headquarters, U.S. Army Corps of Engineers, Civil Works Directorate, under the Structural Engineering Research Program work unit of the Computer-Aided Structural Engineering (CASE) project.

The program and user's guide were written by Dr. William P. Dawkins, P.E., Oklahoma State University, Stillwater, OK, under IPA agreement No. 93-15-M with WES.

The work was managed, coordinated, and monitored in the Information Technology Laboratory (ITL), WES, by Dr. Reed L. Mosher, Acting Chief Computer-Aided Engineering Division (CAED). Mr. H. Wayne Jones, Chief, Scientific and Engineering Applications Center, CAED, is Project Manager for the CASE Project. Dr. N. Radhakrishnan is Director, ITL.

At the time of publication of this report, Director of WES was Dr. Robert W. Whalin. Commander was COL Bruce K. Howard, EN.

Conversion Factors, Non-SI to SI Units of Measurement

Non-SI units of measurement used in this report can be converted to SI units as follows:

Multiply	By	To Obtain
degrees (angle)	0.01745329	radians
feet	0.3048	meters
inches	2.54	centimeters
kips (force)	4448.22	Newtons
pounds (force)	4.4482	Newtons
pounds (force) per square inch	6894.76	Pascals
pounds (force) per square foot	47.8803	Pascals

1 Introduction

General

This report documents a computer program--CBEAMC--for analysis of general beam-column structures supported and/or loaded by components which interact with displacements of the beam and/or column.

Organization of Report

The remainder of this report is divided into the following chapters:

a. *Chapter 2.* Describes the general beam-column system considered and the mathematical model used for analysis.

b. *Chapter 3.* Presents the force-displacement relationships for the mathematical model and describes the computational procedure used for solution.

c. *Chapter 4.* Provides example solution obtained with the program.

Disclaimer

The computer program described in this report has been checked to ensure that the results are accurate within the limitations of the procedures employed. However, there may be unusual situations which were not anticipated, and these situations may cause the program to produce questionable results. It is the responsibility of the user to judge the validity of the results. No responsibility is assumed by the author for the design or behavior of any structure based on results obtained with the program.

2 Beam-Column System

General

The general beam-column system considered for analysis is shown in Figure 1. Characteristics and assumptions employed for each of the system components are described in the following paragraphs.

Beam

The following assumptions of conventional beam bending theory are employed:

a. The beam is composed of a linearly elastic, piecewise, homogeneous material with modulus of elasticity E.

b. The beam is essentially piecewise prismatic, i.e., the cross section of area A and moment of inertia 1 may vary along the length but do not alter the basic assumption that plane cross sections remain plane after loading.

c. All cross sections share an initially straight, common centroidal axis; the x-axis is shown in Figure 1.

d. All cross sections gave a principal axis parallel to the y-axis, Figure 1.

e. Deformations due to shearing strains are negligibly small.

f. Stress concentrations at abrupt changes in cross sections, at concentrated loads or springs, and at supports are negligibly small.

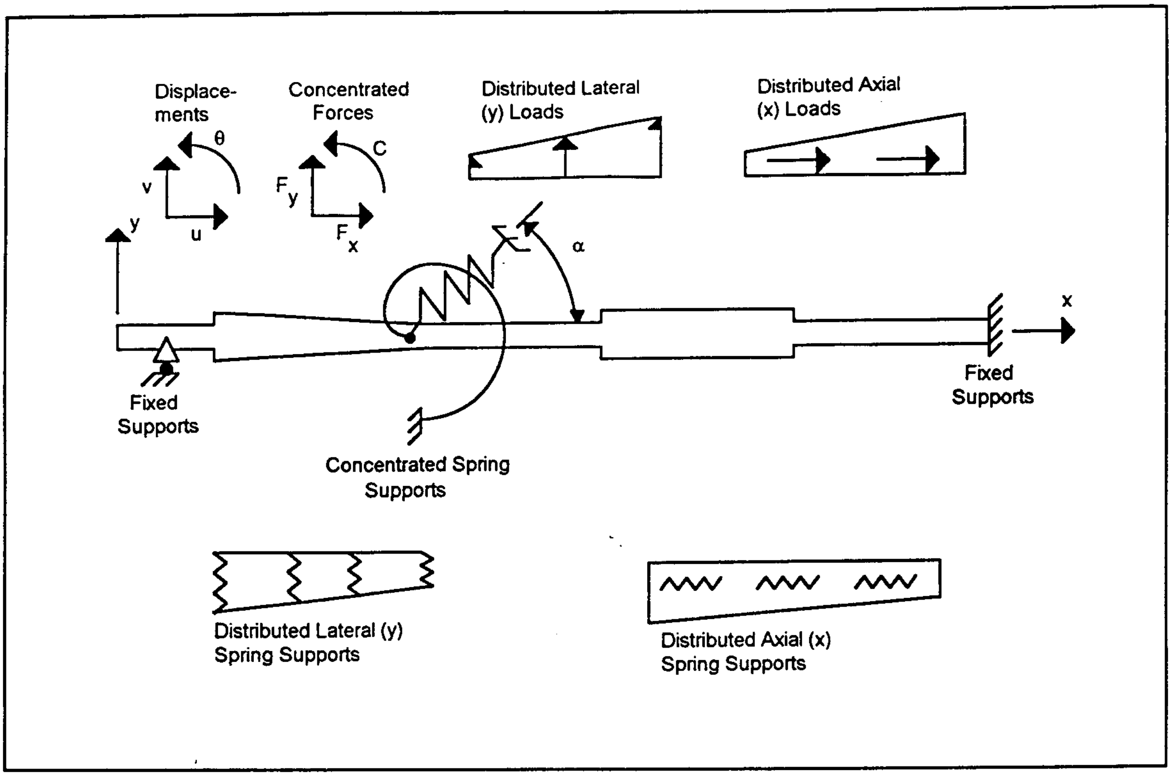


Figure 1. General beam-column system

Displacements

The following assumptions regarding displacement of the system are employed:

a. Displacement of points on the beam centroidal axis are completely defined by translations u (parallel to the x-axis), and v (parallel to the y-axis), and rotation θ about and axis perpendicular to the x-y plane.

b. All displacements are small and do not alter the basic geometry of the system.

c. Rotations θ are small enough to permit the approximations $\cos\theta = 1$ and $\sin\theta = \theta$.

Applied Loads

The following assumptions regarding external applied loads are employed:

a. Applied loads are unaffected either in magnitude, direction, or point of application by the deflections of the system.

b. Concentrated applied loads are assumed to be applied as components parallel to the x- and y-directions or as couples acting about axes perpendicular to the x-y plane.

c. Distributed applied loads are assumed to act parallel to the x and y axes.

Fixed Supports

A fixed support is assumed to be an external influence which results in a specific value (either zero or nonzero) of one or more displacement components at a point on the structure, regardless of other effects.

Concentrated Linear Spring Supports

Concentrated linear springs produce forces on the structure which are directly proportional to the displacements of the point of attachment. The following assumptions are employed:

a. Concentrated linear translational springs may be inclined at an angle (α, Figure 1) to the axis of the beam. The resisting force developed in the spring is directly proportional to the translation of the point of attachment parallel to the line of action of the spring.

b. A concentrated linear rotational spring applies a resisting couple proportional to the rotation of the point of attachment.

Distributed Linear Spring Supports

Distributed linear spring supports produce distributed resisting forces directly proportional to the displacements of the structure. The following assumptions are employed:

a. Distributed linear springs resist only lateral displacements v or axial displacements u.

b. The magnitude of the distributed resisting force depends only on the lateral displacement v or axial displacement u at any point, i.e., the Winkler hypothesis.

Concentrated Nonlinear Spring Supports

Concentrated nonlinear spring supports produce forces on the structure which depend on, but are not directly proportional to, the displacements of the point of attachment. The following assumptions are employed:

a. Concentrated nonlinear translational springs may be inclined at an angle (α, Figure 1) with the axis of the beam. The resisting force developed in the spring is a function of the translation of the point of attachment parallel to the line of action of the spring.

b. Nonlinear rotational springs are not considered.

Distributed Nonlinear Spring Supports

Distributed nonlinear spring supports produce distributed resisting forces which depend on, but are not directly proportional to, the displacements of the structure. The following assumptions are employed:

a. Distributed nonlinear springs resist only lateral displacements v or axial displacements u.

b. The magnitude of the distributed resisting force depends only on the lateral displacement v or axial displacement u at any point, i.e., the Winkler hypothesis.

Characteristics of Nonlinear Springs

The force-deformation relationship for a nonlinear spring (either concentrated or distributed) is assumed to be provided as a piecewise linear curve, as shown in Figure 2. For any deformation Δ, parallel to the spring line of action, the total resisting force (concentrated or distributed) may be expressed as

$$Q \ (or \ q) = Q_o \ (or \ q_o) + k \cdot \Delta \tag{1}$$

Hence, although an iterative solution is required, during any iteration a nonlinear spring may be replaced by a combination of a fixed load (Q_o or q_o) and a linear spring with stiffness k.

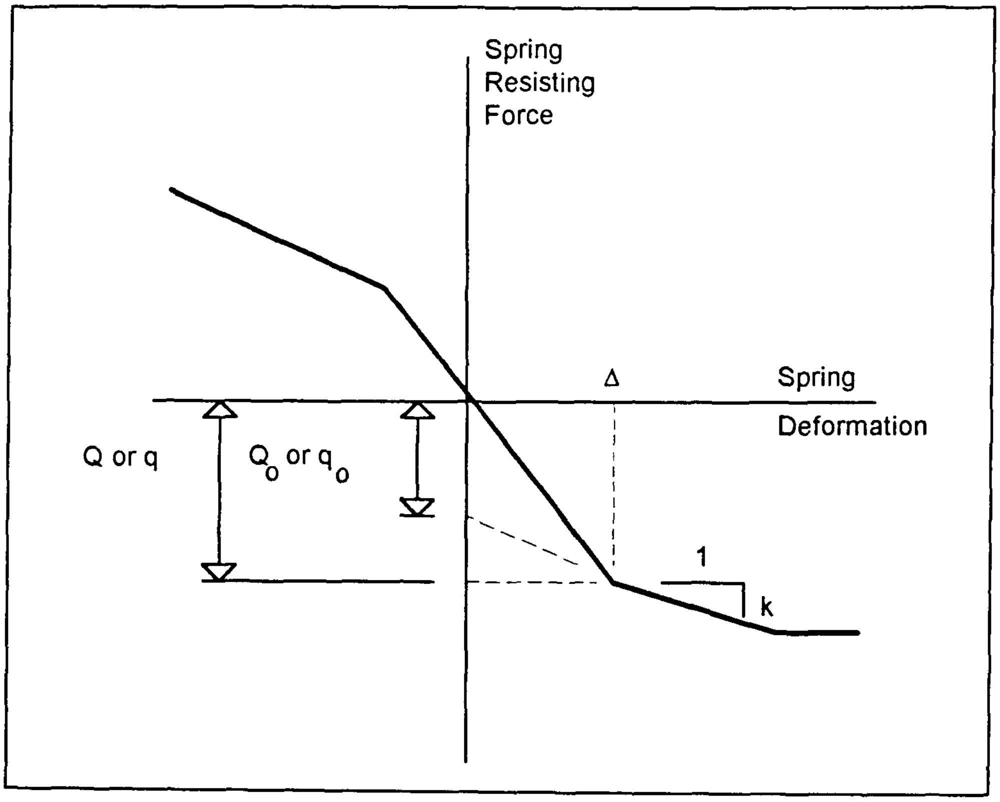


Figure 2. Characteristics of nonlinear springs

3 Finite Element Model

General

Solutions for the general beam-column system are obtained with the finite element model described below.

Nodes

Because the centroidal axis of the beam is initially straight, the location of a node is completely defined by its x-coordinate. For identification in the following paragraphs, nodes are assumed to be numbered sequentially starting with node 1 at the left end of the beam, as shown in Figure 3.

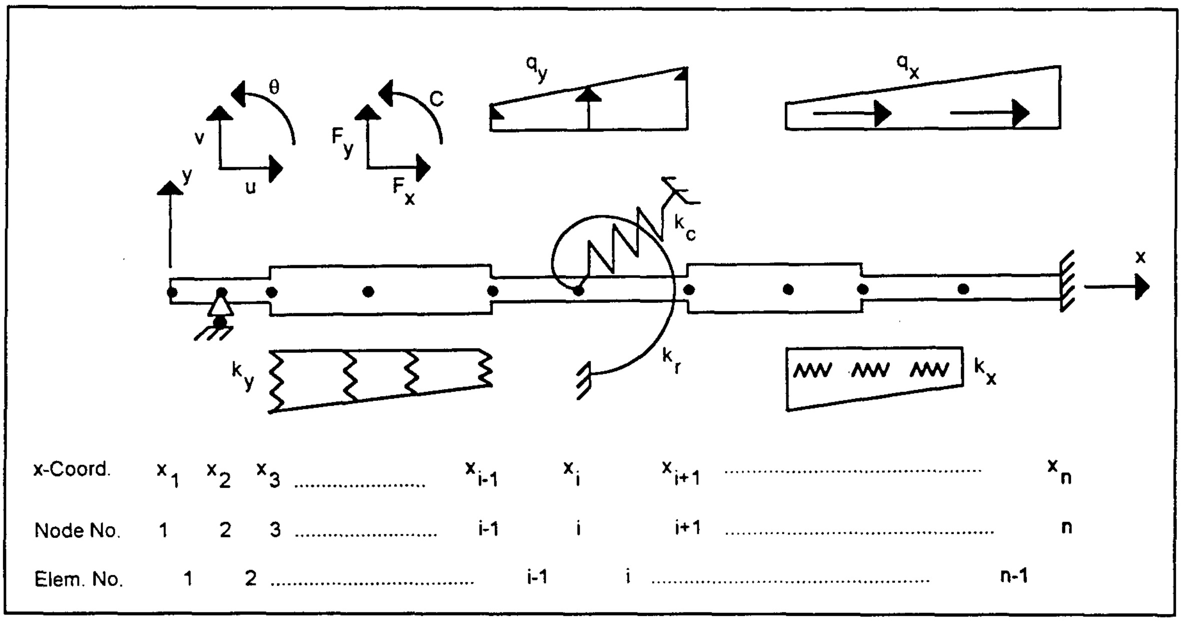


Figure 3. Finite element model

A minimum number of nodes are dictated by the characteristics of the system:

a. At the left and right ends of the beam.

b. At points of change in beam properties (i.e., E, I, and /or A).

c. At the point of application of concentrated loads.

d. At fixed supports.

e. At the point of application of concentrated spring supports.

f. At the beginning and end of distributed (either axial or lateral) loads.

g. At the beginning and end of distributed spring (either axial or lateral) supports.

Displacements, internal forces, and other effects are determined only at the nodes. To provide more detailed description of the variation of effects along the length, nodes may be defined at any other location on the beam. In addition, the accuracy of a solution may be affected by the number of nodes. This aspect will be discussed subsequently.

Variations in System Properties

The following assumptions are employed for system properties which vary along the length of the structure.

a. Properties E, A, and I of beam cross section are assumed to be constant between adjacent nodes. In effect, a tapered beam is replaced by a piecewise prismatic (stepped) member. As discussed later, this replacement is accomplished automatically.

b. Distributed loads (either applied or resulting from the fixed load portion of distributed nonlinear springs) are assumed to vary linearly between adjacent nodes.

c. Distributed spring stiffnesses (either distributed linear springs or the linear stiffness component of distributed nonlinear springs) are assumed to vary linearly between adjacent nodes.

Elements

An element is defined as the portion of the beam between adjacent nodes. For reference in the following paragraphs, an element is identified by the node number at its left end, as shown in Figure 3.

Under the assumptions stated above, each element is characterized by modulus of elasticity E, properties A and I of the cross section, and length h and is subjected to distributed loads and springs, as shown in Figure 4.

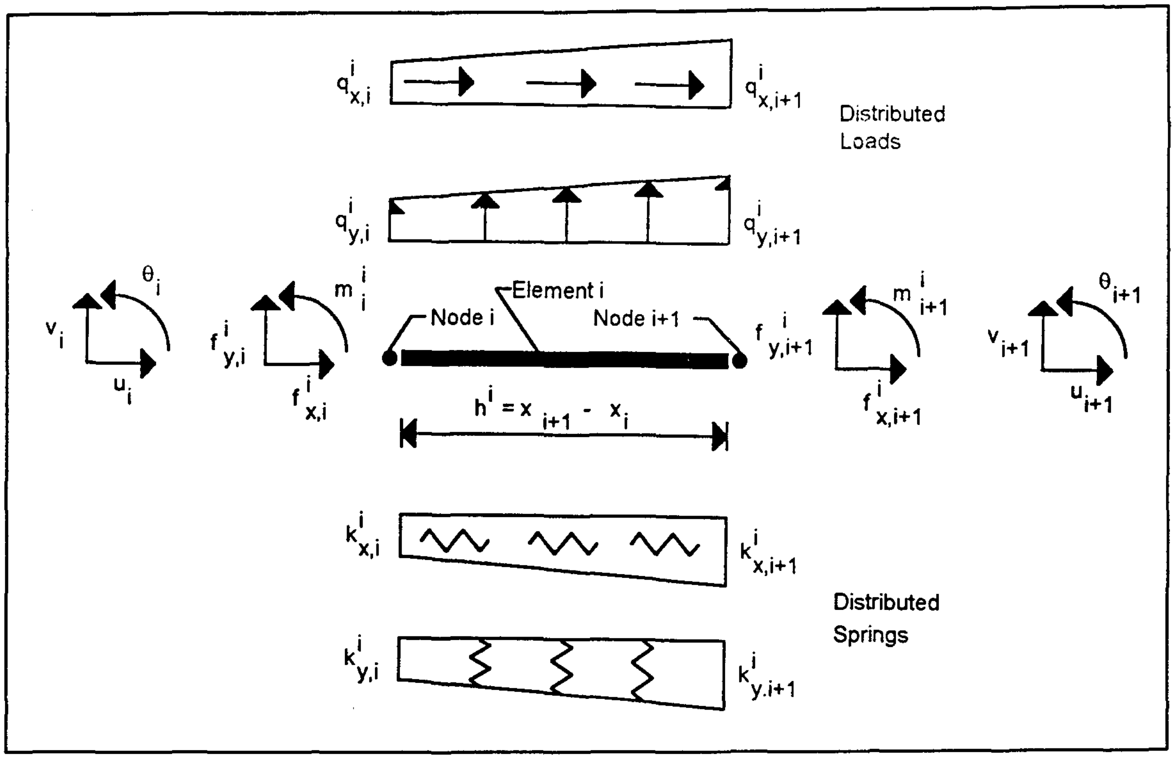


Figure 4. Typical element

Interaction of internal axial stress resultants with lateral deflections will affect the bending resistance of the element. When this effect is included, the bending resistance of the element is developed using a constant average axial stress resultant given by

$$P = \frac{EA}{h}\left(u_{i+1} - u_i\right) \tag{2}$$

The relationship between element end forces and end nodal displacements, obtained by procedures given by Clough and Penzien (1975), Przemieniecki (1968), and Zienkiewicz (1971), is

$$\underline{f}_i = \left[\underline{S}^i_E + \underline{S}^i_K\right] \cdot \underline{U}_i + \underline{f}^i_e \tag{3}$$

where

$\underline{f}^{i} = \left[f_{x,i} \quad f_{y,i} \quad m_{i} \quad f_{x,i+1} \quad f_{y,i+1} \quad m_{i+1}\right]^{T}$

= (6×1) vector of element end forces

$\underline{S}_{E}^{i}$ = (6×) axial and beam bending stiffness matrix including effects of axial stress resultant on bending

$\underline{S}_{K}^{i}$ = (6×6) stiffness matrix representing effects of distributed springs

$\underline{U}^{i} = \left[u_{i} \quad v_{i} \quad \theta_{i} \quad u_{i+1} \quad v_{i} \quad \theta_{i+1}\right]^{T}$

= (6×1) vector end of nodal displacements, and

$\underline{f}_{e}^{i} = \left[f_{ex,i}^{i} \quad f_{ey,i}^{i} \quad m_{e,i}^{i} \quad f_{ex,i+1}^{i} \quad f_{ey,i+1}^{i} \quad m_{e,i+1}^{i}\right]$

= (6×1) vector of fixed end forces due to distributed loads.

Coefficients for matrices $\underline{S}_{E}^{i}$, $\underline{S}_{K}^{i}$, and $\underline{f}_{e}^{i}$ are given in Figures 5, 6, and 7 and Table 1.

Node Equilibrium

A typical node, shown in Figure 8, is subjected to :

a. Concentrated forces $F_{x,i}$, $F_{y,i}$, and C_{i}, where $F_{x,i}$ and $F_{y,i}$ include both applied external concentrated loads and the fixed force component of all concentrated nonlinear springs attached to the node.

b. One or more concentrated linear translation springs which include the linear spring components of concentrated nonlinear springs attached to the node.

c. One or more linear rotation springs.

d. Element end forces from elements on either side of the node.

Equilibrium of node i is expressed by

$$\underline{S}_E^i = \frac{E}{h}\begin{bmatrix} A & 0 & 0 & -A & 0 & 0 \\ & a_1\dfrac{I}{h^2} & a_2\dfrac{I}{h} & 0 & -a_1\dfrac{I}{h^2} & a_2\dfrac{I}{h} \\ & & a_3 I & 0 & -a_2\dfrac{I}{h} & a_4 I \\ & & & A & 0 & 0 \\ & \text{SYM} & & & a_1\dfrac{I}{h^2} & -a_2\dfrac{I}{h} \\ & & & & & a_3 I \end{bmatrix}$$

where:

E = modulus of elasticity

A = area of cross section

I = moment of inertia of cross section

a_1 - a_4 = constants given in Table 1.

a. Explicit

$$\underline{S}_E^i = \begin{bmatrix} \underline{S}_{E,i,i}^i & \underline{S}_{E,i,i+1}^i \\ \underline{S}_{E,i+1,i}^i & \underline{S}_{E,i+1,i+1}^i \end{bmatrix}$$

b. Partitioned symbolic

Figure 5. Element axial and bending beam stiffness matrix

$$\underline{S}_K^i = \frac{h}{840}\begin{bmatrix} (210k_{x,j} + 70k_{x,j+1}) & 0 & 0 & 70(k_{x,j} + k_{x,j+1}) & 0 & 0 \\ & (240k_{y,j} + 72k_{y,j+1}) & h(30k_{y,j} + 14k_{y,j+1}) & 0 & 54(k_{y,j} + k_{y,j+1}) & -h(14k_{y,j} + 12k_{y,j+1}) \\ & & h^2(k_{y,j} + k_{y,j+1}) & 0 & h(12k_{y,j} + 14k_{y,j+1}) & -3h^2(k_{y,j} + k_{y,j+1}) \\ & & & (70k_{y,j} + 210k_{y,j+1}) & 0 & 0 \\ & \text{SYM} & & & (72k_{y,j} + 240k_{y,j+1}) & -h(14k_{y,j} + 30k_{y,j+1}) \\ & & & & & h^2(3k_{y,j} + 5k_{y,j+1}) \end{bmatrix}$$

a. Explicit

$$\underline{S}_K^i = \begin{bmatrix} \underline{S}_{K,j,j}^i & \underline{S}_{K,j,j+1}^i \\ \underline{S}_{K,j+1,j}^i & \underline{S}_{K,j+1,j+1}^i \end{bmatrix}$$

b. Partitioned symbolic

Figure 6. Contribution of distributed springs to element stiffness matrix

$$\underline{f}_e^i = \left\{ \begin{matrix} f_{ex,i}^i \\ f_{ey,i}^i \\ m_{e,i}^i \\ f_{ex,i+1}^i \\ f_{ey,i+1}^i \\ m_{e,i+1}^i \end{matrix} \right\} = \left\{ \begin{matrix} -(20q_{x,i} + 10q_{x,i+1}) \\ (b_1 q_{y,i} + b_2 q_{y,i+1}) \\ h(b_3 q_{y,i} + b_4 q_{y,i+1}) \\ -(10q_{x,i} + 20q_{x,i+1}) \\ (b_2 q_{y,i} + b_1 q_{y,i+1}) \\ -h(b_4 q_{y,i} + b_3 q_{y,i+1}) \end{matrix} \right\}$$

where b_1, b_2, b_3, b_4 are constants given in Table 1

a. Explicit

$$\underline{f}_e^i = \left\{ \begin{matrix} \underline{f}_{e,i}^i \\ \underline{f}_{e,i+1}^i \end{matrix} \right\}$$

b. Partitioned symbolic

Figure 7. Element fixed end forces due to distributed loads

$$\left\{ \begin{matrix} f_{x,i}^{i-1} \\ f_{y,i}^{i-1} \\ m_i^{i-1} \end{matrix} \right\} + \left\{ \begin{matrix} f_{x,i}^{i} \\ f_{y,i}^{i} \\ m_i^{i} \end{matrix} \right\} + \sum \begin{bmatrix} k_{e,i} \cos^2 \alpha & k_{c,i} \sin \alpha \cos \alpha & 0 \\ k_{e,i} \sin \alpha \cos \alpha & k_{c,i} \sin^2 \alpha & 0 \\ 0 & 0 & k_{r,i} \end{bmatrix} \left\{ \begin{matrix} u_i \\ v_i \\ \theta_i \end{matrix} \right\} = \left\{ \begin{matrix} F_{x,i} \\ F_{y,i} \\ C_i \end{matrix} \right\} \tag{4}$$

or

$$\underline{f}_i^{i-1} = \underline{f}_i^i + \left(\Sigma \underline{K}_{c,i}\right) \cdot \underline{U}_i = \underline{F}_i$$

where the summation Σ extends over all concentrated springs attached to node i.

Table 1
Constants for Element Stiffness Matrix and Fixed End Forces
(P = Axial Internal Stress Resultant)

Coefficient	P = 0	P = Compression	P = Tension
a_1	12	$\Phi^5 s/\Delta$	$\Phi^5 s/\Delta$
a_2	6	$\Phi^4(1-c)/\Delta$	$-\Phi^4(1-c)/\Delta$
a_3	4	$\Phi^3(s-\Phi c)/\Delta$	$-\Phi^3(s-\Phi c)/\Delta$
a_4	2	$\Phi^3(\Phi-s)/\Delta$	$-\Phi^3(\Phi-s)/\Delta$
b_1	-21	$10[-3(4+\Phi^2)(1-c)$ $+\Phi s(6+2\Phi^2)]/\Delta$	$10[3(4-\Phi^2)(1-c)$ $+\Phi s(6-2\Phi^2)]/\Delta$
b_2	-9	$10[3(4-\Phi^2)(1-c)$ $-\Phi s(6-\Phi^2)]/\Delta$	$10[-3(4+\Phi^2)(1-c)$ $-\Phi s(6+\Phi^2)]/\Delta$
b_3	-3	$10[9\Phi s-\Phi^2(1+2c)$ $-12(1-c)]/\Delta$	$10[9\Phi s-\Phi^2(1+2c)$ $+12(1-c)]/\Delta$
b_4	-2	$10[3\Phi s-\Phi^2(2+c)]/\Delta$	$10[3\Phi s-\Phi^2(2+c)]/\Delta$
Δ	---	$\Phi^2(2-2c-\Phi s)$	$\Phi^2(2-2c+\Phi s)$
Φ	---	$\sqrt{Ph^2/EI}$	$\sqrt{Ph^2/EI}$
s	---	$\sin\Phi$	$\sinh\Phi$
c	---	$\cos\Phi$	$\cosh\Phi$

Substitution for end force vectors f_i^{i-1} and f_i^{i} in terms of element stiffness matrices using Equation 3 and Figures 5, 6, and 7 results in the following governing equation which must be satisfied at every node.

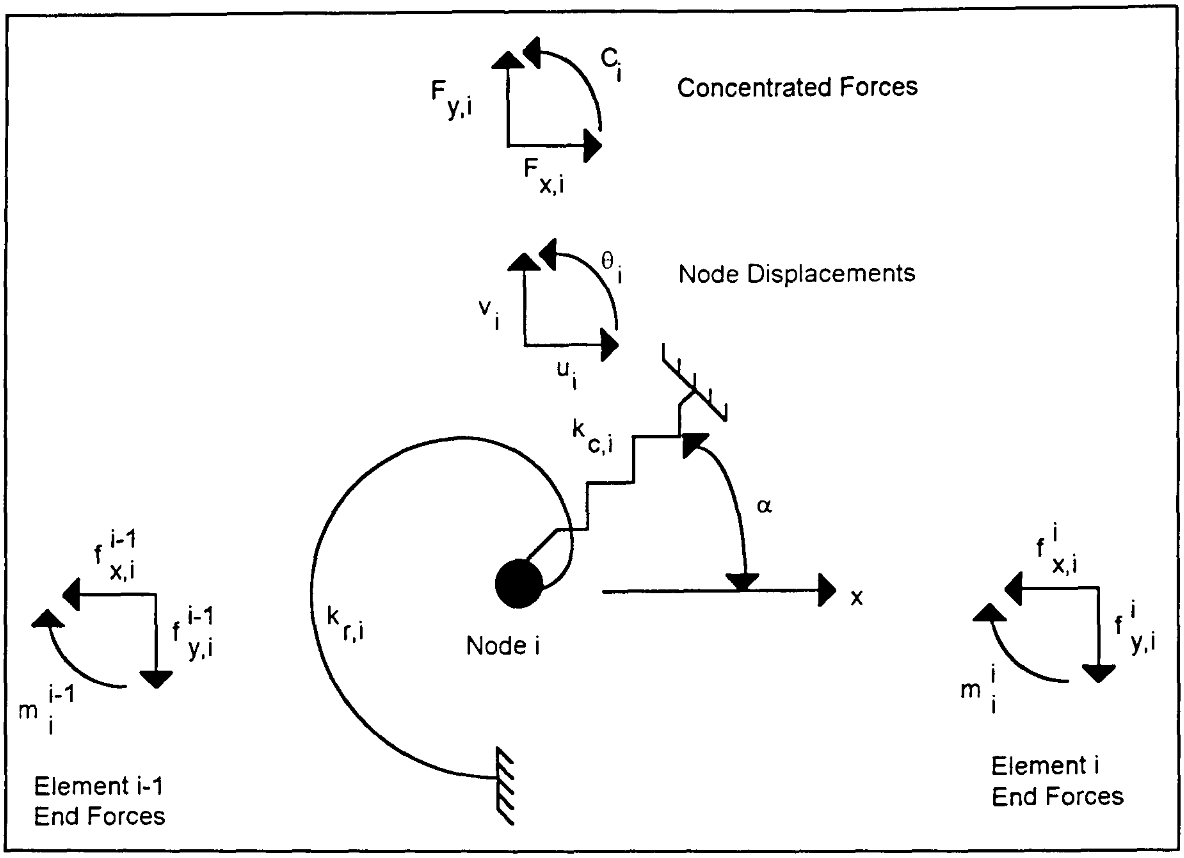


Figure 8. Typical node

$$
\begin{aligned}
&[\underline{S}^{i-1}_{E,i,i-1} + \underline{S}^{i-1}_{K,i,i-1}] \cdot \underline{U}_{i-1} \\
&\quad + [\underline{S}^{i-1}_{E,i,i} + \underline{S}^{i-1}_{K,i,i} + \underline{S}^{i}_{E,i,i} + \underline{S}^{i}_{K,i,i} + \Sigma \underline{K}_{c.i}] \cdot \underline{U}_i \\
&\quad + [\underline{S}^{i}_{E,i,i+1} + \underline{S}^{i}_{K,i,i+1}] \cdot \underline{U}_{i+1} = \underline{F}_i + \underline{f}^{i-1}_{e,i} + \underline{f}^{i}_{e,i}
\end{aligned}
\tag{5}
$$

Evaluation of Equation 5 at every node in the model results in a system of simultaneous equations (3n equations for a model with n nodes) which are solved for the nodal displacements. When nodal displacements are known, internal forces, spring forces, and reactions may be determined for the system.

Iteration

A single solution of the simultaneous equations produces final results for systems possessing the following characteristics:

a. Only linear spring supports are present; and *b.* The effect of axial stress resultant on element bending stiffness is excluded; or

c. The effect of axial stress resultant on element bending stiffness is included, axial distributed springs are absent, and all concentrated translational springs are perpendicular to the beam axis.

All other systems require an iterative solution to evaluate axial stress resultants and/or fixed force-linear spring components for nonlinear springs. The iterative solution process is initiated by evaluating axial stress resultants and nonlinear spring characteristics for zero displacements. The simultaneous equations are evaluated and solved for a new estimate of system displacements. On each iteration, the solution of the preceding iteration is used for evaluation of system properties. This process is continued until the results of two successive iterations differ by an acceptably small amount.

