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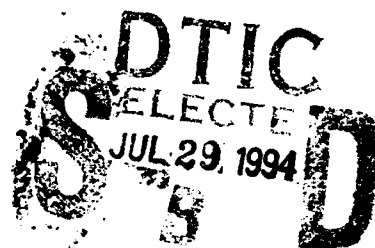
**Experimental Study Of An Automatic Pitch Control System
On A Swath Model**

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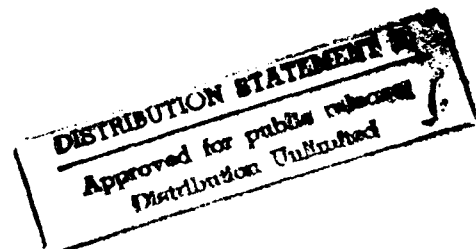
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Technical Report Documentation Page

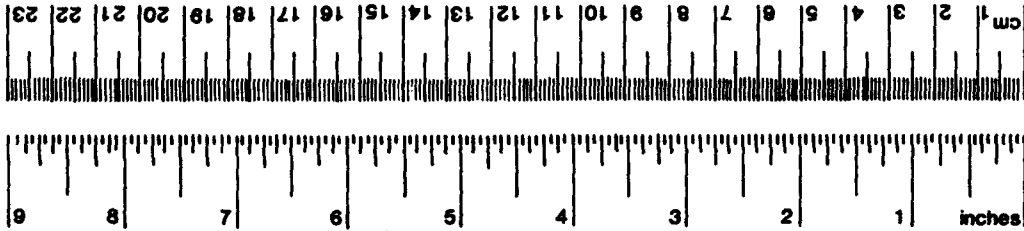
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16. Abstract <p>The report presents the results of a series of tests which was carried out in order to develop a small physical model of an automatic pitch control system for a SWATH ship, and to study the interaction of the control surface with the twin hulls in calm water and regular waves.</p> <p>A four phase test program was followed to assess the control system: fixed trim, free to heave tests of the unappended hull, tests of isolated canards, fixed trim tests in calm water and in waves with instrumented canards, and free to trim and heave tests in regular waves with and without automatic control. Drag, pitching moment, canard lift and drag, etc., were measured and analyzed. The maximum lift on the canards in waves was found to be significantly larger than that measured in static tests. Results are presented in graphic and tabular form. Tests in regular waves with pitch control showed that a 50% reduction in pitch amplitude is possible in following seas.</p>					
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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply By	To Find	Symbol
LENGTH				
in	inches	* 2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
AREA				
in ²	square inches	6.5	square centimeters	cm ²
ft ²	square feet	0.09	square meters	m ²
yd ²	square yards	0.8	square meters	m ²
mi ²	square miles	2.6	square kilometers	km ²
	acres	0.4	hectares	ha
MASS (WEIGHT)				
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
VOLUME				
tsp	teaspoons	5	milliliters	ml
tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft ³	cubic feet	0.03	cubic meters	m ³
yd ³	cubic yards	0.76	cubic meters	m ³
TEMPERATURE (EXACT)				
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly).



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply By	To Find	Symbol
LENGTH				
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
in	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
AREA				
cm ²	square centimeters	0.16	square inches	in ²
m ²	square meters	1.2	square yards	yd ²
km ²	square kilometers	0.4	square miles	mi ²
ha	hectares (10,000 m ²)	2.5	acres	
MASS (WEIGHT)				
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
VOLUME				
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	0.125	cups	c
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m ³	cubic meters	35	cubic feet	ft ³
m ³	cubic meters	1.3	cubic yards	yd ³
TEMPERATURE (EXACT)				
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

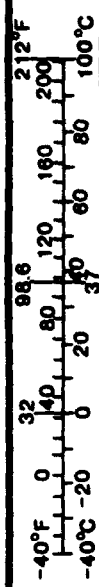


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ABSTRACT

A series of tests was carried out to develop a model of an automatic pitch control system for a SWATH ship, and to study the interaction of control surface (canard) with the ship in calm water and in regular waves. In one series of tests, a canard was instrumented to measure lift and drag; maximum lift on the canard in waves was found to be significantly larger than that measured in static tests. Tests in regular waves with pitch control showed that a 50% reduction in pitch amplitude is possible in following seas.

KEYWORDS

SWATH
Automatic Control
Control Surfaces
Unsteady Lift
Seakeeping

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist.	Avail and/or
A-1	Class 01

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NOMENCLATURE and SIGN CONVENTION

A	A constant in a Fourier series
a	Wave amplitude, ft; also acceleration ft/sec ²
B	a constant in a Fourier series
C _{D1} , C _{L1}	Amplitude of first harmonic of drag or lift coefficient
C _{D2} , C _{L2}	Amplitude of second harmonic of drag or lift coefficient
CTM	Model total resistance coefficient
CTS	Ship total resistance coefficient
c	Fin chord, ft
D	Drag, lb
d	the differential operator
EHP	Effective horsepower
g	Acceleration of gravity, ft/sec ²
g ₁	Pitch displacement gain factor, deg/deg
g ₂	Pitch angular velocity gain factor, deg/(deg/sec)
h	Water depth, ft, or height of towpoint, ft
I	moment of inertia, slug ft ²
L	Lift, lb
LBP	Ship length between perpendiculars, ft
l	Ship length between perpendiculars, ft
M	Pitching moment, ton ft (ship) or lb ft (model), positive bow up
M _p	Pitching moment measured about an axis passing through the towpoint, ton ft (ship) or lb ft (model), positive bow up
m	Mass, slugs
N	The highest harmonic used in a finite harmonic series

NOMENCLATURE and SIGN CONVENTION (continued)

n	Frequency coefficient in a harmonic series
Phase	Phase lag, degrees
R	A calibration matrix
Re	Reynolds number, Vc/ν
R_{ij}	An element in a calibration matrix
rps	Radians per second
RS	Ship resistance (full scale), lb
S	Projected area of fin, ft^2
T	Wave period, sec
T_e	Wave encounter period, sec
t	Time, sec
u	Wave particle horizontal velocity component, fps
V	Velocity, knots (ship) or fps (model)
v	Wave particle vertical velocity component, fps, or voltage reading
W	A weight, lb
w	A tare weight, lb
Y	Vertical coordinate of canard relative to calm water surface, ft (below water surface is negative)
z	Heave, positive upward, ft
z_1, z_2	Amplitudes of first and second harmonics of heave, ft
α	Angle of attack of canard relative to hull, deg, positive leading edge up
α_1, α_2	Amplitudes of first and second harmonics of canard angle, deg
ϕ	A phase angle, radians
θ	Pitch angle, deg, positive bow up

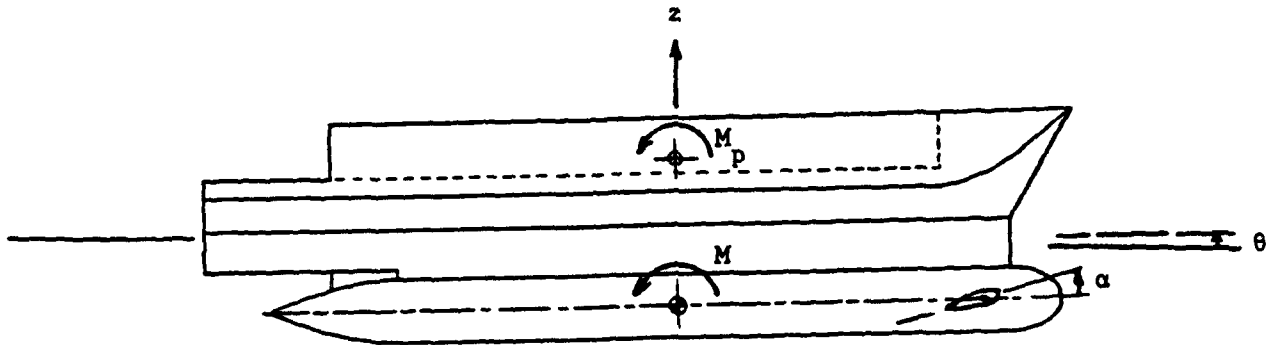
NOMENCLATURE and SIGN CONVENTION (Continued)

θ_1, θ_2	Amplitudes of first and second harmonic of pitch, deg
$\dot{\theta}$	Pitch angular velocity, deg/sec
λ	Wavelength, ft
ν	Kinematic viscosity, ft ² /sec
ρ	Density, slug/ft ³
Ω	Frequency ratio, ω/ω_0
ω	Frequency, radians/sec
ω_e	Encounter frequency, radians/sec
ω_0	Natural frequency, radians/sec

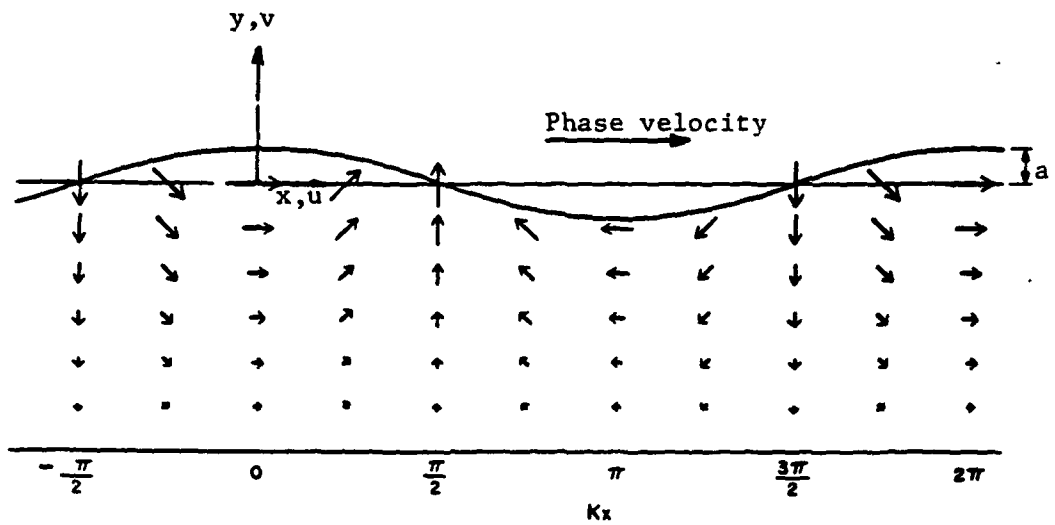
Subscripts

1	First harmonic, also first channel
2	Second harmonic, also second channel
e	Encounter
i	An index, e.g. the i th channel or the i th harmonic
m	Model
n	Number associated with a harmonic in a harmonic series
o	The lowest order harmonic, or mean, in a harmonic series; also an amplitude
s	Ship

NOMENCLATURE AND SIGN CONVENTION (Concluded)



SIGN CONVENTION (Positive senses shown)



Sign convention for wave particle motions
(Positive senses shown)

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The author is indebted to Dr. T. Glenn McKee, Chief of the Mathematical Services Division at Davidson Laboratory, for the calculation of the Fourier series expansion of the canard angle of attack in waves and for preparation of Appendix G.

INTRODUCTION

The advantages of Small Waterplane Twin Hull (SWATH) ships over conventional hulls in applications requiring minimal motions and high sustained speeds in heavy seas are well documented. Because of its reduced waterplane area, the natural periods of the motions of the SWATH are longer than those of conventional hulls, effectively detuning it from the modal periods of the most commonly occurring seaways. Additionally, the wave forces on the submerged hulls of the SWATH are smaller than those on a monohull ship of equivalent displacement, because the wave-induced velocity field decreases exponentially with depth¹⁵ [Superscripts refer to References on page 22].

SWATH hulls are typically fitted with fins to enhance stability at high speed in calm water. The fins also provide increased damping of motions in waves. Further reductions in the motions when the vessel is underway are possible by actively controlling the fins. Theoretical analyses^{11,12} and limited full-scale experience¹³ have indicated that an active control system would significantly increase the already substantial operating envelope of the SWATH vessel and would certainly contribute to increased comfort of passengers and crew in all sea states. However, prior to the present work no model tests have been carried out to validate the theoretical work and to study the positive benefits and limitations of active motion control.

As part of its extensive research program on SWATH hydromechanics being conducted at Davidson Laboratory, the U.S. Coast Guard has undertaken the development of a towing tank model of a possible SWATH pitch control system and a study of the associated modeling laws. This pioneering work is the subject of the present report.

To study the hydrodynamics of a SWATH with automatic pitch control, the following four phase test program was developed in which the major components of the system - hull and control surfaces - were first tested independently and then assembled for the final evaluation of the control system:

1. Fixed trim, free-to-heave tests of the unappended hull.
2. Tests of isolated canards.
3. Fixed trim tests in calm water and in waves with instrumented canards.
4. Free-to-trim and heave tests in regular waves with and without automatic control.

The unappended hull was tested fixed in trim, in calm water, in the first phase. This was done to determine the relationship between pitch moment and trim at various speeds. In the second part of the program, the canards were tested alone on a

groundboard at various speeds and angles of attack. The third phase consisted of tests with an instrumented canard mounted on the model. The tests were conducted in calm water and in regular waves with the model fixed in trim and heave. In the final phase of testing the model was run free to pitch and heave in regular waves with and without active control. Phase 2 is reported separately in Reference 1; phases 1, 3 and 4 are reported herein.

Tests were carried out in the High Speed Test Facility (Tank 3) of the Davidson Laboratory in January, March, May and August, 1987; some of the tests were observed by Mr. James A. White of the U.S. Coast Guard. Testing was funded under Contract N00014-84-C-0644 (Task 7), Office of Naval Research.

MODEL

An existing 1/24-scale model of a U.S. Coast Guard SWATH design, designated as SWATH 10, was used in these tests. With the exception of the cylindrical midbodies of the lower hulls and the wet deck, the model was constructed from pine. The wet deck was made from 1/2 inch marine plywood and the hull midbodies were made from foam-packed ABS plastic tubing. Figure 1 is a drawing of the configuration, which gives all major dimensions (full-scale). Particulars are listed in Table 1.

Canard and stabilizer fins, fitted for phases three and four of the tests, were made from plexiglass. Based on the results of Reference 1, NACA 0015 section canards, and existing NACA 0015 section stabilizers with an aspect ratio of 1.195 were used for these tests. The canards were fixed at various angles in phase 3 and active in phase 4. Figure 2 is a drawing of the canards.

To help induce a turbulent boundary layer, Hama strips were placed on the lower hulls, struts and appendages of the model. The strips consist of a double layer of electrical tape cut with pinking shears to form a serrated leading edge. They were placed on the hulls and struts at five percent of the hull length aft of the nose, and five percent of the strut length aft of the leading edge. On appendages, the strips were placed five percent of the local chord length aft of the leading edge. The Hama strips can be seen on Figure 3, which is a photograph of the model running in calm water.

The model was ballasted to the 14.5 foot waterline, corresponding to a full scale displacement of 591 long tons. For the fixed trim tests (phases 1 and 3), the center of moments was located 51.25 feet aft of the strut leading edge (the nominal LCG of the SWATH 10 configuration, as reported in Reference 16) and 27.25 feet above the baseline (a convenient location for the deck-mounted instrumentation, also consistent with the tests reported in Reference 16). Measured pitch moments were transferred to a point on the full-scale thrust axis, which was taken to be on the centerline of the lower hulls, 5 feet above the baseline. This was done to remove the effects of thrust from the moment results.

For the free to pitch and heave tests (phase 4), the model was towed from a point 27.75 feet above the baseline (the "pivot point"). The construction of the existing SWATH 10 model did not permit towing from the thrust line, which would have been a better arrangement. The towpoint was located at the apparent longitudinal center of flotation, 63.25 feet aft of the strut nose. This was done to minimize pitch-heave coupling at the pivot point (the location of the pitch sensor, which produced the input signal to the control system). Because the contract Statement of Work limited the present study to pitch control, it was desirable to eliminate coupling with heave.

An inclining experiment was performed to determine the longitudinal GM, which was 24.45 feet. The natural pitch period at zero speed in the water was 11.76 seconds. Prior to the dynamic tests, the pitch gyradius of the fully equipped model was determined to be 38.4 ft; this is close to the value of 38.2 ft reported in Reference 2 for this model. Other particulars appear in Table 1. Details of the LCF determination and the inclining test are given in Appendix A.

APPARATUS

Tests were conducted in the Davidson Laboratory High Speed Test Facility (Tank 3). Figure 3 shows the apparatus used in the fixed trim tests. A pivot box was mounted in the model at the second deck level. Two screws on the pivot box permit the adjustment of trim to any desired angle. Trim was measured by means of an inclinometer, also located on the deck. Above the pivot box was a moment balance, used to measure pitch moment, and a stainless steel drag balance. These were attached to the crosspiece of a free to heave apparatus. Heave, the vertical motion of the towpoint relative to the static floating location, was also measured.

For the third phase of the tests, the model was instrumented to measure lift and drag on the starboard canard, as well as the total drag and pitch moment on the model. The fin balance was located inside the lower hull as shown on Figure 4a. Figure 4b is a photograph of the installation in the model; the photograph is taken looking aft from a position forward of the model (the hull nose has been removed). The angle of attack of the canard was adjustable by means of a clamp on the shaft. These tests were conducted at zero nominal trim; however, trim was monitored using the inclinometer on deck. The phase three tests were conducted with the model fixed in heave at the 14.5 foot waterline level.

The fourth phase of the tests employed an automatic pitch control system. The system is shown schematically on Figure 5. For these tests the model was free to pitch and heave but fixed in surge, sway, roll and yaw. The rotary differential transformer in the pivot box produced a voltage signal proportional to pitch angle. This signal was passed to the control box, mounted on the towing carriage. The output of the control box to the servo motors was a signal linearly

proportional to pitch angle and rate of change of pitch angle; thus the fin angle was related to pitch angle as follows:

$$\alpha = -g_1\theta + g_2\dot{\theta}$$

where α is the canard angle of attack relative to the hull, θ and $\dot{\theta}$ are the pitch angle and pitch rate, and g_1 and g_2 are the gains (degrees per degree and degrees per (degrees per second), respectively). It should be noted that a positive pitch angle (bow up) gives rise to negative canard angle (leading edge down). The gains were adjustable using potentiometers on the control box, in the following ranges (model scale):

$$0 \leq g_1 \leq 7.07 \text{ degrees/degree}$$

$$0 \leq g_2 \leq 1.97 \text{ degrees/(degrees/sec)}$$

The response of the control system to the input signal was essentially instantaneous, due to a tight feedback loop of fin position and rate to the servo motors. The output signal to the servo motors was used to monitor the canard angle of attack. Mean drag was measured during these tests, as well as pitch, heave and canard angle.

A wave strut attached to the towing carriage, forward of the model near the tank edge, was used in the phase three and four tests to measure encounter period and to establish the phase of the model forces and motions relative to the waves.

Voltage signals from the balances, proportional to forces or moments, and from heave and pitch or trim transducers, proportional to linear or angular displacement, were amplified by signal conditioners on the carriage and transmitted through overhead cables to a shore-based analog to digital converter and MASSCOMP computer for processing and storage. Processed data (for example, mean forces and moments in engineering units) were printed out at tankside. Time histories of transducer signals were monitored at tankside on an oscillograph recorder. All data has subsequently been backed up from the computer disk to one quarter inch magnetic tape.

Regular waves were generated using the dual-flap hydraulically-driven wave machine³ located at the end of the tank. The wavemaker was controlled by a PDP-11 minicomputer; desired wave lengths and wave heights were entered on a tankside console to start a run.

All good runs were recorded on VHS videotape. Videotape scenarios are given in Appendix B. In addition, still color photographs were taken of selected runs.

TEST PROGRAM

As explained above, the test program had four parts. The program for each phase of the test is described below:

Phase 1: Calm water fixed trim tests of the unappended model. Model free to heave but otherwise fixed.

Speeds: 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 knots

Trim angles: -2, -1, 0, 1, 2 degrees

Measure: Drag, pitch moment, heave

Phase 2: Tests of canard fins on a groundboard. These tests are reported in Reference 1.

Phase 3: Calm water and regular wave tests of model with instrumented canard. Model fixed in all degrees of freedom, including trim and heave.

a) Calm water tests

Speeds: 10, 15, 20 knots

Canard angles: 0, 5, 10, 15, 20 degrees

Canard aspect ratios: 1, 2

Measure: Drag, pitch moment, fin lift and drag

b) Regular wave tests: Head seas

Speeds: 10, 20 knots

Wave length/Ship length (LWL): 1, 1.5, 2, 2.5, 3, 3.5

Wave heights: 4, 8 ft (8, 12 ft for two longest waves)

Canard aspect ratios: 1, 2

Canard angle: 0 degrees

Measure: Mean drag and pitch moment; mean and oscillatory canard lift and drag

c) Regular wave tests: Following seas

Speeds: 10, 20 knots

Wave length/Ship length (LWL): 1, 2 (10 knots)

1, 2.5, 3.5 (20 knots)

Wave heights: 4, 8 ft (8, 12 ft for two longest waves)

Canard aspect ratios: 1, 2

Canard angle: 0 degrees

Measure: Mean drag and pitch moment; mean and oscillatory canard lift and drag

The number of conditions in following seas was reduced because of the impossibility of encountering a sufficient number of waves for proper analysis in a tank of finite length.

Phase 4: Regular wave tests with automatic pitch control system. Model free to pitch and heave but restrained in surge, sway, roll and yaw.

a) Tests with inactive fins (baseline)

Speeds: 15, 20 knots
 Headings: Head seas, following seas
 Wave length/Ship length (LWL): 1.5, 2.5, 3.5
 Wave heights: 4, 8 ft (8, 12 ft for longest wave)
 Canard aspect ratio: 1
 Measure: Mean drag; heave, pitch

b) Tests with active fins

Speeds: 15, 20 knots
 Headings: Head seas, following seas
 Wave length/Ship length (LWL): 1.5, 2.5, 3.5, 4.0
 Wave heights: 4, 8 ft (8, 12 ft for two longest waves)
 Canard aspect ratios: 1, 2
 Measure: Mean drag; heave, pitch, canard angle

Part b) of the phase four tests was conducted with the control system optimized, that is, with the gains set to minimize the amplitude of the pitch motion. To find the optimum settings, 10 runs were made in head seas and 15 runs were made in following seas with various gain settings (see Results).

It was not possible to run all of the conditions in the table above in following seas, in some cases because a sufficient number of waves was not encountered for proper analysis. In other cases a dangerous condition arose in which the model attained a pronounced bow down attitude, eventually plunging into the water. These results will be described in more detail later.

Wave heights listed above are nominal values. Actual wave heights (given in the data tables) were determined from the wave machine calibration.

In addition to this test matrix, a series of runs was made in calm water at two speeds with the canards oscillating, over a range of frequencies, to study the response of the system to sinusoidal excitation. These tests are described and the results are presented in Appendix C.

TEST PROCEDURE

Calibration

All transducers were calibrated immediately prior to the test. The drag and pitch moment balances were calibrated by applying known forces and moments, taking readings, and fitting a straight line to the results, the slope of which is the calibration rate. The heave and pitch transducers and the inclinometer used for steady trim were calibrated by setting

known heave or trim displacements and taking readings. All calibrations were well represented by straight line fits.

The two component fin balance was calibrated by applying known weights in the direction of lift, drag, and combinations of the two, taking voltage readings on both channels, and using a multivariate least squares fit to express the digitized voltage readings as linear functions of both the lift and the drag. The resulting matrix of coefficients was inverted to obtain the calibration rates. In addition, because the balance was to be used to make dynamic force measurements, a dynamic calibration was carried out, which showed the response of the balance to be flat in the range of frequencies to be encountered in the tests. The calibration procedure is explained in detail in Appendix D, which includes calibration results, plots, and a photograph of the dynamic calibration rig.

The wave machine was calibrated by running waves of the nominal length and height past a stationery wave wire located 130 ft from the wave machine; wave heights and periods were measured.

Fixed-Trim Tests in Calm Water

Zero readings were taken on all transducers, with the exception of the trim inclinometer, with the model floating at zero trim. The inclinometer reference (zero voltage) was horizontal. The model trim was next set to the desired value and a run was made. Data were collected in a 50 foot run length after the model had reached steady speed, and the results averaged for the time of the run. The resulting readings, minus the zero readings, were multiplied by the calibration rates to obtain measured quantities in engineering units.

Tests in Regular Waves

Zero readings were again taken on all channels with the model floating at level trim, when the water was sufficiently calm (generally in the morning before the first run and after lunch break). To start a run, the desired wave length and height were entered into the PDP-11 computer and the wave machine was activated. For the head seas tests, the model was started just before the waves reached the beach (located at the opposite end of the tank from the wave machine). Data were collected in a 110 foot run length after the model had reached steady speed. For the following seas tests, the model was started after a sufficient length of the wave train had gone by so that the model would not overtake the waves before the end of the run. Data were collected in an 80 ft run length; the oscillograph traces were then carefully examined for the effects of wave reflections from the beach which often occurred near the end of a run in the longer waves. If necessary the time histories to be processed were truncated to eliminate the apparently corrupted portion.

Preliminary tests in calm water with the model free to pitch and heave showed that the model had a pronounced bow-down tendency. This was due in part to the towing arrangement: The model was towed from a point above the deck, introducing a bow-down moment not present on the ship, where the thrust is applied along the centerline of the lower hulls. In addition, the flow over the hulls introduces a bow-down pitching moment; the high pressure at the nose is indicated by the bow wave. The particle velocities of the bow wave induce a downward force on the canards. To achieve zero mean trim on the ship it was assumed that the SWATH operator would adjust the stabilizers (the aft fins), allowing the canards (controlled automatically) to counteract the oscillatory pitch induced by waves. For the model tests the stabilizers were set to -15° (the downward force on the stern stabilizers brings the bow up). Final adjustment to near zero mean trim was achieved by shifting a small amount of weight on deck:

Speed	Weight	Distance
15 kt (5.17 fps)	1.50 lb	4 ft
20 kt (6.89 fps)	0.50 lb	4 ft

The weight shifts were symmetrical about the CG so that the pitch moment of inertia was unaffected.

Because the weight of the model and apparatus exceeded its displacement at the 14.5 ft waterline, it was necessary to unload. This was accomplished by attaching weights to an arm which applies an upward force, equal to twice the applied weight, at the pivot point. During dynamic tests, this means that the mass being accelerated by the waves was about 7 percent greater than the mass of the model plus hydrodynamic added mass. Since the effectiveness of the pitch control system was judged by comparison of results for this same configuration with and without control, the extra mass was not perceived to be of critical importance in interpretation of the results.

