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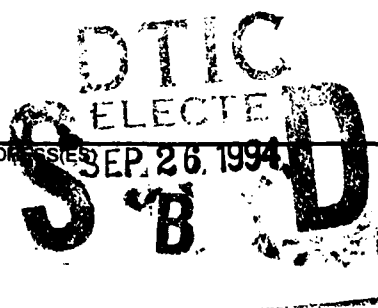
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The fractal properties of a plume dispersing in a turbulent velocity field have been examined using Large-Eddy Simulation results for neutral and convective boundary layers. A fractal generation technique has been developed that correctly matches a specified mean and variance distribution for the plume. The spatial correlation scale of the fractal realizations can also be specified, and the one-point probability density function can be chosen as clipped normal or lognormal. Realizations generated with the fractal technique show reasonably close resemblance to the LES results. The small-scale structure of the plume is further analyzed using multifractal techniques, and a the generation methodology is extended to incorporate unequal partitioning of the random variance during the refinement process. This procedure corresponds to the localization of small-scale energy in the turbulent cascade process which leads to an intermittent dissipation field. The multifractal spectrum of the dissipation field can be adjusted to match observations and the LES calculation results. The visual appearance of the dissipation field from the fractal/multifractal model is much more intermittent than the fractal realization, and the concentration field shows more localized small-scale fluctuations. These features give better correspondence to the LES realizations.

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**A.R.A.P Report No. 710**

**The Small-scale Structure of Dispersing Clouds in the  
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# SUMMARY

The fractal properties of a plume dispersing in a turbulent velocity field have been examined using Large-Eddy Simulation results for neutral and convective boundary layers. Scalar concentration isosurfaces were found to have a fractal dimension of about 1.3 from a two-dimensional plume cross-section, consistent with atmospheric and laboratory results. A fractal generation technique has been developed that correctly matches a specified mean and variance distribution for the plume. The spatial correlation scale of the fractal realizations can also be specified, and the one-point probability density function can be chosen as clipped-normal or lognormal. Realizations generated with the fractal technique show reasonably close resemblance to the LES results.

The small-scale structure of the plume is further analyzed using multifractal techniques, and the generation methodology is extended to incorporate unequal partitioning of the random variance during the refinement process. This procedure corresponds to the localization of small-scale energy in the turbulent cascade process which leads to an intermittent dissipation field. The extended fractal/multifractal model maintains the fractal isosurface properties, but also yields a multifractal dissipation field consistent with laboratory observations. The multifractal spectrum of the dissipation field can be adjusted to match observations and the LES calculation results. The visual appearance of the dissipation field from the fractal/multifractal model is much more intermittent than the fractal realization, and the concentration field shows more localized small-scale fluctuations. These features give better correspondence to the LES realizations.

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# 1. INTRODUCTION

The dispersion of material in the atmosphere is a chaotic process, due to the turbulent nature of the wind field. Molecular diffusivity is very small, and contaminants are transported by the local wind, which induces distortions into the evolving concentration field. The stretching and shearing motions of the turbulent wind field produce an increasingly complex concentration field, and eventually cascade the scale of the variations down to the smallest sizes where molecular diffusivity is effective. The detailed structure of a dispersing plume of material is therefore highly convoluted, and the smooth Gaussian shapes predicted by most dispersion models are only relevant to long term or ensemble averages.

Many applications of atmospheric dispersion modeling are concerned with short-term or near-instantaneous concentration measurements. An example is the obscuration problem, where the ability to see through an obscurant cloud depends on the instantaneous distribution of material. Long-term averages are not an appropriate measure for this problem, since the 'average' cloud does not exist at any instance in time. Instead, we need to characterize the random fluctuations in the cloud to determine the probability of obscuration. We may also need to characterize the temporal and spatial variations of the obscuration, if the duration and extent of periods of visibility are important factors. Other examples where the small-scale statistical structure is important include the dispersion of highly toxic materials, where short-term exposure can cause serious health effects, and chemically reactive dispersion, where interaction between species can occur on very fast timescales and is determined by very localized conditions.

The preceding discussion has introduced the notion of an instantaneous cloud as a random field with complicated structure on all scales down to the molecular mixing scale. This random nature of a dispersing cloud or plume has long been recognized by atmospheric dispersion modelers, and attempts to represent the phenomenon were initiated by Gifford (1959) using the meandering plume concept. Model validation was one of the principal reasons for the study of the statistical fluctuations, since the inherently unpredictable variation in an observed concentration can mask any model errors or differences between models (Fox, 1981). However, direct interest in short-term concentration values for assessing either toxic effects, flammability limits, or visibility estimates has also prompted recent research in this area (e.g., Durbin, 1980; Fackrell and

Robins, 1982a,b; Sawford and Hunt, 1986; Dinar et al., 1988; Chatwin and Sullivan, 1990; Mylne and Mason, 1991).