Effect of Node Spacing on Solution

The finite element procedure described produces "exact" solutions for systems possessing the following characteristics:

a. The beam is piecewise prismatic.

b. Only concentrated external loads and linearly varying lateral (y) loads are present. Or, if the effect of axial stress resultant on element bending stiffness is excluded, distributed axial (x) loads vary linearly.

c. Only concentrated springs are present and , if the effect of axial stress resultant on element bending stiffness is included, all concentrated translation springs are perpendicular to the beam axis. (Note: The accuracy of a solution including the effect of axial stress resultant in the presence of inclined translational springs may be affected by the iteration convergence tolerance but not by node spacing.)

d. Adjacent elements are approximately equal in length. Adjacent elements having drastically different lengths may result in significant round-off errors in the solution of simultaneous equations.

In all other systems, the number of nodes (and elements) used in the finite element model affects the accuracy of the solution. In general, as the number of nodes (and elements) is increased, the solution tends to converge to the "exact" solution. There is no "rule-of-thumb" to provide the necessary number of nodes for acceptable results. It may be necessary to perform several solutions for various numbers of nodes and node spacings to ensure that an adequate solution has been obtained.

4 Computer Program

General

A computer program implementing the analytical process described in the previous chapter has been written in the FORTRAN programming language with all arithmetic operations performed in double precision.

Input Data

The program has been written to generate automatically intermediate data values from a minimum of user input data. Details of required user input data are presented in the Input Guide, Appendix A. The processes and assumptions used in converting user input data to intermediate data values are described below.

Global Coordinate System

A horizontal beam centroidal axis (the x-axis, positive to the right) is assumed. The global y-axis is positive upward. The origin of the x-y coordinate system is arbitrary. A local coordinate system associated with nonlinear springs is described later.

Displacement and Load Sign Conventions

Positive directions for displacements and loads are as follows:

a. Nodal displacements u and v are positive if the node translates in the positive x- and y-directions, respectively.

b. Nodal rotation is positive if the node rotates counterclockwise.

c. External loads (concentrated F_x and F_y; or distributed q_x and q_y) are positive if they act in the positive x- and y-directions, respectively.

d. A concentrated applied couple (C) is positive if it tends to rotate the node counterclockwise.

e. Concentrated or distributed linear spring stiffnesses are always positive.

Nonlinear Spring Conventions

As stated previously, the characteristics of nonlinear springs are assumed to be described by a piecewise linear curve giving spring resisting force as a function of spring deformation. The sign conventions assumed for each of the three types of nonlinear springs permitted by the program are as follows.

Concentrated Nonlinear Springs

Concentrated nonlinear springs are assumed to resist nodal translation displacement components parallel to the spring line of action. Positive resisting forces and spring deformations for concentrated springs are defined for a local x'-coordinate, as shown in Figure 9. The origin of the local x'-coordinate is at the point of attachment (node); the positive x'-direction extends along the spring line of action away from the point of attachment. The inclination of the spring is given by the angle α between the global x-axis and the local x'-axis (positive counterclockwise).

Spring resisting forces are positive if they act in the positive local x'-direction. To illustrate, consider a spring in an initial state of compression at zero nodal displacement. The spring therefore would produce a force acting on the node in the negative x'-direction (i.e., negative force), $-F_1$, at zero spring deformation, $\Delta_1 = 0$, as illustrated by point (1) in Figure 9b. If the point of attachment undergoes nodal displacements u and v to produce a positive deformation $+\Delta_2$, Figure 9a, the compression (i.e., negative spring force), $-F_2$, would increase as illustrated by point (2) in Figure 9b. Further positive deformation to Δ_3 would result in a negative resisting force, $-F_3$, as shown by point (3). If the nodal displacements produce deformation in the negative x'-direction, e.g., $-\Delta_4$, the compressive (negative) force in the spring would reduce to, $-F_4$, point (4). At some negative deformation, $-\Delta_5$, the initial compression might be totally relieved, i.e., $F_5 = 0$, as shown by point (5). Further negative deformation, $-\Delta_6$, might produce tension in the spring ($+F_6$), as illustrated by point (6). The coordinate values, Δ, F, of each point on the deformation-resistance curve are provided as input. A linear variation between input deformation-resistance coordinates is assumed.

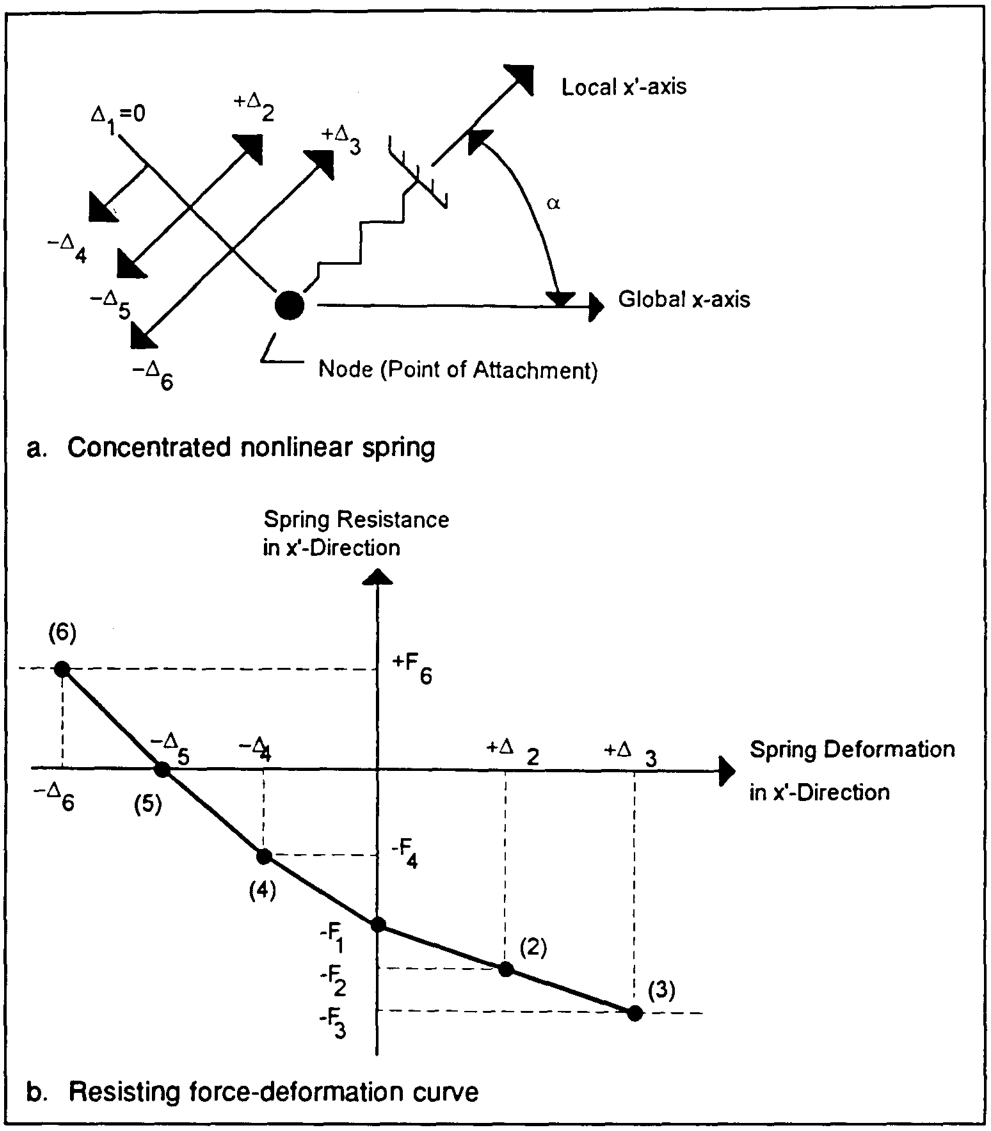


Figure 9. Characteristics of concentrated nonlinear spring

Distributed Nonlinear Springs

Distributed nonlinear springs produce distributed forces proportional to the translation displacements of the nodes. The deformation of the spring is equal to and has the same sign as the nodal displacement. The distributed force produced on the beam by a distributed nonlinear spring is positive if the force acts in the positive global x- or y-coordinate directions. Distributed spring characteristics are assumed to be described by pairs resisting force-deformation coordinate values similar to those for concentrated springs.

Data Generation

Concentrated applied loads, concentrated springs, and fixed support occur at isolated points on the structure. All other data (beam properties, E, A, and I, and distributed loads and springs) are distributed over a range of x-coordinates. The user is required to provide the locations (x-coordinates) at which concentrated effects occur and the terminal values (beginning and end) of distributed quantities. These define the "dictated" nodes in the model. The program also provides for defining nodes intermediate to the dictated nodes. The following paragraphs describe the procedures and assumptions used to obtain system characteristics at the intermediate nodes.

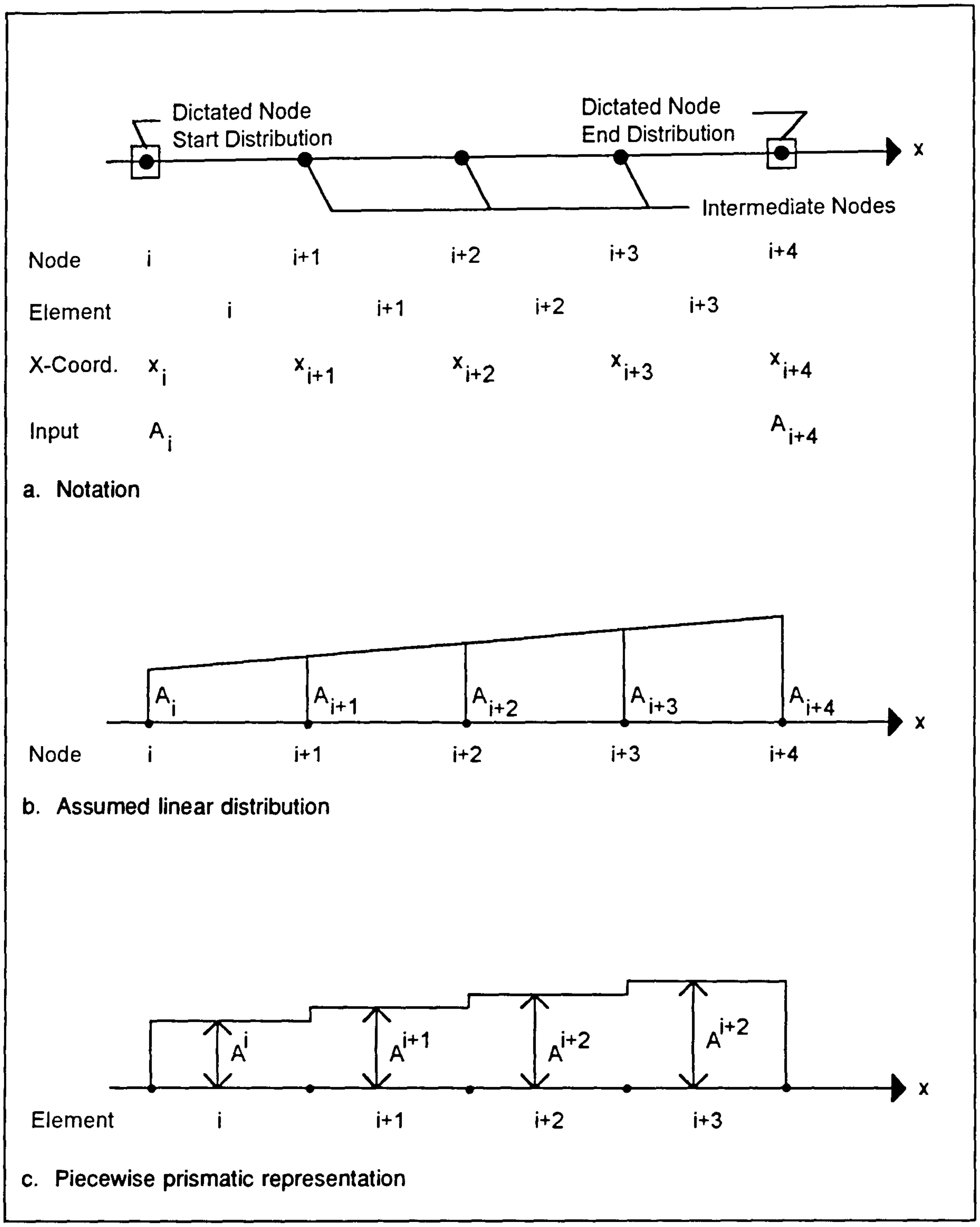
Properties of Beam Cross Section

Properties A and I of beam cross section are provided at the terminals of a distribution. Figure 10 illustrates for cross section area the steps used to convert input data to element properties. The user has specified values of cross section area A_i and A_{i+4} at x_i and x_{i+4}, respectively, resulting in "dictated" nodes i and i+4. Through node spacing data (see below), intermediate nodes have been defined at i+1, i+2, and i+3. The initial assumption is made that cross section area varies linearly from A_i at x_i to A_{i+4}, as shown in Figure 10b, to result in values of cross section areas A_{i+1}, A_{i+2}, and A_{i+3} at x_{i+1}, x_{i+2}, and x_{i+3}, respectively. The distribution is further converted to the piecewise prismatic representation shown in Figure 10c by assigning to each element covered by the distribution a constant area equal to the average of the areas at each end of the element, e.g., $A^{i+1} = (A_{i+1} + A_{i+2})/2$. Note that E is required to be constant over the distribution.

Properties data of beam cross section are assumed to define the extreme (left and right) ends of the beam-column system to be analyzed. Nonzero values of section properties E, A, and I must be supplied for every point on the structure between these extremes. Any other distributed or concentrated data which fall beyond these extremes are ignored by the program.

Node Spacing Data

Node spacing data provide for defining nodes intermediate to the "dictated" nodes. The user specifies a maximum node spacing, H_{max}, to be used in a range of x-coordinates. For example, consider the range of x-coordinates between "dictated" nodes x_i and x_{i+4} in Figure 10a. The range of coordinates x_i to x_{i+4} will be subdivided into elements of equal length as follows. The number of elements is equal to the largest integer given by



a. Notation

b. Assumed linear distribution

c. Piecewise prismatic representation

Figure 10. Distribution of cross-sectional properties of beam

$$n = (x_{i+4} - x_i)/H_{max} + 0.9 \tag{6}$$

and the length of elements in this region will be

$$h = (x_{i+4} - x_i)/n \tag{7}$$

If H_{max} is greater than or equal to $(x_{i+4} - x_i)$, then the entire region will be treated as a single element.

If a "dictated" node falls between the limits of a range of node spacing data, intervals to the left and right of the dictated node are treated as separate ranges for intermediate node spacing.

Distributed Loads

Distributed loads vary linearly between the values at the beginning and end of the distribution provided as input.

Distributed Linear Springs

The program provides for variations in linear spring stiffness coefficients over an interval x_1 to x_2 given by the general expression

$$k = a + bz^c \tag{8}$$

where

k = stiffness coefficient (force/length/displacement) of distributed x or y spring at any point x_1 to x_2

a = stiffness at k_1, always positive

b = constant with units such that bz^c has units of stiffness; b may be positive or negative; however, only positive values of k are permitted

z = distance measured from x_1

c = dimensionless constant

For the input values x_i, x_{i+3}, a, b, and c, the program calculates at each intermediate node the distributed spring stiffness from the general expression given in Equation 8, as illustrated in Figure 11.

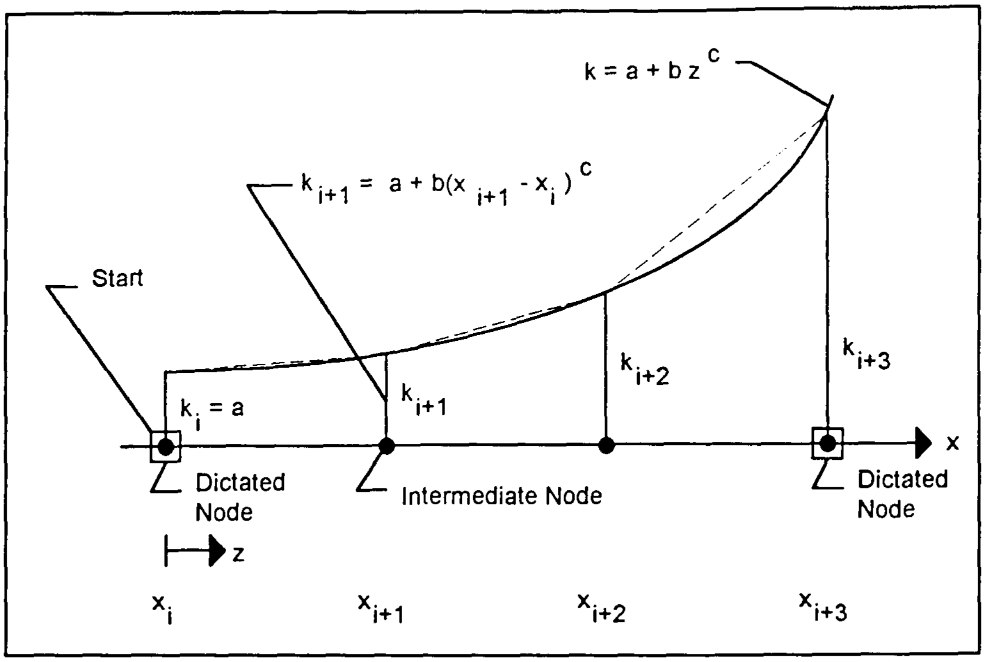


Figure 11. Distributed linear spring stiffnesses

Distributed Nonlinear Springs

Distributed nonlinear springs are described by resisting force-deformation curves for each end of the distribution, as illustrated in Figure 12. As stated previously, a nonlinear spring is represented by a combination of a fixed force and a linear spring. The components of the nonlinear spring at intermediate nodes are obtained as follows. Consider node i+1 in Figure 12. The spring deformation at node i+1, Δ_{i+1} (either u_{i+1} or v_{i+1}) is used with the input curve at node i to obtain $q_{o,\ell}$, and k_ℓ; and with the curve at node i+3 to obtain $q_{o,r}$ and k_r. The values at node 1+1 are then calculated from linear interpolation as

$$q_{o,i+1} = [q_{o,\ell} \cdot (x_{i+3} - x_{i+1}) + q_{o,r} \cdot (x_{i+1} - x_i)]/(x_{i+3} - x_i) \qquad (9)$$

and

$$k_{i+1} + [k_\ell \cdot (x_{i+3} - x_{i+1}) + k_r \cdot (x_{i+1} - x_i)]/(x_{i+3} - x_i) \qquad (10)$$

Output Data

Output data are provided in three parts. Output may be directed to the user terminal, to a data file, to both or some parts of the output may be omitted entirely.

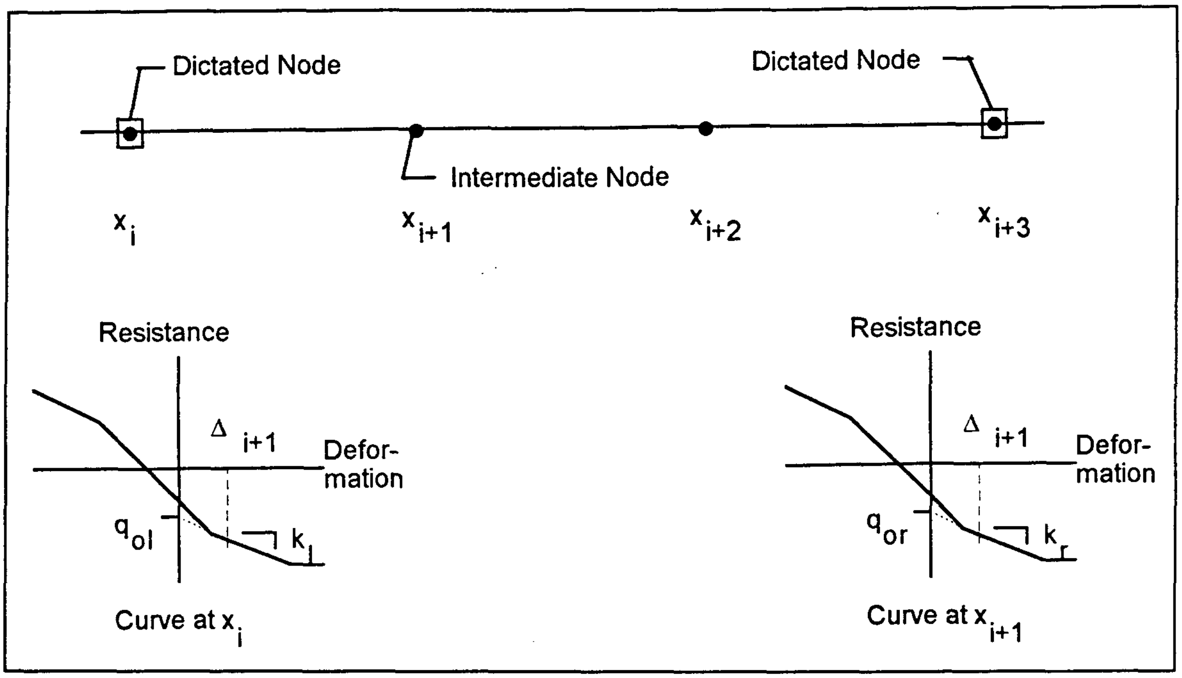


Figure 12. Distributed nonlinear spring

Echoprint of Input Data

This section contains a tabular listing of all input data currently available to the program. This section is output at the option of the user.

Summary of Results

This section contains: maximum positive and maximum negative values of displacements and internal forces and the x-coordinates at which the maxima occur; reactions at fixed supports; and forces in concentrated linear and/or nonlinear springs. This section is always output to the user terminal, to an output file, or both.

Complete Results

This section contains a tabulation of displacements, internal stress resultants, and forces in distributed springs at nodes contained within range of x-coordinates specified by the user. This section may be omitted entirely.

Graphics Output

Graphical displays of input data consist of:

a. Schematic of variation of axial and bending stiffnesses (EA and EI, respectively).

b. Schematic showing locations of fixed and/or concentrated spring supports.

c. Schematic showing locations and magnitudes of applied loads.

d. Schematic showing ranges of linear and/or nonlinear distributed springs.

Graphical displays of results include:

a. Axial and lateral displacement diagrams.

b. Shear and bending moment diagrams.

c. Forces in distributed springs.

Sign Conventions for Output

Sign conventions used for output data are presented in Table 2.

Table 2
Output Sign Conventions

Quantity	Sign Convention
Translations u and v	Positive if node moves in positive x- or y-coordinate direction, respectively
Rotation	Positive if node rotates counterclockwise
Axial Internal Stress Resultant	Positive if tension
Shear Stress Resultant	Positive if shear force tends to move left end of segment of beam in positive y-direction (i.e., up on left, down on right end of segment)
Bending Moment	Positive if produces compression in plus y-face (top) of beam
X-, Y-Reaction at Fixed Support	Positive if acts in positive x- or y-coordinate direction, respectively
Moment Reaction at Fixed Support	Positive if acts counterclockwise on beam
Force in Concentrated Linear Translation Spring	Positive if tension
Moment in Linear Rotation Spring	Positive if acts counterclockwise on beam
Force in Concentrated Nonlinear Translation Spring	Sign determined according to input resisting force-deformation curve coordinates
Force in Distributed Linear Spring	Positive if tension
Force in Distributed Nonlinear Spring	Sign determined according to input resisting force-deformation curve coordinates

5 Example Solutions

The examples presented below are intended only to illustrate the use of the program and are not to be interpreted as a guide for application of the program.

Example 1: Fixed End Beam

The fixed end beam shown in Figure 13 was analyzed for a succession of loading conditions described in the input data file shown in Figure 14.

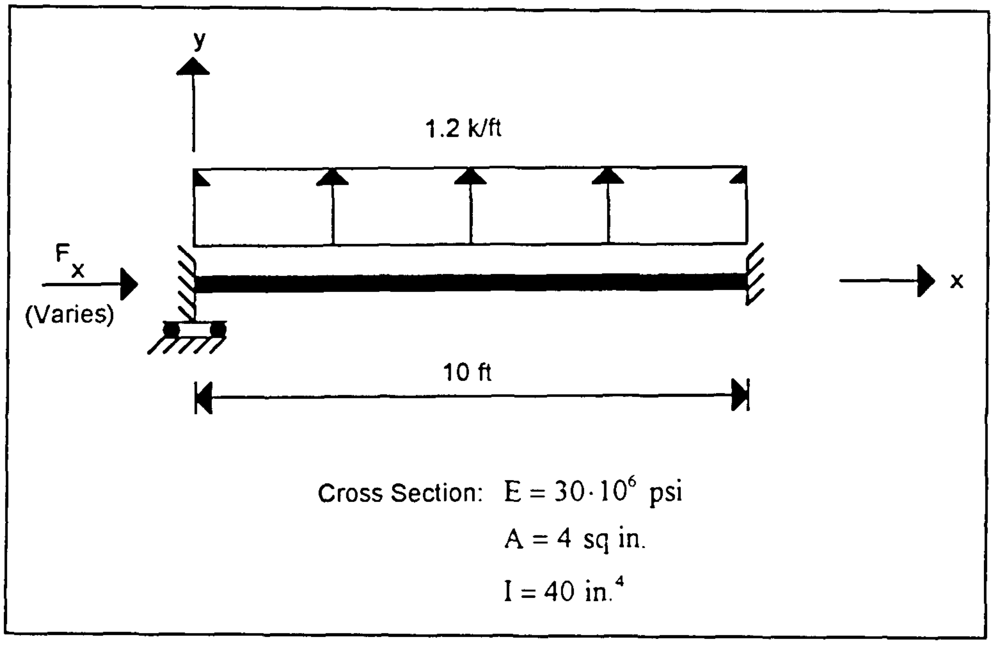


Figure 13. Fixed end beam for Example 1

a. The basic problem is described in lines 1000 through 1120 of the file. During execution, effects of axial load on bending were excluded for the solution of this part.

```
1000 'EXAMPLE 1 -- FIXED END BEAM ANALYSIS
1010 'EFFECTS OF AXIAL LOAD ON BENDING EXCLUDED
1020 BEAM
1030 0 120 30.E6 4 40
1040 NODES F
1050 -10 20 5
1060 LOADS F K
1070 D Y 0 1.2 10
1080 C 0 40 0 0
1090 FIXED F
1100 0 FREE 0 0
1110 10 0 0 0
1120 FINISH RERUN
1130 'EXAMPLE 1A -- SAME AS EXAMPLE 1
1140 'EXCEPT INCLUDE EFFECTS OF AXIAL LOAD ON BENDING
1150 'AXIAL LOAD = 40 KIPS
1160 FINISH RERUN
1170 'EXAMPLE 1B -- INCREASE AXIAL LOAD TO 2000 KIPS
1180 LOADS ADD
1190 C 0 1960 0 0
1200 FINISH RERUN
1210 'EXAMPLE 1C -- INCREASE AXIAL LOAD TO 3000 KIPS
1220 LOADS ADD
1230 C 0 1000 0 0
1240 FINISH RERUN
1250 'EXAMPLE 1D -- INCREASE AXIAL LOAD TO 3289 KIPS
1260 'THIS AXIAL LOAD IS SLIGHTLY LESS THAN BUCKLING LOAD
1270 LOADS ADD
1280 C 0 289 0 0
1290 FINISH RERUN
1300 'EXAMPLE 1E -- INCREASE AXIAL LOAD TO 3290 KIPS
1310 'THIS AXIAL LOAD IS SLIGHTLY GREATER THAN BUCKLING LOAD
1320 LOADS ADD
1330 C 0 1 0 0
1340 FINISH
```

Figure 14. Input file for Example 1

b. On the first rerun, Example 1A, only the problem heading was changed to that shown on lines 1140 through 1160 of the data file. All other data remain unchanged. During execution, the effects on bending due to the 40-kip[1] axial load were included in the solution.

c. On subsequent reruns, lines 1170 through 1340, the axial load was increased by adding concentrated load data to each preceding data set.

Interactive control of the program is shown in Figure 15.

---

[1] A table of factors for converting non-SI units of measurement to SI units is presented on page vi.

```
    PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
    DATE: 21-MAR-1994                                  TIME: 10.35.51

    ARE INPUT DATA TO BE READ FROM YOUR TERMINAL OR A FILE?
    ENTER 'TERMINAL' OR 'FILE'.
f
    ENTER INPUT FILE NAME (64 CHARACTERS MAXIMUM).
x0050d1
    INPUT COMPLETE.
    DO YOU WANT INPUT DATA ECHOPRINTED TO YOUR TERMINAL,
    TO A FILE, TO BOTH, OR NEITHER?
    ENTER 'TERMINAL', 'FILE', 'BOTH', OR 'NEITHER'.
f
    ENTER OUTPUT FILE NAME (64 CHARACTERS MAXIMUM).
x0050d1.out
    INPUT COMPLETE.
    DO YOU WANT TO EDIT INPUT DATA?
    ENTER 'YES' OR 'NO'.
n
    DO YOU WANT TO PLOT INPUT DATA?
    ENTER 'YES' OR 'NO'.
y
    PLOT OPTIONS:
    OPTION      PLOT GENERATED
       1........AXIAL AND BENDING STIFFNESSES
       2........FIXED AND/OR CONCENTRATED SPRING SUPPORTS
       3........LOADS
       4........DISTRIBUTED LINEAR SPRINGS
       5........SCHEMATIC OF DISTRIBUTED NONLINEAR SPRINGS

    ENTER PLOT OPTION (1 TO 5) OR 'FINISHED'.
1      (Note: Plots generated by options 1, 2 and 3 are shown in
              Figures 17 through 19.  Items covered by options 4
              and 5 are not present in this example.)

    INPUT COMPLETE.
    DO YOU WANT TO CONTINUE WITH THE SOLUTION?
    ENTER 'YES' OR 'NO'.
y
    DO YOU WANT AXIAL FORCE EFFECTS ON BENDING STIFFNESS
    INCLUDED IN THE SOLUTION?
    ENTER 'YES' OR 'NO'.
n
    SOLUTION COMPLETE.
    DO YOU WANT RESULTS PRINTED TO YOUR TERMINAL,
    TO FILE
    X0050D1.OUT
    OR BOTH?
    ENTER 'TERMINAL', 'FILE', OR 'BOTH'.
f
```

Figure 15. Interactive program control for Example 1 (Sheet 1 of 7)

Echoprints of input data were requested only for the first and last in the succession of problems, Figures 16 and 23, respectively. Plots of input data for the initial run of the sequence are shown in Figures 17 through 19.

```
        DESIRED OUTPUT UNITS?  ENTER:
        'DEFAULT', 'CONSISTENT', OR 'SELECTIVE'.
d          (Note:  Summary of results is shown in Figure 20.)
        DO YOU WANT COMPLETE RESULTS OUTPUT?
        ENTER 'YES' OR 'NO'.
n
        DO YOU WANT TO PLOT RESULTS?
        ENTER 'YES' OR 'NO'.
y
        PLOT OPTIONS:
        OPTION      PLOT GENERATED
           1........AXIAL AND LATERAL DISPLACEMENTS
           2........SHEAR FORCES AND BENDING MOMENTS
           3........FORCES IN DISTRIBUTED LINEAR SPRINGS
           4........FORCES IN DISTRIBUTED NONLINEAR SPRINGS

        ENTER PLOT OPTION (1 TO 4) OR 'FINISHED'.
1          (Note:  Plots for options 1 and 2 are shown in Figures
                   21 and 22.  Items covered by options 3 and 4
                   are not present in this example.)

        DO YOU WANT OUTPUT WITH DIFFERENT UNITS?
        ENTER 'YES' OR 'NO'.
n
        OUTPUT COMPLETE.
        INPUT COMPLETE.
        DO YOU WANT INPUT DATA ECHOPRINTED TO YOUR TERMINAL,
        TO FILE
        X0050D1.OUT
        TO BOTH OR NEITHER?
        ENTER 'TERMINAL', 'FILE', 'BOTH', OR 'NEITHER'.
n
        DO YOU WANT TO EDIT INPUT DATA?
        ENTER 'YES' OR 'NO'.
n
        DO YOU WANT TO PLOT INPUT DATA?
        ENTER 'YES' OR 'NO'.
n
        INPUT COMPLETE.
        DO YOU WANT TO CONTINUE WITH THE SOLUTION?
        ENTER 'YES' OR 'NO'.
y
        DO YOU WANT AXIAL FORCE EFFECTS ON BENDING STIFFNESS
        INCLUDED IN THE SOLUTION?
        ENTER 'YES' OR 'NO'.
y
```

Figure 15. (Sheet 2 of 7)

The summary of results was output for each problem, Figures 20 and 23. Because the model used to represent the beam contains only three nodes and two elements, no additional information would have been provided by a complete tabulation.

