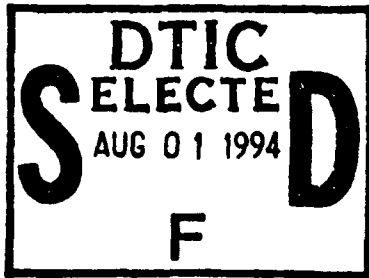


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On Performability Theory and  
the Inverse Sliding Problem



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## Abstract

For a given robot, we would like to know its capabilities and limitations. The question of what tasks a robot is capable of executing has been asked with increasing frequency as researchers explore the central issues of robotics. I shall refer to the theory that would answer this question as *Performability Theory*. One approach to Performability Theory is to examine the mechanics of the robot and the task. However, a complete model of the mechanics of a real robot and its environment quickly becomes intractable. The *Inverse Sliding Problem* involves simple mechanics; it deals with a planar world in which objects are free to translate and rotate but are subject to the forces of Coulomb friction. The problem is to determine the initial translational and rotational velocities required for an object to slide to a given final configuration. In this paper, I present a solution to a portion of the Inverse Sliding Problem, which I propose as the basis for a physically realistic yet simple world in which we can start to formulate the basic ideas of Performability Theory.

## 1. Introduction

Today we have no general purpose planners. Instead, we custom design a planner for a particular robot operating in a particular environment. The reason that we do this, of course, is that it would be far too complicated to write a planner that took into account all the assumptions we make about a robot and its world, such as compliance in the links of a robot; friction and backlash in its actuators; quasi-static assumptions; and inaccuracies in world models and sensors.

Consequently, our planner does not fully know the capabilities or the limitations of our robot. Our planner may then formulate a plan which fails to complete the task, or our planner may declare that our robot cannot perform a task which it in fact can.

Just as there is Computability Theory for Computer Science, there should be a Performability Theory for Robotics. A key question of Performability<sup>1</sup> would be, "For a given robot and a given task, can the robot successfully complete (perform) this task?" A fully developed Performability Theory should be able to answer questions about what classes of tasks a robot can or cannot perform.

One approach to a Theory of Performability is to examine the task mechanics of the robot and its world. However, as noted above, this can become quite complicated. In order to find a domain with simpler task mechanics, I have explored the Inverse Sliding Problem.

The Inverse Sliding Problem deals with a planar world in which objects can translate and rotate in the plane but are subject to the forces of Coulomb friction. Suppose an object with known frictional properties is given some initial velocity and moment. Acted upon only by the forces of friction, it slides until it comes to rest. Given the initial conditions, it is fairly straightforward to integrate the equations of motion forward in time to determine where the object will stop; the Inverse Sliding Problem is to determine the initial translational and rotational velocities required for an object to slide from its initial configuration to a given final configuration. In such a simplified yet still physically realistic world, we can better examine the issues and results of Performability.

In section 2, I will discuss some motivations for a theory of Performability and the notion of using the Inverse Sliding Problem as the basis for a world in which to explore Performability. I will describe related work in section 3.

Section 4 will present the Inverse Sliding Problem and outline the portions which I have solved. Sections 5 and 6 give the details of these solutions.

## 2. The World of Performability

We would like to know whether a robot is capable of performing a task and how easily it can perform that task. There may be certain features of a robot that enable it to perform a task or to perform a task more reliably. Currently this knowledge is empirically derived. Definitively knowing what these features are would help shape the design of future robots.

Many of the assumptions we make in formulating a planner are made on an ad hoc basis. We may make a round robot square, make quasi-static assumptions, or presume that the links of a robot are rigid. When we push the limits of our robots, we often see these assumptions break down.

A Theory of Performability should characterize and quantify this empirical knowledge of what the capabilities of a robot are, how easily it can perform a task, and what assumptions are permissible.

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<sup>1</sup>I will use the terms Performability, Performability Theory, and Theory of Performability interchangeably in this paper.

When we ask what the capabilities of a robot are and how easily it can perform a task, we must look at both the robot and the task mechanics. For example, suppose a robot does not have precise control over the direction its velocity. There is a velocity cone that describes the set of velocities the robot may take. For a given robot, the size of its velocity cone may preclude the possibility of doing certain tasks, particularly tasks involving intricate maneuvering. For two different robots with two different velocity cones, the one with better control over its velocity (a narrower velocity cone) will be able to formulate simpler plans to accomplish a task and complete the task more reliably.

When we ask what assumptions are permissible in formulating a planner, we must also look at both the robot and the task.

For example, suppose a robot is asked to manipulate an object. A robot which knows the stable grasping region for its gripper knows it can tolerate inaccuracies in sensing and positioning, so long as the object will still be in the stable grasping region. Under these circumstances, we may assume that the robot has perfect sensing and perfect actuation.

As another example, suppose a robot is asked to perform an alignment task. Now, if we assume perfect sensing and perfect actuation, inaccuracies can cause the robot to fail. The robot might think the alignment is complete when in fact it is not, or the robot could get stuck in a cycle in which it is continually trying to correct its position.

