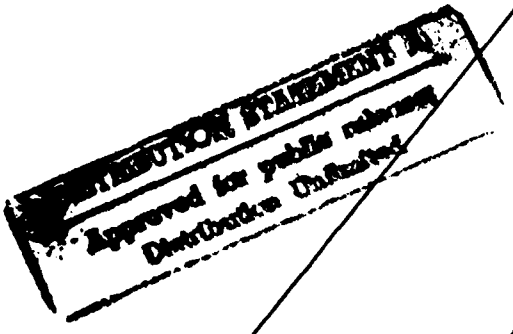
NBS MONOGRAPH 6

Properties of High-Temperature Ceramics and Cermets

Elasticity and Density at Room Temperature



U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials devices, and structures; advisory services to government agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. Research projects are also performed for other government agencies when the work relates to and supplements the basic program of the Bureau or when the Bureau's unique competence is required. The scope of activities is suggested by the listing of divisions and sections on the inside of the back cover.

Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers. These papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three periodicals available from the Government Printing Office: The Journal of Research, published in four separate sections, presents complete scientific and technical papers; the Technical News Bulletin presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: Monographs, Applied Mathematics Series, Handbooks, Miscellaneous Publications, and Technical Notes.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($1.50), available from the Superintendent of Documents, Government Printing Office, Washington 25, D.C.

UNITED STATES DEPARTMENT OF COMMERCE • Frederick H. Mueller, *Secretary*
NATIONAL BUREAU OF STANDARDS • A. V. Astin, *Director*

Properties of High-Temperature Ceramics and Cermets

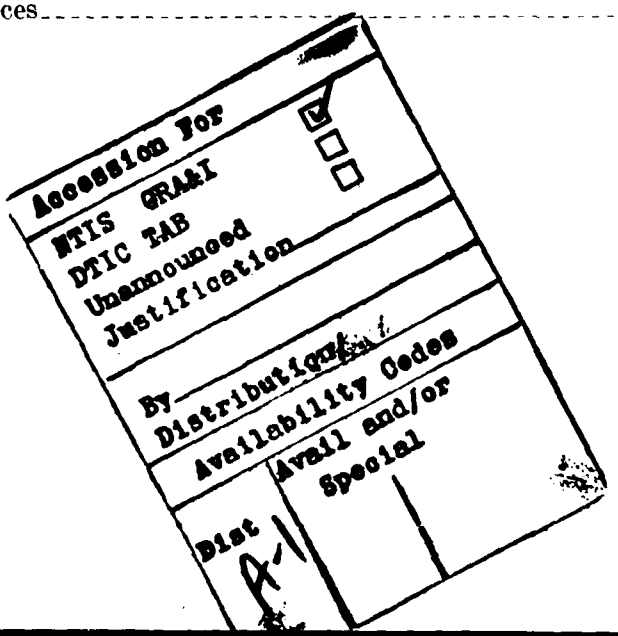
Elasticity and Density at Room Temperature

S. M. Lang

National Bureau of Standards Monograph 6
Issued March 1, 1960

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington 25, D.C. - Price 20 cents

Contents



Properties of High-Temperature Ceramics and Cermets

Elasticity and Density at Room Temperature [1]

S. M. Lang[2]

In order to provide some of the basic data necessary for the effective utilization of ceramics and cermets in various high-temperature applications, a specimen "bank" of such materials, mainly commercially fabricated, was established for the measurement of physical properties and constants. This Monograph describes: (1) The materials and some of their fabrication data; (2) bulk densities; (3) theoretical densities; and (4) the dynamic room-temperature elastic constants. Data are given for 46 sets of specimens, representing 20 different materials; these include oxides, carbides, borides, cermets, and an intermetallic compound. A statistical evaluation was used for analyzing the data.

Results of the room-temperature measurements show that: (1) Significant variations are common both in the specimens of one group and from group to group of specimens prepared of the same material; (2) the largest variations occur for specimens formed by hot-pressing, although average values are higher for hot-pressed specimens; and (3) measurements of the dynamic elastic constants by the sonic method are more sensitive as indicators of homogeneity and group uniformity than bulk-density measurements.

1. Introduction

Recent outstanding advances in the field of jet, rocket, and atomic-powered heat engines have strongly stimulated an accompanying development in high-temperature ceramic and cermet materials to withstand the high temperatures and corrosive atmospheres involved. The new ceramics have been well described by E. J. Runck [1] [3] who states that they are commercially produced by "radical departure from orthodox processes and materials", being "ceramic bodies . . . that possess no silica or clay in their structure. The coarse nonplastic portions of these new refractories have high purity and are processed at high temperatures, even up to fusion . . . . The mixtures are compacted and fired . . . usually above 1,600° C. In firing, these materials do not form glassy bonds, but sinter or recrystallize by solid reactions."

Designers have been severely handicapped by a lack of sufficient data for the high-temperature ceramics and, in addition, the available data have not always been consistent [2]. This lack of consistency could result from differences in experimental conditions, or could be a real variation of properties of nominally the same material caused by uncontrolled differences in fabrication or batch composition. To fill the critical need for more complete and reliable engineering data for these ceramic materials, a "bank" of these products has been established. This bank contains samples from the leading manufacturers as well as some fabricated at the National Bureau of Standards. Also included are a number of cermets. These are metal-ceramic combinations designed for use at elevated temperatures.

The long-range goal of the investigation is to supply reliable engineering data for these new classes of materials. The properties to be investigated include mechanical strength, elastic and anelastic characteristics, the temperature dependence of these properties, and thermal properties generally. Since all these properties will be determined on the same set of specimens by the same test procedure, valid comparisons will then be possible.

The particular purposes of the present investigation were: (1) To determine the room-temperature elastic properties by a dynamic (sonic) method; (2) to determine both the bulk and theoretical densities; (3) to evaluate from a statistical treatment of the data the variability of fabrication of the different types of materials, using the above-determined elastic and bulk-density data; and (4) to compare measurements of the elastic properties and densities for their usefulness as indicators of significant variations in fabrication.

Density and elastic properties were determined first because, in addition to being important properties in themselves, the experimental method for their determination is nondestructive. It thus becomes possible to perform further measurements of other properties without diminishing the number of available specimens.

[1] Financial support for this investigation was supplied by the Division of Research of the U.S. Atomic Energy Commission.

[2] Present address, Owens-Illinois Technical Center, Toledo, Ohio. Since Mr. Lang's departure from NBS, this paper has been revised by M. D. Burdick and S. Spinner.

[3] Figures in brackets indicate the literature references at the end of this Monograph.

The present paper contains detailed descriptions of the methods used in calculating both the elastic constants and the statistical parameters. These detailed descriptions have been included for two reasons: (1) To leave no doubt as to the exact procedure, and (2) to act as a guide to other workers who might wish to make similar determinations.

2. Materials and Procedure

2.1. Materials

A complete list of the materials along with pertinent fabrication data is given in table 1 (appendix II). Most of these materials are commercially available; some are experimental (at the time of this investigation) including all those from source G. Some additional information concerning these materials that could not be listed in table 1 conveniently is included under results.

2.2. Preparation of Specimens

A convenient size and shape of specimen for the elastic modulus measurements, and one that was used whenever possible, was a rectangular prism 6 in. by ½ in. by ¼ in. However, all of the specimens that were fabricated by the hot-pressing technique were supplied as approximately 3-in. long by ½-in. wide by ¼-in. thick bars, rather than 6-in. long bars, because the shorter specimens could be made denser and with greater uniformity. In order to achieve this with available equipment, the blanks were hot-pressed perpendicular rather than parallel to the length. Pressing parallel to the length would have produced a low-density zone near the center of the specimens.

All specimens were finish ground to the final dimensions. For the harder specimens, such as B_4C, B_4C+TiB_2, and $SiC+B_4C$, the machining operation was slow and tedious, even with diamond tools.

2.3. Elastic Moduli

a. Method

The equipment and technique for determining the elastic moduli by the sonic method were the same as those reported by Spinner [3], and previously described by Hornibrook [4]. The method consists of inducing the mechanical resonant frequency of the specimen, usually by means of a tweeter-type speaker, driven by an audio oscillator. The resultant oscillations are detected by means of a crystal pickup which, together with the signal from the audio oscillator, produces a Lissajou pattern on a cathode ray oscilloscope.

One of the basic characteristics of the dynamic method is that the elastic moduli are determined at very low-stress levels. Thus, the possibility of the occurrence of creep, elastic hysteresis, plastic flow, or similar effects is reduced to a minimum [5].

Whenever possible, the fundamental resonant frequency of four types of vibration was determined for each specimen. These were the longitudinal, F_l; flexural vibrating flatwise, F_{fw}; flexural vibrating edgewise, F_{fe}; and torsional F_t. The first three types of vibration were used to determine Young's modulus, whereas the torsional mode yielded the shear modulus. The reason for determining two elastic moduli is that all the elastic constants for isotropic materials are interrelated by well-known equations in such a way that, if any two are known, the others may be calculated. In this investigation, Poisson's ratio, μ, and the bulk modulus, K, were calculated after Young's and the shear moduli had been obtained.

For some specimens, usually the shorter ones, the longitudinal resonant frequency was too high to be detected with the equipment used. In these cases, only the flexural and torsional resonance frequencies were determined.

In three instances, Code 44 (B_4C), Code 43 (B_4C+TiB_2), and Code 37 ($SiC+B_4C$) the combination of size and density were such that the resonant frequency of the test specimens could not be detected either in the longitudinal or in the torsional modes. One specimen of each of these materials was reshaped so that the resonant torsional frequency could be obtained. The remainder of the specimens in the group were not reshaped because (1) the grinding and polishing operations were difficult and time consuming due to the extreme hardness of the B_4C constituent, and (2) the reshaping operation made the specimens unsuitable for other testing.

The shear moduli and values of Poisson's ratio were calculated for each of these reshaped specimens. These values of Poisson's ratio were then assumed to be representative of the entire group of specimens of that mixture and were used to calculate the shear and bulk moduli for the remainder of the specimens. Because only one specimen was measured in shear, the data for these groups were not treated statistically.

It may be noted that it is usually possible to obtain adequate responses even when frequencies are considerably higher than what is usually regarded as the upper limit of the audiofrequency range (20 kc). This is because the frequency response of a driver or pickup is usually higher than its rated value. The upper frequency limit for a good crystal pickup is ordinarily given as around 10 to 11 kc. Although this is the frequency above which the response falls off from being flat, it does not drop immediately to zero. There continues to be a reduced response at much higher frequencies. With the apparatus used, frequencies well above 20 kc could usually be detected. The actual upper limit of frequency response was between 25 to 30 kc.

b. Calculations

All of the following equations for calculating the elastic constants from the various resonant frequencies are based on the assumption that the specimens are isotropic and homogeneous. Although the individual crystals comprising the specimens are elastically anisotropic, their distribution and orientation within the specimen are random so that the assumption of isotropy would appear to be valid. However, as will be seen later, certain differences in Young's modulus, when calculated from flexural and longitudinal frequencies, indicate that either or both of these conditions (isotropy and homogeneity) were not completely satisfied in all cases.

The following well-known equations were used to calculate the speed of sound, V_c, and Young's modulus, E_l, from the longitudinal resonant frequency, F_l

$$V_c = 2lF_l, \tag{1}$$

where l=length in cm, F_l is in cps, and V_c is in cm/sec, and

$$E_l = V_c^2 \rho, \tag{2}$$

where ρ=density in g/cm^3. If V_c is in cm/sec, then E_l will be in dynes/cm^2. All elastic moduli are given in kilobars where

$$10^9 \text{ dynes/cm}^2 = 1 \text{ kilobar.}^4$$

Correction for cross-sectional effect was neglected as too small to be significant. The following equation from Giebe and Scheibe [6] gives the amount by which the fundamental longitudinal frequency, F_l, of a prism of given rectangular cross section is less than that of an infinitely thin rod of the same length

$$F_l \approx \frac{F_m}{1+\frac{\pi^2\mu^2(w^2-e^2)}{24l^2}}, \tag{3}$$

where F_m=longitudinal resonant frequency of an infinitely thin rod,

μ=Poisson's ratio,

w=width,

e=edge or thickness,

l=length of the specimen.

Substituting the dimensions for the long (6 in.) specimen and, assuming a μ value of 1/4

$$F_m = 1.0002\ F_l \tag{4}$$

or, the resonant longitudinal frequencies of bars of the dimensions used here are 2 parts in 10,000 less than for an infinitely thin rod of the same length. This is a higher order of precision than the resonant frequency determination itself (1 part in 3,000), and therefore may be neglected. Since the entire lateral correction is neglible and, since Poisson's ratio is only one of the factors entering into the correction, the value of E_l is, within the precision used here, independent of Poisson's ratio.

To calculate Young's modulus from the flexural frequencies and the shear modulus from the torsional frequency, the relations developed by Pickett [7] were used. These equations have been modified to conform to the cgs system.

The following pair of equations relate Young's modulus to the flatwise and edgewise flexural frequencies,

$$E_{fw} = 9.464\times10^{-10}\left(\frac{l}{e}\right)^3 \frac{T_1}{w}\, mF_{fw}^2 \tag{5}$$

$$E_{fe} = 9.464\times10^{-10}\left(\frac{l}{w}\right)^3 \frac{T_1}{e}\, mF_{fe}^2 \tag{6}$$

E_{fw}=Young's modulus as determined from flatwise flexural vibration, F_{fw}; E_{fe}=Young's modulus as determined from edgewise flexural vibration, F_{fe}; and m=mass of specimen in grams. The numerical constant in eqs (5) and (6) is chosen so that Young's modulus will be in kilobars. The factor, T_1, depends upon r/l, the ratio of the radius of gyration of the cross-sectional area in the direction of vibration (=0.288675$\times e$ or 0.288675$\times w$, depending on whether the vibration is flatwise or edgewise) to the length of the specimen. Pickett gives algebraic relations, graphs based on these relations, and also a table of selected numerical values from which T_1 can be determined as a function of Poisson's ratio for values of 0, 1/6, and 1/3. Subsequently, the following equation has been offered [8] from which T_1 can be evaluated for Poisson's ratio values other than those given by Pickett,

$$T_1 = T\left[\frac{1+(0.26\mu+3.22\mu^2)r/l}{1+0.1328\times r/l}\right] \tag{7}$$

where μ is the particular value of Poisson's ratio and T is Pickett's value of T_1 for μ=1/6 for the corresponding value of r/l.

The nature of the function relating T_1 to r/l is such that not only does T_1 increase as r/l increases, but also the values of T_1 diverge from each other more rapidly for different values of Poisson's ratio. Therefore, the accuracy of μ becomes more critical in the accurate determination of E_{fw} and E_{fe}, as r/l increases.

---

[4] The cgs system is used throughout as being a more desirable form in which to develop the equations and present the final data. However, for the convenience of those who are more familiar with the English system of units, the final data, in appendix II, are presented in the English as well as the metric system.
Kilobars may be converted to psi by means of the following equation,

Kilobars$\times$14,503.8=psi.

(This conversion factor assumes a value for the acceleration of gravity, g,= 980.1 cm/sec^2.)

The shear modulus, G, was calculated from the torsional resonant frequency F_t, by means of the following equation,

$$G=BmF_t^2 \tag{8}$$

where B is defined by the following relation

$$B=\frac{4lR}{a} \tag{9}$$

where a is the cross-sectional area and R is the ratio of the polar moment of inertia of this cross-sectional area to the "shape factor" [9] for the same cross section. The following approximation for rectangular cross section is given by Pickett [7] and based on Roark's Monograph [9],

$$R=\frac{e/w+w/e}{4(e/w)-2.52(e/w)^2+0.21(e/w)^6}. \tag{10}$$

A reexamination of the accuracy of the equation for R for the dynamic shear modulus calculations has been presented [10]. The revised relation for the specimen sizes used in this study, all with width-to-thickness ratios of about 2, is not significantly different from those calculated from the relation given.

Once Young's modulus and the shear modulus were determined, Poisson's ratio was calculated from the following equation,

$$\mu=\frac{E}{2G}-1. \tag{11}$$

For the 3-in. specimens, where E_l could not be determined, E_{fw} or E_{fe} (eq (5) or (6)) was used in eq (11). However, the importance of an accurate value of μ for the determination of E_{fw} or E_{fe} has already been mentioned. The procedure that was followed, then, was to assume a reasonable value of μ in calculating E_{fw} or E_{fe} from eq (5) or (6); then by successive approximation increasingly accurate values of E_{fw} or E_{fe} and μ were obtained. The process ceased when two successive calculations of E_{fw} or E_{fe} did not vary by more than about 2 in the fourth significant figure. Usually, no more than two recalculations were necessary.

The subscript in the symbol for Poisson's ratio (μ_l, μ_{fw}, or μ_{fe}) indicates whether the longitudinally or flexurally determined values of Young's modulus were used in eq (11).

For those specimens where F_l could not be determined, the speed of sound was calculated from the equation

$$V_c=\sqrt{\frac{E_{fw}}{\rho}}. \tag{12}$$

The bulk modulus, K, is obtained from the following equation

$$K=\frac{E}{3(1-2\mu)}. \tag{13}$$

As with Poisson's ratio, the symbols K_l or K_{fw} indicated whether E_l and μ_l or E_{fw} and μ_{fw} were used in the equation for the bulk modulus.

2.4. Density

a. Bulk Density

The bulk density was determined from the mass and volume which was calculated from the dimensions.

In order to determine the mass accurately, the specimens were cleaned with soap and water, then with trichloroethylene, and dried to constant weight either by heating at 800° C in an electrically heated muffle furnace or, if there was any doubt about the oxidation resistance of the materials at 800° C, they were vacuum-dried at an absolute pressure of about 1×10^{-5} mm of Hg. A few of the specimens were vacuum-dried in a desiccator at about 30 microns pressure.

After cleaning and drying, all of the specimens were stored in desiccators until they could be weighed on an analytical balance. The mass was corrected for the air buoyancy referred to a barometric pressure of 760 mm of Hg and 20° C by the equation

$$\text{Corr. mass}=[\text{mass in air}\times0.99986]+[\text{volume}\times0.0012]. \tag{14}$$

b. Theoretical Density

The theoretical density, P, was determined from X-ray diffraction examination, using the following relation

$$\mathrm{P}=\frac{NM}{VA} \tag{15}$$

where N is the number of molecules per unit cell, M is the molecular weight, V is the volume of the unit cell in angstrom units, and A is Avogadro's number (6.024×10^{23}) used in conjunction with the newly adopted angstrom length unit.

The molecular weights of solid solutions were calculated assuming that electrostatically neutral structures exist. No theoretical density computations were made in those instances where the type of solid solution was not known, where a number of solid solutions and compounds could exist, and where the reactions between these phases were not known, and when the structures of the material were of various low orders of symmetry and the angular values of the intersecting axes were not readily available. When it could reasonably be assumed that no reactions occurred between the two or more phases present in some of the test specimens, the theoretical densities were calculated as though the specimen were composed of a "mechanical" mixture of the component phases and that the densities of each were additive according to the amounts present.

2.5. Precision

The precision of the resonant frequency easurements has been given as about 1 part in)00 (2.3b). The speed of sound, when calcu-ed from the longitudinal frec ency, is the most ecise of all the constants given since it depends ly on the length, known to about 1 part in)00, and the longitudinal frequency. The ror in this determination is less than 0.1 percent. hen combined with the other factors involved its determination the precision of E_l is estimated be better than 0.4 percent. The precision of E_{fw}, E_{fe}, and V_c from eq (12) are estimated be about 0.4 percent. The precision of the)isson's ratio and bulk modulus values are, from e nature of their determination, reduced by factor of 10, from 0.4 percent to 4 percent. The precision of the bulk density measurements estimated to be about 0.2 percent.

2.6. Statistical Treatment

A detailed description of the statistical techques employed and their application to the rticular problems of this investigation is pre-nted in appendix I. The basic features of this atistical approach were as follows:

In most cases two or more groups, usually consting of 10 specimens each, of a single type of aterial, were supplied by the manufacturer. he different groups were either fabricated at ifferent times, using the same batch, or fabri-ated using batches prepared at different times. he first condition was designed to test the uni-)rmity of fabrication, whereas the latter condi-on tested the uniformity of batch preparation. s mentioned in the introduction, the elastic onstants and densities were used independently s indicators of the uniformity of production.

The specimens of a single group were desig-ated "acceptable" if the coefficient of variation as 1 percent or less. This rather arbitrary cri-erion of acceptability seemed reasonable on the basis of earlier testing [11, 12] and the data ported here. This standard of acceptability could favor some materials at the expense of others, and it should not be interpreted as a rigid criterion of the quality of any material or manufacturer's product.

The F-test was to determine whether or not two or more groups of specimens were significantly different in the degree of scatter. The t-test was used to determine whether or not the averages of two groups of data were alike. For both these tests, the 95 percent confidence level was used. The t-test was applied *only* if the F-test showed the two groups to have the same degree of scatter within the specified 95 percent confidence level. The following table illustrates how these two tests were employed to evaluate the parameters of fabrication and batch preparation for the two or more groups of specimens of each material.

Statistical test	Entire batch mixed at one time; specimens fabricated at different times.	Batches mixed at different times; specimens fabricated all at one time.
F	No significant difference means that the separate fabrication procedures result in products of comparable variability.	No significant difference means that both groups of batch materials were of comparable variability as were the mixing procedures.
t	No significant difference means that not only were the fabrication procedures of comparable variability but they were sufficiently uniform to permit production of products with the same properties.	No significant difference means that not only were the batches of comparable variability but that they were sufficiently uniform to permit fabrication of products with the same properties.

3. Results and Discussion

3.1. Presentation of Data

All tabular data are presented in appendix II. For illustration only, complete data and calcu-tions are given for one material, Code 4, Al_2O_3, n tables 2, 3, and 4.

Table 5 gives results of the X-ray diffraction xaminations. This table includes descriptions f the phases present, the unit cell parameters, nd the theoretical densities calculated from the iffraction data.

Table 6 presents a summary of the elastic roperties and densities for all the materials udied.

