CORNELL UNIVERSITY

Center for Radiophysics and Space Research

RESEARCH REPORT RS 33

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F. W. Perkins, Jr,

THEORETICAL STUDIES—ARECIBO, Scientific Report No. 3

[Contract No. AF19(604)-8804]

AFCRL-62-27

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Contract No. AF19(604)-8804 ARPA Order No. 106-61

30 November 1961

Prepared for
ELECTRONICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

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ACKNOWLEDGMENTS

Professor E. E. Salpeter read the manuscript and his suggestions for its improvement are greatly appreciated.

CONTENTS

I. INTRODUCTION

In the radar experiment at Arecibo, some power is returned in a line separated from the transmitted frequency by the plasma frequency of the electrons. This is called the plasma line. The electron density at a given altitude can be determined if it is possible to detect the plasma line and measure its separation from the transmitted frequency. This note shows that for reasonable ionospheric data, the plasma line is just at the limit of detectability for the radar at Arecibo (see Table I). The electron density can be determined to ± 5 per cent by this technique and provides a check on other measurements. Some deviations from thermodynamic equilibrium and their effect on the plasma line are also discussed.

Table I. Signal-to-Noise Ratio for the Plasma Line of the Radar at Arecibo.

Altitude (km)	S/N*	ν_p = expected plasma frequency (Mc/s)
100	4η**	3
200	3.2η	9
300	3η	12.5
400	$.8\eta$	9
600	1.1η	3

* Factor of 10^2 obtained from averaging pulses is included.

**η = gain of antenna system at $430 \pm \nu_p$ Mc/s relative to 430 Mc/s. ($\eta = 1/2$ at $\nu_p = 5$ Mc/s).

II. GENERAL FEATURES OF PLASMA-LINE SCATTERING

The intensity of the backscattered plasma line signal will be shown equal to the intensity of a central line signal scattered from an electron density of approximately 3×10^3 electrons/cm^3 (see Section II). The bandwidth of the returned signal is determined principally by the change in plasma frequency over the scattering volume (see Section III) and is given by

$$B = \nu_p \frac{c\tau}{2H} ,$$

where $c\tau$ is the pulse length, H is the scale height for the electron density, and ν_p is the plasma frequency. Since the bandwidth is proportional to the plasma frequency, regions with low electron density are more easily detected for a given scale height and pulse length. Also since both the bandwidth and the intensity of the backscattered signal are proportional to the pulse length, the signal-to-noise ratio is independent of the pulse length. Thus short pulses can be used and the electron density determined accurately. It is suggested, however, that the pulse length be no shorter than 10 km; otherwise local electron density fluctuations might exceed the change in electron density resulting from the general electron density gradient.

III. STRENGTH OF THE PLASMA-LINE SIGNAL

Salpeter[1] and others have shown that the power, P_r backscattered in the central line by incoherent scattering in a singly ionized gas is proportional to

$$P_r = (\text{const.}) \times \frac{Ne^2}{2} ,$$

when $\alpha = \frac{\lambda}{2D} >> 1$, where D is the Debye Length and N is the number of electrons in the scattering volume. Salpeter shows there is also a much weaker signal returned in two lines displaced from the central line by an amount equal to the plasma frequency of the electrons. The power returned in one of these lines, P_r^P , is

$$P_r^P \alpha \frac{Ne^2}{2\alpha^2} = \frac{n V e^2}{2} \cdot \frac{4kT}{\lambda^2 4\pi n e^2}$$

$$= \frac{1}{2} \frac{4kT}{4\pi e^2 \lambda^2} V e^2 \quad .$$

This expression shows that the power returned in the plasma line is independent of the electron density; $4kT/4\pi e^2 \lambda^2$ can be thought of as an effective electron density, n_{eff} , whose value in the F-layer is

$$n_{eff} = \frac{4kT}{4\pi e^2 \lambda^2} = 3 \times 10^3 \text{ cm}^{-3}$$

for $\lambda = \frac{70}{2\pi}$ cm and T = 2000° K.

IV. BANDWIDTH OF THE PLASMA-LINE SIGNALS

The bandwidth of the plasma line signal is determined by three factors: (1) the natural width of the plasma line, (2) the frequency spread resulting from the bandwidth of the transmitted pulse, and (3) the frequency spread resulting from the change in the electron density over the pulse length.

The dominant contribution comes from the changes in electron density.

According to Salpeter,[1] the natural width of the plasma line, $\Delta\nu_n$, is

$$\Delta\nu_n = \nu_e \sqrt{\pi}\, \frac{\alpha^4}{2}\, e^{-\alpha^2/2}$$

where

$$\nu_e = \frac{1}{2\pi}\left(\frac{8kT}{\lambda^2 m_e}\right)^{1/2} \sim 4.10^5$$

in the ionosphere and $\alpha \sim 20$; therefore $\Delta\nu_n \overset{\sim}{<} 1$ cps for $\alpha > 6$.

Since the natural line width is negligible, one can write the bandwidth as

$$B \approx \frac{1}{2\pi}\left(\frac{1}{\tau} + \omega_p \frac{c\tau}{2H}\right)$$

where ω_p is the plasma angular frequency, and H is the scale height. Here $1/\tau$ is due to the bandwidth of the transmitted pulse, and $\omega_p \frac{c\tau}{2H} \overset{\sim}{=} \frac{\partial\omega_p}{\partial x}$; $c\tau$ represents the contribution of the change in electron density over the pulse length.

