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Analysis of Infrared Scaling Laws

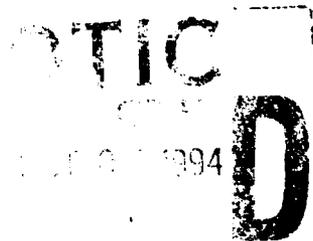
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ABSTRACT

The scaling laws appropriate to the IR radiation from ships are displayed. The heat equation is rendered in dimensionless form so that its solutions manifestly apply to ships of arbitrary scale. The important variables for IR thermodynamic similarity are shown, viz. the dimensionless quantities, also called π variables, associated with temperature, position, time, power per unit volume, and heat transfer coefficient. The purpose of scale modeling is to duplicate the ship values of these dimensionless quantities in the scale models studied. In so doing, the IR signals are duplicated as well and may be studied in a laboratory setting with all the advantages of repetition and control thus implied.

Questions of compatibility of the dimensionless variables are discussed. It is concluded that no apparent inherent incompatibilities among the dimensionless quantities relevant to the IR present themselves.

A relative variance of the dimensionless temperature is derived, and it is shown how its errors depend upon the experimental variables controlling the IR scaling experiment.

A test program for IR scale models is outlined. A mathematical criterion for the objective comparison of the experimentally determined dimensionless temperature of a scale model and its prototype ship is given.

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INTRODUCTION

Recent work has indicated that scale models will be useful in advancing the field of ship IR radiation analysis. This is in keeping with their utility, long recognized, in the engineering of radar, hydro/aero dynamics, acoustics, etc. The scaling laws for IR are known, but several questions related to their laboratory application remain.

For example, given that the IR scaling laws are in hand, are they mutually compatible, and can they all be satisfied simultaneously? The answers are not immediately obvious, but are certainly important for purposes of planning laboratory experimentation. It is common experience in other scaling applications to find conflicts between scaling laws; engineering choices are often required to sacrifice one in favor of the other. A well-known illustration of the incompatibility of dimensionless numbers is that of mechanical similarity of fluid flow around two spheres. If only inertial forces and frictional forces are considered, mechanical similarity requires the constancy of the Reynolds number, i.e.,

$$Re = \frac{\rho v L}{\eta} = \text{constant}, \quad (1)$$

where:

- ρ = fluid mass density
- v = relative velocity of fluid and sphere
- L = a characteristic length (the radius)
- η = kinematic viscosity of fluid

On the other hand, if only inertial forces and gravitational forces are considered, mechanical similarity requires the constancy of the Froude number, i.e.,

$$Fr = \frac{v^2}{Lg} = \text{constant} \quad (2)$$

where g is the gravitational acceleration constant.

It is evident that for one fluid, constancy of the Re and Fr puts conflicting demands on the fluid's velocity. In the case of the Reynolds number, a decrease in L , as would occur for a small-scale model, demands an increase in v ; whereas, in the Froude number, a decrease in L demands a decrease in v . But the same fluid cannot both increase and decrease v simultaneously. Both requirements for constancy in Re and Fr cannot be met. This illustrates what is meant by incompatibility of dimensionless numbers.

Notice, however, if we remove the restriction that one fluid be used for both values of L , then a new possibility enters, viz., that a change in kinematic viscosity could remove the previous incompatibility. Constancy of both Re and Fr would require of the fluids a ratio

$$\frac{\eta_1}{\eta_2} = \left[\frac{L_1}{L_2} \right]^{3/2} = \left[\frac{v_1}{v_2} \right]^3 \quad (3)$$

where subscripts 1 and 2 refer to properties associated with different scales. Mathematically, the possibility for compatibility of Re and Fr lies in satisfying this last equation. Whether the mathematical possibility can be physically realized, however, depends upon the actual existence of fluids with the required viscosities. It is important to note that the mathematical possibility of compatibility, and its actual physical realization with real materials, are different matters. We investigate these matters for the dimensionless

numbers of IR scaling and find plausible the physical realization of their compatibility using ordinary structural materials and thermodynamic conditions.

Another very practical question is what kind of accuracy is to be expected of IR scaling technology, and upon what does the accuracy depend? Does one expect to replicate ship temperatures by scale modeling experiments to within 0.1°F, 1°F, 10°F or 100°F? We investigate these matters by identifying the variables that control ship surface temperatures (dimensionless), and then straightforwardly propagate the variable errors into the errors of surface temperatures, of which they are the cause. We present an expression that relates the relative variance of dimensionless temperature in terms of the experimental errors which underlie such variance.

Finally, we outline a test program for verification of the validity of scale model results. In what way can one ascertain that the IR scale models work? Is there a systematic manner of testing the models? We emphasize that the fundamental test concerns a comparison of temperatures from an IR scale model and the ship it represents. We discuss the comparison in terms of dimensionless quantities, emphasizing the efficiency they bring to the experimental testing process.

REVIEW AND INTERPRETATION OF THE IR LAWS OF SIMILITUDE

The scaling laws appropriate for ships in the IR have previously been derived.* The following is an outline of this derivation.

The physical problem is that of a naval vessel whose thermal properties are allowed to vary with position. Some of the compartments may be filled with fluids. Throughout the ship various processes (conductive, convective, and radiative) are allowed to occur either separately or in concert. Boundary conditions describe the heat exchanged between the ship and its surroundings.

The heat equation and its boundary conditions describe the thermodynamic system present in a naval vessel. These take the form:

$$\nabla \cdot [k(x)\nabla T(x,t)] + H(x,t) = \rho(x)c(x)\frac{\partial T}{\partial t}(x,t) \quad (4)$$

$$k(x)\nabla_n T(x,t) = h(x)\Delta T(x,t) \quad \text{for all } x \in b \quad (5)$$

* Cervenka, P.O. and L. Massa, "Laws of Infrared Similitude," CDNSWC report CARDIVNSWC-TR-94/002 (Jan 1994).

$$T(x,t)|_{t=0} = T_o(x) \quad \text{for all } x \quad (6)$$

where:

- b** = boundary
- ∇ = three-dimensional gradient operator
- ∇_n = operator yielding gradient magnitude in direction normal to boundary
- k** = thermal conductivity (power/distance/temperature)
- T** = temperature at position **x**
- x** = coordinates of a point in three-dimensional position space
- t** = coordinates of a point in time
- H** = rate of heat production per unit volume at position, **x**, and time, **t**
(power/volume)
- ρ = mass density at the position **x** (mass/volume)
- c** = specific heat at the position **x** (heat/temperature/mass)
- h** = heat transfer coefficient at the position, **x**, belonging to the boundary, **b**
(power/area/temperature differential)
- ΔT = temperature difference prevailing between the ship and its surroundings,
at position, **x**, and time, **t**
- T_o = initial temperature distribution that holds at every position, **x**,
throughout the ship

These equations have as solution the temperature function of the vessel

$$T = T(x, t, H, k, \rho, c, h, \Delta T). \quad (7)$$

Note that this is a function of position and time and depends upon heat sources, material thermodynamic properties, and system-surrounding temperature differences. The description encapsulated by the heat equation is general enough to describe complicated processes of heat transfer between a naval vessel and its usual surroundings.

By applying the heat equation to both a ship and its scale model, the IR scaling laws may be derived. The relation which holds between analogous positions on a model and a full-scale ship is

$$K_x = \frac{x^M}{x^S}. \quad (8)$$

where K_x is the scaling constant of position. If such a scaling of positions holds, geometrical similarity of model and ship prevails. One could assume that geometrical similarity is all that is required of a model in order to be used in infrared scaling studies.

However, geometrical similarity simply forces the model to "look like" the ship; there is no requirement based on geometrical similarity alone that it behave thermodynamically like the ship. That is what is required of a useful IR scale model. It must go through temperature variations analogous to those of the ship it models. A simple "look alike" model does not. For this, geometrical similarity is only the first step. Complete thermodynamic-IR similarity requires imposition of all the scaling relations which flow from simultaneous application of the heat equation and its boundary conditions to both model and ship. We define the scaling constants of time, temperature, conductivity, mass density, specific heat, heat transfer coefficient, temperature difference, and initial temperature, respectively, as

$$K_t = \frac{t^M}{t^S} \quad (9)$$

$$K_T = \frac{T^M}{T^S} \quad (10)$$

$$K_k = \frac{k^M}{k^S} \quad (11)$$

$$K_\rho = \frac{\rho^M}{\rho^S} \quad (12)$$

$$K_c = \frac{c^M}{c^S} \quad (13)$$

$$K_h = \frac{h^M}{h^S} \quad (14)$$

$$K_{\Delta T} = \frac{\Delta T^M}{\Delta T^S} \quad (15)$$

$$K_{T_o} = \frac{T_o^M}{T_o^S} \quad (16)$$

This collection of scaling relations connects all the model variables (M superscript) to those of the ship (S superscript). This allows the heat equation and its boundary conditions for the ship to be rewritten in terms of variables for the model. By comparing the pairs of equations in one set of variables applied to both ship and model, we have:

$$K_k \frac{K_t}{\rho c K_x^2} = 1 \quad (17)$$

$$\frac{K_H K_t}{K_{\rho c} K_T} = 1 \quad (18)$$

$$K_h K_x \frac{K_{\Delta T}}{K_T} = 1 \quad (19)$$

$$\frac{K_T}{K_{T_o}} = 1 \quad (20)$$

These are the IR scaling laws for the time-dependent case. They apply to realistic naval vessels in scenarios that include conduction, convection, and radiation heat transfer mechanisms.

