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By A. I. Chistyakov

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THE BEHAVIOR OF A GYROSCOPIC STABILIZER ON A ROCKING BASE

[Following is the translation of an article by Senior Lecturer A. I. Chistyakov of Kazan' Aviation Institute which was published in Izvestiya Vysshikh Uchebnykh Zavedeniy, Priborostroyeniye (News of Institutions of Higher Education, Equipment Manufacture), No. 3, 1959.]

This work is an investigation of an unloaded gyroscopic stabilizer on a rocking base.

A diagram of this stabilizer is given in Fig. 1.

The conventionally investigated stabilizer has been named "uniaxial," although actually stabilization may be produced about two axes, $X_0 - X_0$ and $Y_0 - Y_0$. This can be done because in the usual scheme of a gyroscopic stabilizer we take advantage of the preliminary incline of the rotor axis in a Cardan suspension. In the scheme under consideration the gyroscope axis is inclined in the XY plane at an angle β of considerable magnitude.

Connected with the outer gimbal of the gyroscope is the object to be stabilized and the rotor of the stabilizing motor which is controlled by a relay switch placed on axis Z - Z of the inner suspension gimbal. In addition, the motions of the system about axis X - X during rotations of the base about axis $Y_0 - Y_0$ are utilized in stabilizing the motion of the base about this axis.

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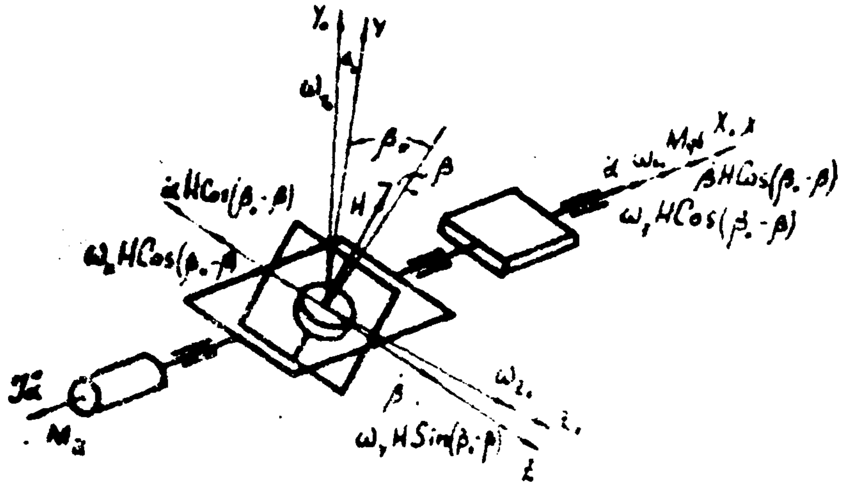

Fig. 1

During oscillations of the base about axes $Y_0 - Y_0$ and $Z_0 - Z_0$, the system decsribed has a tendency to deviate from the assigned position along axis X - X, even in the absence of a load on the part of the object to be stabilized. The velocity of deviation is such that this phenomenon can be explained by the nonuniformity of friction torque along the precession axis Z - Z (during motion in one and the other direction about this axis), [1] or by the connection of the inner gimbal and the outer by means of the contact control system. [2]

There is, however, fuller explanation for this phenomenon, if we take into account the nature of motion of the system along the axis of precession, the reaction of the system to oscillations about axis $Y_0 - Y_0$, and the effect of the forces of inertia and friction torque which appear during the oscillations of the system about axis X - X.

Stating the Problem

The requirement is to determine the motion of the system along axis X - X if the base makes periodic oscillations in two planes, such that the angular velocities of motion of the base, relative to axes $OX_0Y_0Z_0$ which are connected with them, have the following values:

$$\omega_{y_0} = \dot{\psi} = \gamma_0 p \sin pt, \quad \omega_{z_0} = \dot{\vartheta} = -\gamma_0 p \cos pt, \tag{1}$$

where γ_0 is the amplitude of angular oscillations; and
p is the circular frequency of oscillations.

