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Fields and waves in elliptic waveguides

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Licht elliptische golfpijpen kunnen gebruikt worden voor het maken van hoog vermogen millimeter-golf polarisers. Een andere toepassing wordt gevonden in de constructie van een magnetron voorverwarmde auto-katalysator. Een nadeel van elliptische golfpijpen is de complexiteit van het berekenen van de elektromagnetische eigenschappen.

Teneinde de afsnij-golflengten te berekenen van elliptische golfpijpen zonder de wortels van Mathieu functies te bepalen, wordt een eerste orde benadering afgeleid.

De elliptische golfpijp wordt ontbonden in twee cirkelvormige golfpijpen voor even en oneven mode propagatie. De afsnijgolflengten van deze golfpijpen worden berekend en gebruikt als benaderde afsnijgolflengten van de elliptische golfpijp.

Afsnijgolflengten voor de dominante TE_{11}-mode kunnen benaderd worden met een maximale fout van 5 procent voor alle eccentriciteiten. De benadering is ook geldig voor de andere modes voor kleine eccentriciteiten.

De benadering is met succes toegepast in het ontwerp en de realisatie van hoog vermogen millimetergolf polarisers.

Contents

1 INTRODUCTION

Slightly elliptic waveguides can, for example, be used for making high power millimetre-wave polarisers [1]. Another application is found in the construction of a magnetron pre-heated catalytic converter [2]. The problem with elliptic waveguides, however, is the difficulty in calculating the electromagnetic field properties.

In order to be not completely dependent on experimental methods, an easy to understand approximate method for calculating cut-off wavelengths is developed. The idea, originating from Frans A. Nennie, is explained and results are compared with exact solutions, yielding the field of reliability of the approximate cut-off wavelength formula.

2 SOLUTION OF MAXWELL'S EQUATIONS FOR AN ELLIPTIC CYLINDER

For an elliptic coordinate system as shown in figure 1, the following relations apply:

$$Z = Z \tag{1a}$$

$$X = q\cosh\zeta\cos\eta \tag{1b}$$

$$Y = q\sinh\zeta\sin\eta \tag{1c}$$

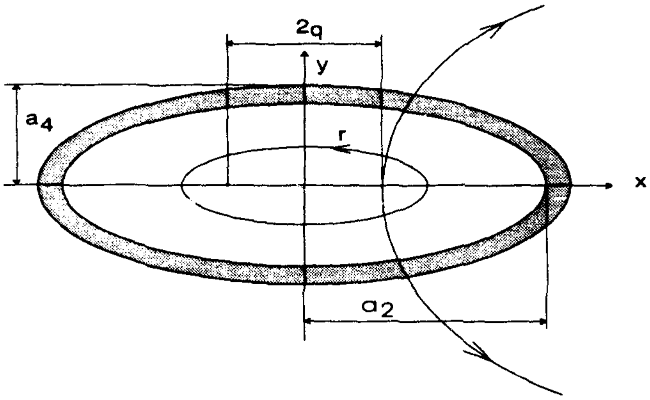

Fig. 1: Elliptic coordinate system

The following expressions for the axial electric field E_z and the axial magnetic field H_z are found [3]:

$$\left.\begin{matrix} E_z \\ H_z \end{matrix}\right\} = \left[B_1 S_{en}(\eta) R_{en}(\zeta) + B_2 S_{on}(\eta) R_{on}(\zeta)\right] e^{j\omega t - jkz} \tag{2}$$

where B_1 and B_2 are complex constants, S_{en} and S_{on} are the even and odd angular Mathieu functions of order n, and R_{en} and R_{on} are the even and odd radial Mathieu functions of order n, where n is a positive integer.

Applying the boundary conditions for TM waves leads to [3]:

$$\left.\begin{matrix} R_{en}(\zeta_o) \\ R_{on}(\zeta_o) \end{matrix}\right\} = 0 \tag{3}$$

Applying the boundary conditions for TE waves leads to [3]:

$$\left.\begin{matrix} R'_{en}(\zeta_o) \\ R'_{on}(\zeta_o) \end{matrix}\right\} = 0 \tag{4}$$

The prime denotes the derivative with respect to ζ.

Calculating the roots of Mathieu functions is far from easy [4, 5], so that the search for an approximate method to calculate electromagnetic field properties in elliptic waveguides is appropriate

3 APPROXIMATE METHOD

The dominant mode of an elliptic waveguide is the TE_{11}-mode [6]. Figure 2 shows the even and odd TE_{11}-mode patterns for elliptic waveguides with axes $2a_1$ (long) and $2a_2$ (short). The TE_{11}-mode patterns for circular waveguides with radii a_1 and a_2 are shown in the same figure.