Plots of the results are shown in Figures 21 and 22. The plots of lateral deflection, Figure 21, and bending moment, Figure 22, emphasize the fact that

```
       SOLUTION COMPLETE.
       DO YOU WANT RESULTS PRINTED TO YOUR TERMINAL,
       TO FILE
       X0050D1.OUT
       OR BOTH?
       ENTER 'TERMINAL', 'FILE', OR 'BOTH'.
f
       DESIRED OUTPUT UNITS?  ENTER:
       'DEFAULT', 'CONSISTENT', OR 'SELECTIVE'.
d         (Note:  Summary of results is shown in Figure 20.)
       DO YOU WANT COMPLETE RESULTS OUTPUT?
       ENTER 'YES' OR 'NO'.
n
       DO YOU WANT TO PLOT RESULTS?
       ENTER 'YES' OR 'NO'.
n
       DO YOU WANT OUTPUT WITH DIFFERENT UNITS?
       ENTER 'YES' OR 'NO'.
n
       OUTPUT COMPLETE.
       INPUT COMPLETE.
       DO YOU WANT INPUT DATA ECHOPRINTED TO YOUR TERMINAL,
       TO FILE
       X0050D1.OUT
       TO BOTH OR NEITHER?
       ENTER 'TERMINAL', 'FILE', 'BOTH', OR 'NEITHER'.
n
       DO YOU WANT TO EDIT INPUT DATA?
       ENTER 'YES' OR 'NO'.
n
       DO YOU WANT TO PLOT INPUT DATA?
       ENTER 'YES' OR 'NO'.
n
       INPUT COMPLETE.
       DO YOU WANT TO CONTINUE WITH THE SOLUTION?
       ENTER 'YES' OR 'NO'.
y
       DO YOU WANT AXIAL FORCE EFFECTS ON BENDING STIFFNESS
       INCLUDED IN THE SOLUTION?
       ENTER 'YES' OR 'NO'.
y
       SOLUTION COMPLETE.
       DO YOU WANT RESULTS PRINTED TO YOUR TERMINAL,
       TO FILE
       X0050D1.OUT
       OR BOTH?
       ENTER 'TERMINAL', 'FILE', OR 'BOTH'.
f
```

Figure 15. (Sheet 3 of 7)

results are calculated only at the nodes of the model. The use of a larger number of nodes would have resulted in more appropriately curved diagrams.

The deflections and forces presented are "exact" solutions for each loading. The effect of axial load on bending is to increase the deflections and bending

```
      DESIRED OUTPUT UNITS?  ENTER:
      'DEFAULT', 'CONSISTENT', OR 'SELECTIVE'.
d        (Note:  Summary of results is shown in Figure 20.)
      DO YOU WANT COMPLETE RESULTS OUTPUT?
      ENTER 'YES' OR 'NO'.
n
      DO YOU WANT TO PLOT RESULTS?
      ENTER 'YES' OR 'NO'.
n
      DO YOU WANT OUTPUT WITH DIFFERENT UNITS?
      ENTER 'YES' OR 'NO'.
n
      OUTPUT COMPLETE.
      INPUT COMPLETE.
      DO YOU WANT INPUT DATA ECHOPRINTED TO YOUR TERMINAL,
      TO FILE
      X0050D1.OUT
      TO BOTH OR NEITHER?
      ENTER 'TERMINAL', 'FILE', 'BOTH', OR 'NEITHER'.
n
      DO YOU WANT TO EDIT INPUT DATA?
      ENTER 'YES' OR 'NO'.
n
      DO YOU WANT TO PLOT INPUT DATA?
      ENTER 'YES' OR 'NO'.
n
      INPUT COMPLETE.
      DO YOU WANT TO CONTINUE WITH THE SOLUTION?
      ENTER 'YES' OR 'NO'.
y
      DO YOU WANT AXIAL FORCE EFFECTS ON BENDING STIFFNESS
      INCLUDED IN THE SOLUTION?
      ENTER 'YES' OR 'NO'.
y
      SOLUTION COMPLETE.
      DO YOU WANT RESULTS PRINTED TO YOUR TERMINAL,
      TO FILE
      X0050D1.OUT
      OR BOTH?
      ENTER 'TERMINAL', 'FILE', OR 'BOTH'.
f
      DESIRED OUTPUT UNITS?  ENTER:
      'DEFAULT', 'CONSISTENT', OR 'SELECTIVE'.
d        (Note:  Summary of results is shown in Figure 20.)
      DO YOU WANT COMPLETE RESULTS OUTPUT?
      ENTER 'YES' OR 'NO'.
n
```

Figure 15. (Sheet 4 of 7)

moments. As the axial load approaches the theoretical buckling load (3,289.87 kips), deflections and moments attain extremely large values.

However, for an axial load below buckling, deflections and moments are consistent with the direction of the applied lateral load. For an axial load

```
         DO YOU WANT TO PLOT RESULTS?
         ENTER 'YES' OR 'NO'.
    n
         DO YOU WANT OUTPUT WITH DIFFERENT UNITS?
         ENTER 'YES' OR 'NO'.
    n
         OUTPUT COMPLETE.
         INPUT COMPLETE.
         DO YOU WANT INPUT DATA ECHOPRINTED TO YOUR TERMINAL,
         TO FILE
         X0050D1.OUT
         TO BOTH OR NEITHER?
         ENTER 'TERMINAL', 'FILE', 'BOTH', OR 'NEITHER'.
    n
         DO YOU WANT TO EDIT INPUT DATA?
         ENTER 'YES' OR 'NO'.
    n
         DO YOU WANT TO PLOT INPUT DATA?
         ENTER 'YES' OR 'NO'.
    n
         INPUT COMPLETE.
         DO YOU WANT TO CONTINUE WITH THE SOLUTION?
         ENTER 'YES' OR 'NO'.
    y
         DO YOU WANT AXIAL FORCE EFFECTS ON BENDING STIFFNESS
         INCLUDED IN THE SOLUTION?
         ENTER 'YES' OR 'NO'.
    y
         SOLUTION COMPLETE.
         DO YOU WANT RESULTS PRINTED TO YOUR TERMINAL,
         TO FILE
         X0050D1.OUT
         OR BOTH?
         ENTER 'TERMINAL', 'FILE', OR 'BOTH'.
    f
         DESIRED OUTPUT UNITS?  ENTER:
         'DEFAULT', 'CONSISTENT', OR 'SELECTIVE'.
    d       (Note:  Summary of results is shown in Figure 20.)
         DO YOU WANT COMPLETE RESULTS OUTPUT?
         ENTER 'YES' OR 'NO'.
    n
         DO YOU WANT TO PLOT RESULTS?
         ENTER 'YES' OR 'NO'.
    n
         DO YOU WANT OUTPUT WITH DIFFERENT UNITS?
         ENTER 'YES' OR 'NO'.
    n
```

Figure 15. (Sheet 5 of 7)

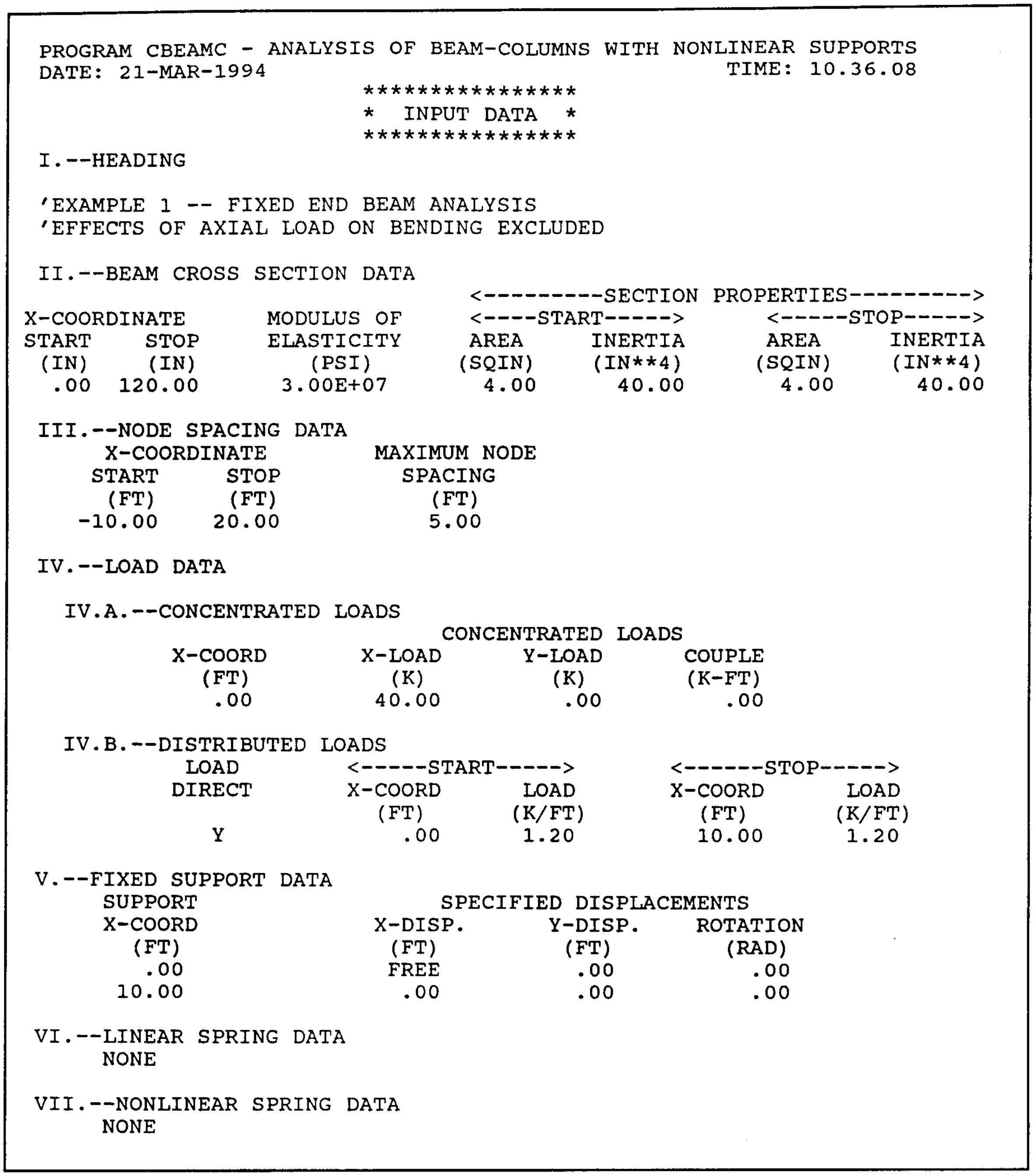
slightly greater than the buckling load, deflections and moments undergo a reversal in sign (i.e., are inconsistent with lateral load direction) indicating that buckling has occurred.

```
          OUTPUT COMPLETE.
          INPUT COMPLETE.
          DO YOU WANT INPUT DATA ECHOPRINTED TO YOUR TERMINAL,
          TO FILE
          X0050D1.OUT
          TO BOTH OR NEITHER?
          ENTER 'TERMINAL', 'FILE', 'BOTH', OR 'NEITHER'.
      y
          DO YOU WANT TO EDIT INPUT DATA?
          ENTER 'YES' OR 'NO'.
      n
          DO YOU WANT TO PLOT INPUT DATA?
          ENTER 'YES' OR 'NO'.
      n
          INPUT COMPLETE.
          DO YOU WANT TO CONTINUE WITH THE SOLUTION?
          ENTER 'YES' OR 'NO'.
      y
          DO YOU WANT AXIAL FORCE EFFECTS ON BENDING STIFFNESS
          INCLUDED IN THE SOLUTION?
          ENTER 'YES' OR 'NO'.
      y
          SOLUTION COMPLETE.
          DO YOU WANT RESULTS PRINTED TO YOUR TERMINAL,
          TO FILE
          X0050D1.OUT
          OR BOTH?
          ENTER 'TERMINAL', 'FILE', OR 'BOTH'.
      f
          DESIRED OUTPUT UNITS?  ENTER:
          'DEFAULT', 'CONSISTENT', OR 'SELECTIVE'.
      d      (Note:  Summary of results is shown in Figure 20.)
          DO YOU WANT COMPLETE RESULTS OUTPUT?
          ENTER 'YES' OR 'NO'.
      n
          DO YOU WANT TO PLOT RESULTS?
          ENTER 'YES' OR 'NO'.
      n
          DO YOU WANT OUTPUT WITH DIFFERENT UNITS?
          ENTER 'YES' OR 'NO'.
      n
          OUTPUT COMPLETE.
          INPUT COMPLETE.
          DO YOU WANT INPUT DATA ECHOPRINTED TO YOUR TERMINAL,
          TO FILE
          X0050D1.OUT
          TO BOTH OR NEITHER?
          ENTER 'TERMINAL', 'FILE', 'BOTH', OR 'NEITHER'.
      f      (Note:  Echoprint for this part is shown in Figure 23.)
```

Figure 15. (Sheet 6 of 7)

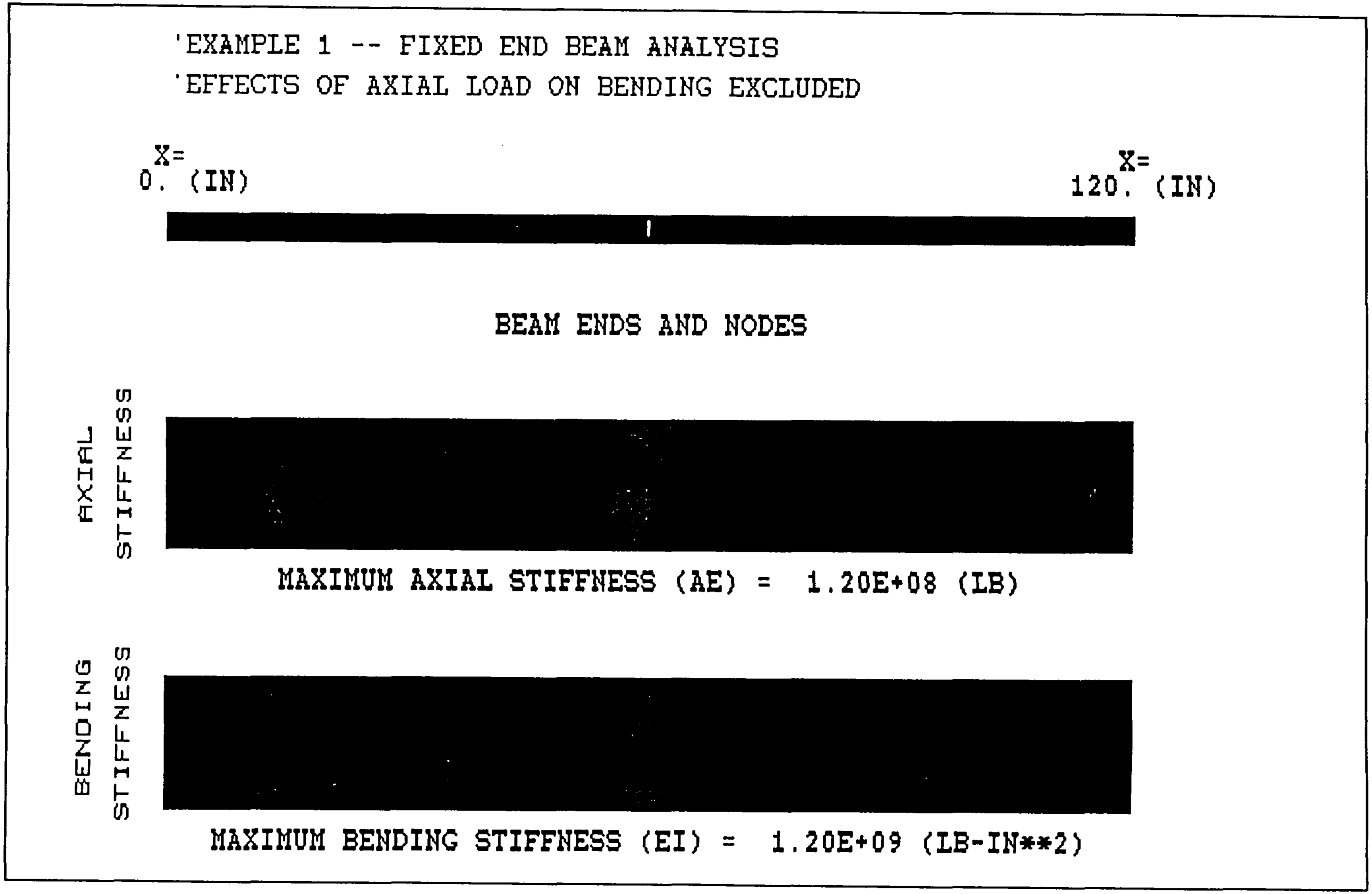
```
         DO YOU WANT TO EDIT INPUT DATA?
         ENTER 'YES' OR 'NO'.
    n
         DO YOU WANT TO PLOT INPUT DATA?
         ENTER 'YES' OR 'NO'.
    n
         INPUT COMPLETE.
         DO YOU WANT TO CONTINUE WITH THE SOLUTION?
         ENTER 'YES' OR 'NO'.
    y
         DO YOU WANT AXIAL FORCE EFFECTS ON BENDING STIFFNESS
         INCLUDED IN THE SOLUTION?
         ENTER 'YES' OR 'NO'.
    y
         SOLUTION COMPLETE.
         DO YOU WANT RESULTS PRINTED TO YOUR TERMINAL,
         TO FILE
         X0050D1.OUT
         OR BOTH?
         ENTER 'TERMINAL', 'FILE', OR 'BOTH'.
    f
         DESIRED OUTPUT UNITS?  ENTER:
         'DEFAULT', 'CONSISTENT', OR 'SELECTIVE'.
    d       (Note:  Summary of results is shown in Figure 23.)
         DO YOU WANT COMPLETE RESULTS OUTPUT?
         ENTER 'YES' OR 'NO'.
    n
         DO YOU WANT TO PLOT RESULTS?
         ENTER 'YES' OR 'NO'.
    n
         DO YOU WANT OUTPUT WITH DIFFERENT UNITS?
         ENTER 'YES' OR 'NO'.
    n
         OUTPUT COMPLETE.
         DO YOU WANT TO EDIT INPUT DATA?
         ENTER 'YES' OR 'NO'.
    n
         LAST INPUT FILE PROCESSED = X0050D1

         OUTPUT SAVED IN FILE X0050D1.OUT
         DO YOU WANT TO MAKE ANOTHER RUN?
         ENTER 'YES' OR 'NO'.
    n

         ***** NORMAL TERMINATION *****
```

Figure 15. (Sheet 7 of 7)



```
  PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
  DATE: 21-MAR-1994                                    TIME: 10.36.08
                              ****************
                              *  INPUT DATA  *
                              ****************
  I.--HEADING

  'EXAMPLE 1 -- FIXED END BEAM ANALYSIS
  'EFFECTS OF AXIAL LOAD ON BENDING EXCLUDED

  II.--BEAM CROSS SECTION DATA
                                  <---------SECTION PROPERTIES--------->
 X-COORDINATE      MODULUS OF     <----START----->      <-----STOP----->
 START    STOP     ELASTICITY      AREA     INERTIA      AREA     INERTIA
 (IN)     (IN)        (PSI)       (SQIN)    (IN**4)     (SQIN)    (IN**4)
  .00   120.00      3.00E+07       4.00      40.00       4.00      40.00

  III.--NODE SPACING DATA
      X-COORDINATE         MAXIMUM NODE
     START      STOP          SPACING
      (FT)      (FT)           (FT)
    -10.00     20.00           5.00

  IV.--LOAD DATA

    IV.A.--CONCENTRATED LOADS
                                CONCENTRATED LOADS
            X-COORD       X-LOAD       Y-LOAD       COUPLE
             (FT)          (K)          (K)         (K-FT)
              .00         40.00         .00           .00

    IV.B.--DISTRIBUTED LOADS
             LOAD        <-----START----->        <------STOP----->
            DIRECT       X-COORD      LOAD        X-COORD      LOAD
                          (FT)       (K/FT)        (FT)       (K/FT)
               Y           .00        1.20         10.00       1.20

  V.--FIXED SUPPORT DATA
      SUPPORT                 SPECIFIED DISPLACEMENTS
      X-COORD             X-DISP.       Y-DISP.     ROTATION
       (FT)                (FT)          (FT)         (RAD)
        .00                FREE          .00           .00
      10.00                 .00          .00           .00

  VI.--LINEAR SPRING DATA
      NONE

  VII.--NONLINEAR SPRING DATA
      NONE
```

Figure 16. Echoprint of input data for Example 1



'EXAMPLE 1 -- FIXED END BEAM ANALYSIS
'EFFECTS OF AXIAL LOAD ON BENDING EXCLUDED

MAXIMUM AXIAL STIFFNESS (AE) = 1.20E+08 (LB)

MAXIMUM BENDING STIFFNESS (EI) = 1.20E+09 (LB-IN**2)

Figure 17. Axial and bending stiffnesses for Example 1

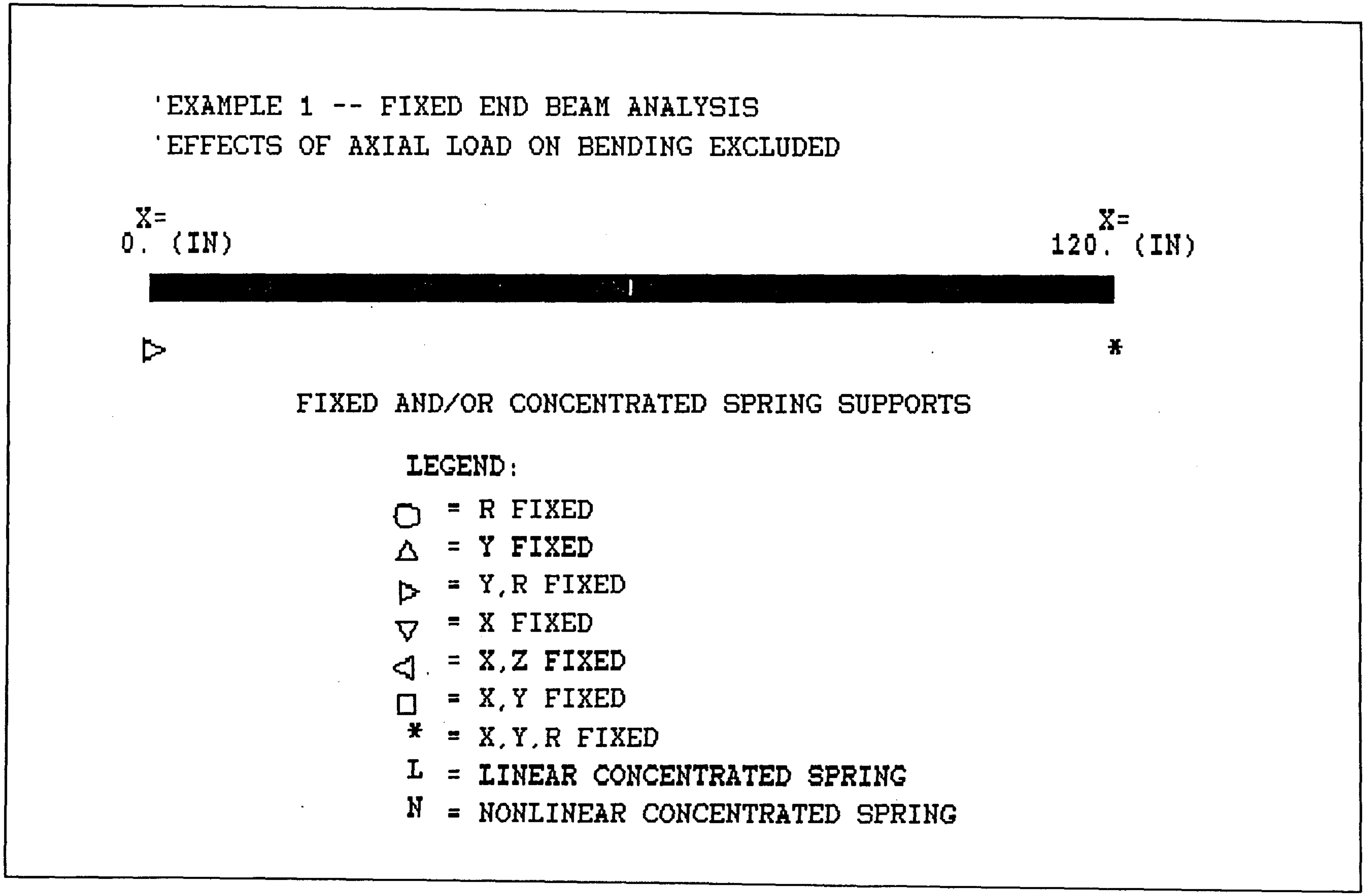


Figure 18. Schematic of supports for Example 1

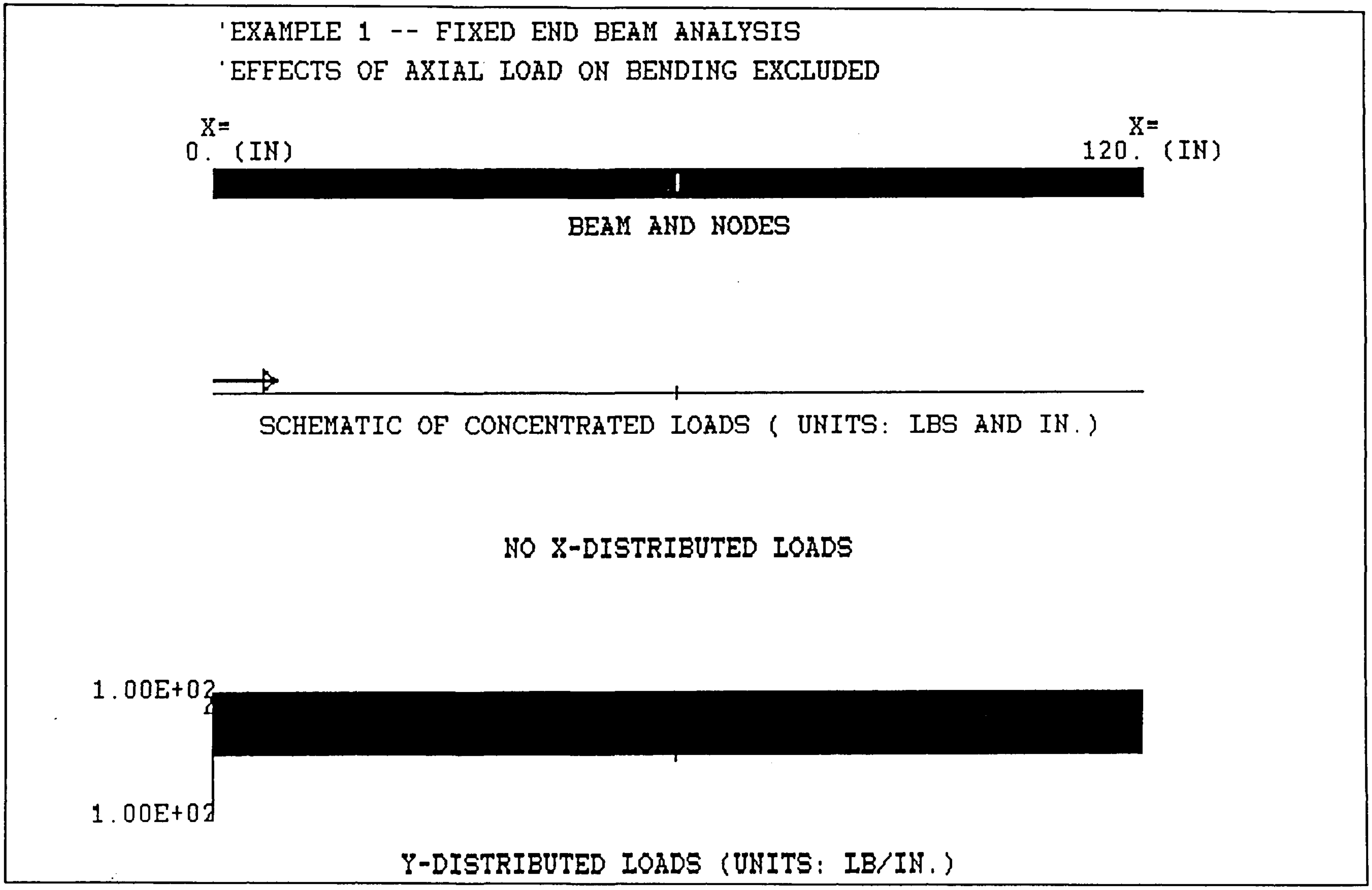
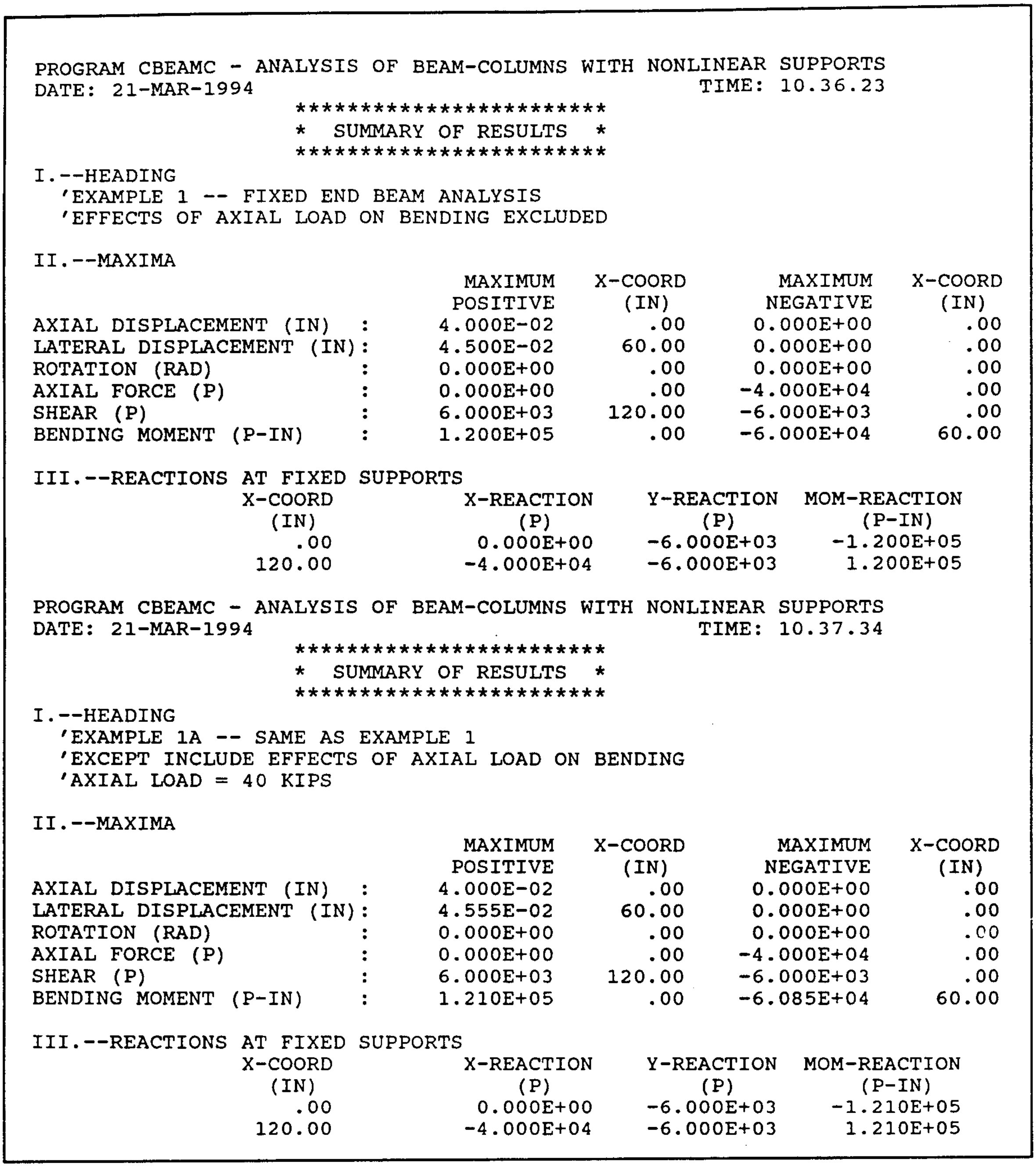