Axis OX_0, being rigidly connected with axes OY_0 and OZ_0, will have a composite motion. In accordance with expressions (1), axis OX_0 along with axis OY_0 will complete oscillating motion relative to axis OZ_0, and along with axis OZ_0 will oscillate relative to axis OY_0. Both of these oscillating motions of axis OX_0 take place with a phase shift of 90°. Thus, in accordance with expressions (1), axis OX_0 moves along the surface of a right circular cone with the vertex at the origin of the coordinates. Here, axes OY_0 and OZ_0 do not turn about axis OX_0. Taking into consideration what has been said above,

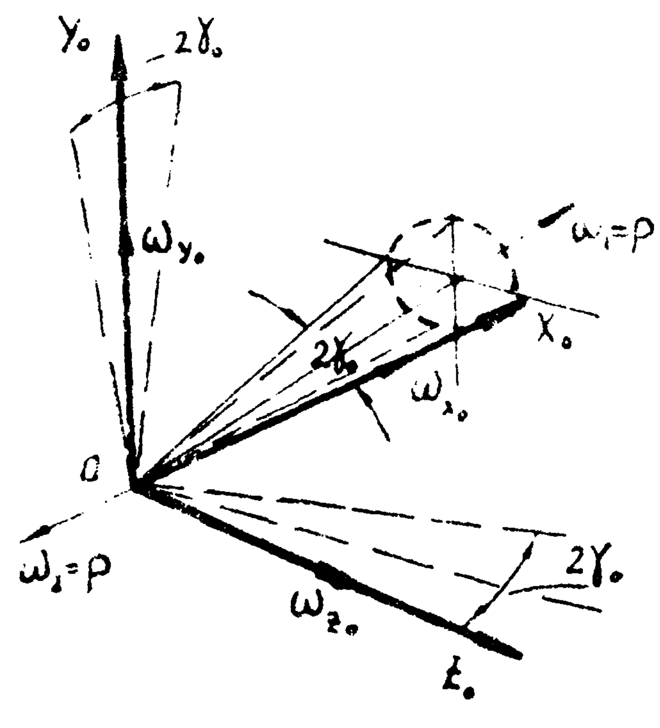

Fig. 2

we can write (Fig. 2)

$$\omega_{x_0} = p\cos\gamma_0 - p = -(1-\cos\gamma_0)p = -2p\sin^2\frac{\gamma_0}{2}$$

Or, for small angles γ_0

$$\omega_{x_0} = -\frac{1}{2}\gamma_0^2 p \qquad (3)$$

Equations of Motion of the System

Assuming that the gyroscope is astatic and neglecting friction troque and the gyroscope's inertia about the axis of precession Z - Z, we will write the equations of the stabiliser within the system OXYZ connected with it, in the following form:

$$\dot\beta H\cos(\beta_v-\beta) + \omega_z H\cos(\beta_v-\beta) = M_a\,\mathrm{sign}\,\dot\alpha + J\ddot\alpha - M_m[\beta],$$

$$\dot\alpha H\cos(\beta_v-\beta) + \omega_x H\cos(\beta_v-\beta) - \omega_y H\sin(\beta_v-\beta) = 0.$$

The equations are explained in Fig. 1.

Here M_a is the friction torque along the axis of stabilization X - X;

$M_m[\beta]$ is the torque of the stabilizing motor

J is the moment of inertia of the stabilizer relative to the axis of atbilization X - X plus the moment of inertia of the object to be stabilized which has been brought to this axis;

H is the kinetic moment of the gyroscope; and $\omega_x, \omega_y, \omega_z$ are the projections of transfer angular velocity of the OXYZ system onto the corresponding axes of this system.

