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Modeling of Split-Core Transformers For Power Transmission

Presented at the International Magnetics Conference, Stockholm, Sweden, 13-16 April 1993

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PREFACE

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13. ABSTRACT (Maximum 200 words)

This paper was presented in poster format at the International Magnetics Conference in Stockholm, Sweden, 13-16 April 1993. It describes how the magnetic vector potential finite element method is used to predict the performance of a pot-core transformer for power transmission across a variable air gap. Performance predictions computed by the model include mutual and leakage inductance for various air gaps between 1 and 9 mm. Magnetic flux density external to the pot-core was also computed as a function of air gap. The predicted performance was then compared with measurements taken on a prototype core. Experimental and numerical results were shown to be in good agreement.

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MODELING OF SPLIT-CORE TRANSFORMERS FOR POWER TRANSMISSION

I. INTRODUCTION

The magnetic vector potential finite element (FE) method is well known for solving magnetostatic and eddy current problems [1,2]. This paper describes how the magnetic vector potential FE method is used to predict the performance of a split-core transformer for power transmission across an air gap. An initial study of core geometry and the physical constraints of the intended application required the selection of a pot-core as the optimum design geometry.

Typical applications of pot-core transformers for power transmission normally maintain gap spacing much smaller than 1 mm. For the application described here, both halves of the pot-core are entirely separated by air gaps up to 9 mm. The large air gap reduces magnetic coupling efficiency and increases leakage inductance, thereby limiting power transmission across the gap.

Performance modeling predictions of inductances and magnetic fields were computed by the FE model. The FE predictions of leakage and mutual inductance for each of the gaps were then used as input to an equivalent circuit model that predicted performance parameters such as real and apparent power. The predicted performances from both the FE and circuit models were next compared with measurements taken on the prototype core. For simplicity, both the model and prototype core consisted of identical primary and secondary core halves, each wound with Litz wire coils of equal size and turns.

II. THEORY OF FINITE ELEMENTS

The FE method solves for the fields by minimizing an energy functional. The functional is

$$F = \int_V \frac{\mathbf{B}^2}{2\mu} dv + \int_V \frac{\varepsilon \mathbf{E}^2}{2} dv - \int_V \frac{\mathbf{J} \bullet \mathbf{A}}{2} dv + \mathrm{W_d}, \tag{1}$$

where the first term is the stored magnetic energy, the second term is the stored electric energy, and the third term accounts for the energy input through sources. The final term, W_d, contains other energy terms such as that lost to dissipation. In equation (1), **A** is the magnetic vector potential, defined in terms of the magnetic flux density **B** by

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{2}$$

Minimizing equation (1) ensures that the energy is conserved. It has been shown that the variational method is exactly equivalent to a direct solution of the integral equation by Galerkin's method [3].

III. NUMERICAL MODELING

A. The FE Model

The introduction of an air gap increases leakage inductance and introduces a concern for the distribution of **B** fields external to the core. In particular, simple design equations are inadequate to accurately predict the leakage inductance, the fringing effects due to the air gap, and the externally generated **B** field. A two-dimensional axisymmetric model was constructed to model the pot-core transformer with varying gaps. Figure 1 shows the FE model of a 70-mm diameter MnZn ferrite pot-core transformer, including a 5-mm air gap. The model included an 8.5-mm-diameter post hole through the center of the core. The Z-axis is the axis of rotation. The core is assumed to operate in the linear range with an initial permeability of 1900. Hysteresis loss is assumed to be small and is neglected in this analysis. The primary winding was excited with 1 ampere-turn (At) of direct current in the θ direction. Performing the analysis with direct current is valid because the smallest dimension of the core cross section (5.5 mm) is much less than $\lambda/2$ at 100 kHz [4].

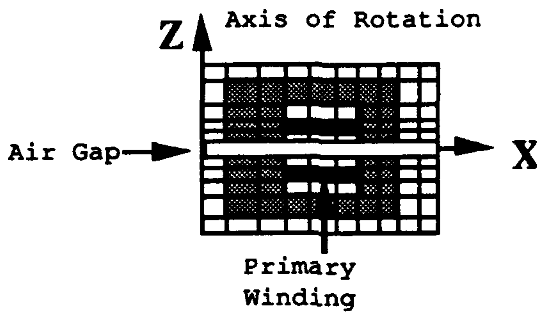

Figure 1. The 70-mm Pot-Core Transformer Model (Not to Scale).