Figure 19. Schematic of applied loads for Example 1



```
PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                   TIME: 10.36.23
                                 ************************
                                 *  SUMMARY OF RESULTS  *
                                 ************************
I.--HEADING
   'EXAMPLE 1 -- FIXED END BEAM ANALYSIS
   'EFFECTS OF AXIAL LOAD ON BENDING EXCLUDED

II.--MAXIMA
                                  MAXIMUM   X-COORD       MAXIMUM   X-COORD
                                 POSITIVE    (IN)        NEGATIVE    (IN)
AXIAL DISPLACEMENT (IN)  :      4.000E-02     .00       0.000E+00     .00
LATERAL DISPLACEMENT (IN):      4.500E-02   60.00       0.000E+00     .00
ROTATION (RAD)           :      0.000E+00     .00       0.000E+00     .00
AXIAL FORCE (P)          :      0.000E+00     .00      -4.000E+04     .00
SHEAR (P)                :      6.000E+03  120.00      -6.000E+03     .00
BENDING MOMENT (P-IN)    :      1.200E+05     .00      -6.000E+04   60.00

III.--REACTIONS AT FIXED SUPPORTS
                     X-COORD          X-REACTION    Y-REACTION  MOM-REACTION
                      (IN)               (P)           (P)         (P-IN)
                       .00            0.000E+00    -6.000E+03    -1.200E+05
                     120.00          -4.000E+04    -6.000E+03     1.200E+05

PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                   TIME: 10.37.34
                                 ************************
                                 *  SUMMARY OF RESULTS  *
                                 ************************
I.--HEADING
   'EXAMPLE 1A -- SAME AS EXAMPLE 1
   'EXCEPT INCLUDE EFFECTS OF AXIAL LOAD ON BENDING
   'AXIAL LOAD = 40 KIPS

II.--MAXIMA
                                  MAXIMUM   X-COORD       MAXIMUM   X-COORD
                                 POSITIVE    (IN)        NEGATIVE    (IN)
AXIAL DISPLACEMENT (IN)  :      4.000E-02     .00       0.000E+00     .00
LATERAL DISPLACEMENT (IN):      4.555E-02   60.00       0.000E+00     .00
ROTATION (RAD)           :      0.000E+00     .00       0.000E+00     .00
AXIAL FORCE (P)          :      0.000E+00     .00      -4.000E+04     .00
SHEAR (P)                :      6.000E+03  120.00      -6.000E+03     .00
BENDING MOMENT (P-IN)    :      1.210E+05     .00      -6.085E+04   60.00

III.--REACTIONS AT FIXED SUPPORTS
                     X-COORD          X-REACTION    Y-REACTION  MOM-REACTION
                      (IN)               (P)           (P)         (P-IN)
                       .00            0.000E+00    -6.000E+03    -1.210E+05
                     120.00          -4.000E+04    -6.000E+03     1.210E+05
```

Figure 20. Summary of results for initial reruns of Example 1 (Sheet 1 of 3)

```
PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                  TIME: 10.38.08
                    ************************
                    *  SUMMARY OF RESULTS  *
                    ************************
I.--HEADING
  'EXAMPLE 1B -- INCREASE AXIAL LOAD TO 2000 KIPS

II.--MAXIMA
                                 MAXIMUM   X-COORD       MAXIMUM   X-COORD
                                POSITIVE     (IN)       NEGATIVE     (IN)
AXIAL DISPLACEMENT (IN)  :     2.000E+00       .00     0.000E+00       .00
LATERAL DISPLACEMENT (IN):     1.138E-01     60.00     0.000E+00       .00
ROTATION (RAD)           :     0.000E+00       .00     0.000E+00       .00
AXIAL FORCE (P)          :     0.000E+00       .00    -2.000E+06       .00
SHEAR (P)                :     6.000E+03    120.00    -6.000E+03       .00
BENDING MOMENT (P-IN)    :     2.373E+05       .00    -1.703E+05     60.00

III.--REACTIONS AT FIXED SUPPORTS
                X-COORD          X-REACTION    Y-REACTION  MOM-REACTION
                  (IN)               (P)           (P)         (P-IN)
                    .00           0.000E+00    -6.000E+03    -2.373E+05
                 120.00          -2.000E+06    -6.000E+03     2.373E+05

PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                  TIME: 10.38.41
                    ************************
                    *  SUMMARY OF RESULTS  *
                    ************************
I.--HEADING
  'EXAMPLE 1C -- INCREASE AXIAL LOAD TO 3000 KIPS

II.--MAXIMA
                                 MAXIMUM   X-COORD       MAXIMUM   X-COORD
                                POSITIVE     (IN)       NEGATIVE     (IN)
AXIAL DISPLACEMENT (IN)  :     3.000E+00       .00     0.000E+00       .00
LATERAL DISPLACEMENT (IN):     5.041E-01     60.00     0.000E+00       .00
ROTATION (RAD)           :     0.000E+00       .00     0.000E+00       .00
AXIAL FORCE (P)          :     0.000E+00       .00    -3.000E+06       .00
SHEAR (P)                :     6.000E+03    120.00    -6.000E+03       .00
BENDING MOMENT (P-IN)    :     8.818E+05       .00    -8.103E+05     60.00

III.--REACTIONS AT FIXED SUPPORTS
                X-COORD          X-REACTION    Y-REACTION  MOM-REACTION
                  (IN)               (P)           (P)         (P-IN)
                    .00           0.000E+00    -6.000E+03    -8.818E+05
                 120.00          -3.000E+06    -6.000E+03     8.818E+05
```

Figure 20. (Sheet 2 of 3)

```
PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                   TIME: 10.39.03
                    ************************
                    *  SUMMARY OF RESULTS  *
                    ************************
I.--HEADING
  'EXAMPLE 1D -- INCREASE AXIAL LOAD TO 3289 KIPS
  'THIS AXIAL LOAD IS SLIGHTLY LESS THAN BUCKLING LOAD

II.--MAXIMA
                                 MAXIMUM   X-COORD     MAXIMUM   X-COORD
                                POSITIVE    (IN)      NEGATIVE    (IN)
AXIAL DISPLACEMENT (IN)   :    3.289E+00     .00     0.000E+00     .00
LATERAL DISPLACEMENT (IN):     1.681E+02   60.00     0.000E+00     .00
ROTATION (RAD)            :    0.000E+00     .00     0.000E+00     .00
AXIAL FORCE (P)           :    0.000E+00     .00    -3.289E+06     .00
SHEAR (P)                 :    6.000E+03  120.00    -6.000E+03     .00
BENDING MOMENT (P-IN)     :    2.765E+08     .00    -2.764E+08   60.00

III.--REACTIONS AT FIXED SUPPORTS
                X-COORD          X-REACTION     Y-REACTION  MOM-REACTION
                 (IN)               (P)            (P)         (P-IN)
                  .00           0.000E+00     -6.000E+03    -2.765E+08
                120.00         -3.289E+06     -6.000E+03     2.765E+08
```

Figure 20. (Sheet 3 of 3)

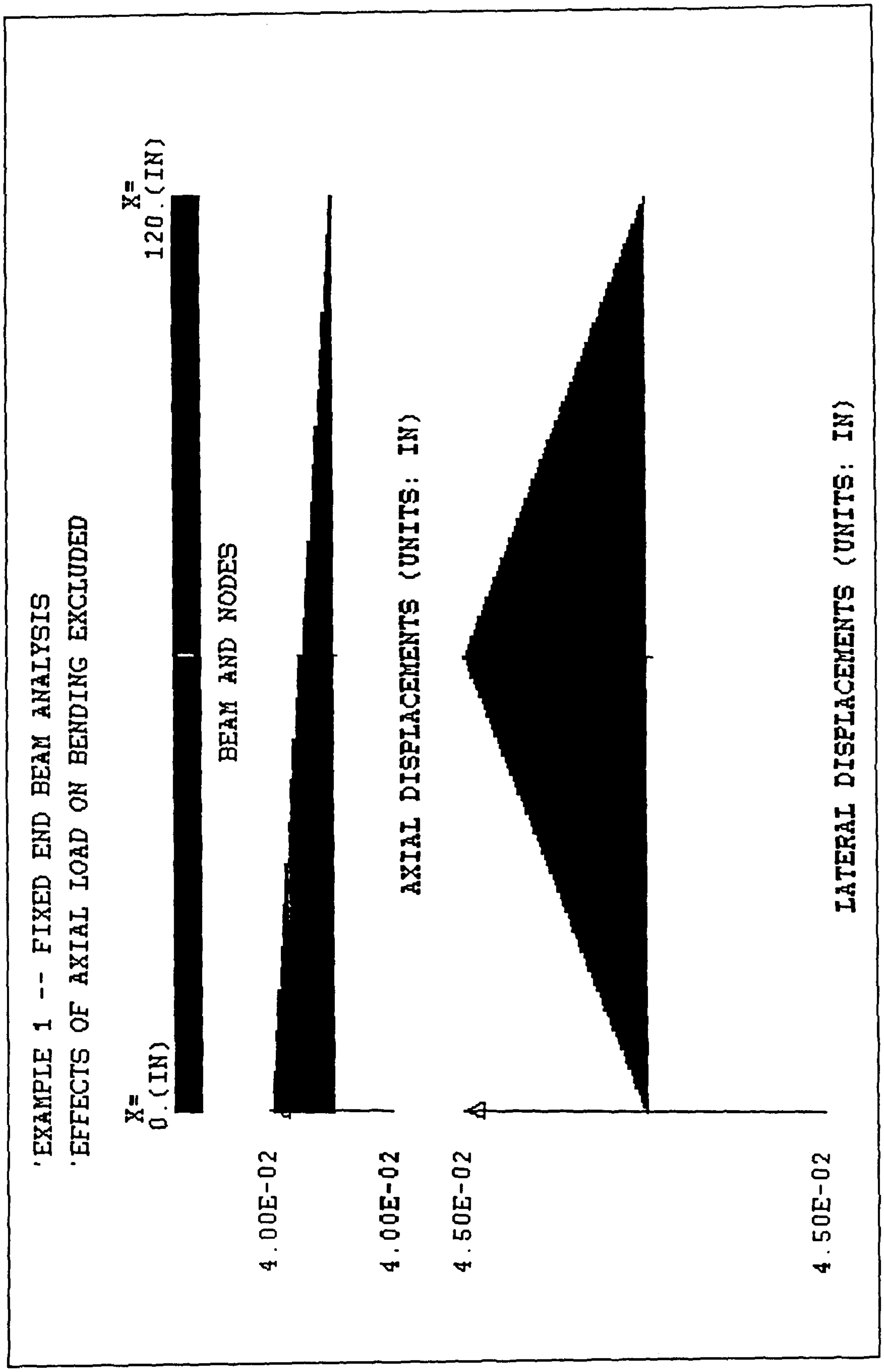


Figure 21. Axial and lateral displacements for Example 1A

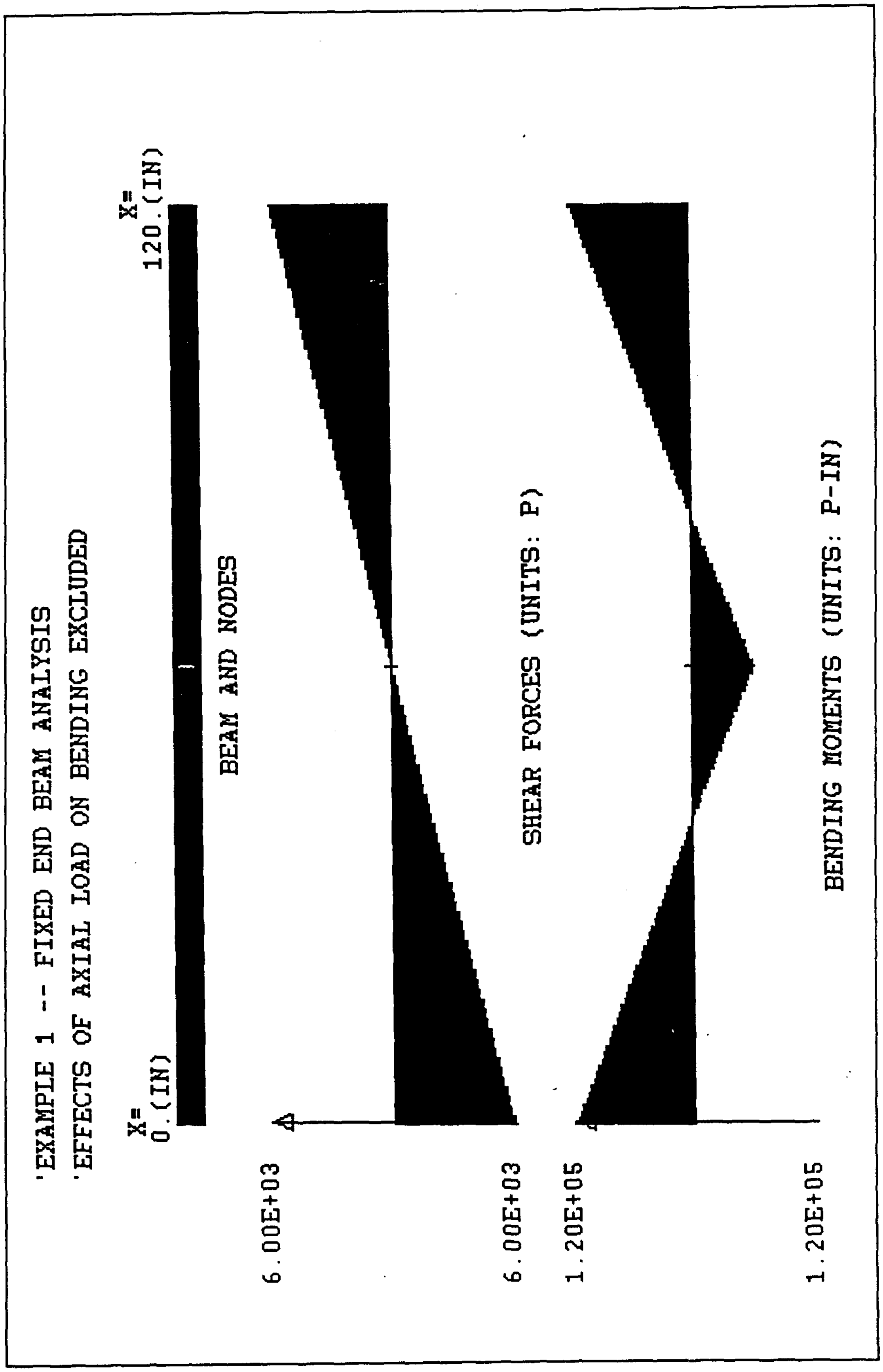


Figure 22. Shear force and bending moment diagrams for Example 1A

```
PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                   TIME: 10.39.12
                                ****************
                                *  INPUT DATA  *
                                ****************
I.--HEADING
'EXAMPLE 1E -- INCREASE AXIAL LOAD TO 3290 KIPS
'THIS AXIAL LOAD IS SLIGHTLY GREATER THAN BUCKLING LOAD
II.--BEAM CROSS SECTION DATA
                                  <---------SECTION PROPERTIES--------->
X-COORDINATE      MODULUS OF     <----START----->     <-----STOP----->
START    STOP     ELASTICITY      AREA     INERTIA     AREA     INERTIA
 (IN)    (IN)       (PSI)        (SQIN)    (IN**4)    (SQIN)    (IN**4)
  .00  120.00     3.00E+07        4.00      40.00      4.00      40.00
III.--NODE SPACING DATA
     X-COORDINATE          MAXIMUM NODE
    START      STOP          SPACING
     (FT)      (FT)           (FT)
   -10.00     20.00           5.00
IV.--LOAD DATA
  IV.A.--CONCENTRATED LOADS
                             CONCENTRATED LOADS
           X-COORD      X-LOAD      Y-LOAD      COUPLE
            (FT)          (K)         (K)       (K-FT)
             .00        40.00         .00         .00
             .00      1960.00         .00         .00
             .00      1000.00         .00         .00
             .00       289.00         .00         .00
             .00         1.00         .00         .00
  IV.B.--DISTRIBUTED LOADS
            LOAD      <-----START----->       <------STOP----->
           DIRECT     X-COORD     LOAD        X-COORD     LOAD
                       (FT)      (K/FT)        (FT)      (K/FT)
             Y          .00       1.20         10.00      1.20
V.--FIXED SUPPORT DATA
     SUPPORT                 SPECIFIED DISPLACEMENTS
     X-COORD            X-DISP.      Y-DISP.     ROTATION
      (FT)               (FT)         (FT)        (RAD)
       .00               FREE          .00          .00
     10.00                .00          .00          .00
VI.--LINEAR SPRING DATA
     NONE
VII.--NONLINEAR SPRING DATA
     NONE
```

Figure 23. Echoprint of input data and summary of results for final run of Example 1 (Continued)

```
PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                   TIME: 10.39.30

                    ************************
                    *  SUMMARY OF RESULTS  *
                    ************************

I.--HEADING
  'EXAMPLE 1E -- INCREASE AXIAL LOAD TO 3290 KIPS
  'THIS AXIAL LOAD IS SLIGHTLY GREATER THAN BUCKLING LOAD


II.--MAXIMA
                              MAXIMUM    X-COORD       MAXIMUM    X-COORD
                              POSITIVE    (IN)        NEGATIVE     (IN)
AXIAL DISPLACEMENT (IN)   :   3.290E+00    .00       0.000E+00      .00
LATERAL DISPLACEMENT (IN):    0.000E+00    .00      -1.106E+03    60.00
ROTATION (RAD)            :   0.000E+00    .00       0.000E+00      .00
AXIAL FORCE (P)           :   0.000E+00    .00      -3.290E+06      .00
SHEAR (P)                 :   6.000E+03  120.00     -6.000E+03      .00
BENDING MOMENT (P-IN)     :   1.820E+09   60.00     -1.820E+09      .00

III.--REACTIONS AT FIXED SUPPORTS
                X-COORD         X-REACTION     Y-REACTION   MOM-REACTION
                 (IN)              (P)            (P)          (P-IN)
                  .00           0.000E+00     -6.000E+03     1.820E+09
                120.00         -3.290E+06     -6.000E+03    -1.820E+09
```

Figure 23. (Concluded)

Example 2: Beam on Uniform Elastic Foundation

Accuracy of results produced by the finite element model used to represent the system shown in Figure 24 depends on node spacing. Solutions were obtained for two node spacings as indicated in the data file shown in Figure 25.

The echoprint of input data, summary of results, and the (trivial) complete results table for the first case (lines 1000 through 1140 of the input file) are shown in Figure 26. Maximum lateral deflection and bending moment for the model with three nodes and two elements differ only slightly from the "exact" solution (0.19097 in. and -1.52025E+5 lb-in., respectively).

Increasing the model to 11 nodes and 10 elements produces results, Figure 27, which are exact to four significant figures.

It should be emphasized that node spacing may have a more significant effect on the accuracy of the solution than is indicated by this problem. This is demonstrated in Example 3.

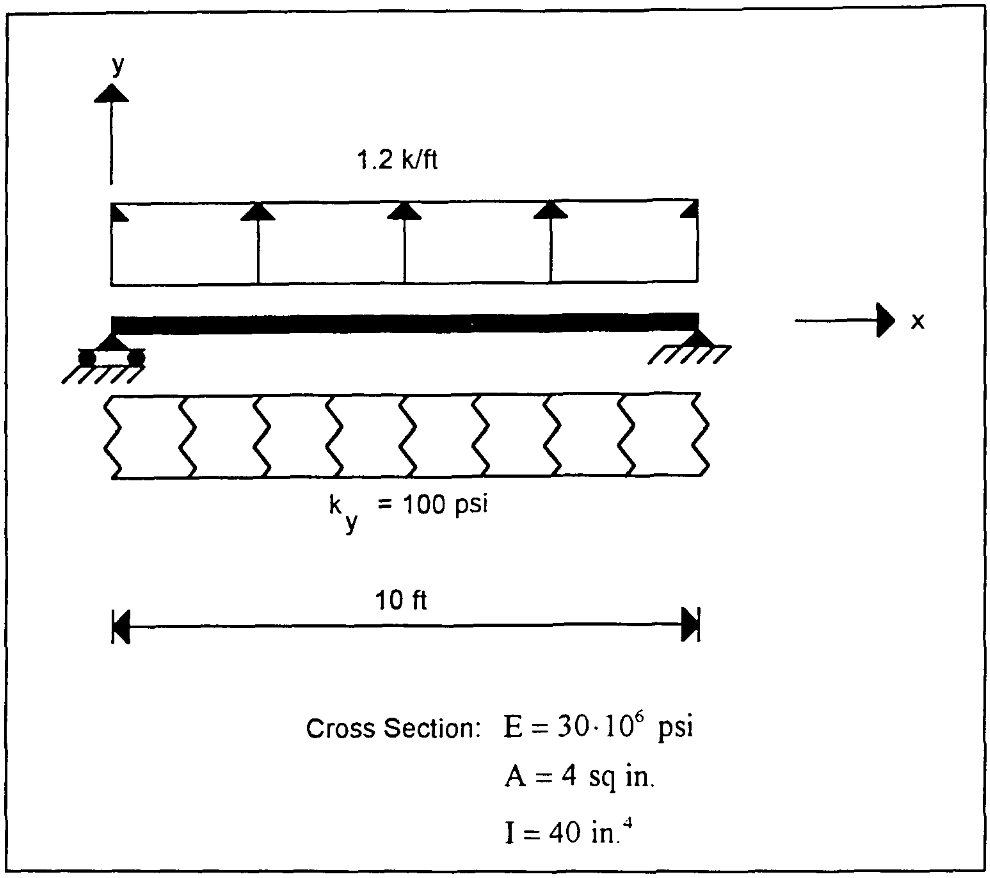


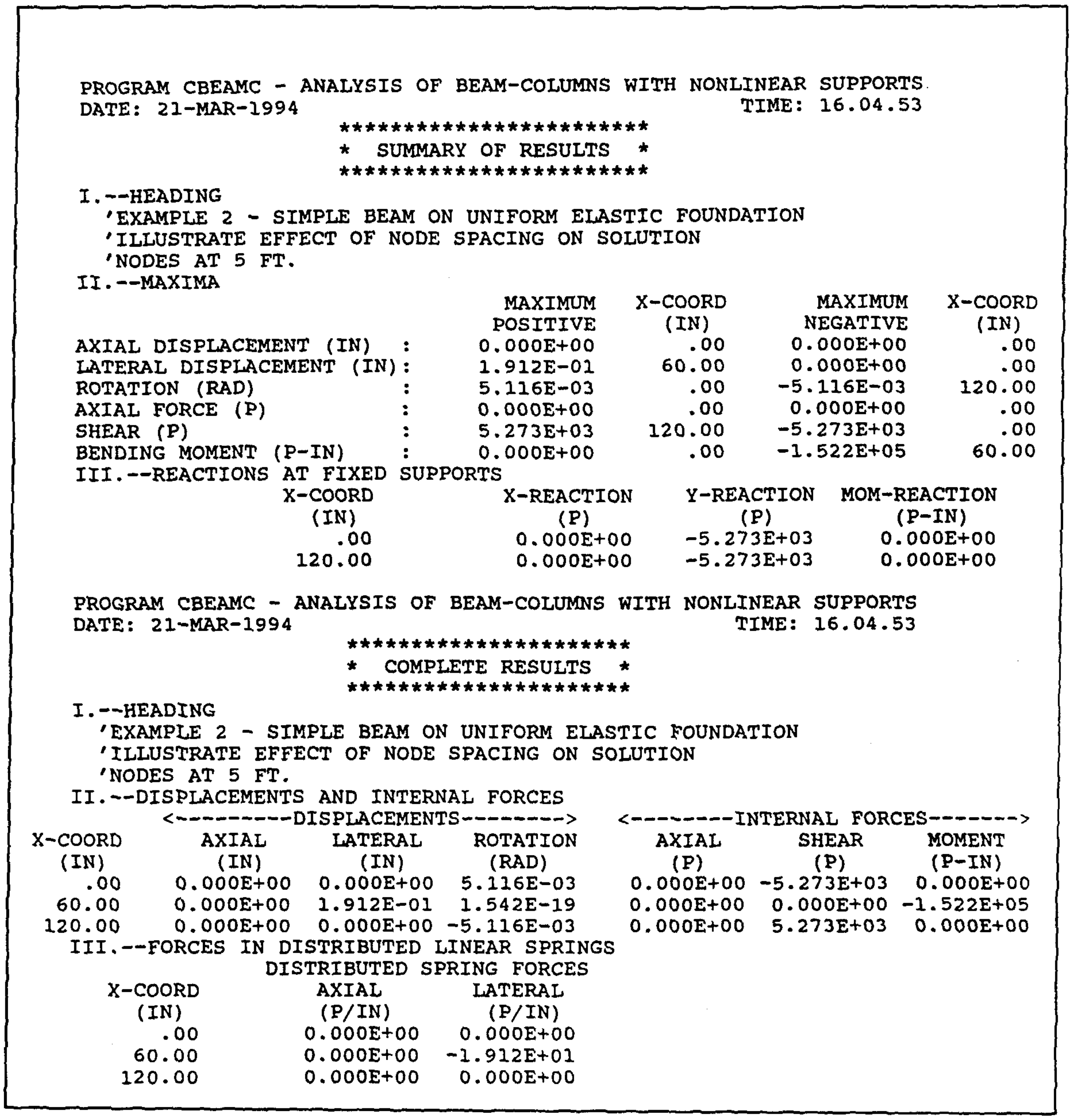
Figure 24. Beam on elastic support for Example 2

```
1000 'EXAMPLE 2 - SIMPLE BEAM ON UNIFORM ELASTIC FOUNDATION
1010 'ILLUSTRATE EFFECT OF NODE SPACING ON SOLUTION
1020 'NODES AT 5 FT.
1030 BEAM
1040 0 120 30.E6 4 40
1050 NODES F
1060 0 10 5
1070 LOADS F K
1080 D Y 0 1.2 10
1090 FIXED F
1100 0 FREE 0 FREE
1110 10 0 0 FREE
1120 LINEAR
1130 D Y 0 120 100 0 0
1140 FINISH RERUN
1150 'EXAMPLE 2A -- SAME AS EXAMPLE 2 EXCEPT NODES AT 1.25 FT.
1160 NODES N F
1170 0 10 1.25
1180 FINISH
```

Figure 25. Input file for Example 2

```
 PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
 DATE: 21-MAR-1994                                  TIME: 16.04.36
                        ****************
                        *  INPUT DATA  *
                        ****************
 I.--HEADING
 'EXAMPLE 2 - SIMPLE BEAM ON UNIFORM ELASTIC FOUNDATION
 'ILLUSTRATE EFFECT OF NODE SPACING ON SOLUTION
 'NODES AT 5 FT.

 II.--BEAM CROSS SECTION DATA
                                   <---------SECTION PROPERTIES--------->
X-COORDINATE      MODULUS OF       <----START----->       <-----STOP----->
START    STOP     ELASTICITY        AREA     INERTIA       AREA     INERTIA
 (IN)    (IN)        (PSI)         (SQIN)    (IN**4)      (SQIN)    (IN**4)
  .00  120.00      3.00E+07         4.00      40.00        4.00      40.00

 III.--NODE SPACING DATA
      X-COORDINATE          MAXIMUM NODE
     START      STOP          SPACING
      (FT)      (FT)           (FT)
       .00     10.00           5.00

 IV.--LOAD DATA
   4.A.--CONCENTRATED LOADS
          NONE
   IV.B.--DISTRIBUTED LOADS
            LOAD        <-----START----->       <------STOP----->
           DIRECT       X-COORD      LOAD       X-COORD      LOAD
                          (FT)      (K/FT)        (FT)      (K/FT)
             Y             .00       1.20        10.00       1.20

 V.--FIXED SUPPORT DATA
      SUPPORT                   SPECIFIED DISPLACEMENTS
      X-COORD              X-DISP.       Y-DISP.     ROTATION
       (FT)                 (FT)          (FT)         (RAD)
        .00                 FREE           .00         FREE
      10.00                  .00           .00         FREE

 VI.--LINEAR SPRING DATA
   VI.A.--CONCENTRATED LINEAR SPRINGS
          NONE
   VI.B.--DISTRIBUTED LINEAR SPRINGS
      SPRING            X-COORD           SPRING STIFFNESS COEFFICIENTS
     DIRECTION       START      STOP            A          B           C
                     (IN)       (IN)         (P/I/I)    (P/I/I)
         Y            .00      120.00        100.00       .00         .00

 VII.--NONLINEAR SPRING DATA
      NONE
```

Figure 26. Echoprint of input and results for Example 2A (Continued)

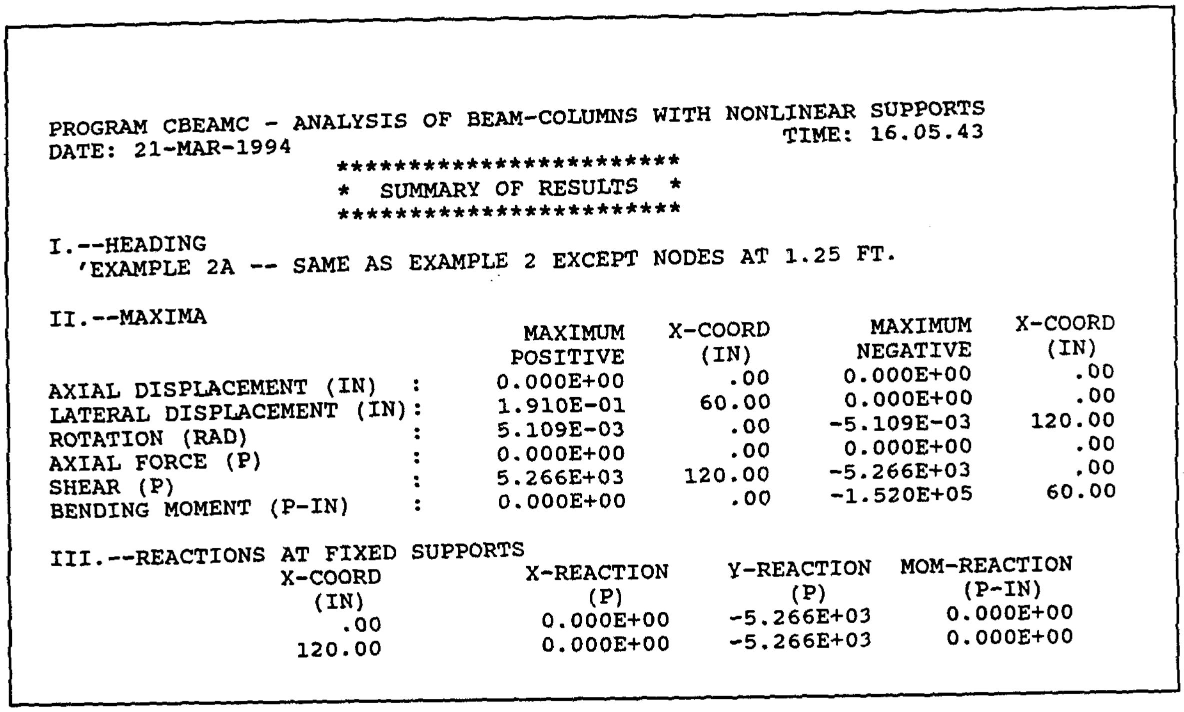


```
PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                   TIME: 16.04.53
                          *************************
                          *  SUMMARY OF RESULTS   *
                          *************************
I.--HEADING
  'EXAMPLE 2 - SIMPLE BEAM ON UNIFORM ELASTIC FOUNDATION
  'ILLUSTRATE EFFECT OF NODE SPACING ON SOLUTION
  'NODES AT 5 FT.
II.--MAXIMA
                                MAXIMUM    X-COORD       MAXIMUM   X-COORD
                               POSITIVE     (IN)        NEGATIVE    (IN)
AXIAL DISPLACEMENT (IN)   :   0.000E+00      .00       0.000E+00      .00
LATERAL DISPLACEMENT (IN):    1.912E-01    60.00       0.000E+00      .00
ROTATION (RAD)            :   5.116E-03      .00      -5.116E-03   120.00
AXIAL FORCE (P)           :   0.000E+00      .00       0.000E+00      .00
SHEAR (P)                 :   5.273E+03   120.00      -5.273E+03      .00
BENDING MOMENT (P-IN)     :   0.000E+00      .00      -1.522E+05    60.00
III.--REACTIONS AT FIXED SUPPORTS
                  X-COORD          X-REACTION    Y-REACTION   MOM-REACTION
                   (IN)               (P)           (P)          (P-IN)
                    .00            0.000E+00    -5.273E+03     0.000E+00
                 120.00            0.000E+00    -5.273E+03     0.000E+00

PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                   TIME: 16.04.53
                          ***********************
                          *  COMPLETE RESULTS   *
                          ***********************
I.--HEADING
  'EXAMPLE 2 - SIMPLE BEAM ON UNIFORM ELASTIC FOUNDATION
  'ILLUSTRATE EFFECT OF NODE SPACING ON SOLUTION
  'NODES AT 5 FT.
II.--DISPLACEMENTS AND INTERNAL FORCES
          <---------DISPLACEMENTS-------->   <--------INTERNAL FORCES------->
X-COORD      AXIAL      LATERAL    ROTATION      AXIAL      SHEAR      MOMENT
 (IN)        (IN)        (IN)       (RAD)         (P)        (P)       (P-IN)
   .00    0.000E+00  0.000E+00  5.116E-03    0.000E+00 -5.273E+03  0.000E+00
  60.00   0.000E+00  1.912E-01  1.542E-19    0.000E+00  0.000E+00 -1.522E+05
 120.00   0.000E+00  0.000E+00 -5.116E-03    0.000E+00  5.273E+03  0.000E+00
III.--FORCES IN DISTRIBUTED LINEAR SPRINGS
                      DISTRIBUTED SPRING FORCES
     X-COORD          AXIAL       LATERAL
      (IN)            (P/IN)       (P/IN)
        .00         0.000E+00    0.000E+00
      60.00         0.000E+00   -1.912E+01
     120.00         0.000E+00    0.000E+00
```