Gifford's (1959) model only accounts for the large-scale, meandering motions which move the entire plume in a coherent fashion. These motions are a major contributor to the variance in the concentration close to the plume source, but the meandering plume model neglects the small-scale fluctuations entirely. A practical methodology for predicting the total concentration fluctuation variance was developed by Sykes et al. (1986) under the sponsorship of the Electric Power Research Institute (EPRI). The basis of the modeling is second-order turbulence closure, and the variance prediction has been incorporated into a Gaussian plume and a Gaussian puff model. The model development is described in a series of reports by Lewellen et al. (1988).

For some applications, a prediction of the variance or one-point probability distribution function is insufficient. In order to represent the visual appearance of a plume, for example, a complete realization of the instantaneous concentration field as a function of time and space is necessary. The one-point information is an important part of the fluctuation description, but a more complete representation of the spatial and temporal structure is needed. Some useful information is obtainable from photographic data or other experimental measurements (Sreenivasan, 1991), but complete quantitative descriptions are very difficult to obtain. A complementary data source is numerical simulation, and the Large-Eddy Simulation (LES) studies of dispersing plumes by Sykes and Henn (1992) and Henn and Sykes (1992) provide detailed results on the three-dimensional, time-dependent behavior of a plume dispersing in a neutral and a convective boundary layer, respectively.

While LES provides a complete description of the instantaneous plume over the range of resolved calculation scales, the technique demands large computational resources since the flow dynamics must be computed in addition to the plume dispersal. A more practical scheme is required that can utilize the statistical information from an ensemble average prediction to generate realistic spatial and temporal variability without explicit dynamics. The most appropriate description of the turbulent field geometry is in terms of fractal structures, which embody the self-similar characteristics of the Kolmogorov inertial range and have been extensively applied to turbulent flows, e.g. Sreenivasan and Meneveau (1986), Prasad and Sreenivasan (1990a,b), Lovejoy and Mandelbrot (1985).

The objective of the study reported here is the development of techniques for generating representations of dispersing plumes that properly describe the random small-scale structure of the fluctuating concentration field. Fractal geometry provides the framework for analyzing and constructing the complex random fields; we analyze the LES plume data of Sykes and Henn (1992) and Henn and Sykes (1992), and incorporate the information into new fractal generation methods. We should emphasize that our objective in generating a fractal plume realization is, not simply to adjust parameters of a fractal model to obtain a field similar to an atmospheric plume, but rather to develop a methodology that can utilize the quantitative predictions of models such as that described by Sykes et al. (1986). We therefore require a fractal field that is consistent with a predicted mean concentration, the fluctuation variance, and a spatial/temporal correlation scale. In addition, the generated realization should match the statistical structure of the small-scale variations as closely as possible, and the fractal analysis of the LES fields will be employed to determine the important characteristics of the small scales. Section 2 describes the results of the analysis and the development of a fractal generation technique based on a recursive or iterative refinement.

Recently, much research on random fields has focused on the multifractal nature of singular quantities such as energy dissipation (e.g., Meneveau and Sreenivasan, 1987). The concept of a fractal is actually only applicable to a set of points, as distinct from a continuous field, and is generally applied to isosurfaces or contour levels. This does not provide a complete description of the entire concentration field, and there is evidence that the fractal properties may depend somewhat on the choice of level (Lane-Serff, 1993). The multifractal definition is based on a functional description and therefore gives a more complete description of a field, but the definition is couched in terms of the singularities of the field and describes the measure of the sets of spatial points where the function displays a particular rate of divergence as the sampling scale is reduced. This is clearly appropriate for dissipation fields, which are concentrated in very small regions when the Reynolds number is high and the molecular diffusivity is very small. In Section 3, we discuss the multifractal properties of the dispersing plume and incorporate some of these properties into the fractal generation model. The extended fractal/multifractal model gives an improved representation of the small-scale plume structure and in particular provides a proper dissipation field.

Section 4 describes the application of our fractal generation methodology to the animation of a plume. The time-dependence can be considered as an independent

dimension, similar to the spatial dimensions, but the localized nature of the generation technique allows the realization to be constructed locally in time. This avoids the requirement that the entire time series be pre-computed and stored before display. Instead, storage is only needed for the spatial field, which is modified as time is advanced. The small scales are modified more rapidly than the large scales and the proper fractal/multifractal is preserved. This technique could form the basis of a real-time display with relatively small storage requirements. Concluding remarks are presented in Section 5.