The characteristics of $M_m[\beta]$ are shown in Fig. 3; here angle β is measured beginning at the value of β_v. Assuming that angle β is small in comparison with angle β_v and

$$\cos(\beta_v-\beta) = \cos\beta_v,$$

and

$$\mathrm{tg}(\beta_v-\beta) = k-\beta, \text{where } k = \mathrm{tg}\,\beta_v.$$

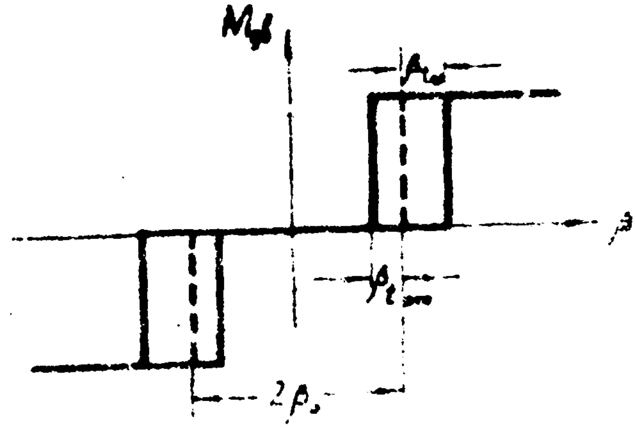

Fig. 3

introducing also designations

$$\frac{M_{\dot{\alpha}}}{H\cos\beta_v} = m_{\dot{\alpha}}, \quad \frac{J}{H\cos\beta_v} = a,$$

$$\frac{M_{тр}|\beta|}{H\cos\beta_v} = m_{тр}|\beta|$$

we will write equations (3) for the motion of the system in the form

$$\begin{aligned} \dot{\beta} + \omega_z &= m_{\dot{\alpha}}\,\mathrm{sign}\,\dot{\alpha} + a\ddot{\alpha} - m_{тр}|\beta|, \\ \dot{\alpha} + \omega_x - \omega_y(\varkappa - \beta) &= 0. \end{aligned} \qquad (4)$$

The position of the stabilizer relative to the base is determined by two angles: angle β within the system OXYZ and angle α between the system OXYZ and the system $OX_0Y_0Z_0$ which is connected with the base. It is not difficult to see in this connection that

$$\omega_x = \omega_{x_0},$$

$$\omega_y = \omega_{y_0}\cos\alpha + \omega_{z_0}\sin\alpha, \qquad (5)$$

$$\omega_z = \omega_{z_0}\cos\alpha - \omega_{y_0}\sin\alpha.$$

Hence, taking into account Equations (1) and (2), we get

$$\omega_x = -\frac{1}{2}\gamma_0^2 p,$$

$$\omega_y = p\gamma_0 \sin(pt - \alpha), \qquad (6)$$

$$\omega_z = -p\gamma_0 \cos(pt - \alpha)$$

Thus, the relations (6) are determined by the laws of motion of the base. Substituting them into the original system of Equations (4), we finally obtain

$$\beta - p\gamma_0\cos(pt - \alpha) = a\alpha + m_s\,\mathrm{sign}\,\alpha - m_{\omega}|\beta|,$$
$$\alpha - \frac{1}{2}\gamma_0^2 p - (k - \beta)p\gamma_0\sin(pt - \alpha) = 0 \qquad (7)$$

Here the first equation characterizes the motion along the axis of precession, and the second, that along the axis of stabilization.

Solution of the Problem

An accurate solution of these equations presents considerable difficulties. Therefore the solution will proceed by the method of successive approximations. First, however, we will simplify the equations.

It can be assumed with a sufficient degree of accuracy that the presence of a moderate amount of dry friction has hardly any effect on the harmonic nature of the forced oscillations. In this case dry friction can be likened to some equivalent viscous friction. Here the coefficient of viscosity should be a function of the frequency and amplitude of the forced oscillations.