Figure 26. (Concluded)

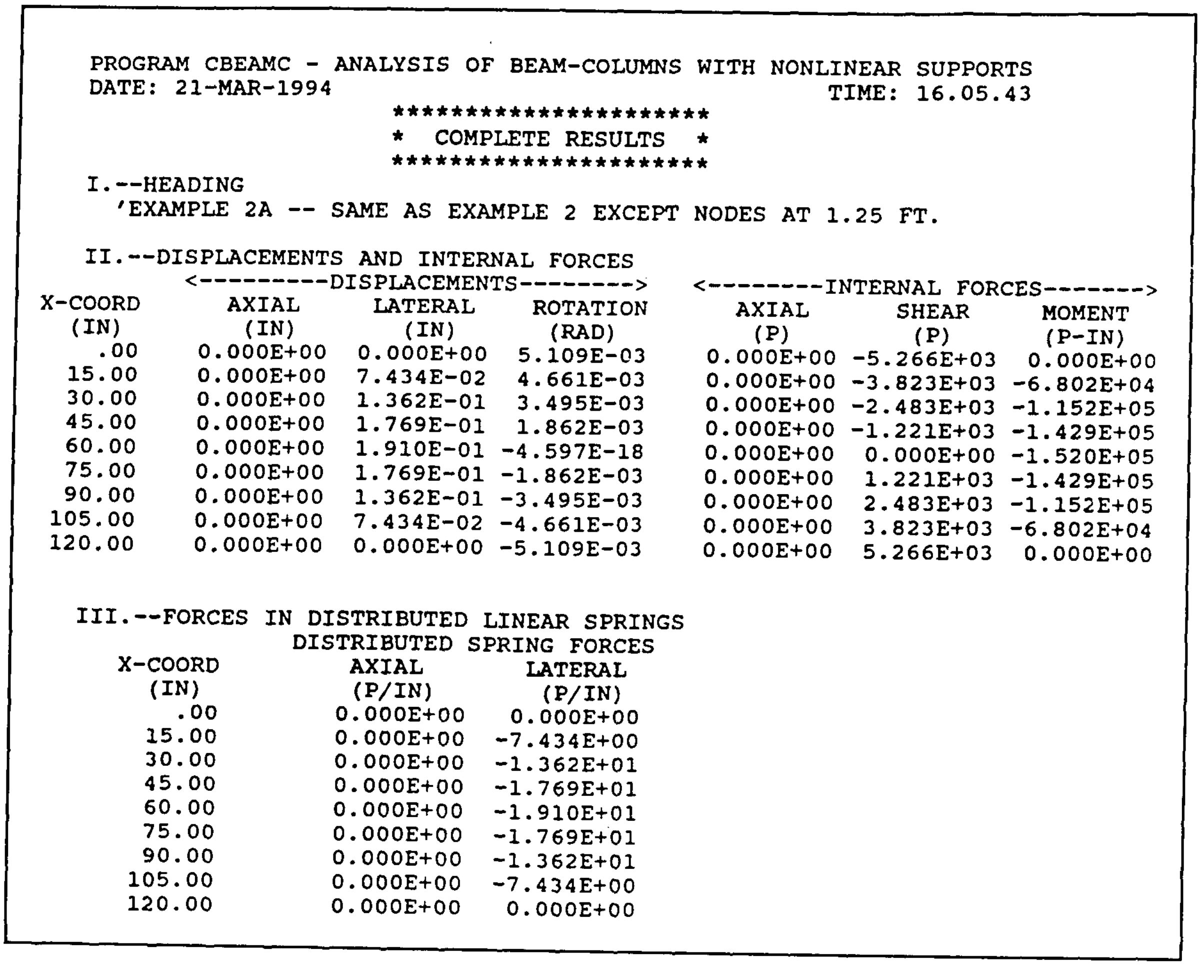
```
 PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
 DATE: 21-MAR-1994                                  TIME: 16.05.35
                         ****************
                         *  INPUT DATA  *
                         ****************
 I.--HEADING
 'EXAMPLE 2A -- SAME AS EXAMPLE 2 EXCEPT NODES AT 1.25 FT.

 II.--BEAM CROSS SECTION DATA
                                 <---------SECTION PROPERTIES--------->
X-COORDINATE      MODULUS OF     <----START----->     <-----STOP----->
START    STOP     ELASTICITY     AREA     INERTIA      AREA     INERTIA
 (IN)    (IN)        (PSI)      (SQIN)    (IN**4)     (SQIN)    (IN**4)
  .00  120.00      3.00E+07      4.00      40.00       4.00      40.00

 III.--NODE SPACING DATA
      X-COORDINATE          MAXIMUM NODE
     START     STOP            SPACING
      (FT)     (FT)              (FT)
       .00    10.00              1.25

 IV.--LOAD DATA
   4.A.--CONCENTRATED LOADS
          NONE

   IV.B.--DISTRIBUTED LOADS
            LOAD        <-----START----->       <------STOP----->
           DIRECT        X-COORD      LOAD       X-COORD      LOAD
                          (FT)       (K/FT)        (FT)      (K/FT)
              Y            .00        1.20        10.00       1.20

 V.--FIXED SUPPORT DATA
     SUPPORT                  SPECIFIED DISPLACEMENTS
     X-COORD             X-DISP.      Y-DISP.     ROTATION
       (FT)               (FT)         (FT)         (RAD)
        .00               FREE          .00          FREE
      10.00                .00          .00          FREE

 VI.--LINEAR SPRING DATA
   VI.A.--CONCENTRATED LINEAR SPRINGS
         NONE

   VI.B.--DISTRIBUTED LINEAR SPRINGS
     SPRING          X-COORD          SPRING STIFFNESS COEFFICIENTS
    DIRECTION      START     STOP           A          B          C
                    (IN)     (IN)        (P/I/I)    (P/I/I)
        Y            .00    120.00       100.00        .00        .00

 VII.--NONLINEAR SPRING DATA
     NONE
```

Figure 27. Echoprint of input and results for Example 2B (Sheet 1 of 3)



```
PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 21-MAR-1994                                      TIME: 16.05.43
                               ************************
                               *  SUMMARY OF RESULTS  *
                               ************************
I.--HEADING
  'EXAMPLE 2A -- SAME AS EXAMPLE 2 EXCEPT NODES AT 1.25 FT.

II.--MAXIMA
                                  MAXIMUM    X-COORD      MAXIMUM    X-COORD
                                 POSITIVE     (IN)       NEGATIVE     (IN)
AXIAL DISPLACEMENT (IN)   :     0.000E+00      .00      0.000E+00      .00
LATERAL DISPLACEMENT (IN):      1.910E-01    60.00      0.000E+00      .00
ROTATION (RAD)            :     5.109E-03      .00     -5.109E-03   120.00
AXIAL FORCE (P)           :     0.000E+00      .00      0.000E+00      .00
SHEAR (P)                 :     5.266E+03   120.00     -5.266E+03      .00
BENDING MOMENT (P-IN)     :     0.000E+00      .00     -1.520E+05    60.00

III.--REACTIONS AT FIXED SUPPORTS
                  X-COORD          X-REACTION     Y-REACTION   MOM-REACTION
                   (IN)               (P)            (P)          (P-IN)
                    .00            0.000E+00     -5.266E+03     0.000E+00
                  120.00           0.000E+00     -5.266E+03     0.000E+00
```

Figure 27. (Sheet 2 of 3)



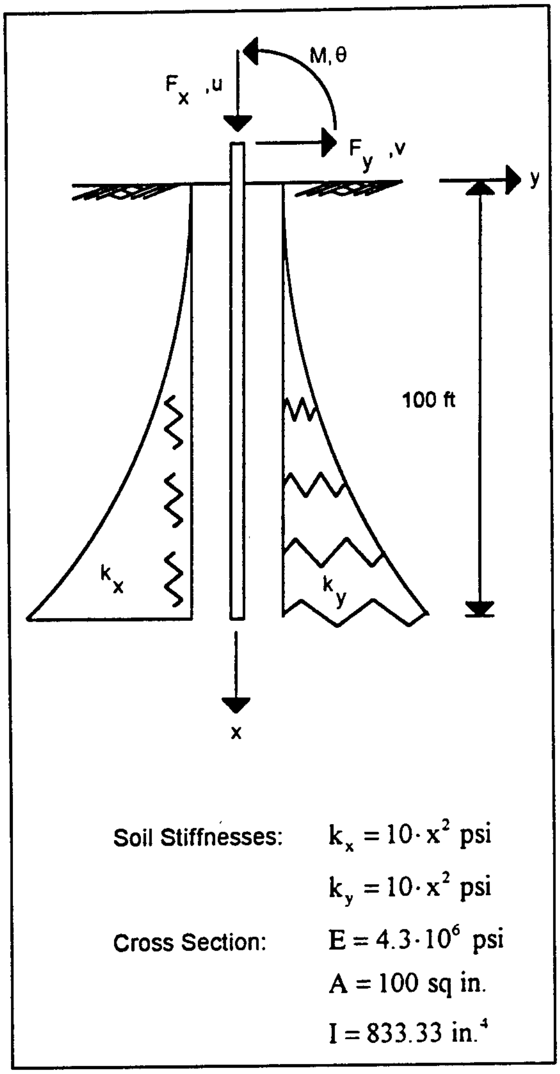
```
   PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
   DATE: 21-MAR-1994                                  TIME: 16.05.43
                             **********************
                             *  COMPLETE RESULTS  *
                             **********************
   I.--HEADING
     'EXAMPLE 2A -- SAME AS EXAMPLE 2 EXCEPT NODES AT 1.25 FT.

   II.--DISPLACEMENTS AND INTERNAL FORCES
             <---------DISPLACEMENTS-------->   <--------INTERNAL FORCES------->
X-COORD      AXIAL      LATERAL    ROTATION      AXIAL      SHEAR      MOMENT
 (IN)        (IN)        (IN)       (RAD)         (P)        (P)       (P-IN)
   .00    0.000E+00  0.000E+00  5.109E-03    0.000E+00 -5.266E+03  0.000E+00
 15.00    0.000E+00  7.434E-02  4.661E-03    0.000E+00 -3.823E+03 -6.802E+04
 30.00    0.000E+00  1.362E-01  3.495E-03    0.000E+00 -2.483E+03 -1.152E+05
 45.00    0.000E+00  1.769E-01  1.862E-03    0.000E+00 -1.221E+03 -1.429E+05
 60.00    0.000E+00  1.910E-01 -4.597E-18    0.000E+00  0.000E+00 -1.520E+05
 75.00    0.000E+00  1.769E-01 -1.862E-03    0.000E+00  1.221E+03 -1.429E+05
 90.00    0.000E+00  1.362E-01 -3.495E-03    0.000E+00  2.483E+03 -1.152E+05
105.00    0.000E+00  7.434E-02 -4.661E-03    0.000E+00  3.823E+03 -6.802E+04
120.00    0.000E+00  0.000E+00 -5.109E-03    0.000E+00  5.266E+03  0.000E+00

  III.--FORCES IN DISTRIBUTED LINEAR SPRINGS
                     DISTRIBUTED SPRING FORCES
      X-COORD           AXIAL       LATERAL
       (IN)             (P/IN)       (P/IN)
          .00         0.000E+00   0.000E+00
        15.00         0.000E+00  -7.434E+00
        30.00         0.000E+00  -1.362E+01
        45.00         0.000E+00  -1.769E+01
        60.00         0.000E+00  -1.910E+01
        75.00         0.000E+00  -1.769E+01
        90.00         0.000E+00  -1.362E+01
       105.00         0.000E+00  -7.434E+00
       120.00         0.000E+00   0.000E+00
```

Figure 27. (Sheet 3 of 3)

Example 3: Pile Head Stiffness Matrix

The program was used to generate the pile head stiffness matrix for the system shown in Figure 28. By definition, coefficients of the pile head stiffness matrix are the reactions generated by imposing unit values of deflection components at the pile head.

The problem was run using the input data file shown in Figure 29. Because axial load effects on bending were excluded, axial and lateral coefficients for the stiffness matrix are obtained simultaneously by imposing unit axial (x) and lateral (y) deflections at the pile head with rotation at that point equal to zero. Two solutions are presented for this case: for nodes spaced at 5 ft and nodes spaced at 1 ft. Moment coefficients for the pile head stiffness matrix were obtained for a 1-ft node spacing by specifying zero axial and lateral displacements and a unit rotation at the pile head.



Soil Stiffnesses: $k_x = 10 \cdot x^2$ psi

$k_y = 10 \cdot x^2$ psi

Cross Section: $E = 4.3 \cdot 10^6$ psi

$A = 100$ sq in.

$I = 833.33$ in.^{4}

Figure 28. Pile/soil system for Example 3

Results for each case are given in Figures 30 and 31.

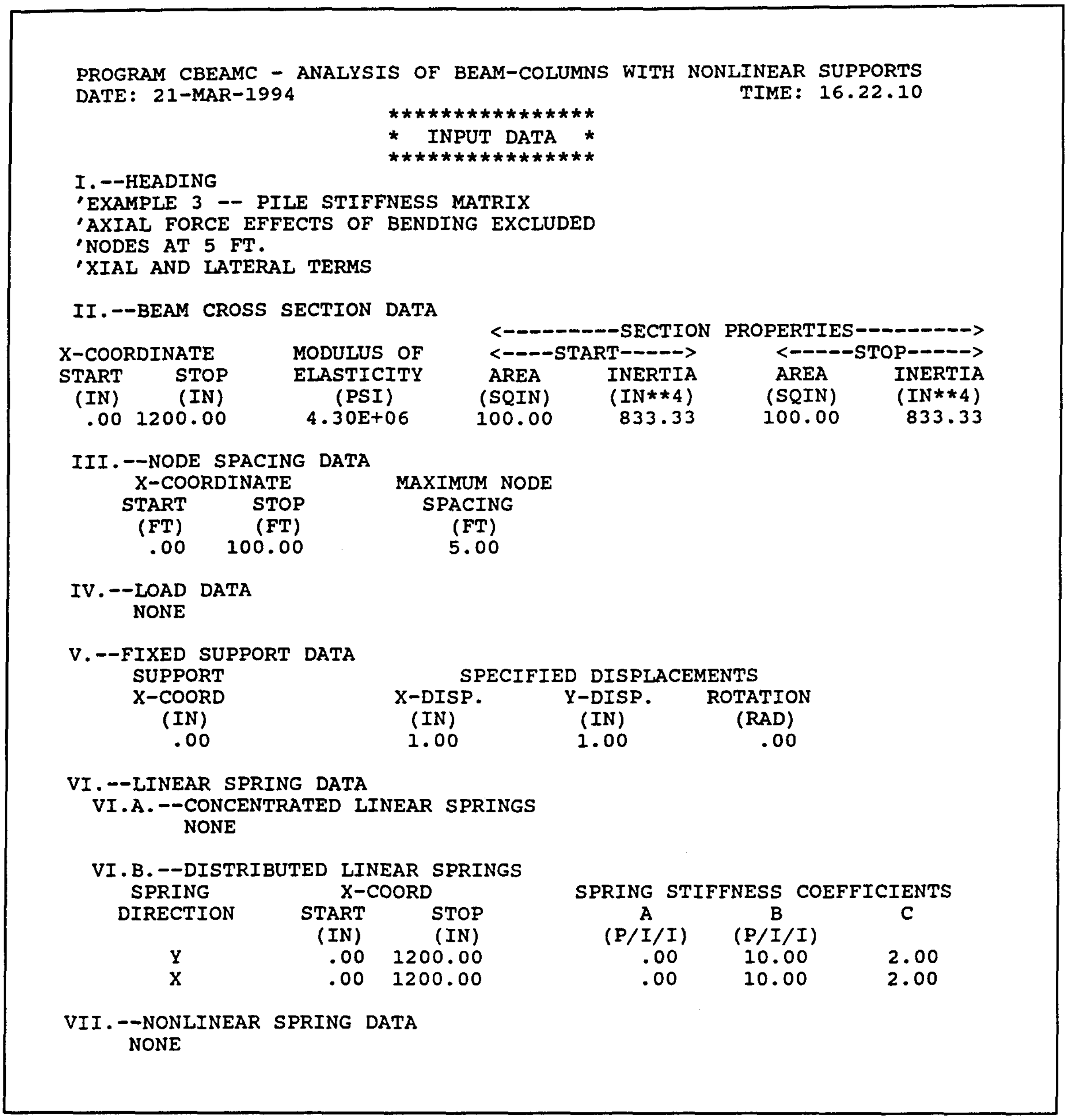
Values of stiffness coefficients obtained for the 5-ft node spacing are significantly in error. Decreasing the node spacing to 1 ft produced the pile head stiffness matrix shown as follows. Values shown in parentheses were obtained by another approximate method reported by Reese and Matlock (1956).

$$\begin{Bmatrix} F_x \\ F_y \\ M \end{Bmatrix} = \begin{bmatrix} 3.602E6 & 0 & 0 \\ & 2.077E5 & 5.424E6 \\ & (2.014E5) & (5.356E6) \\ & & 2.148E8 \\ SYM & & (2.137E8) \end{bmatrix} \begin{Bmatrix} u \\ v \\ \theta \end{Bmatrix}$$

Reducing the node spacing to 0.5 ft had little effect on the results.

```
1000 'EXAMPLE 3 -- PILE STIFFNESS MATRIX
1010 'AXIAL FORCE EFFECTS OF BENDING EXCLUDED
1020 'NODES AT 5 FT.
1030 'AXIAL AND LATERAL TERMS
1040 BEAM
1050 0 1200 4.3E6 100 833.33
1060 NODES F
1070 0 100 5
1080 FIXED I
1090 0 1 1 0
1100 LINEAR
1110 D Y 0 1200 0 10 2
1120 D X 0 1200 0 10 2
1130 FINISH RERUN
1140 'EXAMPLE 3 -- CONTINUED
1150 'AXIAL AND LATERAL TERMS
1160 'NODES AT 1 FT.
1170 NODES NEW F
1180 0 100 1
1190 FINISH RERUN
1200 'EXAMPLE 3 -- CONTINUED
1210 'MOMENT TERMS
1220 FIXED NEW
1230 0 0 0 1
1240 FINISH
```

Figure 29. Input file for Example 3



```
  PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
  DATE: 21-MAR-1994                                    TIME: 16.22.10
                         ****************
                         *  INPUT DATA  *
                         ****************
  I.--HEADING
  'EXAMPLE 3 -- PILE STIFFNESS MATRIX
  'AXIAL FORCE EFFECTS OF BENDING EXCLUDED
  'NODES AT 5 FT.
  'XIAL AND LATERAL TERMS

  II.--BEAM CROSS SECTION DATA
                                  <---------SECTION PROPERTIES--------->
 X-COORDINATE       MODULUS OF    <----START----->       <-----STOP----->
 START     STOP     ELASTICITY      AREA     INERTIA       AREA     INERTIA
  (IN)     (IN)        (PSI)       (SQIN)    (IN**4)      (SQIN)    (IN**4)
   .00 1200.00      4.30E+06      100.00     833.33      100.00     833.33

  III.--NODE SPACING DATA
     X-COORDINATE         MAXIMUM NODE
     START      STOP        SPACING
      (FT)      (FT)          (FT)
       .00    100.00          5.00

  IV.--LOAD DATA
      NONE

  V.--FIXED SUPPORT DATA
      SUPPORT                 SPECIFIED DISPLACEMENTS
      X-COORD             X-DISP.      Y-DISP.     ROTATION
        (IN)               (IN)          (IN)        (RAD)
         .00               1.00          1.00          .00

  VI.--LINEAR SPRING DATA
    VI.A.--CONCENTRATED LINEAR SPRINGS
          NONE

    VI.B.--DISTRIBUTED LINEAR SPRINGS
      SPRING          X-COORD            SPRING STIFFNESS COEFFICIENTS
     DIRECTION     START      STOP            A          B           C
                    (IN)      (IN)         (P/I/I)    (P/I/I)
         Y           .00   1200.00            .00      10.00        2.00
         X           .00   1200.00            .00      10.00        2.00

  VII.--NONLINEAR SPRING DATA
       NONE
```

Figure 30. Output for Example 3, model with 5-ft node spacing (Continued)

```
   PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
   DATE: 21-MAR-1994                                  TIME: 16.22.23
                        ************************
                        *  SUMMARY OF RESULTS  *
                        ************************
   I.--HEADING
    'EXAMPLE 3 -- PILE STIFFNESS MATRIX
    'AXIAL FORCE EFFECTS OF BENDING EXCLUDED
    'NODES AT 5 FT.
    'XIAL AND LATERAL TERMS
   II.--MAXIMA
                                    MAXIMUM   X-COORD       MAXIMUM   X-COORD
                                   POSITIVE     (IN)       NEGATIVE     (IN)
   AXIAL DISPLACEMENT (IN)  :     1.000E+00       .00    -8.297E-06    300.00
   LATERAL DISPLACEMENT (IN):     1.000E+00       .00    -1.547E-04    180.00
   ROTATION (RAD)           :     3.749E-05    120.00    -6.680E-03     60.00
   AXIAL FORCE (P)          :     2.234E+02    300.00    -3.892E+06       .00
   SHEAR (P)                :     3.537E+05       .00    -5.108E+04     60.00
   BENDING MOMENT (P-IN)    :     1.690E+06     60.00    -7.331E+06       .00
   III.--REACTIONS AT FIXED SUPPORTS
                   X-COORD         X-REACTION    Y-REACTION  MOM-REACTION
                     (IN)              (P)          (P)          (P-IN)
                        .00          3.892E+06     3.537E+05     7.331E+06

   PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
   DATE: 21-MAR-1994                                  TIME: 16.22.23
                        **********************
                        *  COMPLETE RESULTS  *
                        **********************
   I.--HEADING
    'EXAMPLE 3 -- PILE STIFFNESS MATRIX
    'AXIAL FORCE EFFECTS OF BENDING EXCLUDED
    'NODES AT 5 FT.
    'AXIAL AND LATERAL TERMS
   II.--DISPLACEMENTS AND INTERNAL FORCES
          <---------DISPLACEMENTS-------->   <--------INTERNAL FORCES------->
X-COORD      AXIAL     LATERAL    ROTATION      AXIAL      SHEAR     MOMENT
  (IN)        (IN)       (IN)       (RAD)        (P)        (P)      (P-IN)
    .00    1.000E+00  1.000E+00  0.000E+00   -3.892E+06  3.537E+05 -7.331E+06
  60.00    4.944E-01  4.959E-02 -6.680E-03   -3.176E+06 -5.108E+04  1.690E+06
 120.00    1.580E-01  7.693E-04  3.749E-05   -1.596E+06  4.434E+03 -5.210E+04
   III.--FORCES IN DISTRIBUTED LINEAR SPRINGS
                  DISTRIBUTED SPRING FORCES
      X-COORD         AXIAL       LATERAL
        (IN)        (P/IN)         (P/IN)
          .00        0.000E+00   0.000E+00
        60.00       -1.780E+04  -1.785E+03
       120.00       -2.276E+04  -1.108E+02
```

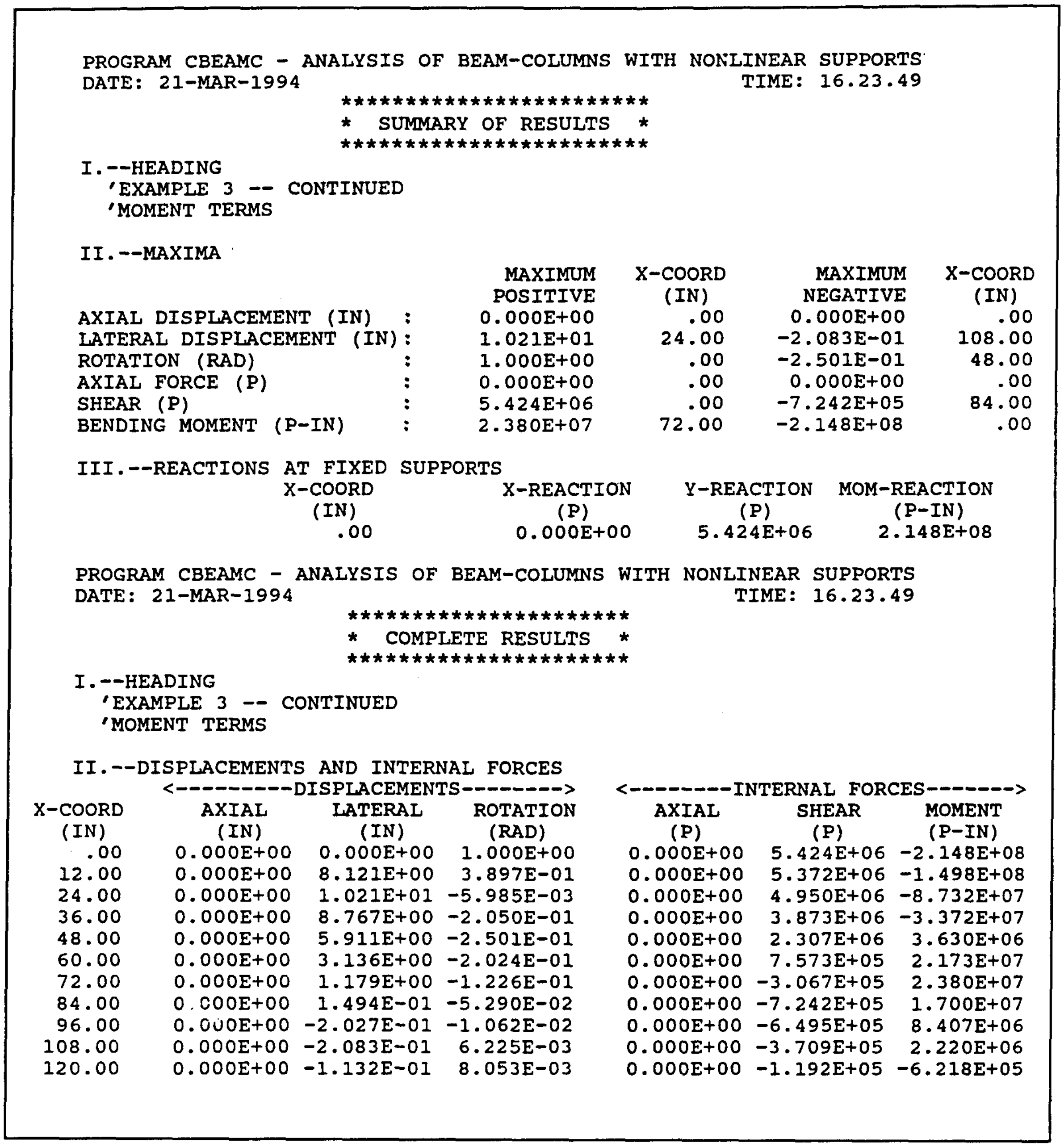
Figure 30. (Concluded)

```
  PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
  DATE: 21-MAR-1994                                  TIME: 16.23.32
                          *************************
                          *  SUMMARY OF RESULTS   *
                          *************************
  I.--HEADING
    'EXAMPLE 3 -- CONTINUED
    'AXIAL AND LATERAL TERMS
    'NODES AT 1 FT.
  II.--MAXIMA
                                   MAXIMUM    X-COORD       MAXIMUM   X-COORD
                                  POSITIVE     (IN)        NEGATIVE     (IN)
  AXIAL DISPLACEMENT (IN)   :    1.000E+00      .00       0.000E+00      .00
  LATERAL DISPLACEMENT (IN):     1.000E+00      .00      -1.716E-02    96.00
  ROTATION (RAD)            :    6.306E-04   108.00      -2.012E-02    24.00
  AXIAL FORCE (P)           :    0.000E+00      .00      -3.602E+06      .00
  SHEAR (P)                 :    2.077E+05      .00      -4.836E+04    72.00
  BENDING MOMENT (P-IN)     :    1.709E+06    60.00      -5.424E+06      .00
  III.--REACTIONS AT FIXED SUPPORTS
                    X-COORD          X-REACTION     Y-REACTION  MOM-REACTION
                     (IN)               (P)             (P)         (P-IN)
                      .00           3.602E+06       2.077E+05     5.424E+06

  PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
  DATE: 21-MAR-1994                                  TIME: 16.23.32
                          ***********************
                          *  COMPLETE RESULTS   *
                          ***********************
  I.--HEADING
    'EXAMPLE 3 -- CONTINUED
    'AXIAL AND LATERAL TERMS
    'NODES AT 1 FT.
  II.--DISPLACEMENTS AND INTERNAL FORCES
        <---------DISPLACEMENTS-------->  <--------INTERNAL FORCES------->
X-COORD      AXIAL      LATERAL    ROTATION       AXIAL      SHEAR      MOMENT
 (IN)        (IN)        (IN)       (RAD)          (P)        (P)       (P-IN)
   .00    1.000E+00  1.000E+00  0.000E+00    -3.602E+06  2.077E+05 -5.424E+06
 12.00    8.996E-01  9.076E-01 -1.402E-02    -3.594E+06  1.994E+05 -2.966E+06
 24.00    7.997E-01  6.953E-01 -2.012E-02    -3.557E+06  1.655E+05 -7.442E+05
 36.00    7.015E-01  4.511E-01 -1.967E-02    -3.474E+06  1.029E+05  8.879E+05
 48.00    6.063E-01  2.398E-01 -1.511E-02    -3.334E+06  3.139E+04  1.689E+06
 60.00    5.159E-01  9.371E-02 -9.233E-03    -3.136E+06 -2.371E+04  1.709E+06
 72.00    4.318E-01  1.465E-02 -4.205E-03    -2.888E+06 -4.836E+04  1.245E+06
 84.00    3.551E-01 -1.476E-02 -1.026E-03    -2.600E+06 -4.545E+04  6.602E+05
 96.00    2.869E-01 -1.716E-02  3.740E-04    -2.288E+06 -2.845E+04  2.104E+05
108.0C    2.275E-01 -1.036E-02  6.306E-04    -1.967E+06 -1.106E+04 -2.207E+04
120.00    1.770E-01 -3.864E-03  4.187E-04    -1.652E+06 -4.502E+02 -8.321E+04
```

Figure 31. Results for Example 3, model with 1-ft node spacing (Sheet 1 of 4)

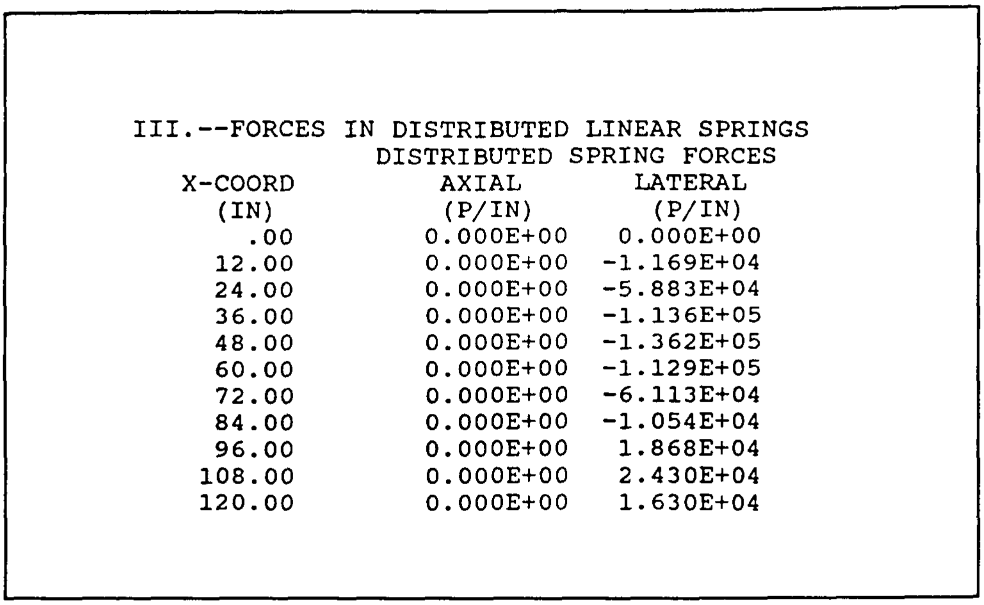
```
III.--FORCES IN DISTRIBUTED LINEAR SPRINGS
                    DISTRIBUTED SPRING FORCES
    X-COORD            AXIAL       LATERAL
     (IN)              (P/IN)       (P/IN)
      .00            0.000E+00   0.000E+00
    12.00           -1.295E+03  -1.307E+03
    24.00           -4.606E+03  -4.005E+03
    36.00           -9.091E+03  -5.847E+03
    48.00           -1.397E+04  -5.525E+03
    60.00           -1.857E+04  -3.373E+03
    72.00           -2.238E+04  -7.596E+02
    84.00           -2.506E+04   1.042E+03
    96.00           -2.644E+04   1.582E+03
   108.00           -2.654E+04   1.208E+03
   120.00           -2.549E+04   5.564E+02
```