Tables 7 to 25 inclusive give the following data for each type of material separately: (a) the average value for each physical property; (b) the 95 percent confidence limits are given by the numbers following the average values; (c) the coefficient of variation $\left(=\frac{\text{standard deviation}}{\text{average}}\times 100\right)$; (d) the calculated and critical F-test number; and, (e) the calculated and critical t-test number when applicable.

The discussions of the 20 materials, generally, are given in the following order: (1) Description of each type of material, fabrication, heat treatment, and general comments on their appearance;

(2) acceptability with respect to the coefficients of variation of the bulk densities and elastic constants; (3) comments on the significance of the statistical comparison calculations; and, (4) other discussion of data, when appropriate.

3.2. Results for Different Materials

a. Aluminum Oxide—Al_2O_3 (Tables 7 and 7a)

Code 1: The six groups of hot-pressed fused Al_2O_3 were made from the same batch and heat-treated under the same conditions but at different times. These specimens were gray in color suggesting that they contained some carbon or graphite. However, one specimen of group I was heated for 30 minutes at 1,200° C in an oxidizing atmosphere with no appreciable loss in weight (0.01%) and no color change.

Based upon their measured bulk densities, all of the groups would be considered "acceptable." With the exceptions of the specimens of groups IV and V, the specimens would be "acceptable" according to the variations of the elastic constants. As a single group, however, these specimens would not be "acceptable" when based upon the coefficients of variation of either their bulk densities or the values of the elastic constants.

Code 27: This group of seven specimens is one of a series fabricated to produce a high density, polycrystalline material, by cold-pressing and sintering. Although a relatively high density was achieved with very small variability in the bulk density measurements, the specimens would not be "acceptable", on the basis of their elastic constants variations.

Code 26: Two additional groups of specimens were fabricated using the best method developed for the Code 27 specimens. Both of these groups would be "acceptable" according to their variations in bulk densities and group II would be "acceptable" with respect to the elastic constants variations. Comparisons of the properties of both groups show that the batches and the fabrication procedures that were established were satisfactory to provide specimens having reproducible physical properties.

Code 4: Two groups of 10 specimens each were prepared from a very high-purity Al_2O_3. The specimens were semitranslucent and appeared to be impervious. Each group was cold-pressed and sintered at the same temperature but at different times. Both groups would be "acceptable" on the basis of the coefficients of variation of the determined properties. Statistical comparisons of the properties of both groups indicate that this material and the particular fabrication process can be used to produce uniformly reproducible specimens.

Code 3: This group of five specimens had a composition and fabrication similar to those of Code 4.

The group would be "acceptable" on the basis of the variation of any physical property. No statistical comparison calculations were made for this and either of the groups of the Code 4 material because examination of the values of the physical properties shows that there is little, if any, difference.

Code 2: Two groups of 10 specimens per group were fabricated from the same material, but group I was heat-treated in a production kiln and group II was heat-treated in a laboratory kiln. Those heated in the production kiln attained a slightly higher temperature. Both groups would be "acceptable" on the basis of their bulk density variations; only the group I specimens could be considered almost "acceptable" with regard to their elastic constants variation. More important, when the various property values of both groups, with the exception of the values for Poisson's ratio, are compared statistically, the two groups are significantly different. In this instance, then, one could not predict the characteristics of a production product if that prediction is based upon the characteristics of a laboratory product.

Code 14: Twelve specimens of a high-purity Al_2O_3 were cold-pressed and sintered to produce a product that was said to be impervious to gases at elevated temperatures, however, the densities and elastic moduli given in table 7 were relatively low. These specimens would be "acceptable" on the basis of the coefficients of variation of any of the physical properties.

Code 15: These eight specimens were fabricated in the same manner as that used for the Code 26 and 27 specimens, but using a different supplier's materials. Neither high density nor high values for the elastic constants were obtained, and the specimens would be "acceptable" only on the basis of the bulk density variations.

Code 5: Eleven specimens of a very high-purity Al_2O_3 were cold-pressed and sintered in a "high-temperature" production kiln. A low-density product resulted that would be "acceptable" only on the basis of the small variation of the bulk density values

General Comments: Figure 1, based on the data in table 7, shows the relationship between the average bulk density, the average values of speed of sound, Young's modulus, and the shear modulus for the Al_2O_3 specimens. These results are in general agreement with those of Coble and Kingery [13] who found the elastic moduli of Al_2O_3 specimens to increase with decreasing porosity. When one considers that the data represent the products of three different fabricators and some five different batches, the regularity of the results is quite surprising. When the curves are extrapolated to the value of the theoretical X-ray density, the values of the elastic constants compare very favorably with those determined for the hot-pressed specimens that attained almost theoretical density.

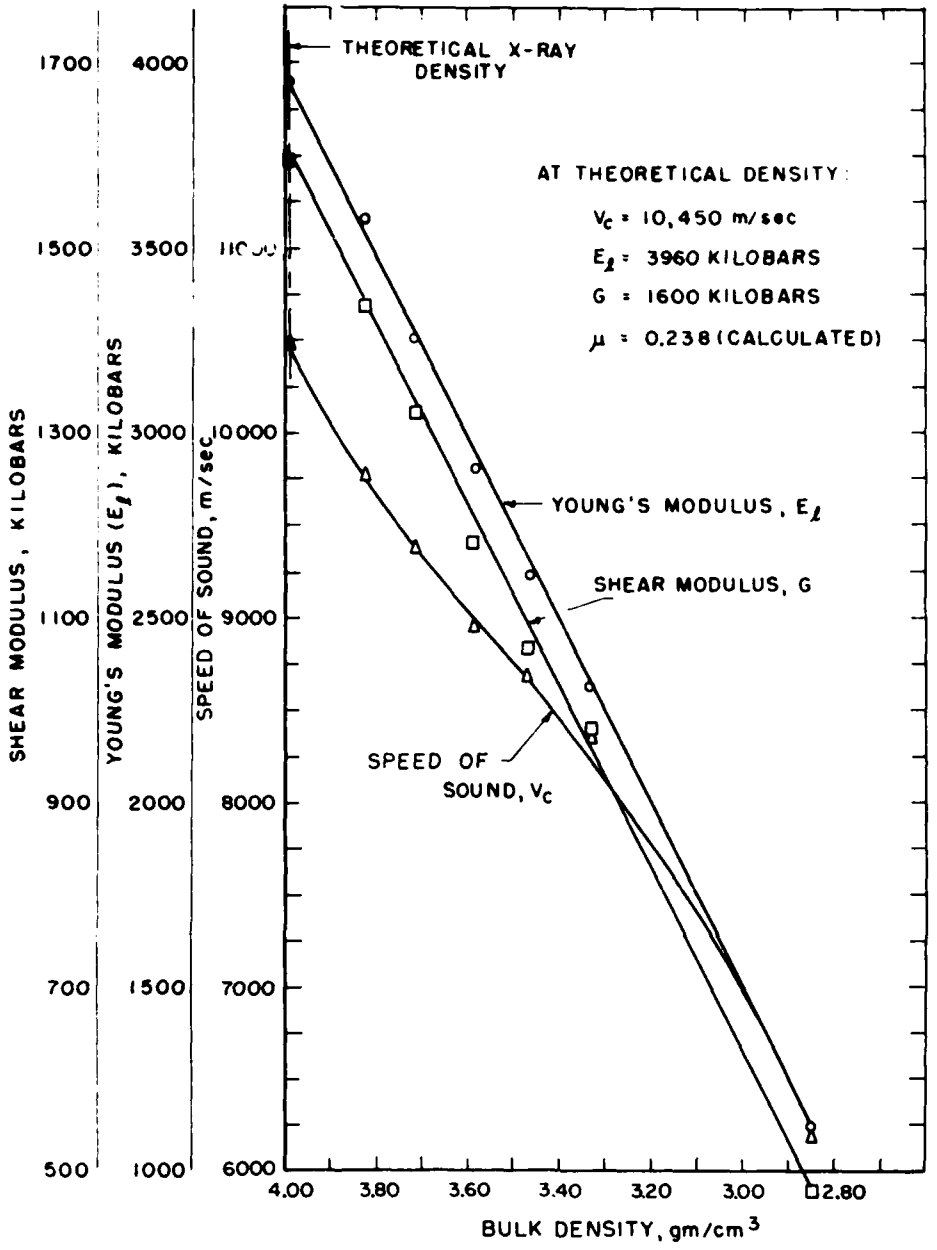


FIGURE 1. *Data for* Al_2O_3.

b. Ruby Alumina—Al_2O_3+Cr_2O_3 (Table 8)

Two groups of five specimens each were obined on loan. One group contained ½ weight ercent Cr_2O_3 (Code 6) and the other (Code 7) ntained 1½ weight percent Cr_2O_3 simulating one the compositions of natural ruby. The alumina ed in compounding these specimens is the same that used for the Code 5 specimens (sec. a). s received, the specimens were extremely porous the result of the very low-temperature treatent that they had received. Because of small e and density, none of the resonant longitudinal bration frequencies could be obtained. After e data given in table 8 were obtained, the specens were heat-treated at 1,800° C, reshaped and ain tested.

The results, which are given in table 8 as codes 6B and 7B showed that, when these specimens were heated at a sufficiently high temperature, they would be considered "acceptable" only on the basis of the variation of the bulk density values. Although a considerable decrease was noted in the coefficients of variation of the elastic constants for the group containing the ½ weight percent Cr_2O_3 "impurity" when they were reheated, no appreciable changes occurred in the coefficients of variation for the group containing 1½ weight percent Cr_2O_3. It was more interesting, however, that additional heat treatment caused only small changes in the calculated values of Poisson's ratio. The change in value for the Code 6 specimens is the reverse of the bulk density-Poisson's ratio

trend shown for practically all of the other data available for other materials.

c. Magnesium Oxide—MgO (Tables 9 and 9a)

Code 24: Two groups of specimens were fabricated from the same batch but heat-treated at different times to produce high-purity, high-density test specimens by cold-pressing and sintering. Both groups would be "acceptable" because of the small variation of the bulk density values, but only group II would be "acceptable" on the basis of variations in the elastic constants. Statistical comparison of the property values (*t*-test) of the two groups indicates that this material and fabrication procedure may be considered satisfactory for producing, from time to time, specimens of about the same characteristics.

Code 25: These two groups of specimens were fabricated similarly to the Code 24 specimens. Some slight differences occurred during the preparation.

Either group would be "acceptable" on the basis of the variation of their bulk densities. Statistical analyses of both groups showed that, although no really significant difference in the variability of the material was introduced because of the slight procedural change, as indicated by the *F*-test results, the change did affect the average values significantly, as indicated by the *t*-test results.

Code 23: These two groups were the first trials at producing a high-purity, high-density product. Each was made from a different source of magnesium carbonate. Each would be considered "acceptable" only on the basis of their bulk density variations.

Code 9: Two groups of 10 specimens each were fabricated from a fused MgO material by cold-pressing and sintering. This material has a purity of +95 percent. The test specimens were extremely porous, somewhat friable, and tan in color, speckled with light-brown areas. Both groups were prepared from the same material and both were heat-treated simultaneously, but in different parts of the same kiln.

The only basis for "acceptability" of both groups would be the low coefficients of variation of the bulk density values. Comparison of data obtained for each of the physical properties of both groups shows that this material, treatment, and location in the particular kiln were such that with two exceptions (μ_{fw} and K_t), there was no really significant difference according to the *F*- and *t*-tests.

d. Mullite—$3Al_2O_3 \cdot 2SiO_2$ (Tables 10 and 10a)

Code 16: Five groups of hot-pressed mullite, totaling 20 specimens, were made from the same material but heat-treated at different times, presumably under the same pressure-temperature conditions. Although the purities of the starting materials were about the same in Codes 16, 17, and 18, those of Code 16 were contaminated with graphite from the arc-fusion process and the graphite mold of the hot-pressing apparatus. In addition, X-ray diffraction examination of one specimen showed that it contained mullite and at least 10 percent of free Al_2O_3.

With the exception of group IV, all would be "acceptable" on the basis of their bulk density variations; however, the entire lot as a group would be "unacceptable" on the same basis. Only group II, containing but 2 specimens, would be "acceptable", according to the coefficients of variation of the elastic properties. There appears to be some difference among most of the groups listed. Because all specimens were made from the same material, it would seem that variations in the control and operation of the hot-pressing facility were sufficient to cause significant changes in the product in three of the five groups processed.

Codes 17 and 18: Two groups of ten cold-pressed and sintered specimens each were obtained separately. The Code 17 specimens were subjected to a "short burn", while the Code 18 specimens sustained a "long burn", both at the same temperature. X-ray diffraction examination of one specimen of each group indicated that both were single-phase materials.

Both groups would be considered "acceptable" on the basis of their bulk density variations and the Code 18 specimens would be "acceptable" with respect to the elastic constants variation. Comparison of the physical properties of both groups by the *t*-test shows that the heat treatment significantly affected the characteristics of the test specimens. It is evident that the longer heating period, although it did not materially affect the bulk density, or elastic modulus values, did result in a more uniform product.

e. Mullite + ZrO_2 (Table 11)

Code 22: One group of 10 cold-pressed and sintered specimens were obtained that were compounded from a mixture of mullite (the same as that used for the Code 17 and 18 mullite specimens) and zircon. These were fabricated and heat-treated in a manner similar to that used for the Code 17 and 18 mullite specimens. X-ray diffraction examination of one of the test specimens showed no zircon present and also that the specimen contained mullite and about 20 percent of monoclinic ZrO_2. No hypothesis is advanced for the loss of silica from the zircon, but the SiO_2 formed from the decomposition is assumed to have entered the mullite phase [14].

This group could be considered "acceptable" both on the basis of the coefficients of variation for bulk density and elastic constants. If it can be assumed that these specimens and those of Code 17 received the same fabrication and heat treatments, then it appears that the addition of zircon is almost as beneficial as a "long burn" in producing more uniform property characteristics, but that the densities and elastic moduli are reduced.

f. Spinel—$MgO \cdot Al_2O_3$ (Tables 12 and 12a)

Code 20: Five groups of hot-pressed spinel, totaling 20 specimens, were fabricated from the same batch composition but heat-treated at different times. The specimens appeared to be coarsely crystalline, grains of ⅛ in. diam being visible. The specimens were freely speckled with black areas. These areas are believed to be contaminations from both the arc-fusion process and the hot-pressing operation. X-ray diffraction examination of one specimen showed that it contained magnesia spinel and less than 5 percent of free MgO.

All of the groups, individually or as a lot, were "acceptable" according to their bulk density variations, but "unacceptable" from the standpoint of variation of the elastic constants. When the properties of each of the groups are compared with those of all of the specimens, a surprising consistency is noted. In fact, this spinel is one of the few hot-pressed materials that gave a fairly consistent product.

Code 21: Two groups of 8 and 10 specimens each were made from the same material as was used for the Code 20 specimens, except that they were fabricated by cold-pressing and sintering. Each group was heat-treated at different times. The test specimens were snow-white in color, very porous, somewhat friable, and rather weak. Two of group I were broken in handling. Unlike those of Code 20, the specimens were fine-grained.

The group II specimens could be considered "acceptable" on the basis of the bulk densities but not on the basis of the elastic properties. The group I specimens were not "acceptable" on any basis. When the two groups were compared with each other by the *F*- and *t*-test, it can be seen that the differences that occurred from processing at different times caused a significant difference in all properties, except μ_1.

g. Thorium Dioxide—ThO_2 (Tables 13 and 13a)

Both types of thoria described in this report, Codes 10 and 51, contain ½ weight percent of CaO (usually added as $CaCO_3$) for densification.

The thoria used in the preparation of Code 10 specimens was electrically-fused, while that used for the Code 51 specimens was a very pure (99.9+%), low-temperature calcined material.

Code 10: Two groups of 10 specimens each were cold-pressed from the same mixture and simultaneously sintered at the same furnace temperature, but in different parts of the furnace. The specimens were a light-brown with a pink cast.

Statistically, both groups would be considered "acceptable", although some of the calculated elastic constants had variations that exceeded the acceptability limits. The *F*-test showed significant differences for bulk density and bulk modulus; the only property that showed a significant difference by the *t*-test was the shear modulus. This uniformity indicates that the position in the furnace did not seriously affect the characteristics of the products.

Code 51: This group of 10 specimens were fabricated from a mixture of very pure, low-temperature calcined ThO_2 and $CaCO_3$ by cold-pressing and sintering. The specimens were off-white in color. The statistical treatment showed that the group would be "acceptable" on the basis of both the bulk densities and the elastic constants.

h. Uranium Dioxide—UO_2 (Table 14)

Code 19: The five specimens of this group, which were prepared by cold-pressing followed by sintering in a hydrogen atmosphere, had a bulk density about 95 percent of theoretical. The uranium oxide used in fabricating the specimens had 2.05 moles rather than 2.00 moles of oxygen. This ratio changed during fabrication to 2.02.

The group would be "acceptable" on the basis of both the bulk density and on all elastic constants, except K_1 and K_{fw}.

Code 19a: One test specimen was fabricated by cold-pressing and sintering an "ammonia-precipitated" UO_2 material. The measured values are included in table 14.

i. Stabilized Zirconia—ZrO_2+CaO (Tables 15 and 15a)

All of the materials described in this section contain about 5 weight percent of CaO. When such mixtures are heated they form cubic solid solutions which are free from the discontinuous volume changes associated with the monoclinic-tetragonal inversions that occur between 800° C and 1,200° C in pure ZrO_2 [15, 16].

Code 11: Four groups of hot-pressed stabilized zirconia, totaling 18 specimens, were made from the same mixture but heat-treated at different times.

All groups would be "acceptable" on the basis of their coefficients of variation for the bulk density, but none would be "acceptable" according to the variations of the elastic constants. A comparison between groups seems unjustified because of the extremely large variations (about 30%) of all of the specimens, considered as a group.

Anticipating data to be presented later in this section, inspection of the data in table 15 shows that, although these hot-pressed specimens attained a very high bulk density, the values of the elastic constants were as low as (and in some instances lower than) the values determined for the cold-pressed and sintered specimens. It is believed that this anomaly is due to internal laminations and cracks in the specimens. Such faults in these test specimens could sometimes be shown to exist, although their full extent could not be readily evaluated, by judicious "probing" during the resonant frequency determinations. At times, the direction and magnitude of the flaw can be approximated, but it does not appear feasible to attempt a quantitative evaluation of the effects. Therefore, all of these, and the later, data given for stabilized

zirconia are suspect. Although there was a small spread in the bulk density measurements, apparently indicating production uniformity, the very large spread of the elastic constants indicated that actually this was not the case. This is an excellent example of the value of the dynamic measurements in determining the variability of specimens.

Code 13: Two groups of 10 specimens, which were fabricated by cold-pressing and sintering, were made from the same material and heat-treated simultaneously at the same furnace temperature, but the groups were located in different parts of the furnace.

Either group would be "acceptable" only with respect to the low coefficients of variation for the bulk densities; they would not be "acceptable" with respect to the elastic constants. With the exception of the bulk density values, statistical comparison of the elastic constants of both groups showed that the location in the furnace (assuming all else equivalent) significantly affected the characteristics of the products.

Code 12: These two groups of 10 specimens each were submitted by the fabricator with the comment that "considerable fabrication difficulty was experienced with longitudinal seams and transverse cracks." Both groups were made by cold-pressing and sintering. They were prepared from the same mixture and heat-treated at the same temperature, but at different times, in a laboratory furnace.

These two groups showed the lowest bulk densities of any of the stabilized zirconias. On the other hand, the elastic constants are not only the highest but also the most uniform. For example, the group II specimens would be considered "acceptable" on the basis of most of the physical properties. Statistical comparison of the elastic properties of the two groups indicates that the fabricator supplied this material with uniform characteristics in spite of his fabrication difficulties.

j. Alumina+Chromium—Al_2O_3+Cr (Tables 16 and 16a)

Code 29: The group numbers of the 19 specimens of this cermet were considered as one group. Another two groups of the same material are described in the next section. With the exception of the low variation of the bulk density values, the Code 29 specimens as a single group would not be considered "acceptable".

Code 30: These two groups of 10 specimens each were cold-pressed and sintered. They have the same composition as the Code 29 specimens. The two groups were made from the same mixture but heat-treated at different times.

Both groups could be considered "acceptable" on the basis of bulk density or the elastic constants, with the possible classification of group I as a borderline case when considering the elastic properties. Again, because of the borderline nature of one of the groups, it is difficult to say, statistically, that the mixture can or cannot be fabricated with uniform characteristics from time to time.

Code 28: Two groups of cold-pressed and sintered Al_2O_3+Cr cermet, 10 specimens per group, were prepared from the same mixture but heat-treated at different times. The composition is similar to, but not identical with, the Code 30 specimens. As was the case for all of the Al_2O_3+Cr mixtures, the X-ray diffraction examinations showed only a single chromium-metal phase. A possible explanation for the absence of Al_2O_3 reflections is that the chromium-metal became "smeared" over the surface during polishing, thus masking the alumina phase.

Only the group I specimens could be considered "acceptable" on the basis of their low coefficients of variation for the bulk densities and elastic constants. When the physical properties of each group are compared statistically, it is apparent that variations in heat-treatment caused a significant change in all of the properties with the possible exception of the values for Poisson's ratio.

k. Ni-bonded Titanium Carbide—TiC+Ni (Tables 17 and 17a)

Four TiC+Ni mixtures containing 10 to 30 weight percent of Ni were fabricated by cold-pressing and sintering. The TiC had about 6 weight percent of tantalum and niobium carbides in solid solution. Each of the four codes was prepared and heat-treated separately under identical conditions. The nickel content of these mixtures are:

Code 31—about 10 wt % Ni
Code 32—about 20 wt % Ni
Code 33—about 30 wt % Ni
Code 34—about 30 wt % Ni (a modified Code 33 composition)

Code 31: Both groups of specimens would be considered "acceptable" on the basis of the coefficients of variation for both the bulk densities and elastic constants, although group I might be borderline. Comparison of the physical properties by the *t*-test of the two groups shows that the mixing and general fabrication controls are such that materials having about the same variability can be produced at different times; also, the *t*-test shows that this cermet can be reproduced with substantially the same physical properties.