The equivalency is determined from the condition that during one fourth of the period, the paths traveled by the gyroscope about axis Z - Z at velocity m_i depending on the action of dry friction along the axis X - X, and at velocity $f_T \cdot \dot{\alpha}$ depending on the action of its equivalent viscous friction, are identical.

Then, on the one hand,

$$\beta_T^{(1)} = f_T \cdot \int_0^{\frac{\pi}{2}} \dot{\alpha}\, dt = f_T \int_0^{\frac{\pi}{2}} \gamma_0 p \sin pt \cdot dt = -\gamma_0 \cdot f_T$$

and on the other,

$$\beta_T^{(2)} = -m_a \frac{T}{4}$$

Whence it follows that

$$f_T = m_a \frac{T}{4\gamma_0} = m_a \frac{\pi}{2p\gamma_0} \qquad (8)$$

Taking into account the substitutions made, the system of equations (7) is written in the form

$$\begin{aligned} &\dot{\beta} - p\gamma_0 \cos(pt - \alpha) = a\ddot{\alpha} + f_T \dot{\alpha} - m_{кв}|\beta|, \\ &\dot{\alpha} - \frac{1}{2}\gamma_0^2 p - (k - \beta) p\gamma_0 \sin(pt - \alpha) = 0. \end{aligned} \qquad (9)$$

We will further note that the mean values of angle α change comparatively slowly. Then we will assume that the change of angle α takes place within a small range about a certain slowly changing value of α_0, so that $\alpha = \alpha_0 + \Delta\alpha$, where $\Delta\alpha$ is a small angle. Further, in the first approximation we will neglect the value $\Delta\alpha$ in comparison with pt in trigonometric expressions.

Let us first consider the problem without taking into account friction and inertia, i. e., by substituting into system (9)

$$a = f_T = 0.$$

Then, from the first equation of system (9) we will find the law of motion along the axis of precession, neglecting at first the action of the motor ($m_m[\beta] = 0$):

$$\beta(t) = \gamma_0 \sin(pt - a_0) + c, \qquad (10)$$

where at t = 0, $\beta = -\gamma_0 \sin a_0$, and then c = 0.

It is quite obvious that with the motor acting on the system, the oscillations along the axis of precession $\beta[t]$ will be circumscribed by the range of insensitivity of the relay switch. Here the motion from one contact of the switch to the other will be determined by the motion of the base, and there will be oscillations on the contacts with amplitudes $\beta_{t_{cp}}$ and $\beta_{t_{om}}$, caused by the connection and disconnection of the motor (Fig. 4 $\beta[t]$, also Fig. 3).

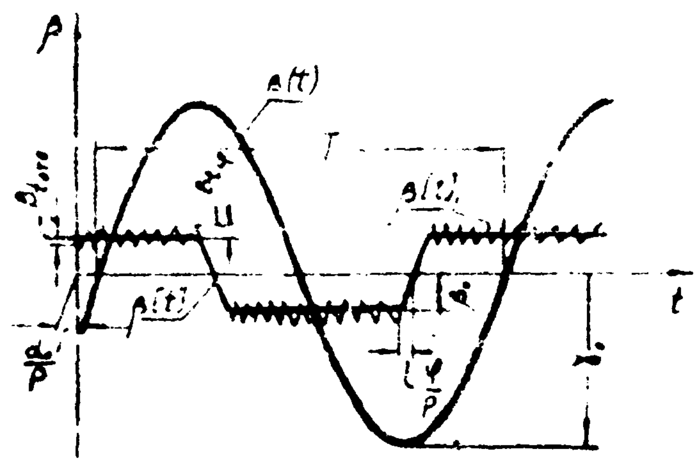

Fig. 4

Thus, the solution of the first equation of system (9) will be presented in the form of a periodic function $\beta(t)$ with the phase displaced by $\frac{\pi}{2}-\varphi$ relative to the function $\beta(t)$. Expanding the function $\beta(t)$ into a Fourier series and limiting ourselves to the first term of the series, we will write the final equation in the form

$$\beta(t)=\frac{8}{\pi^2}A\sin\varphi\cdot\sin\left(pt+\frac{\pi}{2}-\alpha_0-\varphi\right)=\frac{8}{\pi^2}\gamma_0\sin\varphi\cdot\cos(pt-\alpha_0-\varphi) \quad (11)$$

where $A=\gamma_0$ and $\varphi=\frac{\pi}{2}\frac{\beta_0}{\gamma_0}$.