Water temperature was monitored daily during all tests. A tabulation of water temperatures is included as Appendix E.

DATA PROCESSING

Calm Water

For the calm water tests, the data, sampled at a rate of 100 Hz, were averaged over the duration of the run. The results, minus the zero readings, were multiplied by the calibration rates to obtain forces, moments and displacements in engineering units.

The effect on the pitch moment of the height of the towpoint above the full-scale thrust line was accounted for by

mathematically transferring the thrust to the centerline of the lower hulls:

$$M = M_p + D h$$

where M is the reported pitch moment, M_p is the moment measured about the axis passing through the towpoint, and h is the height of the towpoint above the centerline of the lower hulls, 1.8542 ft model scale. This is equivalent to transferring the moment reference to the thrust line. All reported pitch moment data contain this correction.

Regular Waves

In the regular wave tests, time histories of voltage signals on all channels were recorded on the computer disk. The fundamental period of the excitation, the wave encounter period, was determined by counting the number of zero crossings in the time history of the wave strut signal, as described in Reference 4. The signals from each channel were then fit to an expression of the form

$$v_i = a_{i0} + \sum_{n=n_0}^N a_{in} \cos(2\pi nt/T_e) + \sum_{n=n_0}^N b_{in} \sin(2\pi nt/T_e)$$

by the method of least squares, where T_e is the encounter period, N and n_0 are the orders of the highest and lowest harmonics used in the fit, and the coefficients a_{in} , b_{in} are the amplitudes determined by the fit. The expression is then written in the form

$$v_i = C_{i0} + \sum_{n=n_0}^N C_{in} \cos(2\pi nt/T_e - \phi_{in})$$

where C_{i0} is the mean, C_{in} is the amplitude of the n th harmonic, and ϕ_{in} is its phase relative to a specified reference channel, which is the pitch channel in this report unless otherwise noted.

For certain channels only mean quantities are of interest; thus, for drag and pitching moment, only the means C_{i0} (multiplied by appropriate calibration rates) are reported. For other channels it was found that only the amplitudes of the first and second harmonics, C_{i1} and C_{i2} , were significant, so that even though a four-term series ($N = 4$) was used in the fits, only the amplitudes of the first and second harmonics are reported in addition to the means.

Expansion to Full Scale

Mean drag and pitch moment, heave, wave height, and wave period and frequency have all been converted to full scale as noted in the Results section. Resistance expansion has been carried out according to the method described in Appendix A of Reference 2. Pitch moment was scaled up according to the following formula:

$$M_S = (\rho_S / \rho_M) M_M (LWL_S / LWL_M)^4$$

where ρ_S is the density of salt water at 59°F and ρ_M is the density of the tank water. Full scale heave and wave height were obtained by multiplying model quantities by the length ratio, and period is scaled by multiplication by the square root of the length ratio.

RESULTS

Results of this investigation are presented in Tables 3 through 10. Table 2 is a brief directory of the data tables.

Fixed Trim Tests in Calm Water

Table 3 contains the results of the calm water fixed trim tests of the unappended model (phase 1). The table contains run number; model speed, drag and pitch moment; measured trim; full-scale heave; model and ship resistance coefficients; and ship speed, resistance, EHP and pitch moment. Figure 6a is a plot of the behavior of pitch moment with speed; on Figure 6b the pitch moment is plotted against trim at several speeds. The slopes of the pitch moment vs. trim contours are plotted against speed on Figure 7.

Fixed Trim and Heave Tests with Instrumented Canard

Results of the tests with the instrumented canard in calm water are given in Table 4. The table lists run number; canard angle of attack relative to the hull; model speed, drag and pitch moment; measured trim (nominal trim was zero degrees); model scale lift and drag on the canard; model and ship resistance coefficients; and ship speed, resistance, EHP and pitch moment. The fin lift and drag coefficients are plotted against angle of attack on Figure 8, which also shows the results of the tests of the fin on a groundboard from Reference 1. Pitch moment is plotted against fin lift for two speeds and two canard aspect ratios on Figure 9.

Tables 5 and 6 contain the data from the model fixed in regular waves tests, with the aspect ratio 1 and 2 canards, respectively. The tables have three parts. Tables 5.1 and 6.1

contain all measured data in model scale. The tables list run number; model speed; wave height, length and period; encounter period and frequency; mean drag and pitch moment on the model; mean, first and second harmonics of fin lift and drag; and the phase lag of the oscillatory forces with respect to the arrival of the wave crest at the fin shaft. Thus a phase of zero degrees for L_1 , say, would indicate that the first harmonic of fin lift coincides with the wave crest at the shaft (actual values are near 270° , as would be expected; see Appendix F, page F2).

Tables 5.2 and 6.2 contain mean quantities from Table 5.1 and 6.1, expanded to full scale as described above. The tables list run number; ship speed; wave length to ship length ratio; wave height and period; encounter period and frequency; total resistance coefficient; resistance; EHP; and pitch moment.

Tables 5.3 and 6.3 include quantities pertaining to the instrumented canard, presented in coefficient form or in full-scale units. Listed are run number; ship speed; canard Reynolds number based on mean chord length; wave length/ship length; wave height; encounter period and frequency; and mean, first and second harmonics of canard lift and drag coefficients and their phases with respect to the wave crest at the canard shaft.

Behavior of the first harmonic of canard lift (which was the quantity of primary interest in this phase of the tests) with incident wave length and height are shown on Figures 10 and 11 for the aspect ratio 1 and 2 fins, respectively. A comparison is made with the results of a simple theory which is described in the Discussion.

Free to Pitch and Heave Tests with Control System

Results of the fourth phase of the tests, which include the automatic pitch control system, are presented in Tables 7 through 10. The first seven columns of each of these tables contain (in full scale units where applicable): Run number; ship speed; wave length/ship length; wave period; encounter period and frequency; and wave height. Negative encounter periods in following seas indicate overtaking waves. All phase angles in these tables represent phase lags relative to the pitch signal; thus the phase of the first harmonic of pitch is always zero and is not listed.

Table 7 lists results for baseline tests with the control system inactive; canards were fixed at zero degrees relative to the hull. In addition to quantities described above, the table gives the phase of the wave (arrival of the wave crest at the pivot point); mean drag (model); mean trim and heave; mean ship resistance and EHP; and first and second harmonics of pitch and heave, with phases.

Table 8 contains the results of tests of various control system gains to minimize pitch motion. In addition to quantities described above, this table contains the first harmonic of the fin angle and its phase, and the gains g_1 (degrees per degree) and g_2 (degrees per (degrees per second)),

full scale). These tests showed that the optimum gains (settings resulting in the lowest pitching motion amplitudes) were as tabulated below:

TABLE A. PITCH CONTROL OPTIMUM GAIN FACTORS.

	g_1 (displacement)	g_2 (rate, full-scale)
Head seas	0	9.65 deg/(deg/sec)
Following seas	6.36 deg/deg	6.76 deg/(deg/sec)

Results of tests in waves with active control are given in Tables 9 and 10 for the aspect ratio 1 and 2 canards, respectively, with the gain settings in Table A. Tables 9.1 and 10.1 are for the head seas condition; Tables 9.2 and 10.2 are for following seas. The upper portion of the tables list mean quantities: model drag; trim; mean heave; mean canard angle; and mean ship resistance and EHP. The lower portion of the tables gives oscillatory quantities: first and second harmonics of pitch, heave and canard angle, and phases. Phase angles are again with respect to pitch motion.

Effectiveness of the pitch control system is shown on Figures 12 and 13, which show nondimensional pitch amplitude (normalized by maximum wave slope $\pi H/\lambda$), against wave length, for speeds of 15 and 20 knots, respectively. The effect of the control system on heave motion (which no attempt was made to control) is shown for 20 knots on Figure 14. Figures 15 and 16 compare motion results for the two canard aspect ratios used in the tests.

DISCUSSION

Tests in Calm Water

Figure 6 shows that the pitch moment is an oscillatory function of speed; this is evidently due to the influence of the ship wave system. It can be seen that moment minima occur near speeds of 10 and 15 knots; maxima are observed at 12 and 20 knots. Thus an increase in speed from 15 to 20 knots results in a large change in pitching moment, from bow-down to bow-up. The SWATH operator must be alert to this behavior and adjust the canards and/or stabilizers accordingly.

Figure 6b shows that the pitch moment on the unappended SWATH is linear with trim in the range -2° to 2° and that the ship is statically stable in pitch at least up to 20 knots. Static stability is indicated by the slopes of the lines on the plot, which are negative, indicating that the moment tends to counteract any change in trim. The trend of static stability, as indicated by pitch moment per degree of trim, with speed is shown on Figure 7; extrapolation would indicate that the unappended vessel could become statically unstable between 25 and 30 knots.

Figure 8 contains the most important results of the instrumented canard tests in calm water. The figure indicates that the lift curve slope of the fin on the body is virtually identical to that for the fin on a groundboard; thus the combined effects of the free surface and the curvature of the hull on the lift rate are apparently small. Due to the bow wave there is a downward flow at the canards, inducing a speed dependent negative angle of attack. The downward force on the canards produces a bow-down moment which would add to the generally bow down moment on the hull at zero trim indicated on Figures 6a and 6b. Thus the canards, if not at least passively controlled, will tend always to pull the bow down. This would be destabilizing if the ship develops a negative trim (which is the tendency of the unappended hull below 20 knots).

If the lift force on a canard is known, it might be assumed that the pitch moment induced by this lift force is simply the force multiplied by the distance from the pitch axis to an "effective center of pressure" on the canard. This approach does not account for fin-body interaction. To investigate the validity of such an approach, a plot of pitch moment against fin lift was prepared (Figure 9). It can be seen that the moment increases linearly with the fin lift, and straight lines have been fitted as indicated. The slopes of the lines are equal to twice the distance from the moment reference point (on the hull centerline, 51.25 ft aft of the strut leading edge) to the effective center of pressure, since the lift is that on a single canard and the moment contains the effects of both canards. Results are summarized below:

TABLE B. LOCATION OF APPARENT CENTER OF PRESSURE

Speed knots	Aspect ratio	CP location (ft)
15	1	65.77
15	2	50.08
20	1	76.50
20	2	59.23

The distance from the moment axis to the canard shafts is 45 ft; thus the CP locations given above are forward of the canards. It can be concluded that fin-body interaction cannot be neglected when estimating the pitch moment due to the canards. The fact that the apparent lever arm is larger than the geometric one indicates that some of the low pressure induced above the canard when at an angle of attack is "spilling over" onto the hull, so that the upward force induced by the fin is larger than the force on the fin itself (this is the essence of fin-body interaction). Additionally, the downwash induced by the canards may interact with the stabilizers to induce an increased bow-up moment.

Tests Fixed in Regular Waves

An interesting aspect of the instrumented fin results in regular waves was that the maximum oscillatory lift coefficients measured were substantially higher than the stall values found in the steady state tests. The data of Reference 1 show the maximum lift coefficients of the NACA 0015 fins tested on a groundboard to be between 0.7 and 0.8. Figures 10 and 11 show that oscillatory lift coefficients approached 1.0 in the 12 foot waves at 10 knots. In Run 111, for example, Table 6.3 shows that the mean and first harmonic of lift coefficient were -0.316 and 0.968, respectively, so that during this run C_L oscillated between -1.284 and 0.652. No indication of stall was evident in the oscillograph records, and for this particular run the amplitude of the first harmonic of lift (0.49 from Table 6.1) was practically identical to the RMS of the signal multiplied by $\sqrt{2}$:

$$0.3549 \times 1.414 = 0.50$$

indicating that almost all of the energy of the signal was contained in the first harmonic. Thus the sinusoidal fit is a good representation of the signal, which would not be the case if stalling was taking place.* It would therefore seem that the oscillatory flow somehow delays the onset of stall.

Though data for airfoils or hydrofoils operating in oscillating flows is relatively scarce (particularly for cases in which flow angles are near the stall angle of the foils), there is a fair amount of data for foils oscillating in pitch and/or heave in a steady stream. The situations are different, but it is expected that general trends with frequency of oscillation, for example, will be similar, particularly at low frequencies where the effects of unsteadiness should be small.

In the case of pitching oscillation, Halfman et al⁵ point out that "several investigators...have noted that...the stall may occur at an angle of attack considerably above the static stalling angle". More recently Wickens⁶, in an investigation of an NACA 0018 airfoil oscillating in pitch, found that "dynamic stall...occurred about 5 degrees later than for the equivalent steady flow case. This phenomenon resulted in an increase in normal force of about 20%...when the wing was pitching to 30 degrees". This phenomenon is attributed by Ericsson and Reding⁷ to the "accelerating flow on the leeward side of [the] pitching airfoil [which] causes a decrease in the adversity of the pressure gradient, resulting in a large overshoot of the static stall." They express the dynamic stall overshoot $\Delta\alpha_S$ as⁸

$$\Delta\alpha_S = (c\dot{\alpha}/V) \Delta\xi$$

*Examples of oscillograph records of lift on an oscillating airfoil can be found in Reference 5: Records taken when stall occurred were definitely non-sinusoidal and in one case not even periodic.

where c is the chord length, $\dot{\alpha}$ is the rate of change of angle of attack, V is the free stream velocity, and $\Delta\xi$ is a dimensionless time lag due to the oscillation. The quantity in parentheses is equal to the product of the pitching amplitude and the "reduced frequency", $c\omega/V$, where ω is the frequency of oscillation.[†] Ericsson and Reding's analysis of the data of Reference 5 indicates that the value of $\Delta\xi$ is 2. The increase in maximum lift coefficient would then be

$$(dC_L/d\alpha) \Delta\alpha_s = (dC_L/d\alpha) (c\dot{\alpha}/V) \Delta\xi.$$

Referring again to the example of Run 111, the lift curve slope of the aspect ratio 2 fin is given in Reference 1 as 0.060/degree. The measured C_L amplitude would thus correspond to an angle of attack amplitude of 16.13° , and a maximum rate $\alpha = 79.52$ degrees/second. The formula above then gives the result

$$\Delta C_{Lmax} = 0.41$$

for this run, so that the stall would be delayed up to a C_L of 1.2. This is roughly the magnitude observed in Run 111.

In Reference 1 it was concluded that the only important Reynolds number related difference between ship and between ship and scale model appendage lift was a reduction of maximum lift coefficient at model scale. The present results indicate that this reduction is counteracted by the effects of oscillatory flow. The data from the tests in waves would thus appear to be free of any scale effects on lift, particularly in head seas where the encounter frequencies were high. In following seas, the encounter frequencies were low but Tables 5.3 and 6.3 show that the lift coefficients were generally well below the static stall values.

The theory of two dimensional airfoils in non-uniform motion has been applied to sinusoidal oscillations by Sears⁹, who treats both the case of a foil undergoing pitching and/or heaving oscillations and the case of a foil penetrating a sinusoidal gust. The problem of a fin moving in waves is similar to the latter case but not identical, since in the gust problem only the vertical velocity of the fluid is assumed to vary, whereas in waves both velocity components vary. In the tests described here the particle velocity was as much as 28% of the model velocity, which cannot be considered negligible. Hence Sears' results are not directly applicable to the case of a foil in waves.

A three-dimensional theory for predicting hydrodynamic forces and moments on a hydrofoil moving in waves has been presented by Tsakonas and Henry¹⁰; the theory leads to an integral equation which must be solved numerically. However, they note that their results agree quite well with a "quasi-

[†]The reduced frequency parameter occurs frequently in unsteady airfoil theory, and is a dimensionless measure of the angular excursion per chord length of travel at speed V .

steady" prediction in which a time-dependent angle of attack is computed based on particle velocities calculated from linear wave theory, and simply multiplied by the steady-flow lift curve slope.^{††} Their quasi-steady theory is strictly only applicable to the gust problem, since the above-mentioned effect of particle velocity on the horizontal fluid velocity component (more precisely, the horizontal component of the total fluid velocity relative to the foil) does not seem to have been considered. A unique, simplified quasi-steady prediction is developed in Appendix F for the case in which the ratio of particle velocity to ship speed is small (high speeds or low frequencies) but which does not completely neglect the horizontal particle velocities. It is emphasized that "quasi-steady" refers to the computation of angle of attack and not to the maximum lift coefficient, which is affected by unsteadiness as discussed above.

The results of the "quasi-steady" prediction of canard lift in the high speed (20 knot) tests are shown on Figures 10 and 11 as broken lines. It can be seen that the theory generally underestimates the lift, by an amount that increases with wave height. The simple theory thus yields a conservative estimate of canard lift at 20 knots. A more sophisticated theory, not subject to the "small particle velocity" restriction, is required for accurate predictions at lower speeds.

In following seas, the waves overtook the ship in all runs except for the shortest waves and the highest model speed. The overtaking waves become distorted they pass the ship, so it is reasonable to expect that particle velocities associated with the waves would be reduced near the bow (at the location of the canards). Videotapes of the phase three tests show that considerable distortion of the waves does occur at 10 knots; the waves appear to break just aft of the strut nose. At 20 knots, the waves pass by without much distortion. This would account for the lower canard lift coefficient in following seas at 10 knots and the near agreement of the lift coefficients in head and following seas at 20 knots.

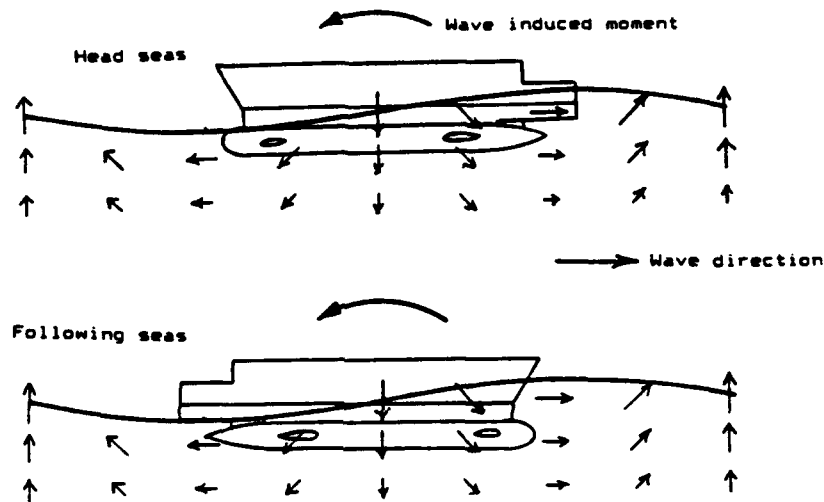
It should be reemphasized that these tests were carried out with the model fixed in heave. When free to heave, the model undergoes considerable sinkage (in excess of 5 feet under some conditions) so that the measured fin forces are not necessarily what would be expected were the model free to pitch and heave.

Tests with Automatic Pitch Control

The effectiveness of the automatic pitch control is shown on Figures 12 and 13, for speeds of 15 and 20 knots, respectively. The figures show a moderate reduction in pitch motion in head seas, and a large reduction in following seas. This is in accord with previous theoretical predictions^{11,12} and

^{††}It should be noted that Tsakonas and Henry were surprised by this agreement, and suggest that it might be somewhat fortuitous.

full scale trial data¹³, but is somewhat surprising in light of the force data (Figures 10 and 11) which indicate lower canard lift in following seas. The increased effectiveness of the canards in following seas may be due in part to the phase of the particle velocities at the canards relative to the pitching motion. For waves longer than about twice the ship length, the wave-induced pitch moment on the hull lags the wave by approximately 90° . In head seas the flow over the canard acts to reinforce this pitch moment, pulling the bow up on the face of the wave and pushing it down on the back side. In following seas, the situation is reversed as shown on Sketch A below.



SKETCH A. PARTICLE VELOCITIES AND PITCH MOMENT IN HEAD AND FOLLOWING SEAS.

It might also be noticed that these same observations apply to the stabilizers; this would tend to worsen the pitch motion in following seas. More will be said about this later.

The magnitude of the encounter frequency relative to the natural frequency of the vessel in pitch also has an important effect on the magnitude of the motions. For an exciting moment of a given amplitude, such as that induced by the moving canards, the amplitude of the response will be greater at frequencies below the natural frequency than at frequencies above the natural frequency (in fact, the motion will approach zero at high frequency). Reference to Tables 9 and 10 shows that the encounter frequencies in head seas were in the range of 1 to 2 rps, whereas those in following seas were in the range of 0.2 to 0.3 rps. The zero speed natural pitch period of the vessel is 11.76 seconds (Table 1), corresponding to a frequency of 0.53 rps. The natural pitch period of a SWATH typically increases with speed¹⁵; thus the encounter frequencies in head seas were well above the natural frequency of pitch motion. Smaller motions would then be expected in head seas than in

following seas, where encounter frequencies were near or below the natural frequency of pitch.

Figure 14 shows that although no attempt was made to reduce heaving motion, heave was also reduced by the action of the control system, again to a greater extent in following seas. Further reductions would be possible by using heave and rate of change of heave as additional inputs to the control system.

The effect of aspect ratio of the canards on the effectiveness of the control system is shown on Figures 15 and 16 to be small. Differences are insignificant at 15 knots; at 20 knots the aspect ratio 2 fins reduce heaving to a slightly greater extent.

Large Motions in Following Seas

During several runs in following seas the model attained a pronounced bow-down attitude, taking on water over the main deck at the bow. This occurred only at the high speed (20 knots) in the 8 foot waves, when the wave speed was close to the model speed. When the model speed slightly exceeded the wave speed, the model seemed to ride up over the first wave crest encountered in a run, and plow into the back of the next wave. Figure 17 shows one such occurrence. In longer waves, just overtaking the model, the model survived the first encounter but near the end of the run the stern seemed to be picked up by the second overtaking wave, plunging the bow into the water. Conditions under which these phenomena were observed are tabulated below:

TABLE C. CONDITIONS FOR MODEL SINKINGS IN FOLLOWING SEAS

Run	λ/LWL	Ship speed, kt	Wave speed, kt	Wave height, ft	Automatic control
85	1.0	20	15.1	8.24	no
88	1.5	20	18.3	8.26	no
97	1.0	20	15.1	8.24	no
101	2.5	20	23.5	8.66	not opt.
216	1.0	20	15.1	8.24	yes
217	1.5	20	18.3	8.26	yes
219	1.5	20	18.3	8.26	yes
220	2.5	20	23.5	8.66	yes

After run 101 a large coaming was placed on the deck to keep the deck mounted electronic equipment dry during the swappings. The coaming is visible in Figure 17 and in the videotapes.

A similar phenomenon was observed by Fein et al during self-propelled model tests of the SWATH SSP Kaimalino:

"Largest motions were found in following seas when ship speed was close to the wave speed. In that case a large bow-down static trim occurred due to the

action of the wave on the full span aft foil. This condition, which could lead to the upper structure bow being buried in the wave and propeller broaching, was later observed in full scale trials."¹³

The following passage pertains to the full scale trials:

"The only case where deck wetness occurred was in following seas when the wave speed approached the ship speed and a large amplitude but gentle bow 'plow-in' occurred. Propeller broaching occurred in similar conditions in quartering seas."¹³

The severity of the "plowing in" in the present tests may have been exaggerated by the towing method. As explained in Test Procedure, the mean hydrodynamic moment on the hull and the couple due to the height of the towpoint above the thrustline were compensated for by setting an angle on the stabilizers and shifting a small amount of weight on deck. When the bow begins to plunge, the drag increases, giving rise to an increase in the bow-down couple which is not compensated for. However, the tendency for "plowing in" is in accord with the self-propelled model and full scale observations quoted above.

In the discussion under Automatic Pitch Control above, it was pointed out that in waves longer than about twice the ship length, the flow over the stabilizers gives rise to a moment acting to reinforce the wave-induced pitch moment on the hull. This is the mechanism alluded to by Fein et al in the quote above as a cause of the plow-in phenomenon. However, no plow-in occurred in a 12 foot wave at a wave length to ship length ratio of 3.5, in which case wave-induced stabilizer angles of attack are slightly larger than for the shorter waves in which the problem occurred. Clearly, further study of this potentially dangerous phenomenon is warranted.

CONCLUSIONS

This hydrodynamic study of a SWATH vessel has provided much unique data and has demonstrated the effectiveness of activating the canards in reducing the pitching motion in regular waves. Several other important conclusions can be drawn from the data and discussions above:

1. The active control system employed in this study is most effective in following seas, where reductions in pitch motion of more than 50% were realized. This is in accord with previous theoretical predictions and full-scale trial data for other SWATH configurations.
2. When the vessel operates in following seas, at speeds nearly equal to the wave speed, large amplitude pitching motions can develop, possibly leading to the bow plowing into the waves.

This tendency has also been observed during full-scale trials of SSP Kaimalino¹³.

3. Measurements of unsteady canard lift in waves indicate that a higher maximum lift coefficient is reached than during static tests. No evidence of stall was detected in the data from these tests; the lift produced by the fins in oscillatory flow is thus expected to be fully representative of full scale canard lift.

4. The lift curve slopes of the canards on the hull are the same as those found previously for the canards tested on a groundboard¹. Pitch moment data indicates that fin-body interaction, and possibly canard-stabilizer interaction, cannot be neglected in predictions of the moment due to canard deflection, however.

5. Measured canard lift in regular waves is larger than that predicted by a simple quasi-steady approach in which the angle of attack is computed using particle velocities from linear wave theory. The prediction improves with increasing speed and decreasing wave height (that is, decreasing angle of attack).

6. The aspect ratio of the canards has little effect on the performance of the SWATH in waves.

RECOMMENDATIONS

The results of this study have indicated that further work is necessary in order to gain a better understanding of the hydrodynamics of a SWATH in waves:

1. A potentially dangerous phenomenon has been observed in following seas. In a situation where the ship just overtakes the waves, bow plow-in took place quite quickly, occurring just after the first encountered wave. A SWATH operator thus would have a limited amount of time in which to take corrective action if such a wave component were present in a seaway (the automatic control system was not adequate to prevent this phenomenon). It is recommended that a careful study of the behavior of a SWATH in following seas be carried out, in regular and irregular waves, to establish the conditions under which bow plow-in occurs, and to explore ways to alleviate the problem. For these tests, the model should be towed from a point on the propeller shaft line, as recommended by the 18th ITTC¹⁴, and should be free to surge. This will minimize any influence of the towing system on the behavior of the model.