Perhaps the ultimate question for a Theory of Performability is, "Given the sensing and action capabilities of a robot system, for what class of tasks can you formulate a complete planner?" That is, a planner which is guaranteed to find a plan if one exists. (See [3] and [8] for discussion on completeness of robot motion planners.) For now, I will stick with the slightly simpler question, "For a given robot and a given task, can the robot successfully complete (perform) this task?"

Answering this question still requires examining the task and the robot. It can be quite complicated to analyze a full model of the task mechanics for a real robot; one would have to include such things as actuator dynamics, compliance in the links of the robot, friction, etc. A world with simpler task mechanics and a robot with a simpler action would be much easier.

Imagine a planar world where objects can translate and rotate in the plane but are subject to Coulomb friction. A robot in this world might manipulate objects by striking them or shooting two-dimensional BBs at them. In either case, objects acted upon by this robot would be given a velocity and moment instantaneously. They would then slide until the frictional forces brought them to a stop.

The planning problem for a robot in this world is to determine how hard it should hit/shoot an object in order to move it from configuration A to configuration B. The first step in answering this question is to determine what initial velocity and moment are required to effect the desired translation and rotation. This is the Inverse Sliding Problem, which I present in this paper. The second step is to determine where and how hard to strike the object in order to effect that velocity and moment. Note that the two steps in this problem serve to decouple the actuator characteristics from the task mechanics.

Once we have some experience with performability in a physically real albeit simple world, then perhaps we can find ways to draw general conclusions about more complicated robots and tasks without working through detailed task mechanics each time.

### 3. Related Work

It is difficult to trace the development of ideas that relate to Performability because such notions are often mentioned only in passing in conjunction with results in robot motion planning. The following summary will certainly not be as extensive or as far reaching as it should be.

In [10], Mason poses the question, "...for a given robot, what class of tasks can be performed?" and suggests that "precise answers require precise models of tasks and robots..." He describes results in pushing, squeeze-grasp, and tray-tilting operations for planar parts.

In [1], Donald discusses the idea of quantifying the amount of information that is encoded in a robot's environment. He also asks the question: "What are the capabilities of a given robot (in a given environment or class of environments)?", though this paper focuses on a theory of information invariants — how to quantify the trade off between information, communication, state and execution time.

In some sense, all of robot motion planning— path planning, grasp planning, etc. — relates to Performability; all robot motion planners attempt to answer the question whether or not a robot can perform a task. Recently, Goldberg has addressed the issue of completeness in robot motion planning in [3]. He identifies three levels of completeness to characterize planners. As a footnote, I would add that a planner is complete insofar as its world model accurately models the world.

Latombe, in [7], argues that robot algorithms transcend the hardware (robot) on which they are executed but notes that different robots will have different 'instruction sets'. Within this framework, I believe the central issue of Performability is examining the characteristics of robots (their instruction sets) to determine what capabilities enable a robot to accomplish a class of tasks.

In regard to the Inverse Sliding Problem, Voyerli and Eriksen have examined the motion of a homogeneous circular ring or disk sliding on a plane under the action of Coulomb friction in [11]. They make some interesting observations regarding the evolution of this system in state space which I build upon in this paper.

In [4] and [5], Goyal et al. formulate a limit surface representation which is useful for visualizing the relationship between the force and torque due to friction and the translational and angular velocities of an object.

As far as potential applications of the Inverse Sliding Problem, see [6]. In this paper, Higuchi presents work using an electromagnetic coil to deliver an impulse to an object in order to do linear micropositioning. This technique could presumably be extended to do micropositioning in position and orientation.

#### **4. The Inverse Sliding Problem**

I propose the Inverse Sliding Problem as the basis for a world in which the task mechanics are simple enough that we can draw some conclusions about what a robot in this world can or cannot do. The Inverse Sliding Problem may be stated as follows:

A rigid planar object of known geometry, support distribution, and frictional properties slides on a uniform surface, slowing down and coming to rest only due to Coulomb friction. Given an initial configuration (position and orientation) of the object, the problem is to determine the initial translational and angular velocities required in order for it to come to rest at a desired final configuration.

When the object is rotationally symmetric— its pressure distribution, and friction properties are invariant with respect to rotation—there are several simplifications that can be made. I will look at this class of objects in section 5 and return to the general problem in section 6.