The form of the IR scaling laws suggests a set of dimensionless variables which may be used to advantage in the design and practice of IR modeling experiments. Such dimensionless variables are:

$$\bar{T} = \frac{T - T_{\infty}}{T_o - T_{\infty}} \quad (21)$$

$$\bar{x} = \frac{x}{L} \quad (22)$$

$$\bar{t} = \frac{k}{\rho c} \frac{t}{L^2} \quad (\text{Fourier number}) \quad (23)$$

$$\bar{H} = \frac{HL^2}{k(T_o - T_\infty)} \quad (24)$$

$$\bar{h} = \frac{hL}{k} \quad (\text{Biot number}) \quad (25)$$

where the bar above a symbol indicates a dimensionless quantity, L is a "typical" length, T_∞ is a convective/radiative background temperature, and all other symbols retain their previously defined meaning.

When the above dimensionless variables are used to transform the heat equation and its companion boundary conditions, they take on the dimensionless form

$$\bar{\nabla}^2 \bar{T} + \bar{H} = \frac{\partial \bar{T}}{\partial \bar{t}} \quad (26)$$

$$\bar{\nabla}_n \bar{T} = \bar{h} \bar{T} \quad \text{for all } \bar{x} \subset b \quad (27)$$

$$\bar{T}|_{\bar{t}=0} = 1 \quad \text{for all } \bar{x} . \quad (28)$$

These last equations in manifestly dimensionless form are the full information equivalents of the previously given heat equation and boundary conditions, but now apply to any ship (or model) in the same way regardless of size. They have the same solution for ship or model, viz.,

$$\bar{T} = \bar{T}(\bar{x}, \bar{t}, \bar{H}, \bar{h}) \quad (29)$$

a function of position \bar{x} , and time \bar{t} , depending parametrically on the heat source \bar{H} , and the transfer coefficient \bar{h} . Notice \bar{T} , when compared to T , displays a reduction in the number of variables upon which it depends, an important advantage in the overall design of an IR scale model test program.

The study of ship IR radiation may be based upon the dimensionless heat equation and its dimensionless boundary conditions. They apply equally well to a prototype ship or to its scale model. This must be so because they are written in a form that is manifestly scale invariant. The same solution \bar{T} applies to ship and model. For \bar{T} to have the same magnitudes for ship and model it is only required that the variables upon which \bar{T} depends also have the same magnitudes on ship and model. These variables are the dimensionless quantities \bar{x} , \bar{t} , \bar{H} , \bar{h} . That scale model is a thermodynamic replicate of its prototype ship which has the same magnitudes as the ship for these four dimensionless variables. The purpose of an IR scale modeling laboratory is to arrange that \bar{x} , \bar{t} , \bar{H} , \bar{h} for a scale model have the same numerical values as on a prototype ship, and to measure \bar{T} for the model, which then is assured to achieve the same numerical values as \bar{T} on the ship. And \bar{T} controls the IR emissions which therefore can be studied directly with the scale model.

A further short discussion of \bar{h} is in order. The Biot number regulates the heat exchange everywhere on the boundary. In general it can have a convective and a radiative component because the regular dimensional heat transfer coefficient contains such components, i.e.,

$$h = h_c + h_r. \quad (30)$$

Obviously, to fix \bar{h} to a particular value, one must adjust h_c and h_r to appropriate magnitudes. It is of interest, therefore, to know what variables these depend upon.

We discuss h_c first. The Nusselt number, important to the description of convective processes, is defined as

$$Nu = \frac{h_c L_f}{k_f}, \quad (31)$$

where L_f is a typical length across which the convecting fluid sweeps, and k_f is the conductivity of the convecting fluid.

Rearranging this last equation, an expression for h_c is obtained of the form

$$h_c = \frac{k_f}{L_f} Nu. \quad (32)$$

In the most general case, the Nusselt number is a function that depends on three other dimensionless numbers

$$Nu = Nu(Re, Pr, Gr) \quad (33)$$

where:

$$Re = \frac{\rho L_f v}{\eta} \quad (\text{Reynolds number}) \quad (34)$$

$$Pr = \frac{\eta c}{k_f} \quad (\text{Prandtl number}) \quad (35)$$

$$Gr = \frac{\beta g \Delta T L^3 \rho^2}{\eta^2} \quad (\text{Grashof number}). \quad (36)$$

where:

- ρ = fluid mass density
- L_f = a representative length over which the fluid flows
- v = fluid free-stream velocity
- η = fluid viscosity
- c_f = fluid heat capacity
- k_f = fluid thermal conductivity
- β = fluid thermal expansion coefficient
- g = acceleration constant due to gravity
- ΔT = temperature differential between fluid and surface

Interestingly, the Nusselt number can assume various simplifications depending upon the type of convection under study. For pure free convection Re does not affect Nu significantly. For pure forced convection, Nu is insensitive to Gr. Further simplifications occur if the fluid considered is a gas, in which case for either free or forced convection Nu is insensitive to Pr. Summarizing, we have

- Nu = Nu (Pr, Gr) free convection
- Nu = Nu (Re, Pr) forced convection
- Nu = Nu (Gr) free convection (gas)
- Nu = Nu (Re) forced convection (gas)

For many flow conditions, reasonably accurate functional forms of Nu are known. In such cases these forms may be used to determine h_c through Equation (32).

Now we discuss h_r . Its form can be obtained using the Stefan-Boltzmann description of radiation exchange, from which we have

$$\frac{\dot{q}}{A} = \epsilon\sigma(T^4 - T_\infty^4), \quad (37)$$

where:

$$\begin{aligned} \frac{\dot{q}}{A} &= \text{radiation power per unit emission area} \\ \epsilon &= \text{surface emissivity} \\ \sigma &= \text{Stefan constant} \\ T_\infty &= \text{temperature of radiation background} \end{aligned}$$

We can rewrite the equation for radiation exchange as

$$\frac{\dot{q}}{A} = \epsilon\sigma(T^3 + T^2T_\infty + T T_\infty^2 + T_\infty^3)(T - T_\infty). \quad (38)$$

By definition h_r is simply the coefficient of $(T - T_\infty)$ in this last equation, thus

$$h_r = \epsilon\sigma(T^3 + T^2T_\infty + T T_\infty^2 + T_\infty^3). \quad (39)$$

However, for temperature differences $(T - T_\infty)$ that are fairly small compared to T (absolute units) an approximation utilizing T' , the average temperature is

$$T' = \frac{T + T_\infty}{2}, \quad (40)$$

and yields

$$h_r = 4\epsilon\sigma T'^3, \quad (41)$$

which is the form most often used.

The Stefan number is defined as

$$St = \epsilon \sigma T^3 \frac{L}{k}. \quad (42)$$

As a result,

$$h_r = \frac{4k}{L} St. \quad (43)$$

Having discussed both h_c and h_r , we recognize they will both contribute to the Biot number, so that

$$\bar{h} = \frac{L}{k} \left[\frac{k_f}{L_f} Nu + \frac{4k}{L} St \right]. \quad (44)$$

The heat equation discussed thus far has been the time-dependent equation. The time-independent equation is less general and is included as a special case of what has already been discussed. The relevant dimensionless variables are the same, except for the Fourier number, which is not included since time plays no role in the description.

COMPATIBILITY OF DIMENSIONLESS VARIABLES

The function \bar{T} depends upon the dimensionless variables \bar{x} , \bar{t} , \bar{H} , and \bar{h} . The temperature behavior of a model matches that of a ship prototype on condition that the magnitude of all the dimensionless variables take the same values on both model and ship. But is it possible to make all the dimensionless variables match simultaneously? The answer to this question is not obvious. In the practice of using models in other fields, it is commonly found that not all relevant dimensionless variables may be simultaneously matched. In such cases, part of the scale modeler's task is to decide which dimensionless variables are to be matched and which are to be sacrificed. At first glance, a similar situation might be expected in the case of scaling the IR. For example, the representative length L , appears in all four dimensionless variables \bar{x} , \bar{t} , \bar{H} , and \bar{h} . When a particular value of L is assigned to the first of these, it might be that the same value used in one of the remaining dimensionless variables contradicts the possibility of their taking matching magnitudes in both a ship and its model. A closer examination reveals there are no inherent incompatibilities among the dimensionless pairs, listed in column one of Table 1. For example, imagine all five π 's (or dimensionless quantities) listed are to be held constant. Imagine that L is reduced, as would occur in building a model of scale smaller than a ship.

As we examine the π 's in pairs, we find that reduction of L introduces no contradictions in the response required of any of the variables. The kind of contradiction we refer to would be an analog to that discussed in the Introduction, where constancy of

Re and Fr for one fluid demanded v to increase as well as to decrease. No such mathematical incompatibilities occur with the IR π 's. For this to occur, it would require one physical property, such as α , k , h , etc., to appear in two π 's in such a way that as L decreased, the property would be forced to respond by increasing in one π and decreasing in the other. In the list of π 's we examine, this does not happen.

Table 1. Experimental relations associated with compatibility of IR dimensionless variables. (M and S indicate model and ship values, respectively)

Dimensionless Variables	Matching Conditions	Experimental Model Variables
1. $\bar{T} = \frac{T - T_\infty}{T_o - T_\infty}$	$\frac{T^M - T_\infty^M}{T_o^M - T_\infty^M} = \frac{T^S - T_\infty^S}{T_o^S - T_\infty^S}$	$T_o^M = T_o^S$; $T_\infty^M = T_\infty^S$
2. $\bar{x} = \frac{x}{L}$	$\frac{x^M}{L^M} = \frac{x^S}{L^S}$	$x^M = \frac{L^M}{L^S} x^S = K_x x^S$ †
3. $\bar{t} = \frac{\alpha t}{L^2}$ #	$\frac{\alpha^M t^M}{L^{M2}} = \frac{\alpha^S t^S}{L^{S2}}$	$t^M = K_x^2 \frac{\alpha^S}{\alpha^M} t^S$
4. $\bar{H} = \frac{HL^2}{kT_o}$	$\frac{H^M L^{M2}}{k^M (T_o^M - T_\infty^M)} = \frac{H^S L^{S2}}{k^S (T_o^S - T_\infty^S)}$	$H^M = K_x^2 \frac{k^M}{k^S} H^S$
5. $\bar{h} = \frac{hL}{k}$	$\frac{h^M L^M}{k^M} = \frac{h^S L^S}{k^S}$	$h^M = K_x \frac{k^M}{k^S} h^S$

* The temperature equation in column 2 indicates that if the temperatures of model and ship are to be the same, then the model must be initialized so that its surface starts out at the same temperature as the full-scale ship.