2. Measurement of the stabilizer forces in calm water and in waves is recommended, in light of their possible role in causing bow plow-in in following seas. Tests with and without canards should be conducted to assess the possible effect of the canard trailing vortex system on the stabilizer forces. Flow

visualization tests could also be conducted to study the trajectories and strength of these vortices.

3. It is possible that further reductions in pitching and heaving motions could be achieved by activating the stabilizers. This possibility also warrants further study.

4. The control system should be extended to include heave motion control. This would involve the use of heave amplitude and rate as inputs to the control system in addition to pitch amplitude and rate.

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TABLE 1

Ship Particulars

Strut length (LWL), ft	125
Hull length, ft	124
Hull diameter, ft	10
Maximum beam, ft	59
Displacement (14.5 ft WL), LT	591
Cross structure clearance, ft (to 14.5 ft WL)	10
Strut wetted area, sq ft	2305
Hull wetted area, sq ft	6710
VCG, ft above baseline	18.98
LCG, ft aft of strut nose	51.72
GM _L , ft	24.45
Pitch period (zero speed), sec	11.76
Pitch gyradius, ft	38.4

TABLE 2

Directory of Data Tables

Table	Description
3	Unappended, fixed-trim tests in calm water
4	With instrumented canard in calm water
Tests Fixed in Trim and Heave in Regular Waves:	
5.1	All data in model scale; canard aspect ratio 1
5.2	Mean quantities expanded to full scale; canard aspect ratio 1
5.3	Canard lift and drag expressed in coefficient form; canard aspect ratio 1
6.1	All data in model scale, canard aspect ratio 2
6.2	Mean quantities expanded to full scale; canard aspect ratio 2
6.3	Canard lift and drag expressed in coefficient form; canard aspect ratio 2
Tests Free to Pitch and Heave in Regular Waves:	
7	Baseline tests without active control; canard aspect ratio 1
8	Varying control system gains to minimize pitching motion; canard aspect ratio 1
9.1	Head seas with optimal control gains; canard aspect ratio 1
9.2	Following seas with optimal control gains; canard aspect ratio 1
10.1	Head seas with optimal control gains; canard aspect ratio 2
10.2	Following seas with optimal control gains; canard aspect ratio 2

TABLE 3

CALM WATER TESTS OF UNAPPENDED MODEL FIXED IN TRIM

Run no.	Vel fps	Drag lb	M ft-lb	Trim deg	Heave ft	CTM (x1000)	CTS (x1000)	Vel knots	RS lb	EHP	M ton-ft
Nominal trim = -2 degrees											
83	0.00	0.00	3.66	-1.94	-0.36						
84	1.72	0.23	3.22	-1.95	-0.46	5.091	2.891	5.00	1846.	28.3	490.4
85	2.41	0.49	2.96	-1.96	-0.46	5.509	3.549	7.00	4445.	95.6	450.8
86	3.10	1.02	2.59	-1.97	-0.60	6.883	5.084	9.01	10540.	291.5	394.5
87	3.45	1.13	1.73	-1.98	-0.62	6.152	4.416	10.02	11324.	348.4	263.5
88	3.79	2.02	1.44	-2.01	-0.68	9.179	7.497	11.00	23202.	784.0	219.3
89	4.14	2.38	2.58	-2.00	-1.06	9.077	7.444	12.02	27474.	1013.9	393.0
90	4.48	2.08	1.95	-2.00	-1.04	6.756	5.166	13.00	22327.	891.6	297.0
91	4.83	2.15	0.50	-2.02	-0.96	5.987	4.437	14.03	22306.	960.8	76.2
92	5.17	2.77	0.18	-2.04	-1.20	6.735	5.220	15.01	30066.	1386.1	27.4
93	5.52	3.84	0.68	-2.06	-1.58	8.214	6.732	16.03	44187.	2174.6	103.6
94	5.86	4.84	1.20	-2.02	-1.82	9.205	7.752	17.02	57382.	2998.9	182.8
95	6.22	5.64	1.72	-2.02	-1.86	9.535	8.111	18.04	67481.	3739.0	262.0
96	6.22	5.70	1.78	-2.04	-1.90	9.624	8.202	18.05	68309.	3786.7	271.1
97	6.55	6.40	1.93	-1.99	-2.12	9.746	8.349	19.01	77151.	4505.2	294.0
98	6.91	7.00	1.95	-2.00	-2.24	9.589	8.216	20.05	84375.	5194.0	297.0
Nominal trim = -1 degree											
35	0.00	0.00	1.93	-1.04	-0.12						
36	1.72	0.24	1.64	-1.06	-0.22	5.202	3.000	5.00	1914.	29.4	249.8
37	2.41	0.50	1.23	-0.97	-0.26	5.544	3.583	7.00	4488.	96.5	187.3
50	2.41	0.49	1.19	-0.93	-0.22	5.456	3.496	7.00	4379.	94.2	181.2
38	3.10	1.00	1.12	-0.98	-0.36	6.750	4.949	9.00	10247.	283.2	170.6
39	3.45	1.12	0.28	-1.00	-0.30	6.117	4.380	10.02	11232.	345.5	42.6
40	3.79	2.01	0.53	-1.01	-0.46	9.143	7.460	11.00	23087.	780.2	80.7
41	4.14	2.33	1.64	-1.01	-0.74	8.855	7.223	12.03	26696.	985.9	249.8
42	4.48	2.02	0.49	-1.01	-0.68	6.550	4.960	13.00	21427.	855.5	74.6
43	4.83	2.09	-0.85	-1.03	-0.66	5.801	4.251	14.03	21391.	921.7	-129.5
44	5.17	2.70	-1.16	-1.04	-0.86	6.548	5.033	15.02	29012.	1338.0	-176.7
45	5.52	3.68	-0.53	-1.06	-1.18	7.873	6.391	16.02	41915.	2062.0	-80.7
46	5.86	4.69	0.22	-1.02	-1.52	8.908	7.456	17.02	55187.	2884.2	33.5
47	6.21	5.48	0.67	-1.03	-1.84	9.267	7.843	18.03	65190.	3610.3	102.0
48	6.55	6.11	1.00	-1.05	-1.96	9.294	7.895	19.01	72958.	4260.3	152.3
49	6.91	6.66	1.16	-1.06	-2.06	9.120	7.745	20.05	79541.	4896.4	176.7

TABLE 3
(Continued)

CALM WATER TESTS OF UNAPPENDED MODEL FIXED IN TRIM

Run no.	Vel fps	Drag lb	M ft-lb	Trim deg	Heave ft	CTM (x1000)	CTS (x1000)	Vel knots	RS lb	EHP	M ton-ft
Nominal trim = 0 degrees											
16	1.72	0.24	-0.04	-0.06	0.00	5.171	2.970	5.00	1896.	29.1	-6.1
99	1.72	0.24	0.02	-0.04	0.04	5.221	3.024	5.00	1929.	29.6	3.0
100	2.06	0.35	-0.01	-0.04	0.02	5.337	3.271	5.98	2992.	55.0	-1.5
17	2.41	0.49	-0.09	-0.06	-0.02	5.407	3.446	7.00	4320.	92.9	-13.7
101	2.41	0.50	-0.05	-0.05	0.00	5.547	3.588	6.99	4484.	96.3	-7.6
102	2.75	0.66	-0.26	0.02	0.00	5.687	3.815	7.98	6217.	152.4	-39.6
18	3.10	1.00	-0.11	-0.07	-0.06	6.746	4.947	9.01	10264.	284.0	-16.8
103	3.10	0.99	-0.14	0.01	-0.06	6.721	4.924	9.01	10208.	282.4	-21.3
19	3.45	1.08	-0.89	-0.07	-0.02	5.896	4.159	10.03	10685.	329.0	-135.6
20	3.80	1.89	-0.68	-0.09	-0.22	8.583	6.900	11.02	21399.	723.9	-103.6
21	4.14	2.32	0.35	-0.09	-0.50	8.816	7.183	12.03	26547.	980.4	53.3
22	4.49	2.02	-0.55	-0.10	-0.42	6.539	4.949	13.02	21436.	857.0	-83.8
23	4.83	2.01	-1.85	-0.11	-0.36	5.596	4.045	14.02	20319.	874.8	-281.8
24	5.17	2.54	-2.23	-0.12	-0.54	6.172	4.656	15.02	26840.	1237.9	-339.6
25	5.53	3.48	-1.74	-0.13	-0.84	7.425	5.943	16.04	39077.	1924.8	-265.0
26	5.86	4.44	-0.88	-0.14	-1.26	8.432	6.978	17.02	51654.	2699.5	-134.0
27	6.22	5.24	-0.19	-0.15	-1.52	8.847	7.422	18.05	61813.	3426.6	-28.9
28	6.55	5.89	0.33	-0.16	-1.68	8.947	7.547	19.03	69831.	4080.2	50.3
29	6.91	6.44	0.56	-0.16	-1.88	8.813	7.437	20.05	76379.	4701.8	85.3
Nominal trim = 1 degree											
51	0.00	0.00	-2.02	1.03	0.26						
52	1.72	0.24	-1.90	1.03	0.24	5.212	3.012	5.00	1921.	29.5	-289.4
53	2.41	0.53	-1.75	1.03	0.20	5.862	3.902	7.00	4888.	105.1	-266.5
67	2.75	0.70	-1.64	1.07	0.18	5.986	4.111	7.98	6699.	164.3	-249.8
54	3.10	1.04	-1.64	1.03	0.14	7.021	5.221	9.01	10825.	299.4	-249.8
55	3.45	1.15	-2.38	1.01	0.18	6.258	4.522	10.03	11617.	357.7	-362.5
56	3.79	1.89	-2.22	1.00	0.04	8.580	6.898	11.01	21358.	721.9	-338.1
57	4.14	2.36	-1.20	0.98	-0.26	8.988	7.355	12.03	27186.	1004.0	-182.8
58	4.48	2.08	-2.04	0.97	-0.16	6.735	5.145	13.00	22236.	888.0	-310.7
59	4.83	2.04	-3.29	1.00	-0.10	5.674	4.123	14.03	20731.	892.9	-501.1
60	5.17	2.51	-3.47	1.02	-0.24	6.099	4.584	15.01	26405.	1217.3	-528.5
61	5.52	3.42	-2.90	1.01	-0.58	7.314	5.831	16.02	38247.	1881.6	-441.7
62	5.86	4.32	-2.06	1.00	-0.92	8.208	6.755	17.01	49948.	2609.0	-313.8
63	6.22	5.05	-1.36	0.98	-1.22	8.532	7.108	18.04	59137.	3276.7	-207.1
65	6.55	5.67	-0.69	0.96	-1.42	8.618	7.218	19.01	66706.	3895.3	-105.1
66	6.91	6.22	-0.39	0.98	-1.52	8.502	7.127	20.06	73281.	4513.7	-59.4

TABLE 3
(Concluded)

CALM WATER TESTS OF UNAPPENDED MODEL FIXED IN TRIM

Run no.	Vel fps	Drag lb	M ft-lb	Trim deg	Heave ft	CTM (x1000)	CTS (x1000)	Vel knots	RS lb	EHP	M ton-ft
Nominal trim = 2 degrees											
32	0.00	0.00	-3.58	1.94	0.40						
68	0.00	0.00	-3.82	2.09	0.38						
33	1.72	0.24	-3.24	1.93	0.38	5.269	3.068	5.00	1957.	30.0	-493.5
69	1.72	0.25	-3.42	2.07	0.42	5.371	3.171	5.00	2023.	31.0	-520.9
34	2.41	0.57	-3.18	2.03	0.40	6.312	4.351	7.01	5460.	117.5	-484.3
70	2.41	0.57	-3.20	2.07	0.44	6.314	4.354	7.00	5459.	117.4	-487.4
71	3.10	1.06	-3.15	2.06	0.40	7.181	5.381	9.00	11142.	308.0	-479.8
72	3.45	1.23	-3.66	2.06	0.44	6.707	4.971	10.02	12754.	392.5	-557.4
73	3.79	1.88	-3.63	2.05	0.34	8.542	6.860	11.00	21230.	717.4	-552.9
74	4.14	2.44	-2.55	2.04	0.00	9.291	7.659	12.03	28334.	1046.9	-388.4
75	4.48	2.17	-3.25	2.04	0.12	7.048	5.458	13.01	23610.	943.2	-495.0
76	4.83	2.05	-4.52	2.03	0.20	5.708	4.158	14.03	20907.	900.5	-688.4
77	5.17	2.50	-4.72	2.02	0.06	6.075	4.560	15.02	26289.	1212.4	-718.9
78	5.53	3.38	-4.05	2.01	-0.24	7.213	5.731	16.04	37684.	1856.2	-616.8
79	5.87	4.23	-3.15	2.01	-0.52	8.020	6.567	17.03	48661.	2544.4	-479.8
80	6.22	5.00	-2.25	2.00	-0.84	8.442	7.018	18.04	58387.	3235.1	-342.7
81	6.55	5.62	-1.64	1.99	-1.08	8.537	7.138	19.03	66041.	3858.8	-249.8
82	6.90	6.11	-1.26	1.98	-1.22	8.365	6.990	20.03	71704.	4411.4	-191.9

TABLE 4
TESTS IN CALM WATER WITH INSTRUMENTED CANARD; FIXED TRIM AND HEAVE

Run no.	α deg	Vel fps	Drag lb	PM ft-lb	Trim deg	L-fin lb	D-fin lb	CTM (x1000)	CTS (x1000)	C_L	C_D	Vel knots	RS lb	EHP	M ton-ft
Fin Aspect Ratio = 1															
10	0	3.45	1.28	-0.05	0.04	-0.15	0.01	7.023	5.473	-0.285	0.022	10.01	14003.	430.3	-7.6
11	0	5.17	3.03	-0.78	-0.02	-0.22	0.02	7.402	6.032	-0.190	0.019	14.99	34666.	1596.3	-118.8
12	0	6.90	6.27	8.82	-0.03	-0.20	0.05	8.583	7.330	-0.099	0.023	20.04	75213.	4628.0	1343.3
13	5	3.45	1.25	0.58	0.05	-0.05	0.01	6.847	5.297	-0.103	0.023	10.01	13568.	417.1	88.3
14	5	5.17	3.01	0.20	-0.02	-0.04	0.02	7.334	5.965	-0.032	0.022	15.00	34306.	1580.3	30.5
15	5	6.90	6.38	11.01	-0.02	0.11	0.05	8.746	7.893	0.052	0.024	20.02	76799.	4722.8	1676.9
16	10	3.45	1.28	1.47	0.05	0.05	0.02	6.974	5.425	0.083	0.047	10.01	13903.	427.6	223.9
17	10	5.17	3.00	1.42	-0.01	0.16	0.05	7.301	5.931	0.136	0.041	15.00	34111.	1571.4	216.3
18	10	6.90	6.46	13.36	0.01	0.47	0.09	8.846	7.593	0.230	0.046	20.03	77887.	4791.8	2034.8
20	15	3.45	1.35	2.05	0.06	0.14	0.05	7.394	5.845	0.273	0.091	10.01	14971.	460.3	312.2
21	15	5.17	3.06	2.47	0.01	0.36	0.09	7.467	6.098	0.316	0.078	15.00	35083.	1616.4	376.2
22	15	6.90	6.71	15.71	0.03	0.84	0.17	9.200	7.947	0.418	0.085	20.03	81471.	5010.9	2392.7
23	20	3.45	1.40	2.70	0.08	0.25	0.08	7.629	6.079	0.480	0.152	10.02	15590.	479.6	411.2
24	20	5.17	3.16	3.57	0.03	0.58	0.15	7.699	6.329	0.507	0.134	15.00	36415.	1677.8	543.7
25	20	6.90	7.03	17.93	0.05	1.23	0.28	9.637	8.384	0.604	0.138	20.03	85979.	5288.9	2730.9
Fin Aspect Ratio = 2															
34	0	3.45	1.29	0.01	0.05	-0.19	0.01	7.082	5.532	-0.378	0.017	10.01	14170.	435.6	1.5
35	0	5.17	3.08	-0.52	-0.02	-0.28	0.03	7.509	6.140	-0.247	0.025	15.00	35296.	1625.6	-79.2
36	0	6.90	6.37	9.12	-0.02	-0.27	0.06	8.722	7.469	-0.132	0.029	20.03	76597.	4711.8	1389.2
37	5	3.45	1.27	0.86	0.01	-0.07	0.01	6.911	5.362	-0.135	0.025	10.02	13759.	423.4	131.0
38	5	5.16	2.99	0.69	-0.07	-0.02	0.03	7.287	5.918	-0.020	0.031	14.99	33993.	1565.0	105.1
39	5	6.90	6.39	11.67	-0.07	0.18	0.07	8.748	7.495	0.090	0.037	20.03	76883.	4730.1	1777.4
46	10	3.45	1.29	1.62	0.06	0.09	0.03	7.022	5.473	0.179	0.067	10.02	14052.	432.5	246.7
48	10	5.17	3.00	1.86	0.03	0.08	0.03	7.027	5.477	0.160	0.068	10.00	14004.	430.2	254.4
47	10	5.17	3.03	2.03	-0.02	0.27	0.07	7.315	5.946	0.269	0.063	14.99	34169.	1573.4	283.3
59	10	6.90	6.58	14.40	0.02	0.77	0.14	9.024	7.771	0.379	0.064	15.02	34546.	1593.3	309.2
60	10	6.90	6.60	14.63	-0.01	0.73	0.14	9.041	7.789	0.362	0.068	20.02	79576.	4891.5	2193.2
42	15	3.45	1.32	2.25	0.07	0.21	0.07	7.220	5.670	0.424	0.131	10.01	14525.	446.5	342.7
45	15	5.17	3.05	2.28	0.07	0.21	0.06	7.131	5.581	0.406	0.122	10.01	14287.	439.1	347.3
55	15	6.90	6.76	16.67	0.01	1.17	0.22	9.301	8.048	0.579	0.111	20.00	82265.	5052.4	2539.0
43	15	5.17	3.05	2.93	0.02	0.53	0.13	7.428	6.059	0.468	0.119	10.02	14357.	441.9	354.9
56	15	5.17	3.07	3.04	-0.01	0.54	0.12	7.484	6.115	0.478	0.115	15.01	34871.	1607.0	446.3
44	15	6.90	6.78	16.61	0.05	1.16	0.23	9.304	8.051	0.573	0.116	20.02	82468.	5070.0	2529.8
57	15	6.89	6.76	16.67	0.01	1.17	0.22	9.301	8.048	0.579	0.111	20.00	82265.	5052.4	2539.0
49	20	3.45	1.40	2.98	0.07	0.39	0.13	7.639	6.090	0.770	0.250	10.01	15608.	480.0	453.9
54	20	5.17	3.24	4.08	0.02	0.82	0.27	7.894	6.524	0.664	0.255	10.02	15521.	477.4	479.8
50	20	5.17	3.15	4.06	0.02	0.80	0.27	7.894	6.524	0.726	0.235	15.00	37521.	1728.5	621.4
53	20	6.89	7.41	16.94	0.03	1.35	0.50	10.186	8.933	0.704	0.235	14.99	36327.	1672.5	618.4
51	20	6.89	7.41	16.94	0.03	1.35	0.50	10.186	8.933	0.668	0.247	20.01	91420.	5617.9	2580.1
52	20	6.90	7.36	17.02	0.02	1.32	0.50	10.104	8.851	0.653	0.247	20.03	90768.	5583.5	2892.3

TABLE 5.1
TESTS WITH INSTRUMENTED ASPECT RATIO 1 CANARD IN REGULAR WAVES. FIXED TRIM AND HEAVE. MODEL SCALE UNITS.

Run no.	Vel fps	H in	λ ft	T sec	T _e sec	ω ₀ rps	mean drag	mean M	mean lift	Head seas				Fin forces				phase
										L ₁ lb	phase	L ₂ lb	phase	mean drag	D ₁ lb	phase	D ₂ lb	
117	3.45	2.10	5.26	1.01	0.608	10.33	1.40	-1.90	-0.15	0.15	273	0.01	131	0.00	0.02	185	0.01	147
119	3.45	4.02	5.31	1.02	0.613	10.25	1.90	-2.36	-0.16	0.30	260	0.05	113	-0.01	0.04	158	0.03	120
127	3.45	2.08	7.84	1.24	0.801	7.84	1.36	-2.12	-0.16	0.16	267	0.01	315	0.01	0.02	332	0.01	114
129	3.45	4.08	7.87	1.24	0.803	7.82	1.60	-2.37	-0.16	0.32	266	0.03	302	0.00	0.04	323	0.02	100
131	3.45	2.26	10.42	1.43	0.970	6.48	1.35	-2.13	-0.16	0.14	270	0.00	208	0.01	0.02	231	0.00	264
155	3.45	4.42	10.43	1.43	0.971	6.47	1.44	-2.50	-0.17	0.31	268	0.02	185	-0.01	0.04	224	0.02	257
135	3.45	2.12	13.01	1.61	1.126	5.58	1.38	-2.09	-0.18	0.13	270	0.00	143	0.01	0.02	171	0.00	136
154	3.45	4.28	13.01	1.61	1.126	5.58	1.41	-2.53	-0.17	0.28	270	0.01	122	-0.01	0.03	165	0.01	134
152	3.45	4.32	15.55	1.77	1.274	4.94	1.42	-2.57	-0.17	0.26	270	0.01	96	-0.01	0.03	126	0.01	56
153	3.45	6.32	15.58	1.77	1.274	4.93	1.51	-3.33	-0.16	0.43	268	0.01	87	-0.01	0.03	108	0.02	44
150	3.45	4.26	18.12	1.94	1.416	4.44	1.47	-2.58	-0.17	0.24	271	0.01	80	0.00	0.03	99	0.01	359
151	3.45	6.40	18.12	1.94	1.416	4.44	1.51	-3.36	-0.16	0.40	269	0.01	71	-0.01	0.03	85	0.02	346
118	6.90	2.10	5.25	1.01	0.435	14.14	6.53	-2.67	-0.22	0.27	279	0.00	122	0.04	0.01	228	0.00	186
120	6.90	4.02	5.41	1.03	0.445	14.12	6.85	-2.69	-0.27	0.45	271	0.01	161	0.02	0.02	195	0.01	145
128	6.90	2.08	7.77	1.23	0.588	10.69	6.49	-2.67	-0.23	0.27	266	0.01	1	0.05	0.02	349	0.01	150
130	6.88	4.08	7.90	1.24	0.597	10.52	6.66	-2.63	-0.26	0.56	268	0.04	357	0.03	0.03	345	0.02	136
132	6.90	2.26	10.43	1.43	0.735	8.55	6.44	-2.64	-0.24	0.27	267	0.01	175	0.05	0.01	239	0.01	245
134	6.89	4.42	10.44	1.43	0.736	8.54	6.57	-2.35	-0.26	0.52	267	0.04	185	0.04	0.03	238	0.02	253
136	6.89	2.12	12.97	1.60	0.866	7.26	6.44	-2.52	-0.24	0.24	265	0.00	119	0.05	0.01	176	0.00	118
138	6.89	4.22	13.04	1.61	0.869	7.23	6.55	-2.30	-0.28	0.48	267	0.03	119	0.04	0.03	176	0.02	121
140	6.90	4.22	14.28	1.69	0.930	6.76	6.53	-2.24	-0.28	0.01	229	0.00	18	0.04	0.00	251	0.00	22
142	6.91	6.32	15.58	1.77	0.993	6.33	6.53	-2.36	-0.33	0.68	267	0.06	91	0.02	0.04	134	0.03	55
144	6.90	4.26	18.10	1.94	1.114	5.64	6.43	-1.72	-0.30	0.42	266	0.02	58	0.04	0.02	107	0.01	356
146	6.90	6.40	18.09	1.94	1.114	5.64	6.43	-2.14	-0.35	0.63	267	0.06	57	0.03	0.03	106	0.03	355
Following seas																		
169	3.45	2.10	5.27	1.01	-2.985	-2.10	1.32	-2.08	-0.13	0.13	258	0.00	75	0.00	0.01	261	0.00	352
170	3.45	4.02	5.17	1.01	-3.106	-2.02	1.31	-2.14	-0.11	0.19	260	0.01	37	0.00	0.01	261	0.01	353
171	3.45	2.26	10.44	1.43	-2.715	-2.31	1.35	-2.11	-0.13	0.10	276	0.00	40	0.00	0.01	269	0.00	5
172	3.45	4.42	10.50	1.43	-2.703	-2.32	1.45	-2.71	-0.12	0.19	278	0.01	69	0.00	0.01	269	0.01	17
173	6.90	2.10	5.26	1.01	3.106	2.02	6.45	-2.53	-0.21	0.25	98	0.01	241	0.04	0.01	123	0.00	16
174	6.88	4.02	5.33	1.01	3.316	1.89	6.33	-1.61	-0.19	0.49	95	0.01	270	0.03	0.01	125	0.01	13
175	6.90	2.12	13.09	1.61	-10.690	-0.59	6.42	-2.67	-0.21	0.23	262	0.00	80	0.04	0.01	252	0.00	1
176	6.90	4.28	13.09	1.61	-10.690	-0.59	6.48	-2.19	-0.23	0.46	257	0.01	135	0.03	0.02	241	0.01	340
177	6.89	4.26	18.27	1.94	-7.222	-0.87	6.60	-1.99	-0.22	0.38	263	0.01	301	0.03	0.02	239	0.01	340
178	6.90	6.40	18.34	1.94	-7.176	-0.88	6.53	-0.06	-0.25	0.59	264	0.01	316	0.02	0.02	238	0.02	345

TABLE 5.2

TESTS WITH INSTRUMENTED ASPECT RATIO 1 CANARD IN REGULAR WAVES.
FULL-SCALE EXPANSION OF MEAN QUANTITIES.