#### **5. Rotationally Symmetric Objects**

Rotationally symmetric objects, such as a ring or a disk, have the special properties:

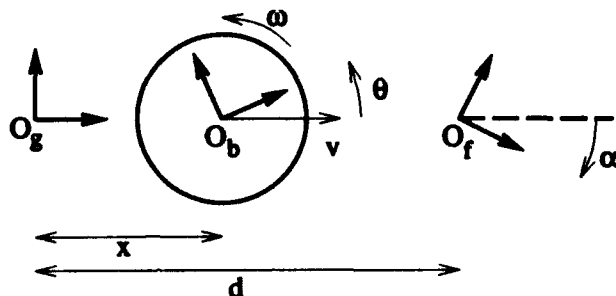


Figure 1: Rotationally Symmetric Case Notation: The global coordinate frame  $\mathcal{O}_g$  is positioned at the center of mass (COM) of the object in the initial configuration. Its  $x$  axis points at the COM of the object in the final configuration. The body frame  $\mathcal{O}_b$  is attached to the object so that it coincides with the global frame in the initial configuration. The final frame  $\mathcal{O}_f$  is placed so that it coincides with the body frame in the final configuration. The configuration of the object is given by its position along the  $x$  axis and its orientation,  $\theta$ , with respect to the global frame. The final configuration is specified by its position  $d$  and orientation  $\alpha$  with respect to the global frame.

- The net force due to friction is independent of orientation.
- The net force due to friction is always parallel to the translational velocity.

Though the second property above may seem obvious, it is only true for rotationally symmetric objects. Non-rotationally symmetric objects will generally have some net frictional force acting perpendicular to the velocity.

### 5.1. Notation

Given an initial and final configuration, place the global coordinate frame  $\mathcal{O}_g$  so that its origin is at the center of mass (COM) of the object in the initial configuration, and so that its  $x$  axis lies along the line connecting the COM in the initial configuration to the COM in the final configuration.

Attach the body frame  $\mathcal{O}_b$  to the object so that it coincides with the global frame  $\mathcal{O}_g$  in the initial configuration. Place the final coordinate frame  $\mathcal{O}_f$  so that it coincides with the body frame when the object is in the final configuration. See figure 1.

If the object has an initial velocity along the  $x$  axis of  $\mathcal{O}_g$ , then the force due to friction will be parallel to this velocity, so the COM of the object will travel along the  $x$  axis in a straight line. Therefore, the state of the object can be represented by the  $x$  coordinate of the COM of the object in the global frame,  $\theta$ , the orientation of the object with respect to the global frame, and their derivatives,  $v = \dot{x}$  and  $\omega = \dot{\theta}$ .

The final configuration can be represented as translation of  $d$  along the  $x$  axis and an orientation of  $\alpha$  with respect to the global frame.

The object is of mass  $M$ , and has a moment of inertia  $I$  about the COM. Since the object is rotationally symmetric, the coefficient of friction  $\mu(|\vec{r}|)$  is a function of only the magnitude of  $\vec{r}$ , where  $\vec{r}$  is a vector in the body frame.



## 5.2. Preliminaries

The net velocity of a point  $\vec{r}$  on the object is given by:

$$\vec{u} = v\hat{x} + \omega\hat{z} \times \vec{r} \quad (1)$$

The direction of this velocity is:

$$\hat{u} = \frac{v\hat{x} + \omega\hat{z} \times \vec{r}}{|v\hat{x} + \omega\hat{z} \times \vec{r}|} = \frac{\hat{x} + \frac{\omega}{v}\hat{z} \times \vec{r}}{|\hat{x} + \frac{\omega}{v}\hat{z} \times \vec{r}|} \quad (2)$$

Note that the direction of the net velocity is dependent upon only the the ratio of angular velocity to translational velocity,  $\frac{\omega}{v}$ , and the vector  $\vec{r}$ .

The force due to friction at this point (adopting the convention that  $-\vec{d}\vec{f}$  is the force that acts on this point) is:

$$\vec{d}\vec{f} = \mu(|\vec{r}|) dm g \hat{u} \quad (3)$$

Since the net force will lie along the  $x$  axis, we can write:

$$F\left(\frac{\omega}{v}\right) = \left| \int \vec{d}\vec{f} \right| = \left| g \int \mu(|\vec{r}|) \hat{u} dm \right| \quad (4)$$

The net torque is given by:

$$T\left(\frac{\omega}{v}\right) = \int \vec{r} \times \vec{d}\vec{f} = g \int \mu(|\vec{r}|) \vec{r} \times \hat{u} dm \quad (5)$$

## 5.3. Equations of Motion

The equations of motion that govern this system are:

$$M\dot{v} = -F\left(\frac{\omega}{v}\right) \quad (6)$$

$$I\dot{\omega} = -T\left(\frac{\omega}{v}\right) \quad (7)$$

$$\dot{x} = v \quad (8)$$

$$\dot{\theta} = \omega \quad (9)$$

## 5.4. Monotonicity of Force and Torque

As noted in subsection 5.2, (net) force and torque are functions of only  $\frac{\omega}{v}$ . As shown in [9], as a function of  $\frac{\omega}{v}$ , force is strictly monotonic decreasing, and torque is strictly monotonic increasing. See figure 2 for an example.

Intuitively, we can see this from the following argument. The magnitude of the frictional force at any given point is fixed. Its direction is determined by the ratio  $\frac{\omega}{v}$ . When  $\omega = 0$  ( $\frac{\omega}{v} = 0$ ), all the frictional forces oppose translation, so there will be maximum force and zero torque. As  $\omega$  increases, the frictional forces will increasingly oppose the rotational motion. This results in increased torque and decreased force. Thus, force is strictly monotonic decreasing; torque, strictly monotonic increasing.