Code 32: Both groups of specimens would be "acceptable" except for Poisson's ratio and bulk modulus. Comparison of the physical properties of the two groups shows that, although the mixing and general fabrication controls were such that a material with the same scatter in values can be reproduced from time to time, the materials were not consistent in their physical properties.

Codes 33 and 34: The same statistical comments that were given for the Code 32 material are applicable to these materials.

General Comments: Figure 2 shows the extent of the variation of the values of the elastic constants and the bulk density with the nominal nickel content. The average values of the two groups of each mixture for codes 31, 32, and 33 were averaged to provide the data for these curves.

l. Boron Carbide—B_4C (Table 18)

Two groups of 10 hot-pressed specimens (Code 44) were made from the same batch but each was prepared and heat-treated at a different time. For the reasons stated in section 2.3(a), torsional frequencies were determined for only one specimen of a group; and consequently, no statistical comparisons were made.

m. Boron Carbide+Titanium Diboride—B_4C+TiB_2 (Table 19)

Code 43: These four groups of five specimens were hot-pressed from the same mixture of 82 parts (volume) of B_4C and 18 parts of TiB_2, but each group was heat-treated at a different time. The same comments that were given for specimen size, reshaping, and calculation method for boron carbide (sec. 2.3(a)) are applicable to these specimens. The addition of titanium diboride increased the bulk density but did not significantly affect the values of the elastic constants.

n. Silicon Carbide—SiC (Tables 20 and 20a)

Code 45: "High-purity" materials, probably less than 3 percent of uncombined silicon or carbon excess, were used for both Codes 45 and 35. Two groups of cold-pressed and sintered specimens, 9 and 8 specimens, respectively, were prepared from the same material but heat-treated at different times. The resonant longitudinal vibration frequencies could not be determined with the available equipment on the 15-cm long specimens because of the high values of the speed of sound. Therefore, the elastic moduli were calculated only from the flexural mode of vibration.

Both groups would be "acceptable" on the basis of their variation in bulk density values, and group

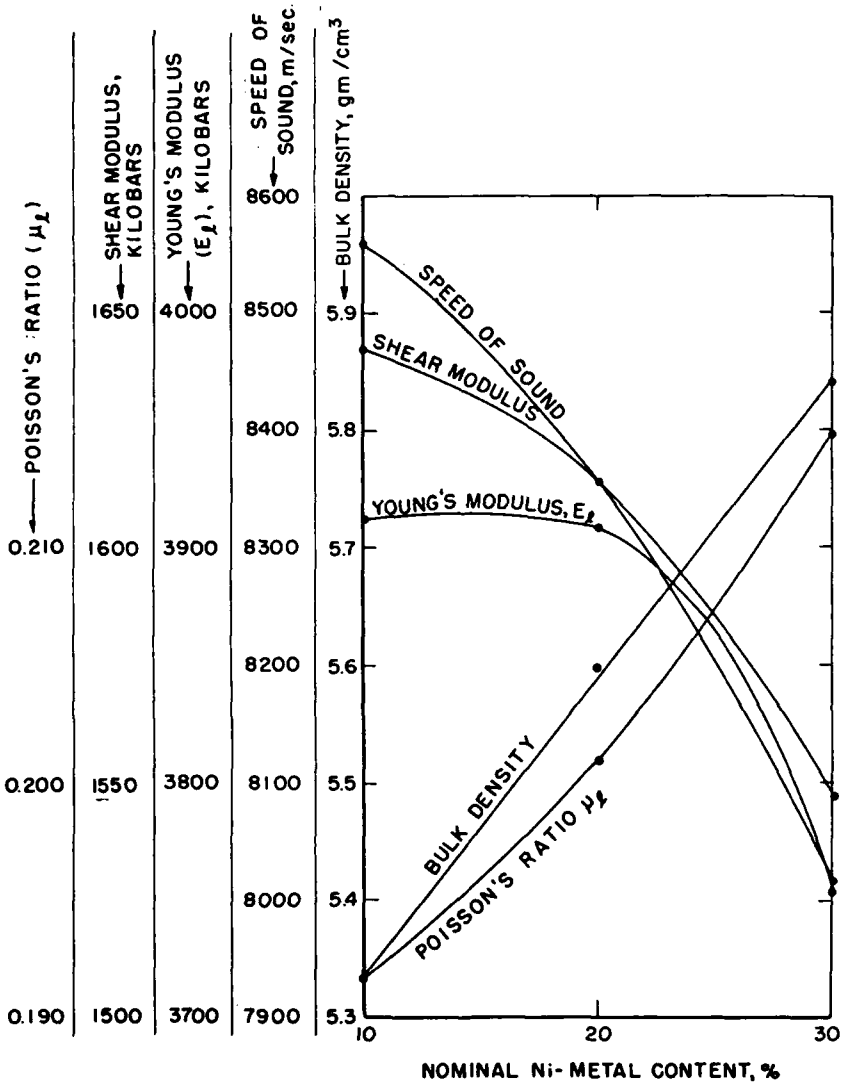


FIGURE 2. *Compositional dependence of the elastic constants and bulk density for* TiC+Ni *(codes 31 through 34).*

II would be "acceptable" with respect to the variability in elastic constants. When the physical properties of the two groups are considered, it is apparent that the two groups, except for E_{fc}, are significantly different.

Code 35: Two cold-pressed and sintered groups, were prepared from the same material but heat-treated at different times. X-ray diffraction examination on one specimen revealed that it was composed of a mixture of cubic crystals with several hexagonal SiC phases. It has been shown [17] that there are some 15 or more hexagonal polytypes of SiC, all or any of which may coexist with or without the cubic form.

On the basis of the coefficients of variation for their bulk densities only, both groups could be considered "acceptable". When the other property values of the two groups are compared by by the *F*- and *t*-test, it appears that the materials and fabrication methods can, in general, provide reproducible products, but with rather poor uniformity of physical property characteristics; the uniformity of bulk density appears to be the exception.

o. Silicon Carbide+Boron Carbide—SiC+B_4C (Table 21)

Code 37: Each of the 4 groups of 5 specimens of this material (designated as "boron carbide bonded silicon carbide") were made from the same mixture but hot-pressed at different times. The SiC was the same as that used for the Code 35 specimens; the B_4C was of "commercial" grade. The same comments given for specimen size, reshaping, and method of calculation for boron carbide (sec. 2.3(a)) apply here. However, it will be noted that the addition of the 10 parts of B_4C very substantially increased the bulk density and elastic moduli values.

p. Zirconium Carbide—ZrC (Tables 22 and 22a)

Code 38: Six groups of hot-pressed zirconium carbide, totaling 20 specimens, were made from the same material but pressed and heat-treated at different times. The material was of commercial grade. X-ray diffraction examination of one specimen showed it to be essentially a single phase material (ZrC) but that it contained a very small amount of free graphite.

Groups III, IV, V, and VI would be "acceptable" on the basis of the low variation for the bulk density values; however, the entire lot would not be "acceptable" according to the coefficients of variation of the elastic constants. Assuming all else equal, the results in table 22 show that considerable variations in heat-treatment must have occurred during the fabrication of these test specimens. As was the case with many other materials, there was very little change in Poisson's ratio with heat-treatment.

q. Zirconium Diboride—ZrB_2 (Tables 23 and 23a)

Code 41: Four groups of five hot-pressed specimens were made from the same material but heat-treated at different times.

With the exception of the specimens of group III, all other groups would be considered "acceptable" because of their low coefficients of variation for both the bulk density and elastic constants values. When the values of each of the groups are compared with the values for the entire code, it is apparent that the values for the physical properties were not reproduced from time to time.

Code 42: Two groups of 10 hot-pressed specimens, whose composition is a modification of Code 41, were fabricated at different times.

Comparison of the values of both groups shows that nonuniform specimens can be fabricated with reproducible properties, although neither of the groups would be "acceptable" under the criteria established.

r. Molybdenum Disilicide—$MoSi_2$ (Tables 24 and 24a)

Code 39: Six groups of hot-pressed specimens, 19 in all, were prepared from the same material designated as of "high purity" but heat-treated at different times.

All of the groups, with the exceptions of II and III, would be "acceptable" on the basis of the low variation of their bulk densities, although the entire code would not be "acceptable"; groups IV and VI would be "acceptable" according to the coefficients of variation for the elastic constants.

s. Nickel Aluminide—NiAl (Tables 25 and 25a)

Code 40: Two groups of cold-pressed specimens were fabricated from the same mixture but heat-treated at different times. The mixture was a proprietary modification of the basic composition.

Only the specimens of group I would be "acceptable" and then only on the basis of the bulk densities. When all of the physical properties of the two groups are compared, it should be noted that the indicated similarity has little meaning because of the large variability of the physical properties.

3.3. Discussion

One of the interesting observations was that Young's moduli obtained from the longitudinal mode of vibration were 1 to 2 percent less in most cases than those obtained from the flexural modes.

Similar measurements made with glasses [3] and other homogeneous isotropic materials [18] have shown good agreement for Young's moduli calculated from the flexural frequencies with Young's moduli calculated from the longitudinal frequencies using the same equations for both calculations as in this investigation. The observed lack of agreement for many of the materials studied here may be attributed to one or more of the following causes arising from some part of the fabrication process: (1) Variations in density in the specimen; (2) The presence of cracks within the specimen. If these cracks are not randomly distributed, their

gross effect will be that of a structural inhomogeneity; (3) The segregation of grain sizes during fabrication [19]; and, (4) The lack of complete randomness in the orientation of the crystalline particles composing the specimen.

If the particles forming the structure assume some preferred orientation, then macroscopically the specimen will not be completely isotropic. Roth [20] has definitely found evidence of such crystalline orientation in small pressed pellets of ceramic materials from X-ray diffraction examination. The presence of some orientation is shown by a different intensity of certain lines than would be the case for a completely random orientation. Although no such clear-cut evidence was found for the specimens studied here, the possibility is not ruled out that such small preferred orientations were present.

From the few isolated instances where information was available for laboratory and production specimens, less variation occurred when the same material was fabricated as a production item than when it was fabricated as a laboratory item. A possible explanation is that, in each of these instances, a higher heat-treatment temperature was used for the production product. There were also some data available for materials heat-treated in the same furnace at the same temperature but for different time intervals. In these, appreciably smaller variations in density and elastic constants occurred for those specimens heat-treated for a longer period of time. It would seem, therefore, that, although production economies would dictate a minimum time at minimum heat-treating temperatures, a more uniform and reproducible product would result from increasing both firing time and temperature. This improvement in the product appears to be readily achievable at a slightly increased production cost. The value of a statistical analysis for selecting the optimum temperature and time should be apparent.

It is anticipated that significant advances will be achieved in the uniformity of commercially available materials only recently developed, such as hot-pressed stabilized zirconia. As more general experience in the hot-pressing techniques is accumulated, one can reasonably expect to realize the advantages of both higher density and of optimum uniformity of the products.

The dynamic elastic constants, as measured here, appear to be more sensitive indicators of product uniformity in quality control than are bulk density measurements. However, in the case of cold-pressed and sintered specimens in which the bulk densities are within a few percent of the theoretical value, for example, thorium and uranium dioxide, Codes 10 and 19, bulk density appears to be as sensitive a criterion as the dynamic elastic constants.

The reported information for the various types of "stabilized" zirconia (sec. 3.2.9) gives one of the very few examples available of the relative values of bulk density and dynamic elastic constants as production-control measurements for materials that are difficult to fabricate. The bulk density values of each of the four groups of hot-pressed specimens (Code 11) show little variation. The coefficient of variation of the bulk density of the entire code of 18 specimens is only ½ percent, yet the Young's modulus values for the same specimens vary from 1,100 to 2,050 kilobars, and the coefficient of variation is on the order of 30 percent. Although the calculations are not shown in table 15, the two groups of Code 13, when considered as a whole, show a coefficient of variation of less than ½ percent in bulk densities. On the other hand, the average E_{fw} value for the 20 specimens as a whole was 1,380 kilobars with a coefficient of variation of only 8.3 percent. In the set of specimens in which difficulties were encountered from laminations and fissures (Code 12), the coefficients of variation of both the bulk density and elastic modulus values are low, although the average value of Young's modulus was 1,483 kilobars compared to 1,380 for the average of Code 13. Although it might appear facetious, it seems possible that the Code 12 specimens were fabricated with uniform imperfections.

Inasmuch as the precision of the elastic modulus measurements was estimated to be about 0.4 percent whereas that of the density measurements was estimated as about 0.2 percent, it is pertinent to inquire to what extent the coefficients of variation of these two properties were affected by the precision of the measurements themselves. Or, stating the problem in another way, it is necessary to ascertain whether the greater variability found for the elastic modulus measurements represents a real variation from specimen to specimen, thus supporting the claim of greater sensitivity for this method as an indicator of specimen uniformity, or whether this increased variability was not merely a reflection of the lower precision of the dynamic measurements.

The equation relating the contribution to the total variability, expressed here as σ_T, in a property measurement, from that due to the variability in the measurement itself, σ_m and that due to the variation from specimen to specimen, σ_s, is as follows,

$$\sigma_T = \sqrt{\sigma_m^2 + \sigma_s^2} \tag{16}$$

or, in terms of the coefficient of variation, V,

$$V_T = \sqrt{V_m^2 + V_s^2}. \tag{17}$$

In this investigation, $V_T = 1$ percent was chosen as the criterion of acceptability, $V_m = 0.2$ percent for density measurements, and $V_m = 0.4$ percent for elastic modulus measurements.[5]

[5] Actually, V_m for elastic and density measurements was better than the values given. In a normal distribution, the coefficient of variation includes about 34 percent of the cases, whereas the estimated measures of precision would probably include more than 80 percent of the cases. The results, then, make the contribution of the precision of the measurements to V_T even smaller than shown in the text.

Therefore, for the elastic determinations

$$1=\sqrt{(0.4)^2+V_s^2}$$

and $$V_s=0.92 \text{ percent}$$

for density measurements,

$$1=\sqrt{(0.2)^2+V_s^2}$$

and

$$V_s=0.98 \text{ percent.}$$

Thus, it is seen that for a coefficient of variation of 1 percent, the difference in precision of the elastic modulus and density measurements is a negligible factor since in both instances, by far the greater contribution to the coefficient of variation, V_r, stems from a real difference from specimen to specimen. Furthermore, as the coefficient of variation increases, the contribution from the precision of the measurement grows increasingly smaller. Conversely, the contribution from the precision of the measurements grows larger for coefficients of variation less than 1 percent. However, this is of no practical importance since all coefficients of variation of 1 percent or less were considered "acceptable" without regard to degree. However, for those elastic constants which were more indirectly computed, such as μ and K, and hence of decreased precision, the 1 percent coefficient of variation as a criterion becomes much less significant as, in these cases, the contribution of the precision of the measurement becomes more important and may overshadow any real specimen variability.

The problem may also be approached in another way, as follows: If a 1 percent coefficient of variation is set as a criterion of acceptability for density measurements and the precision of this measurement is 0.2 percent and the contribution of V_s is found to be 0.98 percent then what would be an equivalent coefficient of variation to set for elastic modulus measurements? These have a precision of 0.4 percent and V_s is also 0.98 percent. The coefficient of variation under these conditions is obtained from

$$V_r=\sqrt{(0.4)^2+(0.98)^2}=1.05\%.$$

This value is seen not to be significantly different from the coefficient of variation actually set and groups of specimens having such a coefficient of variation would indeed be classed as borderline cases of acceptability.

In addition to its use as an indicator for determining the uniformity of production for particular groups of specimens, it is interesting to estimate the variability of the elastic moduli of all the materials of this investigation, taken as a whole; and to compare this variability with that of a different material, steel, which is generally considered to be much more uniform with respect to this same property.

Taking E_l whenever available, and E_{fw} otherwise, the following summary was prepared for all the materials studied;

Range in coefficient of variation	Percentage of specimens
0 to 1%	44
0 to 2%	62
0 to 2.5%	72
0 to 3.0%	81

Analagous data for steel is not readily found. However, one study [21] gives the following results for specimens of "black sheet steel": V, for specimens cut in direction of rolling=2.4 percent. Average of E=1,962 kilobars; V, for specimens cut transverse to direction of rolling=2.6 percent, average of E=2,053 kilobars; V, for specimens taken as a single group, 3.3 percent, average value of E=2,006 kilobars. Thus, it is seen that, if either value of V for steel is taken for comparison, about 70 percent of all the specimens of this investigation would be included, and, if the coefficient of variation of all the steel specimens without regard to direction of rolling had been taken for comparison (which seems more reasonable), then more than 80 percent of all the materials of this investigation would be included. These data [21] were for only one type of steel and may not be representative. But, if they are at all indicative, then the materials of this investigation, as a whole, compare favorably with a material which has traditionally been regarded as quite uniform.

It is believed that the present study shows that a relatively simple statistical approach can be valuable not only to the designers and engineers, but also to the fabricators and suppliers. One of the main deterrents to the use of ceramic and cermet materials in many applications where they seem to be potentially useful is the lack of knowledge of the physical properties and constants of these materials, and, when such information is available, the lack of confidence in the uniformity or reliability of the reported values. A great deal of information on product uniformity could be provided by the fabricators and suppliers, usually without additional expense, by statistically analyzing the data that are already available to the manufacturer.

The author thanks Jack Shartsis for performing most of the computations, Nancy Tighe for performing some of the later experiments and calculations, and Robert S. Roth for performing the X-ray analysis.

Appendix I. Statistical Treatment of Data

The following description is not intended as a basic exposition of the statistical concepts involved but rather as an aid to those who might desire to perform the same (or similar) calculations and wish to have some understanding of the significance of the results. All of the computations used in this report are discussed and described in detail by Youden [22] and Dixon and Massey [23].

Let "x" represent a value of any property in one group of specimens, "y" represent a value for the same property of another group, "$\bar{x}$" represent the average value of any property of the first group, "$\bar{y}$" represent the average value of the same property of the second group, "n" represent the number of values determined for any property in the first group, and "m" represent the number of values determined for the same property of the second group; then statistical calculations (based upon the assumption that the sets of data of all of the determined properties of the materials follow follow the normal distribution law) are performed according to formulas (A), (B), (C), and (D) which apply to the first group. Similar formulas with x replaced by y apply to the second group. Parts (E) and (F) apply to both groups.

(A) Standard deviation of an individual determination $= S = \sqrt{\text{variance}}$

or

$$= \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}}.$$

$$= \sqrt{\frac{(x_1^2 + x_2^2 + \ldots + x_n^2) - (x_1 + x_2 + \ldots + x_n)^2/n}{n-1}}$$

(B) Standard deviation of the average of "n" individual values $= S' = \frac{S}{\sqrt{n}}$

(C) Coefficient of variation V, in $\% = \frac{S}{\bar{x}}(100)$

(D) Ninety-five percent confidence limits (C. L.) for the average are given by

$$\bar{x} \pm S't \text{ in } \% = \frac{S}{\sqrt{n}}.$$

Where t is the upper 2.5 percent point of the t distribution for $n-1$ degrees of freedom. If such limits are calculated for many sets of data, there will be approximately 95 percent of the sets for which the limits enclose the true average.

(E) The F-test, essentially, is a method for comparing the "precision" of two sets of data for the same property. It provides one with a criterion for determining at a selected confidence level whether a significant difference exists between the scatter of two (or more) sets of data; that is, whether the groups (or materials, etc.) used to obtain that data are of a different degree of variability. In this study, the 95 percent confidence level was used. The F-number that is calculated from the expression,

$$F = \frac{S_a^2}{S_b^2}$$

where S_a^2 is always the larger number of the two being examined, is compared to tabulated critical F-values. The tabulated critical F-value selected, is in this situation that for the upper 2.5 percent point of F. If the calculated value is higher than the critical value, a significant difference does exist.

(F) The t-test, essentially, is a method for comparing the "accuracy" of two sets of data for the same property. The t-test should be applied to the data of two groups only when the F-test has shown that these data sets are of comparable variability. The t-test provides one with a criterion for determining whether a significant difference exists at a selected confidence level between two averages, $\bar{x}$ and $\bar{y}$, on the basis of the spread of the individual values, S_x and S_y, used to compute those averages. This test, however, does not allow one to determine the accuracy of either $\bar{x}$ or $\bar{y}$ unless, of course, the true value is known.

As is the case with the F-test, the t-value that is calculated is compared to tabulated critical t-values and, if the calculated value is higher than the critical value, a significant difference does exist (see the similar discussion of significance under subsection (E), preceding). The calculated value is obtained from the expression

$$t = \frac{\bar{x} - \bar{y}}{S_p} \sqrt{\frac{n \times m}{n+m}},$$

with $n+m-2$ degrees of freedom, where S_p is the pooled standard deviation of the individual values of both groups, S_x and S_y, and is obtained from

$$S_p = \sqrt{\frac{[\sum x_i^2 - (\sum x_i)^2/n] + [\sum y_i^2 - (\sum y_i)^2/m]}{(n-1)+(m-1)}}.$$

During the early part of this study, it was a part of the computation procedure to perform the F and t-tests to compare the values of E_t, E_{fw}, and E_{fs}, in pairs, and those of μ_t and μ_{fw}. Very rarely were significant differences found to exist and, therefore, the time-consuming calculations were neglected for at least the latter half of the data obtained. However, occasional check calculations were made, but the results of such comparisons within a group are not included in this compilation of data.

The preceding relations could be applied to the data obtained in this investigation in several ways. The approach selected is given as follows. Each of the suppliers of materials for the "bank" was asked to prepare his specimens in either of two ways: (1) Compound the same starting composition at two different times but fabricate each group identically; or, (2) compound one large quantity of the starting composition and fabricate each group separately. All fabrications were to achieve the maximum practical bulk density and it was to be uniform throughout the specimens.