Substituting the solution obtained into the second equation of (9), we get

$$\dot\alpha=\frac{1}{2}\gamma_0^2p+\left[\kappa-\frac{8}{\pi^2}\gamma_0\sin\varphi\cos(pt-\alpha_0-\varphi)\right]p\gamma_0\sin(pt-\alpha_0),$$

and after transposition

$$\dot\alpha=\frac{1}{2}\gamma_0^2p-\frac{4}{\pi^2}p\gamma_0^2\sin^2\varphi+\kappa p\gamma_0\sin(pt-\alpha_0)-$$

$$-\frac{2}{\pi^2}p\gamma_0^2\sin 2\varphi\sin 2(pt-\alpha_0)+\frac{4}{\pi^2}p\gamma_0^2\sin^2\varphi\cos 2(pt-\alpha_0)$$

Here the ultimate and penultimate terms characterize oscillations of an amplitude small in comparison with γ_0 and of double the frequency. In principle, the motion along the axis of stabilization will be determined by the expression

$$\dot\alpha=\kappa p\gamma_0\sin(pt-\alpha_0)+\frac{1}{2}\gamma_0^2p\left(1-\frac{8}{\pi^2}\sin^2\varphi\right),$$

where the first term characterizes oscillatory motion, and the second, the velocity of the constant displacement. This velocity is relatively small in magnitude, and therefore the effect of forces of inertia and friction will consist primarily of oscillatory motion about axis X - X. For this reason we will assume, for the purpose of deriving α of the second approximation, that

$$\dot\alpha=pk\gamma_0\sin(pt-\alpha_0), \quad (12)$$

and its integral

$$\alpha=-k\gamma_0\cos(pt-\alpha_0)+C.$$

where at $t = 0$, $u = k\gamma_0 \cos \alpha_0$ and, consequently, ...

Integrating the first equation of (9) and substituting into it the values of a and c of the first approximation, we find $\beta(t)$ of the second approximation right away, without taking into account the effect of the motor ($m_m [\beta] = 0$):

$$\beta(t) = \gamma_0 \sin(pt - \alpha_0) + ua + f\ a + c =$$

$$= \gamma_0 \sin(pt - \alpha_0) + ak\gamma_0 p \sin(pt - \alpha_0) - kf_c\gamma_0 \cos(pt - \alpha_0) + c,$$

where c is a constant of integration which is negligible in subsequent analysis.

The expression obtained can be reduced to the form

$$\beta(t) = A \sin(pt - \alpha_0 - \varphi_0). \qquad (13)$$

where

$$A = \gamma_0 \sqrt{(1 + a\kappa p)^2 + f_c^2 \kappa^2}$$

$$\varphi_0 = \operatorname{arc\,tg} \frac{f_c \kappa}{1 + a\kappa p} \qquad (14)$$

Taking into consideration the effect of the motor we plot, according to the function $\beta(t)$, the graphically averaged function $\beta[t]$ (Fig. 4). Expanding it into a Fourier series and limiting ourselves to its first term, we have the solution of the first equation of the system to a second approximation:

$$\beta[t] = \frac{8}{\pi^2} A \sin\varphi_0 \sin(pt + \frac{\pi}{2} - \alpha_c - \varphi_0 - \varphi_c) + \frac{8}{\pi^2} A \sin\varphi_c \cos(pt - \alpha_0 - \varphi),$$