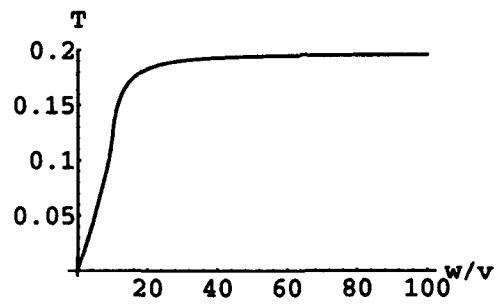
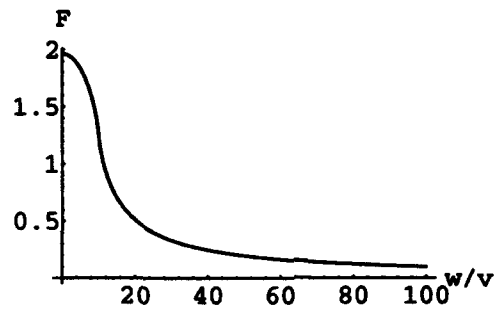


Figure 2: Force and Torque functions for a ring: The force and torque for a ring are functions of only  $\frac{w}{v}$ . Note that  $F$  is strictly monotonic decreasing;  $T$  is strictly monotonic increasing.

## 5.5. More Notation

We now turn our attention to the total distance and angle the object traverses as it translates and rotates, slowing down due to friction. Let the total distance and angle traveled by the object be denoted by  $x_f$  and  $\theta_f$ , which will be functions of the initial translational and angular velocities,  $v_0$  and  $\omega_0$ . Mathematically, this is:

$$x_f(v_0, \omega_0) = \int_0^{t_f} v dt \quad (10)$$

$$\theta_f(v_0, \omega_0) = \int_0^{t_f} \omega dt \quad (11)$$

where  $v(t)$  and  $\omega(t)$  are solutions to the equations of motion, subject to the initial conditions:

$$v(0) = v_0 \quad \omega(0) = \omega_0 \quad (12)$$

and where  $t_f$  is defined as the time such that:

$$v(t_f) = 0 \quad \omega(t_f) = 0 \quad (13)$$

Although  $x_f$  and  $\theta_f$  cannot be computed analytically, we can deduce certain properties about them.

## 5.6. Monotonicity of $x_f$ and $\theta_f$

The functions  $x_f(v_0, \omega_0)$  and  $\theta_f(v_0, \omega_0)$  are monotonic in  $v_0$  and  $\omega_0$ . That is, for  $v_{01}, v_{02}, \omega_{01}, \omega_{02} > 0$ :

$$v_{01} > v_{02}, \omega_{01} = \omega_{02} \implies x_f(v_{01}, \omega_{01}) > x_f(v_{02}, \omega_{02}) \quad (14)$$

$$v_{01} > v_{02}, \omega_{01} = \omega_{02} \implies \theta_f(v_{01}, \omega_{01}) > \theta_f(v_{02}, \omega_{02}) \quad (15)$$

$$v_{01} = v_{02}, \omega_{01} > \omega_{02} \implies x_f(v_{01}, \omega_{01}) > x_f(v_{02}, \omega_{02}) \quad (16)$$

$$v_{01} = v_{02}, \omega_{01} > \omega_{02} \implies \theta_f(v_{01}, \omega_{01}) > \theta_f(v_{02}, \omega_{02}) \quad (17)$$

We can prove this by reasoning about a pair of trajectories. If for some  $t_1$  we have

$$v_1 > v_2 \quad \omega_1 > \omega_2 \quad (18)$$

then for all  $t > t_1$ ,

$$v_1 \geq v_2 \quad \omega_1 \geq \omega_2 \quad (19)$$

The only way that equation 19 could be violated is if  $v_1(t)$  crossed  $v_2(t)$  or  $\omega_1(t)$  crossed  $\omega_2(t)$ . Both of these crossings cannot happen simultaneously because the derivatives  $\dot{v}$  and  $\dot{\omega}$  are uniquely defined by the equations of motion. However it could be possible for  $v_1(t)$  to cross  $v_2(t)$  while  $\omega_1(t) > \omega_2(t)$ , or vice versa.

But in order for  $v_1$  to cross  $v_2$ , at some point we must have  $v_1(t) = v_2(t)$ . Since  $\omega_1(t) > \omega_2(t)$ , we know that  $\frac{\omega_1}{v_1} > \frac{\omega_2}{v_2}$ . From monotonicity of  $F(\frac{\omega}{v})$ , we know that

$$F(\frac{\omega_1}{v_1}) < F(\frac{\omega_2}{v_2}) \quad (20)$$

This means there will be more force to decelerate object 2, so we will once again have the condition that  $v_1(t) > v_2(t)$ .