The calculations of the standard deviation of an individual measurement of each group and the coefficient of variation for all of the properties permits one to use the reliability of the measurements of any property as a criterion for determining the product uniformity or acceptability. That acceptability may be based upon an arbitrary or specified standard variation. If average values were determined repeatedly and if in each instance 95 percent confidence limits were calculated for the average value, then in the long run we would expect that 95 percent of the confidence limits would include the true average value.

Application of the *F*- and *t*-tests to the data obtained from specimens prepared under the first condition allows one to state that there was or was not a significant difference in the compounding procedures of the same nominal composition at different times. In a similar way the *F*- and *t*-tests can show whether or not the fabrication differences (such as heating at different times, or in different types of furnaces, or at different temperatures in the same furnace) had a significant effect on either the scatter of the values obtained or the average of the property values determined.

This series of statistical computations has another important value. Some of the suppliers of the specimens for this "bank" took the opportunity to fabricate specimens, from the same batch composition, both in the laboratory and on the production line. Here, then, was an excellent opportunity to evaluate the results of a laboratory or pilot-plant experiment and a production run. It seemed, therefore, that the statistical methods described should be of considerable interest for many types of laboratory and production-control evaluations.

When it was shown that a significant difference existed for the *F*-test, it was concluded that the materials were not of comparable variability due to a variation either during the compounding or the fabrication stages depending upon the production conditions. Under these conditions, one must logically conclude that the *t*-test is not applicable because the materials of each group were not of comparable scatter (*F*-test results).

Briefly then, under the conditions imposed, the results of the *F*-test show whether material of comparable variability is produced, and the *t*-test shows whether it can be supplied with reproducible properties from time to time. It must be re-emphasized that one must not blindly apply such conclusions to groups of data for which other conditions may have been varied. The *F*- and *t*-tests are not as restrictive as may be implied from this discussion.

Appendix II. Tabular Information

The locations of the tabular data for each material discussed in the previous sections of this report are presented below.

The symbols in the tables have the following significance:

E_l = Young's modulus from the longitudinal resonant frequency.
E_{fw} = Young's modulus from the flexural resonant frequency in flatwise vibration.
E_{fe} = Young's modulus from the flexural resonant frequency in edgewise vibration.
G = Shear Modulus.
μ_l = Poisson's ratio using E_l and G.
μ_{fw} = Poisson's ratio using E_{fw} and G.
μ_{fe} = Poisson's ratio using E_{fe} and G.
K_l = Bulk modulus, using E_l and μ_l.
K_{fw} = Bulk modulus, using E_{fw} and μ_{fw}.

Underlined *F* and *t* values indicate that a significant difference exists between the compared groups.

The *F* and *t* ratios marked with an asterisk indicate that the comparison has been made with the critical value similarly marked. Two critical values derive from the fact that in choosing this tabulated (critical) value different numbers of specimens are involved.

TABLE 1. *Materials, source, and general fabrication data*

Material	Nominal composition	Code No.	Source	No. of specimens	No. of groups	Reported purity	Manufacturer's		Fabrication method [a]	Temperature of heating [b]
							Designation	Control No.		
						%				°C.
Alumina	Al_2O_3	1	H	20	6	99+		C-5633	HP	(2,000+)
		2	H	20	2	98+		LA7365	CP	1,725
		3	H	5	1	99+		LA 603	CP	1,750
		4	H	20	2	99+		LA 603	CP	1,750
		5	F	11	1		Pure	562V	CP	1,650
		14	F	12	1		Vitreous	555Q	CP	1,680
		15	G	8	1	99.9			CP	1,800
		26	G	14	2	99.9			CP	1,800
		27	G	7	1	99.9			CP	1,800
Ruby alumina	99.5% Al_2O_3+0.5 Cr_2O_3	6	F	5	1			A33A	CP	1,800
	98.5 Al_2O_3+1.5 Cr_2O_3	7	F	5	1			A34A	CP	1,800
Magnesia	MgO	9	H	20	2		Fused	M-202	CP	1,780
		23	G	10	2	99.9			CP	1,800
		24	G	11	2	99.9			CP	1,800
		25	G	14	2	99.9			CP	1,800
Mullite	$3Al_2O_3 \cdot 2SiO_2$	16	H	20	5		Fused	C-5657	HP	(1,750+)
		17	F	8	1				CP	1,650
		18	F	10	1				CP	1,650
Mullite+zirconia	$3Al_2O_3 \cdot 2SiO_2$+ZrO_2	22	F	10	1			397Z	CP	1,650
Spinel	$MgO \cdot Al_2O_3$	20	H	20	5		Fused	C-5633	HP	(1,850+)
		21	H	18	2			431927	CP	1,785
Thoria	ThO_2+0.5% CaO	10	H	17	2			431924	CP	1,785
		51	G	10	[c]				CP	1,800
Urania	UO_2	19	C	5	1	99.9		MCW	CP	1,750
		19a	G	1	1	99.9		NH_3-ppt	CP	1,750
Stabilized zirconia	ZrO_2+5% CaO	11	H	18	4			C-5633	HP	(2,000+)
		12	H	20	2			LZ7590	CP	1,720
		13	H	20	2			LZ604	CP	(1,700+)
Chromium bonded alumina	Al_2O_3+Cr	28	D	20	2		LT-1	MT895/823	Cast	(1,600)
Chromium bonded alumina (modified).	Al_2O_3+Cr+Mo+TiO_2	29	D	19	1		LT-1B	Mixed	Cast	(1,600)
		30	D	20	2		LT-1B	4168/169	Cast	(1,600)
Nickel bonded titanium carbide.	TiC+Ni	31	E	20	2		K150B	3010/3011	CP	1,300 to 1,500
		32	E	20	2		K151B	2459/SC297	CP	1,300 to 1,500
		33	E	20	2		K152B	2697/2739	CP	1,300 to 1,500
		34	E	20	2		K162B	2752/2889	CP	1,300 to 1,500
Boron carbide	B_4C	44	H	20	2			X431925	HP	
Boron carbide — titanium boride.	B_4C+TiB_2 (82/18 vol)	43	H	20	4			B5592	HP	
Silicon carbide	SiC	35	H	19	2			5347/5343	CP	
		45	B	17	2		KT		CP	(1,800+)
Silicon carbide—boron carbide.	SiC+B_4C (90/10 wt)	37	H	20	4			C-5572	HP	
Zirconium carbide	ZrC	38	H	20	6			C-5633	HP	
Zirconium di-boride	ZrB_2	41	H	20	4			5572/5592	HP	
		42	A	20	2		101	178-10/20	CP	(2,000+)
Molybdenum disilicide	$MoSi_2$	39	H	19	6			2372/5592	HP	
Nickel aluminide	NiAl	40	A	14	2		1505	927 A/B	CP	(1,500+)

[a] HP represents hot-pressed in graphite mold, CP represents cold-pressed and sintered, and Cast represents slipcast and sintered.
[b] The heating temperatures given in parenthesis are approximate.

TABLE 2. *Room temperature dynamic elastic constants for* Al_2O_3—*Code 4*

Specimen	ρ	V_s	E_l	$E_{f\omega}$	E_{fs}	G	μ_l	$\mu_{f\omega}$	K_l	$K_{f\omega}$
	g/cm³	*m/sec.*	*Kilobars*	*Kilobars*	*Kilobars*	*Kilobars*			*Kilobars*	*Kilobars*
4.1	3.829	9699	[a]3601.3	3584	3593	1447	0.244	0.238	2348	2280
4.2	3.813	9649	3550.2	3523	3537	1425	.246	.236	2330	2226
4.3	3.831	9696	3601.7	3578	3583	1445	.246	.238	2363	2273
4.4	3.810	9635	3537.3	3509	3546	1421	.245	.235	2311	2206
4.5	3.819	9679	3577.3	3555	3563	1437	.245	.237	2334	2251
4.6	3.828	9682	3588.3	3566	3571	1439	.247	.239	2361	2276
4.7	3.825	9673	3579.3	3550	3562	1437	.245	.235	2340	2233
4.8	3.824	9684	3586.5	3566	3573	1441	.244	.237	2338	2263
4.9	3.831	9714	3615.2	3599	3608	1449	.248	.242	2386	2324
4.10	3.829	9691	3596.5	3574	3584	1441	.248	.240	2378	2292
Average	3.824	9680	3583.35	3561	3572	1438	0.246	0.238	2349	2263
95% C.L.	0.005	17	18.3	20	15	7	0.001	0.002	17	25
Std. dev	0.008	23	25.54	28	21	9	0.001	0.002	23	35
Coef. var	0.2%	0.2%	0.71%	0.8%	0.6%	0.6%	0.5%	0.9%	1.0%	1.5%
4.11	3.832	9694	[a]3600.7	3583	3592	1446	0.245	0.239	2356	2291
4.12	3.816	9653	3555.7	3532	3546	1428	.245	.237	2323	2235
4.13	3.815	9683	3577.3	3552	3564	1436	.241	.233	2307	2215
4.14	3.830	9688	3594.4	3572	3582	1444	.245	.237	2346	2263
4.15	3.819	9673	3572.7	3542	3556	1433	.247	.236	2351	2236
4.16	3.821	9677	3577.8	3549	3566	1436	.246	.236	2347	2239
4.17	3.817	9667	3567.7	3536	3550	1433	.245	.234	2329	2214
4.18	3.819	9668	3569.3	3546	3551	1432	.246	.238	2342	2255
4.19	3.834	9691	3600.5	3580	3588	1446	.245	.238	2352	2275
4.20	3.820	9677	3577.1	3550	3555	1437	.245	.236	2338	2239
Average	3.822	9677	3579.32	3554	3565	1437	0.245	0.236	2339	2246
95% C.L.	0.005	9	10.6	13	12	4	0.001	0.001	11	18
Std. dev	0.007	12	14.84	18	17	6	0.001	0.002	15	25
Coef. var	0.2%	0.1%	0.41%	0.5%	0.5%	0.4%	0.6%	0.8%	0.7%	1.1%
F-test:										
Critical	4.03	4.03	4.03	4.03	4.03	4.03	4.03	4.03	4.03	4.03
Calculated	1.16	3.59	2.96	2.37	1.62	2.20	1.13	1.30	2.26	2.01
t-test:										
Critical	2.10	2.10	2.10	2.10	2.10	2.10	2.10	2.10	2.10	2.10
Calculated	0.79	0.39	0.43	0.60	0.84	0.35	1.30	1.64	1.13	0.42

[a] These data are reported to give figures only for the purpose of illustrating the statistical computations shown in table 3.

TABLE 3. *Statistical calculations for* Al_2O_3

Young's modulus E_l—Code 4[a]

Computation	Group I (Specimens 1 to 10)	Group II (Specimens 11 to 20)
Average $=\bar{x}=$ total/no. values	35833.5/10=3583.35	35793.2/10=3579.32
$\Sigma x_i^2=x_1^2+x_2^2+\ldots x_n^2$	128,409,842.1	128,117,298.2
$(\Sigma x_i)^2/n=(x_1+x_2\ldots x_n)^2/n$	128,403,972.2	128,115,316.6
$\Sigma x_i^2-(\Sigma x_i)^2/n=A$	5869.9	1981.6
Deg. of freedom = No. determinations − 1	10−1=9	10−1=9
$S^2=A/n-1$	5869.9/9=652.2	1981.6/9=220.2
Standard deviation $=S=\sqrt{S^2}=$	25.54	14.84
Coefficient of variation $=\frac{S\times 100}{\bar{x}}$	$\frac{25.54\times 100}{3583.35}=0.71\%$	$\frac{14.84\times 100}{3579.32}=0.41\%$
95% c.l. $=Sk$	25.54×0.7153=18.3	14.84×0.7153=10.6

F-test $F=S_a^2/S_b^2$, where S_a^2 is larger of the two.

$F=652.2/220.2=2.96$
Critical value of $F=4.03$ } There is no significant difference between the scatter of group I and group II.

t-test $t=\frac{\bar{x}-\bar{y}}{S_p}\sqrt{\frac{n\times m}{n+m}}$, with $n+m-2$ deg. freedom

where $S_p=\sqrt{\frac{[\Sigma x_i^2-(\Sigma x_i)^2/n]+[\Sigma y_i^2-(\Sigma y_i)^2/m]}{(n-1)+(m-1)}}$

$$S_p=\sqrt{\frac{5869.9+1981.6}{9+9}}=\sqrt{436}=20.89$$

$t=\frac{3583.35-3579.32}{20.89}\sqrt{\frac{10\times 10}{10+10}}=0.43$
Critical value of $t=2.10$ } There is no significant difference between the average values of group I and group II.

[a] The actual data records each value to the least one significant figure more than those given in table 2.

TABLE 4. *Bulk density and dynamic elastic constants calculations for specimen 4.1—see table 2*

Length = l = 15.253 cm; width = w = 1.270 cm; edge = e = 0.6359 cm.

Weight = 47.1555 grams.

Resonant frequencies: longitudinal, F_l = 31,792 cps.

flexural flatwise, F_{fw} = 2,700 cps.

flexural edgewise, F_{fe} = 5,302 cps.

torsional, F_t = 14,948 cps.

Volume = (15.253)(1.270)(0.6359) = 12.318 cm³.

Corrected mass = (mass × 0.99986) + (volume × 0.0012) = (47.1555 × 0.99986) + (12.318 × 0.0012) = 47.1633 grams.

Bulk density, $\rho = M/V = \frac{47.1633}{12.318} = 3.829$ gm/cm³.

Young's modulus, $E_l = V_e^2 \rho$, where $V_e = 2lF_l = 2(15.253)(31,792)(10^{-3})^a = 9,699$ m/sec.

$$E_l = (9,699)^2(3.829)(10^{-6})\text{fn}^a = 3,601 \text{ kilobars}$$

or $$E_l = 4l^2F_l^2\rho = 4(15.253)^2(31,792)^2(3.829)(10^{-9})^a = 3,601 \text{ kilobars}$$

Shear modulus, $G = MF_t^2B$ where $B = \frac{4l\left(\frac{e/w + w/e}{4(e/w) + 2.52(e/w)^2 + 0.21(e/w)^6}\right)}{ew} = 137.307 \text{ cm}^{-1}$

$$G = (47.1633)(14,948)^2(137.307)(10^{-9})^a = 1,447 \text{ kilobars}$$

Poisson's ratio, $\mu_l = \frac{E_l}{2G} - 1 = \frac{3,601}{2(1,447)} - 1 = 0.244$

(Average value of μ_l for this group is 0.246)

Bulk modulus, $K_l = \frac{E_l}{3(1-2\mu_l)} = \frac{3,601}{3[1-2(0.244)]} = 2,348$ kilobars

Young's modulus, $E_{fw} = 9.464 \times 10^{-10}\,\text{MF}^2\text{fw}\,\frac{1}{w}\left(\frac{l}{e}\right)^3 T_1$

where $T_1 = T\left[\frac{1 + (0.26\mu + 3.22\mu^2)r/l}{1 + 0.1328r/l}\right] = 1.0136$

where $T = 1.01182$

$$r/l = \frac{(0.28867)(0.6359)}{15.253} = 0.012035$$

$$\mu = 0.246$$

$$E_{fw} = (9.464 \times 10^{-10})(47.1633)(2,700)^2\left(\frac{1}{1.270}\right)\left(\frac{15.253}{0.6359}\right)^3(1.0136) = 3,584 \text{ kilobars}$$

Young's modulus, $E_{fe} = 9.464 \times 10^{-10}\,\text{MF}^2\text{fe}\,\frac{1}{e}\left(\frac{l}{w}\right)^3 T_1$

where $T_1 = 1.01182\left[\frac{1 + (0.26)(0.246) + (3.22)(0.246)^2(0.024036)}{1 + (0.1328)(0.024036)}\right] = 1.0511$

$$E_{fe} = (9.464 \times 10^{-10})(47.1633)(5,302)^2\left(\frac{1}{0.6359}\right)\left(\frac{15.253}{1.270}\right)^3(1.0511) = 3,593 \text{ kilobars}$$

Poisson's ratio, $\mu_{fw} = \frac{E_{fw}}{2G} - 1 = \frac{3,584}{2(1,447)} - 1 = 0.238$

Bulk modulus, $K_{fw} = \frac{E_{fw}}{3(1-2\mu_{fw})} = \frac{3,584}{3[1-2(0.238)]} = 2,280$ kilobars

[a] The factors 10^{-3}, 10^{-5}, and 10^{-9} are necessary for the conversion of the units given, such as $\frac{\text{cm}}{\text{sec}}$ to $\frac{\text{meters}}{\text{sec}}$ for V_e and $\frac{\text{dynes}}{\text{cm}^2}$ to kilobars for elastic moduli.

TABLE 5. *Summary of X-ray diffraction and bulk density data*

Material	Code	X-ray examination			Theoretical density	Bulk density			Densification
		Phases present	Lattice constants	Structure		Group	No. spec.	Avg.	
			Å		g/cm³			g/cm³	%
Al_2O_3	1	Single phase	a=4.758₉ c=12.99₃	Hex.	3.986	All	20	3.942	99
	2	Single phase	a=4.759₆ c=12.99₃	Hex.	3.985	I	10	3.714	93
						II	10	3.584	90
Ruby Al_2O_3	6	Single phase solid solution	a=4.760₁ c=12.99₃	Hex.	3.991	----	5	3.728	93
	7	Single phase solid solution	a=4.758₆ c=13.00₃	Hex.	4.004	----	5	3.661	91
MgO	24	MgO	a=4.2125	Cubic	3.600	I	6	3.502	97
Mullite: $3Al_2O_3 \cdot 2SiO_2$	16	Mullite plus at least 10% Al_2O_3	----	Ortho.+ Hex.	----	All	20	2.963	----
	17	Single phase	----	Ortho.	----	----	8	2.771	----
	18	Single phase	----	Ortho.	----	----	10	2.779	----
Mullite+ZrO_2	22	Mullite plus about 20% monoclinic ZrO_2, no zircon.	----	Ortho.+ Mon.	----	----	10	2.768	----
Spinel: $MgO \cdot Al_2O_3$	20	Spinel plus small excess MgO.	a=8.082₀	Cubic	3.580	All	20	3.510	98
	21	Spinel plus small excess MgO	a=8.085₀	Cubic	3.576	I	8	2.451	69
						II	10	2.522	71
ThO_2[c]	10	Single phase solid solution	a=5.596₈	Cubic	9.821	I	9	9.722	99
						II	8	9.664	98
	51	Single phase solid solution	a=5.59₇	Cubic	9.820	----	10	9.702	99
UO_2	19	UO_2	a=5.471	Cubic	10.949	I	5	10.37	95
Stab. ZrO_2	11	Single phase solid solution	a=5.119₅	Cubic	5.754	All	18	5.634	98
	12	Solid solution plus small amount monoclinic ZrO_2.	a=5.117₃	Cubic	5.762	I	10	4.966	86
						II	10	4.971	86
Al_2O_3+Cr	28	Cr+faint peak; poor pattern because Cr smear.	Cr, a=2.88+	----	----	I	10	5.958	----
						II	10	5.944	----
	29	Cr+Al_2O_3 (see Code 28)	Cr, a=2.936	----	----	----	19	6.053	----
TiC+Ni	31	TiC+(Ni)	TiC, a=4.325₄	Cubic	[b] 5.430	I	10	5.341	98
						II	10	5.343	98
	32	TiC+Ni	TiC, a=4.330₁	Cubic	[b] 5.800	I	10	5.654	97
						II	10	5.541	96
	33	TiC+Ni	TiC, a=4.332₈	Cubic	[b] 6.174	I	10	5.862	95
						II	10	5.821	94
	34	TiC+Ni	TiC, a=4.331₁	Cubic	[b] 6.367	I	10	5.723	90
						II	10	5.882	92
B_4C	44	B_4C+Cg	----	Hex.	----	I	10	2.506	----
B_4C+TiB_2	43	B_4C+TiB_2	B_4C {a=5.61 c=12.07; TiB_2 {a=3.028 c=3.228	Hex.	----	II	5	2.815	----
SiC	35	Mixture of cubic and several hexagonal poly types.	----	----	----	I	10	2.576	----
						II	9	2.596	----
	45	----	----	----	----	I	9	3.103	----
						II	8	3.128	----
SiC+B_4C	37	Same as Code 35 plus B_4C	----	----	----	All	20	3.082	----
ZrC	38	ZrC plus faint Cg peak	a=4.686₅	Cubic	6.661	All	20	6.118	92
ZrB_2	41	ZrB_2 and few unknown peaks	a=3.166₆ c=3.536₅	Hex.	6.102	All	20	5.585	92
	42	ZrB_2 and few unknown peaks	----	----	----	I	10	4.557	----
						II	10	4.524	----
$MoSi_2$	39	$MoSi_2$	a=3.20 c=7.85	Tetragonal	6.29	I	4	5.987	95
NiAl	40	NiAl and few unknown peaks	a=4.082₄	Cubic	8.382	I	10	5.763	69
						II	4	5.656	68

[a] Values based upon the calculated bulk density and the theoretical density calculated from the NBS lattice constant determination.
[b] Values based upon the assumption that no reaction occurs between the two phases present and, therefore, that they are "mechanical" mixtures.
[c] These high density materials contain ½ wt % CaO.