## 2. FRACTAL ANALYSIS AND REPRESENTATION

As a preliminary step towards a realistic description of a turbulent plume in the atmosphere, we have examined some simple fractal field generation methods to determine their consistency with known plume characteristics. The detailed description of this work is included as Appendix A to this report and is briefly summarized here. Fractal geometry has been used to analyze turbulent fields in several contexts (Sreenivasan and Meneveau, 1986; Prasad and Sreenivasan, 1990a,b; Lovejoy and Mandelbrot, 1985), and provides a natural method for describing the self-similar nature of the turbulent cascade process. The characterization of scalar isosurfaces in terms of monofractal properties is imperfect, but is a reasonable step toward a more general approach. Fractal fields can be generated with the appropriate degree of complexity and spatial structure and can be used to produce 'cloud-like' fields (Lovejoy and Mandelbrot, 1985). Our objective here is not simply to produce a visual image reminiscent of a dispersing plume, but rather to generate a 'realization' that is statistically consistent with a given plume and can therefore be used for quantitative analysis.

We have also analyzed the LES (Large-Eddy Simulation) plume calculation data of Sykes and Henn (1992) and Henn and Sykes (1992), which provide a number of time-dependent, three-dimensional 'realizations' of the instantaneous plume. The numerical results have previously been analyzed to obtain statistical measures such as concentration moments, spatial and temporal correlations, and one-point probability distribution functions, and these parameters will be required for a fractal generation scheme. We performed fractal dimension analyses of the concentration fields in these 'realizations' in order to quantify a fractal description of the LES plumes. Perimeter-area relations and box counting methods were utilized, and the dimension of a concentration contour in a two-dimensional cross-section was found to be roughly 1.30 - 1.35 using the two different estimation methods. This is slightly smaller than observational estimates from natural clouds.

We next examined techniques for generating a fractal field with the correct ensemble mean and variance values. A simple method for producing a random fractal realization was demonstrated, giving either a 'clipped-normal' probability distribution (Lewellen and Sykes, 1986) or a lognormal probability distribution. The recursive refinement technique was adapted to provide a consistent representation of the mean and



variance of an inhomogeneous concentration field. The interpolation methodology of the standard method was replaced by a randomized pulse approach to obtain a good match with the ensemble statistics. It was found that the concentration statistics were modified by the interpolation step, which gives the concentration value on the successively refined grid before the addition of the new random component. The interpolation effectively reduces the local variance, since it averages two random numbers, so the ensemble statistics of the realizations becomes increasingly inaccurate as each new level of grid points is added. The randomization of the pulse position in the new method avoids the implicit averaging problem and gives correct ensemble statistics for inhomogeneous fields, such as a dispersing plume. The fractal generation scheme can be used to generate a concentration field with the desired fractal dimension and accurate ensemble mean moments of first and second order; the scheme also requires the specification of a spatial correlation scale as the outer scale for the fractal field.

Finally, we compared the characteristics of the fractal-generated plume cross-sections with the LES realizations. The application of the resulting technique using ensemble statistics produced reasonably good 'realizations' of the LES plume.

### 3. MULTIFRACTAL ANALYSIS AND REPRESENTATION

The representation of a concentration field, which is a function of space and/or time, as a fractal is known to be incomplete since fractal concepts strictly only apply to sets of points. The generation technique described in Section 2 (and Appendix A) produces a concentration field with the appropriate mean and variance and also with isosurfaces that exhibit the correct fractal dimension. Recently, the multifractal concept has been introduced to characterize the behavior of intermittent fields such as the energy dissipation. The definition of a multifractal involves the functional behavior of the field, and therefore gives a more complete description than conventional fractal analysis. The fractal fields generated by the methods of Section 2 were therefore analyzed to determine their multifractal properties and compared with those of the LES realizations and laboratory data. The detailed description of this work is given in Appendix B, and is summarized here.

We analyzed the pseudo-dissipation field from the fractal realizations, defined as the square of the concentration gradient, for comparison with the observational data on true scalar dissipation fields. The scalar pseudo-dissipation field derived from the fractal generation scheme of Section 2 fails to exhibit the intermittent, multifractal behavior observed in experimental measurements of scalar dissipation fields (Prasad and Sreenivasan, 1990b). The homogeneous nature of the random pulse addition gives small scale energy everywhere with the same likelihood. Since the gradient operator emphasizes the smallest scales, we find a dense distribution of dissipation peaks rather than intermittent spikes. The multifractal analysis shows that the dissipation field is essentially monofractal, that is the multifractal spectrum consists of a single point.