Similarly, if we consider  $\omega_1$  crossing  $\omega_2$  while  $v_1(t) > v_2(t)$ , at the instant that  $\omega_1(t) = \omega_2(t)$ , there will be more torque to decelerate the rotation of object 2, so we will once again have the condition that  $\omega_1(t) > \omega_2(t)$ .

When  $v_{0_1} > v_{0_2}$  and  $\omega_{0_1} = \omega_{0_2}$ , we know that at  $t = 0^+$ , we will have  $v_{0_1} > v_{0_2}$  and  $\omega_{0_1} > \omega_{0_2}$ . For all time after this, we will have  $v_{0_1} \geq v_{0_2}$  and  $\omega_{0_1} \geq \omega_{0_2}$ . These inequalities, taken altogether, imply:

$$\int_0^{\infty} v_1(t) dt > \int_0^{\infty} v_2(t) dt \quad (21)$$

$$\int_0^{\infty} \omega_1(t) dt > \int_0^{\infty} \omega_2(t) dt \quad (22)$$

or

$$x_f(v_{0_1}, \omega_{0_1}) > x_f(v_{0_2}, \omega_{0_2}) \quad (23)$$

$$\theta_f(v_{0_1}, \omega_{0_1}) > \theta_f(v_{0_2}, \omega_{0_2}) \quad (24)$$

which proves equations 14 and 15. Similar reasoning can be employed to prove equations 16 and 17.

### 5.7. Level Curves & Solutions

In addition to knowing that the functions  $x_f$  and  $\theta_f$  are monotonic increasing, we can easily determine their values when  $v_0 = 0$  or  $\omega_0 = 0$ . For these cases, the coupling between translational and rotational motion disappears, the problem is reduced to elementary physics, and we get:

$$x_f(0, \omega_0) = 0 \quad (25)$$

$$\theta_f(v_0, 0) = 0 \quad (26)$$

$$x_f(v_0, 0) = \frac{\frac{1}{2} M v_0^2}{g \int \mu(|\vec{r}|) dm} \quad (27)$$

$$\theta_f(0, \omega_0) = \frac{\frac{1}{2} I \omega_0^2}{g \int \mu(|\vec{r}|) |\vec{r}| dm} \quad (28)$$

Although we cannot solve for level curves of  $x_f$  and  $\theta_f$  algebraically, we can deduce some facts about them from monotonicity and from equations 25 – 28. To find a level curve of  $x_f$  at a height  $d$ , we start on the  $v_0$  axis. There, we can solve  $d = x_f(v_{0_d}, 0)$  for  $v_{0_d}$  using equation 27. We know that  $x_f$  increases as we move upwards, parallel to the  $\omega_0$  axis, and since  $x_f$  is 0 along the  $\omega_0$  axis and is increasing along the positive  $v_0$  axis, the level curve must lie to the left of the line  $v_0 = v_{0_d}$ . Furthermore, we also can deduce that this curve, given by  $d = x_f(v_{0_d}, 0)$ , is monotonic in the sense that as its  $v_0$  coordinate decreases, its  $\omega_0$  coordinate increases. See figure 3.

By similar reasoning, we can deduce that the level curve of  $\theta_f$  at some height  $\alpha$  must lie below the line  $\omega_0 = \omega_{0_\alpha}$  where we can solve the equation  $\alpha = \theta_f(0, \omega_{0_\alpha})$  for  $\omega_{0_\alpha}$  using equation 28.

These two level curves must cross somewhere in the region below the line  $\omega_0 = \omega_{0_\alpha}$  and to the left of the line  $v_0 = v_{0_d}$ . The coordinates of this intersection point are the desired initial conditions to achieve a displacement of  $d$  and a rotation of  $\alpha$ . See figure 4. Note that if we are trying to achieve some final *orientation*  $\alpha$ , there are an infinite number of solutions, corresponding to  $\theta_f(v_0, \omega_0) = \alpha + 2\pi n$  for any  $n \in \mathcal{Z}$ .

We can find the coordinates of this intersection point to any accuracy by employing a variation on bisection. For any point, we can do forward integration of the equations of motion. Comparing  $x_f$  with  $d$  and  $\theta_f$  with  $\alpha$  tells us which side of the level curves this point is on. We know that the intersection point must lie in the rectangle defined by the  $\omega_0$  and  $v_0$  axes and the lines  $\omega_0 = \omega_{0_\alpha}$  and  $v_0 = v_{0_d}$ . We can subdivide this rectangle into four smaller rectangles, do forward integration at each of the vertices, and eliminate rectangles from consideration based on whether or how the level curves enter and leave each rectangle. In this manner, we can zero in on the coordinates of the intersection point. See figure 5.