TABLE 6. *Summary of the dynamic elastic constants and other physical constants* [a]

Material	Code	Fabrication [b]	Bulk density		Speed of sound		Young's modulus		Shear modulus		Poisson's ratio	Bulk modulus	
			g/cm³	*lb/ft³*	*m/sec*	*ft/sec*	*Kilobars* [c]	*psi*	*Kilobars*	*psi*		*Kilobars*	*psi*
Al_2O_3	1	HP	3.942	246.1	(d10,020)	(32,870)	(3,958)	(57.41×10⁶)	1,565	22.70×10⁶	(0.254)	(2,645)	(38.36×10⁶)
	27	CP	3.904	243.7	(9,540)	(31,300)	(3,555)	(51.56)	1,453	21.07	(.221)	(2,120)	(30.88)
	26	CP	3.902	243.6	(9,900)	(32,480)	(3,824)	(55.46)	1,548	22.45	(.236)	(2,415)	(35.03)
	4	CP	3.824	238.7	9,680	31,760	3,583	51.97	1,438	20.86	.246	2,349	34.07
	3	CP	3.825	238.8	9,675	31,740	3,580	51.92	1,445	20.96	.239	2,289	33.20
	2	CP	3.714	231.9	9,361	30,710	3,255	47.21	1,321	19.16	.232	2,018	29.27
	14	CP	3.470	216.6	8,676	28,460	2,612	37.88	1,067	15.48	.224	1,579	22.90
	15	CP	3.335	208.2	8,340	27,360	2,317	33.61	977	14.17	.187	1,235	17.91
	5	CP	2.850	177.9	6,236	20,460	1,109	16.08	179	6.95	.170	533	7.73
Ruby Al_2O_3	6	CP	3.728	232.7	(9,570)	(31,400)	(3,412)	(49.49)	1,368	19.84	(0.247)	(2,251)	(32.65)
	7	CP	3.661	228.6	(9,470)	(31,070)	(3,284)	(47.63)	1,285	18.64	(.258)	(2,234)	(32.40)
MgO	24	CP	3.506	218.9	(9,170)	(30,090)	(2,947)	(42.74)	1,243	18.03	(0.186)	(1,564)	(22.68)
	25	CP	3.483	217.4	(9,080)	(29,790)	(2,873)	(41.67)	1,242	18.01	(.157)	(1,401)	(20.32)
	23	CP	3.479	217.2	(9,090)	(29,820)	(2,872)	(41.65)	1,207	17.51	(.191)	(1,564)	(22.68)
	9	CP	2.648	165.3	5,732	18,810	870	12.62	374	5.42	.163	430	6.24
Mullite: $3Al_2O_3 \cdot 2SiO_2$	16	HP	2.963	185.0	(7,840)	(25,720)	(1,819)	(26.38)	704	10.21	(0.293)	(1,501)	(21.77)
	18	CP	2.779	173.5	7,176	23,540	1,431	20.75	578	8.38	.238	910	13.20
	17	CP	2.771	173.0	7,144	23,440	1,415	20.52	573	8.31	.233	883	12.81
Mullite + ZrO_2	22	CP	2.768	172.8	6,767	22,200	1,268	18.39	524	7.60	0.211	732	10.62
Spinel: $MgO \cdot Al_2O_3$	20	HP	3.510	219.1	(8,670)	(28,440)	(2,636)	(38.23)	1,019	14.78	(0.294)	(2,173)	(31.52)
	21	CP	2.451	153.0	5,219	17,120	665	9.64	271	3.93	.228	408	5.92
ThO_2 [e]	10	CP	9.722	606.9	4,972	16,310	2,404	34.87	942	13.66	0.275	1,785	25.89
	51	CP	9.702	605.7	4,957	16,260	2,384	34.58	930	13.49	.282	1,819	26.38
UO_2	19	CP	10.37	647.4	4,314	14,150	1,929	27.98	741	10.75	0.302	1,620	23.50
	19a	CP	10.19	636.1	4,230	13,880	1,823	26.44	706	10.25	.291	1,457	21.13
Stabilized ZrO_2	11	HP	5.634	351.7	(4,940)	(16,210)	(1,376)	(19.96)	510	7.40	(0.337)	[f] (810)	(11.75)
	13	CP	5.149	321.4	5,216	17,110	1,401	20.32	585	8.48	.255	955	13.85
	12	CP	4.971	310.3	5,481	17,980	1,493	21.65	584	8.47	.279	1,125	16.32
Al_2O_3+Cr	29	Cast	6.053	377.9	6,667	21,870	2,690	39.02	1,074	15.58	0.253	1,813	26.30
	30	Cast	5.958	371.9	6,787	22,270	2,585	37.49	1,032	14.97	.252	1,732	25.12
	28	Cast	5.691	355.3	6,870	22,540	2,686	38.96	1,114	16.16	.205	1,512	21.93
TiC+Ni	31	CP	5.343	333.6	8,549	28,050	3,905	56.64	1,631	23.66	0.197	2,150	31.18
	32	CP	5.654	353.0	8,467	27,780	4,053	58.78	1,681	24.38	.206	2,295	33.29
	33	CP	5.862	366.0	8,023	26,320	3,773	54.72	1,560	22.63	.210	2,168	31.44
	34	CP	5.882	367.2	8,056	26,430	3,817	55.36	1,586	23.00	.204	2,147	31.14
B_4C	44	HP	2.058	128.5	(14,700)	(48,230)	(4,467)	(64.79)	[f] (1,850)	26.83	[f] (0.207)	[f] (2,542)	(36.87)
B_4C+TiB_2	43	HP	2.816	175.8	(12,600)	(41,340)	(4,485)	(65.05)	[f] (1,860)	26.98	[f] (0.206)	[f] (2,539)	(36.82)
SiC	45	CP	3.128	195.3	(11,300)	(37,070)	(4,013)	(58.20)	1,683	24.41	(0.192)	(966)	(14.01)
	35	CP	2.596	162.1	8,744	28,690	1,985	28.79	836	12.12	.187	1,057	15.33
SiC+B_4C	37	HP	3.082	192.4	(11,600)	(38,060)	(4,151)	(60.21)	[f] (1,699)	24.64	[f] (0.221)	[f] (2,478)	(35.94)
ZrC	38	HP	6.118	381.9	(7,140)	(23,430)	(3,117)	(45.21)	1,240	17.98	(0.257)	(2,142)	31.07
ZrB_2	41	HP	5.585	348.7	(8,880)	(29,130)	(4,399)	(63.80)	1,922	27.88	(0.144)	(2,077)	(30.12)
	42	CP	4.557	284.5	(7,340)	(24,080)	(2,455)	(35.61)	1,085	15.74	(.131)	(1,110)	(16.10)
$MoSi_2$	39	HP	5.966	372.4	(7,980)	(26,180)	(3,795)	(55.04)	1,629	23.63	(0.165)	(1,887)	(27.37)
NiAl	40	CP	5.763	359.8	(5,620)	(18,440)	(1,817)	(26.35)	723	10.49	(0.261)	(1,320)	(19.14)

[a] When two groups of specimens were available, the average values of the more dense group are given. When three or more groups were available, as with the hot-pressed specimens, the average values of all of the specimens are given.

[b] The fabrication code used is: HP = hot-pressed, CP = cold-pressed and sintered, and Cast = slip-cast and sintered.

[c] 1 kilobar = 10^9 dynes/cm² = 14,503.8 lb./in.²

[d] The accuracy of values in parenthesis is less than that of the other values.

[e] Contains ½ percent CaO.

[f] These values are not considered reliable because of an assumption that was made during the calculation.

TABLE 7. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{tr}
1-I	H	Average	4	3.974±0.005 g/cm³ (248.1±0.3 lb/ft³)			3997±42 kilobars (57.97±0.61×10⁶ psi)
		V		0.1%			0.7%
1-II		Average	4	3.983±0.005 g/cm³ (248.7±0.3 lb/ft³)			4047±72 kilobars (58.70±1.04×10⁶ psi)
		V		0.1%			1.1%
1-III		Average	2	3.980±0.004 g/cm³ (248.5±0.2 lb/ft³)			4020±24 kilobars (58.31±0.35×10⁶ psi)
		V		0.0%			0.1%
1-IV		Average	4	3.907±0.063 g/cm³ (243.9±3.9 lb/ft³)			3916±104 kilobars (56.80±1.51×10⁶ psi)
		V		1.0%			1.7%
1-V		Average	4	3.876±0.044 g/cm³ (241.6±2.7 lb/ft³)			3803±132 kilobars (55.16±1.91×10⁶ psi)
		V		0.7%			2.2%
1-VI		Average	2	3.962±0.104 g/cm³ (247.3±6.5 lb/ft³)			4034±75 kilobars (58.51±1.09×10⁶ psi)
		V		0.3%			0.2%
1-All		Average	20	3.942±0.021 g/cm³ (246.1±1.3 lb/ft³)			3958±48 kilobars 57.41±0.70×10⁶ psi)
		V		1.2%			2.6%
27	G	Average	7	3.904±0.003 g/cm³ (243.7±0.2 lb/ft³)			3555±151 kilobars (51.56±2.19×10⁶ psi)
		V		0.1%			4.6%
26-I	G	Average	9	3.902±0.016 g/cm³ (243.6±1.0 lb/ft³)			3824±44 kilobars (55.46±0.64×10⁶ psi)
		V		0.5%			1.4%
26-II		Average	5	3.902±0.017 g/cm³ (243.6±1.1 lb/ft³)			3784±42 kilobars (54.88±0.61×10⁶ psi)
		V		0.3%			0.7%
4-I	H	Average	10	3.824±0.005 g/cm³ (238.7±0.3 lb/ft³)	9680±17.0 m/sec (31.76±0.06×10³ ft/sec)	3583±18.0 kilobars (51.97±0.26×10⁶ psi)	3561±20 kilobars (51.65±0.29×10⁶ psi)
		V		0.2%	0.2%	0.7%	0.8%
4-II		Average	10	3.822±0.005 g/cm³ (238.6±0.3 lb/ft³)	9677±9.0 m/sec (31.75±0.03×10³ ft/sec)	3579±11.0 kilobars (51.91±0.16×10⁶ psi)	3554±13 kilobars (51.55±0.19×10⁶ psi)
		V		0.2%	0.1%	0.4%	0.5%
3	H	Average	5	3.825±0.006 g/cm³ (238.8±0.4 lb/ft³)	9675±26.0 m/sec (31.74±0.09×10³ ft/sec)	3580±25.0 kilobars (51.92±0.36×10⁶ psi)	3576±24 kilobars (51.87±0.35×10⁶ psi)
		V		0.1%	0.2%	0.6%	0.5%
2-I	H	Average	10	3.714±0.005 g/cm³ (231.9±0.3 lb/ft³)	9361±18.0 m/sec (30.71±0.06×10³ ft/sec)	3255±16.0 kilobars (47.21±0.23×10⁶ psi)	3260±22 kilobars (47.28±0.32×10⁶ psi)
		V		0.2%	0.3%	0.7%	1.0%
2-II		Average	10	3.584±0.014 g/cm³ (223.7±0.9 lb/ft³)	8995±38.0 m/sec (29.51±0.12×10³ ft/sec)	2899±34.0 kilobars (42.05±0.49×10⁶ psi)	2926±29 kilobars (42.44 ±0.42×10⁶ psi)
		V		0.6%	0.6%	1.7%	1.4%
14	F	Average	12	3.470±0.002 g/cm³ (216.6±0.1 lb/ft³)	8676±8 m/sec (28.46±0.03×10³ ft/sec)	2612±6 kilobars (37.88±0.09×10⁶ psi)	2613±9 kilobars (37.90±0.13×10⁶ psi)
		V		0.1%	0.1%	0.4%	0.6%
15	G	Average	8	3.332±0.017 g/cm³ (208.0±1.1 lb/ft³)	8349±93 m/sec (27.39±0.31×10³ ft/sec)	2316±63 kilobars (33.59±0.91×10⁶ psi)	2326±67 kilobars (33.74 ±0.97×10⁶ psi)
		V		0.5%	1.2%	2.9%	3.1%
5	F	Average	11	2.850±0.005 g/cm³ (177.9±0.3 lb/ft³)	6236 ±69 m/sec (20.46 ±0.23×10³ ft/sec)	1109±25 kilobars (16.08±0.36×10⁶ psi)	1119 ±20 kilobars (16.23 ±0.29×10⁶ psi)
				0.3%	1.7%	3.3%	2.7%

[a] The torsional frequency of only one or two specimens could be measured.
[b] Based on less than 20 specimens.

TABLE 7a. *Statistical data for the*

Code	Compared groups	Critical value	*F* and *t* values for			
			Bulk density	Speed of sound	Young's modulus	
					E_l	E_{tr}
26	*F*-test: I and II	14.6	2.98			3.94
	t-test: I and II	2.23	0.03			1.29
4	*F*-test: I and II	4.03	1.16	3.59	2.96	2.37
	t-test: I and II	2.10	0.70	0.39	0.43	0.60
2	*F*-test: I and II	4.03	[a] 8.07	4.50	5.81	1.73
	t-test: I and II	2.10				20.5

[a] Underlined figures indicate that a significant difference does exist between the compared groups.

for Al_2O_3

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\theta}$	G	μ_f	$\mu_{f\omega}$	K_f	$K_{f\omega}$
400±41 kilobars (58.03±0.59×10⁶ psi) 0.7%	1,585 kilobars [a] (22.99×10⁶ psi) [a]	----	0.258 [a]	----	2755 kilobars [a] (3996 ×10⁶ psi) [a]
4,012±16 kilobars (58.19±0.23×10⁶ psi) 0.2%	1,591 kilobars [a] (23.08×10⁶ psi) [a]	----	0.264 [a]	----	2841 [a] kilobars (41.21 [a]×10⁶ psi)
4,016±0 kilobars (58.25±0×10⁶ psi) 0.0%	1,595 kilobars [a] (23.13×10⁶ psi) [a]	----	0.261 [a]	----	2797 [a] kilobars (40.57 [a]×10⁶ psi)
3,918±113 kilobars (56.83±1.64×10⁶ psi) 1.8%	1,563±51 kilobars (22.67±0.74×10⁶ psi) 2.1%	----	0.253±0.011 2.7%	----	2645±82 kilobars (38.36±1.19×10⁶ psi) 1.9%
3,846±90 kilobars (55.78±1.31×10⁶ psi) 1.5%	1,518±54 kilobars (22.02±0.78×10⁶ psi) 2.2%	----	0.253±0.003 0.7%	----	2590±10[illegible] kilobars (37.13±1.45×10⁶ psi) 2.5%
4,038±34 kilobars (58.57±0.49×10⁶ psi) 0.1%	1,616±78 kilobars (23.44±1.13×10⁶ psi) 0.5%	----	0.248±0.037 1.7%	----	2671±407 kilobars (38.74±5.90×10⁶ psi) 1.7%
3,961±37 kilobars (57.45±0.54×10⁶ psi) 2.0%	1,565±24 kilobars [b] (22.70±0.35×10⁶ psi) [b] 2.7%	----	0.254 [b]±0.003 2.3%	----	2655 ±59 kilobars [b] (38.65±0.86×10⁶ psi) 3.8%
3,520±125 kilobars (51.05±1.81×10⁶ psi) 3.8%	1,453±69 kilobars (21.07±1.00×10⁶ psi) 4.5%	----	0.221±0.018 7.9%	----	2129±226 kilobars (30.88±3.28×10⁶ psi) 10.1%
3,806±53 kilobars (55.20±0.77×10⁶ psi) 1.7%	1,548±11 kilobars (22.45±0.16×10⁶ psi) 0.9%	----	0.236±0.009 4.6%	----	2415±108 kilobars (35.03±1.57×10⁶ psi) 5.3%
3,772±39 kilobars (54.71±0.57×10⁶ psi) 0.6%	1,533±14 kilobars (22.23±0.20×10⁶ psi) 0.6%	----	0.234±0.003 0.7%	----	2372±48 kilobars (34.40±0.70×10⁶ psi) 1.3%
3,572±15 kilobars (51.81±0.22×10⁶ psi) 0.6%	1,438±7 kilobars (20.86±0.10×10⁶ psi) 0.6%	0.246±0.001 0.5%	0.238±0.002 0.9%	2349±17 kilobars (34.07±0.25×10⁶ psi) 1.0%	2283±25 kilobars (32.82±0.36×10⁶ psi) 1.5%
3,565±12 kilobars (51.71±0.17×10⁶ psi) 0.5%	1,437±4 kilobars (20.84±0.06×10⁶ psi) 0.4%	0.245-±0.001 0.6%	0.236±0.001 0.8%	2339±11 kilobars (33.92±0.16×10⁶ psi) 0.7%	2246±18 kilobars (32.58±0.26×10⁶ psi) 1.1%
3,554±23 kilobars (51.55±0.33×10⁶ psi) 0.5%	1,445±7 kilobars (20.96±0.10×10⁶ psi) 0.4%	0.239±0.002 0.7%	0.237±0.001 0.4%	2287±30 kilobars (33.17±0.44×10⁶ psi) 1.1%	2270±22 kilobars (32.92±0.32×10⁶ psi) 0.8%
3,245±18 kilobars (47.06±0.26×10⁶ psi) 0.8%	1,321±11 kilobars (19.16±0.16×10⁶ psi) 1.2%	0.232±0.005 2.9%	0.234±0.001 0.9%	2018±31 kilobars (29.27±0.45×10⁶ psi) 2.1%	2044±33 kilobars (29.65±0.48×10⁶ psi) 2.2%
2,897±37 kilobars (42.02±0.54×10⁶ psi) 1.8%	1,180±13 kilobars (17.11±0.19×10⁶ psi) 1.5%	0.228±0.003 1.8%	0.240±0.005 2.9%	1787±27 kilobars (25.92±0.39×10⁶ psi) 2.1%	1881±32 kilobars (27.28±0.46×10⁶ psi) 2.4%
2,601±5 kilobars (37.72±0.07×10⁶ psi) 0.3%	1,067±3 kilobars (15.48±0.04×10⁶ psi) 0.5%	0.224±0.001 1.0%	0.225±0.002 1.4%	1579±7 kilobars (22.90±0.10×10⁶ psi) 0.7%	1583±16 kilobars (22.96±0.23×10⁶ psi) 1.6%
2,331±33 kilobars (33.81±0.48×10⁶ psi) 1.5%	975±21 kilobars (14.14±0.30×10⁶ psi) 2.3%	0.188±0.014 7.8%	0.192±0.014 8.1%	1217±84 kilobars (17.65±1.22×10⁶ psi) 7.5%	1264±85 kilobars (18.33±1.23×10⁶ psi) 7.3%
1,141±16 kilobars (16.55±0.23×10⁶ psi) 2.1%	479±7 kilobars (6.95±0.10×10⁶ psi) 2.1%	0.170±0.013 8.3%	0.172±0.005 3.2%	533±45 kilobars (7.73±0.65×10⁶ psi) 9.2%	574±19 kilobars (8.33±0.28×10⁶ psi) 3.5%

analysis of values given in table 7

F and *t* values for—Continued					
Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\theta}$	G	μ_f	$\mu_{f\omega}$	K_f	$K_{f\omega}$
6.89	2.17	----	41.8	----	18.2
0.89	0.87	----	----	----	----
1.62	2.20	1.13	1.30	2.26	2.01
0.84	0.35	1.30	1.64	1.13	0.42
4.12	1.35	2.80	1.15	1.37	1.05
----	18.7	1.63	1.73	12.8	8.10

TABLE 8. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
6-A	F	Average	5	2.780±0.083 g/cm³ (173.6±5.2 lb/ft³)			1217±127 kilobars (17.65±1.84×10⁶ psi)
		V		2.4%			8.4%
6-B		Average	5	3.728±0.024 g/cm³ (232.7±1.5 lb/ft³)			3412±112 kilobars (49.49±1.62×10⁶ psi)
		V		0.5%			2.6%
7-A	F	Average	5	2.682±0.013 g/cm³ (167.4±0.8 lb/ft³)			1031±50 kilobars (14.95±0.73×10⁶ psi)
		V		0.4%			3.9%
7-B		Average	5	3.661±0.035 g/cm³ (228.6±2.2 lb/ft³)			3234±126 kilobars (46.91±1.83×10⁶ psi)
		V		0.8%			3.2%

TABLE 9. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
24-I	G	Average	6	3.502±0.012 g/cm³ (218.6±0.7 lb/ft³)			2900±71 kilobars (42.06±1.03×10⁶ psi)
		V		0.3%			2.3%
24-II		Average	5	3.506±0.006 g/cm³ (218.9±0.4 lb/ft³)			2947±22 kilobars (42.74±0.32×10⁶ psi)
		V		0.1%			0.6%
25-I	G	Average	5	3.120±0.021 g/cm³ (194.8±1.3 lb/ft³)			2115±36 kilobars (30.68±0.52×10⁶ psi)
		V		0.7%			1.8%
25-II		Average	9	3.483±0.015 g/cm³ (217.4±0.9 lb/ft³)			2873±51 kilobars (41.67±0.74×10⁶ psi)
				0.6%			2.3%
23-A*	G	Average	5	3.463±0.034 g/cm³ (216.2±2.1 lb/ft³)			2832±115 kilobars (41.07±1.67×10⁶ psi)
		V		0.8%			3.3%
23-B*		Average	5	3.479±0.010 g/cm³ (217.2±0.6 lb/ft³)			2872±20 kilobars (41.65±0.29×10⁶ psi)
		V		0.2%			0.6%
9-I	H	Average	10	2.644±0.017 g/cm³ (165.1±1.1 lb/ft³)	5710±77 m/sec (18.73±0.25×10³ ft/sec)	862±28 kilobars (12.50±0.41×10⁶ psi)	847±26 kilobars (12.3±0.4×10⁶ psi)
		V		0.9%	1.9%	4.6%	4.3%
9-II		Average	10	2.648±0.012 g/cm³ (165.3±0.7 lb/ft³)	5732±40 m/sec (18.81±0.13×10³ ft/sec)	870±15 kilobars (12.62±0.22×10⁶)	851±15 kilobars (12.3±0.2×10⁶ psi)
		V		0.7%	1.0%	2.5%	2.5%

* These two groups were made using different starting materials.