where

$$\varphi = \varphi_0 + \varphi_c \qquad (15)$$

$$\varphi_c = \frac{\pi}{2} \frac{\beta_c}{A}$$

Then $\dot{\alpha}$ of the second approximation, in accordance with the second equation of the system, will be written as follows:

$$\dot{\alpha} = \frac{1}{2}\gamma_0^2 p + \kappa p \gamma_0 \sin(pt - \alpha_0) - \frac{4}{\pi^2} p \gamma_0 A \sin\varphi_0 \sin(pt - \alpha_0) \cdot$$

$$\cdot \cos(pt - \alpha_0 - \varphi).$$

and after transposition

$$\dot{\alpha} = \frac{1}{2}\gamma_0^2 p - \frac{4}{\pi^2} p \gamma_0 A \sin\varphi_0 \sin\varphi + \kappa p \gamma_0 \sin(pt - \alpha_0) -$$

$$- \frac{4}{\pi^2} p A \sin\varphi_0 \sin 2\left(pt - \alpha_0 - \frac{1}{2}\varphi\right). \qquad (16)$$

Thus we have a compound motion along the axis of stabilization. The motion consists of oscillatory motion (basically determinable by the third term) and an added constant component.

The constant component of the displacement rate equals

$$\dot{\alpha} = \frac{1}{2}\gamma_0^2 p \left(1 - \frac{8}{\pi^2}\sqrt{(1 + a\kappa p)^2 + f_\tau^2 \kappa^2} \sin\varphi_0 \sin\varphi\right).$$

After substitution of the values of $\sin\varphi$ and $\sin\varphi_0$ the expression for the constant component of the displacement rate can be reduced to

$$\dot{\alpha} = \frac{1}{2}\gamma_0^2 p \left(1 - \frac{8}{\pi^2}\sin^2\varphi_0 - \frac{8}{\pi^2} a\kappa p \sin^2\varphi_0 - \frac{4}{\pi^2}\kappa f_\tau \sin 2\varphi_0\right)$$

where

$$\varphi_0 = \frac{\pi}{2} \frac{\vartheta_0}{\gamma_0\sqrt{(1 + \kappa a p)^2 + f_\tau^2 \kappa^2}} \qquad (17)$$

The first term of the derived expression $\frac{1}{2}\gamma_0^2 p$ characterizes the departure along the axis of stabilization from the rotation of the base about axis X - X. The sign of displacement changes with the change of direction of the rotation of the base.

The second term

$$\frac{4}{\pi^2}\gamma_0^2 p \sin^2\varphi_0$$

characterizes displacement along the axis of stabilization from oscillatory motion of the base about axes Y_0 - Y along ψ and Z_0 - Z_0 along ϑ with a 90° phase shift. magnitude of displacement is determined, all other conditions being equal, by the magnitude of the insensitive zone and by the amplitude of oscillation A. The sign of displacement changes with the change of direction of rotation of the base, i. e., a phase shift of -90° in the oscillations of the base along ψ and ϑ reverses the sign of displacement.

In the case of a free gyroscope ($m_m|\beta| = 0$) this term will be equal to $\frac{1}{2}\gamma_0^2 p$, if we substitute the expression (10) for $\beta(t)$ into the second equation of the system (9). Then, taking into account the first term of the expression (17), the ultimate value of α would equal zero. In the case under consideration the term

$$\frac{4}{\pi^2}\gamma_0^2 p \sin^2\varphi_0$$

is smaller than $\frac{1}{2}\gamma_0^2$ p. The physical cause lies in the fact that the precession of the gyroscope along axis Z - Z is circumscribed by the boundaries of insensitivity of the switch.