Even for pulses as short as $10\,\mu s$, one has

$$\frac{\omega_p \frac{c\tau}{2H}}{\frac{1}{\tau}} = \frac{5.6 \times 10^7\ \frac{1.5 \times 10^5}{10^7}}{10^5} = 8 \quad ,$$

when $n = 10^6$ electrons/cm^3, and $H = 100$ km. Thus one can write the bandwidth as

$$B = \nu_p \frac{c}{2H} = 9\sqrt{n}\ \frac{c\tau}{2H}\ \text{kc/s} .$$

Using $c\tau = 100$ km, $H = 100$ km, one has

$$B = .45\sqrt{n}\ \text{kc/s}$$

when $n = 10^6$ electrons/cm^3, $B = 450$ kc/s .

V. SIGNAL-TO-NOISE RATIO FOR THE PLASMA LINE SIGNAL

This calculation is uncertain because the frequency response of the antenna system is not known and is left as an arbitrary parameter. The only numbers available indicate a loss of 3 db at 5 Mc/s off the design frequency. The plasma lines will be separated by up to 13 Mc/s from the transmitted frequency.

The signal-to-noise ratio is given by

$$\frac{S}{N} = \frac{\pi}{4}\ \frac{P_T \frac{A}{r^2} \frac{\sigma_e}{2} \frac{1}{L} \frac{c}{2} n_{eff}\ \eta}{kT\ B\ r^2}\ (Av)$$

Numbers used in the calculations are:

$P_T = 2.5 \times 10^6$ watts

$A = \frac{\lambda^2}{4\pi} 10^6 = 3.9 \times 10^4\ m^2$

L = loss factor = 1.5

$(Av) = 10^2$ (obtained from averaging pulses)

$\sigma_e = 8 \times 10^{-30}\ m^2$

$n_{eff} = 3 \times 10^9\ m^{-3}$

T = 400° K (see research report[2])

B = $\nu_p \frac{c}{2H}$, with H = 100 km

η = gain of the antenna system of 430 $\pm \nu_p$ Mc/s relative to the gain at 430 Mc/s

r = altitude

$$S/N = \frac{\pi}{4} \frac{P_T A(\sigma_e/2) H n_{eff}}{k T \nu_p L r^2} \quad . \quad (Av)$$

The results of the calculations of the signal-to-noise ratios are given in Table II.

Table II. Results of Calculation of Signal-to-Noise Ratio

Altitude (km)	Expected electron density (cm^{-3})	B* (kc/s)	ν_p (Mc/s)	S/N	T (°K)
100	10^5	150	3	4η	200
200	10^6	450	9	3.2η	2000
300	2.10^6	210**	12.5	3η	2000
400	10^6	450	9	$.8\eta$	2000
600	10^5	150	3	1.1η	2000

* Based on a pulse length of 10 km.

**Scale height is taken as 300 km near the F_2 maximum.

When $\nu_p = 3$ Mc/s, $\eta = .75$; therefore the plasma line should be detectable at 100 km and 600 km. In at least two regions, therefore, the plasma line results can be used as a check on other methods of measuring the electron density.

VI. PROPOSED EXPERIMENT:

The electron density is to be determined by measuring the frequency separation of the plasma line from the central line of the back-scattered signal. The relation between the electron density and the frequency separation is[1]

$$(\Delta\omega)^2 = \frac{4\pi n_e e^2}{m_e} + \frac{3KT}{m_e}\left(\frac{4\pi}{\lambda}\right)^2$$

where $\Delta\omega$ is the frequency separation between central line and the plasma line, and n_e is the electron density.

For the radar experiment at Arecibo, the second term is at least an order of magnitude smaller than the first, in the ionosphere. A measurement of the electron density by the plasma line technique should aid in resolving any discrepancy between conventional ionosonde measurements and the electron density as measured by the total back-scattered power in the central line. This is because the plasma-line technique is a frequency measurement and not affected by small nonthermodynamic fluctuations in the electron density. Small fluctuations will affect the total power back-scattered into the central line. At present a discrepancy does exist between ionosonde and incoherent scatter methods.[3,4]

It is suspected that more energy than predicted by thermodynamic arguments may be present in the form of plasma waves in the ionosphere, because of their slow decay time and the fact that plasma waves are excited by fast particles and other electric forces. This would increase the total intensity of the plasma-line signal but would not change its frequency separation from the control line. Thus a plasma line intensity substantially greater than that predicted would indicate deviations from thermodynamic equilibrium of plasma waves in the ionosphere.

On the other hand, a departure from thermal equilibrium might take the form of strong fluctuations, $\Delta n/n \sim 10^{-1}$, in the electron density within a pulse length, of 10 km. The frequency of the plasma line signal in this case would be spread over a large bandwidth which would lower the signal-to-noise ratio and make the plasma line appear weaker.

An investigation of the plasma line width would give information on the change in electron density in a region the size of the pulse length. By varying the pulse length, it might be possible to determine the contribution to variation in electron density resulting from the over-all gradient in electron density and the contribution resulting from local changes in the electron density arising, for instance, from the trapping of electrons in magnetic tubes of force.

A possible experimental arrangement consists of recording the plasma-line signal intensity versus height in a 450 kc/s band centered at 421 Mc/s. Significant maxima in this wave, using a 10-km. pulse length, locate regions where the electron density is 10^6 ($\pm$ 5 per cent) electrons/cm^3.

Modifications of the receiving equipment now planned for Arecibo would be necessary to detect the plasma line. A device to measure the bandwidth of the plasma line signal as well as its frequency separation from the central line would be useful.

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