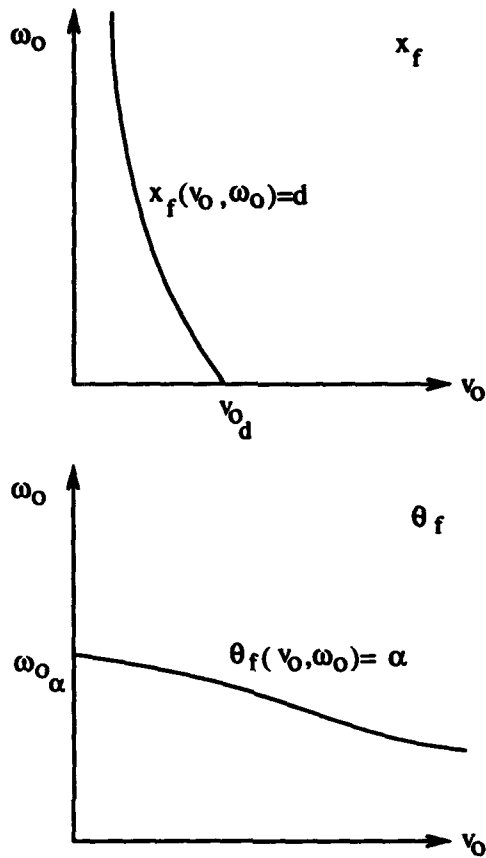


Figure 3: Level Curves of  $x_f$  and  $\theta_f$  (qualitative): We can solve  $d = x_f(v_{0d}, 0)$  for  $v_{0d}$  using equation 27 and use the monotonicity properties of  $x_f$  to determine a level curve. The level curve of  $x_f$  must lie to the left of the line  $v_0 = v_{0d}$ . Similarly, we can find a level curve of  $\theta_f$ , and it must lie below the line  $\omega_0 = \omega_{0\alpha}$ .

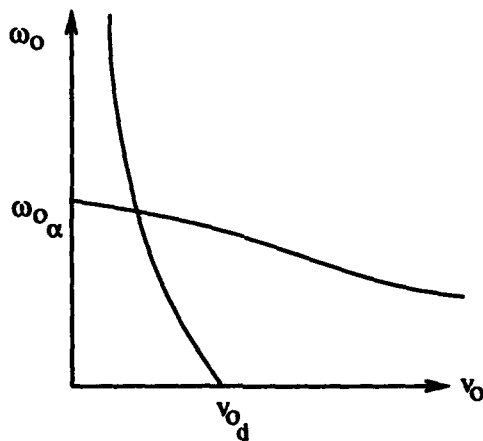


Figure 4: Intersection of Level Curve in the  $v_0\omega_0$  plane: The coordinates of the intersection point of the  $x_f$  level curve and the  $\theta_f$  level curve in the  $v_0\omega_0$  plane are the initial conditions that will result in the corresponding translation and rotation of the object.

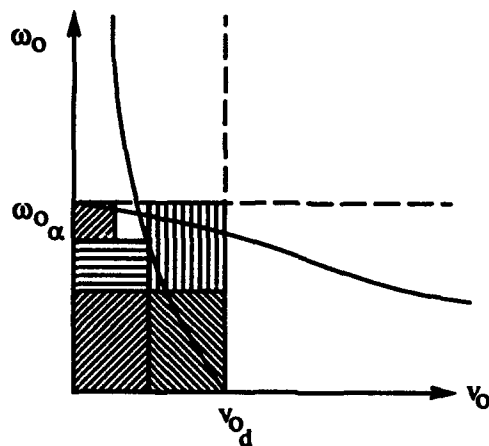


Figure 5: Determining the Initial Conditions: We can find the coordinates of the intersection point of the two level curves by employing a variation on bisection. We know the intersection point must lie in the rectangle below the line  $\omega_0 = \omega_{0\alpha}$  and to the left of the line  $v_0 = v_{0d}$ . We can subdivide this rectangle into 4 smaller rectangles and do forward integration for the initial conditions corresponding to the vertices of these rectangles. This will tell us whether the vertex is above or below each of the level curves, allowing us to eliminate certain rectangles from consideration and zero in on the coordinates of the intersection point.

We can also conclude that all configurations are reachable, so long as the initial velocity and moment can be provided. For any given  $d$  and  $\alpha$ , we can find level curves of  $x_f$  and  $\theta_f$ , and these level curves are guaranteed to intersect.

We can consider this our first result of Performability: a robot in this world which has perfect sensing and is able to deliver any velocity and moment can position a rotationally symmetric object at any configuration in the plane.

## 6. Non-Rotationally Symmetric Objects

The case of non-rotationally symmetric objects is much more complicated because:

- The net force and torque due to friction is now dependent on orientation.
- There will generally be a component of the net friction perpendicular to the velocity.

For these reasons, the COM of the object will not move in a straight line, so 6 state variables are required instead of the 4 used for rotationally symmetric objects.

I have not yet been able to formulate a solution for non-rotationally symmetric objects; in the following sections, I will present the progress I have made to date.