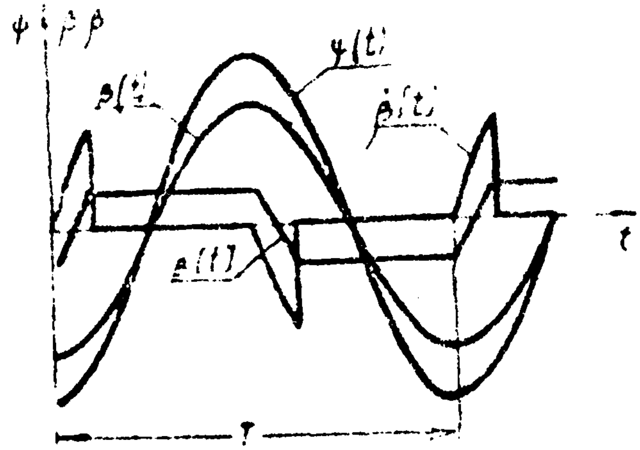

Fig. 5

The third term

$$\frac{4}{\pi^2} a k \gamma_0^2 p^2 \sin^2 \varphi_0$$

characterizes displacement along the axis of stabilization as a function of forces of inertia which appear during oscillations of the system about this axis. However, the oscillations of the system about axis X - X are basically determined by the inclination of the axis of the gyroscope's rotor and by directional oscillations of the base. The magnitude of displacement is determined by the system parameters β_0 , A and β . The sign is a function of both the direction of rotation of the base and the direction of the vector of the gyroscope's kinetic moment. Thus, depending on the direction of the vector of the gyroscope's kinetic moment, the effect of the forces of inertia will either decrease or increase the displacement rate along the axis of stabilization.

In Figs. 5 and 6 is illustrated the physical cause of the effect of inertia along the axis of stabilization upon the accuracy of stabilization of the system relative to this axis.

Figure 5 shows curves of the change of the directional position of the base $\psi(t)$ and of the velocity of precession $\dot\beta(t)$ as functions of inertia forces only. In Fig. 6a we present a case where the base turns in the right hand direction. Here the precession vector of angular velocity $\dot\beta(t)$, as a function of inertia forces of the system's turn to the right along the axis X - X, will have a direction along axis OZ.

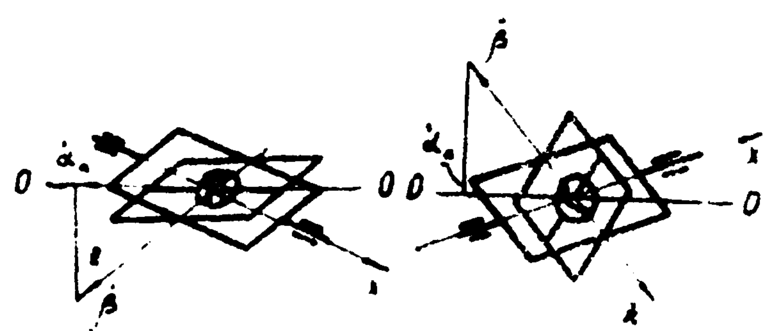

Fig. 6

Let us expand the angular velocity $\beta(t)$ along the direction 0 - 0 relative to which the base undergoes [illegible] tional oscillations, and perpendicular to it. The component in the 0 - 0 direction characterizes the direction of the departure of the gyroscope's axis in space during the half period of oscillation of the base under consideration. The second half of this period is illustrated in Fig. 6b. The precession vector of angular velocity $\beta(t)$ will now be directed along the negative axis OZ, and its component along the original position of axis 0 - 0 will assume its former direction. Thus, during the course of oscillations of the base as functions of inertial forces, there will be a directionally constant displacement of the gyroscope's axis in space. It is not difficult to see (Fig. 5) that the presence of a motor limits the period of the effect of and hence diminishes the effect of inertia forces on the behavior of the system.