† Note that by definition $K_x = \frac{L^M}{L^S}$.

$\alpha = \frac{k}{\rho c}$ = diffusivity

Even though there is no mathematical incompatibility, that does not establish the actual existence of materials and conditions adequate to satisfy the constancy requirement on the π 's. Whether or not this is the case depends on the range of variability in the physical properties controlling all the dimensionless variables. Qualitatively one may say that the more narrow the range of variability, the more difficult it is to enforce the matching of dimensionless variables of model to ship; and conversely the wider the range of variability, the easier to enforce such a match. As we show in an example below, in

the case of IR scaling no physical prohibition seems to preclude all dimensionless variables from simultaneously taking matching magnitudes on a ship and its scaled model. The consequence of this is that all dimensionless variables controlling the IR are simultaneously compatible. The ship values can all be simultaneously replicated in one physical scale model, using available structural materials and thermodynamic conditions.

The general way to enforce the equality between ship and model dimensionless variables is as follows. First, it is recognized that the thermal properties of the ship are fixed and given. That is, if a ship is to be scaled, we assume to know the materials used in its construction, for this is a requirement if scaling is to be feasible. Moreover, these materials have known thermal properties, k , ρ , c . The heat sources, H , on the ship are assumed to be known as are the conditions under which the ship operates, e.g., the ship's relative velocity with respect to water and wind, and the temperatures of water and wind. Given the information of the ship in its operating conditions, one may calculate the magnitude of the π 's from their respective definitions which apply to the ship. The analogous model π 's are to be adjusted to these same values. This is done by choosing thermal properties in accordance with the IR scaling laws and physical variables for the model which allow the correct π values to hold. Simply building a small model to look exactly like the ship certainly will not suffice. Consider the π variables one at a time in succession.

Once the scale of length K_x is chosen, all values of \bar{x} will automatically be equal on model and ship; i.e., geometrical similarity is ensured. Suppose one chooses materials to build the model based upon structural and cost considerations. The thermal properties of the material (k , ρ , c) must be known (or measured). Now, equality between Fourier numbers \bar{t} of ship and model is enforced by controlling the time variable, i.e., the time at which thermal events are measured on the model. Equality of the dimensionless heat sources is ensured by regulating the magnitude of the ordinary heat sources built into the model. The dimensionless transfer coefficients \bar{h} are made equal on model and ship, by appropriate selection of the ordinary heat transfer coefficient of the model. This, in turn, depends on control of the relative velocities of water and wind in the experiment, and also upon the choice of emissivity attached to the model surface. In summary, each of the dimensionless variables can be appropriately controlled by bringing under experimental regulation for given geometrical scale and thermal properties k , ρ , c the measurement times, heat sources, and heat transfers associated with a model.

This process of equalizing analogous π 's of ship and model through experimental control of model properties may be clarified by referring to Table 1. In the first column of the table each of the dimensionless variables is defined. The second column lists the equations which follow from matching dimensionless variables of model and ship. The third column rearranges these equations to display the magnitude that model experimental variables must assume in order for all of the π relations to be simultaneously compatible. The right side of the equations contain the choice of geometrical scale factor K_x , and the thermal properties k , α which follow from a choice of model building materials. All other variables on the right side of the equations refer to and are fixed by the ship. Notice in so far as each model variable on the left side of the equations, i.e., x^M , t^M , T_o^M , H^M , and h^M , is capable of experimental control, it follows that in principle the

compatibility of the π 's can be experimentally ensured. This, of course, assumes a range of experimental variability in the left side variables which allows the equations to hold, given the choices made on the right sides. By examining specific cases of model scales and typical ship and model materials, one can ensure that such a range of experimental variability is within realistic expectations. This is illustrated in the following example.

For convenience we reformulate two of the quantities discussed above, viz., H , the magnitude of the heat source, and h , the heat transfer coefficient. We want to rewrite H and h in terms of quantities that are directly controlled in modeling experiments, viz., power of model heat sources such as electrical resistors, velocity of fluid flow, and surface emissivity. Then we will rewrite the π , matching conditions using these quantities and examine whether they are physically realizable.

Recall that

$$H = \frac{\dot{q}}{V} \quad (45)$$

where \dot{q} equals power emitted, and V equals volume element.

The model/ship ratio of heat sources is therefore

$$\frac{H^M}{H^S} = \frac{\dot{q}^M}{\dot{q}^S} \frac{V^S}{V^M} \quad (46)$$

If we consider the scaling of volumes to obey

$$V^M = K_x^3 V^S, \quad (47)$$

then

$$\frac{H^M}{H^S} = \frac{1}{K_x^3} \frac{\dot{q}^M}{\dot{q}^S}, \quad (48)$$

a ratio we shall use presently. When we consider the experimental control of h , the heat transfer coefficient, we must account for both the convective and radiative contributions; i.e., h_c and h_r , respectively, as previously defined. The convective contribution h_c is determined by the Nusselt number, which is considered a function of the Prandtl number Pr , and the Reynolds number Re . For our purposes, it is adequate to take the specific functional form of the Nusselt number to be

$$Nu = 0.664 Pr^{1/3} Re^{1/2}, \quad (49)$$

which if used to enforce the equality of model-ship convective contributions to the Biot number gives

$$\left(\frac{V^M}{V^S} \right) = \frac{1}{K_x} \left(\frac{k^M}{k^S} \right)^2. \quad (50)$$

This is the condition for similarity between ship and model of convective exchanges.

We turn now to the radiative contribution h_r to the Biot number. Recall h_r is controlled by the Stefan number, which if used to enforce the equality of model-ship radiative contributions to the Biot number gives

$$\frac{\epsilon^M}{\epsilon^S} = \frac{L^S}{L^M} \frac{k^M}{k^S}. \quad (51)$$

For a given ship and choice of scale K_x and model material thermal properties k and α , we may arrange the information contained in the equality of model-ship π 's (Table 1) as follows:

$$\frac{t^M}{t^S} = K_x \frac{\alpha^S}{\alpha^M} \quad (52)$$

$$\frac{q^M}{q^S} = K_x \frac{k^M}{k^S} \quad (53)$$

$$\frac{V^M}{V^S} = \left(\frac{k^M}{k^S} \right)^2 \frac{1}{K_x} \quad (54)$$

$$\frac{\epsilon^M}{\epsilon^S} = \frac{1}{K_x} \frac{k^M}{k^S}. \quad (55)$$

These equations have been used to compute the information listed in Table 2, where it has been assumed that a ship is made of aluminum and the model is made of fiberglass. Using the thermal properties of these materials, the model-ship ratios given by the above equations are listed as a function of scale. The principal conclusion drawn from the table

is that conditions for simultaneous compatibility of all the dimensionless variables considered is practical, at least with the materials of this one example.

Consider, for example, the Table 2 results associated with the scale = 1/48. A patrol gunboat of length 160 ft would be modeled at a length of 3 1/3 ft. A thermal event occurring over the course of 1 hr on the aluminum gunboat would occur in 3.8 min on the fiberglass model. An engine on the gunboat emitting 5×10^6 Btu/hr of heat into the engine room would correspond to 1.77 watts on the model. Navy haze gray paint with emissivity = 0.95 on the gunboat would correspond to a model painted to emissivity = 0.26. These magnitudes for the model experimental variables appear to be practical. For this one example, the dimensionless variables associated with IR scaling are simultaneously compatible with material properties that are physically realizable. It is a reasonable expectation that similar results would ensue with a variety of other structural materials. This remains to be investigated. The equations used to calculate the results listed in Table 2, however, could be used as the basis for a wider study of the influence of material thermal properties on π -compatibility.

Table 2. Property ratios versus scale of length.*

Scale	t^M/t^S	\dot{q}^M/\dot{q}^S	V^M/V^S	ϵ^M/ϵ^S
1/48	0.063	1.2×10^{-4}	16.0×10^{-4}	0.280
2/48	0.250	2.4	8.1	0.140
3/48	0.570	3.6	5.4	0.093
4/48	1.000	4.8	4.1	0.070
5/48	1.500	6.1	3.2	0.056

* Ratios are calculated based on matching π 's in model and ship. Calculations are for an aluminum prototype and a fiberglass model, whose ratio of diffusivity = 0.0069 and ratio of conductivity = 0.0058.

ACCURACY CRITERION FOR IR SCALING TECHNOLOGY

The point of IR scale modeling is that the temperature behavior of a ship may be replicated and conveniently studied in a laboratory setting. In order for the scaled temperatures to be faithfully reproduced, the scaling laws must be obeyed, i.e., the relevant dimensionless variables of ship and scaled model must be equivalent. In this way both geometric and thermodynamic similarity are ensured. However, in any practical case similarity is never totally achieved, because of experimental uncertainties of the properties of both ship and scaled model. Therefore, quite naturally, questions arise concerning just how these uncertainties will impact the IR scaling results. Does one

expect temperature agreement between ship and model surfaces to match to within 0.1°F, 1°F, 10°F, 100°F? In what way does the expected temperature error depend upon experimental errors in the problem-defining variables? These include background temperatures and thermodynamic properties of structural materials. Given a certain measured error in a surface temperature, what experimental variable must be improved to reduce the error? Or, given known errors in experimental conditions and thermodynamic properties, what error can be expected in matching temperatures? What is an objective, quantitative measure of how good IR scaling may be considered to be? This array of related questions addresses essentially the accuracy of results reasonably expected in implementation of IR scaling technology.