We therefore generalized the fractal model to produce a multifractal dissipation field while maintaining the simple fractal nature of the scalar field. Noting that a simple binomial multiplicative cascade process produces intermittent distributions and has been shown to agree well with measurements of turbulent kinetic energy dissipation (Meneveau and Sreenivasan, 1987), a model was constructed which randomly allocates variance unequally in a fixed ratio to successive levels of refinement. The unequal partition of the variance as the cascade proceeds concentrates variance in localized regions and actually produces a multifractal distribution of the variance. Since the scalar

field is made up of a summation of random pulses from many refinement levels with decreasing variance, it is dominated by the larger scale structures that contain most of the variance. The fractal behavior of the concentration field is maintained since the average variance at each iteration level is still controlled by co-dimension,  $H$ , and, hence, the power spectra is unchanged. The fractal behavior of the scalar field remains unchanged since the total variance at each level is still controlled by the given codimension. However, the gradient operator involved in calculating the pseudo-dissipation field emphasizes small scales. In fact, the summation of pulse *gradients* diverges, so that the pseudo-dissipation field becomes highly intermittent and exhibits multifractal behavior. The partition parameter can be chosen to match observational dissipation data, while the concentration field still exhibits the appropriate fractal isosurface behavior. An important feature of the model is that it is still completely local and may be applied to inhomogeneous fields.

Idealized time series were generated with the new fractal/multifractal model using clipped-normal and lognormal distributions to define the pulses. An analysis of these show that the resulting multifractal spectra agree very well with the (assumed) universal spectrum derived from measurements of turbulent jets and wakes, even though the pseudo-dissipation fields from the two distributions are somewhat different in character. Analysis of LES neutral boundary layer plumes shows that the pseudo-dissipation fields reveal multifractal behavior, although the scaling range is limited. Application of the fractal/multifractal model using the LES statistics yields plumes realizations which are similar in appearance to the LES realizations. The fractal dimensions of plume isosurfaces from the LES and model are close (1.36 and 1.30 respectively); the model dimension is unchanged from the simple fractal model results given in Section 2 (Appendix A). The LES multifractal spectrum is close to the model and experimental spectra, although it may indicate less intermittency in the LES pseudo-dissipation fields. However, given the uncertainty resulting from the small scaling range, the match with the presumed universal spectrum is reasonably good.

## 4. SCALAR FIELD CONSTRUCTION FOR ANIMATION

A fractal animation model has been developed which simulates the time evolution of scalar fields in one, two, or three dimensions. This animation model is simply an extension of our successive refinement technique by considering time as an independent dimension. In this fractal animation scheme, 'time pulses' are generated in addition to the 'spatial pulses' of the static fractal models.

Consider a one-dimensional fractal generation. In the static model we add random pulses on successively smaller spatial scales with appropriately scaled variance. The location of the pulses is randomized in order to give an accurate representation of the scalar field variance. The method has also been adapted for inhomogeneous fields and provides either clipped-normal or lognormal one-point probability density functions. The concentration field,  $c(x)$ , can thus be written schematically in the form

$$c(x) = \bar{c} + \sum_n \sum_i a_{ni} P_n(x - x_{ni}) \quad (4.1)$$

where the overbar denotes the ensemble average,  $n$  is the refinement level, and  $i$  is the range of overlapping spatial pulse functions that contribute to the concentration fluctuation at the location  $x$ . The spatial pulse function,  $P_n(x - x_{ni})$ , with width  $\Delta$  and centered at  $x_{ni}$ , is defined in one dimension as

$$P(x - x_0) = \begin{cases} 1 - \frac{|x - x_0|}{\Delta}, & |x - x_0| < \Delta \\ 0, & |x - x_0| \geq \Delta \end{cases} \quad (4.2)$$

The pulse amplitude  $a_{ni}$  is chosen randomly from a Gaussian distribution with zero mean and standard deviation  $\sigma_n$ , where,  $\sigma_n = 2^{-nH} \sigma_0$ . The constant  $H$  controls the fractal dimension of the generated field. Now, consider time as an added dimension. The local concentration can now be written

$$c(x, t) = \bar{c} + \sum_n \sum_i \sum_m a_{nim} P(x - x_{ni}) T(t - t_{nm}) \quad (4.3)$$

where  $m$  is the range of overlapping time pulse functions at a given iteration level  $n$ . The time pulse function,  $T(t - t_{nm})$ , with width  $\Delta_t$  at time  $t_0$  is defined as

$$T(t-t_0) = \begin{cases} 1 - \frac{|t-t_0|}{\Delta_r}, & |t-t_0| < \Delta_r \\ 0, & |t-t_0| \geq \Delta_r \end{cases} \quad (4.4)$$