TABLE 9a. *Statistical data for the*

Code	Compared groups	Critical value	*F* and *t* values for			
			Bulk density	Speed of sound	Young's modulus	
					E_l	E_{fw}
24	*F*-test: I and II	9.36	5.35			*14.24
	t-test: I and II	2.23	0.88			1.79
25	*F*-test: I and II	8.98	1.19			3.04
	t-test: I and II	2.18	*32.8			23.2
9	*F*-test: I and II	4.03	1.92	3.69	3.36	2.87
	t-test: I and II	2.10	1.29	1.70	0.98	0.83

* Underlined figures indicate that a significant difference does exist between the compared groups.

for ruby Al_2O_3

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
E_{fs}	G	μ_f	μ_{fw}	K_f	K_{fw}
1241±150 kilobars (18.00±2.17×10⁶ psi) 9.8%	480±55 kilobars (6.96±0.80×10⁶ psi) 9.3%		0.269±0.041 12.4%		865±74 kilobars (12.55±1.07×10⁶ psi) 6.9%
3339±106 kilobars (48.43±1.54×10⁶ psi) 2.6%	1368±44 kilobars (19.84±0.64×10⁶ psi) 2.6%		0.247±0.009 3.0%		2251±120 kilobars (32.65±1.74×10⁶ psi) 4.3%
1060±82 kilobars (15.37±1.19×10⁶ psi) 6.3%	411±19 kilobars (5.96±0.28×10⁶ psi) 3.8%		0.254±0.012 3.9%		700±54 kilobars (10.15±0.78×10⁶ psi) 6.2%
3111±126 kilobars (45.12±1.83×10⁶ psi) 3.3%	1285±50 kilobars (18.64±0.73×10⁶ psi) 3.2%		0.258±0.015 4.8%		2234±167 kilobars (32.40±2.42×10⁶ psi) 6.0%

for MgO

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
E_{fs}	G	μ_f	μ_{fw}	K_f	K_{fw}
2888±48 kilobars (41.89±0.70×10⁶ psi) 1.6%	1227±34 kilobars (17.80±0.49×10⁶ psi) 2.6%		0.182±0.012 6.4%		1522±64 kilobars (22.07±0.93×10⁶ psi) 4.0%
2937±31 kilobars (42.60±0.45×10⁶ psi) 0.9%	1243±14 kilobars (18.03±0.20×10⁶ psi) 0.9%		0.186±0.007 3.1%		1564±34 kilobars (22.68±0.49×10⁶ psi) 1.8%
2137±30 kilobars (30.99±0.44×10⁶ psi) 1.5%	893±14 kilobars (13.0±0.2×10⁶psi) 1.7%		0.184±0.016 9.1%		1117±67 kilobars (16.20±0.97×10⁶ psi) 6.3%
2935±40 kilobars (42.57±0.58×10⁶ psi) 1.8%	1242±22 kilobars (18.01±0.32×10⁶ psi) 2.3%		0.157±0.017 14.1%		1401±82 kilobars (20.32±1.19×10⁶ psi) 7.6%
2870±53 kilobars (41.63±0.77×10⁶ psi) 1.5%	1204±30 kilobars (17.46±0.44×10⁶ psi) 2.0%		0.177±0.078 35+%		1537±583 kilobars (22.29±8.46×10⁶ psi) 30+%
2904±31 kilobars (42.12±0.45×10⁶ psi) 0.9%	1207±48 kilobars (17.51±0.70×10⁶ psi) 3.2%		0.191±0.043 18+%		1564±218 kilobars (22.68±3.16×10⁶ psi) 11.2%
845±28 kilobars (12.3±0.4×10⁶ psi) 4.6%	371±12 kilobars (5.38±0.17×10⁶ psi) 4.4%	0.161±0.004 3.5%	0.140±0.006 5.7%	424±16 kilobars (6.15±0.23×10⁶ psi) 5.4%	393±14 kilobars (5.70±0.20×10⁶ psi) 4.8%
854±15 kilobars (12.4±0.2×10⁶ psi) 2.4%	374±7 kilobars (5.42±0.10×10⁶ psi) 2.6%	0.163±0.002 2.0%	0.137±0.004 3.6%	430±7 kilobars (6.24±0.10×10⁶ psi) 2.1%	390±8 kilobars (5.66±0.12×10⁶ psi) 3.0%

analysis of values given in table 9

F and *t* values for—Continued					
Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
E_{fs}	G	μ_f	μ_{fw}	K_f	K_{fw}
3.36	8.63		4.16		4.96
2.51	1.25		3.48		1.66
2.83	3.56		1.74		2.27
31.1	25.6		2.34		5.31
3.48	2.88	2.96	2.56	6.29	3.17
2.00	1.40	2.04	3.76		1.13

TABLE 10. *Data for*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
16-I	H	Average	4	2.737±0.032 g/cm³ (170.9±2.0 lb/ft³)			1,510±52 kilobars (21.90±0.75×10⁶ psi)
		V		0.7%			2.2%
16-II		Average	2	2.966±0.040 g/cm³ (185.2±2.5 lb/ft³)			1,942±115 kilobars (28.17±1.67×10⁶ psi)
		V		0.2%			0.7%
16-III		Average	4	3.039±0.033 g/cm³ (189.7±2.1 lb/ft³)			1,908±78 kilobars (27.67±1.13×10⁶ psi)
		V		0.7%			2.6%
16-IV		Average	5	3.041±0.063 g/cm³ (189.8±3.9 lb/ft³)			1,911±97 kilobars (27.72±1.41×10⁶ psi)
		V		1.7%			4.1%
16-V		Average	5	3.003±0.026 g/cm³ (187.5±1.6 lb/ft³)			1,856±35 kilobars (26.92±0.51×10⁶ psi)
		V		0.7%			1.5%
16-All		Average	20	2.963±0.057 g/cm³ (185.0±3.6 lb/ft³)			1,819±79 kilobars (26.38±1.15×10⁶ psi)
		V		4.1%			9.2%
18 [a]	F	Average	10	2.779±0.006 g/cm³ (173.5±0.4 lb/ft³)	7,176±11 m/sec (23.54±0.04×10³ ft/sec)	1,431±7 kilobars (20.75±0.10×10⁶ psi)	1,428±9 kilobars (20.71±0.13×10⁶ psi)
		V		0.3%	0.2%	0.6%	0.9%
17 [a]	F	Average	8	2.771±0.008 g/cm³ (173.0±0.5 lb/ft³)	7,144±39 m/sec (23.44±0.13×10³ ft/sec)	1,415±21 kilobars (20.52±0.30×10⁶ psi)	1,420±21 kilobars (20.60±0.30×10⁶ psi)
		V		0.4%	0.5%	1.4%	1.8%

[a] These two materials, after having been coded and examined, were found to be the same except that Code 18 was heat-treated for a longer time.

TABLE 10a. *Statistical data for*

Code	Compared groups	Critical value	*F* and *t* values for			
			Bulk density	Speed of sound	Young's modulus	
					E_l	E_{fw}
[a] 17-18	*F*-test: 17 and 18	[a] 4.20-4.48[a]	1.43	[a] [b] 5.59[a]	5.00[a]	3.66
	t-test: 17 and 18	2.12	2.01			0.09

[a] *F* and *t* values marked with an asterisk indicate comparison with the critical value similarly marked.
[b] Underlined figures indicate that a significant difference does exist between the compared groups.

TABLE 11. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
22	F	Average	10	2.768±0.010 g/cm³ (172.8±0.6 lb/ft³)	6,767±11 m/sec (22.20±0.04×10³ ft/sec)	1,268±9 kilobars (18.39±0.13×10⁶ psi)	1,265±9 kilobars (18.35±0.13×10⁶ psi)
		V		0.5%	0.2%	0.9%	1.0%

Mullite: $3\ Al_2O_3 \cdot 2\ SiO_2$

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\infty}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
1,553±68 kilobars (22.52±0.99×10⁶ psi) 2.8%	580±39 kilobars (8.41±0.57×10⁶ psi) 4.2%	----------	0.301±0.049 10%	----------	1,244±258 kilobars (18.04±3.74×10⁶ psi) 13.%
1,992±50 kilobars (28.89±0.73×10⁶ psi) 0.3%	731±33 kilobars (10.6±0.5×10⁶ psi) 0.5%	----------	0.327±0.018 0.6%	----------	1,920±76 kilobars (27.85±1.10×10⁶ psi) 0.4%
1,896±66 kilobars (27.50±0.96×10⁶ psi) 2.2%	739±32 kilobars (10.7±0.46×10⁶ psi) 2.7%	----------	0.291±0.004 0.8%	----------	1,551±68 kilobars (22.50±0.99×10⁶ psi) 2.7%
1,923±103 kilobars (27.89±1.49×10⁶ psi) 4.3%	740±41 kilobars (10.73±0.59×10⁶ psi) 4.4%	----------	0.291±0.009 2.3%	----------	1,525±86 kilobars (22.12±1.25×10⁶ psi) 4.5%
1,869±20 kilobars (27.11±0.29×10⁶ psi) 0.9%	730±71 kilobars (10.59±1.03×10⁶ psi) 7.8%	----------	0.277±0.092 27.%	----------	1,474±424 kilobars (21.38±6.15×10⁶ psi) 23.%
1,837±73 kilobars (26.64±1.06×10⁶ psi) 8.5%	704±34 kilobars (10.21±0.49×10⁶ psi) 10.2%	----------	0.293±0.018 13.%	----------	1,501±37 kilobars (21.77±0.54×10⁶ psi) 5.2%
1,425±7 kilobars (20.67±0.10×10⁶ psi) 0.6%	578±3 kilobars (8.38±0.04×10⁶ psi) 0.7%	0.238±0.002 1.3%	0.235±0.003 1.6%	910±5 kilobars (13.2±0.1×10⁶ psi) 0.7%	899±5 kilobars (13.0±0.1×10⁶ psi) 0.7%
1,409±14 kilobars (20.44±0.20×10⁶ psi) 1.2%	573±7 kilobars (8.31±0.10×10⁶ psi) 1.5%	0.233±0.005 2.2%	0.240±0.009 4.5%	883±15 kilobars (12.8±0.2×10⁶ psi) 2.0%	911±39 kilobars (13.2±0.6×10⁶ psi) 5.2%

the analysis of values given in table 10

F and t values for—Continued					
Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\infty}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
3.38	3.80	2.81*	8.29*	6.86*	51.2
2.53	1.87	22.9	----------	----------	----------

for mullite plus ZrO_2

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\infty}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
1,273±9 kilobars (18.46±0.13×10⁶ psi) 1.0%	524±4 kilobars (7.60±0.06×10⁶ psi) 0.9%	0.211±0.003 1.9%	0.208±0.001 0.8%	732±8 kilobars (10.62±0.12×10⁶ psi) 1.5%	724±11 kilobars (10.50±0.16×10⁶ psi) 2.1%

TABLE 12. *Data for*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{l_w}
20-I	H	Average	2	3.487±0.084 g/cm³ (217.7±5.2 lb/ft³)			2,573±821 kilobars (37.32±11.91 ×10⁶ psi)
		V		0.3%			3.6%
20-II		Average	5	3.502±0.043 g/cm³ (218.6±2.3 lb/ft³)			2,625±62 kilobars (38.07±0.90×10⁶ psi)
		V		1.0%			1.9%
20-III		Average	4	3.539±0.019 g/cm³ (220.9±1.2 lb/ft³)			2,685±116 kilobars (38.94±1.68×10⁶ psi)
		V		0.3%			2.7%
20-IV		Average	5	3.533±0.022 g/cm³ (220.6±1.4 lb/ft³)			2,674±45 kilobars (38.78±0.65×10⁶ psi)
		V		0.5%			1.4%
20-V		Average	4	3.473±0.044 g/cm³ (216.8±2.7 lb/ft³)			2,584±65 kilobars (37.48±0.94×10⁶ psi)
		V		0.8%			1.6%
20-All		Average	20	3.510±0.016 g/cm³ (219.1±1.0 lb/ft³)			2,636±31 kilobars (38.23±0.45×10⁶ psi)
		V		1.0%			2.6%
21-I	H	Average	8	2.451±0.025 g/cm³ (153.0±1.6 lb/ft³)	5,219±155 m/sec (17.12±0.51×10³ ft/sec)	665±39 kilobars (9.65±0.57×10⁶ psi)	652±47 kilobars (9.46±0.68×10⁶ psi)
		V		1.2%	3.6%	7.1%	8.6%
21-II		Average	10	2.522±0.010 g/cm³ (157.4±0.6 lb/ft³)	5,647±66 m/sec (18.53±0.22×10³ ft/sec)	805±22 kilobars (11.68±0.32×10⁶ psi)	785±27 kilobars (11.39±0.39×10⁶ psi)
		V		0.6%	1.6%	3.7%	4.8%

TABLE 12a. *Statistical data for the*

Code	Compared groups	Critical value	*F* and *t* values for—			
			Bulk density	Speed of sound	Young's modulus	
					E_l	E_{l_w}
21	F-test: I and II	4.20	4.13	4.10	2.46	2.22
	t-test: I and II	2.12	• 2.85	6.42	7.64	6.06

• Underlined figures indicate that a significant difference does exist between the compared groups.

Spinel: $MgO \cdot Al_2O_3$

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\bullet}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
2,547±713 kilobars (36.94±10.34×10⁶ psi) 3.1%	1,017±71 kilobars (14.75±1.03×10⁶ psi) 0.8%	.....	0.265 21.%	.....	1,916±600 kilobars (27.79±8.70×10⁶ psi) 35.%
2,611±59 kilobars (37.87±0.86×10⁶ psi) 1.8%	1,017±76 kilobars (14.75±1.10×10⁶ psi) 6.0%	.....	0.293±0.066 18.%	.....	2,191±427 kilobars (31.78±6.19×10⁶ psi) 16.%
2,659±131 kilobars (38.57±1.90×10⁶ psi) 3.1%	1,041±12 kilobars (15.10±0.17×10⁶ psi) 0.7%	.....	0.290±0.054 12.%	.....	2,158±568 kilobars (31.30±8.24×10⁶ psi) 16.%
2,657±43 kilobars (38.54±0.62×10⁶ psi) 1.3%	1,028±15 kilobars (14.91±0.22×10⁶ psi) 1.2%	.....	0.300±0.008 2.1%	.....	2,249±113 kilobars (32.62±1.64×10⁶ psi) 4.0%
2,619±54 kilobars (37.99±0.78×10⁶ psi) 1.3%	990±26 kilobars (14.36±0.38×10⁶ psi) 1.6%	.....	0.305±0.008 1.6%	.....	2,198±56 kilobars (31.88±0.81×10⁶ psi) 1.6%
2,628±28 kilobars (38.12±0.41×10⁶ psi) 2.3%	1,019±16 kilobars (14.78±0.23×10⁶ psi) 3.4%	.....	0.294±0.015 11.%	.....	2,173±132 kilobars (31.52±1.91×10⁶ psi) 13.%
655±48 kilobars 9.50±0.70×10⁶ psi) 8.8%	271±20 kilobars (3.93±0.29×10⁶ psi) 8.8%	0.228±0.017 9.0%	0.202±0.007 4.2%	408±13 kilobars (5.92±0.19×10⁶ psi) 3.7%	365±33 kilobars (5.29±0.48×10⁶ psi) 11.%
791±20 kilobars (11.47±0.29×10⁶ psi) 3.6%	329±8 kilobars (4.77±0.12×10⁶ psi) 3.4%	0.221±0.008 4.8%	0.191±0.030 22.%	482±23 kilobars (6.99±0.33×10⁶ psi) 6.5%	428±42 kilobars (6.21±0.61×10⁶ psi) 14.%

analysis of values given in table 12

F and *t* values for—Continued

Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\bullet}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
4.23	4.59	3.72	24.7	4.45	3.94
6.59	.....	0.93	.....	.....	2.77

TABLE 13.[a]

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
10-I	H	Average	9	9.722±0.034 gm/cm³ (606.9±2.1 gm/cm³)	4,972±9 m/sec (16.31±0.03×10³ ft/sec)	2,404±13 kilobars (34.87±0.19×10⁶ psi)	2,434±23 kilobars (35.30±0.33×10⁶ psi)
		V		0.5%	0.2%	0.7%	1.2%
10-II	----------	Average	8	9.664±0.012 gm/cm³ (603.3±0.7 gm/cm³)	4,960±12 m/sec (16.27±0.04×10³ ft/sec)	2,378±8 kilobars (34.49±0.12×10⁶ psi)	2,406±18 kilobars (34.90±0.26×10⁶ psi)
		V		0.2%	0.3%	0.4%	0.9%
51	G	Average	10	9.702±0.008 gm/cm³ (605.7±0.5 gm/cm³)	4,957±3 m/sec (16.26±0.01×10³ ft/sec)	2,384±5 kilobars (34.58±0.07×10⁶ psi)	2,395±6 kilobars (34.74±0.09×10⁶ psi)
		V		0.1%	0.1%	0.3%	0.3%

[a] These compositions contain ½ weight percent CaO as a densifying agent.

TABLE 13-a. *Statistical data for the*

Code	Compared groups	Critical value	*F* and *t* values for			
			Bulk density	Speed of sound	Young's modulus	
					E_l	E_{fw}
10	*F*-test: I and II	[a] 4.90–4.53*	[b] 9.01	1.38*	3.14	1.90
	t-test: I and II	2.13	----------	0.32	0.62	0.37

[a] *F* and *t* values marked with an asterisk indicate comparison with the critical value similarly marked.
[b] Underlined figures indicate that a significant difference does exist between the compared groups.

TABLE 14

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
19	C	Average	5	10.368±0.021 g/cm³ (647.3±1.3 lb/ft³)	4,314±9 m/sec (14.15±0.03×10³ ft/sec)	1,929±10 kilobars (27.98±0.15×10⁶ psi)	1,936±11 kilobars (28.08±0.16×10⁶ psi)
		V		0.2%	0.2%	0.4%	0.5%
19a	G	Average	1	10.188 g/cm³ (636.0 lb/ft³)	4.230 m/sec (13.88×10³ ft/sec)	1,823 kilobars (26.44×10⁶ psi)	1,737 kilobars (25.19×10⁶ psi)

Data for ThO_2

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
E_{fs}	G	μ_f	μ_{fw}	K_f	K_{fw}
2,424±12 kilobars (35.16±0.17×10⁶ psi) 0.6%	943±9 kilobars (13.68±0.13×10⁶ psi) 1.2%	0.275±0.006 3.0%	0.291±0.003 1.4%	1,785±46 kilobars (25.89±0.67×10⁶ psi) 3.4%	1,945±37 kilobars (2,821±0.54×10⁶ psi) 2.5%
2,399±11 kilobars (34.79±0.16×10⁶ psi) 0.5%	930±7 kilobars (13.49±0.10×10⁶ psi) 0.9%	0.279±0.006 2.7%	0.294±0.009 3.6%	1,793±48 kilobars (26.01±0.70×10⁶ psi) 3.2%	1,948±94 kilobars (28.25±1.36×10⁶ psi) 5.8%
2,384±3 kilobars (34.58±0.04×10⁶ psi) 0.2%	930±3 kilobars (13.49±0.04×10⁶ psi) 0.5%	0.282±0.004 1.3%	0.288±0.004 2.0%	1,819±7 kilobars (26.38±0.10×10⁶ psi) 0.5%	1,881±35 kilobars (27.28±0.51×10⁶ psi) 2.6%

analysis of values given in table 13

F and t values for—Continued					
Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
E_{fs}	G	μ_f	μ_{fw}	K_f	K_{fw}
1.47	1.95	1.23	5.43*	1.10	5.43*
0.59	2.72	0.90	0.73	0.27	----------

Data for UO_2

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
E_{fs}	G	μ_f	μ_{fw}	K_f	K_{fw}
1,930±15 kilobars (27.99±0.22×10⁶ psi) 0.6%	741± 4 kilobars (10.75±0.06×10⁶ psi) 0.4%	0.302±0.003 0.8%	0.306±0.003 0.7%	1,620±26 kilobars (23.50±0.38×10⁶ psi) 1.3%	1,662±30 kilobars (24.11±0.44×10⁶ psi) 1.4%
1,842 kilobars (26.72×10⁶ psi)	706 kilobars (10.24×10⁶ psi)	0.291	----------	1,457 kilobars (21.13×10⁶ psi)	----------

TABLE 15. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_1	E_{fw}
11-I	H	Average	5	5.634±0.026 g/cm³ (351.7±1.6 lb/ft³) 0.4%			1157±83 kilobars (16.78±2.65×10⁶ psi) 5.8%
		V					
11-II		Average	4	5.639±0.027 g/cm³ (352.0±1.7 lb/ft³) 0.3%			1072±44 kilobars (15.55±0.64×10⁶ psi) 2.6%
		V					
11-III		Average	5	5.654±0.045 g/cm³ (353.0±2.8 lb/ft³) 0.6%			2051±81 kilobars (29.75±1.17×10⁶ psi) 3.2%
		V					
11-IV		Average	4	5.603±0.024 g/cm³ (349.8±1.5 lb/ft³) 0.3%			1111±385 kilobars (16.11±5.58×10⁶ psi) 22%
		V					
11-All		Average	18	5.634±0.015 g/cm³ (351.7±0.9 lb/ft³) 0.5%			1376±222 kilobars (19.96±3.22×10⁶ psi) 32%
		V					
13-I	H	Average	10	5.149±0.016 g/cm³ (321.4±1.0 lb/ft³) 0.4%	5216±51 m/sec (17.11±0.17×10³ ft/sec) 1.4%	1401±23 kilobars (20.32±0.33×10⁶ psi) 2.3%	1391±26 kilobars (20.17±0.38×10⁶ psi) 2.6%
		V					
13-II		Average	10	5.162±0.013 g/cm³ (322.3±0.8 lb/ft³) 0.4%	4950±184 m/sec (16.24±0.60×10³ ft/sec) 5.1%	1268±93 kilobars (18.39±1.35×10⁶ psi) 10.3%	1269±92 kilobars (18.41±1.33×10⁶ psi) 10.1%
		V					
12-I	H	Average	10	4.967±0.017 g/cm³ (310.1±1.1 lb/ft³) 0.5%	5456±20 m/sec (17.90±0.07×10³ ft/sec) 0.5%	1478±15 kilobars (21.44±0.22×10⁶ psi) 1.4%	1483±15 kilobars (21.51±0.22×10⁶ psi) 1.4%
		V					
12-II		Average	10	4.971±0.011 g/cm³ (310.3±0.7 lb/ft³) 0.3%	5481±10 m/sec (17.98±0.03×10³ ft/sec) 0.3%	1493±8 kilobars (21.65±0.12×10⁶ psi) 0.8%	1499±10 kilobars (21.74±0.15×10⁶ psi) 1.0%
		V					

[a] This composition contains about 5 wt % of CaO as the stabilizing agent.
[b] Because of the very high and diverse values calculated for Poisson's ratio, the calculated bulk modulus values have little significance.