### 6.1. Notation & Preliminaries

The notation used for the non-rotationally symmetric case is the same as in the rotationally symmetric case except that the quantities  $\vec{x}$ ,  $\vec{v}$ , and  $\vec{d}$  are now vectors. See figure 6. The coefficient of friction,  $\mu(\vec{r})$ , is no longer restrained to be a function of  $|\vec{r}|$ . Also, we will have to be more careful about transforming vectors from one coordinate frame to another.

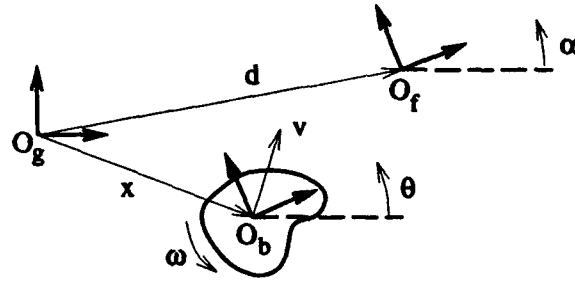


Figure 6: Non-Rotationally Symmetric Case Notation: Notation for the non-rotationally symmetric case is the same as in the symmetric case, except that the position  $\vec{x}$ , velocity  $\vec{v}$ , and the final configuration position  $\vec{d}$ , are now vectors.

The net velocity of a point  $\vec{r}$  on the object is given by:

$$\vec{u} = \vec{v}_b + \omega \hat{z} \times \vec{r} \quad (29)$$

where  $\vec{v}_b$  is the velocity of the object in the body frame  $\mathcal{O}_b$ .

The direction of this velocity is:

$$\hat{u} = \frac{\vec{v}_b + \omega \hat{z} \times \vec{r}}{|\vec{v}_b + \omega \hat{z} \times \vec{r}|} = \frac{\hat{v}_b + \frac{\omega}{|\hat{v}|} \hat{z} \times \vec{r}}{|\hat{v}_b + \frac{\omega}{|\hat{v}|} \hat{z} \times \vec{r}|} \quad (30)$$

The direction of the net velocity is now dependent upon the direction of the translational velocity  $\hat{v}_b$ , and the ratio of angular velocity to the magnitude of the translational velocity  $\frac{\omega}{|\hat{v}|}$ .

The force due to friction at this point is:

$$d\vec{f} = \mu(\vec{r}) dm g \hat{u} \quad (31)$$

The total force and torque due to friction are then given by:

$$\vec{F}_b\left(\frac{\omega}{|\hat{v}|}, \hat{v}_b\right) = \int d\vec{f} = g \int \mu(\vec{r}) \hat{u} dm \quad (32)$$

$$T_b\left(\frac{\omega}{|\hat{v}|}, \hat{v}_b\right) = \int \vec{r} \times d\vec{f} = g \int \mu(\vec{r}) \vec{r} \times \hat{u} dm \quad (33)$$

However, since the integration is done in the body frame, the net force is also in the body frame. Thus, we must rotate  $\vec{F}_b$  by  $-\theta$  in order to use it in the global frame. Since  $\vec{v}_b$  is a function of  $\theta$  and  $\hat{v}$ , we can conveniently write:

$$\vec{F}\left(\frac{\omega}{|\hat{v}|}, \hat{v}, \theta\right) = \text{Rot}\left(\vec{F}_b\left(\frac{\omega}{|\hat{v}|}, \hat{v}_b\right), -\theta\right) \quad (34)$$

The torque is, of course, invariant to orientation, but for uniformity, we can write:

$$T\left(\frac{\omega}{|\hat{v}|}, \hat{v}, \theta\right) = T_b\left(\frac{\omega}{|\hat{v}|}, \hat{v}_b\right) \quad (35)$$

## 6.2. Equations of Motion

The equations of motion for the non-rotationally symmetric case are:

$$M \dot{\vec{v}} = -\vec{F}\left(\frac{\omega}{|\hat{v}|}, \hat{v}, \theta\right) \quad (36)$$

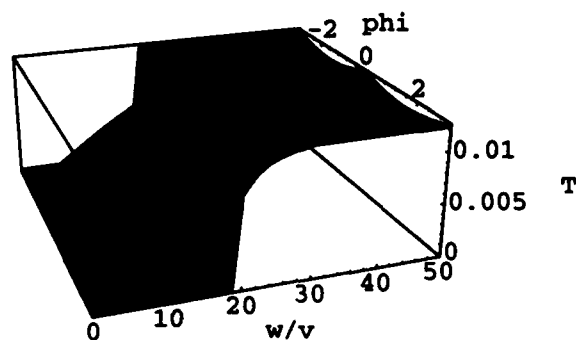
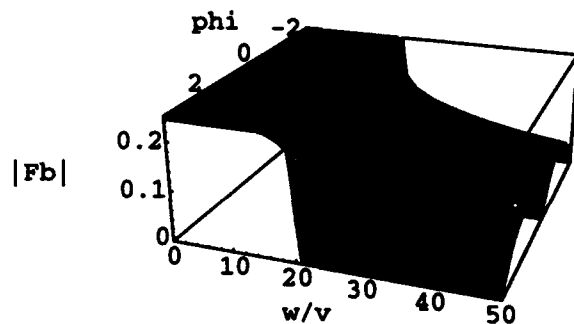