The quantitative approach to answering these questions is that of error analysis and the propagation of errors. If a function is defined in terms of other variables, then the uncertainties in the variables lead in a well defined way to corresponding uncertainties in the evaluated function. In the case of IR scaling technology, the function of interest is the dimensionless temperature, \bar{T} , and the variables of interest are the dimensionless quantities \bar{x} , \bar{t} , \bar{H} and \bar{h} . We now consider how errors in the latter dimensionless quantities propagate into the former dimensionless temperature. We write the dimensionless temperature as

$$\bar{T} = \bar{T}(\bar{x}, \bar{t}, \bar{H}, \bar{h}). \quad (56)$$

We take the most probable value, $\langle \bar{T} \rangle$, to be

$$\langle \bar{T} \rangle = \bar{T}(\langle \bar{x} \rangle, \langle \bar{t} \rangle, \langle \bar{H} \rangle, \langle \bar{h} \rangle). \quad (57)$$

Taylor-expanding \bar{T} about its average value $\langle \bar{T} \rangle$, we obtain

$$\bar{T} - \langle \bar{T} \rangle = (\bar{x} - \langle \bar{x} \rangle) \frac{\partial \bar{T}}{\partial \bar{x}} + (\bar{t} - \langle \bar{t} \rangle) \frac{\partial \bar{T}}{\partial \bar{t}} + (\bar{H} - \langle \bar{H} \rangle) \frac{\partial \bar{T}}{\partial \bar{H}} + (\bar{h} - \langle \bar{h} \rangle) \frac{\partial \bar{T}}{\partial \bar{h}} + \dots \quad (58)$$

The variance of \bar{T} , i.e.,

$$\sigma_{\bar{T}}^2 = \frac{1}{N} \sum (\bar{T} - \langle \bar{T} \rangle)^2, \quad N \gg \quad (59)$$

becomes

$$\sigma_{\bar{T}}^2 = \frac{1}{N} \sum \left[(\bar{x} - \langle \bar{x} \rangle) \frac{\partial \bar{T}}{\partial \bar{x}} + (\bar{t} - \langle \bar{t} \rangle) \frac{\partial \bar{T}}{\partial \bar{t}} + (\bar{H} - \langle \bar{H} \rangle) \frac{\partial \bar{T}}{\partial \bar{H}} + (\bar{h} - \langle \bar{h} \rangle) \frac{\partial \bar{T}}{\partial \bar{h}} \right]^2 + \dots \quad (60)$$

or

$$\begin{aligned} \sigma_{\bar{T}}^2 = & \frac{1}{N} \sum \left[(\bar{x} - \langle \bar{x} \rangle)^2 \left[\frac{\partial \bar{T}}{\partial \bar{x}} \right]^2 + (\bar{t} - \langle \bar{t} \rangle)^2 \left[\frac{\partial \bar{T}}{\partial \bar{t}} \right]^2 \right] + \\ & \left[(\bar{H} - \langle \bar{H} \rangle)^2 \left[\frac{\partial \bar{T}}{\partial \bar{H}} \right]^2 + (\bar{h} - \langle \bar{h} \rangle)^2 \left[\frac{\partial \bar{T}}{\partial \bar{h}} \right]^2 \right] + \\ & \frac{2}{N} \sum \left[(\bar{x} - \langle \bar{x} \rangle) \frac{\partial \bar{T}}{\partial \bar{x}} \left[(\bar{t} - \langle \bar{t} \rangle) \frac{\partial \bar{T}}{\partial \bar{t}} + (\bar{H} - \langle \bar{H} \rangle) \frac{\partial \bar{T}}{\partial \bar{H}} + (\bar{h} - \langle \bar{h} \rangle) \frac{\partial \bar{T}}{\partial \bar{h}} \right] \right] + \\ & (\bar{t} - \langle \bar{t} \rangle) \frac{\partial \bar{T}}{\partial \bar{t}} \left[(\bar{H} - \langle \bar{H} \rangle) \frac{\partial \bar{T}}{\partial \bar{H}} + (\bar{h} - \langle \bar{h} \rangle) \frac{\partial \bar{T}}{\partial \bar{h}} \right] + \\ & \left[(\bar{H} - \langle \bar{H} \rangle) \frac{\partial \bar{T}}{\partial \bar{H}} \left[(\bar{h} - \langle \bar{h} \rangle) \frac{\partial \bar{T}}{\partial \bar{h}} \right] \right] + \dots \quad (61) \end{aligned}$$

In terms of the variances and covariances of the variables on the right side above, we have for the variance of \bar{T} ,

$$\sigma_{\bar{T}}^2 = \sigma_x^2 \left[\frac{\partial \bar{T}}{\partial \bar{x}} \right]^2 + \sigma_t^2 \left[\frac{\partial \bar{T}}{\partial \bar{t}} \right]^2 + \sigma_H^2 \left[\frac{\partial \bar{T}}{\partial \bar{H}} \right]^2 + \sigma_h^2 \left[\frac{\partial \bar{T}}{\partial \bar{h}} \right]^2 +$$

$$2 \left\{ \sigma_{xt}^2 \frac{\partial \bar{T}}{\partial x} \frac{\partial \bar{T}}{\partial t} + \sigma_{xH}^2 \frac{\partial \bar{T}}{\partial x} \frac{\partial \bar{T}}{\partial H} + \sigma_{xh}^2 \frac{\partial \bar{T}}{\partial x} \frac{\partial \bar{T}}{\partial h} + \sigma_{tH}^2 \frac{\partial \bar{T}}{\partial t} \frac{\partial \bar{T}}{\partial H} + \sigma_{th}^2 \frac{\partial \bar{T}}{\partial t} \frac{\partial \bar{T}}{\partial h} + \sigma_{Hh}^2 \frac{\partial \bar{T}}{\partial H} \frac{\partial \bar{T}}{\partial h} \right\} + \dots \quad (62)$$

The covariances may be assumed to vanish if the errors in the variables are independent. In such a case, we have the simplified expression

$$\sigma_T^2 = \sigma_x^2 \left[\frac{\partial \bar{T}}{\partial x} \right]^2 + \sigma_t^2 \left[\frac{\partial \bar{T}}{\partial t} \right]^2 + \sigma_H^2 \left[\frac{\partial \bar{T}}{\partial H} \right]^2 + \sigma_h^2 \left[\frac{\partial \bar{T}}{\partial h} \right]^2 + \dots \quad (63)$$

Here the variance in measurements of \bar{T} is given in terms of the variance in measurements of the dimensionless variables and the derivatives of the dimensionless temperature with respect to these variables. The importance of this last equation is that it allows one to quantitatively assess the expected accuracy in the dimensionless temperature.

Notice σ_T^2 depends on the variable variances and derivatives of \bar{T} with respect to them. Consider these factors one at a time. First, the variances can be analyzed in terms of the definitions of the dimensionless variables. From these definitions, we derive the following variance behaviors:

$$\frac{\sigma_x^2}{\bar{x}^2} = \frac{\sigma_x^2}{x^2} + \frac{\sigma_{K_x}^2}{K_x^2} \quad (64)$$

$$\frac{\sigma_t^2}{\bar{t}^2} = \frac{\sigma_t^2}{t^2} + \frac{\sigma_\alpha^2}{\alpha^2} + \frac{\sigma_L^2}{L^2} \quad (65)$$

$$\frac{\sigma_{\bar{H}}^2}{\bar{H}^2} = \frac{\sigma_H^2}{H^2} + \frac{\sigma_{L^2}^2}{L^2} + \frac{\sigma_k^2}{k^2} + \frac{\sigma_{T_o - T_\infty}^2}{(T_o - T_\infty)^2} \quad (66)$$

$$\frac{\sigma_h^2}{\bar{h}^2} = \frac{\sigma_h^2}{h^2} + \frac{\sigma_L^2}{L^2} + \frac{\sigma_k^2}{k^2} \quad (67)$$

Each of the experimental variables on the right side of Equations (64) through (67) can also depend upon additional variables whose errors must then be propagated. For example, we have for the heat transmission coefficient h , a convective and radiative contribution

$$h = h_c + h_r \quad (68)$$

and thus

$$\sigma_h^2 = \sigma_{h_c}^2 + \sigma_{h_r}^2 \quad (69)$$

The convective coefficient has the dependence

$$h_c = \frac{k_f}{L_f} Nu(Gr, Pr, Re), \quad (70)$$

so that

$$\frac{\sigma_{h_c}^2}{h_c^2} = \frac{\sigma_{k_f}^2}{k_f^2} + \frac{\sigma_{L_f}^2}{L_f^2} + \frac{\sigma_{Nu}^2}{Nu^2} \quad (71)$$

Of course by reapplying previously used arguments, the errors in the Nusselt number obey

$$\frac{\sigma_{Nu}^2}{Nu^2} = \frac{\sigma_{Gr}^2}{Gr^2} + \frac{\sigma_{Pr}^2}{Pr^2} + \frac{\sigma_{Re}^2}{Re^2} \quad (72)$$