Run no.	Vel knots	λ/l	H ft	T sec	T_e sec	w_e rps	CTS	RS lb	EHP	M ton-ft
Head seas										
117	10.	1.0	4.20	4.95	2.98	2.11	0.00610	15638.	481.	-289.4
119	10.	1.0	8.04	5.00	3.00	2.09	0.00887	22745.	699.	-359.4
127	10.	1.5	4.16	6.07	3.92	1.60	0.00588	15069.	463.	-322.9
129	10.	1.5	8.16	6.07	3.93	1.60	0.00721	18481.	568.	-361.0
131	10.	2.0	4.52	7.01	4.75	1.32	0.00582	14927.	459.	-324.4
155	10.	2.0	8.84	7.01	4.76	1.32	0.00632	16206.	498.	-380.8
135	10.	2.5	4.24	7.89	5.52	1.14	0.00599	15353.	472.	-318.3
154	10.	2.5	8.56	7.89	5.52	1.14	0.00616	15780.	485.	-385.3
152	10.	3.0	8.44	8.67	6.23	1.01	0.00621	15922.	489.	-391.4
153	10.	3.0	12.64	8.67	6.24	1.01	0.00671	17201.	529.	-507.2
150	10.	3.5	8.52	9.50	6.94	0.91	0.00649	16633.	511.	-393.0
151	10.	3.5	12.80	9.50	6.94	0.91	0.00671	17201.	529.	-511.8
118	20.	1.0	4.20	4.95	2.13	2.95	0.00774	79081.	4853.	-406.7
120	20.	1.0	8.04	5.05	2.18	2.88	0.00816	83694.	5144.	-409.7
128	20.	1.5	4.16	6.03	2.88	2.18	0.00768	78512.	4818.	-406.7
130	20.	1.5	8.16	6.07	2.92	2.15	0.00793	80865.	4955.	-400.6
132	20.	2.0	4.52	7.01	3.60	1.75	0.00760	77866.	4786.	-402.1
134	20.	2.0	8.84	7.01	3.61	1.74	0.00779	79649.	4888.	-357.9
136	20.	2.5	4.24	7.84	4.24	1.48	0.00760	77866.	4786.	-383.8
138	20.	2.5	8.44	7.89	4.26	1.48	0.00776	79365.	4871.	-350.3
140	20.	2.7	8.44	8.28	4.56	1.38	0.00772	79145.	4864.	-341.2
142	20.	3.0	12.64	8.67	4.86	1.29	0.00770	79209.	4875.	-359.4
144	20.	3.5	8.52	9.50	5.46	1.15	0.00758	77723.	4777.	-262.0
146	20.	3.5	12.80	9.50	5.46	1.15	0.00758	77723.	4777.	-325.9
Following seas										
169	10.	1.0	4.20	4.95	-14.62	-0.43	0.00566	14501.	446.	-316.8
170	10.	1.0	8.04	4.95	-15.22	-0.41	0.00560	14358.	441.	-325.9
171	10.	2.0	4.52	7.01	-13.30	-0.47	0.00582	14927.	459.	-321.4
172	10.	2.0	8.84	7.01	-13.24	-0.47	0.00638	16348.	502.	-412.8
173	20.	1.0	4.20	4.95	15.22	0.41	0.00761	78008.	4794.	-385.3
174	20.	1.0	8.04	4.95	16.25	0.39	0.00747	76174.	4668.	-245.2
175	20.	2.5	4.24	7.89	-52.37	-0.12	0.00757	77581.	4768.	-406.7
176	20.	2.5	8.56	7.89	-52.37	-0.12	0.00765	78434.	4820.	-333.6
177	20.	3.5	8.52	9.50	-35.38	-0.18	0.00783	80076.	4914.	-303.1
178	20.	3.5	12.80	9.50	-35.16	-0.18	0.00772	79145.	4864.	-9.1

TABLE 5.3
TESTS WITH INSTRUMENTED ASPECT RATIO 1 CANARD IN REGULAR WAVES. CANARD LIFT AND DRAG COEFFICIENTS.

Run no.	Vel knots	Re $\times 10^{-6}$	λ/L	H ft	T_e sec	ω_e rps	C_{L1} mean	C_{L1} phase	C_{L2} phase	C_D mean	C_{D1} phase	C_{D2} phase	phase
Head seas													
117	10.	0.071	1.0	4.20	2.98	2.11	-0.296	0.296	0.020	0.000	0.039	0.020	147.
119	10.	0.071	1.0	8.04	3.00	2.09	-0.316	0.592	0.099	-0.020	0.079	0.059	120.
127	10.	0.071	1.5	4.16	3.92	1.60	-0.316	0.316	0.020	0.020	0.039	0.020	114.
129	10.	0.071	1.5	8.16	3.93	1.60	-0.316	0.632	0.059	0.000	0.079	0.039	100.
131	10.	0.071	2.0	4.52	4.75	1.32	-0.316	0.276	0.000	0.020	0.039	0.000	264.
155	10.	0.071	2.0	8.84	4.76	1.32	-0.336	0.612	0.039	-0.020	0.079	0.039	257.
135	10.	0.071	2.5	4.24	5.52	1.14	-0.355	0.257	0.000	0.020	0.039	0.000	136.
154	10.	0.071	2.5	8.56	5.52	1.14	-0.336	0.553	0.020	-0.020	0.059	0.020	134.
152	10.	0.071	3.0	8.44	6.23	1.01	-0.336	0.513	0.020	0.020	0.059	0.020	56.
153	10.	0.071	3.0	12.64	6.24	1.01	-0.316	0.849	0.020	-0.020	0.059	0.020	44.
150	10.	0.071	3.5	8.52	6.94	0.91	-0.336	0.474	0.020	0.000	0.059	0.020	359.
151	10.	0.071	3.5	12.80	6.94	0.91	-0.316	0.790	0.020	-0.020	0.059	0.039	346.
118	20.	0.141	1.0	4.20	2.13	2.95	-0.109	0.134	0.000	0.020	0.005	0.000	186.
120	20.	0.141	1.0	8.04	2.18	2.88	-0.133	0.222	0.005	0.010	0.010	0.005	145.
128	20.	0.142	1.5	4.16	2.88	2.18	-0.114	0.134	0.005	0.025	0.010	0.005	150.
130	20.	0.141	1.5	8.16	2.92	2.15	-0.129	0.278	0.020	0.015	0.015	0.010	136.
132	20.	0.142	2.0	4.52	3.60	1.75	-0.118	0.133	0.005	0.025	0.005	0.005	245.
134	20.	0.142	2.0	8.84	3.61	1.74	-0.129	0.257	0.020	0.020	0.015	0.010	253.
136	20.	0.142	2.5	4.24	4.24	1.48	-0.118	0.118	0.000	0.025	0.005	0.000	118.
138	20.	0.142	2.5	8.44	4.26	1.48	-0.139	0.238	0.015	0.020	0.015	0.010	121.
140	20.	0.142	2.7	8.44	4.56	1.38	-0.138	0.005	0.000	0.020	0.000	0.000	22.
142	20.	0.142	3.0	12.64	4.86	1.29	-0.162	0.335	0.030	0.010	0.020	0.015	55.
144	20.	0.142	3.5	8.52	5.46	1.15	-0.148	0.207	0.010	0.020	0.010	0.005	356.
146	20.	0.142	3.5	12.80	5.46	1.15	-0.173	0.311	0.030	0.015	0.015	0.015	355.
Following seas													
169	10.	0.071	1.0	4.20	-14.62	-0.43	-0.257	0.257	0.000	0.000	0.020	0.000	352.
170	10.	0.071	1.0	8.04	-15.22	-0.41	-0.217	0.375	0.020	0.000	0.020	0.020	353.
171	10.	0.071	2.0	4.52	-13.30	-0.47	-0.257	0.197	0.000	0.000	0.020	0.000	5.
172	10.	0.071	2.0	8.84	-13.24	-0.47	-0.237	0.375	0.020	0.000	0.020	0.020	17.
173	20.	0.142	1.0	4.20	15.22	0.41	-0.104	0.123	0.005	0.020	0.005	0.000	16.
174	20.	0.142	1.0	8.04	16.25	0.39	-0.094	0.243	0.005	0.015	0.005	0.005	13.
175	20.	0.142	2.5	4.24	-52.37	-0.12	-0.104	0.114	0.000	0.020	0.005	0.000	1.
176	20.	0.142	2.5	8.56	-52.37	-0.12	-0.114	0.227	0.005	0.015	0.010	0.005	340.
177	20.	0.142	3.5	8.52	-35.38	-0.18	-0.109	0.188	0.005	0.015	0.010	0.005	340.
178	20.	0.142	3.5	12.80	-35.16	-0.18	-0.123	0.291	0.005	0.010	0.010	0.010	345.

TABLE 6.1
TESTS WITH INSTRUMENTED ASPECT RATIO 2 CANARD IN REGULAR WAVES. FIXED TRIM AND AVE. MODEL SCALE UNITS.

Run no.	Vel fps	H in	λ ft	T sec	T _e sec	ω _e rps	mean drag	mean M	mean lift	Head seas				Pin forces				D ₂ lb	phase
										L ₁ lb	phase	L ₂ lb	phase	mean drag	D ₁ lb	phase	D ₂ lb		
93	3.45	2.10	5.23	1.01	0.607	10.35	1.39	-2.15	-0.18	0.21	273	0.00	155	-0.01	0.02	184	0.01	165	
95	3.45	4.02	5.31	1.02	0.612	10.27	1.84	-2.55	-0.17	0.39	267	0.04	175	0.00	0.03	157	0.03	138	
97	3.45	2.08	7.82	1.24	0.799	7.86	1.33	-2.23	-0.18	0.20	270	0.01	355	0.01	0.02	333	0.01	122	
99	3.45	4.08	7.86	1.24	0.802	7.83	1.51	-2.35	-0.18	0.41	269	0.05	356	0.01	0.04	315	0.02	94	
101	3.45	2.26	10.40	1.43	0.969	6.48	1.36	-2.27	-0.19	0.19	271	0.01	243	0.01	0.02	230	0.01	262	
103	3.45	4.42	10.42	1.43	0.970	6.48	1.48	-2.53	-0.18	0.38	268	0.03	239	0.01	0.03	209	0.02	232	
105	3.45	4.28	13.02	1.61	1.127	5.58	1.42	-2.54	-0.18	0.34	268	0.03	179	0.01	0.03	151	0.02	104	
107	3.45	6.32	13.04	1.61	1.128	5.57	1.72	-3.36	-0.17	0.53	265	0.07	167	0.00	0.03	118	0.03	93	
109	3.45	4.22	15.53	1.77	1.271	4.94	1.41	-2.50	-0.19	0.32	268	0.03	138	0.00	0.03	114	0.01	23	
111	3.45	6.32	15.59	1.77	1.274	4.93	1.63	-3.32	-0.16	0.49	266	0.07	130	0.00	0.02	81	0.03	12	
113	3.45	4.26	18.09	1.94	1.414	4.44	1.45	-2.31	-0.19	0.29	269	0.03	103	0.01	0.03	89	0.01	324	
115	3.45	6.40	18.13	1.94	1.417	4.43	1.57	-2.98	-0.17	0.46	266	0.07	99	0.01	0.02	54	0.02	310	
94	6.89	2.10	5.24	1.01	0.434	14.48	6.53	-2.86	-0.25	0.38	286	0.00	30	0.06	0.01	249	0.01	207	
96	6.90	4.02	5.34	1.02	0.440	14.28	6.79	-2.76	-0.29	0.72	280	0.02	182	0.04	0.03	213	0.03	168	
98	6.90	2.08	7.82	1.24	0.591	10.63	6.54	-2.56	-0.26	0.39	275	0.00	334	0.06	0.02	358	0.01	132	
100	6.90	4.08	7.87	1.24	0.594	10.58	6.62	-2.34	-0.30	0.74	273	0.01	57	0.04	0.04	351	0.02	121	
102	6.89	2.26	10.41	1.43	0.734	8.56	6.50	-2.61	-0.26	0.35	271	0.01	194	0.06	0.02	237	0.01	259	
104	6.90	4.42	10.46	1.43	0.736	8.54	6.58	-2.43	-0.30	0.70	269	0.04	207	0.04	0.04	233	0.03	260	
106	6.89	2.12	12.97	1.60	0.866	7.24	6.43	-2.27	-0.27	0.31	269	0.01	148	0.06	0.02	174	0.01	127	
108	6.90	4.28	13.03	1.61	0.868	7.24	6.53	-2.27	-0.30	0.64	269	0.03	146	0.04	0.03	173	0.02	128	
110	6.90	4.22	15.57	1.77	0.993	6.33	6.50	-2.11	-0.29	0.59	269	0.03	101	0.04	0.03	133	0.02	53	
112	6.90	6.32	15.60	1.78	0.994	6.32	6.25	-2.59	-0.32	0.89	268	0.09	98	0.02	0.05	130	0.04	53	
114	6.89	4.26	18.16	1.94	1.118	5.62	6.53	-1.58	-0.28	0.55	269	0.03	53	0.05	0.03	104	0.01	356	
116	6.90	6.40	18.12	1.94	1.116	5.63	6.42	-1.83	-0.32	0.83	268	0.09	62	0.02	0.04	102	0.03	353	
Following seas																			
179	3.45	2.10	5.31	1.01	-2.942	-2.14	1.29	-2.04	-0.16	0.16	266	0.01	316	0.00	0.01	259	0.00	327	
180	3.45	4.02	5.39	1.01	-2.854	-2.20	1.39	-2.07	-0.14	0.29	263	0.01	294	0.00	0.01	234	0.01	308	
181	3.45	2.26	10.52	1.43	-2.702	-2.33	1.29	-2.05	-0.17	0.12	277	0.00	160	0.00	0.01	268	0.00	11	
182	3.45	4.42	10.58	1.43	-2.684	-2.34	1.44	-2.52	-0.15	0.23	278	0.01	286	0.00	0.01	252	0.00	321	
183	6.90	2.10	5.36	1.01	3.260	1.93	6.52	-2.65	-0.27	0.35	99	0.01	213	0.04	0.01	122	0.00	24	
184	6.89	4.02	5.38	1.01	3.453	1.82	6.36	-1.02	-0.24	0.65	97	0.01	288	0.03	0.02	122	0.02	18	
185	6.89	2.12	13.19	1.61	-10.180	-0.62	6.51	-2.81	-0.28	0.29	260	0.01	187	0.04	0.01	235	0.00	349	
186	6.89	4.28	13.11	1.61	-10.534	-0.60	6.28	-1.71	-0.24	0.59	254	0.03	348	0.03	0.02	234	0.01	335	
187	6.89	4.26	18.42	1.94	-7.086	-0.89	6.49	-2.49	-0.28	0.50	264	0.00	56	0.03	0.02	243	0.01	355	
188	6.90	6.40	18.39	1.94	-7.121	-0.88	5.92	-0.95	-0.23	0.75	260	0.01	93	0.03	0.03	237	0.02	339	

TABLE 6.2

TESTS WITH INSTRUMENTED ASPECT RATIO 2 CANARD IN REGULAR WAVES.
FULL-SCALE EXPANSION OF MEAN QUANTITIES.

Run no.	Vel knots	λ/l	H ft	T sec	T_e sec	ω_e rps	CTS	RS lb	EHP	M ton-ft
Head seas										
93	10.	1.0	4.20	4.95	2.97	2.11	0.00605	15496.	476.	-327.5
95	10.	1.0	8.04	5.00	3.00	2.10	0.00854	21892.	673.	-388.4
97	10.	1.5	4.16	6.07	3.91	1.60	0.00571	14643.	450.	-339.6
99	10.	1.5	8.16	6.07	3.93	1.60	0.00671	17201.	529.	-357.9
101	10.	2.0	4.52	7.01	4.75	1.32	0.00588	15069.	463.	-345.7
103	10.	2.0	8.84	7.01	4.75	1.32	0.00655	16775.	515.	-385.3
105	10.	2.5	8.56	7.89	5.52	1.14	0.00621	15922.	489.	-386.9
107	10.	2.5	12.64	7.89	5.53	1.14	0.00788	20186.	620.	-511.8
109	10.	3.0	8.44	8.67	6.23	1.01	0.00616	15780.	485.	-380.8
111	10.	3.0	12.64	8.67	6.24	1.01	0.00738	18907.	581.	-505.7
113	10.	3.5	8.52	9.50	6.93	0.91	0.00638	16348.	502.	-351.8
115	10.	3.5	12.80	9.50	6.94	0.90	0.00704	18054.	555.	-453.9
94	20.	1.0	4.20	4.95	2.13	2.96	0.00774	79081.	4853.	-435.6
96	20.	1.0	8.04	5.00	2.16	2.91	0.00808	82841.	5091.	-420.4
98	20.	1.5	4.16	6.07	2.90	2.17	0.00773	79287.	4873.	-389.9
100	20.	1.5	8.16	6.07	2.91	2.16	0.00784	80424.	4943.	-356.4
102	20.	2.0	4.52	7.01	3.60	1.75	0.00769	78654.	4827.	-397.5
104	20.	2.0	8.84	7.01	3.61	1.74	0.00779	79856.	4908.	-370.1
108	20.	2.5	4.24	7.84	4.24	1.48	0.00760	77659.	4766.	-382.3
106	20.	2.5	8.56	7.89	4.25	1.48	0.00772	79145.	4864.	-345.7
110	20.	3.0	8.44	8.67	4.86	1.29	0.00768	78719.	4838.	-321.4
112	20.	3.0	12.64	8.72	4.87	1.29	0.00733	75165.	4620.	-394.5
114	20.	3.5	8.52	9.50	5.48	1.15	0.00774	79081.	4853.	-240.6
116	20.	3.5	12.80	9.50	5.47	1.15	0.00757	77581.	4768.	-278.7
Following seas										
179	10.	1.0	4.20	4.95	-14.41	-0.44	0.00549	14074.	432.	-310.7
180	10.	1.0	8.04	4.95	-13.98	-0.45	0.00605	15496.	476.	-315.3
181	10.	2.0	4.52	7.01	-13.24	-0.48	0.00549	14074.	432.	-312.2
182	10.	2.0	8.84	7.01	-13.15	-0.48	0.00632	16206.	498.	-383.8
183	20.	1.0	4.20	4.95	15.97	0.39	0.00771	79003.	4855.	-403.6
184	20.	1.0	8.04	4.95	16.92	0.37	0.00750	76664.	4705.	-155.4
185	20.	2.5	4.24	7.89	-49.87	-0.13	0.00771	78797.	4836.	-428.0
186	20.	2.5	8.56	7.89	-51.61	-0.12	0.00739	75527.	4635.	-260.4
187	20.	3.5	8.52	9.50	-34.71	-0.18	0.00768	78512.	4818.	-379.2
188	20.	3.5	12.80	9.50	-34.89	-0.18	0.00687	70474.	4331.	-144.7

TABLE 6.3

TESTS WITH INSTRUMENTED ASPECT RATIO 2 CANARD IN REGULAR WAVES. CANARD LIFT AND DRAG COEFFICIENT.

Run no.	Vel knots	Re $\times 10^{-6}$	λ/L	H ft	T_e sec	ω_e rps	C_L mean	C_{L1} phase	C_{L2} phase	C_D mean	C_{D1} phase	C_{D2} phase
Head seas												
93	10.	0.071	1.0	4.20	2.97	2.11	-0.355	0.415	273.	0.000	155.	0.020
95	10.	0.071	1.0	8.04	3.00	2.10	-0.336	0.770	267.	0.079	175.	0.059
97	10.	0.071	1.5	4.16	3.91	1.60	-0.355	0.395	270.	0.020	355.	0.020
99	10.	0.071	1.5	8.16	3.93	1.60	-0.355	0.810	269.	0.099	356.	0.039
101	10.	0.071	2.0	4.52	4.75	1.32	-0.375	0.375	271.	0.020	243.	0.020
103	10.	0.071	2.0	8.84	4.75	1.32	-0.355	0.750	268.	0.059	239.	0.039
105	10.	0.071	2.5	8.56	5.52	1.14	-0.355	0.671	268.	0.059	179.	0.039
107	10.	0.071	2.5	12.64	5.53	1.14	-0.336	1.047	265.	0.138	167.	0.059
109	10.	0.071	3.0	8.44	6.23	1.01	-0.375	0.632	268.	0.059	138.	0.020
111	10.	0.071	3.0	12.64	6.24	1.01	-0.316	0.968	266.	0.138	130.	0.020
113	10.	0.071	3.5	8.52	6.93	0.91	-0.375	0.573	269.	0.059	103.	0.020
115	10.	0.071	3.5	12.80	6.94	0.90	-0.336	0.908	266.	0.138	99.	0.039
94	20.	0.142	1.0	4.20	2.13	2.96	-0.124	0.188	286.	0.000	30.	0.005
96	20.	0.142	1.0	8.04	2.16	2.91	-0.143	0.355	280.	0.010	182.	0.015
98	20.	0.142	1.5	4.16	2.90	2.17	-0.128	0.193	275.	0.000	334.	0.005
100	20.	0.142	1.5	8.16	2.91	2.16	-0.148	0.365	273.	0.005	57.	0.010
102	20.	0.142	2.0	4.52	3.60	1.75	-0.129	0.173	271.	0.005	194.	0.005
104	20.	0.142	2.0	8.84	3.61	1.74	-0.148	0.346	269.	0.020	207.	0.015
106	20.	0.142	2.5	4.24	4.24	1.48	-0.134	0.153	269.	0.005	148.	0.005
108	20.	0.142	2.5	8.56	4.25	1.48	-0.148	0.316	269.	0.015	146.	0.010
110	20.	0.142	3.0	8.44	4.86	1.29	-0.143	0.291	269.	0.015	101.	0.010
112	20.	0.142	3.0	12.64	4.87	1.29	-0.158	0.439	268.	0.044	98.	0.020
114	20.	0.142	3.5	8.52	5.48	1.15	-0.139	0.272	269.	0.015	53.	0.005
116	20.	0.142	3.5	12.80	5.47	1.15	-0.158	0.410	268.	0.044	62.	0.015
Following seas												
179	10.	0.071	1.0	4.20	-14.41	-0.44	-0.316	0.316	266.	0.020	316.	0.000
180	10.	0.071	1.0	8.04	-13.98	-0.45	-0.276	0.573	263.	0.020	294.	0.020
181	10.	0.071	2.0	4.52	-13.24	-0.48	-0.336	0.237	277.	0.000	160.	0.000
182	10.	0.071	2.0	8.84	-13.15	-0.48	-0.296	0.454	278.	0.020	286.	0.000
183	20.	0.142	1.0	4.20	15.97	0.39	-0.133	0.173	99.	0.005	213.	0.000
184	20.	0.142	1.0	8.04	16.92	0.37	-0.119	0.322	97.	0.005	288.	0.010
185	20.	0.142	2.5	4.24	-49.87	-0.13	-0.139	0.144	260.	0.005	187.	0.000
186	20.	0.142	2.5	8.56	-51.61	-0.12	-0.119	0.292	254.	0.015	348.	0.005
187	20.	0.142	3.5	8.52	-34.71	-0.18	-0.139	0.248	264.	0.015	56.	0.005
188	20.	0.142	3.5	12.80	-34.89	-0.18	-0.114	0.370	260.	0.005	93.	0.010

TABLE 7
TESTS IN REGULAR WAVES WITHOUT ACTIVE CONTROL. CANARD ASPECT RATIO 1.

Run no.	Vel knots	λ/L	T sec	T_e sec	ω_e rps	H ft	mean drag lb	mean trim deg	mean heave ft	RS lb	EHP	wave phase deg	θ_1 deg	θ_2 deg	phase ft	z_1 ft	phase ft	z_2 ft	phase
Head seas																			
63	15.00	1.50	6.07	3.331	1.89	4.08	3.49	0.41	-2.74	41397	1905	317	0.27	0.01	329	0.14	108	0.00	215
62	15.00	1.51	6.07	3.351	1.88	8.26	3.88	0.04	-2.72	46942	2160	313	0.53	0.02	300	0.28	112	0.00	76
60	15.00	2.49	7.84	4.781	1.31	4.28	3.45	0.43	-2.66	40828	1879	100	0.51	0.01	91	0.32	102	0.00	292
61	15.00	2.50	7.84	4.796	1.31	8.66	3.75	-0.03	-2.68	45094	2075	95	1.14	0.04	75	0.74	109	0.02	246
58	15.00	3.51	9.50	6.129	1.02	8.68	3.98	-0.16	-2.60	48364	2225	184	2.56	0.14	33	2.48	120	0.08	154
59	15.00	3.51	9.50	6.129	1.02	12.92	5.23	-0.80	-2.78	66139	3043	184	4.19	0.39	343	4.16	108	0.14	101
71	20.00	1.50	6.07	2.895	2.17	4.08	10.40	0.30	-6.32	134353	8243	264	0.27	0.04	290	0.12	43	0.02	314
70	20.00	1.51	6.07	2.915	2.16	8.26	11.51	0.22	-6.82	150176	9213	276	1.13	0.12	293	0.44	70	0.04	270
68	20.00	2.50	7.84	4.243	1.48	4.28	10.38	0.03	-6.26	134069	8225	72	0.84	0.05	298	0.40	89	0.04	257
69	20.00	2.50	7.84	4.247	1.48	8.66	11.28	0.00	-6.04	146867	9010	81	2.09	0.27	293	1.00	88	0.04	282
168	20.00	2.50	7.84	4.247	1.48	8.66	11.07	0.53	-5.68	143920	8829	15	1.83	0.26	297	0.92	87	0.02	297
65	20.00	3.51	9.50	5.477	1.15	8.68	11.21	0.63	-5.54	145871	8949	170	3.33	0.34	304	2.68	100	0.04	66
67	20.00	3.51	9.50	5.482	1.15	12.92	11.28	0.49	-4.98	146867	9010	179	4.68	0.63	309	4.20	88	0.20	48
149	20.00	4.00	10.29	6.065	1.04	12.50	11.72	0.25	-4.54	153123	9394	37	5.52	0.70	309	5.40	101	0.20	57
161	20.00	4.00	10.29	6.065	1.04	12.50	11.89	0.17	-4.58	155580	9545	40	5.65	0.72	311	5.40	102	0.22	61
Following seas																			
75	15.00	1.50	6.07	-34.004	-0.19	4.08	3.48	0.90	-9.34	41191	1895	146	3.32	0.46	287	1.04	318	0.14	299
76	15.00	1.50	6.07	-31.775	-0.20	8.26	3.34	0.84	-9.22	39264	1807	133	5.47	0.72	229	1.70	302	0.12	283
77	15.00	2.50	7.84	-21.810	-0.29	4.28	3.31	0.90	-9.32	38869	1788	75	2.42	0.18	13	1.56	317	0.10	281
78	15.00	2.50	7.84	-21.668	-0.29	8.66	3.40	0.76	-9.36	40117	1846	74	4.23	0.59	338	3.02	309	0.28	294
79	15.00	3.50	9.46	-21.178	-0.30	8.68	3.34	0.49	-9.44	39296	1808	27	2.85	0.35	331	3.64	301	0.16	245
82	15.00	3.50	9.46	-21.090	-0.30	12.92	3.41	0.57	-9.20	40227	1851	29	4.04	0.55	335	5.36	299	0.28	242
83	20.00	0.89	4.65	12.003	0.52	4.26	10.75	-0.13	-12.98	139487	8557	99	1.01	0.11	53	0.20	11	0.10	38
84	20.00	1.00	4.95	15.711	0.40	4.22	11.40	-0.56	-13.16	148573	9115	114	1.17	0.18	63	0.32	9	0.12	328
124	20.00	1.34	5.73	39.226	0.16	4.20	10.53	0.67	-5.72	136202	8356	182	3.36	1.11	30	1.32	38	0.60	82
87	20.00	1.50	6.07	65.142	0.10	4.08	8.60	1.98	-10.80	108718	6670	212	4.51	0.34	172	0.58	47	0.34	94
96	20.00	2.50	7.84	-54.139	-0.12	3.66	9.33	1.49	-5.50	119177	7311	68	4.51	0.55	284	3.52	317	0.42	287
92	20.00	2.50	7.84	-52.517	-0.12	4.28	8.72	1.58	-5.84	110542	6782	56	1.92	0.21	265	1.56	305	0.06	253
93	20.00	3.50	9.46	-34.156	-0.18	8.68	8.72	1.70	-5.68	110464	6777	24	2.38	0.25	248	3.44	297	0.10	202
95	20.00	3.50	9.46	-35.121	-0.18	12.92	8.97	1.27	-5.66	114019	6995	23	3.96	0.65	272	5.28	298	0.38	229

TABLE 8

RUNS IN REGULAR WAVES TO OPTIMIZE CONTROL SYSTEM. CANARD ASPECT RATIO 1.