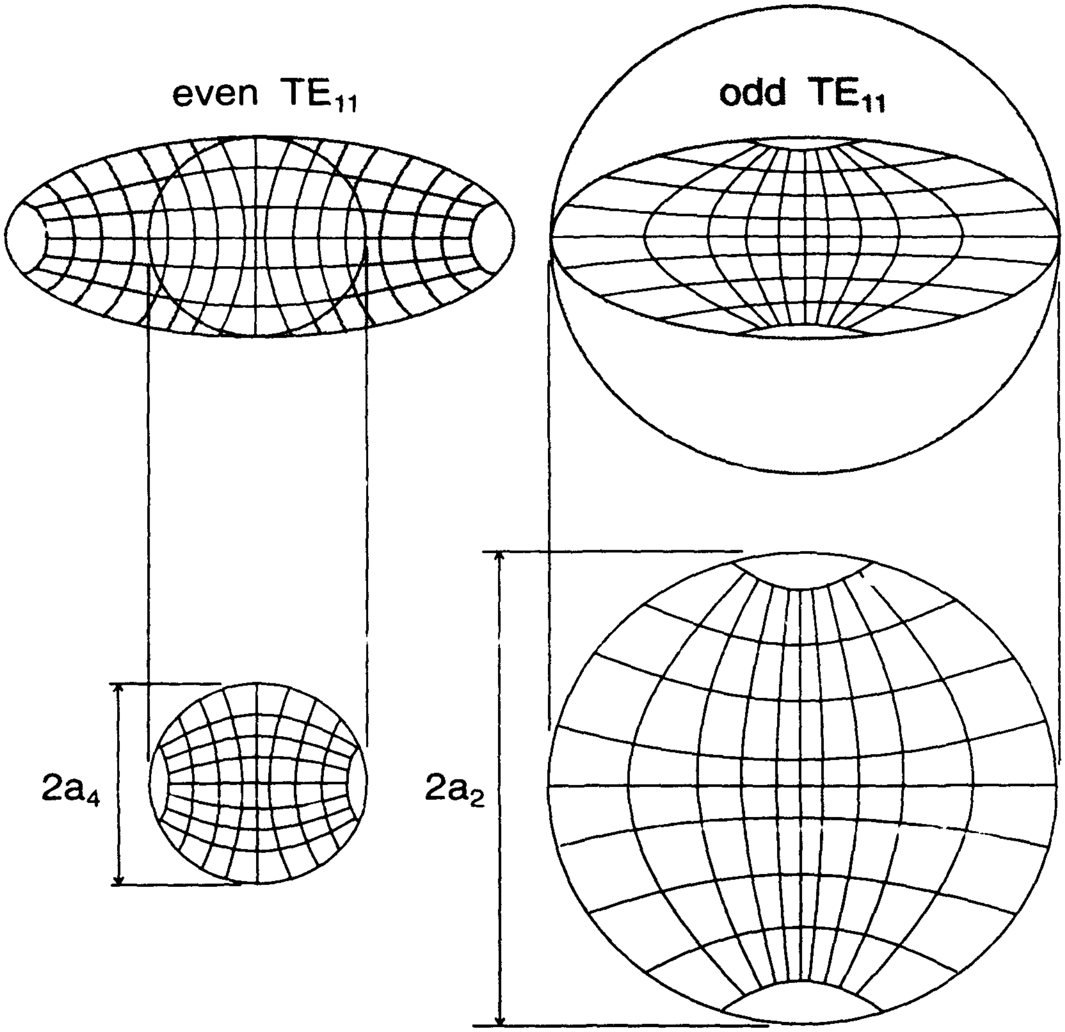

Fig. 2: TE_{11}-mode patterns for elliptic and circular waveguides

Since the mode patterns of an elliptic waveguide are very similar to those of a circular waveguide with a diameter equal to the ellipse axis length parallel to the E-field, it is likely that the cut-off wavelengths for even and odd mode can be approximated by the cut-off wavelengths of the corresponding circular waveguides. The cut-off wavelength of a circular waveguide is easy to calculate. Naturally, the error in the approximation increases with increasing eccentricity of the elliptic waveguide.

The cut-off wavelength is thus approximated by, for TE-waves [6]:

$$\lambda_c = \frac{2\pi}{P_{nl}} a_i \qquad (i = 1,2) \tag{5a}$$

$$J_n(P_{nl}) = 0 \tag{5b}$$

with p_{nl} root of the Bessel function of order n.

For TM-waves:

$$\lambda_c^* = \frac{2\pi}{P_{nl}} a_i \qquad (i = 1,2) \tag{6a}$$

$$J_n{}^*\left(P_{nl}{}^*\right) = 0 \tag{6b}$$

with $p_{nl}{}^*$ root of the Bessel function of order n.

The roots of the Bessel functions [5] are substituted in (5a) and (6a) and shown in table 1.

Table 1: Cut-off wavelengths circular waveguide

mode	TM_{01}	TM_{11}	TE_{01}	TE_{11}
	$\frac{2\pi a_i}{2.40482}$	$\frac{2\pi a_i}{3.83171}$	$\frac{2\pi a_i}{3.83171}$	$\frac{2\pi a_i}{1.84118}$

4 COMPARISON EXACT AND APPROXIMATE METHOD

For comparison of the approximate cut-off frequencies with the exact ones, use is made of the formulae of Kretzschmar for calculating the roots of Mathieu functions and the roots of the derivatives of Mathieu functions [7]. These formulae are shown in table 2.
In these formulae, e is the eccentricity of the elliptic waveguide, defined as [4]:

$$e = 1/\cosh\zeta_0 = \sqrt{1-(a_2/a_1)^2} \tag{7}$$

The cut-off wavelengths are given by [7]:

$$\lambda_c = \frac{\pi a e}{\sqrt{q_c}} \tag{8}$$

a is half the length of the long axis of the elliptic waveguide.