TABLE 15a. *Statistical data for the*

Code	Compared groups	Critical value	*F* and *t* values for			
			Bulk density	Speed of sound	Young's modulus	
					E_1	E_{fw}
13	*F*-test: I and II	4.03	1.51	[a] 13.2	15.9	12.4
	t-test: I and II	2.10	1.48	.....	.....	.....
12	*F*-test: I and II	4.03	2.58	3.95	3.15	2.04
	t-test: I and II	2.10	1.56	2.46	1.98	2.02

[a] Underlined figures indicate that a significant difference does exist between the compared groups.

for "stabilized" ZrO_2[a]

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
E_{f_e}	G	μ_l	μ_{f_w}	K_l	K_{f_w}
1202±80 kilobars (17.43±1.16×10⁶ psi) 5.4%	455±39 kilobars (6.60±0.57×10⁶ psi) 6.9%		0.274±0.022 6.4%		857±43 kilobars (12.43±0.62×10⁶ psi) 4.0%
1122±58 kilobars (16.27±0.84×10⁶ psi) 3.2%	423±13 kilobars (6.14±0.19×10⁶ psi) 2.0%		0.268±0.02 4.6%		772±93 kilobars (11.20±1.33×psi) 7.5%
2031±68 kilobars (29.46±0.99×10⁶ psi) 2.7%	725±48 kilobars (10.52±0.70×10⁶ psi) 5.3%		0.416±0.052 10.1%		[b]
1132±372 kilobars (16.42±5.40×10⁶ psi) 21%	398±70 kilobars (5.77±1.02×10⁶ psi) 11.0%		0.386±0.218 36%		[b]
1399±208 kilobars (20.29±3.02×10⁶ psi) 30%	510±71 kilobars (7.40±1.03×10⁶ psi) 28%		0.337±0.046[b] 27%		[b]
1394±22 kilobars (20.22±0.32×10⁶ psi) 2.2%	558±7 kilobars (8.09±0.10×10⁶ psi) 1.9%	0.255±0.006 3.3%	0.246±0.008 4.6%	955±37 kilobars (13.85±0.54×10⁶ psi) 5.3%	906±59 kilobars (13.14±0.86×10⁶ psi) 9.1%
1259±94 kilobars (18.26±1.36×10⁶ psi) 10.5%	514±33 kilobars (7.45±0.48×10⁶ psi) 9.1%	0.232±0.012 7.0%	0.234±0.010 6.2%	796±92 kilobars (11.54±1.33×10⁶ psi) 16%	802±87 kilobars (11.63±1.26×10⁶ psi) 15%
1479±12 kilobars (21.45±0.17×10⁶ psi) 1.2%	575±6 kilobars (8.34±0.09×10⁶ psi) 1.4%	0.285±0.005 2.6%	0.289±0.005 2.4%	1144±26 kilobars (16.59±0.38×10⁶ psi) 3.2%	1168±12 kilobars (16.94±0.17×10⁶ psi) 1.4%
1492±10 kilobars (21.64±0.15×10⁶ psi) 0.9%	584±4 kilobars (8.47±0.06×10⁶ psi) 1.0%	0.279±0.002 1.1%	0.284±0.002 0.8%	1125±11 kilobars (16.32±0.16×10⁶ psi) 1.4%	1157±30 kilobars (16.78±0.44×10⁶ psi) 3.6%

analysis of values given in table 15

F and t values for—Continued					
Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
E_{f_e}	G	μ_l	μ_{f_w}	K_l	K_{f_w}
19.1	20.2	3.83	1.64	6.41	2.19
		3.90	2.07		2.24
1.57	2.08	5.31	1.08	5.94	6.67
1.88	2.65		1.53		

TABLE 16. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
29	D	Average	19	6.053±0.021 g/cm³ (377.9±1.3 lb/ft³)	6667±22 m/sec (21.87±0.07×10³ ft/sec)	2690±25 kilobars (39.01±0.36×10⁶ psi)	2711±19 kilobars (39.32±0.28×10⁶ psi)
		V		0.7%	0.7%	1.9%	1.5%
30-I	D	Average	10	5.958±0.017 g/cm³ (371.9±1.1 lb/ft³)	6587±18 m/sec (21.61±0.06×10³ ft/sec)	2585±21 kilobars (37.49±0.30×10⁶ psi)	2611±25 kilobars (37.87±0.36×10⁶ psi)
		V		0.4%	0.4%	1.1%	1.4%
30-II		Average	10	5.944±0.015 g/cm³ (371.1±0.9 lb/ft³)	6563±11 m/sec (21.53±0.04×10³ ft/sec)	2560±15 kilobars (37.13±0.22×10⁶ psi)	2589±17 kilobars (37.55±0.25×10⁶ psi)
		V		0.4%	0.2%	0.8%	0.9%
28-I	D	Average	10	5.691±0.006 g/cm³ (355.3±0.4 lb/ft³)	6870±12 m/sec (22.54±0.04×10³ ft/sec)	2686±12 kilobars (38.96±0.17×10⁶ psi)	2713±20 kilobars (39.35±0.29×10⁶ psi)
		V		0.2%	0.2%	0.6%	1.0%
28-II	----------	Average	10	5.564±0.041 g/cm³ (347.4±2.6 lb/ft³)	6712±47 m/sec (22.02±0.15×10³ ft/sec)	2507±53 kilobars (36.36±0.77×10⁶ psi)	2543±56 kilobars (36.88±0.81×10⁶ psi)
		V		1.0%	1.0%	2.9%	3.1%

[a] Code 28 composition is slightly different than compositions of codes 29 and 30 specimens.

TABLE 16a. *Statistical data for the*

Code	Compared groups	Critical value	*F* and *t* values for			
			Bulk density	Speed of sound	Young's modulus	
					E_l	E_{fw}
30	*F*-test: I and II	4.03	1.23	2.56	1.95	1.70
	t-test: I and II	2.10	1.43	2.59	2.25	1.80
28	*F*-test: I and II	4.03	[a]40.8	14.9	18.3	7.67
	t-test: I and II	2.10	----------	----------	----------	----------

[a] Underlined figures indicate that a significant difference does exist between the compared groups.

for $Al_2O_3 + Cr$ [a]

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
E_{f_e}	G	μ_1	μ_{f_w}	K_1	K_{f_w}
2694±28 kilobars (39.07±0.33×10[6] psi) 1.8%	1074±8 kilobars (15.58±0.12×10[6] psi) 1.5%	0.253±0.003 2.3%	0.262±0.001 0.9%	1813±35 kilobars (26.30±0.51×10[6] psi) 4.0%	1900±14 kilobars (27.56±0.20×10[6] psi) 1.5%
2602±28 kilobars (37.74±0.41×10[6] psi) 1.5%	1032±8 kilobars (14.97±0.12×10[6] psi) 1.0%	0.252±0.003 1.5%	0.264±0.003 1.5%	1732±22 kilobars (25.12±0.32×10[6] psi) 1.8%	1845±29 kilobars (26.76±0.42×10[6] psi) 2.2%
2578±17 kilobars (37.39±0.25×10[6] psi) 0.9%	1023±6 kilobars (14.84±0.09×10[6] psi) 0.8%	0.252±0.001 0.6%	0.266±0.002 1.2%	1724±9 kilobars (25.00±0.13×10[6] psi) 0.8%	1848±22 kilobars (26.80±0.32×10[6] psi) 1.7%
2682±20 kilobars (38.90±0.29×10[6] psi) 1.0%	1114±6 kilobars (16.16±0.09×10[6] psi) 0.8%	0.205±0.001 1.0%	0.217±0.004 2.2%	1512±8 kilobars (21.93±0.12×10[6] psi) 0.8%	1601±30 kilobars (23.22±0.44×10[6] psi) 2.7%
2514±56 kilobars (36.46±0.81×10[6] psi) 3.2%	1042±27 kilobars (15.11±0.39×10[6] psi) 3.0%	0.203±0.001 0.8%	0.221±0.004 2.6%	1434±29 kilobars (20.80±0.42×10[6] psi) 2.8%	1517±39 kilobars (22.00±0.57×10[6] psi) 3.6%

analysis of values given in table 16

F and *t* values for—Continued					
Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
E_{f_e}	G	μ_1	μ_{f_w}	K_1	K_{f_w}
1.75	1.90	5.74	1.62	5.48	1.77
1.94	2.27	----------	0.15	----------	0.20
8.45	13.8	1.34	1.39	12.3	1.65
----------	----------	2.16	1.33	----------	3.83

TABLE 17. *Data for* TiC

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
31-I	E	Average	10	5.341±0.012 g/cm³ (333.4±0.7 lb/ft³)	8576±23 m/sec (28.14±0.08×10³ ft/sec)	3928±29 kilobars (56.97±0.42×10⁶ psi)	4035±30 kilobars (58.52±0.44×10⁶ psi)
		V		0.3%	0.4%	1.0%	1.0%
31-II	.....	Average	10	5.343±0.008 g/cm³ (333.6±0.5 lb/ft³)	8549±15 m/sec (28.05±0.05×10³ ft/sec)	3905±19 kilobars (56.64±0.28×10⁶ psi)	3986±16 kilobars (57.96±0.23×10⁶ psi)
		V		0.2%	0.3%	0.7%	0.6%
32-I	E	Average	10	5.654±0.003 g/cm³ (353.0±0.2 lb/ft³)	8467±8 m/sec (27.78±0.03×10³ ft/sec)	4053±8 kilobars (58.78±0.12×10⁶ psi)	4086±9 kilobars (59.26±0.13×10⁶ psi)
		V		0.1%	0.1%	0.3%	0.3%
32-II	.....	Average	10	5.541±0.004 g/cm³ (345.9±0.2 lb/ft³)	8249±9 m/sec (27.06±0.03×10³ ft/sec)	3770±9 kilobars (54.68±0.13×10⁶ psi)	3869±15 kilobars (56.07±0.22×10⁶ psi)
		V		0.1%	0.2%	0.3%	0.5%
33-I	E	Average	10	5.862±0.002 g/cm³ (366.0±0.1 lb/ft³)	8023±5 m/sec (26.32±0.02×10³ ft/sec)	3773±5 kilobars (54.72±0.07×10⁶ psi)	3808±7 kilobars (55.23±0.10×10⁶ psi)
		V		0.0%	0.1%	0.2%	0.3%
33-II	.....	Average	10	5.821±0.005 g/cm³ (363.4±0.3 lb/ft³)	7998±9 m/sec (26.24±0.03×10³ ft/sec)	3749±9 kilobars (54.37±0.13×10⁶ psi)	3757±12 kilobars (54.49±0.17×10⁶ psi)
		V		0.1%	0.2%	0.3%	0.5%
34-I	E	Average	10	5.723±0.003 g/cm³ (357.3±0.2 lb/ft³)	8267±7 m/sec (27.12±0.02×10³ ft/sec)	3911±7 kilobars (56.72±0.10×10⁶ psi)	3917±11 kilobars (56.81±0.16×10⁶ psi)
		V		0.1%	0.1%	0.3%	0.4%
34-II	.....	Average	10	5.882±0.006 g/cm³ (367.2±0.4 lb/ft³)	8056±5 m/sec (26.43±0.02×10³ ft/sec)	3817±6 kilobars (55.36±0.09×10⁶ psi)	3904±9 kilobars (56.62±0.13×10⁶ psi)
		V		0.1%	0.1%	0.2%	0.3%

[a] Code 31 contains about 10 wt % Ni. Code 32 contains about 20 wt % Ni. Code 33 contains about 30 wt % Ni. Code 34 contains about 30 wt % Ni (Modified).

TABLE 17a. *Statistical data for the*

Code	Compared groups	Critical value	*F* and *t* values for			
			Bulk density	Speed of sound	*Young's modulus*	
					E_l	E_{fw}
31	*F*-test: I and II	4.03	1.94	2.25	2.35	3.48
	t-test: I and II	2.10	0.30	2.22	1.53	2.60
32	*F*-test: I and II	4.03	1.71	1.15	1.22	2.53
	t-test: I and II	2.10	[a]47.2	41.2	52.4	28.1
33	*F*-test: I and II	4.03	6.11	3.72	2.78	3.29
	t-test: I and II	2.10	.....	49.7	5.47	8.31
34	*F*-test: I and II	4.03	5.14	2.20	1.50	1.58
	t-test: I and II	2.10	.....	56.5	24.0	2.20

[a] Underlined figures indicate that a significant difference does exist between the compared groups.

TABLE 18. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
44-I	H	Average	10	2.506±0.002 g/cm³ (156.4±0.1 lb/ft³)	.....	.....	4467±24 kilobars (64.79±0.35×10³ psi)
		V		0.1%			0.7%
44-II	.....	Average	10	2.505±0.002 g/cm³ (156.4±0.1 lb/ft³)	.....	.....	4481±19 kilobars (64.99±0.28×10³ psi)
		V		0.1%			0.6%

[a] These values are not considered reliable because of an assumption that was made during the calculation.
[b] μ=0.2074 assumed value for one specimen see sec. 2.3(a).

+10 to 30 wt % Ni[a]

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\sigma}$	G	μ_t	$\mu_{f\omega}$	K_t	$K_{f\omega}$
4027±29 kilobars (58.41±0.42×10⁶ psi) 1.0%	1654±11 kilobars (23.99±0.16×10⁶ psi) 0.9%	0.188±0.003 2.1%	0.220±0.003 2.1%	2099±25 kilobars (30.44±0.36×10⁶ psi) 1.7%	2403±29 kilobars (34.85±0.42×10⁶ psi) 1.7%
4031±15 kilobars (58.46±0.22×10⁶ psi) 0.5%	1631±8 kilobars (23.66±0.12×10⁶ psi) 0.6%	0.197±0.003 1.9%	0.225±0.002 1.5%	2149±18 kilobars (31.17±0.26×10⁶ psi) 1.2%	2427±11 kilobars (35.20±0.16×10⁶ psi) 0.7%
4081±8 kilobars (59.19±0.12×10⁶ psi) 0.3%	1681±3 kilobars (24.38±0.04×10⁶ psi) 0.2%	0.206±0.003 2.0%	0.216±0.003 2.0%	2295±23 kilobars (33.29±0.33×10⁶ psi) 1.4%	2394±4 kilobars (34.72±0.06×10⁶ psi) 0.2%
3851±10 kilobars (55.85±0.15×10⁶ psi) 0.4%	1548±6 kilobars (22.45±0.09×10⁶ psi) 0.5%	0.197±0.004 3.0%	0.228±0.005 3.4%	2078±30 kilobars (30.14±0.44×10⁶ psi) 2.0%	2368±42 kilobars (34.34±0.61×10⁶ psi) 2.5%
3804±8 kilobars (55.17±0.12×10⁶ psi) 0.3%	1560±6 kilobars (22.63±0.09×10⁶ psi) 0.5%	0.210±0.001 0.9%	0.221±0.002 1.1%	2168±5 kilobars (31.44±0.07×10⁶ psi) 0.3%	2275±5 kilobars (33.00±0.07×10⁶ psi) 0.3%
3741±13 kilobars (54.26±0.19×10⁶ psi) 0.5%	1538±6 kilobars (22.31±0.09×10⁶ psi) 0.5%	0.219±0.003 1.8%	0.222±0.002 1.4%	2226±13 kilobars (32.29±0.19×10⁶ psi) 0.8%	22[illegible]±15 kilobars (32.[illegible]±0.22×10⁶ psi) 0.9%
----------	1699±3 kilobars (24.64±0.04×10⁶ psi) 0.3%	0.151±0.003 2.7%	0.153±0.004 3.5%	1871±16 kilobars (27.14±0.23×10⁶ psi) 1.2%	1882±24 kilobars (27.30±0.35×10⁶ psi) 1.8%
3886±9 kilobars (56.36±0.13×10⁶ psi) 0.3%	1586±2 kilobars (23.00±0.03×10⁶ psi) 0.2%	0.204±0.002 1.2%	0.231±0.003 1.7%	2147±3 kilobars (31.14±0.04×10⁶ psi) 0.2%	2417±28 kilobars (35.06±0.41×10⁶ psi) 1.6%

analysis of values given in table 17

F and t values for—Continued					
Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\sigma}$	G	μ_t	$\mu_{f\omega}$	K_t	$K_{f\omega}$
3.62	2.00	1.18	1.69	2.06	6.73
9.24	3.98	5.55	2.89	3.75	----------
1.46	5.60	1.98	3.08	1.62	1.98
40.8	----------	3.64	4.31	13.1	1.41
2.65	1.07	4.22	1.76	7.79	9.57
9.75	5.84	----------	0.68	----------	----------
----------	1.78	2.85	1.81	3.06	1.38
----------	64.8	35.3	36.8	38.3	32.7

for B_4C

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\sigma}$	G	μ_t	$\mu_{f\omega}$	K_t	$K_{f\omega}$
4450±23 kilobars (64.54±0.33×10⁶ psi) 0.7%	1850±10 kilobars[a] (26.83±0.15×10⁶ psi) 0.7%	----------	(b)	----------	2544±13 kilobars[a] (36.90±0.19×10⁶ psi) 0.7%
4457±15 kilobars (64.64±0.22×10⁶ psi) 0.5%	1856±8 kilobars[a] (26.92±0.12×10⁶ psi) 0.6%	----------	(b)	----------	2552±5 kilobars[a] (37.01±0.07×10⁶ psi) 0.6%

TABLE 19. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
43-I	H	Average	5	2.814±0.006 g/cm³ (175.7±0.4 lb/ft³)	----------	----------	4454±58 kilobars (64.60±0.84×10⁶ psi)
		V		0.2%			1.1%
43-II	----------	Average	5	2.815±0.006 g/cm³ (175.7±0.4 lb/ft³)	----------	----------	4513±101 kilobars (65.46±1.46×10⁶ psi)
		V		0.2%			1.9%
43-III	----------	Average	5	2.815±0.006 g/cm³ (175.7±0.4 lb/ft³)	----------	----------	4474±43 kilobars (64.89±0.62×10⁶ psi)
		V		0.2%			0.8%
43-IV	----------	Average	5	2.818±0.004 g/cm³ (175.9±0.2 lb/ft³)	----------	----------	4500±31 kilobars (65.27±0.45×10⁶ psi)
		V		0.1%			0.6%
43-All	----------	Average	20	2.816±0.002 g/cm³ (175.8±0.1 lb/ft³)	----------	----------	4485±25 kilobars (65.05±0.36×10⁶ psi)
		V		0.2%			1.2%

[a] This mixture contains 82 parts (volume) of B_4C and 18 parts TiB_2.
[b] These values are not considered reliable because of an assumption that was made during the calculation.
[c] μ=0.2056 assumed value for one specimen, see sec. 2.3(a).

TABLE 20.

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
45-I	B	Average	9	3.103±0.012 gm/cm³ (193.7±0.7 lb/ft³)	----------	----------	3,948±64 kilobars (57.26±0.93×10⁶ psi)
		V		0.5%			2.1%
45-II	----------	Average	8	3.123±0.002 gm/cm³ (195.3±0.1 lb/ft³)	----------	----------	4,013±15 kilobars (58.20±0.22×10⁶ psi)
		V		0.1%			0.4%
35-I	H	Average	10	2.576±0.004 gm/cm³ (160.8±0.2 lb/ft³)	8,818 ±87 m/sec (28.93±0.29×10³ ft/sec)	2,003±41 kilobars (29.05±0.59×10⁶ psi)	2,021±43 kilobars (29.31±0.62×10⁶ psi)
		V		0.2%	1.4%	2.9%	3.0%
35-II	----------	Average	9	2.596±0.005 gm/cm³ (162.1±0.3 lb/ft³)	8,744±81 m/sec (28.69±0.27×10³ ft/sec)	1,985±39 kilobars (28.79±0.57×10⁶ psi)	1,963±50 kilobars (28.47±0.73×10⁶ psi)
		V		0.2%	1.2%	2.5%	3.3%

TABLE 20a. *Statistical data for the*

Code	Compared groups	Critical value	*F* and *t* values for			
			Bulk density	Speed of sound	Young's modulus	
					E_l	E_{fw}
45	*F*-test: I and II	4.90–[a] 4.53*	[b] 12.2	----------	----------	22.4*
	t-test: I and II	2.13	----------	----------	----------	----------
35	*F*-test: I and II	4.36–4.10*	1.71*	1.34	1.33	1.15*
	t-test: I and II	2.11	7.86	1.92	0.71	2.03

[a] *F* and *t* values marked with an asterisk indicate comparison with the critical value similarly marked.
[b] Underlined figures indicate that a significant difference does exist between the compared groups.

for $B_4C + TiB_2$[a]

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
E_{f_e}	G	μ_l	μ_{f_w}	K_l	K_{f_w}
4442±47 kilobars (64.43±0.68×10⁶ psi) 0.9%	1847±24 kilobars[b] (26.79±0.35×10⁶ psi) 1.1%	----------	(c)	----------	2521±33 kilobars[b] (36.56±0.48×10⁶ psi) 1.1%
4455±46 kilobars (64.61±0.67×10⁶ psi) 0.8%	1872±42 kilobars[b] (27.15±0.61×10⁶ psi) 1.8%	----------	(c)	----------	2545±57 kilobars[b] (36.91±0.83×10⁶ psi) 1.8%
4466±37 kilobars (64.77±0.54×10⁶ psi) 0.7%	1856±18 kilobars[b] (26.92±0.26×10⁶ psi) 0.8%	----------	(c)	----------	2533±25 kilobars[b] (36.74±0.36×10⁶ psi) 0.8%
4478±32 kilobars (64.95±0.46×10⁶ psi) 0.6%	1866±13 kilobars[b] (27.06±0.19×10⁶ psi) 0.6%	----------	(c)	----------	2547±18 kilobars[b] (36.94±0.26×10⁶ psi) 0.6%
4460±16 kilobars (64.69±0.23×10⁶ psi) 0.7%	1860±10 kilobars[b] (26.98±0.15×10⁶ psi) 1.2%	----------	(c)	----------	2539±14 kilobars[b] (36.83±0.20×10⁶ psi) 1.2%

Data for SiC

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
E_{f_e}	G	μ_l	μ_{f_w}	K_l	K_{f_w}
3,997±11 kilobars (57.97±0.16×10⁶ psi) 0.3%	1,665±24 kilobars (24.15±0.36×10⁶ psi) 1.9%	----------	0.182±0.003 2.0%	----------	965±14 kilobars (14.00±0.20×10⁶ psi) 1.8%
3,931±21 kilobars (57.01±0.30×10⁶ psi) 0.6%	1,683±5 kilobars (24.41±0.07×10⁶ psi) 0.4%	----------	0.192±0.005 3.3%	----------	966±5 kilobars (14.01±0.07×10⁶ psi) 0.7%
2,035±30 kilobars (29.52±0.44×10⁶ psi) 2.1%	851±19 kilobars (12.34±0.28×10⁶ psi) 3.1%	0.117±0.003 2.1%	0.187±0.002 1.8%	1,034±18 kilobars (15.00±0.26×10⁶ psi) 2.6%	1,078±24 kilobars (15.64±0.35×10⁶ psi) 3.1%
1,993±42 kilobars (28.91±0.01×10⁶ psi) 2.8%	836±57 kilobars (12.13±0.83×10⁶ psi) 8.9%	0.187±0.003 1.7%	0.173±0.005 3.7%	1,057±15 kilobars (15.33±0.22×10⁶ psi) 1.8%	1,002±38 kilobars (14.53±0.55×10⁶ psi) 5.0%

analysis of values given in table 20

F and *t* values for—Continued					
Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
E_{f_e}	G	μ_l	μ_{f_w}	K_l	K_{f_w}
3.01*	28.9	----------	2.67*	----------	7.82
1.27	----------	----------	4.63	----------	----------
1.10*	7.95*	1.26*	3.78*	1.93	2.23*
1.70	----------	5.58	6.08	2.61	3.97

TABLE 21. *Data for*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
37-I	H	Average	5	3.074±0.007 g/cm³ (191.9±0.4 lb/ft³)			4120±45 kilobars (59.76±0.65×10⁶ psi)
		V		0.2%			0.9%
37-II		Average	5	3.086±0.014 g/cm³ (192.7±0.9 lb/ft³)			4114±37 kilobars (59.67±0.54×10⁶ psi)
		V		0.4%			0.7%
37-III		Average	5	3.083±0.003 g/cm³ (192.5±0.2 lb/ft³)			4121±72 kilobars (59.77±1.04×10⁶ psi)
		V		0.1%			1.4%
37-IV		Average	5	3.084±0.006 g/cm³ (192.5±0.4 lb/ft³)			4246±66 kilobars (61.58±0.96×10⁶ psi)
		V		0.2%			1.3%
37-All		Average	20	3.082±0.004 g/cm³ (192.4±0.2 lb/ft³)			4151±31 kilobars (60.21±0.45×10⁶ psi)
		V		0.3%			1.6%

[a] This mixture contains 90 parts of SiC and 10 parts of B_4C (by weight).
[b] These values are not considered reliable because of an assumption that was made during the calculation.
[c] μ=0.2208 assumed value for one specimen, see sec. 2.3(a).