The definitions of Gr, Pr, and Re can be used to express the errors on the right side of the last equation in more elementary terms. For example, the variances obey, in the case of the Reynolds number,

$$\frac{\sigma_{Re}^2}{Re^2} = \frac{\sigma_v^2}{v^2} + \frac{\sigma_{L_f}^2}{L_f^2} + \frac{\sigma_\nu^2}{\nu^2} \quad (73)$$

of the Prandtl number,

$$\frac{\sigma_{Pr}^2}{Pr^2} = \frac{\sigma_\eta^2}{\eta^2} + \frac{\sigma_{C_f}^2}{C_f^2} + \frac{\sigma_{k_f}^2}{k_f^2} \quad (74)$$

and of the Grashof number,

$$\frac{\sigma_{Gr}^2}{Gr^2} = \frac{\sigma_\beta^2}{\beta^2} + \frac{\sigma_{\Delta T}^2}{\Delta T^2} + 9 \frac{\sigma_L^2}{L^2} + \frac{\sigma_\rho^2}{\rho^2} \quad (75)$$

Returning to the radiative coefficient, we note that

$$h_r = \epsilon \sigma (T + T_\infty) (T^2 + T_\infty^2) \quad (76)$$

For temperature differences

$$\Delta T = T - T_\infty, \quad (77)$$

which may be considered small compared to T and T_∞ (expressed in absolute temperature units), h_r may be simplified using a first-order Taylor expansion, to obtain approximately

$$h_r = 4 \epsilon \sigma T'^3, \quad (78)$$

where T' is the average temperature

$$T' = \frac{T + T_\infty}{2} \quad (79)$$

Therefore, approximately

$$\frac{\sigma_{h_r}^2}{n_r^2} = \frac{\sigma_\epsilon^2}{\epsilon^2} + \frac{\sigma_{T'}^2}{T'^2} \quad (80)$$

We can now collect all of the separate contributions derived for the variance of \bar{T} . Doing so will simplify our final expression if certain reasonable assumptions are invoked. For example, it is often true that errors in measurement of position and time may be neglected compared to errors in thermodynamic properties. In the Nusselt number, if the fluid studied is a gas, its dependence on the Grashof number can be neglected, and if the same gas under similar conditions is applied to prototype and scaled model, all errors in the Prandtl number and those in the Reynolds number due to kinematic viscosity may be neglected. After collecting the remaining contributions, we obtain

$$\sigma_{\bar{T}}^2 = \frac{\sigma_\alpha^2}{\alpha^2} \left(\frac{\partial \bar{T}}{\partial t} \right)^2 + \left(\frac{\sigma_H^2}{H^2} + \frac{\sigma_k^2}{k^2} + \frac{\sigma_{T_o - T_\infty}^2}{(T_o - T_\infty)^2} \right) \left(\frac{\partial \bar{T}}{\partial H} \right)^2 +$$

$$\left(\frac{\sigma_k^2}{k^2} + \frac{\sigma_{k_f}^2}{k_f^2} + \frac{\sigma_v^2}{v^2} + \frac{\sigma_\epsilon^2}{\epsilon^2} + \frac{\sigma_{T'}^2}{T'^2} \right) \left(\frac{\partial \bar{T}}{\partial h} \right)^2 \quad (81)$$

We have now analyzed the variances upon which $\sigma_{\bar{T}}^2$ depends. Recall that $\sigma_{\bar{T}}^2$ also depends upon the derivatives of the dimensionless variables. In order to analyze these derivatives further, it is necessary to know the specific functional dependence of \bar{T} upon each of the variables \bar{x} , \bar{t} , \bar{H} and \bar{h} , and this will differ from one physical problem to another. For example, each class of ship will have its own function \bar{T} specific to that class. In general, the function is not known in advance and hence neither are its derivatives. But as the derivatives are required to evaluate $\sigma_{\bar{T}}^2$, three methods present themselves for discovering the function \bar{T} in its dependence upon \bar{x} , \bar{t} , \bar{H} and \bar{h} : (1) experimental measurement, (2) numerical calculation using computer models, and (3) analytical solution of the heat equation.

For a quick assessment of how errors might be expected to propagate in the IR scaling problem, we utilize the last of these three methods. We consider a simple model problem, for which the exact solution of the heat equation is known. We know therefore in such a case the derivatives demanded in the expression for $\sigma_{\bar{T}}^2$. We use these to evaluate $\sigma_{\bar{T}}^2$ explicitly.

A model problem whose analytical solutions are known is that of a flat plate at temperature T , cooling in a background temperature T_∞ , by convective and radiative processes. A prototype plate is shown with its scaled model in Fig. 1. The prototype plate at the initial temperature T_0 simply cools down to the background temperature T_∞ of the air. The scaled model plate behaves in a similar manner. There are no sources of heat in this model problem, and the surface temperatures are considered independent of position.

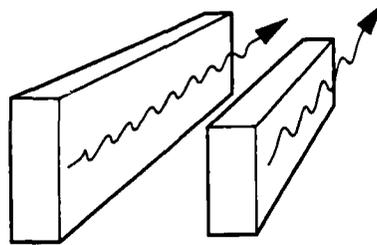


Fig. 1. Flat plate (prototype and scaled model) cooling under convective and radiative exchange.

Conservation of energy implies (for either plate)

$$hA(T - T_\infty)dt = -\rho c V dT \quad (82)$$

which can be rearranged to

$$\frac{dT}{T - T_\infty} = \frac{-hA}{\rho c V} dt, \quad (83)$$

which if integrated as

$$\int_{T_0}^T \frac{dT}{T - T_\infty} = \frac{-hA}{\rho c V} \int_0^t dt \quad (84)$$

yields

$$\ln T - T_\infty \Big|_{T_0}^T = \frac{-hA}{\rho c V} t \Big|_0^t, \quad (85)$$

and after evaluation at the limits of integration we have the solution

$$\frac{T-T_{\infty}}{T_o-T_{\infty}} = e^{-\left[\left(\frac{hA}{\rho cV}\right)t\right]} \quad (86)$$

This solution can be conveniently expressed in terms of dimensionless quantities. The left side is the dimensionless temperature. The exponent on the right side is the product of the Biot and Fourier numbers, since

$$\bar{h}\bar{t} = \left(\frac{hL}{k}\right) \left(\frac{\alpha t}{L^2}\right), \quad (87)$$

which when we use the definition

$$\alpha = \frac{k}{\rho c} \quad (88)$$

becomes

$$\bar{h}\bar{t} = \frac{hA}{\rho cV}t. \quad (89)$$

We have, therefore,

$$\bar{T} = e^{-\bar{h}\bar{t}}, \quad (90)$$

the solution in dimensionless form, and hence it is applicable to either the prototype aluminum plate or the scaled model fiberglass version. Notice the temperature differential between a plate and its background will decrease to e^{-1} of its initial value when

$$\bar{t} = \frac{1}{\bar{h}} \quad (91)$$

thus defining the dimensionless "time constant" for the cooling process. If we recall that the Biot number \bar{h} can be interpreted as a ratio of resistances to heat flow, internal to and external to the plates, then it is quite natural that the system "reaction time" \bar{t} should decrease with increasing \bar{h} .

For our purposes, the value of having an analytical solution of a model problem is that we can obtain the derivatives required to perform an error analysis of the dimensionless temperature, and thus examine the manner in which experimental errors in the IR scaling process propagate into the IR scaling measurements.

Proceeding now to calculate the variance in the dimensionless temperature to be expected as a result of modeling errors in the Biot and Fourier numbers, we have

$$\sigma_{\bar{T}}^2 = \sigma_h^2 \left(\frac{\partial \bar{T}}{\partial h} \right)^2 + \sigma_t^2 \left(\frac{\partial \bar{T}}{\partial t} \right)^2 \quad (92)$$

From the analytical solution for \bar{T} , we obtain the derivatives

$$\frac{\partial \bar{T}}{\partial h} = -\bar{t} \bar{T} \quad (93)$$

and

$$\frac{\partial \bar{T}}{\partial t} = -\bar{h} \bar{T} \quad (94)$$

Using these for the relative variance of \bar{T} , we get

$$\frac{\sigma_{\bar{T}}^2}{\bar{T}^2} = \frac{\sigma_h^2}{\bar{h}^2} + \frac{\sigma_t^2}{\bar{t}^2} \quad (95)$$

It appears that the errors in the Biot and Fourier numbers are equally important, and that the relative variance in \bar{T} is linear in that of both \bar{h} and \bar{t} . Roughly speaking, errors of a given magnitude in either the Biot number or Fourier number cause errors of like magnitude in the dimensionless temperature. Thus, to control the error in \bar{T} , one must give attention to controlling the errors in \bar{h} and \bar{t} and note that they are equally important.

If we assume that errors in position x and time t can be made negligible compared to those involving thermodynamic properties and conditions, we obtain the relative variances

$$\frac{\sigma_h^2}{\bar{h}^2} = \frac{\sigma_h^2}{h^2} + \frac{\sigma_k^2}{k^2}, \quad (96)$$

$$\frac{\sigma_t^2}{\bar{t}^2} = \frac{\sigma_\alpha^2}{\alpha^2} \quad (97)$$

and these yield a relative variance of \bar{T} which is

$$\frac{\sigma_{\bar{T}}^2}{\bar{T}^2} = \frac{\sigma_h^2}{h^2} + \frac{\sigma_k^2}{k^2} + \frac{\sigma_\alpha^2}{\alpha^2} \quad (98)$$

Here the relative variance of nondimensional \bar{T} is given in terms of the relative variance of the regular thermodynamic properties of heat transfer coefficient h , conductivity k , and diffusivity α . Controlling the errors in \bar{T} is reduced to controlling those of h , k , and α , and they are all equally important. Or conversely, knowledge of the errors in h , k , α allows one to know the expected errors in \bar{T} .