Run no.	Vel knots	λ/l	T sec	T_e sec	ω_e rps	H ft	wave phase	θ_1 deg	z_1 ft	phase	α_1 deg	phase	δ_1	δ_2
Head seas														
135	15.00	3.50	9.46	6.119	1.03	8.68	39	2.60	2.08	126	10.42	154	3.54	1.93
137	15.00	3.50	9.46	6.114	1.03	8.68	46	2.42	2.02	135	14.36	131	3.54	4.83
138	15.00	3.50	9.46	6.119	1.03	8.68	49	2.31	2.00	139	17.36	123	3.54	6.76
139	15.00	3.50	9.46	6.119	1.03	8.68	52	2.38	2.00	142	19.84	131	4.95	6.76
140	15.00	3.50	9.46	6.114	1.03	8.68	52	2.17	1.96	143	19.97	118	3.54	8.69
141	15.00	3.49	9.46	6.109	1.03	8.68	54	2.13	1.96	145	21.29	116	3.54	9.65
142	15.00	3.51	9.50	6.124	1.03	8.68	54	2.25	1.94	146	22.16	125	4.95	8.69
143	15.00	3.51	9.50	6.124	1.03	8.68	47	2.01	1.94	140	18.23	96	0.00	9.65
145	20.00	3.50	9.46	5.472	1.15	8.68	35	2.89	2.28	113	22.29	121	3.54	6.76
147	20.00	3.50	9.46	5.472	1.15	8.68	30	2.64	2.30	112	18.71	96	0.00	6.76
Following seas														
98	20.00	1.50	6.07	73.485	0.09	4.08	217	1.99	0.90	51	0.04	181	2.12	0.00
100	20.00	1.50	6.07	68.586	0.09	4.08	222	1.56	0.74	67	5.35	179	3.54	1.45
111	20.00	1.50	6.07	61.727	0.10	4.08	184	2.99	1.92	25	1.36	87	0.00	4.83
113	20.00	1.50	6.07	63.687	0.10	4.08	220	1.38	0.64	76	6.65	174	4.95	4.83
103	20.00	2.50	7.84	-52.968	-0.12	4.28	123	1.72	0.02	316	5.86	181	3.54	0.00
104	20.00	2.50	7.84	-54.678	-0.11	4.28	112	1.75	0.02	328	6.00	175	3.54	2.41
105	20.00	2.50	7.84	-52.718	-0.12	4.28	115	1.93	0.02	331	6.70	171	3.54	4.83
116	20.00	2.50	7.84	-57.823	-0.11	4.28	52	1.10	1.08	300	5.36	172	4.95	6.76
117	20.00	2.50	7.84	-56.559	-0.11	4.28	69	0.97	1.20	314	5.33	173	5.66	6.76
119	20.00	2.50	7.84	-58.175	-0.11	4.28	72	0.96	1.24	318	6.60	175	7.07	6.76
120	20.00	2.50	7.84	-54.031	-0.12	4.28	53	0.99	1.28	298	6.18	173	6.36	7.72
107	20.00	3.50	9.46	-36.978	-0.17	8.68	33	1.81	3.34	306	6.28	174	3.54	2.41
108	20.00	3.50	9.46	-35.870	-0.18	8.68	28	1.55	3.36	297	5.53	166	3.54	4.83
109	20.00	3.50	9.46	-35.714	-0.18	8.68	29	1.31	3.20	299	6.42	169	4.95	4.83
110	20.00	3.50	9.46	-34.092	-0.18	8.68	31	1.38	2.92	299	9.26	173	7.07	4.83

TABLE 9.1
TESTS IN REGULAR HEAD WAVES WITH AUTOMATIC PITCH CONTROL. CANARD ASPECT RATIO 1.

Run no.	Vel knots	λ/l	T sec	T_e sec	ω_e rps	H ft	mean drag lb	mean trim deg	mean heave ft	mean α deg	RS lb	EHP	phase α_1 deg	phase α_2 deg
159	15.00	2.50	7.84	4.796	1.31	8.66	3.79	0.55	-2.28	0.1	45663	2101	258	11.81
158	15.00	3.50	9.46	6.119	1.03	12.92	5.46	-0.28	-2.60	-0.99	69410	3194	88	25.04
157	15.00	4.00	10.29	6.756	0.93	12.50	5.57	-0.52	-2.44	-1.53	70974	3266	94	25.94
154	20.00	1.50	6.07	2.895	2.17	4.08	9.58	0.96	-5.88	0.28	122693	7527	352	2.69
153	20.00	1.51	6.07	2.910	2.16	8.26	11.87	0.44	-6.52	0.07	152256	9525	265	17.21
152	20.00	2.49	7.84	4.238	1.48	4.28	9.07	1.13	-5.60	0.23	115441	7082	228	6.02
151	20.00	2.50	7.84	4.247	1.48	8.66	11.42	0.61	-5.78	0.23	148857	9132	25	19.16
144	20.00	3.50	9.46	5.467	1.15	8.68	11.85	0.91	-5.46	-0.47	154972	9507	21	22.83
148	20.00	3.50	9.46	5.472	1.15	12.92	11.65	-1.18	-4.84	0.01	152167	9335	64	24.48
150	20.00	4.00	10.29	6.060	1.04	12.50	12.27	0.63	-4.84	-0.59	160944	9874	63	24.91

wave phase deg	θ_1 deg	θ_2 deg	phase z_1 ft	phase z_2 ft	phase α_1 deg	phase α_2 deg
39	1.01	0.05	0.56	0.02	127	1.06
42	3.07	0.28	3.28	0.18	122	4.20
56	3.77	0.32	4.98	0.26	127	3.76
59	0.14	0.01	0.08	0.00	75	0.54
55	0.89	0.13	0.38	0.02	76	5.37
14	0.46	0.02	0.26	0.00	121	0.44
26	1.46	0.23	0.74	0.04	102	5.96
35	2.48	0.31	2.26	0.04	117	5.34
37	3.09	0.51	3.22	0.26	109	5.82
44	3.76	0.57	4.38	0.26	111	5.13

TABLE 9.2
 TESTS IN REGULAR FOLLOWING WAVES WITH AUTOMATIC PITCH CONTROL. CANARD ASPECT RATIO 1.

Run no.	Vel knots	λ/L	T sec	T_e sec	w_e rps	H ft	mean		mean		RS		EHP
							drag lb	trim deg	heave ft	α deg	lb	lb	
131	15.00	1.50	6.07	-33.156	-0.19	4.08	3.27	0.64	-2.40	-6.54	38205	1758	
132	15.00	1.50	6.07	-32.289	-0.19	8.26	3.12	0.70	-2.16	-6.67	36072	1660	
125	15.00	2.00	7.01	-24.348	-0.26	8.50	3.32	0.67	-2.32	-6.67	38916	1791	
130	15.00	2.50	7.84	-21.913	-0.29	4.28	3.23	0.56	-2.46	-6.10	37636	1732	
127	15.00	2.50	7.84	-21.683	-0.29	8.66	3.33	0.65	-2.38	-6.57	39058	1797	
128	15.00	3.50	9.46	-21.315	-0.29	8.68	3.24	0.61	-2.42	-6.37	37778	1738	
129	15.00	3.50	9.46	-21.144	-0.30	12.92	3.28	0.67	-2.28	-6.69	38347	1764	
123	20.00	1.34	5.73	37.232	0.17	4.20	10.24	0.20	-6.04	-3.63	132039	8101	
122	20.00	1.50	6.07	60.747	0.10	4.08	10.56	0.10	-6.16	-3.07	136628	8382	
118	20.00	2.50	7.84	-59.008	-0.11	4.28	9.43	0.21	-6.10	-3.94	120521	7394	
121	20.00	3.50	9.46	-38.486	-0.16	8.68	9.98	0.25	-5.94	-3.80	128342	7874	

Run no.	Vel knots	λ/L	T sec	T_e sec	w_e rps	H ft	wave phase		θ_1 deg	θ_2 deg	α_1 deg	α_2 deg	phase				
							z_1 ft	z_2 ft									
131	15.00	1.50	6.07	-33.156	-0.19	4.08	151	1.65	0.13	229	0.88	273	138	170	0.84	30	
132	15.00	1.50	6.07	-32.289	-0.19	8.26	143	2.92	0.24	150	1.74	262	55	18.38	169	1.62	309
125	15.00	2.00	7.01	-24.348	-0.26	8.50	107	2.48	0.14	301	2.34	288	203	15.92	166	0.91	92
130	15.00	2.50	7.84	-21.913	-0.29	4.28	94	1.02	0.10	0	1.28	310	0.16	6.53	164	0.73	154
127	15.00	2.50	7.84	-21.683	-0.29	8.66	87	1.89	0.13	295	2.58	300	0.32	12.21	164	1.03	89
128	15.00	3.50	9.46	-21.315	-0.29	8.68	51	1.26	0.21	351	3.34	309	0.28	8.15	164	1.50	143
129	15.00	3.50	9.46	-21.144	-0.30	12.92	43	2.02	0.29	325	5.28	298	0.52	13.09	164	2.13	115
123	20.00	1.34	5.73	37.232	0.17	4.20	209	1.39	0.37	59	0.22	107	0.12	314	171	2.55	221
122	20.00	1.50	6.07	60.747	0.10	4.08	210	1.00	0.41	29	0.64	85	0.10	294	175	2.52	196
118	20.00	2.50	7.84	-59.008	-0.11	4.28	78	0.92	0.29	68	1.36	321	0.24	342	174	1.87	236
121	20.00	3.50	9.46	-38.486	-0.16	8.68	40	1.67	0.41	72	3.26	306	0.16	333	171	2.69	234

TABLE 10.1
TESTS IN REGULAR HEAD WAVES WITH AUTOMATIC PITCH CONTROL. CANARD ASPECT RATIO 2.

Run no.	Vel knots	λ/l	T sec	T_e sec	w_e rps	H ft	mean drag lb	mean trim deg	mean heave ft	mean α deg	RS lb	EHP
172	15.00	1.51	6.07	3.356	1.87	8.26	3.81	0.57	-2.30	0.17	45947	2114
174	15.00	2.50	7.84	4.796	1.31	4.28	3.37	0.99	-2.26	0.17	39690	1826
173	15.00	2.50	7.84	4.796	1.31	8.66	3.76	0.36	-2.32	0.16	45236	2081
175	15.00	3.51	9.50	6.124	1.03	8.68	3.97	0.11	-2.30	0.17	48222	2219
176	15.00	3.50	9.50	6.119	1.03	12.92	5.49	-0.23	-2.64	-1.05	69836	3213
177	15.00	4.00	10.29	6.756	0.93	12.50	5.64	-0.45	-2.46	-1.76	71969	3311
170	20.00	1.50	6.07	2.895	2.17	4.08	9.80	0.73	-5.98	0.19	125821	7719
169	20.00	1.51	6.07	2.910	2.16	8.26	12.02	0.39	-6.56	-0.13	157389	9656
166	20.00	2.50	7.84	4.252	1.48	4.28	9.31	0.93	-5.64	0.11	118893	7294
167	20.00	2.50	7.84	4.247	1.48	8.66	11.39	0.65	-5.70	0.14	148431	9106
165	20.00	3.50	9.46	5.472	1.15	8.68	11.92	0.35	-5.34	-0.46	156007	9571
163	20.00	3.50	9.46	5.467	1.15	12.92	11.50	1.30	-4.78	-0.31	149995	9202
164	20.00	3.50	9.46	5.472	1.15	12.92	11.33	1.20	-4.78	0.19	147617	9056
171	20.00	3.51	9.50	5.477	1.15	12.92	11.69	1.18	-4.88	-0.29	152697	9368
162	20.00	4.00	10.29	6.065	1.04	12.50	12.41	0.33	-4.74	-0.59	162935	9996

Run no.	wave phase	θ_1 deg	θ_2 deg	z_1 ft	phase z_1	z_2 ft	phase z_2	α_1 deg	phase α_1	α_2 deg	phase α_2
172	90	0.46	0.02	0.20	303	0.00	117	7.62	101	0.62	58
174	47	0.48	0.01	0.22	153	0.00	127	5.54	97	0.33	256
173	43	1.01	0.05	0.58	124	0.02	130	11.70	97	1.05	226
175	50	1.95	0.09	2.06	63	0.06	142	17.75	96	1.60	163
176	44	2.96	0.27	3.20	2	0.18	123	24.54	97	4.08	83
177	57	3.61	0.33	4.92	12	0.24	128	25.51	97	3.89	78
170	54	0.19	0.02	0.10	316	0.00	54	3.69	103	0.95	73
169	59	0.93	0.12	0.40	332	0.02	76	17.92	104	5.37	89
166	18	0.51	0.03	0.28	10	0.02	126	6.77	98	0.74	113
167	29	1.38	0.22	0.70	331	0.04	104	18.06	99	5.80	78
165	36	2.42	0.34	2.22	353	0.06	118	22.63	98	5.91	88
163	37	2.95	0.49	3.08	342	0.26	109	24.17	97	5.74	77
164	35	3.28	0.52	3.26	336	0.26	108	21.95	96	6.39	78
171	36	3.03	0.52	3.10	339	0.26	109	24.28	98	5.82	76
162	42	3.96	0.60	4.44	341	0.22	110	25.30	97	5.13	65

TABLE 10.2
TESTS IN REGULAR FOLLOWING WAVES WITH AUTOMATIC PITCH CONTROL. CANARD ASPECT RATIO 2.

Run no.	Vel knots	λ/L	T sec	T_e sec	ω_e rps	H ft	mean drag lb	mean trim deg	mean heave ft	mean α deg	RS lb	EHP
179	15.00	1.50	6.07	-34.499	-0.18	4.08	3.55	0.58	-2.48	-4.91	42218	1943
178	15.00	1.50	6.07	-32.480	-0.19	8.26	3.38	0.68	-2.10	-5.51	39769	1830
180	15.00	2.00	7.01	-24.363	-0.26	4.25	3.49	0.58	-2.48	-4.89	41365	1903
181	15.00	2.00	7.01	-23.951	-0.26	8.50	3.49	0.72	-2.28	-5.75	41365	1903
183	15.00	2.50	7.84	-21.702	-0.29	4.28	3.54	0.54	-2.56	-4.66	42076	1936
182	15.00	2.50	7.84	-21.693	-0.29	8.66	3.53	0.68	-2.40	-5.52	41966	1931
184	15.00	3.50	9.46	-20.968	-0.30	8.68	3.52	0.56	-2.50	-4.83	41823	1924
185	15.00	3.50	9.46	-20.860	-0.30	12.92	3.58	0.64	-2.44	-5.28	42677	1964
186	20.00	1.34	5.73	37.683	0.17	4.20	11.00	0.15	-6.24	-2.22	142805	8766
187	20.00	1.50	6.07	68.096	0.09	4.08	10.95	-0.04	-6.22	-1.06	142174	8722
188	20.00	1.50	6.07	67.116	0.09	4.08	11.08	-0.08	-6.20	-0.83	144023	8836
189	20.00	2.50	7.84	-52.909	-0.12	4.28	10.32	0.28	-6.10	-3.05	133216	8173
190	20.00	2.50	7.84	-52.419	-0.12	8.66	11.71	-0.62	-6.20	0.77	153020	9388
191	20.00	3.50	9.46	-35.023	-0.18	8.68	11.04	0.19	-5.98	-2.49	143493	8803
192	20.00	3.50	9.46	-36.404	-0.17	12.92	11.72	0.12	-5.98	-2.02	153163	9396

Run no.	Wave phase deg	θ_1 deg	θ_2 deg	phase z_1 ft	phase z_2 ft	phase α_1 deg	phase α_2 deg
179	151	1.55	0.18	263	0.92	269	189
178	143	2.92	0.26	146	1.80	258	47
180	342	1.30	0.10	292	1.18	296	210
181	107	2.37	0.19	273	2.34	288	197
183	90	0.98	0.07	285	1.36	307	215
182	85	1.85	0.13	295	2.68	298	196
184	50	1.05	0.11	336	3.30	313	230
185	34	1.67	0.27	319	5.10	296	210
186	210	1.24	0.29	52	0.40	90	307
187	194	0.99	0.52	14	0.74	62	91
188	204	0.89	0.51	30	0.74	68	96
189	60	0.99	0.31	60	1.26	313	313
190	64	3.51	0.90	157	3.32	315	159
191	41	1.57	0.46	89	3.44	307	100
192	43	2.89	0.69	126	5.22	310	169

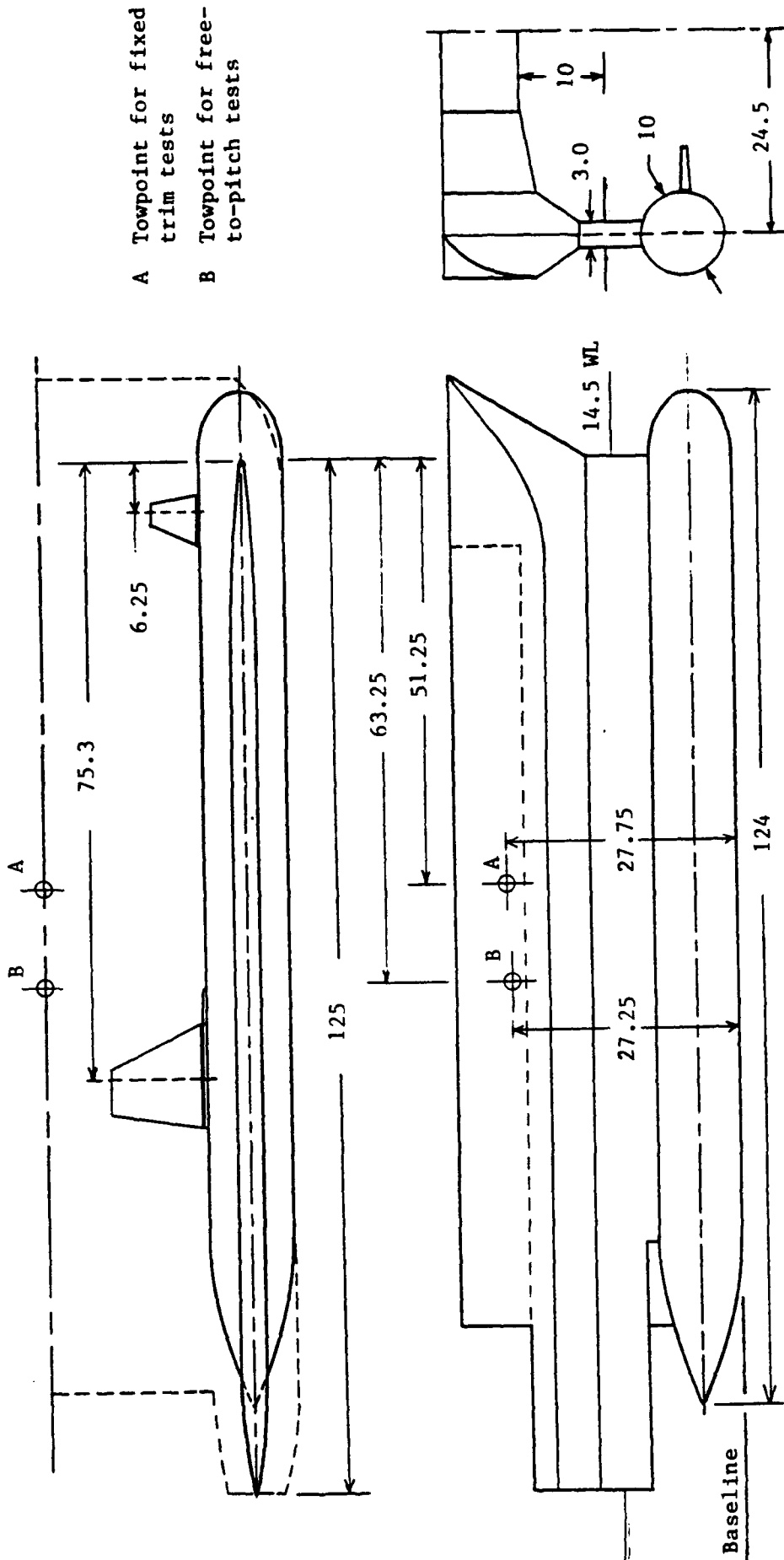
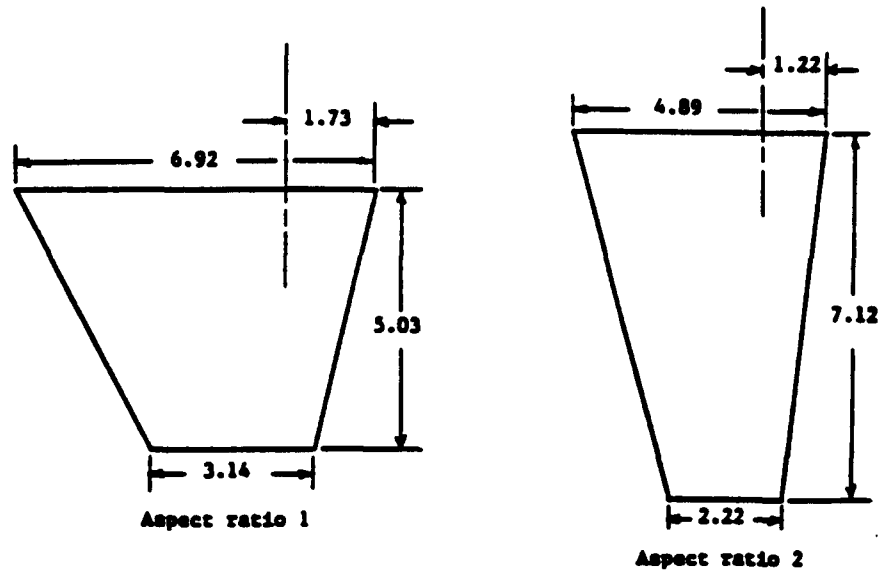


FIGURE 1 SWATH CONFIGURATION. ALL DIMENSIONS IN FEET.



Section view
NACA 0015

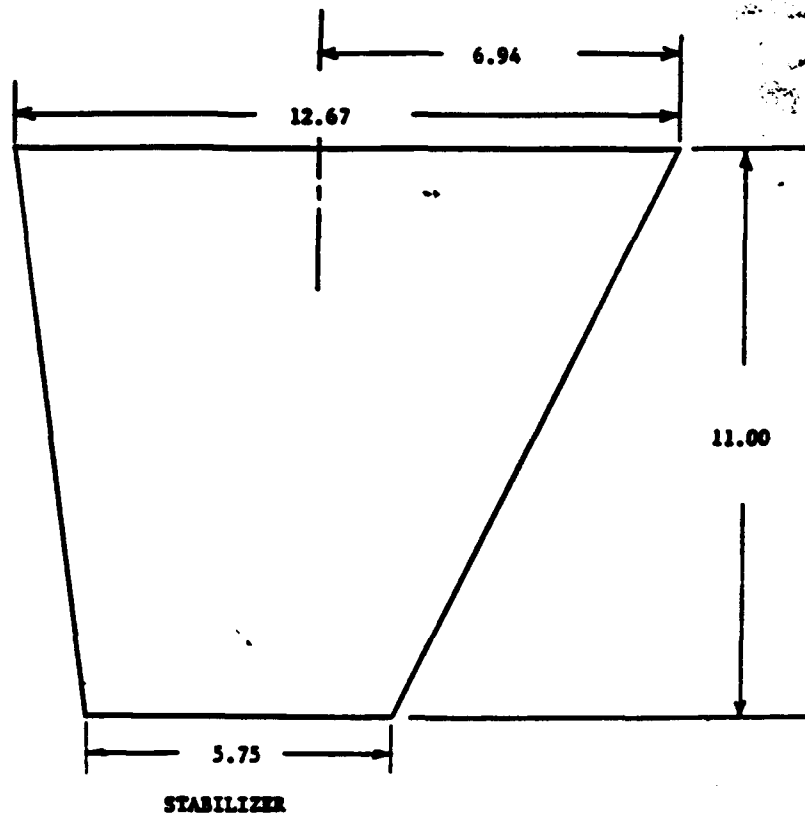


FIGURE 2 PLAN AND SECTION VIEW OF MODEL FINS.
ALL DIMENSIONS IN FEET.