In order to compute fields external to the pot-core, the FE mesh was extended to approximately 6 radii (200 mm) of the pot-core. Mesh density outside the core was selected so that element size was on the order of the measurement loop dimensions. Along the axis of symmetry (Z-axis), the vector potential was forced to zero, meeting a B normal = 0 condition along this edge. The model was terminated at 200 mm with an absorbing boundary element that simulated free space.

B. Equivalent Circuit Model

The well known transformer "T" equivalent circuit model was used for electrical parameter predictions. Mutual and leakage inductances obtained from the FE model for each gap spacing were used to represent the inductances in the "T" equivalent circuit. Winding resistance, as well as distributed and shunt capacitances, was neglected. A 50-Ω resistive load was added across the secondary winding for power transmission predictions.

IV. RESULTS

A. FE Results

Figure 2 shows the results of the FE calculation on the 70-mm pot-core. Flux lines are shown in the core, gap, and surrounding air. The self-inductance and leakage inductance can be calculated from the computed vector potential. The magnetic flux Ψ is related to the vector potential by

$$\Psi = \oint \mathbf{A} \bullet dl \quad \cdot \tag{3}$$

By integrating the vector potential around a closed path on the primary side of the pot-core, the flux in the primary can be computed. The primary self-inductance is related to the magnetic flux by

$$L_S = \frac{N\Psi}{I} , \tag{4}$$

where L_S is the self-inductance, N is the number of turns, and I is the primary current. A similar procedure can be used to compute the mutual inductance. The leakage inductance L_l and coupling coefficient K are given by $L_S - L_m$ and L_m/L_S, respectively.

Measurements were performed on a 70-mm diameter MnZn pot-core transformer wound with 18 turns of Litz wire with a 1:1 turn ratio. The primary winding was excited at 100 kHz with an impedance analyzer. Inductance measurements were made with both open- and short-circuited secondary winding for self-inductance and leakage inductance, respectively.

Figures 3, 4, and 5 depict predicted versus measured results for normalized inductance, coupling coefficient, and magnetic flux density. In figure 5, **B** field results for a 30-At excitation are shown at 140 mm from the core edge.

The B_Z measurements were made with a 10-mm-diameter field probe and recorded with a spectrum analyzer. As the figure shows, the external **B** field increases with decreasing gap. This