Figure 7: Force and Torque functions for the 2D barbell: These graphs show the force and torque due to friction in body coordinates for the 2D barbell.  $|\vec{F}_b|$  and  $T_b$  as functions of  $\frac{w}{v}$  and  $\phi$  (phi), the angle of  $\hat{v}$  with respect to the body frame.

$$I\dot{\omega} = -T\left(\frac{w}{|v|}, \hat{v}, \theta\right) \quad (37)$$

$$\dot{\vec{x}} = \vec{v} \quad (38)$$

$$\dot{\theta} = \omega \quad (39)$$

### 6.3. Monotonicity of Force and Torque

Unfortunately, the net force and torque do not have the same monotonicity properties as in the rotationally symmetric case.

As an example of a non-rotationally symmetric object, I have been looking at the '2D barbell', two point masses connected by a rigid, massless, frictionless rod. The force and torque functions for this object in body coordinates appear in figure 7. It is not clear whether the force and torque functions in global coordinates have any useful monotonicity properties.



#### 6.4. More Notation

We can define the functions  $\bar{x}_f$  and  $\theta_f$ , analogous to their namesakes in the rotationally symmetric case.

$$\bar{x}_f(\bar{v}_0, \omega_0) = \int_0^{t_f} \bar{v} dt \quad (40)$$

$$\theta_f(\bar{v}_0, \omega_0) = \int_0^{t_f} \omega dt \quad (41)$$

where  $\bar{v}(t)$  and  $\omega(t)$  are solutions to the equations of motion, subject to the initial conditions:

$$\bar{v}(0) = \bar{v}_0 \quad \omega(0) = \omega_0 \quad (42)$$

and where  $t_f$  is defined as the time such that:

$$\bar{v}(t_f) = \bar{0} \quad \omega(t_f) = 0 \quad (43)$$

However, it is difficult to draw conclusions about level curves of  $\bar{x}_f$  and  $\theta_f$  because they are now functions of three variables and because the force and torque functions do not seem to have analogous monotonicity properties.

#### 6.5. Discussion

For rotationally symmetric objects, the monotonicity of  $x_f$  and  $\theta_f$  can be explained somewhat intuitively. If the object has a higher initial translational velocity, it will slide further, (and it will also spin more). If the object has a higher initial rotational velocity, it will spin more, (and it will also slide further). These relationships are a guideline to adjusting the initial velocities to achieve the desired translation and rotation.

Analogous relationships for the case of non-rotationally symmetric objects are desirable, but there is an additional complication that there is a third variable: the direction of the initial velocity. Currently, it is unclear how changing the direction of the initial velocity affects  $\bar{x}_f$  and  $\theta_f$ .

In analysis of the 2D barbell, the  $y$  component of  $\bar{x}_f$  has been very small. This means adjustments to the direction of  $\bar{v}_0$  would also be very small, so linearization techniques may be successful.

Another possible path for a solution is to examine the trajectory of the object in some phase or state space. Of course, in order to visualize trajectories in this state space, we are limited to choosing 3 variables, which may not be enough.

#### 7. Conclusions

In this paper, I have discussed the motivations and preliminaries of Performability Theory. In writing planners, we often make assumptions about the robot and its environment which may result in failure to complete the task. By taking into account the mechanics of the robot and its actions on the environment, we can determine what assumptions are appropriate for a particular robot and task. However, for real world robots, analysis of all aspects of the task mechanics is infeasible. In search of a simpler domain, I turned to the Inverse Sliding Problem. I have presented a solution to the Inverse Sliding Problem for the class of rotationally symmetric objects and have shown my work to date and some thoughts on solving the general problem.

The Inverse Sliding Problem has many advantages as a simplified world in which to study Performability. The action in this world is quite simple, does not involve grasping kinematics or dynamics, and isolates

actuator dynamics from planning. On the other hand, though the task mechanics are relatively simple, they do not have an analytic form, and sensitivity analysis will still be difficult.

There are many other related issues that should also be considered in the context of this work. It is not clear how manipulation strategies (such as in the tray-tilting of [2]) generally interact with the task mechanics with regard to Performability. Although a robot may not be capable of directly performing a task, it might accomplish this task through clever use of manipulation strategies or convergence properties of the task mechanics.

In [3], Goldberg raises some issues relating to recursively enumerable sets, computability, and Church's Thesis; it may be the case that there are limitations on Performability results because of computability issues.

Furthermore, it is possible that before a full Performability Theory can be developed, it will be necessary to have a language to describe arbitrary tasks. However, it is not clear whether such a language can possibly exist.

Despite these difficulties, I believe there is potential for developing a general Performability Theory, though it will take some time before conclusions in simplified worlds can be generalized to more complex ones. The increasing interest in the issues such as robot algorithms, planner completeness, and performability will hopefully lay the foundations for a science of robotics.

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