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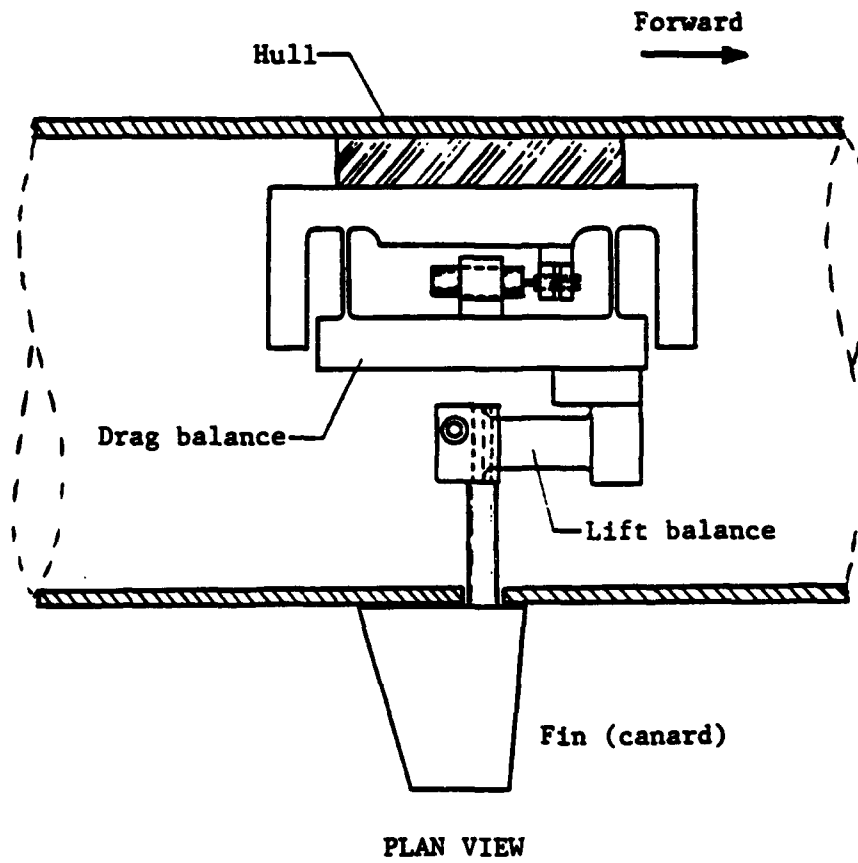
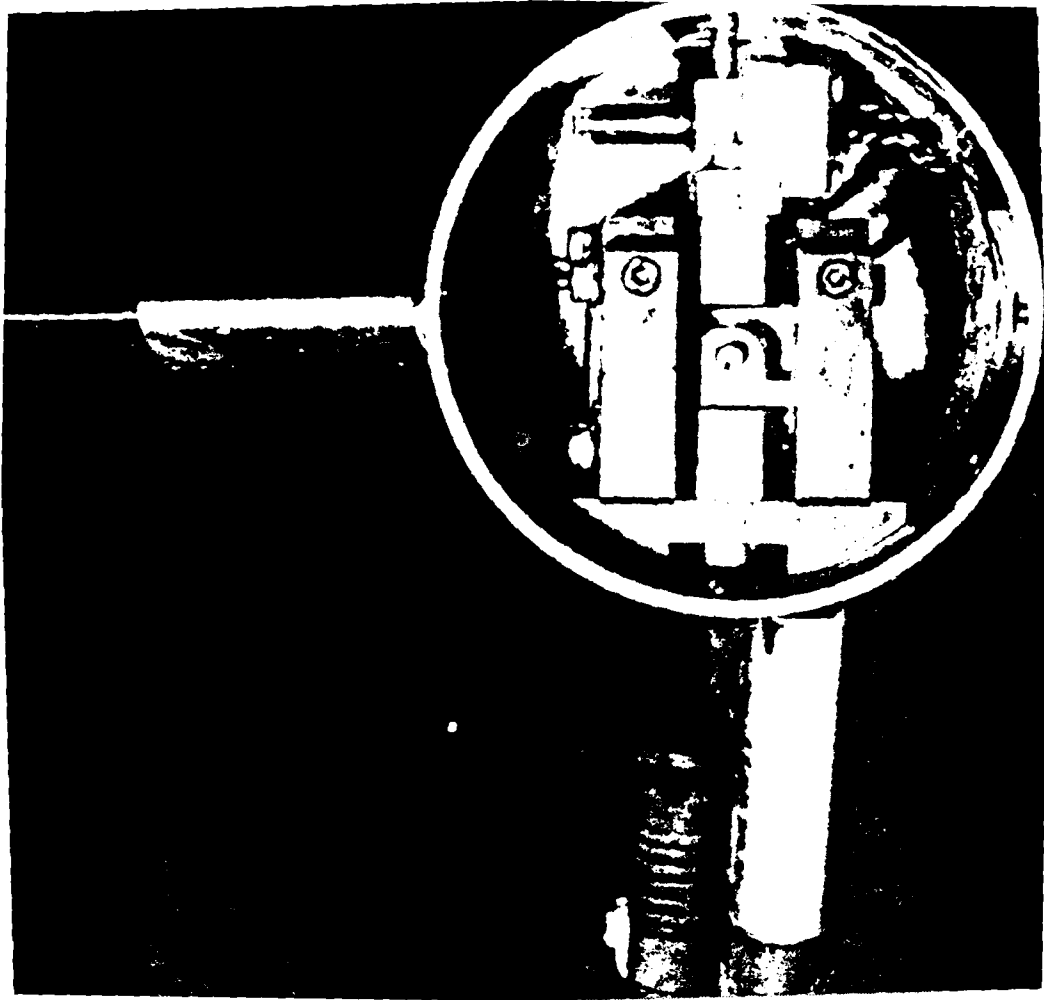


FIGURE 4a LIFT AND DRAG BALANCE INSTALLATION IN MODEL.



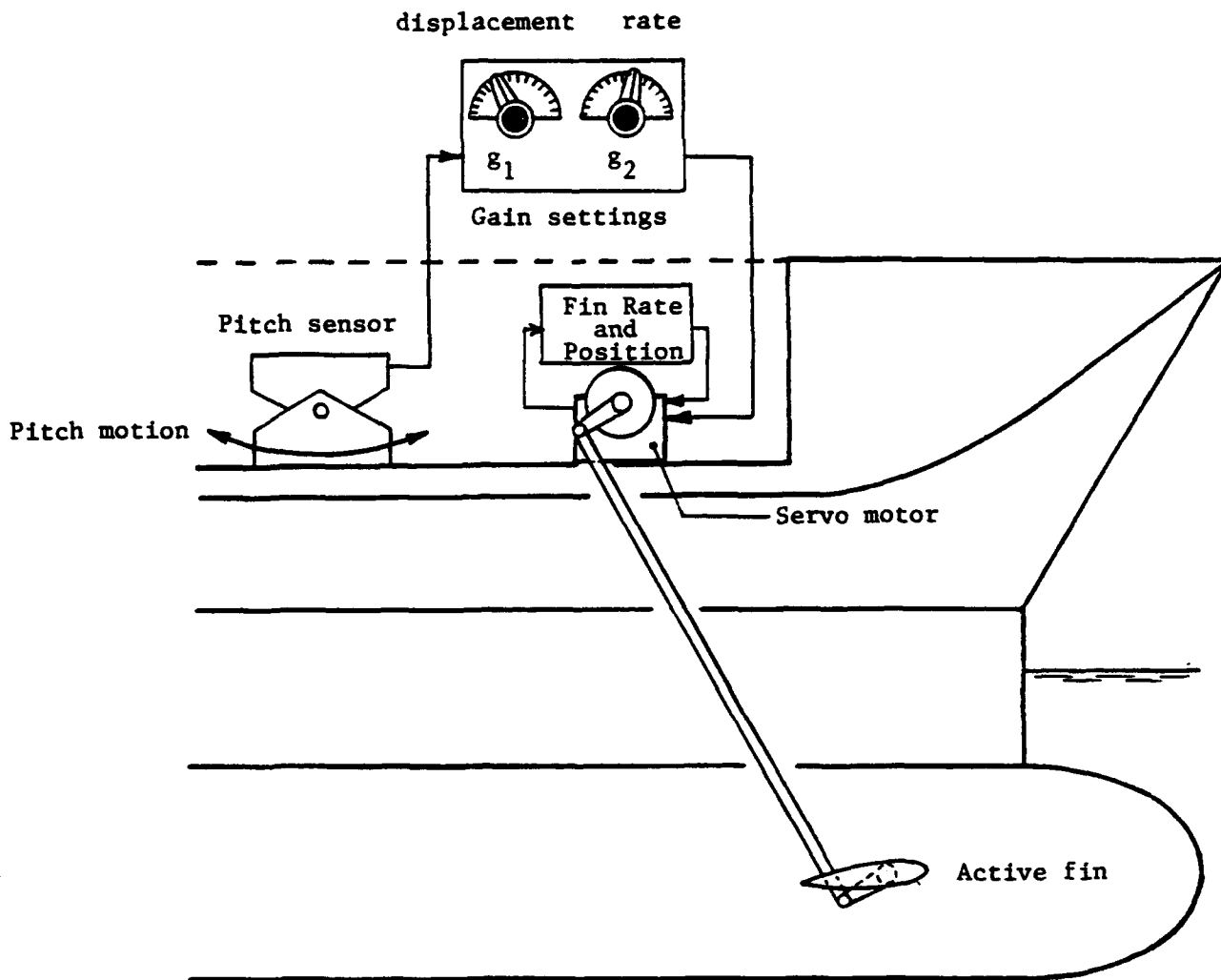


FIGURE 5 SCHEMATIC DIAGRAM OF PITCH CONTROL SYSTEM

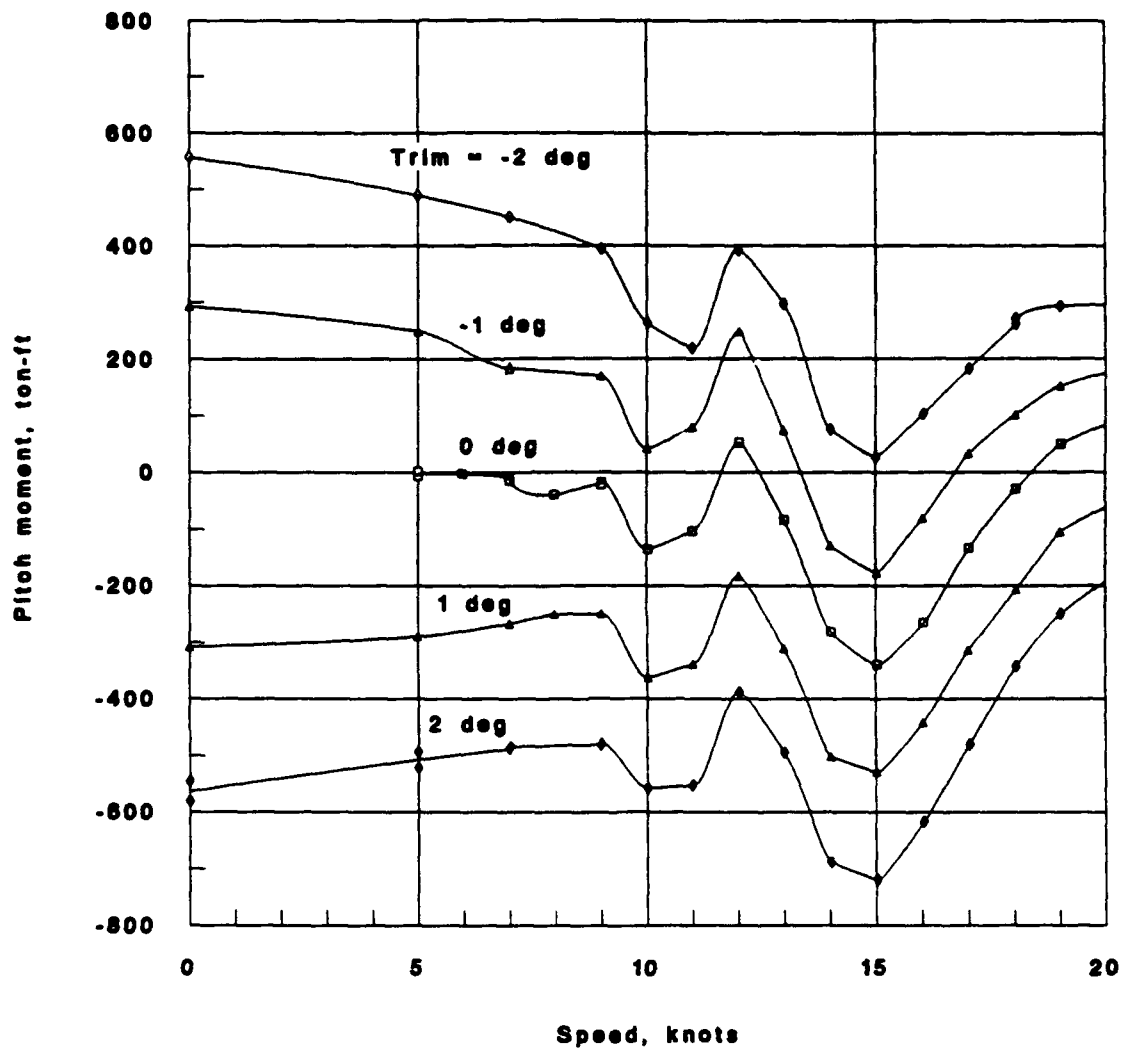


FIGURE 6a BEHAVIOR OF PITCH MOMENT WITH SPEED FOR UNAPPENDED SHIP AT VARIOUS TRIM ANGLES. FIXED TRIM, FREE-TO-HEAVE.

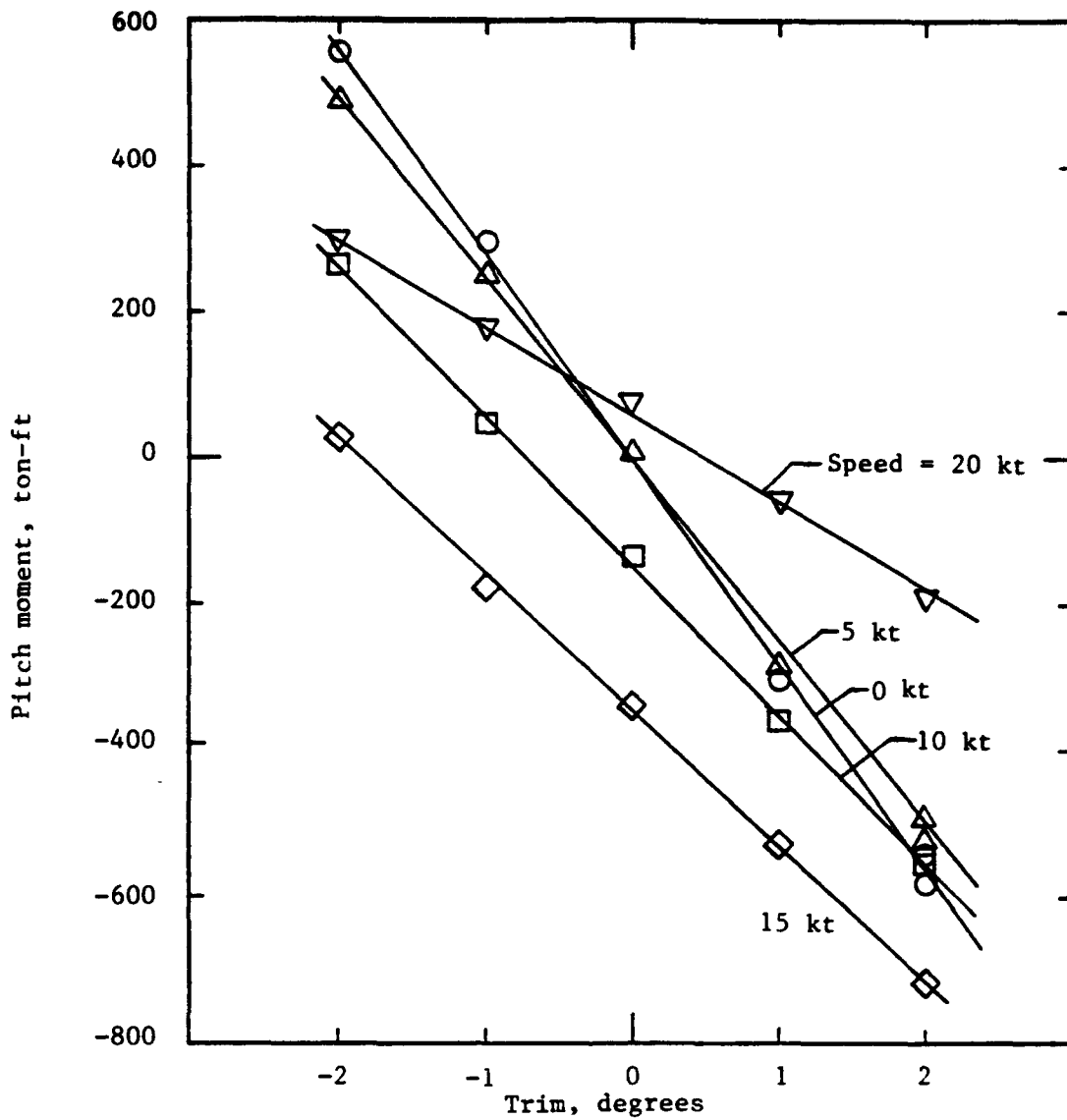


FIGURE 6b BEHAVIOR OF PITCH MOMENT WITH TRIM ANGLE FOR UNAPPENDED SHIP AT VARIOUS SPEEDS (FIXED TRIM, FREE-TO-HEAVE).

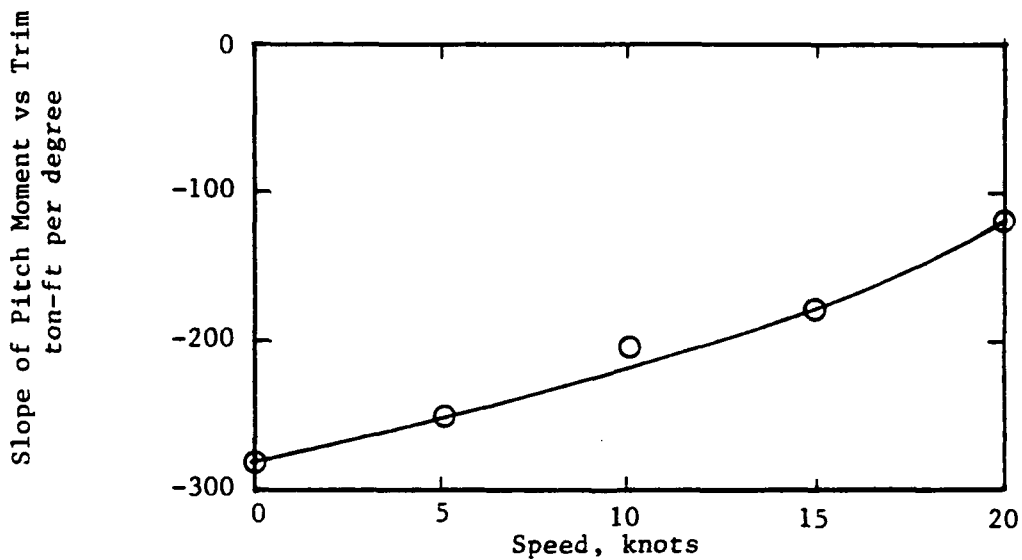


FIGURE 7 PITCH STABILITY VS. SPEED.

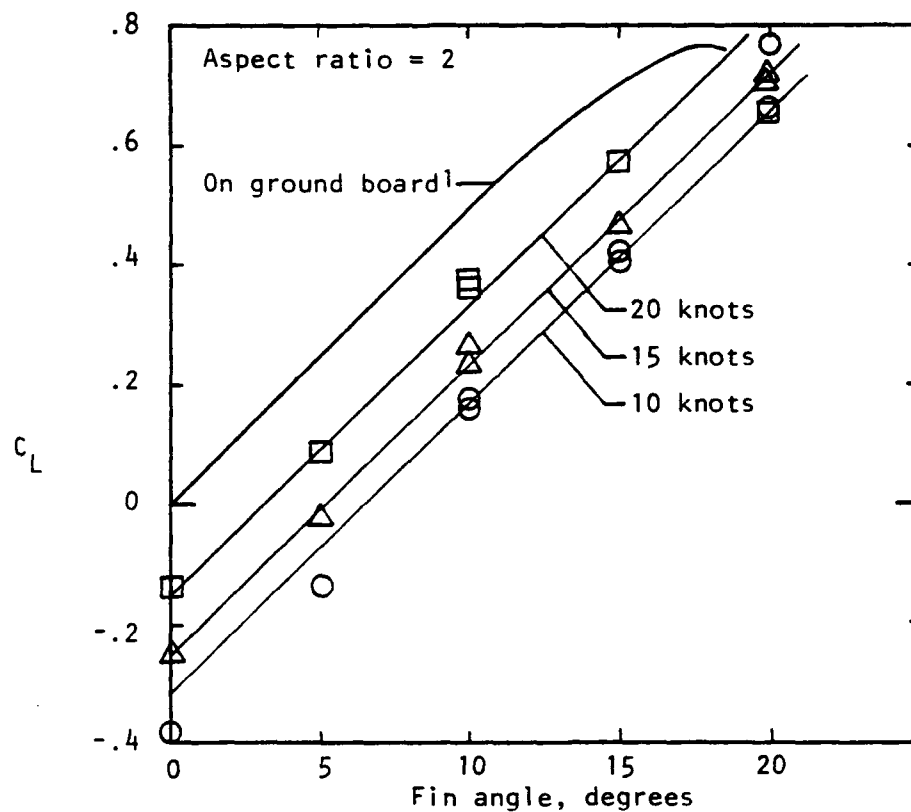
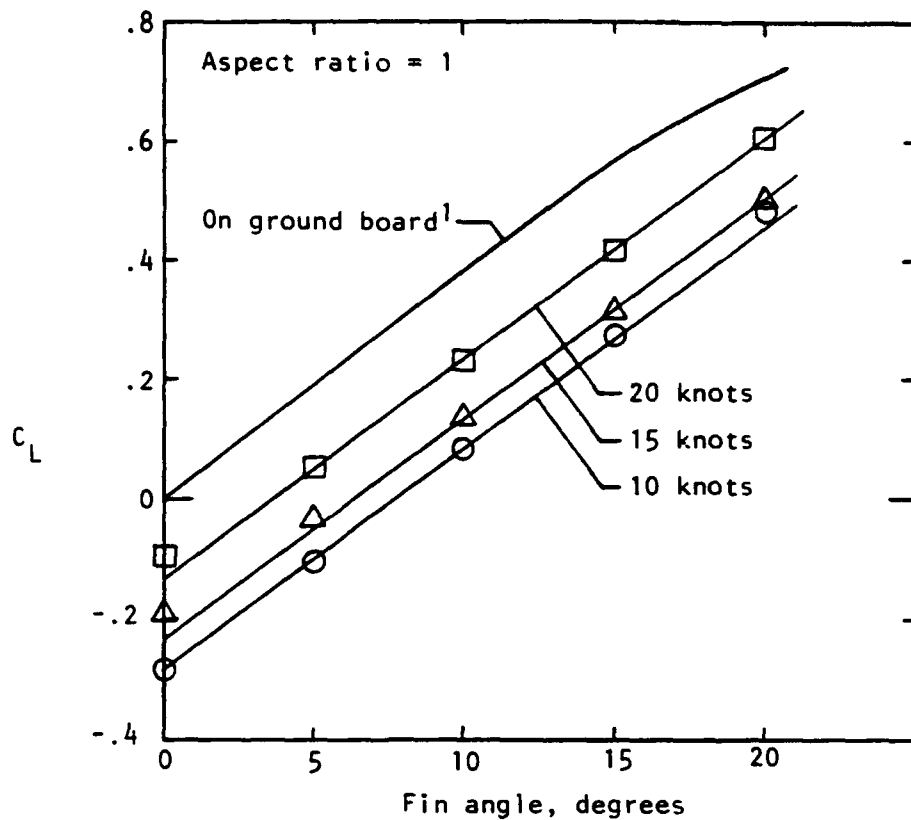
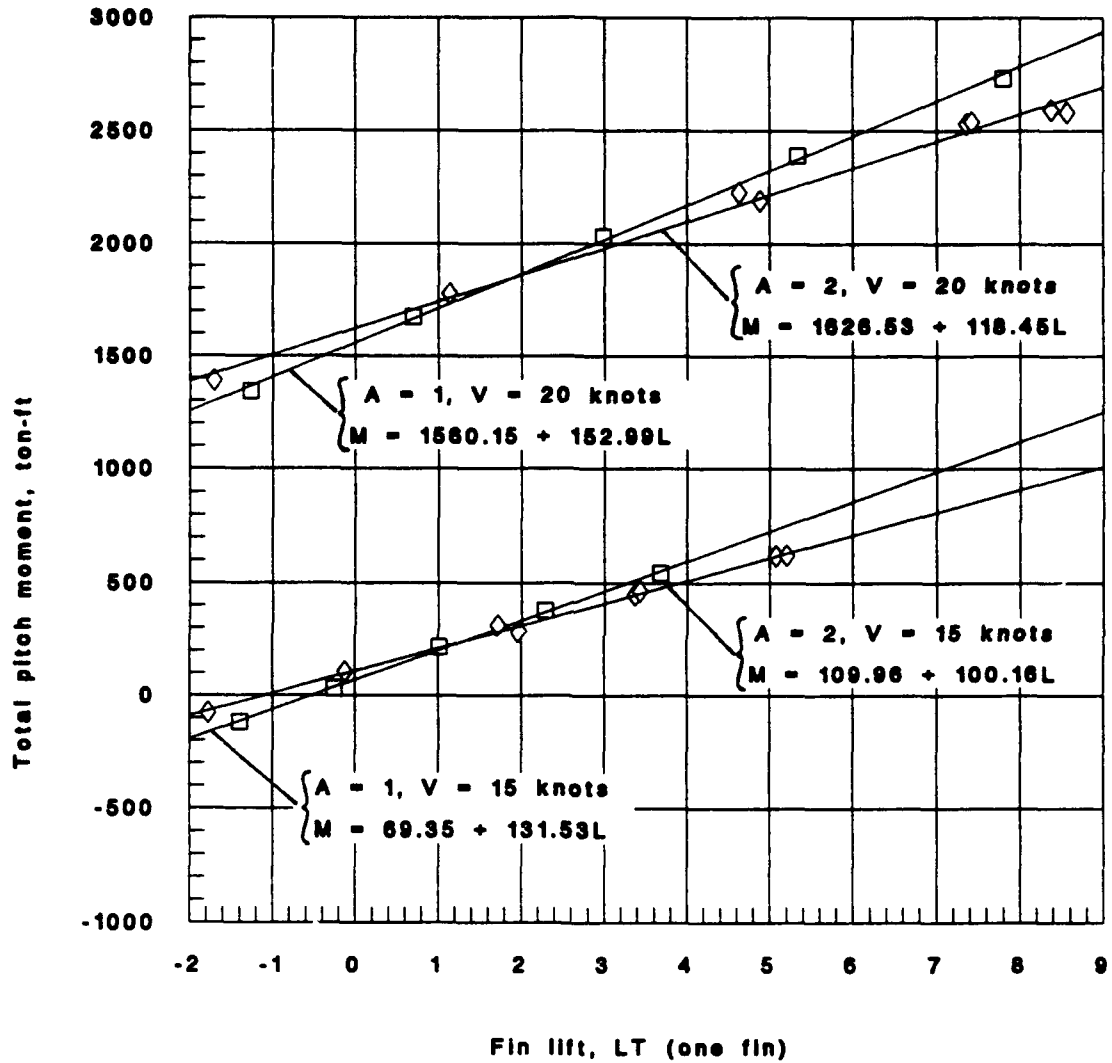
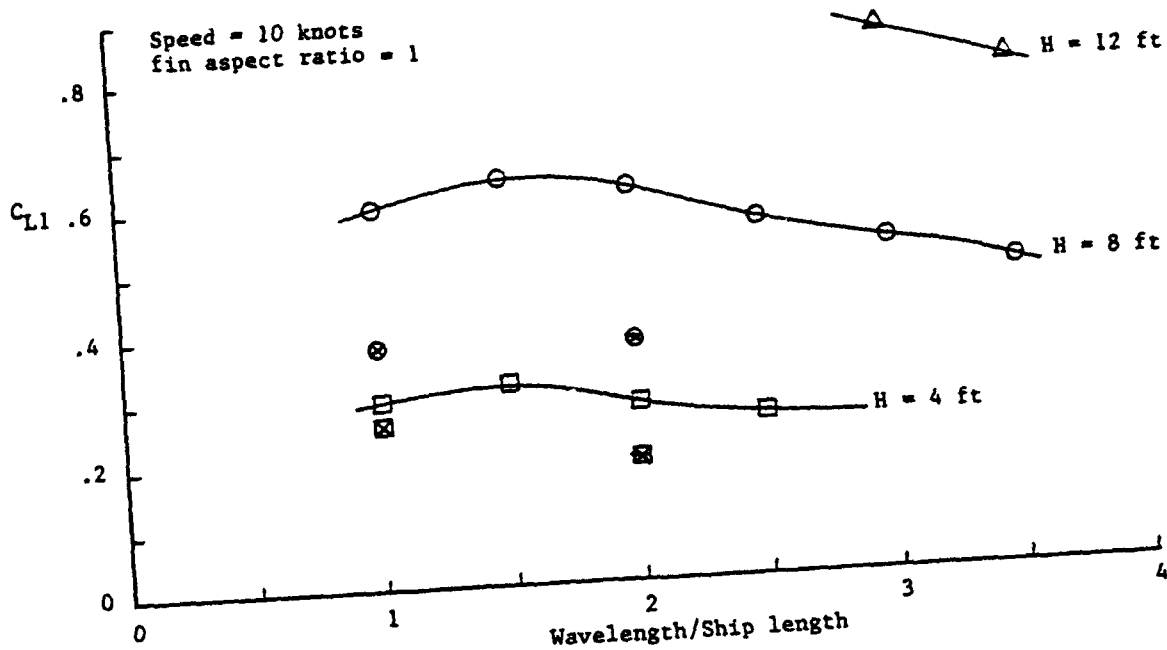