Consider further the errors associated with the heat transfer coefficient h , which is composed of two contributions, one convective and one radiative, thus

$$h = h_c + h_r \quad (99)$$

from which we obtain the variance

$$\sigma_h^2 = \sigma_{h_c}^2 + \sigma_{h_r}^2 \quad (100)$$

How do the convective and radiative processes separately contribute to the errors in \bar{T} ? Taking the convective process first, we recall that

$$h_c = \frac{k_f}{L_f} Nu \quad (101)$$

Now the functional form of the Nusselt number, Nu , has been widely studied for a variety of flow conditions and flow geometries. The average value of the Nusselt number for laminar flow over a flat plate is known to take the form

$$Nu = 0.664 Pr^{1/3} Re^{1/2} \quad (102)$$

under the conditions that

1. $0.6 \leq Pr \leq 50$,
2. $100 \leq Re \leq 5 \times 10^5$, and
3. thermodynamic properties used apply for the average temperature,

$$\frac{T_o + T_\infty}{2}$$

It follows that

$$\frac{\sigma_{h_c}^2}{h_c^2} = \frac{1}{9} \frac{\sigma_{Pr}^2}{Pr^2} + \frac{1}{4} \frac{\sigma_{Re}^2}{Re^2} \quad (103)$$

The relative variance in the convective coefficient is linear in the relative variances of the Prandtl number and the Reynolds number, but the former is less important for a given magnitude with a statistical weight of 1/9, compared to the latter with a statistical weight of 1/4. The Prandtl number depends only upon the fluid under consideration, and for the scaling experiment under discussion, the fluid air can be reasonably assumed to have exactly the same Prandtl number for prototype and model. Thus in such a case,

$$\frac{\sigma_{h_c}^2}{h_c^2} = \frac{1}{4} \frac{\sigma_{Re}^2}{Re^2} \quad (104)$$

The Reynolds number depends upon the kinematic viscosity of air, but again for this experiment, all properties of air can be taken to be identical for prototype and model, implying zero error due to kinematic viscosity. If we assume a negligible error in the measure of distance along the plates, the only error of significance arises from control of the fluid velocity, hence

$$\frac{\sigma_{h_c}^2}{h_c^2} = \frac{1}{4} \frac{\sigma_v^2}{v^2} \quad (105)$$

The relative variance of the convective contribution to the heat transfer coefficient depends only on the relative variance of the fluid velocity, and is one fourth its magnitude.

Taking up the radiative processes, with

$$h_r = \epsilon \sigma T'^3, \quad (106)$$

results in

$$\frac{\sigma_{h_r}^2}{h_r^2} = \frac{\sigma_\epsilon^2}{\epsilon^2} + 9 \frac{\sigma_{T'}^2}{T'^2} \quad (107)$$

The relative variance in h_r is linear in the relative variances of ϵ and T' , but the latter is weighted nine times more than the former.

Summing together the separate convective and radiative contributions to the variance of the overall heat transfer coefficient, we obtain

$$\sigma_h^2 = h_c^2 \left[\frac{1}{4} \frac{\sigma_v^2}{v^2} \right] + h_r^2 \left[\frac{\sigma_\epsilon^2}{\epsilon^2} + 9 \frac{\sigma_{T'}^2}{T'^2} \right], \quad (108)$$

so the relative variance of h is

$$\frac{\sigma_h^2}{h^2} = \left[\frac{h_c}{h} \right]^2 \left[\frac{1}{4} \frac{\sigma_v^2}{v^2} \right] + \left[\frac{h_r}{h} \right]^2 \left[\frac{\sigma_\epsilon^2}{\epsilon^2} + 9 \frac{\sigma_{T'}^2}{T'^2} \right]. \quad (109)$$

Now the relative variance of \bar{T} , accounting for errors in \bar{h} , k , and α is

$$\frac{\sigma_{\bar{T}}^2}{\bar{T}^2} = \left\{ \left[\left[\frac{h_c}{h} \right]^2 \left[\frac{1}{4} \frac{\sigma_v^2}{v^2} \right] + \left[\frac{h_r}{h} \right]^2 \left[\frac{\sigma_\epsilon^2}{\epsilon^2} + 9 \frac{\sigma_{T'}^2}{T'^2} \right] \right] + \frac{\sigma_k^2}{k^2} \right\} + \left[\frac{\sigma_\alpha^2}{\alpha^2} \right]. \quad (110)$$

On the right side, the first curly bracket is the total contribution due to errors in the Biot number, and the second curly bracket that due to errors in the Fourier number. Ultimately the errors in \bar{T} are due to errors in flow velocity v , emissivity ϵ , the average temperature of the background and the initial average temperature T' , the conductivity k , and the diffusivity α . The relative variance of \bar{T} is linear in the relative variance of each variable v , ϵ , T' , k , and α . The importance of the relative variance for each of these variables in the overall error is measured by its statistical weight shown in the last equation.

The importance of the last equation is that it gives the quantitative connection which holds between the errors in the fundamental dimensionless variable \bar{T} , which determines the thermal state of the ship, and the errors in variables controlling the experiment. This equation provides a numerical answer to questions concerning the quality of the IR scaling experiment.

By inspecting the equation giving the relative variance of \bar{T} , a few general conclusions may be drawn. As errors in each of the experimental variables, v , ϵ , T' , k , α , increase so too do errors in \bar{T} . Other factors being equal, the variance of \bar{T} increases with the fraction of h due to radiation. These conclusions are verified by the numerical results, based on the equation for variance of \bar{T} , shown in Table 3.

For example, the first row of Table 3 shows that, for $h_c/h = 1.00$ and $h_r/h=0$, as the relative standard deviation of the experimental variables increases from 0.10% to 30%, that of \bar{T} increases from 0.15% to 45%. The first column shows that, for the relative standard deviation of the experimental values fixed at 0.1%, that of \bar{T} increases from 0.15% to 0.30% as (h_c/h) decreases from one to zero.

Of course, once the variance in the dimensionless temperature \bar{T} is known, this will also determine the variance in the normal temperature T . Since

$$T = \bar{T}(T_o - T_\infty) + T_\infty, \quad (111)$$

Table 3. Relative standard deviation in \bar{T} , $e_{\bar{T}}$, for given (h_c/h) and e_x .*

h_c/h	$e_x = 0.001$	0.010	0.100	0.200	0.300
1.00	15.00×10^{-4}	15.00×10^{-3}	15.00×10^{-2}	30.00×10^{-2}	45.00×10^{-2}
0.75	16.00	16.00	16.00	33.00	50.00
0.50	21.00	21.00	21.00	43.00	64.00
0.25	27.00	27.00	27.00	55.00	83.00
0.00	30.00	34.00	34.00	69.99	104.00

* Relative standard deviation in x , e_x . Here x is any of the variables upon which \bar{T} depends. For simplicity, all x 's are assumed to have the same relative errors.

$$e_{\bar{T}} \equiv \sqrt{\frac{\sigma_{\bar{T}}^2}{\bar{T}^2}}, \quad e_x \equiv \sqrt{\frac{\sigma_x^2}{x^2}}, \quad \text{and} \quad \frac{h_c}{h} + \frac{h_r}{h} = 1$$

it follows that

$$\sigma_{\bar{T}}^2 = \sigma_{\bar{T}(T_o - T_\infty)}^2 + \sigma_{T_\infty}^2 \quad (112)$$

The first term on the right of this equation may be expanded according to

$$\frac{\sigma_{\bar{T}(T_o - T_\infty)}^2}{\bar{T}^2 (T_o - T_\infty)^2} = \frac{\sigma_{\bar{T}}^2}{\bar{T}^2} + \frac{\sigma_{T_o - T_\infty}^2}{(T_o - T_\infty)^2} \quad (113)$$

so that

$$\sigma_{\bar{T}}^2 = (T_o - T_\infty)^2 \frac{\sigma_{\bar{T}}^2}{\bar{T}^2} + \bar{T}^2 \sigma_{T_o - T_\infty}^2 + \sigma_{T_\infty}^2 \quad (114)$$

This gives the variance of T in terms of the variance of \bar{T} . If it is desired, this can also be expressed explicitly in terms of the relative variance of \bar{T} , as

$$\sigma_{\bar{T}}^2 = (T - T_\infty)^2 \frac{\sigma_{\bar{T}}^2}{\bar{T}^2} + \bar{T}^2 \sigma_{T_o - T_\infty}^2 + \sigma_{T_\infty}^2 \quad (115)$$

Now let us turn our attention to a numerical application of the equation for relative variance of the dimensionless temperature. We consider the problem of a flat aluminum plate cooling toward a lower background temperature due to heat transfer by convection and radiation. The data used for the calculation are listed in Table 4.

Table 4. Data for calculation of h_c and h_r .

k_f , Btu/hr·ft·°F	L_f , ft	P_r	v , ft/sec	μ , ft ² /sec	ϵ	σ	T , °R
0.0152	1.04	0.70	0.0352	15.0×10^{-5}	0.27	0.1710×10^{-8}	530.0

In Table 5, we list the relative variance of \bar{T} and its square root, $e_{\bar{T}}$ which we take to be a measure of the fractional error in \bar{T} . For ease of display, we have assumed all relative errors in the thermodynamic properties to be of equal magnitude, i.e., all 1% or all 10%, etc. Obviously they will not, in a practical case, be all the same; but the general nature of the conclusions reached under our assumption will still hold. Table 5 shows that, for the conditions of the experiment considered, as the relative errors of thermodynamic properties grow from 1% to 20%, the \bar{T} relative error $e_{\bar{T}}$ varies from 2.5% to 51%. This corresponds to a standard deviation in the surface temperature which varies from 1.48°F to 2.68°F if it is assumed that the background temperature and the initial state temperatures can be measured to within a variance of 1°F². If instead, these temperatures could only be measured to an accuracy with a variance of 5°F² then the range of surface temperature error would vary from 3.59°F to 3.90°F.