This process can be envisioned as a one-dimensional field sweeping through static time pulses. At each new instant in time, time pulses are interpolated as they lose or gain influence on the field. Consider iteration level  $n_r$  with time pulses of half-width  $\Delta_{n_r} = 2^{-n_r} \Delta_0$ . Once the field has moved through a time  $\Delta_{n_r}$ , an older time pulse loses its influence on the concentration field and a new time pulse gains influence. At any given time, only three pulses affect the field since the pulse centroids fall within  $\Delta_{n_r}$ , while the width of the pulses is twice  $\Delta_{n_r}$ . Therefore, it is only necessary to store three time pulses at each level of iteration, and the summation of overlapping time pulses is carried out over only three pulses.

The animation technique is easily extended to higher dimensions by using the product of spatial triangular functions for each dimension as well as the time pulse function. Therefore, for two dimensions we have

$$c(x, y, t) = \bar{c} + \sum_n \sum_i \sum_j \sum_m a_{nijm} P(x - x_{ni}) P(y - y_{nj}) T(t - t_{nm}) \quad (4.5)$$

where  $y$  is the second space coordinate and  $j$  is the corresponding range of overlapping pulses.

Real plumes generally have some mean flow translation and large scale meandering along with time evolution. Translation is created by adding a further extension to our simple animation procedure, namely using a mean flow velocity to move the streamwise spatial pulse locations downstream. Accordingly, at each time step, new streamwise pulses are created at the domain origin to replace the pulses that move downstream. Unfortunately, accounting for large scale meandering is not a simple task and a complete procedure has yet to be established.

## 5. CONCLUDING REMARKS

The instantaneous structure of a plume of material dispersing in the atmosphere is a random field with detailed structure on a wide range of scales. The self-similar nature of the turbulence spectrum suggests a fractal description of the scalar concentration field, and Large-Eddy Simulation results for a plume in both a neutral and a convective boundary layer have been analyzed to determine appropriate fractal properties. It is found that the LES concentration isosurfaces exhibit a fractal dimension in general agreement with atmospheric observations of water and smoke clouds.

Using a combination of the successive refinement and random pulse techniques, we have developed a method for generating a fractal field with given statistics. The method adds successively smaller scale triangular pulses with random amplitude and random location. The scale of the pulse is halved at each refinement, and the amplitude variance is reduced appropriately. Ensemble mean and variance as well as the spatial correlation scale can be specified arbitrarily and the one-point probability distributions can be specified as clipped-normal or lognormal. The realizations generated using this technique will be consistent with the given statistics.

The fractal generation technique was generalized to incorporate a multifractal aspect in the small-scale fluctuations. It was shown that an unequal partition of the variance during the refinement process yields a multifractal dissipation field but maintains the fractal isosurface properties. The resulting realizations are much improved as representations of a turbulent plume, and the small-scale variations are correctly localized in regions of intermittent dissipation.

We have developed a technique for representing the instantaneous structure of a dispersing plume. The technique requires the specification of the mean and variance of the concentration field, as well as the spatial correlation scale, and the realizations will be consistent with the given statistics. The statistics can be inhomogeneous, so that the plume can be localized in space, and the technique can be efficiently implemented for animation purposes since the pulses are defined in real space and time and only need to be considered locally. The generation method can therefore be used with a dispersion model such as that of Sykes et al. (1986) to produce a representation of a complete plume.

While much progress has been made in developing techniques for representing the instantaneous structure, there are several areas for further investigation. First, we are currently restricted to a choice of clipped-normal or lognormal statistics for the concentration fluctuations, while observations and numerical simulations show intermediate distributions between these two. Early time meandering plumes and plume edges tend to be intermittent and close to a clipped-normal distribution, while the plume interior far downstream from the source is closer to a lognormal distribution. The transition between the two probability distributions needs to be characterized and a *generalized representation technique* needs to be found to improve the fractal realizations.

The second area for future research is in the representation of large scale coherence and spatial anisotropy of the fluctuations. The early plume often contains a meandering component, where the entire plume is moved coherently by the large eddies. The plume also exhibits different characteristics in the streamwise and transverse directions. The techniques described in this report are essentially isotropic, and give the same characteristics on all scales. One can imagine a technique that modifies the simple triangular pulse shape to describe different spatial structures, but it is far from clear how to specify the appropriate shapes for a given plume. Further research is needed to characterize this behavior and develop appropriate representation techniques.

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