FIGURE 8 RESULTS OF INSTRUMENTED FIN ON BODY TESTS IN CALM WATER.



**FIGURE 9 PITCH MOMENT VS FIN LIFT AT TWO SPEEDS IN CALM WATER
(MODEL FIXED IN TRIM AND HEAVE)**



Wave Height	Head Seas	Following Seas
4	□	⊠
8	○	●
12	△	▲

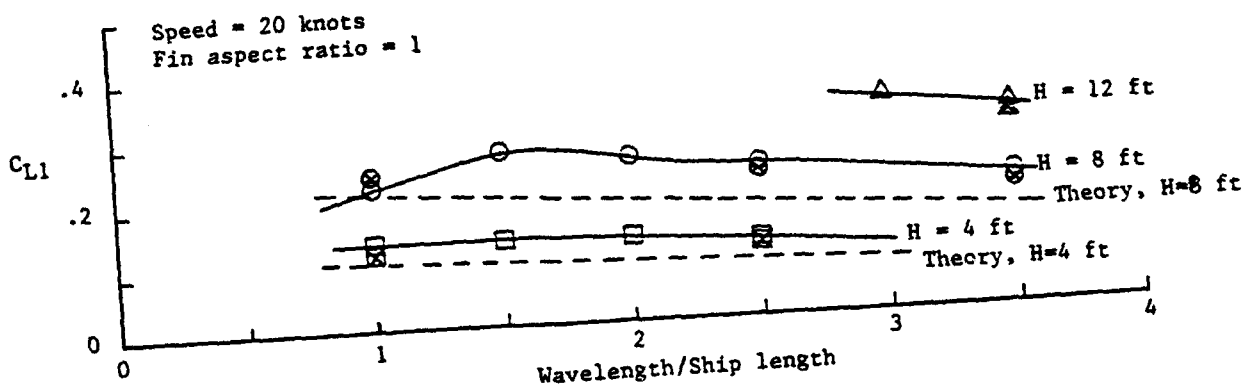


FIGURE 10 BEHAVIOR OF FIRST HARMONIC OF FIN LIFT COEFFICIENT WITH WAVELENGTH AND WAVE HEIGHT MODEL FIXED IN TRIM AND HEAVE

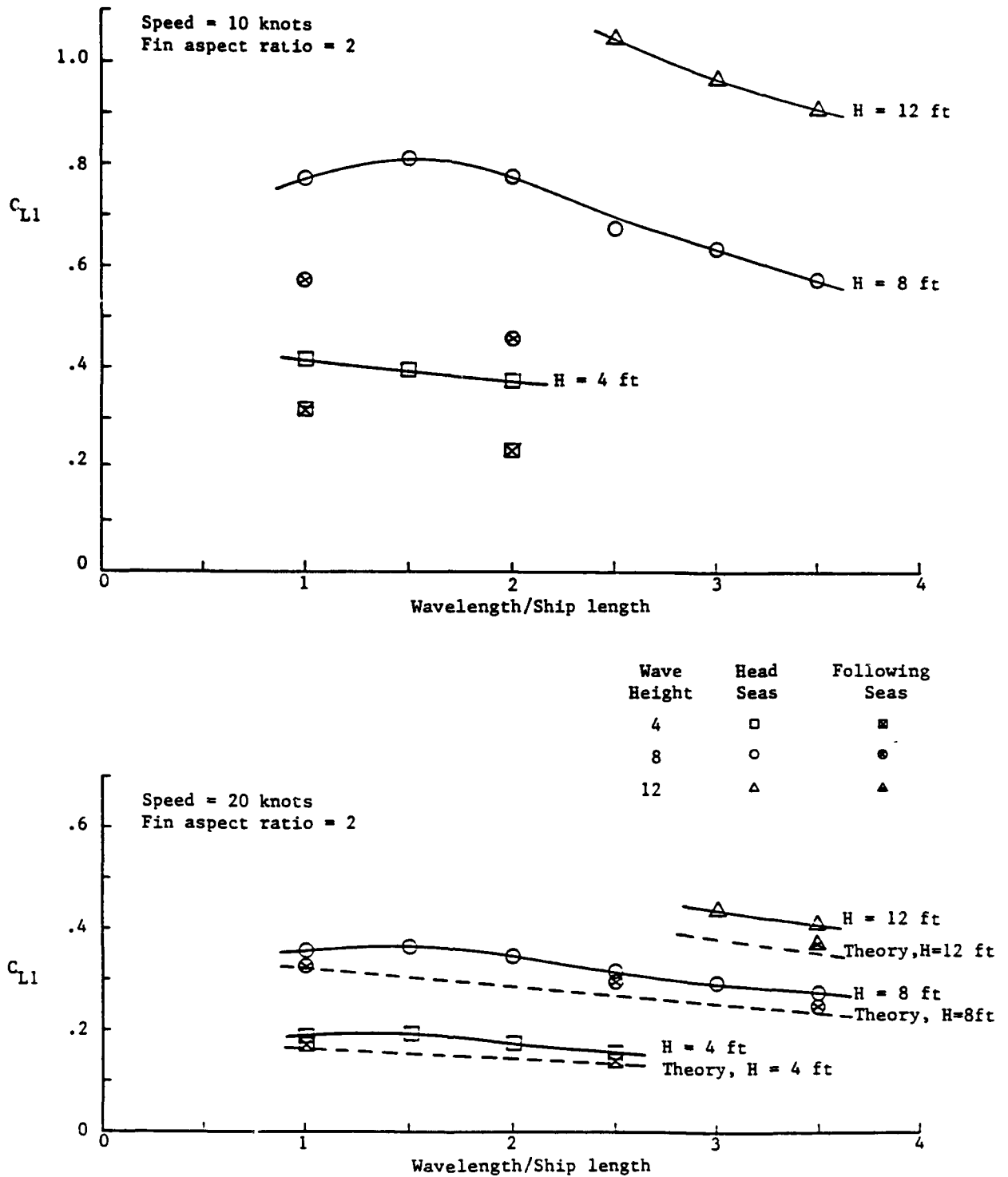


FIGURE 11 BEHAVIOR OF FIRST HARMONIC OF FIN LIFT COEFFICIENT WITH WAVELENGTH AND WAVE HEIGHT MODEL FIXED IN TRIM AND HEAVE

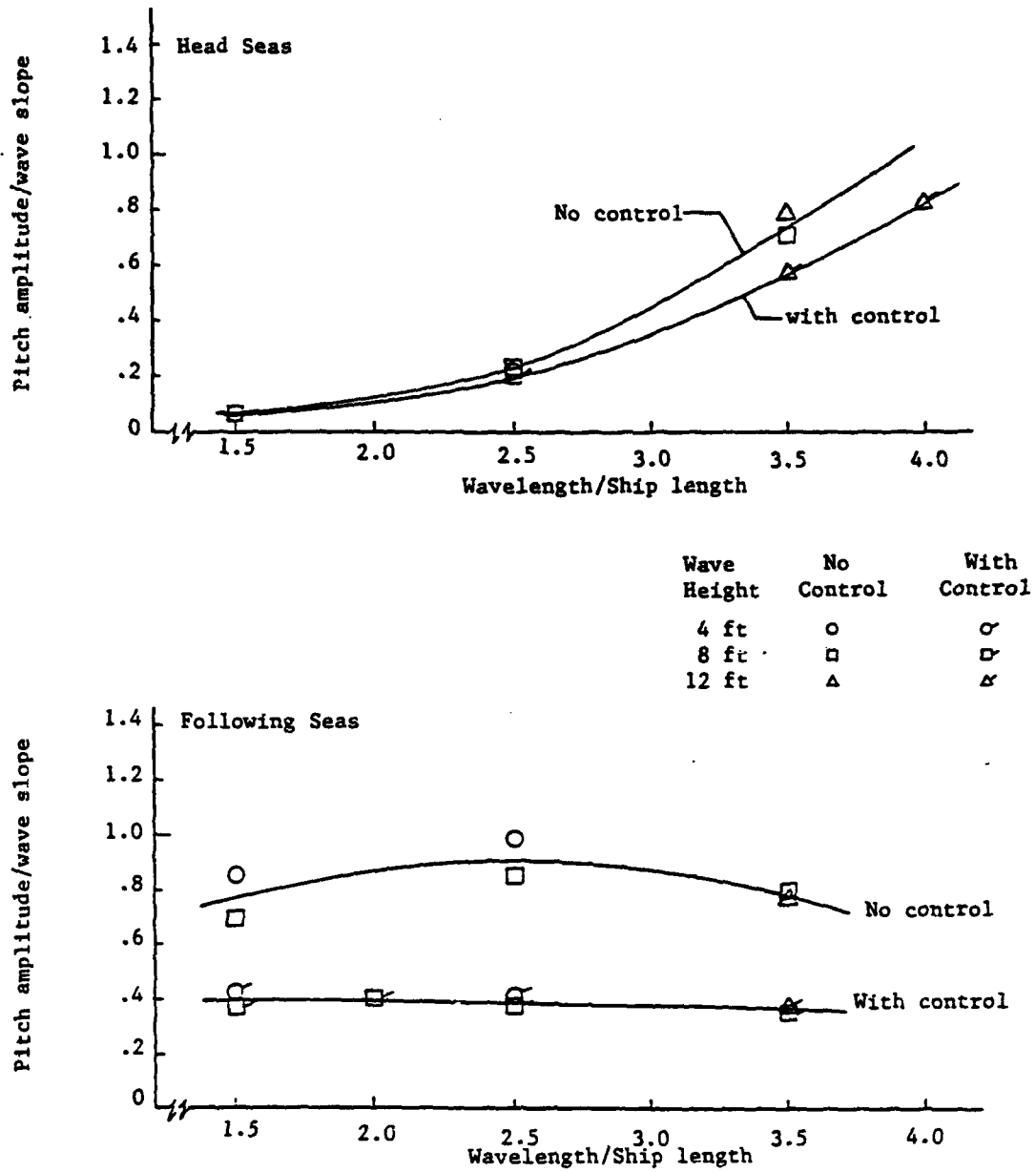
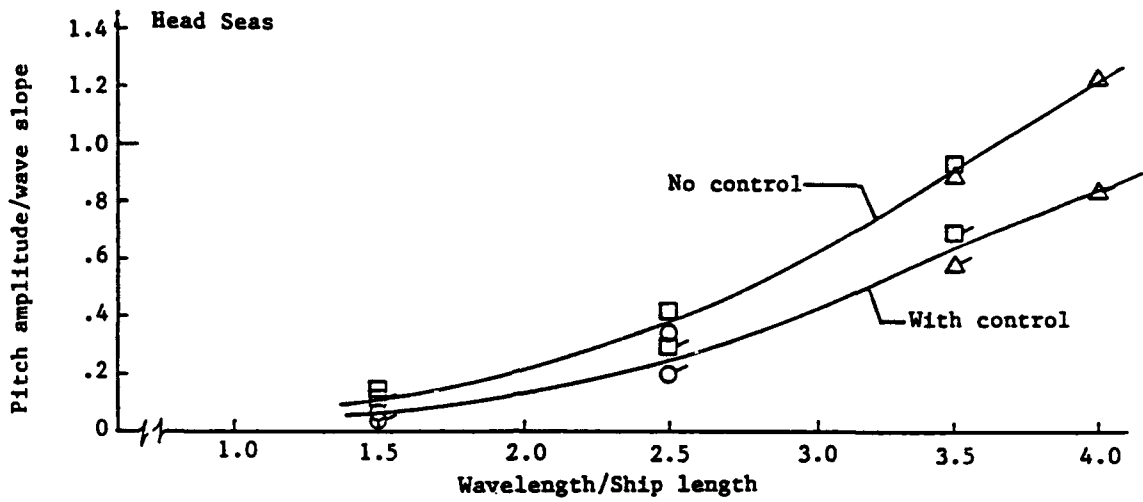


FIGURE 12 EFFECT OF ACTIVE CONTROL ON PITCHING MOTION, SPEED = 15 KNOTS. FIN ASPECT RATIO = 1.



Wave Height	No Control	With Control
4 ft	○	◌
8 ft	□	◻
12 ft	△	◤

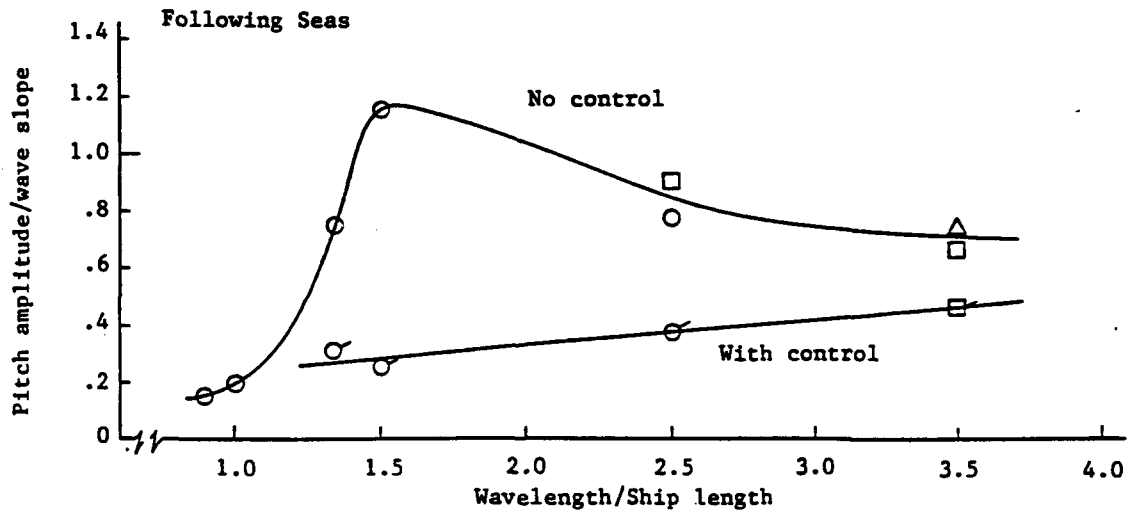
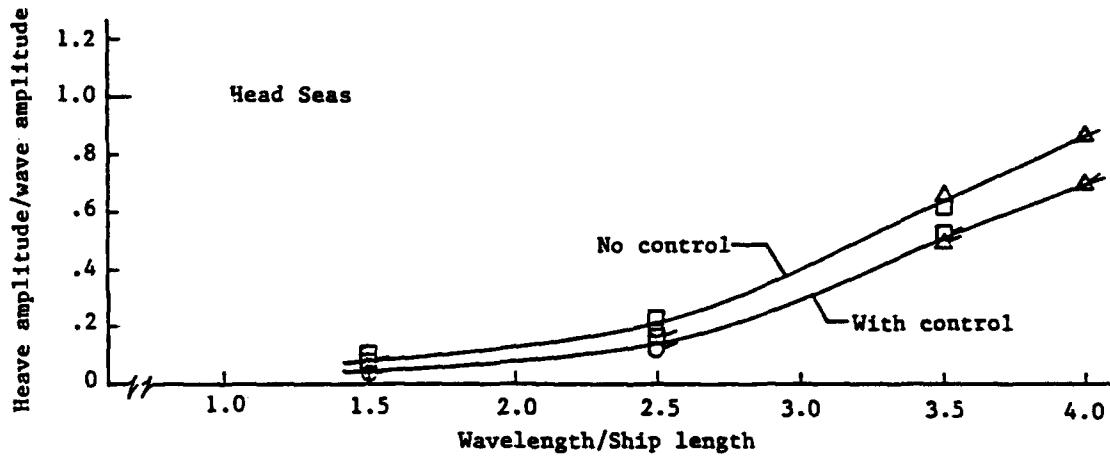


FIGURE 13 EFFECT OF ACTIVE CONTROL ON PITCHING MOTION, SPEED = 20 KNOTS. FIN ASPECT RATIO = 1.



Wave Height	No Control	With Control
4 ft	○	◉
8 ft	□	◻
12 ft	△	◀

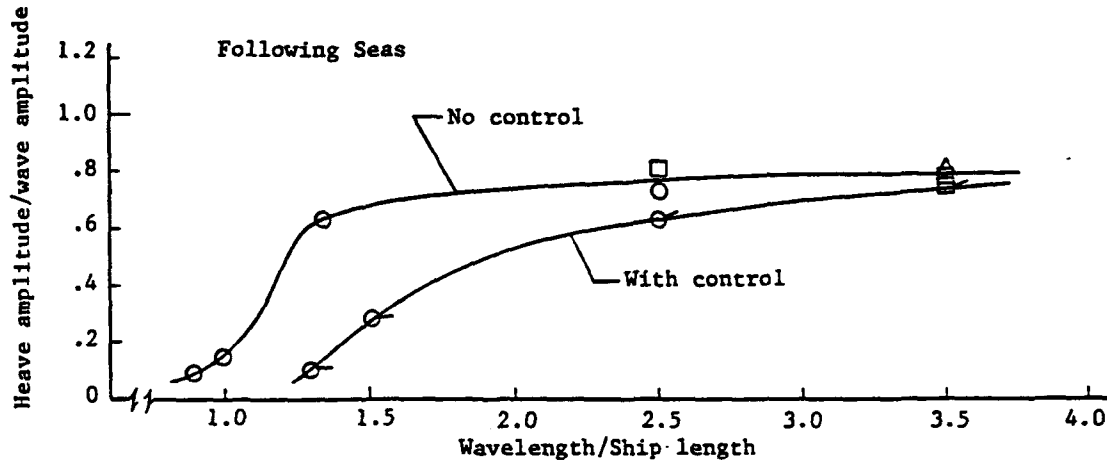


FIGURE 14 EFFECT OF ACTIVE CONTROL ON HEAVE MOTION, SPEED = 20 KNOTS. FIN ASPECT RATIO = 1.

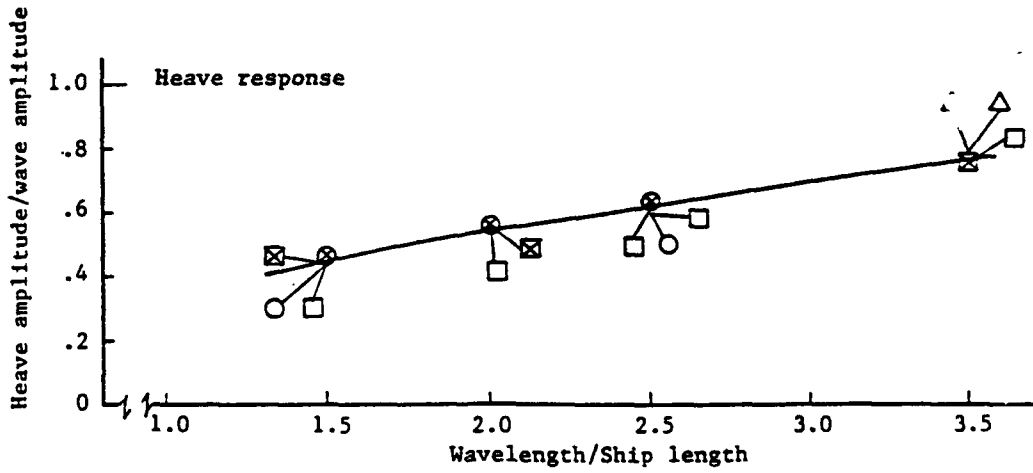
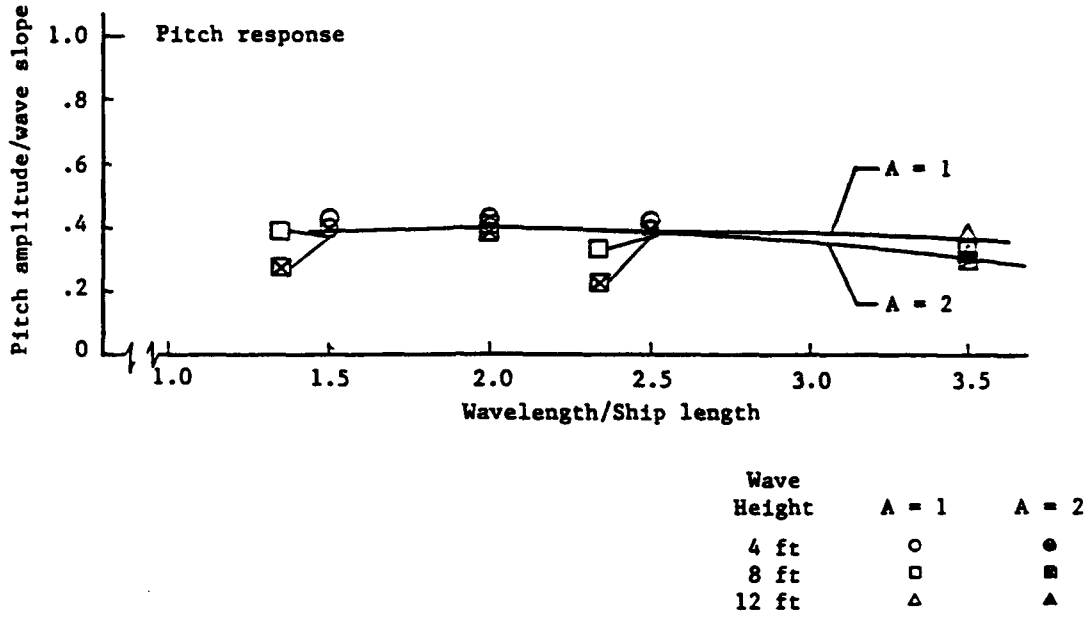


FIGURE 15 EFFECT OF FIN ASPECT RATIO ON MOTION RESPONSES WITH CONTROL, SPEED = 15 KNOTS.