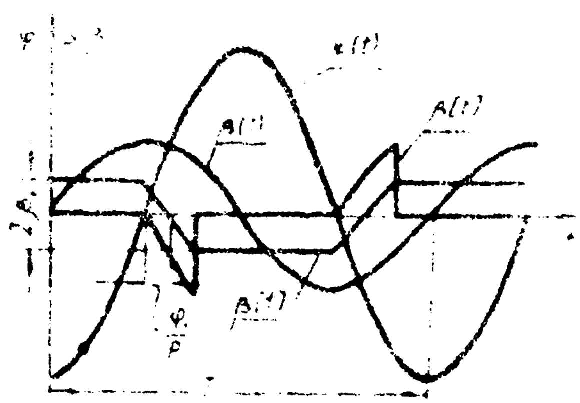

Fig. 7

The fourth term of the expression (17)

$$\frac{2}{\pi^2}\gamma_0^2 p\kappa f_v \sin 2\varphi_0$$

obtained by taking into account Equation (8) can be written as follows:

$$\frac{1}{\pi}\gamma_0 m_a k \sin 2\varphi_0$$

This term characterizes the displacement component along the axis of stabilization as a function of friction torque appearing during the oscillations of the system about this axis. The effect of friction torque is also determined by the inclination of the gyroscope's axis β_v and by the directional oscillations of the base (if we assume that the angle is small). The sign of departure is determined only by the direction of the vector of the gyroscope's kinetic moment.

The physical cause of displacement along the axis of stabilization as a function of the effect of friction torque along this axis is shown in Figs. 7 and 6 and requires no explanation. We only wish to note that at $A = 2\varphi_0$ the magnitude of displacement will be the greatest, $(2\varphi_0 = \frac{\pi}{2})$

and in the case of a free gyroscope the displacement caused by friction along the axis of stabilization will equal zero.

Conclusions

1. During oscillations of the base in two mutually perpendicular planes with a phase shift of 90° along axis X - X of a gyroscopic stabilizer, there will be a constant displacement. The rate of displacement α is not a function of the magnitude of angle α.

2. The phenomenon of displacement is explained by the nature of motion along the axis of precession, caused by the relay-type switch and an insensitive zone, and the effects of inertia and friction developing during oscillations of the system about axis X - X.

3. The magnitude of displacement is a function of the following parameters of the system:

β_v - the incline angle of the gyroscope's axis,
$2\varphi_0$ - the magnitude of the zone of insensitivity of the switch,
J - the torque of the system along axis X - X
- the friction torque along axis X - X.

4. In order to diminish displacement, it is advisable that the direction of the vector of the gyroscope's kinetic moment be such that its projection on the axis of stabilization coincides with the direction of the projection on the same axis of the angular velocity vector of the motion of the base (at $+p$ we must have $+\overline{H}$).

5. The expression (17) at k = 0 characterizes the displacement along the axis of the outer gimbal of a gyroscope with three degrees of freedom, with correction of the perpendicularity of the gimbals from the relay switch

$$\alpha = \frac{1}{2}\gamma_0^2 p - \frac{8}{\pi^2}\gamma_0^2 p \sin^2\varphi_0, \tag{18}$$

where

$$\varphi_0 = \frac{\pi}{2}\cdot\frac{\beta_0}{\gamma_0}.$$

Bibliography

1. D. S. Pel'por, Issledovaniye zakonov dvizheniya gyropolukompasa, ustanovlennogo na gorizontal'nom osnovanii [Investigation of the Laws of Motion of the Directional Gyro Mounted on a Horizontal Base], Trudy MVTU [Transactions of Moscow Higher Technical School]; Elementy rascheta tochnykh priborov [Calculation Components of Precision Instruments], 1954.

2. G. O. Fridlender, O dvizhenii osi [illegible]ursovogo giroskopa avtopilota pri kolebaniyakh osi naruzhnogo kol'tsa vokrug proizvol'noy osi, Sbornik statey po avtomatike i telemekhanike, AN SSSR [On the Motion of the Axis of a Directional Gyroscope of an Automatic Pilot During Oscillation of the Axis of the Outer Gimbal About a Random Axis, Collection of Articles on Automation and Telemechanics, Academy of Sciences USSR], 1956.

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END

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