Figure 31. (Sheet 2 of 4)



```
 PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
 DATE: 21-MAR-1994                                  TIME: 16.23.49
                         ************************
                         *  SUMMARY OF RESULTS  *
                         ************************
 I.--HEADING
   'EXAMPLE 3 -- CONTINUED
   'MOMENT TERMS

 II.--MAXIMA
                                MAXIMUM   X-COORD      MAXIMUM   X-COORD
                                POSITIVE    (IN)      NEGATIVE     (IN)
 AXIAL DISPLACEMENT (IN)  :    0.000E+00     .00     0.000E+00      .00
 LATERAL DISPLACEMENT (IN):    1.021E+01   24.00    -2.083E-01   108.00
 ROTATION (RAD)           :    1.000E+00     .00    -2.501E-01    48.00
 AXIAL FORCE (P)          :    0.000E+00     .00     0.000E+00      .00
 SHEAR (P)                :    5.424E+06     .00    -7.242E+05    84.00
 BENDING MOMENT (P-IN)    :    2.380E+07   72.00    -2.148E+08      .00

 III.--REACTIONS AT FIXED SUPPORTS
                  X-COORD          X-REACTION     Y-REACTION  MOM-REACTION
                   (IN)               (P)            (P)         (P-IN)
                    .00           0.000E+00      5.424E+06     2.148E+08

 PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
 DATE: 21-MAR-1994                                  TIME: 16.23.49
                         ***********************
                         *  COMPLETE RESULTS  *
                         ***********************
 I.--HEADING
   'EXAMPLE 3 -- CONTINUED
   'MOMENT TERMS

 II.--DISPLACEMENTS AND INTERNAL FORCES
           <----------DISPLACEMENTS-------->  <---------INTERNAL FORCES------->
 X-COORD      AXIAL     LATERAL    ROTATION      AXIAL      SHEAR      MOMENT
  (IN)        (IN)       (IN)       (RAD)         (P)        (P)       (P-IN)
     .00   0.000E+00  0.000E+00  1.000E+00    0.000E+00  5.424E+06 -2.148E+08
   12.00   0.000E+00  8.121E+00  3.897E-01    0.000E+00  5.372E+06 -1.498E+08
   24.00   0.000E+00  1.021E+01 -5.985E-03    0.000E+00  4.950E+06 -8.732E+07
   36.00   0.000E+00  8.767E+00 -2.050E-01    0.000E+00  3.873E+06 -3.372E+07
   48.00   0.000E+00  5.911E+00 -2.501E-01    0.000E+00  2.307E+06  3.630E+06
   60.00   0.000E+00  3.136E+00 -2.024E-01    0.000E+00  7.573E+05  2.173E+07
   72.00   0.000E+00  1.179E+00 -1.226E-01    0.000E+00 -3.067E+05  2.380E+07
   84.00   0.000E+00  1.494E-01 -5.290E-02    0.000E+00 -7.242E+05  1.700E+07
   96.00   0.000E+00 -2.027E-01 -1.062E-02    0.000E+00 -6.495E+05  8.407E+06
  108.00   0.000E+00 -2.083E-01  6.225E-03    0.000E+00 -3.709E+05  2.220E+06
  120.00   0.000E+00 -1.132E-01  8.053E-03    0.000E+00 -1.192E+05 -6.218E+05
```

Figure 31. (Sheet 3 of 4)



```
III.--FORCES IN DISTRIBUTED LINEAR SPRINGS
                 DISTRIBUTED SPRING FORCES
  X-COORD           AXIAL         LATERAL
   (IN)            (P/IN)          (P/IN)
     .00         0.000E+00     0.000E+00
   12.00         0.000E+00    -1.169E+04
   24.00         0.000E+00    -5.883E+04
   36.00         0.000E+00    -1.136E+05
   48.00         0.000E+00    -1.362E+05
   60.00         0.000E+00    -1.129E+05
   72.00         0.000E+00    -6.113E+04
   84.00         0.000E+00    -1.054E+04
   96.00         0.000E+00     1.868E+04
  108.00         0.000E+00     2.430E+04
  120.00         0.000E+00     1.630E+04
```

Figure 31. (Sheet 4 of 4)

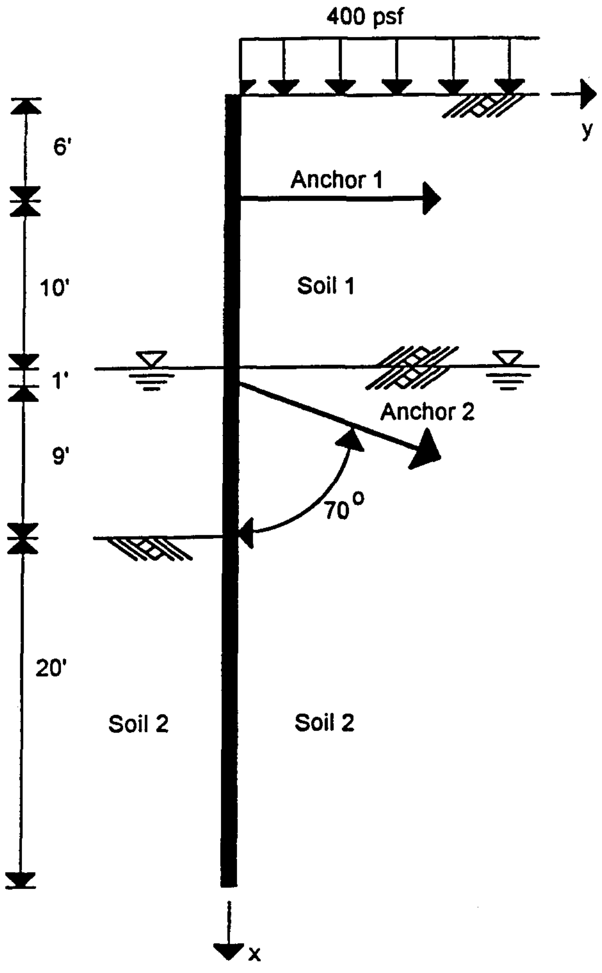
Example 4: Anchored Retaining Wall

A 1-ft strip of the multiple anchored retaining wall described in Figure 32 was analyzed as a beam-column in which the soil and anchors were represented by nonlinear springs.

Procedures described by Haliburton (1971) were used to obtain the force-deformation curves shown in Figure 33 for the soil-resisting lateral (y) displacements of the system. To the writer's knowledge, no procedure has been established for representing wall-soil friction interaction. The force-displacement curves shown in Figure 34 were arbitrarily selected for this effect.

The anchors were represented by concentrated nonlinear springs with characteristics shown in Figure 35.

The input data file used for solution with the program is shown in Figure 36. Output from the program is given in Figure 37. Anchor forces are given by Section III of the Summary of Results. Soil pressures are given by the tabulation of forces in distributed nonlinear springs in Section III of the Complete Results. Plots of the input data are shown in Figures 38 through 40. Plots of the results are shown in Figures 41 through 43.



Beam: PZ38 Sheet Pile

$E = 29 \cdot 10^6$ psi

$A = 11.2$ sq in./ft of wall

$I = 280.0$ in.^{4} /ft of wall

	Soil 1	Soil 2
Unit Weight	100 pcf	62.5 pcf (sub)
Internal Friction	30°	0
Cohesion	0	1000 psf
At Rest, k_0	0.5	1
Soil Modulus	8 pci	50 pci
Interact. Dist.	6 ft	10 ft

	Anchor 1	Anchor 2
E	$29 \cdot 10^6$ psi	$29 \cdot 10^6$ psi
A (sq in./ft)	0.16	0.2
Eff. Length (ft)	25	30

Figure 32. Multiple anchored retaining wall for Example 4

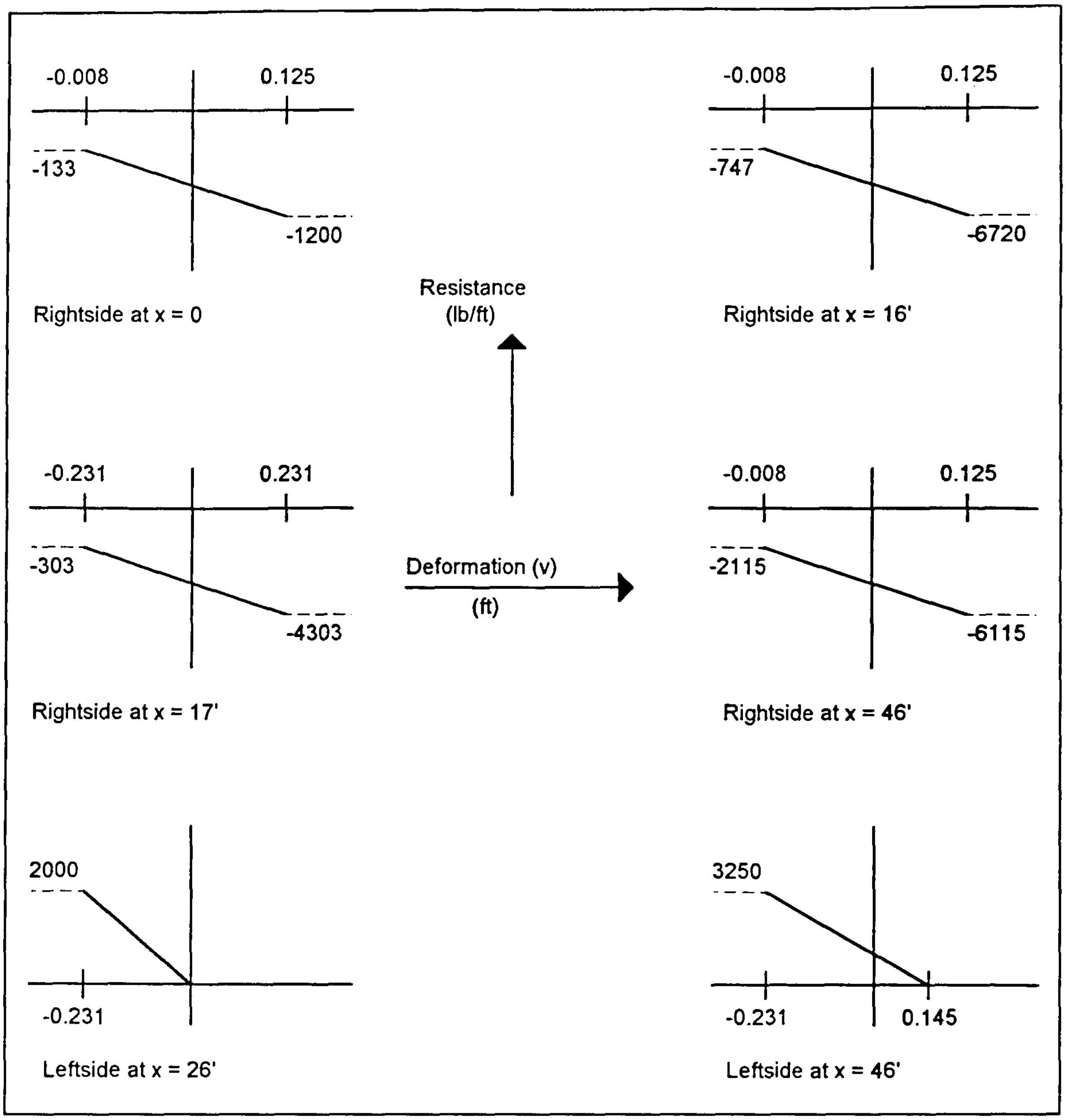


Figure 33. Force-deformation curves for lateral (y) soil resistance

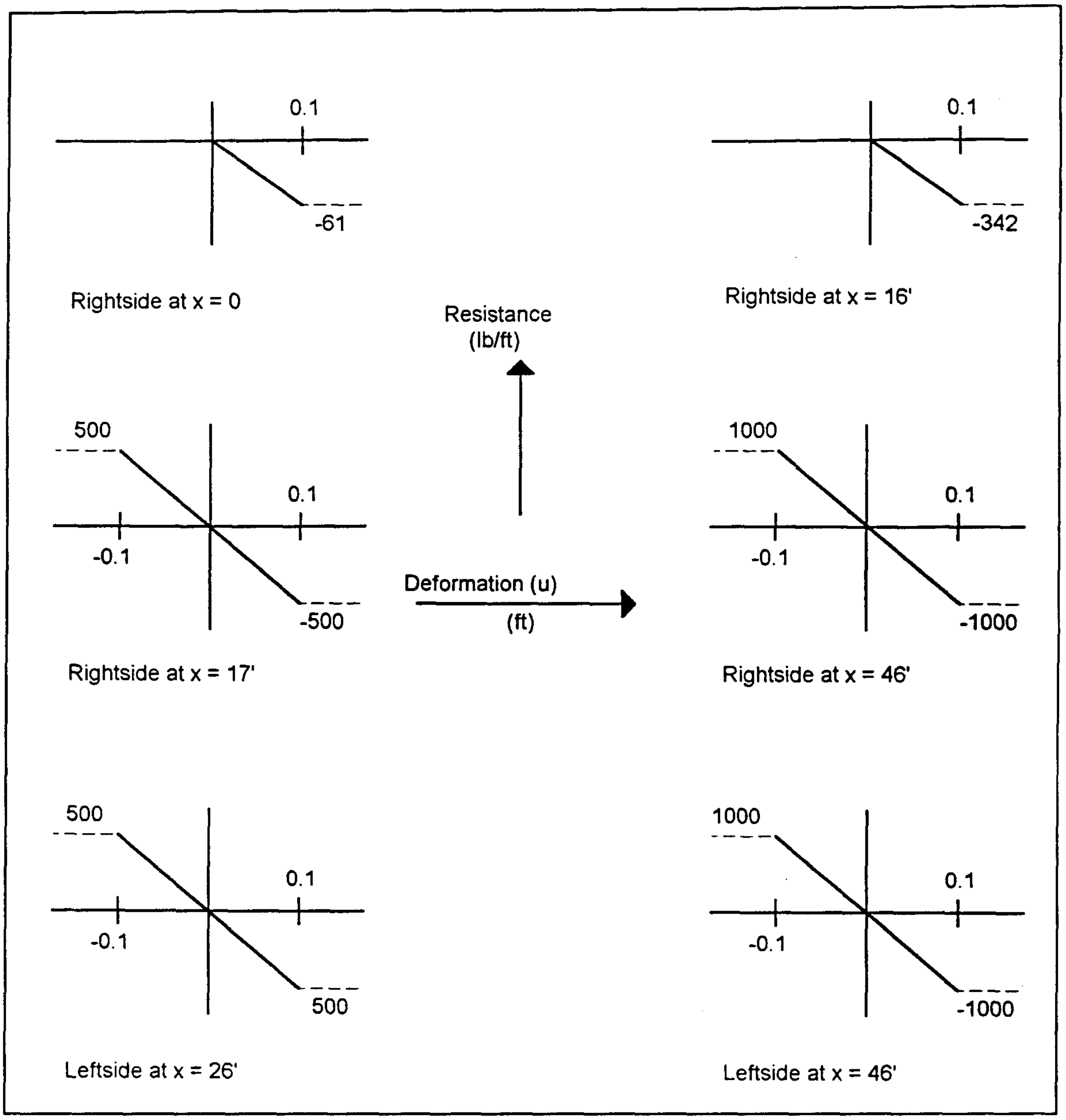


Figure 34. Force-deformation curves for axial (x) soil resistance

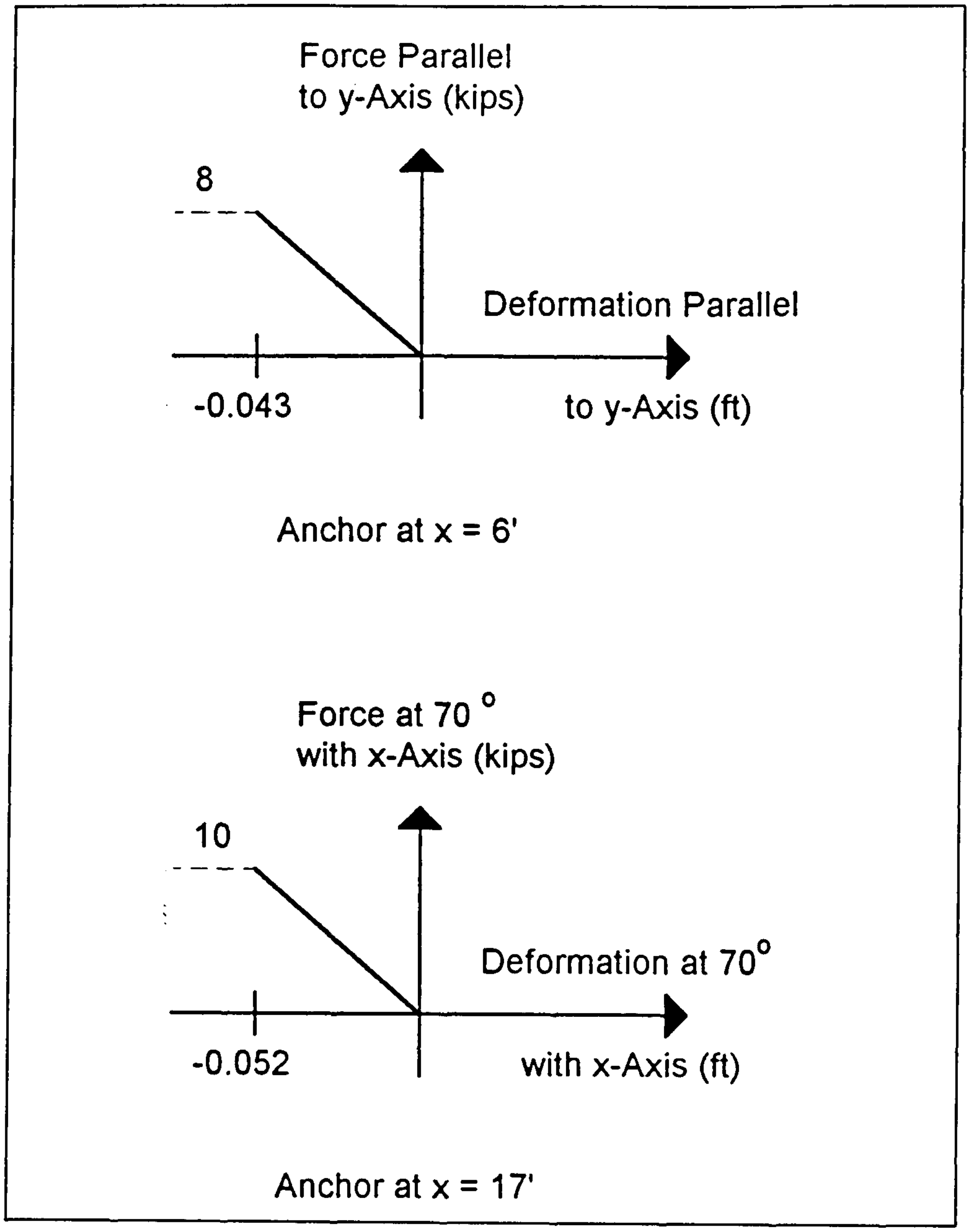
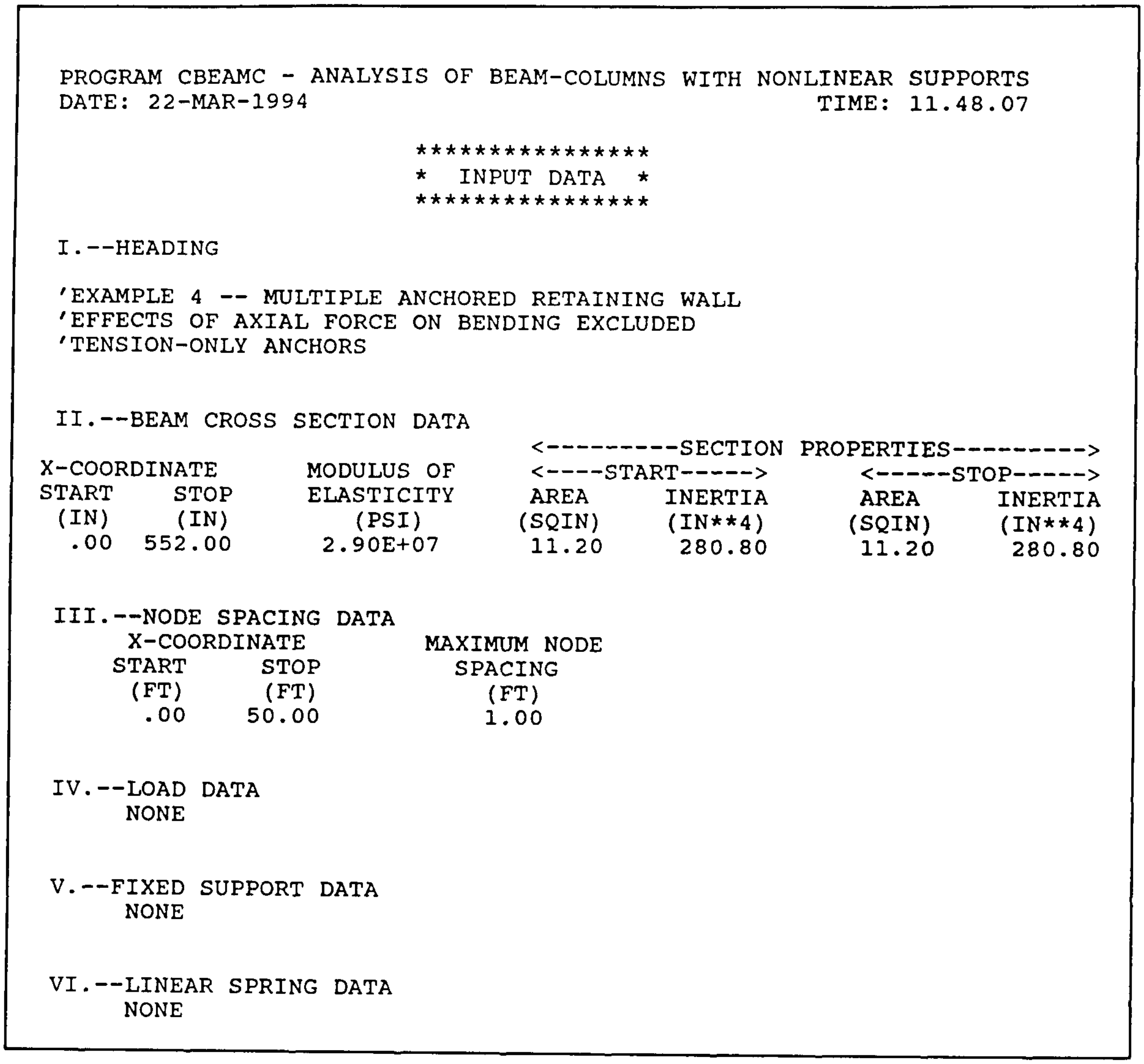


Figure 35. Force-deformation curves for anchors

```
1000 'EXAMPLE 4 -- MULTIPLE ANCHORED RETAINING WALL
1010 'EFFECTS OF AXIAL FORCE ON BENDING EXCLUDED
1020 'TENSION-ONLY ANCHORS
1030 BEAM
1040 (PZ 38 SHEET PILE)
1050 0 552 29.E6 11.2 280.8
1060 NODES F
1070 0 50 1
1080 NONLINEAR F P
1090 (LATERAL SPRINGS FOR RIGHTSIDE SOIL)
1100 D Y 0 2 1 1
1110 -0.008 -133 0.125 -1200
1120 C 16 2 1 1
1130 -0.008 -747 0.125 -6720
1140 C 17 2 1 1
1150 -0.231 -303 0.231 -4303
1160 E 46 2 1 1
1170 -0.231 -2115 0.231 -6115
1180 (LATERAL SPRINGS FOR LEFTSIDE SOIL)
1190 D Y 26 2 1 1
1200 -0.231 2000 0.231 0
1210 E 46 2 1 1
1220 -0.231 3250 0.145 0
1230 (AXIAL SPRINGS FOR RIGHTSIDE SOIL)
1240 D X 0 2 1 1
1250 0 0 0.1 -61
1260 C 16 2 1 1
1270 0 0 0.1 -342
1280 C 17 2 1 1
1290 -0.1 500 0.1 -500
1300 E 46 2 1 1
1310 -0.1 1000 0.1 -1000
1320 (AXIAL SPRINGS FOR LEFTSIDE SOIL)
1330 D X 26 2 1 1
1340 -0.1 500 0.1 -500
1350 E 46 2 1 1
1360 -0.1 1000 0.1 -1000
1370 (ANCHORS)
1380 C 6 90 2 1 1.E3
1390 -0.043 8 0 0
1400 C 17 70 2 1 1.E3
1410 -0.052 10 0 0
1420 FINISH
```

Figure 36. Input file for Example 4



```
PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 22-MAR-1994                                 TIME: 11.48.07

                        ****************
                        *  INPUT DATA  *
                        ****************

 I.--HEADING

 'EXAMPLE 4 -- MULTIPLE ANCHORED RETAINING WALL
 'EFFECTS OF AXIAL FORCE ON BENDING EXCLUDED
 'TENSION-ONLY ANCHORS


 II.--BEAM CROSS SECTION DATA
                                <----------SECTION PROPERTIES--------->
X-COORDINATE        MODULUS OF   <----START----->      <-----STOP----->
START    STOP       ELASTICITY     AREA     INERTIA      AREA     INERTIA
 (IN)     (IN)         (PSI)      (SQIN)    (IN**4)     (SQIN)    (IN**4)
  .00   552.00       2.90E+07      11.20     280.80      11.20     280.80


 III.--NODE SPACING DATA
      X-COORDINATE          MAXIMUM NODE
     START     STOP           SPACING
      (FT)     (FT)             (FT)
       .00     50.00            1.00


 IV.--LOAD DATA
      NONE


 V.--FIXED SUPPORT DATA
      NONE


 VI.--LINEAR SPRING DATA
      NONE
```

Figure 37. Output file for Example 4 (Sheet 1 of 8)

```
VII.--NONLINEAR SPRING DATA

  VII.A.--NONLINEAR CONCENTRATED SPRINGS

     SPRING                      CURVE COORD. MULTIPLIERS
    X-COORD         ANGLE       DISPLACEMENT       FORCE
     (FT)           (DEG)           (FT)            (P)
      6.00          90.00        1.00E+00        1.00E+03

         DISPLACEMENT              FORCE
          COORDINATE             COORDINATE
          -4.300E-02             8.000E+00
           0.000E+00             0.000E+00

     SPRING                      CURVE COORD. MULTIPLIERS
    X-COORD         ANGLE       DISPLACEMENT       FORCE
     (FT)           (DEG)           (FT)            (P)
     17.00          70.00        1.00E+00        1.00E+03

         DISPLACEMENT              FORCE
          COORDINATE             COORDINATE
          -5.200E-02             1.000E+01
           0.000E+00             0.000E+00

  VII.B.--NONLINEAR DISTRIBUTED SPRINGS

                       SPRING       SPRING      CURVE COORD. MULTIPLIERS
    DISTRIBUTION     DIRECTION     X-COORD     DISPLACEMENT        FORCE
                                     (FT)          (FT)             (P)
       STARTS            Y            .00        1.00E+00        1.00E+00

         DISPLACEMENT              FORCE
          COORDINATE             COORDINATE
          -8.000E-03             -1.330E+02
           1.250E-01             -1.200E+03

                       SPRING       SPRING      CURVE COORD. MULTIPLIERS
    DISTRIBUTION     DIRECTION     X-COORD     DISPLACEMENT        FORCE
                                     (FT)          (FT)             (P)
      CONTINUES          Y           16.00       1.00E+00        1.00E+00

         DISPLACEMENT              FORCE
          COORDINATE             COORDINATE
          -8.000E-03             -7.470E+02
           1.250E-01             -6.720E+03
```

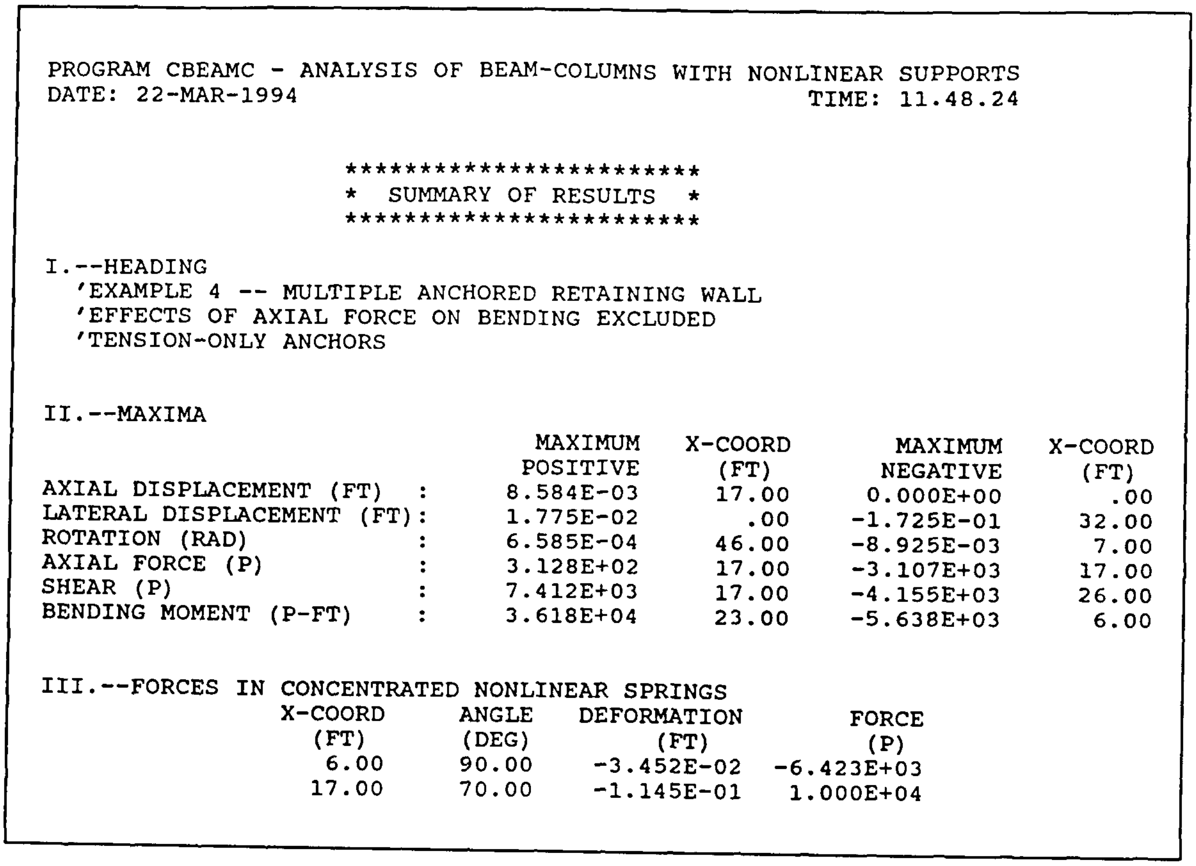
Figure 37. (Sheet 2 of 8)

```
                    SPRING         SPRING        CURVE COORD. MULTIPLIERS
  DISTRIBUTION     DIRECTION       X-COORD       DISPLACEMENT       FORCE
                                    (FT)            (FT)             (P)
    CONTINUES          Y            17.00         1.00E+00        1.00E+00

       DISPLACEMENT               FORCE
        COORDINATE              COORDINATE
        -2.310E-01              -3.030E+02
         2.310E-01              -4.303E+03

                    SPRING         SPRING        CURVE COORD. MULTIPLIERS
  DISTRIBUTION     DIRECTION       X-COORD       DISPLACEMENT       FORCE
                                    (FT)            (FT)             (P)
    ENDS               Y            46.00         1.00E+00        1.00E+00

       DISPLACEMENT               FORCE
        COORDINATE              COORDINATE
        -2.310E-01              -2.115E+03
         2.310E-01              -6.115E+03

                    SPRING         SPRING        CURVE COORD. MULTIPLIERS
  DISTRIBUTION     DIRECTION       X-COORD       DISPLACEMENT       FORCE
                                    (FT)            (FT)             (P)
    STARTS             Y            26.00         1.00E+00        1.00E+00

       DISPLACEMENT               FORCE
        COORDINATE              COORDINATE
        -2.310E-01               2.000E+03
         2.310E-01               0.000E+00

                    SPRING         SPRING        CURVE COORD. MULTIPLIERS
  DISTRIBUTION     DIRECTION       X-COORD       DISPLACEMENT       FORCE
                                    (FT)            (FT)             (P)
    ENDS               Y            46.00         1.00E+00        1.00E+00

       DISPLACEMENT               FORCE
        COORDINATE              COORDINATE
        -2.310E-01               3.250E+03
         1.450E-01               0.000E+00

                    SPRING         SPRING        CURVE COORD. MULTIPLIERS
  DISTRIBUTION     DIRECTION       X-COORD       DISPLACEMENT       FORCE
                                    (FT)            (FT)             (P)
    STARTS             X              .00         1.00E+00        1.00E+00

       DISPLACEMENT               FORCE
        COORDINATE              COORDINATE
         0.000E+00               0.000E+00
         1.000E-01              -6.100E+01
```