With the above and equations (5,6) the deviations of the cut-off wavelengths from the exact ones as function of eccentricity are calculated. The results are shown in table 3.

Table 2: Roots of Mathieu functions

mode	formula	interval e	max. rel. error
TE_{e11}	$\bar{q}_{e11} = 0.8476e^2 - 0.0379e^4$	[0.0, 0.4]	0.01 %
	$\bar{q}_{e11} = -0.0064e + 0.8838e^2 - 0.0696e^3 + 0.0820e^4$	[0.4, 1.0]	0.02 %
	$\bar{q}_{e11} = -0.00012e + 0.8520e^2 - 0.0196e^3 + 0.0573e^4$	[0.0, 0.3]	0.20 %
		[0.3, 1.0]	0.04 %
TE_{e01}	$\bar{q}_{e01} = -0.0073e + 3.8569e^2 - 1.3105e^3 + 4.6802e^4$	[0.05, 0.45]	0.3 %
	$\bar{q}_{e01} = -1.2264 - 1.3936e + 1.5515e^2 + 1.3156/(1-e)$	[0.45, 0.95]	0.3 %
TE_{o11}	$\bar{q}_{o11} = -0.0018e + 0.8974e^2 - 0.3679e^3 + 1.612e^4$	[0.05, 0.50]	0.4 %
	$\bar{q}_{o11} = -0.1483 - 1.0821e + 1.0829e^2 + \frac{0.3493}{(1-e)}$	[0.50, 0.95]	0.5 %
TM_{e01}	$q_{e01} = -0.0016e + 1.488e^2 - 0.314e^3 + 1.425e^4$	[0.05, 0.50]	0.2 %
	$q_{e01} = -0.222 - 0.728e + 1.308e^2 + \frac{0.341}{(1-e)}$	[0.50, 0.95]	0.2 %
TM_{e11}	$q_{e11} = -0.0049e + 3.7888e^2 - 0.7228e^3 + 2.2314e^4$	[0.05, 0.55]	0.3 %
	$q_{e11} = -0.1379 - 1.3138e + 3.9307e^2 + \frac{0.4056}{(1-e)}$	[0.55, 0.95]	0.3 %
TM_{o11}	$q_{o11} = -0.0063e + 3.8316e^2 - 1.1351e^3 + 5.2229e^4$	[0.05, 0.45]	0.3 %
	$q_{o11} = -1.2014 - 1.6271e + 2.1684e^2 + \frac{1.3089}{(1-e)}$	[0.45, 0.95]	0.3 %

Table 3: Deviation approximate cut-off wavelengths

e	TE_{e11}	TE_{e01}	TE_{o11}	TM_{e01}	TM_{e11}	TM_{o11}
0.1	0.15 %	0.10 %	0.23 %	0.14 %	0.27 %	0.04 %
0.2	0.15 %	1.03 %	0.16 %	1.01 %	0.54 %	0.53 %
0.3	0.21 %	2.16 %	0.31 %	2.42 %	1.17 %	1.17 %
0.4	0.33 %	3.57 %	0.26 %	4.74 %	2.34 %	1.99 %
0.5	0.55 %	5.36 %	0.77 %	7.93 %	4.07 %	3.26 %
0.6	0.79 %	7.02 %	1.21 %	13.00 %	6.65 %	4.70 %
0.7	1.09 %	8.64 %	1.59 %	21.51 %	11.11 %	6.47 %
0.8	1.47 %	10.47 %	2.75 %	37.03 %	19.50 %	8.78 %
0.9	1.93 %	12.67 %	5.20 %	75.14 %	41.11 %	11.77 %

The above table shows that the approximate method works well for all modes for small eccentricities. The method works well (within 6 percent) for all eccentricities for the dominant TE_{11}-mode.

This method has been used in the design and realisation of high power millimetre-wave polarisers [1].

5 CONCLUSIONS

Shown is that, in first order approximation, elliptic waveguides can be thought of as being consisting of two circular waveguides with diameters equal to the axis lengths of the elliptic waveguide.

Cut-off wavelengths for the dominant TE_{11}-mode can be approximated with a maximum error of 5 percent for all eccentricities. The approximation is also valid for the other modes for small eccentricities.

The approximation has been successfully applied in the design and realisation of high power millimetre-wave polarisers.

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In order to calculate cut-off wavelengths of elliptic waveguides without trying to solve the roots of Mathieu functions, a first order approximation is derived.

The elliptic waveguide is decomposed in two circular waveguides for even and odd mode propagation. The cut-off wavelengths of these waveguides are calculated and used as approximate cut-off wavelengths of the elliptic waveguide.

Cut-off wavelengths for the dominant TE_{11}-mode can be approximated with a maximum error of 5 percent for all eccentricities. The approximation is also valid for the other modes for small eccentricities.

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