TABLE 22. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
38-I	H	Average	1	6.465 g/cm³ (403.6 lb/ft³)			3881 kilobars (56.29×10⁶ psi)
		V					
38-II		Average	2	6.177±0.602 g/cm³ (385.6±37.6 lb/ft³)			3045±515 kilobars (44.16±7.47×10⁶ psi)
		V		1.1%			1.9%
38-III		Average	4	6.148±0.028 g/cm³ (383.8±1.7 lb/ft³)			3220±72 kilobars (46.70±1.04×10⁶ psi)
		V		0.3%			1.4%
38-IV		Average	4	6.202±0.009 g/cm³ (387.2±0.6 lb/ft³)			3279±24 kilobars (47.56±0.35×10⁶ psi)
		V		0.1%			0.5%
38-V		Average	5	6.016±0.031 g/cm³ (375.6±1.9 lb/ft³)			2961±65 kilobars (42.95±0.94×10⁶ psi)
		V		0.4%			1.8%
38-VI		Average	4	6.017±0.025 g/cm³ (375.6±1.6 lb/ft³)			2890±89 kilobars (41.92±1.29×10⁶ psi)
		V		0.3%			1.9%
38-All		Average	20	6.118±0.055 g/cm³ (381.9±3.4 lb/ft³)			3117±113 kilobars (45.21±1.64×10⁶ psi)
		V		1.9%			7.7%

$SiC + B_4C$[a]

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\infty}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
4090±44 kilobars (59.32±0.64×10⁶ psi) 0.9%	1688±18 kilobars[b] (24.48±0.26×10⁶ psi) 0.9%	----------	(c)	----------	2460±27 kilobars[b] (35.68±0.39×10⁶ psi) 0.9%
4100±67 kilobars (59.47±0.97×10⁶ psi) 1.3%	1685±15 kilobars[b] (24.44±0.22×10⁶ psi) 0.7%	----------	(c)	----------	2456±22 kilobars[b] (35.62±0.32×10⁶ psi) 0.7%
4084±79 kilobars (59.23±1.15×10⁶ psi) 1.6%	1688±29 kilobars[b] (24.48±0.42×10⁶ psi) 1.4%	----------	(c)	----------	2460±43 kilobars[b] (35.68±0.62×10⁶ psi) 1.4%
4321±29 kilobars (62.67±0.42×10⁶ psi) 0.6%	1739±27 kilobars[b] (25.22±0.39×10⁶ psi) 1.3%	----------	(c)	----------	2535±40 kilobars[b] (36.77±0.58×10⁶ psi) 1.3%
4149±52 kilobars (60.18±0.75×10⁶ psi) 2.7%	1699±20 kilobars[b] (24.64±0.29×10⁶ psi) 1.6%	----------	(c)	----------	2478±29 kilobars[b] (35.94±0.42×10⁶ psi) 1.6%

for ZrC

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\infty}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
3870 kilobars (56.13×10⁶ psi)	1540 kilobars (22.34×10⁶ psi)	----------	0.260	----------	2688 kilobars (38.99×10⁶ psi)
3119±503 kilobars (45.24±7.30×10⁶ psi) 1.8%	1223±40 kilobars (17.74±0.58×10⁶ psi) 0.4%	----------	0.245±0.170 7.7%	----------	1999±82 kilobars (31.27±1.19×10⁶ psi) 9.8%
3249±51 kilobars (47.12±0.74×10⁶ psi) 1.0%	1280±27 kilobars (18.56±0.39×10⁶ psi) 1.3%	----------	0.258±0.005 1.2%	----------	2206±115 kilobars (32.00±1.67×10⁶ psi) 3.3%
3291±15 kilobars (47.73±0.22×10⁶ psi) 0.3%	1317±7 kilobars (19.10±0.10×10⁶ psi) 0.3%	----------	0.245±0.010 2.5%	----------	2156±82 kilobars (31.27±1.19×10⁶ psi) 2.4%
3063±45 kilobars (44.43±0.65×10⁶ psi) 1.2%	1165±31 kilobars (16.90±0.45×10⁶ psi) 2.2%	----------	0.271±0.008 2.4%	----------	2136±68 kilobars (30.98±0.99×10⁶ psi) 2.6%
2995±59 kilobars (43.44±0.86×10⁶ psi) 1.2%	1149±35 kilobars (16.66±0.51×10⁶ psi) 1.9%	----------	0.258±0.003 0.7%	----------	2009±78 kilobars (29.14±1.13×10⁶ psi) 2.5%
3178±95 kilobars (46.09±1.38×10⁶ psi) 6.4%	1240±46 kilobars (17.98±0.67×10⁶ psi) 8.0%	----------	0.257±0.005 4.5%	----------	2142±76 kilobars (31.07±1.10×10⁶ psi) 7.6%

TABLE 23.

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	E_{fw}
41-I	H	Average	5	5.968±0.015 g/cm³ (372.6±0.9 lb/ft³)			5069±22 kilobars (73.52±0.32×10³ psi)
		V		0.2%			0.4%
41-II		Average	5	5.308±0.013 g/cm³ (331.4±0.8 lb/ft³)			3582±32 kilobars (51.95±0.46×10³ psi)
		V		0.2%			0.7%
41-III		Average	5	5.271±0.209 g/cm³ (329.1±13.0 lb/ft³)			3405±148 kilobars (49.39±2.15×10³ psi)
		V		3.2%			3.5%
41-IV		Average	5	5.793±0.031 g/cm³ (361.6±1.9 lb/ft³)			4399±52 kilobars (63.80±0.75×10³ psi)
		V		0.4%			1.0%
41-All		Average	20	5.585±0.146 g/cm³ (348.7±9.1 lb/ft³)			4114±321 kilobars (59.67±4.66×10³ psi)
		V		5.6%			17%
42-I	A	Average	10	4.557±0.034 g/cm³ (284.5±2.1 lb/ft³)			2455±85 kilobars (35.61±1.23×10³ psi)
		V		1.1%			4.8%
42-II		Average	10	4.524±0.056 g/cm³ (282.4±3.5 lb/ft³)			2368±143 kilobars (34.34±2.07×10³ psi)
		V		1.7%			8.4%

TABLE 23a. *Statistical data for the analysis*

Code	Compared groups	Critical value	*F* and *t* values for			
			Bulk density	Speed of sound	Young's modulus	
					E_l	E_{fw}
42	*F*-test I and II	4.03	2.68	.....	.....	2.82
	t-test I and II	2.10	0.93	.....	.....	0.97

[a] Underlined figures indicate that a significant difference does exist between the compared groups.

TABLE 24. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						E_l	F_{fw}
39-I	H	Average	4	5.987±0.072 g/cm³ (373.8±4.5 lb/ft³)			3815±106 kilobars (55.33±1.54×10³ psi)
		V		0.8%			1.8%
39-II	.....	Average	5	5.874±0.300 g/cm³ (366.7±18.7 lb/ft³)			3664±125 kilobars (53.14±1.81×10³ psi)
		V		4.1%			2.8%
39-III	.....	Average	1	5.974 g/cm³ (372.9 lb/ft³)			3839 kilobars (55.68×10³ psi)
39-IV	.....	Average	4	6.041±0.022 g/cm³ (377.1±1.4 lb/ft³)			3886±18 kilobars (56.36±0.26×10³ psi)
		V		0.2%			0.3%
39-V	.....	Average	2	5.966±0.485 g/cm³ (372.4±30.3 lb/ft³)			3807±596 kilobars (55.22±8.64×10³ psi)
		V		0.9%			1.7%
39-IV	.....	Average	3	5.991±0.008 g/cm³ (374.0±0.5 lb/ft³)			3839±19 kilobars (55.68±0.28×10³ psi)
		V		0.1%			0.2%
39-All	.....	Average	19	5.966±0.038 g/cm³ (372.4±2.4 lb/ft³)			3795±49 kilobars (55.04±0.71×10³ psi)
		V		1.3%			2.7%

Data for ZrB_2

Young's modulus—Con.	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\theta}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
5090±22 kilobars (73.82±0.32×10⁶ psi) 0.4%	2206±14 kilobars (32.00±0.20×10⁶ psi) 0.5%	----------	0.149±0.003 1.3%	----------	2205±15 kilobars (31.98±0.22×10⁶ psi) 0.5%
3635±27 kilobars (52.72±0.39×10⁶ psi) 0.6%	1527±14 kilobars (22.15±0.20×10⁶ psi) 0.7%	----------	0.173±0.002 0.8%	----------	1825±16 kilobars (26.47±0.23×10⁶ psi) 0.7%
3470±145 kilobars (50.33±2.10×10⁶ psi) 3.4%	1482±62 kilobars (21.49±0.90×10⁶ psi) 3.3%	----------	0.149±0.066 3.0%	----------	1601±66 kilobars (32.22±0.96×10⁶ psi) 3.3%
4363±56 kilobars (63.28±0.81×10⁶ psi) 1.0%	1922±25 kilobars (27.88±0.36×10⁶ psi) 1.0%	----------	0.144±0.003 1.5%	----------	2077±27 kilobars (30.12±0.39×10⁶ psi) 1.0%
4139±310 kilobars (60.03±4.50×10⁶ psi) 16%	1784±143 kilobars (25.87±2.07×10⁶ psi) 17%	----------	0.154±0.006 7.6%	----------	1977±144 kilobars (28.67±2.09×10⁶ psi) 16.0%
2458±95 kilobars (35.65±1.38×10⁶ psi) 5.4%	1065±35 kilobars (15.74±0.51×10⁶ psi) 4.5%	----------	0.131±0.005 5.0%	----------	1110±48 kilobars (16.10±0.70×10⁶ psi 6.1%
2362±150 kilobars (34.26±2.18×10⁶ psi) 8.9%	1037±54 kilobars (15.04±0.78×10⁶ psi) 7.3%	----------	0.141±0.013 12%	----------	1104±100 kilobars (16.01±1.45×10⁶ psi) 13.0%

of values given in table 23

F and *t* values for—Continued					
Young's modulus—Continued	Shear modulus	Poisson's ratio		Bulk modulus	
$E_{f\theta}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
2.51	2.36	----------	7.04	----------	4.32
1.00	1.37	----------	----------	----------	0.13

for $MoSi_2$

Young's modulus	Shear modulus	Poisson's ratio		Bulk modulus	
$F_{f\theta}$	G	μ_l	$\mu_{f\infty}$	K_l	$K_{f\infty}$
3805±67 kilobars (55.19±0.97×10⁶ psi) 1.1%	1636±49 kilobars (23.73±0.71×10⁶ psi) 1.9%	----------	0.166±0.005 1.7%	----------	1910±51 kilobars (27.70±0.74×10⁶ psi) 1.7%
3754±46 kilobars (54.45±0.67×10⁶ psi) 1.0%	1582±44 kilobars (22.95±0.64×10⁶ psi) 2.2%	----------	0.158±0.008 4.1%	----------	1790±102 kilobars (25.96±1.48×10⁶ psi) 4.6%
3807 kilobars (55.22×10⁶ psi)	1638 kilobars (23.76×10⁶ psi)	----------	0.172	----------	1931 kilobars (28.01×10⁶ psi)
3866±13 kilobars (56.07±0.19×10⁶ psi) 0.2%	1665±10 kilobars (24.15±0.15×10⁶ psi) 0.4%	----------	0.167±0.005 2.0%	----------	1935±23 kilobars (28.06±0.33×10⁶ psi) 0.8%
3798±377 kilobars (55.09±5.47×10⁶ psi) 1.1%	1628±272 kilobars (23.61±3.95×10⁶ psi) 1.9%	----------	0.169±0.013 0.8%	----------	1920±324 kilobars (27.85±0.46×10⁶ psi) 3.9%
3804±5 kilobars (55.17±0.07×10⁶ psi) 0.1%	1648±6 kilobars (23.90±0.09×10⁶ psi) 0.1%	----------	0.165±0.006 1.5%	----------	1918±48 kilobars (27.82±0.70×10⁶ psi) 1.0%
3804±23 kilobars (55.17±0.33×10⁶ psi) 1.2%	1629±19 kilobars (23.63±0.28×10⁶ psi) 2.4%	----------	0.165±0.003 3.4%	----------	1887±36 kilobars (27.37±0.52×10⁶ psi) 3.9%

TABLE 25. *Data*

Code and group	Source	Statistical parameters	No. of specimens	Bulk density	Speed of sound	Young's modulus	
						F_t	F_{f_0}
40–I	A	Average	10	5.763±0.034 g/cm³ (359.8±1.5 lb/ft³)			1817±48 kilobars (26.35±0.70×10⁶ psi)
		V		0.6%			3.5%
40–II	.....	Average	4	5.656±0.213 g/cm (353.1±13.3 lb/ft³)			1760±62 kilobars (25.53±0.90×10⁶ psi)
		V		2.4%			2.2%

TABLE 25a. *Statistical data for the*

Code	Compared groups	Critical value	*F* and *t* values for—			
			Bulk density	Speed of sound	Young's modulus	
					F_t	F_{f_0}
40	*F*-test: I and II	5.08—[a]14.5*	[b]15.3	.....	.....	*2.92
	t-test: I and II	2.18	.....	.....	.....	1.58

[a] *F* and *t* values marked with an asterisk indicate comparison with the critical value similarly marked.

[b] Underlined figures indicate that a significant difference does exist between the compared groups.

for NiAl

Young's modulus	Shear modulus	Poisson's ratio		Bulk modulus	
F_{fs}	G	μ_i	μ_{fw}	K_i	K_{fw}
1786±43 kilobars (25.90±0.62×10⁶ psi) 3.4%	723±33 kilobars (10.49±0.48×10⁶ psi) 6.4%	.....	0.261±0.040 21%	.....	1320±177 kilobars (19.15±2.57×10⁶ psi) 19%
1737±194 kilobars (25.19±2.81×10⁶ psi) 7.0%	695±8 kilobars (10.08±0.12×10⁶ psi) 0.7%	.....	0.265±0.039 9.2%	.....	1260±254 kilobars (18.27±3.68×10⁶ psi) 13%

analysis of values given in table 25

		F and t values for—			
Young's modulus	Shear modulus	Poisson's ratio		Bulk modulus	
F_{fs}	G	μ_i	μ_{fw}	K_i	K_{fw}
4.12	*82.2	.....	*5.23	.....	*2.40
1.03	.....	.....	0.14	.....	0.45

4. References

[1] High temperature technology edited by I. E. Campbell, p. 29 (John Wiley & Sons, Inc., New York, N.Y., 1956).

[2] W. R. Sheridan, The use of refractory ceramics in rocket engines, Bull. Am. Ceram. Soc. **37** [2] 91 (1958).

[3] S. Spinner, Elastic moduli of glasses by a dynamic method, J. Am. Ceram. Soc. **37** [5] 229 (1954).

[4] F. B. Hornibrook, Application of sonic method to freezing and thawing studies of concrete, ASTM Bull. No. 101, p. 6 (Dec. 1939).

[5] J. M. Ide, Some dynamic methods of determination of Young's modulus, Rev. Sci. Instr. **6** [10] 296 (1935).

[6] W. G. Cady, Piezoelectricity, p. 102, 1st ed. (McGraw-Hill Book Co., Inc., New York, N.Y., 1946).

[7] G. Pickett, Equations for computing elastic constants from flexural and torsional resonant frequencies of vibration of prisms and cylinders, ASTM Proc. **45,** 846 (1945).

[8] ASTM book of standards, part 3, p. 1355 (C215–55T) (1955).

[9] R. J. Roark, Formulas for stress and strain, 2d ed, p. 154 (McGraw-Hill Book Co., Inc., New York, N.Y., 1938).

[10] S. Spinner, and R. C. Valore, Jr., Comparison of theoretical and empirical relations between the shear modulus and torsional resonance frequencies for bars of rectangular cross section, J. Research NBS **60** [5] 459 (1958) RP2861.

[11] S. M. Lang, R. S. Roth, and C. L. Fillmore, Some properties of beryllia and zirconia with titania, ceria and chromia, J. Research NBS **53** [4] 201 (1954) RP2534.

[12] S. M. Lang and F. P. Knudsen, Some physical properties of high-density thorium dioxide, J. Am. Ceram. Soc. **39** [12] 415 (1956).

[13] R. L. Coble and W. D. Kingery, Effect of porosity on physical properties of sintered alumina, J. Am. Ceram. Soc. **39** [11] 377 (1956).

[14] K. Konopicky, Beitrag zur Untersuchung des Diagramms Al_2O_3–SiO_2, Berichte der Technischeu, Komission und des Forschungs—Institute der Feuerfest Industrie, Berichte 22 (1957).

[15] Otto Ruff and Fritz Ebert, Z. anorg. Chem. **180,** 19 (1929). Also U.S. Patent 1,969,099.

[16] R. F. Geller and P. J. Yavorsky, Effects of some oxide additions on the thermal length changes of zirconia, J. Research NBS **35,** 87 (1945) RP1662.

[17] L. S. Ramsdell and J. A. Kohn, Developments in silicon carbide research, Acta Cryst. **5,** part 2, p. 215 (March 1952).

[18] L. E. Mong and W. L. Pendergast, Dynamic and static tests for mechanical properties of fired plastic refractories and other more resilient materials, J. Am. Ceram. Soc. **39** [9] 301 (1956).

[19] L. E. Mong and D. M. Adelman, Control of factors affecting reproducibility of mechanical properties of refractory semidry press specimens, J. Am. Ceram. Soc. **41** [7] 267 (1958).

[20] R. S. Roth (private communication).

[21] J. M. Frankland and H. L. Whittemore, Test of cellular sheet-steel flooring, J. Research NBS **9,** 131 (1932) RP463.

[22] W. J. Youden, Statistical methods for chemists, (John Wiley & Sons, Inc., New York, N.Y., 1951).

[23] Wilfred J. Dixon and Frank J. Massey, Jr., Introduction to statistical analysis (McGraw-Hill Book Co., Inc., New York, N.Y. 1951).

WASHINGTON, D. C., July 28, 1958

U.S. GOVERNMENT PRINTING OFFICE: 1960

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D.C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Molecular Kinetics. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Radiation Theory. Radioactivity. X-ray. High Energy Radiation. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning, Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Safety Standards. Heat Transfer. Concreting Materials.

Applied Mathematics. Numerical Analysis Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Applications Engineering.

- Office of Basic Instrumentation.
- Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionosphere Research. Regular Prediction Services. Sun-Earth Relationships. VHF Research. Radio Warning Services. Airglow and Aurora. Radio Astronomy and Arctic Propagation.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmospheric Physics.

Radio Standards. High-Frequency Electrical Standards. Radio Broadcast Service. Radio and Microwave Materials. Electronic Calibration Center. Microwave Circuit Standards.

Radio Communication and Systems. Low Frequency and Very Low Frequency Research. High Frequency and Very High Frequency Research. Modulation Systems. Antenna Research. Navigation Systems. Systems Analysis. Field Operations.