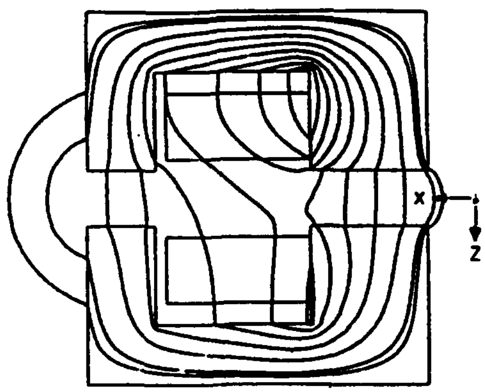

Figure 2. Flux Plot in Core and Air Gap

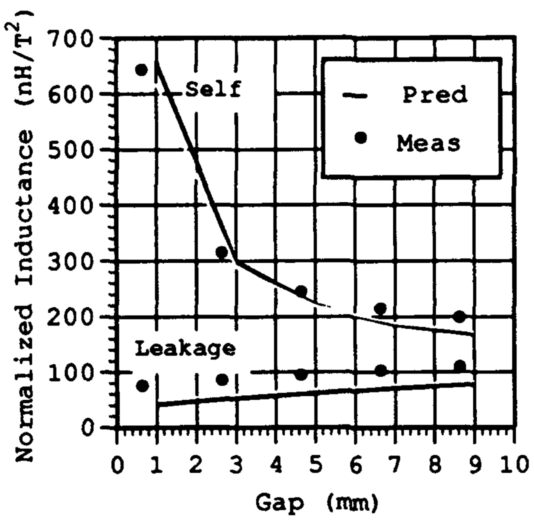

Figure 3. Normalized Inductance Factor as a Function of Gap

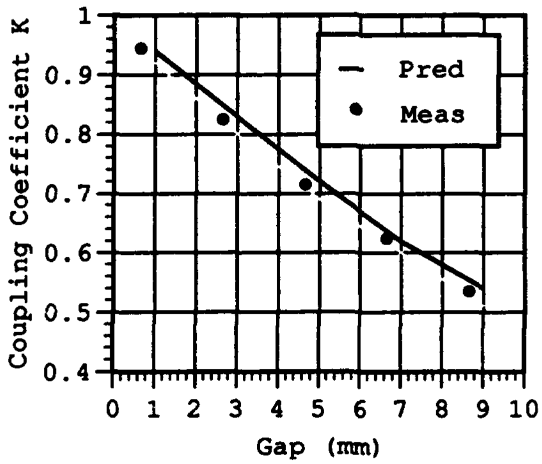

Figure 4. Coupling Coefficient as a Function of Gap

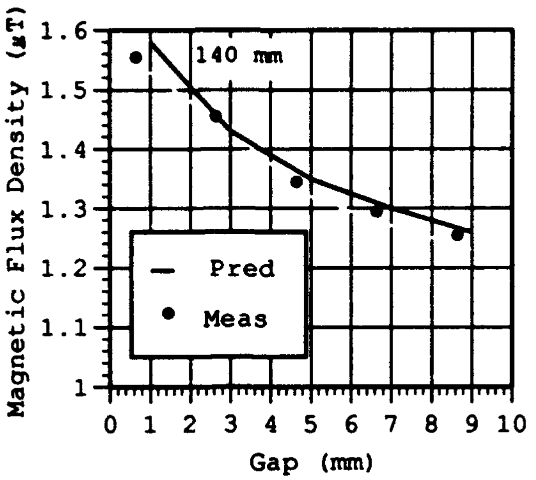

Figure 5. **B** Field as a Function of Gap

result is not unexpected because as the gap increases, the flux density in the gap falls in proportion to the increased reluctance. However, as the gap approaches zero, the field should be entirely contained within the core.

Figure 6 depicts predicted versus measured output power as a function of gap for a constant primary voltage. As can be seen in the figure, the error in the output power is largest at 1 mm, which is in agreement with the error shown in the inductance curves at 1 mm (see figure 3). This error is due to shunt and distributed capacitance and to core losses that are not accounted for in the models.

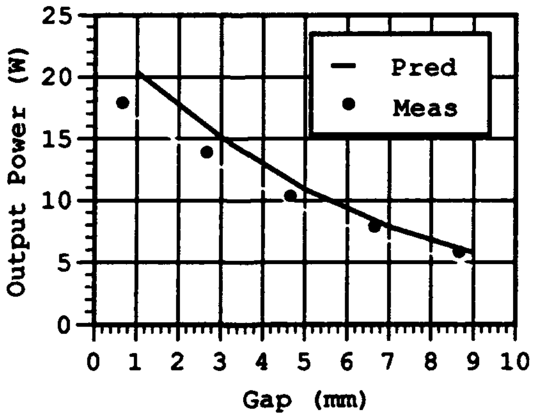

Figure 6. Output Power as a Function of Gap

The plots show excellent agreement between predicted and measured results. Thus, simple FE models can give accurate predictions that are useful for the design of large gap transformers.

REFERENCES

[1] J. Brauer, L. Larkin, V. Overbye, "Finite Element Modeling of Permanent Magnet Devices," *Journal of Applied Physics,* vol. 55, March 1984, pp. 2183-2185.

[2] J. Brauer, "Open Boundary Finite Elements for Axisymmetric Magnetic and Skin Effect Problems," *Journal of Applied Physics,* vol. 53, November 1982, pp. 8366-8368.

[3] D. Jones, "A Critique of the Variational Method in Scattering Problems," *IRE Transactions for Antennas and Propagation*, vol. 4, 1956, pp. 297-301.

[4] E. C. Snelling, *Soft Ferrites Properties and Applications*, 2nd ed., Butterworth and Co., Ltd., 1988, pp. 147-148.

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