Figure 37. (Sheet 3 of 8)

```
                     SPRING        SPRING      CURVE COORD. MULTIPLIERS
   DISTRIBUTION    DIRECTION      X-COORD      DISPLACEMENT        FORCE
                                    (FT)           (FT)             (P)
     CONTINUES         X           16.00        1.00E+00       1.00E+00

       DISPLACEMENT             FORCE
        COORDINATE            COORDINATE
         0.000E+00             0.000E+00
         1.000E-01            -3.420E+02

                     SPRING        SPRING      CURVE COORD. MULTIPLIERS
   DISTRIBUTION    DIRECTION      X-COORD      DISPLACEMENT        FORCE
                                    (FT)           (FT)             (P)
     CONTINUES         X           17.00        1.00E+00       1.00E+00

       DISPLACEMENT             FORCE
        COORDINATE            COORDINATE
        -1.000E-01             5.000E+02
         1.000E-01            -5.000E+02

                     SPRING        SPRING      CURVE COORD. MULTIPLIERS
   DISTRIBUTION    DIRECTION      X-COORD      DISPLACEMENT        FORCE
                                    (FT)           (FT)             (P)
       ENDS            X           46.00        1.00E+00       1.00E+00

       DISPLACEMENT             FORCE
        COORDINATE            COORDINATE
        -1.000E-01             1.000E+03
         1.000E-01            -1.000E+03

                     SPRING        SPRING      CURVE COORD. MULTIPLIERS
   DISTRIBUTION    DIRECTION      X-COORD      DISPLACEMENT        FORCE
                                    (FT)           (FT)             (P)
      STARTS           X           26.00        1.00E+00       1.00E+00

       DISPLACEMENT             FORCE
        COORDINATE            COORDINATE
        -1.000E-01             5.000E+02
         1.000E-01            -5.000E+02

                     SPRING        SPRING      CURVE COORD. MULTIPLIERS
   DISTRIBUTION    DIRECTION      X-COORD      DISPLACEMENT        FORCE
                                    (FT)           (FT)             (P)
       ENDS            X           46.00        1.00E+00       1.00E+00

       DISPLACEMENT             FORCE
        COORDINATE            COORDINATE
        -1.000E-01             1.000E+03
         1.000E-01            -1.000E+03
```

Figure 37. (Sheet 4 of 8)



```
PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
DATE: 22-MAR-1994                                    TIME: 11.48.24


                    ************************
                    *  SUMMARY OF RESULTS  *
                    ************************

I.--HEADING
  'EXAMPLE 4 -- MULTIPLE ANCHORED RETAINING WALL
  'EFFECTS OF AXIAL FORCE ON BENDING EXCLUDED
  'TENSION-ONLY ANCHORS


II.--MAXIMA
                                 MAXIMUM    X-COORD       MAXIMUM    X-COORD
                                POSITIVE     (FT)        NEGATIVE     (FT)
AXIAL DISPLACEMENT (FT)  :     8.584E-03     17.00      0.000E+00       .00
LATERAL DISPLACEMENT (FT):     1.775E-02       .00     -1.725E-01     32.00
ROTATION (RAD)           :     6.585E-04     46.00     -8.925E-03      7.00
AXIAL FORCE (P)          :     3.128E+02     17.00     -3.107E+03     17.00
SHEAR (P)                :     7.412E+03     17.00     -4.155E+03     26.00
BENDING MOMENT (P-FT)    :     3.618E+04     23.00     -5.638E+03      6.00


III.--FORCES IN CONCENTRATED NONLINEAR SPRINGS
                X-COORD     ANGLE   DEFORMATION       FORCE
                 (FT)       (DEG)      (FT)            (P)
                  6.00      90.00   -3.452E-02   -6.423E+03
                 17.00      70.00   -1.145E-01    1.000E+04
```

Figure 37. (Sheet 5 of 8)

```
    PROGRAM CBEAMC - ANALYSIS OF BEAM-COLUMNS WITH NONLINEAR SUPPORTS
    DATE: 22-MAR-1994                                    TIME: 11.48.24
                                 **********************
                                 *  COMPLETE RESULTS  *
                                 **********************
     I.--HEADING
     'EXAMPLE 4 -- MULTIPLE ANCHORED RETAINING WALL
     'EFFECTS OF AXIAL FORCE ON BENDING EXCLUDED
     'TENSION-ONLY ANCHORS

     II.--DISPLACEMENTS AND INTERNAL FORCES
             <---------DISPLACEMENTS-------->   <--------INTERNAL FORCES------->
X-COORD       AXIAL      LATERAL    ROTATION       AXIAL       SHEAR      MOMENT
 (FT)         (FT)        (FT)       (RAD)          (P)         (P)       (P-FT)
   .00     8.578E-03  1.775E-02 -8.660E-03    0.000E+00  0.000E+00  0.000E+00
  1.00     8.578E-03  9.087E-03 -8.661E-03    5.986E+00 -3.470E+02 -1.728E+02
  2.00     8.578E-03  4.236E-04 -8.668E-03    1.348E+01 -6.824E+02 -6.902E+02
  3.00     8.578E-03 -8.253E-03 -8.687E-03    2.248E+01 -9.463E+02 -1.510E+03
  4.00     8.578E-03 -1.696E-02 -8.723E-03    3.298E+01 -1.214E+03 -2.587E+03
  5.00     8.578E-03 -2.571E-02 -8.780E-03    4.499E+01 -1.519E+03 -3.950E+03
  6.00     8.578E-03 -3.452E-02 -8.864E-03    5.851E+01 -1.863E+03 -5.638E+03
  6.00     8.578E-03 -3.452E-02 -8.864E-03    5.851E+01  4.560E+03 -5.638E+03
  7.00     8.579E-03 -4.343E-02 -8.925E-03    7.354E+01  4.177E+03 -1.266E+03
  8.00     8.579E-03 -5.235E-02 -8.912E-03    9.007E+01  3.757E+03  2.704E+03
  9.00     8.579E-03 -6.123E-02 -8.832E-03    1.081E+02  3.297E+03  6.234E+03
 10.00     8.579E-03 -6.999E-02 -8.694E-03    1.277E+02  2.800E+03  9.286E+03
 11.00     8.580E-03 -7.860E-02 -8.506E-03    1.487E+02  2.264E+03  1.182E+04
 12.00     8.580E-03 -8.699E-02 -8.279E-03    1.713E+02  1.690E+03  1.380E+04
 13.00     8.581E-03 -9.515E-02 -8.022E-03    1.953E+02  1.077E+03  1.519E+04
 14.00     8.582E-03 -1.030E-01 -7.746E-03    2.209E+02  4.259E+02  1.594E+04
 15.00     8.582E-03 -1.106E-01 -7.462E-03    2.480E+02 -2.635E+02  1.603E+04
 16.00     8.583E-03 -1.180E-01 -7.183E-03    2.766E+02 -9.913E+02  1.540E+04
 17.00     8.584E-03 -1.250E-01 -6.922E-03    3.128E+02 -1.985E+03  1.395E+04
 17.00     8.584E-03 -1.250E-01 -6.922E-03   -3.107E+03  7.412E+03  1.395E+04
 18.00     8.575E-03 -1.318E-01 -6.613E-03   -3.064E+03  6.189E+03  2.075E+04
 19.00     8.565E-03 -1.382E-01 -6.195E-03   -3.019E+03  4.962E+03  2.633E+04
 20.00     8.556E-03 -1.441E-01 -5.689E-03   -2.972E+03  3.725E+03  3.067E+04
 21.00     8.547E-03 -1.496E-01 -5.118E-03   -2.924E+03  2.475E+03  3.378E+04
 22.00     8.538E-03 -1.544E-01 -4.502E-03   -2.875E+03  1.207E+03  3.562E+04
 23.00     8.529E-03 -1.586E-01 -3.865E-03   -2.824E+03 -8.444E+01  3.618E+04
 24.00     8.521E-03 -1.621E-01 -3.230E-03   -2.772E+03 -1.405E+03  3.544E+04
 25.00     8.512E-03 -1.650E-01 -2.620E-03   -2.718E+03 -2.760E+03  3.336E+04
 26.00     8.504E-03 -1.674E-01 -2.058E-03   -2.663E+03 -4.155E+03  2.991E+04
 27.00     8.496E-03 -1.692E-01 -1.565E-03   -2.563E+03 -3.841E+03  2.591E+04
 28.00     8.488E-03 -1.705E-01 -1.140E-03   -2.460E+03 -3.520E+03  2.223E+04
 29.00     8.481E-03 -1.715E-01 -7.773E-04   -2.353E+03 -3.197E+03  1.887E+04
 30.00     8.474E-03 -1.721E-01 -4.710E-04   -2.243E+03 -2.876E+03  1.583E+04
```

Figure 37. (Sheet 6 of 8)

```
31.00     8.467E-03 -1.724E-01 -2.155E-04    -2.129E+03 -2.561E+03  1.311E+04
32.00     8.460E-03 -1.725E-01 -5.342E-06    -2.011E+03 -2.255E+03  1.071E+04
33.00     8.454E-03 -1.724E-01  1.649E-04    -1.890E+03 -1.962E+03  8.599E+03
34.00     8.449E-03 -1.722E-01  3.005E-04    -1.766E+03 -1.684E+03  6.777E+03
35.00     8.444E-03 -1.719E-01  4.062E-04    -1.638E+03 -1.423E+03  5.225E+03
36.00     8.439E-03 -1.714E-01  4.867E-04    -1.507E+03 -1.181E+03  3.924E+03
37.00     8.434E-03 -1.709E-01  5.464E-04    -1.372E+03 -9.593E+02  2.855E+03
38.00     8.430E-03 -1.703E-01  5.890E-04    -1.233E+03 -7.587E+02  1.998E+03
39.00     8.427E-03 -1.697E-01  6.182E-04    -1.091E+03 -5.803E+02  1.331E+03
40.00     8.424E-03 -1.691E-01  6.370E-04    -9.461E+02 -4.248E+02  8.301E+02
41.00     8.421E-03 -1.684E-01  6.484E-04    -7.972E+02 -2.928E+02  4.733E+02
42.00     8.419E-03 -1.678E-01  6.545E-04    -6.448E+02 -1.849E+02  2.364E+02
43.00     8.417E-03 -1.671E-01  6.573E-04    -4.889E+02 -1.013E+02  9.538E+01
44.00     8.416E-03 -1.665E-01  6.583E-04    -3.295E+02 -4.244E+01  2.557E+01
45.00     8.415E-03 -1.658E-01  6.585E-04    -1.665E+02 -8.572E+00  2.166E+00
46.00     8.415E-03 -1.652E-01  6.585E-04     0.000E+00  0.000E+00  0.000E+00

 III.--FORCES IN DISTRIBUTED NONLINEAR SPRINGS
                DISTRIBUTED SPRING FORCES
   X-COORD            AXIAL        LATERAL
    (FT)             (P/FT)         (P/FT)
        .00         -5.233E+00  -3.396E+02
       1.00         -6.739E+00  -3.479E+02
       2.00         -8.246E+00  -3.162E+02
       3.00         -9.752E+00  -2.481E+02
       4.00         -1.126E+01  -2.865E+02
       5.00         -1.277E+01  -3.249E+02
       6.00         -1.427E+01  -3.633E+02
       7.00         -1.578E+01  -4.016E+02
       8.00         -1.729E+01  -4.400E+02
       9.00         -1.879E+01  -4.784E+02
      10.00         -2.030E+01  -5.168E+02
      11.00         -2.181E+01  -5.551E+02
      12.00         -2.332E+01  -5.935E+02
      13.00         -2.483E+01  -6.319E+02
      14.00         -2.633E+01  -6.703E+02
      15.00         -2.784E+01  -7.086E+02
      16.00         -2.935E+01  -7.470E+02
      17.00         -4.292E+01  -1.221E+03
      18.00         -4.435E+01  -1.224E+03
      19.00         -4.578E+01  -1.231E+03
      20.00         -4.721E+01  -1.242E+03
      21.00         -4.863E+01  -1.258E+03
      22.00         -5.005E+01  -1.279E+03
      23.00         -5.147E+01  -1.305E+03
      24.00         -5.289E+01  -1.337E+03
      25.00         -5.430E+01  -1.374E+03
```

Figure 37. (Sheet 7 of 8)

```
       26.00          -5.571E+01  -1.416E+03
       26.00          -9.823E+01   3.081E+02
       27.00          -1.017E+02   3.182E+02
       28.00          -1.052E+02   3.230E+02
       29.00          -1.087E+02   3.229E+02
       30.00          -1.122E+02   3.187E+02
       31.00          -1.157E+02   3.108E+02
       32.00          -1.192E+02   2.997E+02
       33.00          -1.227E+02   2.859E+02
       34.00          -1.261E+02   2.698E+02
       35.00          -1.296E+02   2.518E+02
       36.00          -1.331E+02   2.323E+02
       37.00          -1.366E+02   2.115E+02
       38.00          -1.401E+02   1.897E+02
       39.00          -1.436E+02   1.671E+02
       40.00          -1.471E+02   1.438E+02
       41.00          -1.506E+02   1.200E+02
       42.00          -1.541E+02   9.582E+01
       43.00          -1.577E+02   7.128E+01
       44.00          -1.612E+02   4.642E+01
       45.00          -1.647E+02   2.127E+01
       46.00          -1.683E+02  -4.171E+00
```

Figure 37. (Sheet 8 of 8)

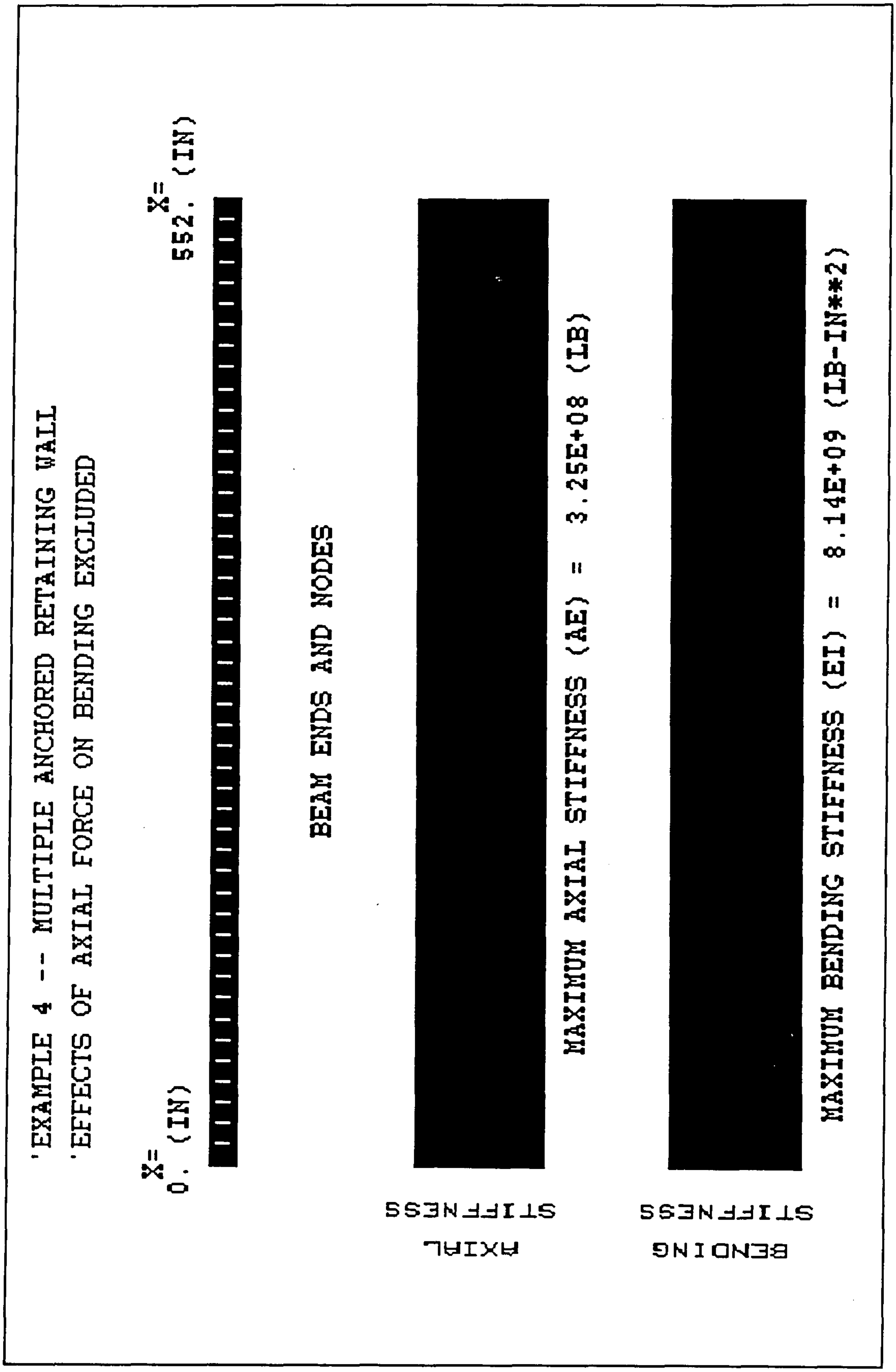


Figure 38. Axial and bending stiffnesses for Example 4

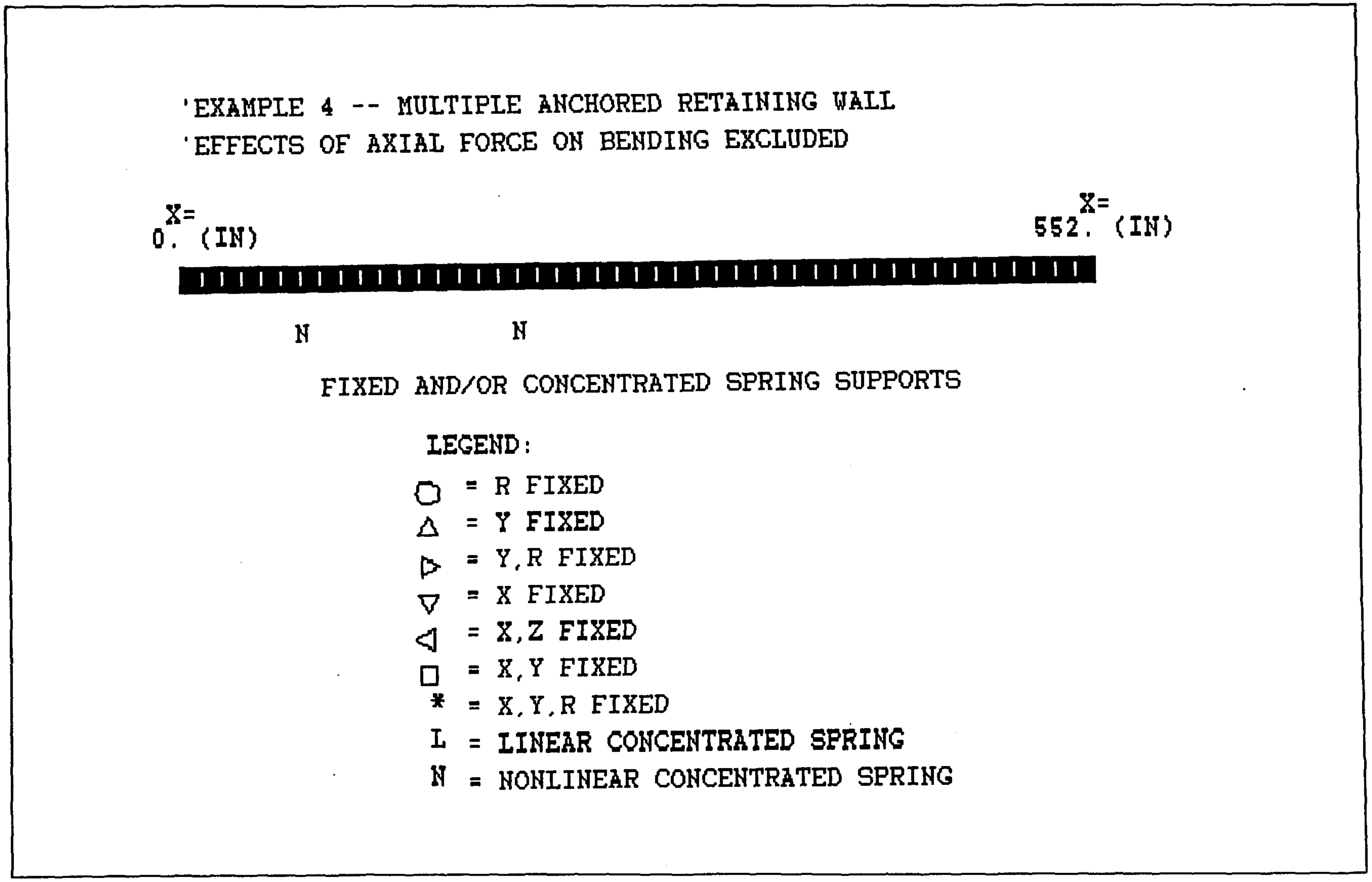


Figure 39. Schematic of supports for Example 4

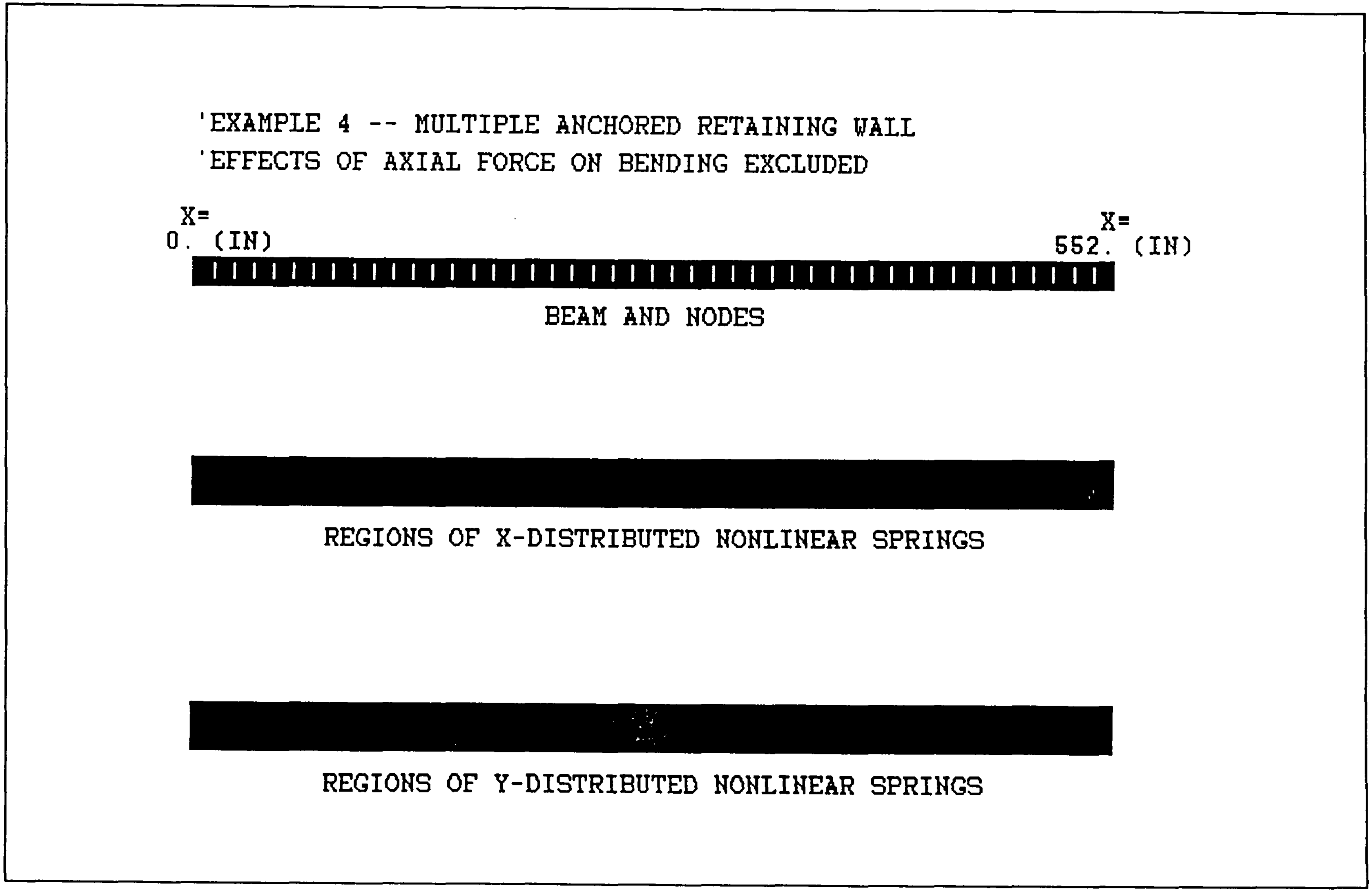


Figure 40. Schematic of nonlinear distributed springs for Example 4

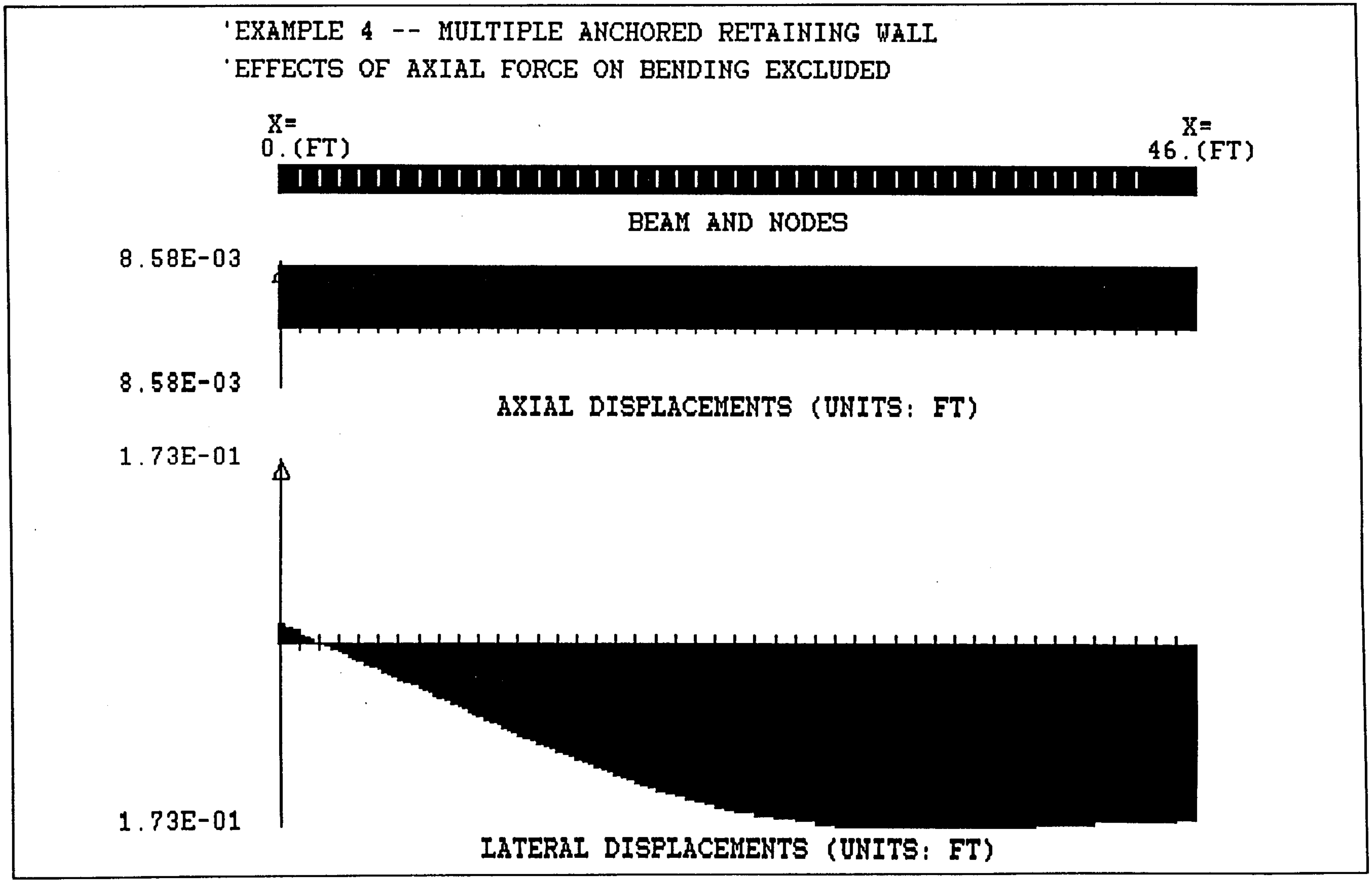


Figure 41. Axial and lateral displacements for Example 4

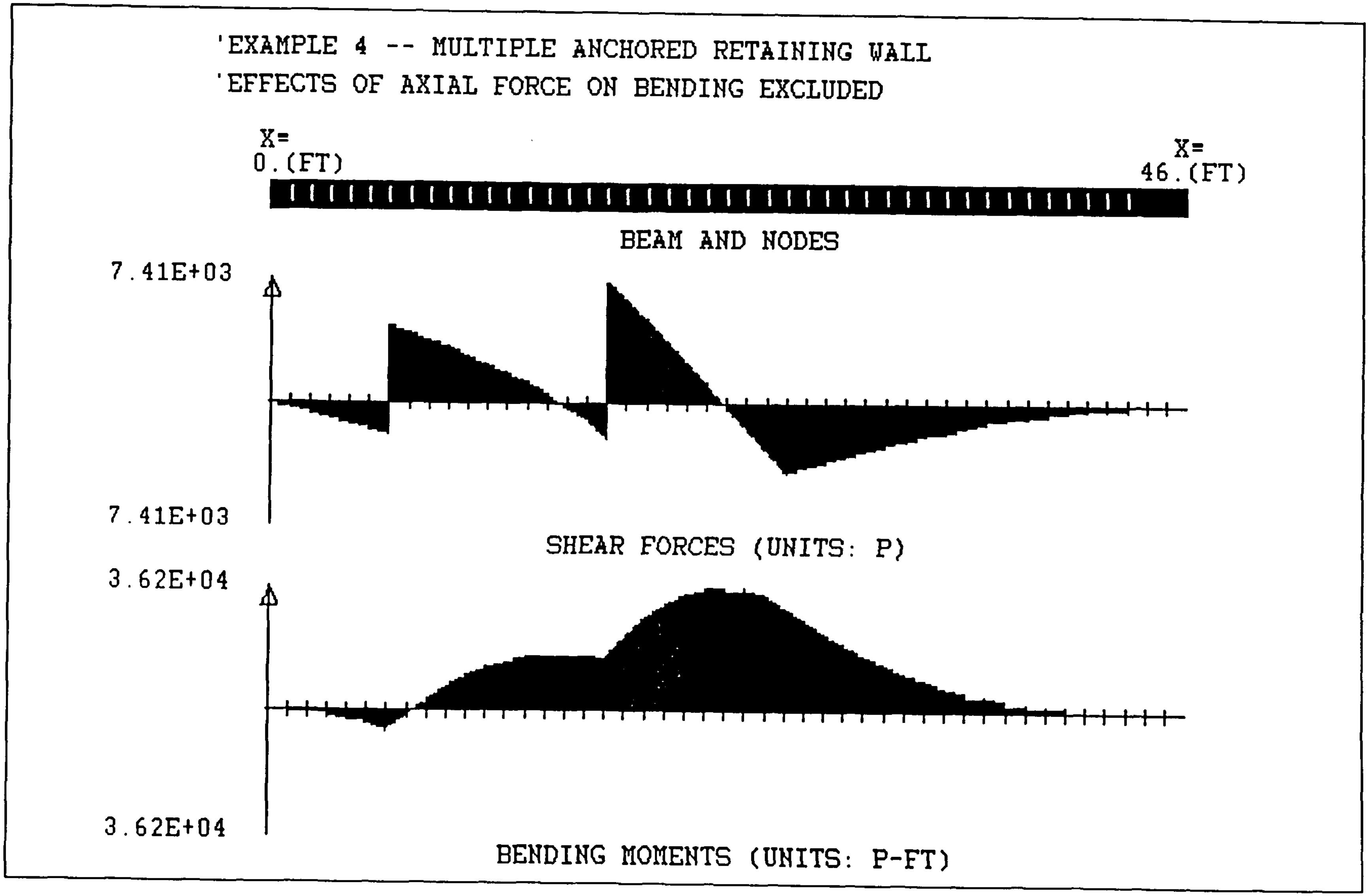


Figure 42. Shear force and bending moment diagrams for Example 4

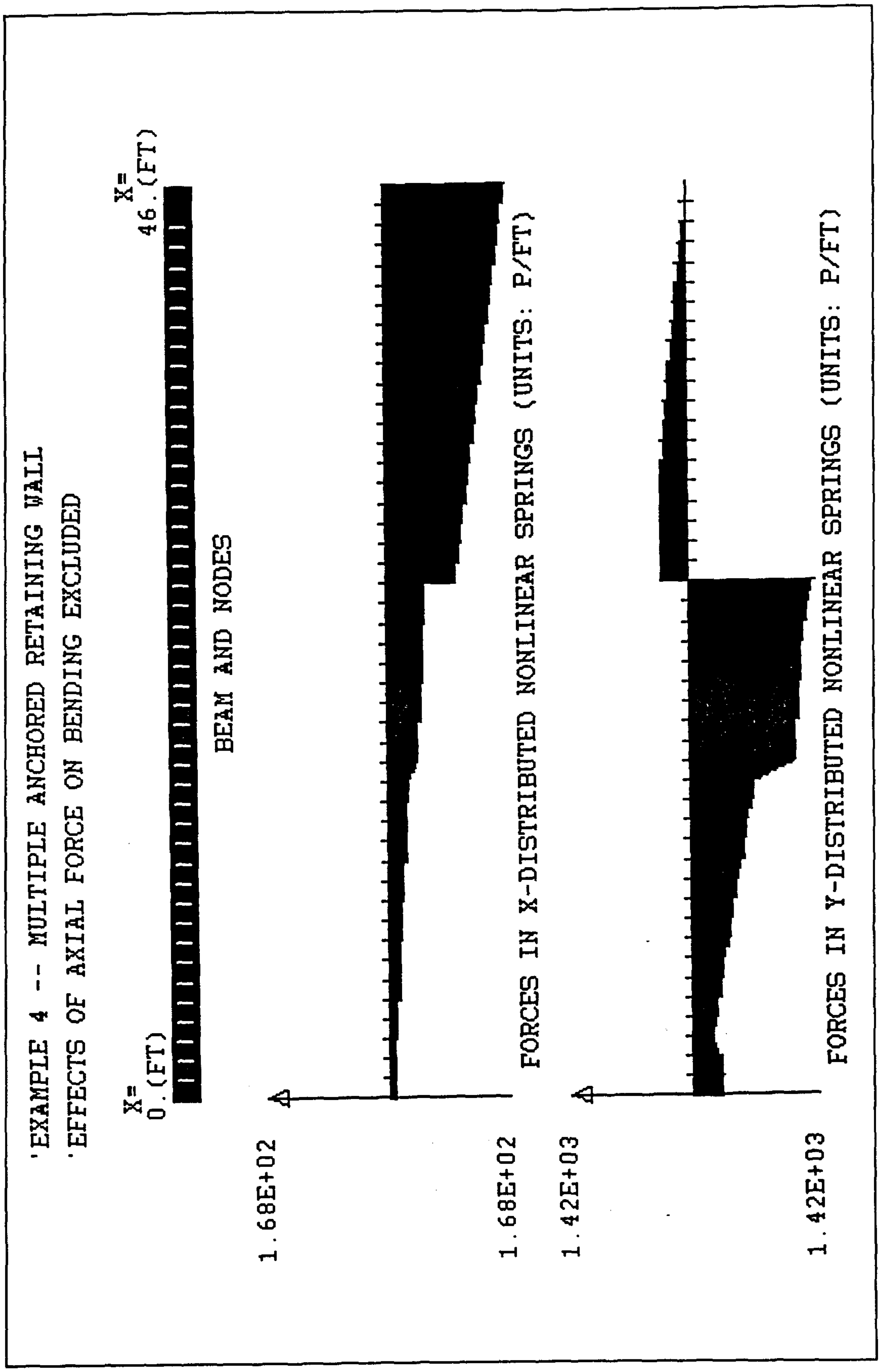


Figure 43. Forces in nonlinear distributed springs for Example 4

References

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Haliburton, T. A. (1971). "Soil structure interaction," Technical Publication No. 14, School of Civil Engineering, Oklahoma State University, Stillwater, OK.