Table 5. Relative standard deviation in \bar{T} and standard deviation in T .
(x and e_x as defined in Table 3)

e_x	0.010	0.100	0.200	$\sigma_{T_o-T_\infty}^2 = \sigma_T^2 = 0$ $= 1$ $= 5$
$e_{\bar{T}}$	0.025	0.260	0.510	
σ_T , °F	0.110	1.100	2.300	
	1.500	1.800	2.700	
	3.500	3.600	3.900	

SCALE MODEL TEST PROGRAM DESIGN

Given an existing IR scale model of a ship, a fair question follows. Does the IR scale model work? Does it do what it is supposed to do, viz., behave thermally, just as does the prototype ship? In what follows, we outline the design of a test program from which answers to the questions posed would naturally emerge.

The underlying idea of IR scale modeling is that the IR contrast is equal to a ship's surface temperature minus its background temperature. An information equivalent to the surface temperature is the dimensionless temperature function \bar{T} . In IR, a ship can be described by its dimensionless temperature $\bar{T} = \bar{T}(\bar{x}, \bar{t}, \bar{H}, \bar{h})$, which is a function of dimensionless variables. One desires knowledge of the ship's function \bar{T} .

A faithful IR scale model has the same dimensionless temperature function as its prototype. To know one is to know the other. According to the theory developed for such scaling, both \bar{T} 's are the same if and only if the set of dimensionless variables \bar{x} , \bar{t} , \bar{H} , \bar{h} are prearranged to be the same on model and ship. How such prearrangement is made to occur concerns the design of the IR scale model and measurements laboratory. Assuming this has been carried out, here we will only discuss the design of a program to test a model.

Figure 2 shows a visible spectrum photograph of a ship at sea and a corresponding laboratory instrumented model. Ultimately the test of a model is to compare its \bar{T} -function to that of its prototype ship. If the pair of \bar{T} -functions is the same, the model works; if not, it does not. Therefore the point of a test program is to compare the model/ship prototype \bar{T} -function pair.

To make the development of our discussion easier, let us assume measurements have been made, and we know the \bar{T} -function of the ship being modeled. For the ship, we know \bar{T} and its companion relative variances $\sigma_{\bar{T}}^2/\bar{T}^2$. What remains is (1) to discover the \bar{T} -function (including its variances $\sigma_{\bar{T}}^2$) for the model and (2) to compare the \bar{T} -functions of model and ship. We consider these two steps one at a time.

First, the model's \bar{T} -function must be determined by measurements. Since the function sought is $\bar{T}(\bar{x}, \bar{t}, \bar{H}, \bar{h})$, the measurements to be made can simply be read out of the function symbolism, i.e., we must measure \bar{T} , \bar{x} , \bar{t} , \bar{H} , and \bar{h} . Indeed, to know the function \bar{T} simply means to know the value of \bar{T} for all values of the variables \bar{x} , \bar{t} , \bar{H} , and \bar{h} . Of course, all of these dimensionless quantities are compounded of sets of other variables and therefore by a measurement of one dimensionless variable, we understand a set of measurements on several regular variables. For example, to measure one value of \bar{T} , as we may see from its definition, is tantamount to a set of measurements on T , T_0 and T_∞ . The function \bar{T} is discovered quite methodically. The result of such measurements is a table of values which contains measured values of \bar{T} in a one-to-one correspondence with measured values of the variables \bar{x} , \bar{t} , \bar{H} , and \bar{h} . Knowledge of such a correspondence is just what is meant by knowledge of the function \bar{T} . By such measurement the function \bar{T} is experimentally determined. Moreover, by repetition of

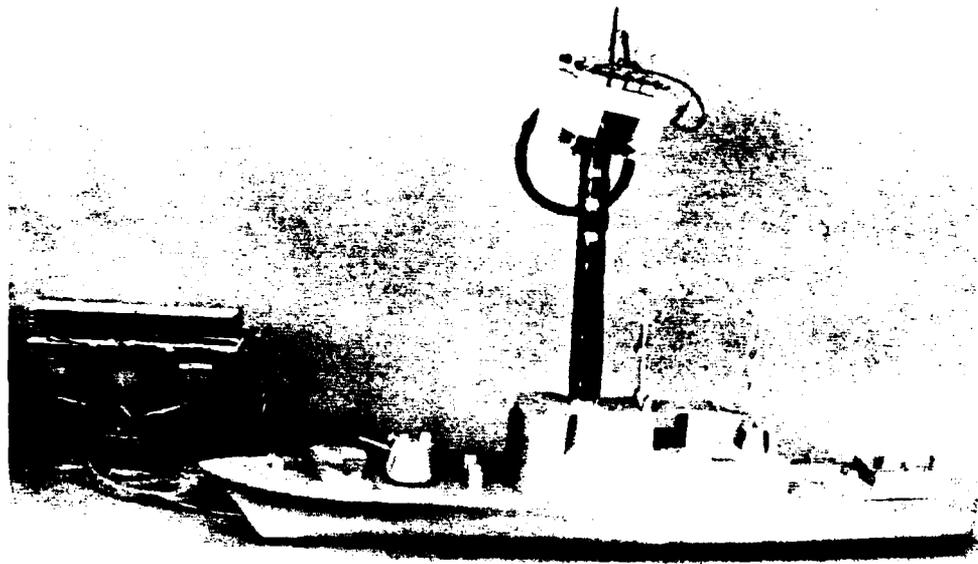


Fig. 2. Visible spectrum photographs of a PG-100 class patrol gunboat at sea (upper) and a corresponding 1/48-scale instrumented model (lower).

such measurements an experimental distribution of values is obtained. From such an experimental distribution, one calculates, in accordance with its definition, the variance $\sigma_{\bar{T}}^2$, which is companion to the experimental function \bar{T} . Similar remarks apply to the calculation of variances for the dimensionless variables \bar{x} , \bar{t} , \bar{H} , \bar{h} , and the regular variables from which they are compounded. That is to say, the distribution of experimental measurements inherent to measurement of \bar{T} leads naturally to experimental knowledge of the relative variances of all the variables upon which \bar{T} depends.

Hence, in the expression

$$\sigma_{\bar{T}}^2 = \sigma_{\bar{x}}^2 \left[\frac{\partial \bar{T}}{\partial \bar{x}} \right]^2 + \sigma_{\bar{t}}^2 \left[\frac{\partial \bar{T}}{\partial \bar{t}} \right]^2 + \sigma_{\bar{H}}^2 \left[\frac{\partial \bar{T}}{\partial \bar{H}} \right]^2 + \sigma_{\bar{h}}^2 \left[\frac{\partial \bar{T}}{\partial \bar{h}} \right]^2, \quad (116)$$

one has independent experimental knowledge of all terms in the equation, left side and right side. Therefore, for a test program check whether or not the theoretical expression above is verified by the experimental results. If it is, the experimental errors on the left may be considered as being due to the experimental errors on the right. Now, with the function \bar{T} and its variances $\sigma_{\bar{T}}^2$ experimentally determined, we turn our attention to comparison of ship and model.

Basically, we want to know whether or not the \bar{T} -functions of model and ship are the same. But we need an objective criterion for deciding the issue, that is, a numerical one. The standard practice to make such a decision is as follows. One views the \bar{T} -function for the model as a "band" of values "arcing" through the space of the dimensionless variables. The width of the "band" is defined by the error measure $\sigma_{\bar{T}}$. The experimental standard is similarly viewed, its own "band" of error having width defined by its own experimental $\sigma_{\bar{T}}$. If the "distance" between the two functions is less than the combined width of the error "bands" which surround the functions, they are the same, i.e., within experimental error; see Fig. 3. In this figure, π stands for any relevant dimensionless variable that is varied over the course of an experiment, while π' stands for any set of dimensionless variables held constant. The measure of the "distance" between the functions is given by the comparison function

$$\chi^2 = \sum \frac{(\bar{T}^M - \bar{T}^S)^2}{\sigma^2}. \quad (117)$$

The sum is over all experimental measurements of the model ship \bar{T} 's, each weighted with its variance σ^2 , reciprocal. This is an unbiased weighting scheme which effectively weights more of those points of lesser uncertainty.

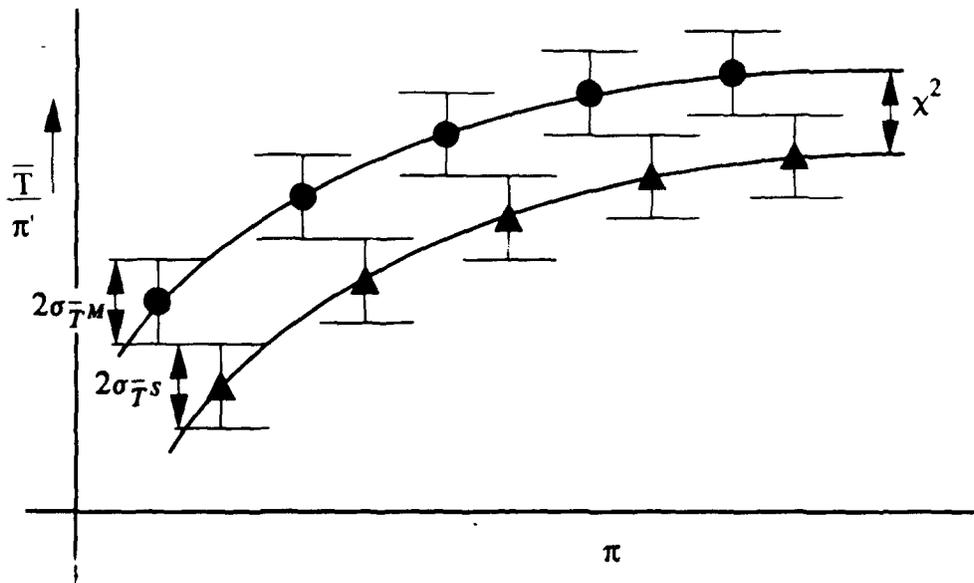


Fig. 3. Comparison criterion $\chi^2 \leq \sigma_{T_M}^2 + \sigma_{T_S}^2$.