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Reese, L. C., and Matlock, H. (1956). "Non-dimensional solutions for laterally loaded piles with soil modulus assumed proportional to depth," *Proceedings, eighth Texas conference on soil mechanics and foundation engineering*, Austin, TX.

Zienkiewicz, O. C. (1971). *The finite element method in engineering science.* McGraw-Hill, New York.

Appendix A
Guide for Data Input

Source of Input

Input data may be supplied from a predefined data file or from the user terminal during execution. If data are supplied from the user terminal, prompting messages are printed to indicate the amount and character of data to be entered.

Data Editing

When all data for a problem have been entered, the user is offered the opportunity to review an echoprint of the currently available input data and to revise any or all sections of the input data before execution is attempted. When editing during execution, each section must be entered in its entirety.

Input Data File Generation

After data have been entered from the terminal, either initially or after editing, the user may direct the program to write the input to a permanent file in input data file format.

Data Format

All input data (whether supplied from the user terminal or from a file) are read in free-field format:

a. Data items must be separated by one or more blanks (comma separators are not permitted).

b. Integer numbers must be of the form NNNN.

c. Real numbers may be of the form ±xxxx, ±xx.xx, or ±xx.xxE±ee.

d. User responses to all requests for control by the program for alphanumeric input may be abbreviated by the first letter of the indicated word response, e.g.,

ENTER 'YES' OR 'NO'--respond Y or N

ENTER 'CONTINUE' OR 'END'--respond C or E.

Sections of Input

Input data are divided onto the following sections:

a. Heading.

b. Beam cross section properties.

c. Node spacing.

d. Applied loads.

e. Fixed supports.

f. Linear spring supports.

g. Nonlinear spring supports.

h. Termination.

When data are entered from the terminal, data sections *a* through *g* may be input in any order. When data are entered from a predefined data file, the heading section *a* must be entered first; other sections *b* through *g* may be entered in any order.

When data are entered from the terminal, the user is cued for Termination. Termination for a data file is discussed later.

Minimum Required Data

Data sections *a*, *b*, and *c* are always required. At least one of sections *e*, *f*, and *g* is required. It is the responsibility of the user to ensure that sufficient supports (fixed or spring supports) are provided to inhibit all rigid body motions of the system, i.e., to ensure a stable structure.

Units

The program recognizes the following units:

*I*nches, *F*eet, *P*ounds, *K*ips

Default is to inches and pounds. Angular units are degrees or radians as explained below.

Each data section may be entered with any combination of units for length and force as specified by the user.

Predefined Data File

In addition to the general format requirements given in paragraphs entitled "Data Format," the following pertain to a predefined data file and to the input data description which follows:

a. Each line must commence with a nonzero, positive line number, denoted *LN* below.

b. A line of input may require both alphanumeric and numeric data items. Alphanumeric data items are enclosed in single quotes in the following paragraphs.

c. A line of input may require a keyword. The acceptable abbreviation for the keyword is indicated by the initial italicized capital letters of the word. The acceptable abbreviation for the keyword '*LIN*ear' is '*LIN*', e.g.

d. Lower case words in single quotes indicate a choice of keywords defined follows.

e. Items designated by upper case letters and numbers without quotes indicate numeric data values. Numeric data values are either real or integer according to standard FORTRAN variable naming conventions.

f. Data items enclosed in brackets [ ] may not be required. Data items enclosed in braces { } indicate special note follows.

g. Input data are divided into the sections discussed in paragraphs entitled "Sections of Input." Except for the heading, each section consists of a header line and one or more data lines. The header line serves the multiple purposes of: indicating the end of the preceding section; identifying the data section to follow; indicating whether the section is '*N*ew' or to be '*A*dded' to the same section of a preceding problem (following page); and indicating the units associated with the section.

h. A data file may contain input data for several problems to be run in succession. On the first problem defined in the file, all data sections must be '*N*ew'. In subsequent problem descriptions, '*N*ew' indicates the section is to replace that section in the preceding problem; '*A*dd' indicates the data are to be appended to that section in the preceding problem. A data section may be deleted from the preceding problem description by indicating '*N*ew' on the section header line and immediately following with another section header.

i. Each section header line contains the information [{'*N*ew' or '*A*dd'}] and [{'units'}]. Either or both of these items may be omitted.

(1) If [{'*N*ew' or '*A*dd'}] is omitted, '*N*ew' is assumed. Omit this item on the first problem description in the file.

(2) If '*A*dd' is indicated, the units for appended data must be the same as for that section in the preceding problem.

(3) [{'units'}] must be '*I*nches' or '*F*eet' and '*P*ounds' or '*K*ips'. If data are to be appended to the preceding problem, omit this item. Data to be appended must be in the same units prescribed for that section in the preceding problem.

(4) If [{'units'}] is omitted in the header for a '*N*ew' section, default is to inches and pounds.

j. If data are added to a preceding section, the number of lines in the preceding section plus the number of lines added must not exceed the limitations on total lines in a section specified in the input description which follows.

k. A data file must begin with a Heading. In a succession of problems, the Heading for the preceding problem may be replaced by entering the new heading at the beginning of the new problem description. Addition to a previous heading is not permitted.

l. Comment lines may be inserted in the input file by enclosing the line following the line number in parentheses. Comment lines are ignored, e.g.,

```
1234   (THIS LINE IS IGNORED)
```

Input Description

Heading--One (1) to four (4) lines for identifying the problem.

a. Line contents.

LN {'heading'}

b. Definition.

'heading' = any alphanumeric information up to 70 characters including LN and any imbedded blanks.

c. Restriction: If a 'heading' line following LN begins with a combination of letters which are permissible abbreviations of any of the keywords described below, the 'heading' must begin with a single quote.

Beam Cross Section Properties--Two (2) to twenty-two (22) lines.

a. Header--One (1) line.

(1) Contents.

LN '*B*eam' [{'*N*ew' or '*A*dd'}] [{'units'}]

(2) Definition.

'*B*eam' = section title.

[{'units'}] = '*I*nches' or '*F*eet' and '*P*ounds' or '*K*ips' for '*N*ew'; default to inches and pounds if omitted for '*N*ew'.

b. Data Lines--One (1) to twenty-one (21) lines.

(1) Contents.

LN X1 X2 E A1 SI1 [A2 SI2]

(2) Definitions.

X1 = x-coordinate at beginning of distribution.

X2 = x-coordinate at end of distribution.

E = modulus of elasticity in region X1 to X2.

A1 = cross section area at X1.

SI1 = cross section moment of inertia at X1.

[A2,SI2] = area and moment of inertia at X2.

(3) Discussion.

(a) If A2 and SI2 are omitted, area and moment of inertia are assumed to be constant from X1 to X2.

(b) If A2 and SI2 are provided, area and moment of inertia are assumed to vary linearly from A1 and SI1 at X1 to A2 and SI2 at X2.

(c) When several distributions are described, the minimum of all X1 values defines the left end of the beam and the maximum of all X2 values defines the right end of the beam. There must be positive, nonzero values of modulus of elasticity, area, and moment of inertia for every x-coordinate between the ends of the beam.

(d) If distributions are specified for overlapping regions, values within the overlap are cumulative.

Node Spacing--One (1) to twenty-two (22) lines.

a. Header--One (1) line.

(1) Contents.

LN '*NOD*e' [{'*N*ew' or '*A*dd'}] [{'units'}]

(2) Definitions.

'*NOD*e' = section title.

[{'units'}] = '*I*nches' or '*F*eet' for '*N*ew'; default to inches if omitted for '*N*ew'.

b. Data Line--One (1) to twenty-one (21) lines.

(1) Contents.

LN X1 X2 HMAX

(2) Definitions.

X1,X2 = x-coordinates at beginning and end of interval, respectively.

HMAX = maximum distance between nodes in interval X1 to X2.

c. Discussion.

(1) Intervals X1 to X2 must proceed sequentially from left to right. Overlapping intervals or gaps between intervals are not permitted.

(2) X1 on the first line input must be less than or equal to the x-coordinate at the left end of the beam.

(3) X2 on the last line input must be greater than or equal to the x-coordinate at the right end of the beam.

(4) Data specified beyond the ends of the beam are ignored.

(5) Node spacing data must result in at least three (3) nodes, (two (2) elements) and not more than two-hundred-one (201) nodes (200 elements) in the model.

Applied Loads--Zero (0) or two (2) to twenty-two (22) lines. Entire section may be omitted.

a. Header--One (1) line.

(1) Contents.

LN '*LO*ads' [{'*N*ew' or *A*dd'}] [{'units'}]

(2) Definitions.

'*LO*ads' = section title.

[{'units'}] = '*I*nches' or '*F*eet' and '*P*ounds' or '*K*ips' for '*N*ew'; default to inches and pounds if omitted.

b. Data Lines for Concentrated Loads--One (1) line for each concentrated load.

(1) Contents.

LN '*C*oncentrated' X1 FX FY C

(2) Definitions for concentrated loads.

'*C*oncentrated' = keyword.

X1 = x-coordinate at point of concentrated load(s).

FX,FY = magnitudes of concentrated x- and y-loads, respectively.

C = magnitude of concentrated couple.

c. Data Lines for Distributed Loads--One (1) line for each distribution.

(1) Contents.

LN '*D*istributed' {'direction'} X1 Q1 X2 [Q2]

(2) Definitions.

'*D*istributed = keyword.

'direction' = 'X' for axial load, or 'Y' for lateral load.

X1 = x-coordinate at start of distribution.

X2 = x-coordinate at end of distribution.

Q1 = magnitude of distributed load at X1.

[Q2] = magnitude of distributed load at X2; assumed to be equal to Q1 if omitted (i.e., uniform load).

d. Discussion.

(1) Multiple loads specified at a single x-coordinate, either concentrated loads or overlapping distributed loads, are cumulative.

(2) Data specified beyond the ends of the beam are ignored.

Fixed Supports--Zero (0) or two (2) to eleven (11) lines. Entire section may be omitted.

a. Header--One (1) line.

(1) Contents.

LN '*FIX*ed' [{'*N*ew' or '*A*dd'}] [{'units'}]

(2) Definitions.

'*FIX*ed' = section title.

[{'units'}] = '*I*nches' or '*F*eet' for '*N*ew'; default to inches if omitted.

b. Data Lines--One (1) to ten (10) lines.

(1) Contents.

LN X1 {XD} {YD} {R}

(2) Definitions.

X1 = x-coordinate at point of support.

{XD} = specified X-displacement; enter '*F*ree' if unspecified.

{YD} = specified Y-displacement; enter '*F*ree' if unspecified.

{R} = specified rotation (in radians); enter '*F*ree' if unspecified.

c. Discussion.

(1) Multiple values of a single displacement component are not permitted.

(2) Data specified beyond the ends of the beam are ignored.

Linear Spring Supports--Zero (0) or two (2) to twenty-two (22) lines. Entire section may be omitted.

a. Header--One (1) line.

(1) Contents.

LN '*LIN*ear' [{'*N*ew' or '*A*dd'}] [{'units'}]

(2) Definitions.

'*LIN*ear' = section title.

[{'units'}] = '*I*nches' or '*F*eet' and '*P*ounds' or '*K*ips' for '*N*ew'; default to inches and pounds if omitted.

b. Data Lines for Concentrated Linear Springs--One (1) line for each concentrated linear spring.

(1) Contents.

LN '*C*oncentrated' X1 ANGLE ST SR

(2) Definitions.

'*C*oncentrated' = keyword.

X1 = x-coordinate at point of attachment.

ANGLE = angle (in degrees) between x-axis and line of action of translation spring; measured positive counter-clockwise from x-axis.

ST = stiffness of translation spring in force per unit displacement.

SR = stiffness of rotation spring in moment per radian.

c. Data Lines for Distributed Linear Springs--One (1) line for each distribution.

(1) Contents.

LN '*D*istributed' {'direction'} X1 X2 A B C

(2) Definitions.

'*D*istributed' = keyword.

{'direction'} = 'X' for axial spring or 'Y' for lateral spring.

X1 = x-coordinate at start of distribution.

X2 = x-coordinate at end of distribution.

A,B,C = spring stiffness coefficients (see following discussion).

d. Discussion.

(1) If several concentrated linear springs are attached to a single point, effects are cumulative.

(2) Stiffness of a distributed spring is assumed to vary according to $S = A + B \cdot Z^C$, where Z is the distance from X1. Units of A are force per unit length per unit displacement; units of B and C must result in the units of force per unit length per unit displacement of the product $B \cdot Z^C$.

(3) Overlapping distributions result in cumulative effects in the overlap.

(4) Data specified beyond the end of the beam are ignored.

Nonlinear Spring Supports--Zero (0) lines or maximum of forty one (41) nonlinear concentrated and/or distributed support curves; entire section may be omitted.

a. Control--One (1) line.

LN '*NON*linear' [{'*N*ew' or '*A*dd'}] [{'units'}]

b. Definitions.

'*NON*linear' = section title.

[{'units'}] = '*I*nches' or '*F*eet' and '*P*ounds' or '*K*ips'; default to inches and pounds if omitted.

c. Data Lines for Concentrated Nonlinear Springs--Two (2) or more lines for each concentrated spring.

(1) Line 1 Contents:

LN '*C*oncentrated' X1 ANGLE NPTS DMUL FMUL

Line 2 Contents:

LN DEF(1) FORCE(1) DEF(2) FORCE(2) [. . . DEF(NPTS) FORCE(NPTS)]

(Pairs of DEF( ) FORCE( ) may be continued on succeeding cards following a line number until NPTS pairs have been entered.)

(2) Definitions.

'*C*oncentrated' = keyword.

X1 = x-coordinate at point of attachment.

ANGLE = angle (in degrees) between x-axis and spring line of action measured positive counterclockwise from positive x-direction.

NPTS = number of points on resisting force-deformation curve; minimum of two (2) points required; maximum of twenty one (21) points permitted.

DMUL = scale factor for curve deformation coordinate values; must be positive and nonzero.

FMUL = scale factor for curve resisting force coordinate values.

DEF( ) = curve deformation coordinate value; must proceed sequentially from most negative to most positive.

FORCE( ) = curve resisting force coordinate value corresponding to DEF( ).

d. Data Lines for Distribution Nonlinear Springs--Minimum of Two (2) groups for each distribution.

(1) Group 1, Line 1 Contents:

LN '*D*istributed' {'direction'} X1 NPTS DMUL FMUL

Group 1, Line 2 Contents:

LN DEF(1) FORCE(1) DEF(2) FORCE(2) [. . . DEF(NPTS) FORCE(NPTS)]

(Pairs of DEF( ) FORCE( ) may be continued on succeeding cards following a line number until NPTS pairs have been entered.)

(2) Definitions for Group 1:

'Distributed' = keyword indicating beginning of distribution.

{'direction'} = '*X*' or '*Y*' indicating distributed spring resists axial (x) or lateral (y) displacements.

X1 = x-coordinate at beginning of distribution.

NPTS = number of points provided on nonlinear resisting force-displacement curve at X1; minimum of two (2) points required; maximum of twenty one (21) points permitted.

DMUL = scale factor for curve deformation coordinate values; positive, nonzero.

FMUL = scale factor for curve resisting force coordinate values.

DEF( ) = deformation coordinate value; must proceed from most negative to most positive.

FORCE( ) = resisting distributed force corresponding to DEF( ).

(3) Group 2, Line 1 Contents:

LN {'control'} X2 NPTS DMUL FMUL

Group 2, Line 2 Contents:

LN DEF(1) FORCE(1) DEF(2) FORCE(2) [. . . DEF(NPTS) FORCE(NPTS)]

(Pairs of DEF( ) Force( ) may be continued on succeeding cards following a line number until NPTS pairs have been entered.)

(4) Definitions for Group 2:

{'control'} = '*C*ontinue' or '*E*nd'; '*C*ontinue' indicates distribution continues and another set of Group 2 lines follows; '*E*nd' indicates this is the last curve in this distribution.

X2 = x-coordinate for this nonlinear curve.

NPTS, DMUL, FMUL, DEF( ), FORCE( ) as in Group 1

e. Discussion.

(1) Final deformation and resisting force curve coordinates are products DMUL·DEF( ) and FMUL·FORCE( ), respectively.

(2) Resisting force-deformation curve must be single valued at every point.

(3) Resisting force-deformation curve is assumed to vary linearly between input points.

(4) Resisting force is assumed to be constant at first or last value provided on curve for deformations beyond the range of deformation coordinates provided.

(5) Several concentrated nonlinear springs may be attached at a single point.

(6) Characteristics of distributed nonlinear springs at intermediate points on a distribution are obtained by linear interpolation between adjacent curves.

(7) Overlapping distributions result in cumulative effects in the overlap.

(8) Data specified beyond the ends of the beam are ignored.

Termination--One (1) line.

a. Contents.

LN '*FIN*ish [{'*R*erun'} [{'*K*eep'}]]

b. Definitions.

'*FIN*ish' = keyword indicating end of input data for a problem. Initiates data checking and solution.

[{'*R*erun'}] = keyword; if omitted, program assumes this is end of data file; if included, after solution of preceding problem, program will immediately read succeeding data as input for a new problem.

[{'*K*eep'}] = keyword; if omitted, program begins with zero displacements for initial iteration of nonlinear problem; if included, iterations begin with displacements from preceding solution in this sequence of problems; omit if [{'Rerun'}] is omitted.

Abbreviated Input Guide

(Data items enclosed in brackets [ ] may be omitted. Braces { } enclosing data lists indicate choose one; arrow indicates default if item is omitted.)

Heading--One (1) to four (4) lines.

LN 'heading'

[LN 'heading']

[LN 'heading']

[LN 'heading']

Beam Cross Section Properties--Two (2) to twenty-two (22) lines.

a. Header--One (1) line.

$$\text{LN }\ \mathit{'\underline{B}eam'}\ \left[\begin{Bmatrix}\rightarrow\mathit{'\underline{N}ew'}\\ \mathit{'\underline{A}dd'}\end{Bmatrix}\right]\left[\begin{Bmatrix}\rightarrow\mathit{'\underline{I}nches'}\\ \mathit{'\underline{F}eet'}\end{Bmatrix}\begin{Bmatrix}\rightarrow\mathit{'\underline{P}ounds'}\\ \mathit{'\underline{K}ips'}\end{Bmatrix}\right]$$

b. Data Lines--One (1) to twenty-one (21) lines.

LN X1 X2 E A1 SI1 [A2 SI2]

Node Spacing--Two (2) to twenty-two (22) lines.

a. Header--One (1) line.

$$\text{LN }\textit{'\underline{NOD}e'}\left[\begin{Bmatrix}\rightarrow\textit{'\underline{N}ew'}\\ \textit{'\underline{A}dd'}\end{Bmatrix}\right]\left[\begin{Bmatrix}\rightarrow\textit{'\underline{I}nches'}\\ \textit{'\underline{F}eet'}\end{Bmatrix}\right]$$

b. Data Lines--One (1) to twenty-one (21) lines.

LN X1 X2 HMAX

Applied Loads--Zero (0) or two (2) to twenty-two (22) lines.

a. Header--One (1) line.

$$\text{LN }\textit{'\underline{LO}ads'}\left[\begin{Bmatrix}\rightarrow\textit{'\underline{N}ew'}\\ \textit{'\underline{A}dd'}\end{Bmatrix}\right]\left[\begin{Bmatrix}\rightarrow\textit{'\underline{I}nches'}\\ \textit{'\underline{F}eet'}\end{Bmatrix}\begin{Bmatrix}\rightarrow\textit{'\underline{P}ounds'}\\ \textit{'\underline{F}eet'}\end{Bmatrix}\right]$$

b. Data Lines for Concentrated Loads--One (1) line for each concentrated load.

LN 'Concentrated' X1 FX FY C

c. Data Lines for Distributed Loads--One (1) line for each distribution.

$$\text{LN }\textit{'\underline{D}istributed'}\begin{Bmatrix}\text{'X'}\\ \text{'Y'}\end{Bmatrix}\text{ X1 Q1 X2 [Q2]}$$

Fixed Supports--Zero (0) or two (2) to eleven (11) lines.

a. Header--One (1) line.

$$\text{LN }\textit{'\underline{FIX}ed'}\left[\begin{Bmatrix}\rightarrow\textit{'\underline{N}ew'}\\ \textit{'\underline{A}dd'}\end{Bmatrix}\right]\left[\begin{Bmatrix}\rightarrow\textit{'\underline{I}nches'}\\ \textit{'\underline{F}eet'}\end{Bmatrix}\begin{Bmatrix}\rightarrow\textit{'\underline{P}ounds'}\\ \textit{'\underline{F}eet'}\end{Bmatrix}\right]$$

b. Data Lines--One (1) to ten (10) lines.

$$\text{LN }\textit{X1}\begin{Bmatrix}\text{XD}\\ \textit{'\underline{F}ree'}\end{Bmatrix}\begin{Bmatrix}\text{YD}\\ \textit{'\underline{F}ree'}\end{Bmatrix}\begin{Bmatrix}\text{R}\\ \textit{'\underline{F}ree'}\end{Bmatrix}$$

Linear Spring Supports--Zero (0) or two (2) to twenty-two (22) lines.

a. Header--One (1) line.

$$\text{LN }\textit{'\underline{LIN}ear'}\left[\begin{Bmatrix}\rightarrow\textit{'\underline{N}ew'}\\ \textit{'\underline{A}dd'}\end{Bmatrix}\right]\left[\begin{Bmatrix}\rightarrow\textit{'\underline{I}nches'}\\ \textit{'\underline{F}eet'}\end{Bmatrix}\begin{Bmatrix}\rightarrow\textit{'\underline{P}ounds'}\\ \textit{'\underline{F}eet'}\end{Bmatrix}\right]$$

b. Data Lines for Concentrated Linear Springs--One (1) line for each spring.

LN 'Concentrated' X1 ANGLE ST SR

c. Data Lines for Distributed Linear Springs--One (1) line for each distribution.

LN $\textit{'\underline{D}istributed'} \left\{\begin{matrix}\textit{'X'}\\ \textit{'Y'}\end{matrix}\right\}$ X1 X2 A B C

Nonlinear Spring Supports--Zero (0) or maximum of forty one (41) nonlinear concentrated and/or distributed support curves.

a. Header--One (1) line.

LN $\textit{'\underline{NON}linear'} \left[\left\{\begin{matrix}\rightarrow\textit{'\underline{N}ew'}\\ \textit{'\underline{A}dd'}\end{matrix}\right\}\right]\left[\left\{\begin{matrix}\rightarrow\textit{'\underline{I}nches'}\\ \textit{'\underline{F}eet'}\end{matrix}\right\}\left\{\begin{matrix}\rightarrow\textit{'\underline{P}ounds'}\\ \textit{'\underline{K}ips'}\end{matrix}\right\}\right]$

b. Data Lines for Concentrated Nonlinear Springs--Two (2) or more lines for each spring.

Line 1:

LN 'Concentrated' X1 ANGLE NPTS DMUL FMUL

Line 2:

LN DEF(1) FORCE(1) DEF(2) FORCE(2) [. . . DEF(NPTS) FORCE(NPTS)]

c. Data Lines for Distributed Nonlinear Springs--Minimum of two (2) groups lines for each distribution.

Group 1, Line 1:

LN $\textit{'\underline{D}istributed'} \left\{\begin{matrix}\textit{'X'}\\ \textit{'Y'}\end{matrix}\right\}$ X1 NPTS DMUL FMUL

Group 1, Line 2:

LN DEF(1) FORCE(1) DEF(2) FORCE(2) [. . . DEF(NPTS) FORCE(NPTS)]

Group 2, Line 1:

LN $\left\{\begin{matrix}\textit{'Continue'}\\ \textit{'End'}\end{matrix}\right\}$ X2 NPTS DMUL FMUL

Group 2, Line 2:

LN DEF(1) FORCE(1) DEF(2) FORCE(2) [. . . DEF(NPTS) FORCE(NPTS)]

Termination--One (1) line.

LN '*F*inish' ['*R*erun' ['*K*eep'}}

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12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This report documents a computer program--CBEAMC--for analysis of general beam-column structures supported and/or loaded by components which interact with displacements of the beam and/or column.

Chapter 2 describes the general beam-column system considered and the mathematical model used for analysis; Chapter 3 presents the force-displacement relationships for the mathematical model and describes the computational procedure used for solution; and Chapter 4 provides example solution obtained with the program.

The computer program described in this report has been checked to ensure that the results are accurate within the limitations of the procedures employed. However, there may be unusual situations which were not anticipated, and these situations may cause the program to produce questionable results. It is the responsibility of the user to judge the validity of the results. No responsibility is assumed by the author for the design or behavior of any structure based on results obtained with the program.

14. SUBJECT TERMS

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	Title	Date
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	Report 1: General Geometry Module	Jun 1980
	Report 3: General Analysis Module (CGAM)	Jun 1982
	Report 4: Special-Purpose Modules for Dams (CDAMS)	Aug 1983
Instruction Report K-80-6	Basic User's Guide: Computer Program for Design and Analysis of Inverted-T Retaining Walls and Floodwalls (TWDA)	Dec 1980
Instruction Report K-80-7	User's Reference Manual: Computer Program for Design and Analysis of Inverted-T Retaining Walls and Floodwalls (TWDA)	Dec 1980
Technical Report K-80-4	Documentation of Finite Element Analyses	
	Report 1: Longview Outlet Works Conduit	Dec 1980
	Report 2: Anchored Wall Monolith, Bay Springs Lock	Dec 1980
Technical Report K-80-5	Basic Pile Group Behavior	Dec 1980
Instruction Report K-81-2	User's Guide: Computer Program for Design and Analysis of Sheet Pile Walls by Classical Methods (CSHTWAL)	
	Report 1: Computational Processes	Feb 1981
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Instruction Report K-81-7	User's Guide: Computer Program for Design or Investigation of Orthogonal Culverts (CORTCUL)	Mar 1981
Instruction Report K-81-9	User's Guide: Computer Program for Three-Dimensional Analysis of Building Systems (CTABS80)	Aug 1981
Technical Report K-81-2	Theoretical Basis for CTABS80: A Computer Program for Three-Dimensional Analysis of Building Systems	Sep 1981
Instruction Report K-82-6	User's Guide: Computer Program for Analysis of Beam-Column Structures with Nonlinear Supports (CBEAMC)	Jun 1982

(Continued)

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	Title	Date
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Instruction Report K-83-1	User's Guide: Computer Program with Interactive Graphics for Analysis of Plane Frame Structures (CFRAME)	Jan 1983
Instruction Report K-83-2	User's Guide: Computer Program for Generation of Engineering Geometry (SKETCH)	Jun 1983
Instruction Report K-83-5	User's Guide: Computer Program to Calculate Shear, Moment, and Thrust (CSMT) from Stress Results of a Two-Dimensional Finite Element Analysis	Jul 1983
Technical Report K-83-1	Basic Pile Group Behavior	Sep 1983
Technical Report K-83-3	Reference Manual: Computer Graphics Program for Generation of Engineering Geometry (SKETCH)	Sep 1983
Technical Report K-83-4	Case Study of Six Major General-Purpose Finite Element Programs	Oct 1983
Instruction Report K-84-2	User's Guide: Computer Program for Optimum Dynamic Design of Nonlinear Metal Plates Under Blast Loading (CSDOOR)	Jan 1984
Instruction Report K-84-7	User's Guide: Computer Program for Determining Induced Stresses and Consolidation Settlements (CSETT)	Aug 1984
Instruction Report K-84-8	Seepage Analysis of Confined Flow Problems by the Method of Fragments (CFRAG)	Sep 1984
Instruction Report K-84-11	User's Guide for Computer Program CGFAG, Concrete General Flexure Analysis with Graphics	Sep 1984
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Technical Report ITL-87-4	Finite Element Studies of a Horizontally Framed Miter Gate Report 1: Initial and Refined Finite Element Models (Phases A, B, and C), Volumes I and II Report 2: Simplified Frame Model (Phase D) Report 3: Alternate Configuration Miter Gate Finite Element Studies–Open Section Report 4: Alternate Configuration Miter Gate Finite Element Studies–Closed Sections Report 5: Alternate Configuration Miter Gate Finite Element Studies–Additional Closed Sections Report 6: Elastic Buckling of Girders in Horizontally Framed Miter Gates Report 7: Application and Summary	Aug 1987
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Instruction Report ITL-90-6	User's Guide: Computer Program for Two-Dimensional Analysis of U-Frame or W-Frame Structures (CWFRAM)	Sep 1990
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Instruction Report ITL-91-1	User's Guide: Computer Program for Design and Analysis of Sheet-Pile Walls by Classical Methods (CWALSHT) Including Rowe's Moment Reduction	Oct 1991
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	Report 1: Scheme A	Jun 1992
	Report 2: Scheme B	Jun 1992
	Report 3: Scheme C	Jun 1992
Instruction Report ITL-92-4	User's Guide: Computer-Aided Structural Modeling (CASM) - Version 3.00	Apr 1992
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Technical Report ITL-92-7	Refined Stress Analysis of Melvin Price Locks and Dam	Sep 1992
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Instruction Report ITL-93-4	Load and Resistance Factor Design for Steel Miter Gates	Oct 1993
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