The criterion for matching \bar{T} 's is then

$$\chi^2 \leq \sigma_{T_M}^2 + \sigma_{T_S}^2. \quad (118)$$

Satisfaction of the inequality is regarded as a match within experimental errors.

In summary, an IR scale modeling test program measures the \bar{T} -functions of model and ship together with their companion error "bands" $\sigma_{\bar{T}}$, and judges the similarity of the functions based on whether or not the "distance" separating the functions χ^2 is less than the combined width of the error "bands" around them.

This discussion of test program design implies that one must measure the variables \bar{T} , \bar{x} , \bar{t} , \bar{H} , \bar{h} ; but this requires, in turn, measurements of the variables T , T_0 , T_∞ , x , L , X , α , t , H , h , k some of which can be measured once and for all for a given model, e.g., L , α , k , but others must be actively monitored over the course of an experiment, e.g., T , T_0 , T_∞ , x , t , H , h . In general, h will depend upon k_f , P_r , ν , and v , and only v need be actively monitored during an actual experimental run.

One of the principal instruments useful for gathering test program data is a FLIR imaging device. A single frame image from a FLIR would correspond to data for which \bar{x} is a variable that changes over the space of the frame, with all other dimensionless variables held constant for the instant of that frame. A sequence of image frames would correspond to data in which the variable \bar{t} would change from frame to frame. Each frame might be thought of as labeled with a different value of \bar{t} .

Building a test program upon the framework of dimensionless variables introduces an important economy into the process. The reason for this is that the number of dimensionless variables required to specify \bar{T} is fewer than the number of regular variables required for T , since as one may recall

$$T = T(x, t, k, H, \alpha, h, \Delta T) \quad (119)$$

whereas,

$$\bar{T} = \bar{T}(\bar{x}, \bar{t}, \bar{H}, \bar{h}) . \quad (120)$$

An argument may be raised in opposition by asserting that although the number of dimensionless variables is reduced, each of them, since they are compounded of regular variables in their very definition, must invoke measurements of all the regular variables for their determination. Therefore, it would be asserted that if the regular variables must be measured anyway, there can be no real experimental economies achieved.

But such arguments fail because they do not take into account the economy of organization introduced by use of dimensionless numbers. We may illustrate this by reference to the actual practice followed in experimentally determining a function. For example, we may seek a function of one variable $y = f(x)$. How does one by measurement determine the rule f ? In the range of x one chooses N_M discrete values of x and at those values determines y . The number N_M might typically be 10, 20, or 100 depending on how well determined we wish to make f , since as N increases so does the accuracy of the determination. The number of experiments required in any event is N_M^1 , i.e., the number of discrete mesh points in the space of the variable raised to the power one, which is the number of variables. But suppose we should have a function of two variables, $y = f(x_1, x_2)$. Now a determination of f which samples the two-variable space with a mesh of test points equally efficient to those in the one variable case would require N_M^2 points, where the power 2 is again the number of variables. By extension, in the general case where the fineness of the experimental mesh along a variable is measured by N_M and the number of variables is counted to be N_V , the number of experiments E_f , to determine the function f becomes

$$E_f = N_M^{N_V} . \quad (121)$$

The experimental "cost" rises exponentially with the number of variables to be tested. It is because of this exponential dependence that the reduction in numbers of variables, even a few variables, by use of dimensionless variables is important. Although it is true that the dimensionless variables depend upon the regular variables, they are organized in such fashion that the number of experiments involved does not depend upon the number of regular variables exponentially. There is a savings inherent in the organization imposed naturally by the use of dimensionless numbers in the description of the experiment. Suppose for example, one is to test a function at N_M points in the space

of a dimensionless variable. The number, as said before, of measurements required is N_M^1 . If the dimensionless variable is compounded of three regular variables, for example, then each determination of the dimensionless variable requires three determinations of regular variables. We obtain in such a case

$$E_f = 3N_M^1, \quad (122)$$

a number which is generally smaller than N_M^3 which would be the number of tests if only the regular variables were used. By extension, if the dimensionless variables in number N_V , each depend upon regular variables in number N_R or less, then in such a case

$$E_f \leq (N_R N_M)^{N_V}, \quad (123)$$

a number which is generally smaller than $N_M^{N_R}$ since $N_V < N_R$.

A more specific example would be one in which $N_M = 10$, $N_{RT} = 4$, where N_{RT} is the total number of regular variables, $N_V = 2$, and $N_R \leq 2$. Notice,

$$(N_R N_M)^{N_V} < N_M^{N_{RT}}, \quad (124)$$

that is,

$$(2 \times 10)^2 < (10)^4. \quad (125)$$

By numerically evaluating the two sides of the above general equality for several values of the parameters involved, one may reach the conclusion that the use of dimensionless numbers reduces the number of experimental measurements required to determine a function of several variables.

SUMMARY AND CONCLUSIONS

The known IR scaling laws that flow from an analysis of the heat equation have been reviewed. The scaling laws lead to the important dimensionless variables relevant to the IR, Equations (21) through (25). These variables are \bar{T} , \bar{x} , \bar{t} , \bar{H} , and \bar{h} . When the last four of these are assumed equal on ship and model, one achieves thermodynamic similarity which results in $\bar{T}^M = \bar{T}^S$. The goal of IR scale modeling is achieved by assuring the constancy of the π 's between ship and scale model.

Now when the constancy of the π 's is so imposed, a question arises whether all the π 's are mutually compatible with one another. Mathematically they prove to be compatible, but this in no way assures they are physically compatible. We have studied this matter by reference to one ideal example, that of an aluminum ship scale modeled in

fiberglass. At a scale of 1/48, it occurs that enforcing the constancy of the π 's would be physically possible under common laboratory conditions. Of course for different materials, and complicated combinations of materials, the π -compatibility might be more or less difficult to achieve than is the case with the simple example studied here. This remains to be investigated. However, the ideas applied here, encapsulated in the requirements of Equations (52) to (55), would form the basis of a wider study. The physically realizable π -compatibility in the case of an aluminum ship and fiberglass scale model in the meantime augurs well for results anticipated with other structural materials.

As stated, if we equate the appropriate set of π 's on ship and model, the \bar{T} -function will be the same for both. But in practice there are always errors in the scaling experiment. Therefore we have addressed the question of how one can get a quantitative estimate of the errors to be expected. The basic answer is arrived at by identifying the variables on which \bar{T} depends, and then propagating the errors in these variables into \bar{T} . In Equation (81) the variance expected in \bar{T} is written in terms of the relative variances of the variables on which it depends and the derivatives of \bar{T} with respect to these variables. In order to evaluate these derivatives, a model problem was chosen for which \bar{T} was known exactly and therefore so were its derivatives. The model problem was that of a plate cooling under combined convection and radiation. A formula for the relative variance was calculated for the cooling plate problem and displayed in Equation (110). Once the errors in \bar{T} are known of course, the errors in T , the regular temperature of the plate, can be obtained as we indicate in Equation (115). Using the expressions for the errors associated with a cooling plate, specific materials and conditions may be chosen to obtain numerical values for the errors. For this purpose, an aluminum plate was modeled by a smaller fiberglass plate. For the particular conditions chosen in the text, if the background and initial temperatures are known to 1°F, as the magnitude of the relative errors in thermal properties grew from 1% to 20%, the standard deviation in the surface temperature associated with the model plate varied from 1.5°F to 2.7°F. The temperature of the plate modeled was 70°F. So for example, at the lower range of error, scale measurements of temperature would predict the result to be 70°F \pm 1.5°F. The percentage error in the scale measurement of surface temperature would be 2.1%. At the higher standard deviation (2.7°F), the same measurement would correspond to a percentage error in the temperature equal to 3.9%. Importantly, if it is desired to improve the errors in the temperature measurement, Equations (110) and (115) show how these errors depend on those in the experimental variables. Improving the experimental errors in any one of the variables would propagate into an improvement in the measured temperatures, and Equations (110) and (115) indicate how big the expected improvement would be. Of course, Equations (110) and (115) are particular results which only apply to a cooling plate of small Biot number.

Such a particular system was studied because this is a case where the temperature solution and its derivatives are known exactly. This allowed numerical values to be obtained for the expected temperature errors associated with scaling. However, the more general equation for the errors expected in the scaling experiment is given in Equation (81). This equation can be used as the basis for a numerical error analysis of complex scaling experiments. The pattern for such analysis, however, is that illustrated here for the simple cooling plate system.

The equations associated with an analysis of experimental error are also useful in the design and implementation of a scale model test program. In such a program, it is helpful to know whether a given scale model in fact thermally replicates a prototype ship. This requires comparing \bar{T} for the model and ship when both are under equivalent thermal conditions. Since the \bar{T} 's will never be the same on ship and model under realistic experimental conditions, it is required to have a measure of "distance" between the dimensionless temperature functions. This is provided in Equation (117). The criterion for whether this "distance" falls within the expected experimental errors discussed above is given by Equation (118). Of course, a test program could be based upon many comparison tests. The comparison suggested here has been based upon the function \bar{T} because it is a function of an optimal number of controlling experimental variables that are dimensionless in form, and thus impose an efficiency upon the organization of measurements.

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