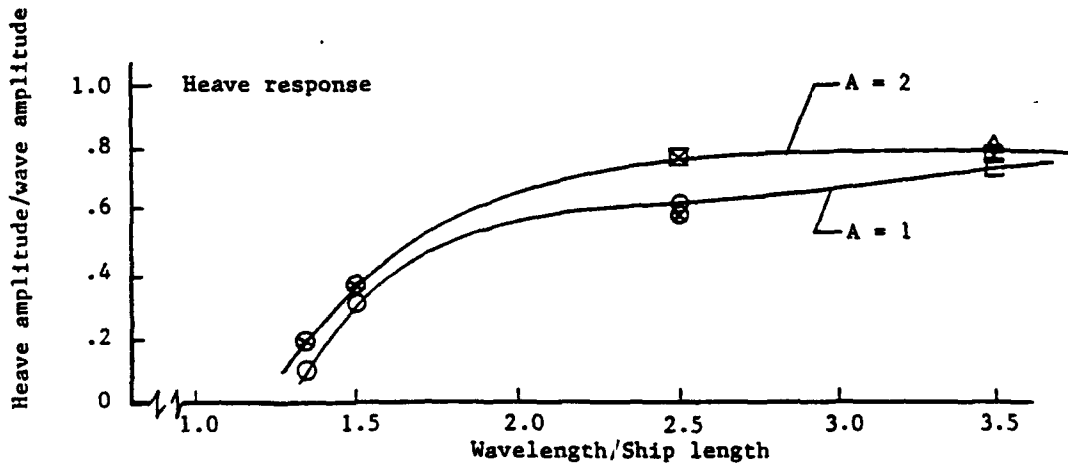
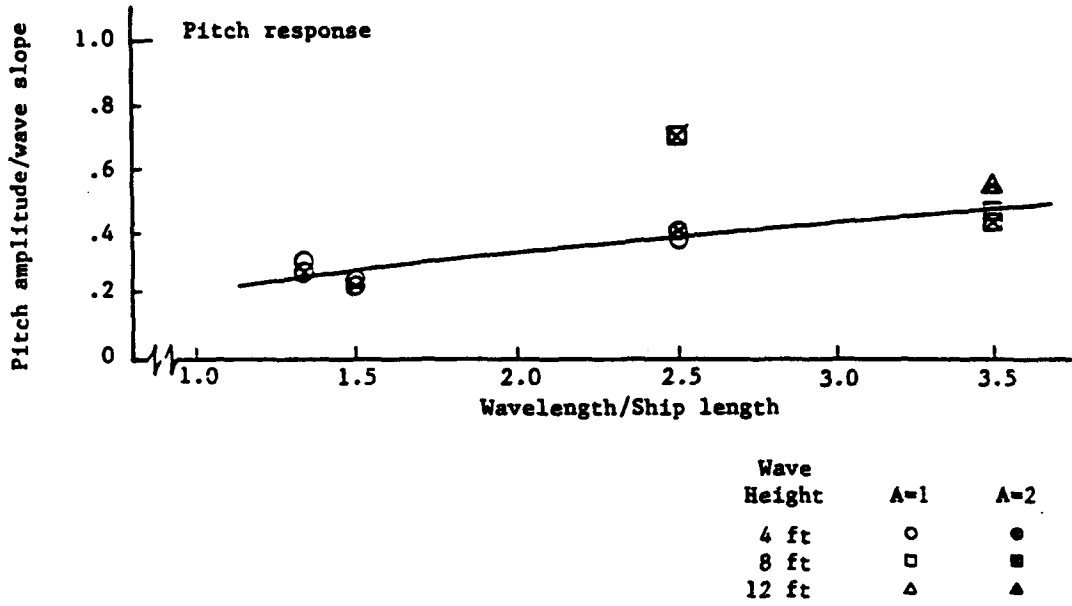
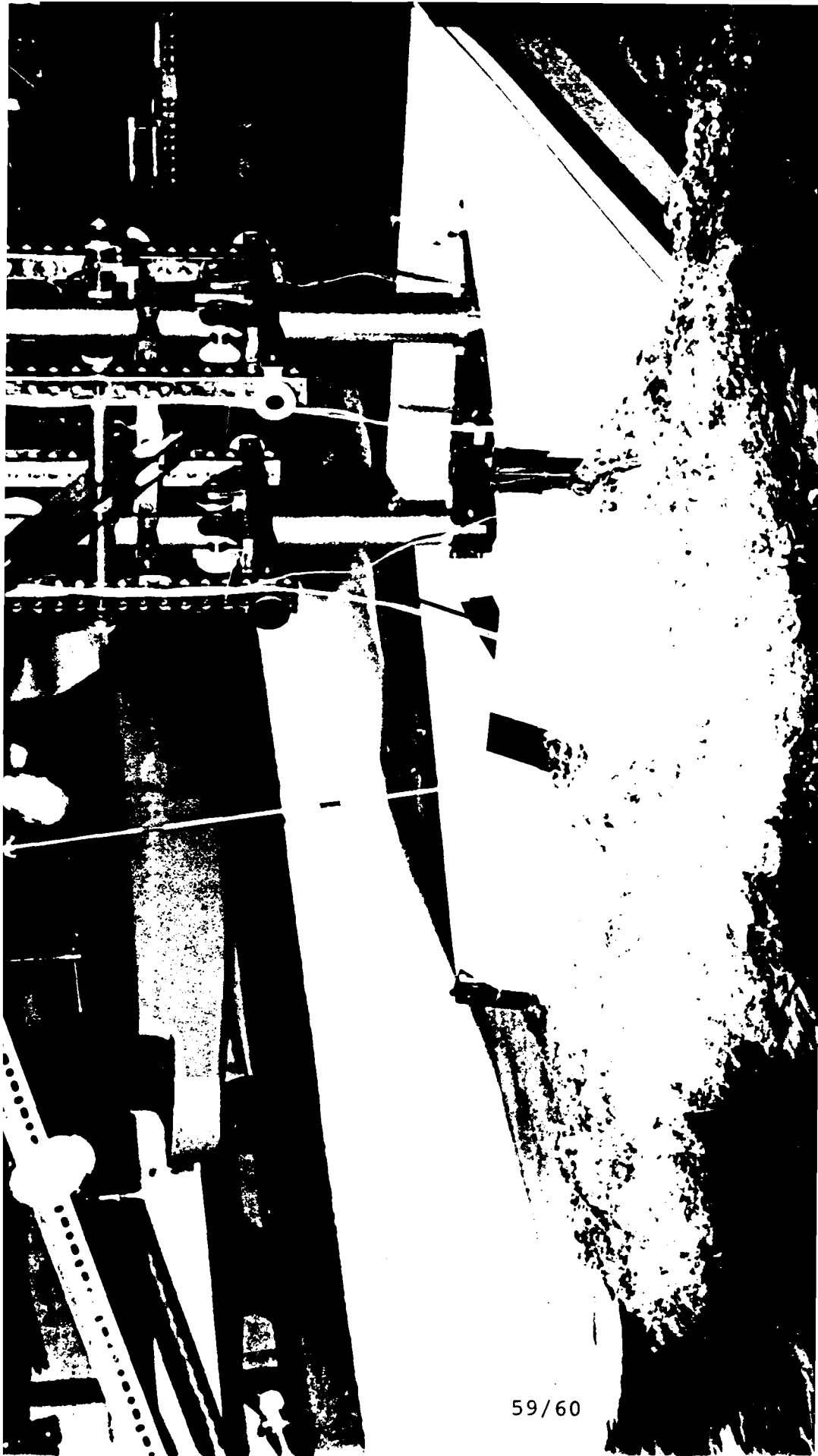


FIGURE 16 EFFECT OF FIN ASPECT RATIO ON MOTION RESPONSES WITH CONTROL, SPEED = 20 KNOTS.



APPENDIX A
Inclining Experiment

An inclining test was done just prior to the tests with the automatic pitch control system, to determine the longitudinal GM. A five pound weight was shifted on the deck to produce a moment about the pivot point. The trim was then read using the inclinometer mounted on the deck. Results are tabulated below.

Weight shift, ft	Moment ft-lb	Trim deg
0	0	.076
0.25	-1.25	-.609
0.50	-2.50	-1.415
0.75	-3.75	-2.170
1.00	-5.00	-2.952
1.25	-6.25	-4.373
-0.25	1.25	.853
-0.50	2.50	1.602
-0.75	3.75	2.345
-1.00	5.00	3.101
-1.25	6.25	3.908
-1.50	7.50	4.580

A straight line was fit to the data in the range $-5 \leq M \leq 7.5$, with the following result:

$$\theta = 0.093 + 0.6033M$$

Using the theoretical relationship between trim and moment,

$$\Delta \cdot GM_L \cdot \theta = M$$

for small angles where Δ is the vessel displacement and θ is in radians, one obtains in the present case that

APPENDIX A
(Continued)

$$(1/\Delta \cdot GM_L)(57.296 \text{ deg/rad}) = 0.6033$$

and using the model displacement $\Delta = 93.22 \text{ lb}$,

$$\begin{aligned} GM_L &= 1.019 \text{ ft (model scale)} \\ &= 24.45 \text{ ft (full scale)} \end{aligned}$$

Determination of LCF

Before the free to pitch tests were conducted, a determination of the longitudinal center of flotation (LCF) was made as follows: with the pivot point at its initial location, 2.135 ft aft of the strut nose, a 5 lb weight was placed at various stations on the deck and the resulting trim change noted:

Location	Trim Change (deg)
Pivot	-1.545
5 in aft	-0.246
6 in aft	0.000
9 in aft	0.813
12 in aft	1.597

The center of flotation is the point at which the addition of a weight causes no trim change, which in this case is 6 in aft of the pivot. To avoid coupling between heave and the measured pitch, which was used as input to the control system, the pivot point was moved to the LCF for the free to pitch tests, to 63.25 ft aft of the strut nose.

APPENDIX B

Videotape Scenarios
Videotape No. 1

Calm Water, Unappended, Fixed Trim

Run	Velocity knots	Trim deg	Video start
16	5	0	1:00
17	7		2:05
18	9		3:17
19	10		4:09
20	11		4:55
21	12		5:39
22	13		6:28
23	14		7:04
24	15		7:40
25	16		8:14
26	17		8:47
27	18		9:18
28	19		9:51
29	20		10:16
33	5	2	10:42
34	7	2	12:08
36	5	-1	13:20
37	7		14:53
38	9		16:00
39	10		16:51
40	11		17:39
41	12		18:21
42	13		19:01
43	14		19:37
44	15		20:11
45	16		20:45
46	17		21:17
47	18		21:47
48	19		22:10
49	20		22:37
51	5	1	23:02
53	7		24:32
54	9	1	25:33
55	10		26:25
56	11		27:07
57	12		27:47
58	13		28:24
59	14		28:56
60	15		29:37
61	16		30:11
62	17		30:40
63	18		31:10
64	19		31:36
66	20		32:05

APPENDIX B
(Continued)Videotape Scenarios
Videotape No. 1

Calm Water, Unappended, Fixed Trim

Run	Velocity knots	Trim deg	Video start
67	8		32:30
69	5	2	33:28
70	7		34:56
71	9		36:00
72	10		36:49
73	11		37:36
74	12		38:16
75	13		38:55
76	14		39:30
77	15		40:03
78	16		40:32
79	17		41:00
80	18		41:28
81	19		41:52
82	20		42:18
84	5	-2	42:41
85	7		44:07
86	9		45:06
87	10		45:54
88	11		46:35
89	12		47:17
90	13		47:56
91	14	-2	48:30
92	15		49:02
93	16		49:32
94	17		50:00
95	18		50:28
97	19		50:50
98	20		51:20

APPENDIX B
(Continued)Videotape Scenarios
Videotape No. 2

Tests with Instrumented Canard; Zero Trim and Heave

Tests in Calm Water Varying Canard Angle

Run	Velocity knots	α deg	Aspect ratio	Video start	Comments
12	20	0	1	2:00	
13	10	5		2:23	
14	15	5		2:52	
15	20	5		3:15	
16	10	10		3:54	
17	15	10		4:33	
18	20	10		5:00	
19	10	15		5:27	
21	15	15		6:10	
22	20	15		6:36	
23	10	20		7:01	
24	15	20		7:41	
25	20	20		8:13	
34	10	0	2	8:32	
35	15	0		9:12	
36	20	0		9:43	
37	10	5		10:09	
38	15	5		10:51	
39	20	5		11:19	
42	10	15		11:44	
43	15	15		12:15	
44	20	15		12:45	
45	10	15		13:09	
46	10	10		14:03	
47	15	10		14:43	
48	20	10		15:12	
49	10	20		15:30	
51	20	20		16:46	
52	20	20		17:06	Closeup of fin
53	15	20		17:30	Closeup of fin
54	10	20		18:00	Closeup of fin

APPENDIX B
(Continued)Videotape Scenarios
Videotape No. 2Tests In Regular Waves with Instrumented Canard. Zero trim and heave; $\alpha=0^\circ$.

Run	Velocity knots	Aspect ratio	λ/l	H ft	Video start	Comments
					Head Seas	
66-74					23:00	Preliminary runs
93	10	2	1	4.20	26:14	
94	20		1	4.20	27:05	
95	10		1	8.04	27:30	
96	20		1	8.40	28:20	
97	10		1.5	4.16	28:44	
98	20		1.5	4.16	29:34	
99	10		1.5	8.16	30:00	
100	20		1.5	8.16	30:47	
101	10		2.0	4.52	31:13	
102	20		2.0	4.52	31:59	
103	10		2.0	8.84	32:27	
104	20		2.0	8.84	33:14	
105	10		2.5	8.56	33:40	
106	20		2.5	12.60	34:11	
107	10		2.5	12.60	34:51	
108	20		2.5	4.24	35:38	
109	10		3.0	8.44	36:08	
110	20		3.0	8.44	36:55	
111	10		3.0	12.64	37:19	
112	20		3.0	12.64	38:07	
113	10		3.5	8.52	38:34	
114	20		3.5	8.52	39:21	
115	10		3.5	12.80	39:46	
116	20		3.5	12.80	40:34	
117	10	1	1.0	4.20	41:03	
118	20		1.0	4.20	41:38	
119	10		1.0	8.04	42:05	
120	20		1.0	8.04	42:32	
127	10		1.5	4.16	43:21	
128	20		1.5	4.16	44:07	
129	10	1	1.5	8.16	44:30	
130	20		1.5	8.16	45:16	
131	10		2.0	4.52	45:44	
132	20		2.0	4.52	46:29	
133	10		2.0	8.84	46:54	
134	20		2.0	8.84	47:46	
135	10		2.5	4.24	48:10	
136	20		2.5	4.24	48:58	
137	10		2.5	8.56	49:23	
138	20		2.5	8.56	50:15	
139	10		3.0	8.44	50:40	

APPENDIX B
(Continued)

Videotape Scenarios
Videotape No. 2

Tests in Regular Waves with Instrumented Canard. Zero trim and heave; $\alpha=0^\circ$.

Run	Velocity knots	Aspect ratio	λ/l	H ft	Video start	Comments
140	20		3.0	8.44	51:27	
141	10		3.0	12.64	51:32	
142	20		3.0	12.64	52:40	
143	10		3.5	8.52	53:08	
144	20		3.5	8.52	53:57	
145	10		3.5	12.80	54:03	
146	20		3.5	12.80	54:52	
150	10		3.5	8.52	57:39	
151	10		3.5	12.80	58:31	
152	10		3.0	8.44	59:18	
153	10		3.0	12.64	1:00:08	
154	10		2.5	8.56	1:00:55	
155	10		2.0	8.84	1:01:46	

Following Seas

163	10	1	1.0	4.20	1:03:02	
164	10		1.0	8.04	1:03:57	
165	20		1.5	8.16	1:04:54	
166	20		2.5	8.56	1:05:26	
167	20		3.5	12.80	1:05:58	
168	10		1.0	4.20	1:06:32	
170	10		1.0	8.04	1:07:26	
171	10		2.0	4.52	1:08:12	
172	10		2.0	8.84	1:09:22	
173	20		1.0	4.20	1:10:14	
174	20	1	1.0	8.04	1:11:05	
175	20		2.5	4.24	1:11:31	
176	20		2.5	8.56	1:12:00	
177	20		3.5	8.52	1:12:35	
178	20		3.5	12.80	1:13:02	
179	10	2	1.0	4.20	1:13:29	
180	10		1.0	8.04	1:14:14	
181	10		2.0	4.52	1:15:02	
182	10		2.0	8.84	1:15:49	
183	20		1.0	4.20	1:16:37	
184	20		1.0	8.04	1:17:02	
185	20		2.5	4.24	1:17:27	
186	20		2.5	8.56	1:17:52	
187	20		3.5	8.52	1:18:16	
188	20		3.5	12.80	1:18:44	

APPENDIX B
(Continued)Videotape Scenarios
Videotape No. 3

Tests with Automatic Pitch Control

Run	Velocity knots	Aspect ratio	λ/l	H ft	Video start	Comments
26-45					0:46	Preliminary runs

Baseline Tests Without Automatic Control. Head Seas.

46	15	1	3.5	8.66	4:17
47			3.5	8.66	5:02
48			3.5	8.66	5:48
49			1.5	4.10	6:38
59			3.5	12.92	7:18
60			2.5	4.28	8:02
61			2.5	8.66	8:37
62			1.5	8.26	9:15
63			1.5	4.08	9:52
64	20		3.5	8.68	10:37
65			3.5	8.68	11:08
66			3.5	12.92	11:44
67			3.5	12.92	12:19
68			2.5	4.28	12:56
69			2.5	8.66	13:33
70			1.5	8.26	14:08
71			1.5	4.08	14:44

Following Seas

72	15	1	0.58	4.08	15:20	No oscillations
74			1.5	4.22	16:34	
75			2.0	4.08	17:13	
76			2.0	8.26	17:56	
77			2.5	4.28	18:47	
78			2.5	8.66	19:37	
79			3.5	8.68	20:26	
80			3.5	12.92	21:15	Need more heave travel
81			3.5	12.92	21:58	Need more heave travel
82			3.5	12.92	22:41	Need more heave travel
83	20		0.89	4.26	23:28	
84			1.0	4.22	24:00	
85			1.0	8.21	24:34	Model nosedives
87			2.5	4.08	25:00	
88			1.5	8.26	25:30	Model nosedives
89	15		2.0	4.25	25:45	4 in coaming added
90	15		2.0	8.50	26:31	
91	20		2.0	4.25	27:20	Zero encounters
92			2.5	4.28	28:00	
93			3.5	8.68	28:33	
94			3.5	12.92	29:07	No data

APPENDIX B
(Continued)

Videotape Scenarios
Videotape No. 3

Following Seas

Run	Velocity knots	Aspect ratio	λ/l	H ft	Video start	Comments
95			3.5	12.92	29:46	
96			2.5	8.66	30:29	
97			1.0	8.24	3:02	Model nosedives

Tests To Find Optimum Gain Settings; Following Seas

Run	Velocity knots	Aspect ratio	λ/l	H ft	Video start	ξ_1	ξ_2	
98	20	1	1.5	4.08	32:00	2.12	0	
99			1.5	4.08	32:45	no data		
100			1.5	4.08	33:33	3.54	1.45	
101			2.5	8.66	34:21	model nosedives		
103			2.5	4.28	34:56	3.54	0	added PVC windshield
104			2.5	4.28	35:35	3.54	2.41	
105			2.5	4.28	36:06	3.54	4.83	
106			3.5	8.68	36:51			
107			3.5	8.68	37:40	3.54	2.41	
108			3.5	8.68	38:08	3.54	4.83	
109			3.5	8.68		4.95	4.83	
110			3.5	8.68		7.07	4.83	
111			3.5	8.68		0	4.83	
112			1.5	4.08		zero	encounters	
113			1.5	4.08		4.95	4.83	
116			2.5	4.28		4.95	6.76	
117			2.5	4.28		5.66	6.76	
118			2.5	4.28		6.36	6.76	
119			2.5	4.28		7.07	6.76	
120			2.5	4.28		6.36	7.72	

APPENDIX B
(Continued)Videotape Scenarios
Videotape No. 3

Tests in Following Seas with Optimal Control Gains

Run	Velocity knots	Aspect ratio	λ/l	H ft	Video start	Comments
121	20	1	2.5	4.28	44:17	
122	20		1.5	4.08	45:15	
123	20		1.34	4.20	46:04	
124	20		1.34	4.20	46:48	
126	15		2.0	8.50	47:30	
127	15		2.0	8.50	48:20	
128	15		2.5	8.68	49:00	
129	15		2.5	12.92	49:38	
130	15		2.5	4.28	50:22	
131	15		1.5	4.08	51:07	
132	15		1.5	8.26	52:07	
133	15		1.5	8.26	53:04	Servos off
134	15		1.5	8.26	54:09	

Tests in Head Seas to Find Optimum Gain Settings

Run	Velocity knots	Aspect ratio	λ/l	H ft	Video start	δ_1	δ_2
135	15	1	3.5	8.68	55:03	3.54	1.93
137	15		3.5	8.68	56:20	3.54	4.83
138	15		3.5	8.68	57:02	3.54	6.76
139	15		3.5	8.68	57:44	4.95	6.76
140	15		3.5	8.68	58:26	3.54	8.69
141	15		3.5	8.68	59:08	3.54	9.65
142	15		3.5	8.68	59:40	4.95	8.69
143	15		3.5	8.68	1:00:33	0	9.65
144	20		3.5	8.68	1:01:15	0	9.65
145	20		3.5	8.68	1:01:48	3.54	6.76
146	20		3.5	8.68	1:02:22	Servos off	
147	20		3.5	8.68	1:02:55	0	6.76

Tests in Head Seas With Optimum Control Gains

148	20	1	3.5	12.92	1:03:27		
149	20		4.0	12.50	1:04:02		
150	20		4.0	12.50	1:04:33		
151	20		2.5	8.66	1:05:05		
152	20	1	2.5	4.28	1:05:40		
153	20		1.5	8.26	1:06:15		
154	20		1.5	4.08	1:06:48		
155	15		4.0	12.50	1:07:20	Servos off	
156	15		4.0	12.50	1:08:03	Servos off	
157	15		4.0	12.50	1:08:47		

APPENDIX B
(Continued)Videotape Scenarios
Videotape No. 3

Tests in Head Seas With Optimum Control Gains

Run	Velocity knots	Aspect ratio	λ/l	H ft	Video start	Comments
158	15		3.5	12.92	1:09:32	
159	15		2.5	8.66	1:10:15	
160	15		2.5	4.28	1:10:57	
161	20	2	4.0	12.50	1:12:26	
162	20		4.0	12.50	1:13:02	
163	20		3.5	12.92	1:13:37	
164	20		3.5	12.92	1:14:09	
165	20		3.5	12.92	1:14:42	
166	20		2.5	4.28	1:15:16	
167	20		2.5	8.66	1:15:53	
168	20		2.5	8.66	1:16:26	Controls not active
169	20		1.5	8.15	1:16:57	
170	20		1.5	4.72	1:17:30	
171	20		3.5	12.92	1:18:05	
172	15		1.5	8.26	1:18:38	
173	15		2.5	8.66	1:19:17	
174	15		2.5	4.28	1:20:00	
175	15		3.5	8.68	1:20:42	
176	15		3.5	12.92	1:21:26	
177	15		4.0	12.50	1:22:06	

Tests in Following Seas With Optimal Control Gain

178	15	2	1.5	8.26	1:22:49	
179	15		1.5	4.08	1:23:31	
180	15		2.0	4.25	1:24:04	
181	15		2.0	8.50	1:24:40	
182	15		2.5	8.66	1:25:20	
183	15		2.5	4.28	1:26:00	
184	15		3.5	8.68	1:26:37	
185	15		3.5	12.92	1:27:17	
186	20		1.34	4.20	1:28:57	
187	20		1.5	4.08	1:28:30	
188	20		1.5	4.08	1:29:25	
189	20		2.5	4.28	1:30:14	
190	20		2.5	8.66	1:30:42	
191	20		3.5	8.68	1:31:14	
191	20		3.5	12.92	1:31:45	

APPENDIX B
(Continued)Videotape Scenarios
Videotape No. 3

Tests In Calm Water With Aspect Ratio 2 Canards Oscillating

Run	Velocity knots	Frequency Hz (model scale)	Video start
193	20	2	1:32:21
194		2	1:33:56
195		1	1:34:25
196		.5	1:34:53
197		.75	1:35:20
198		.625	1:35:47
199		.5	1:36:15
200		.3	1:36:41
201		.4	1:37:11
202		.22	1:37:39
203		.15	1:38:07
204	15	1	1:38:35
205		.75	1:39:08
206		.625	1:39:43
207		.5	1:40:19
208		.4	1:40:54
209		.3	1:41:30
210		.25	1:42:05
211		.2	1:42:37
212		.15	1:43:12
213		.1	1:43:48
214		.45	1:44:24
215		.55	1:45:00

Final Runs in Following Seas With Automatic Control

Run	Velocity knots	Aspect ratio	λ/l	H ft	Video start	Comments
216	20	2	1.0	8.21	1:45:52	
217	20		1.5	8.26	1:46:48	Nosedive
219	20		1.5	8.26	1:47:25	Nosedive
220	20		2.5	8.66	1:48:35	Nosedive

APPENDIX C

FORCED OSCILLATION TESTS

A brief series of tests was conducted in calm water to observe the response of the SWATH model to a sinusoidal pitching moment while underway. A signal generator was used to drive the canards at various frequencies, resulting in constant amplitude pitching motion for the duration of the run. The model was run free to heave and pitch. A range of oscillation periods from 2.47 to 48.63 seconds (full scale) was investigated, at speeds of 15 and 20 knots. Time histories of heave, pitch, and canard angle were measured. A harmonic analysis of these signals was carried out as described in the report under Data Processing. Results are given in Table C1 below. The phase angles in the table are with respect to the canard angle.

Pitch and heave amplitudes per unit canard amplitude and their phases are plotted on Figures C1 and C2 for 15 knots and on Figures C3 and C4 for 20 knots, respectively. Figures C1 and C2 show an apparent heave and pitch resonance peak at a canard period between 12 and 16 seconds. At 20 knots there is a peak in the pitch motion at a canard period of just under 8 seconds and a peak in the heave motion at a period of about 10 seconds. Phase angles are near 90° at the resonance peaks, indicating that the exciting moment (induced in part by the canards but also influenced by the hull) is nearly in phase with the canard angle.

TABLE C1
RESULTS OF FORCED OSCILLATION TESTS

Run no.	Vel kts	T sec	ω rps	mean α deg	mean θ deg	θ_1 deg	phase θ_1 deg	mean θ_2 deg	phase θ_2 deg	mean heave ft	z_1 ft	phase z_1	z_2 ft	phase z_2
204	15	4.875	1.289	0.24	-0.43	0.39	158	0.00	36	-2.64	0.20	156	0.00	301
204	15	4.899	1.283	0.27	-0.43	0.38	158	0.01	38	-2.64	0.20	156	0.00	40
205	15	6.437	0.976	0.22	-0.40	0.92	158	0.00	299	-2.66	0.14	140	0.00	215
206	15	7.633	0.823	0.22	-0.33	1.50	142	0.00	267	-2.66	0.38	199	0.00	48
215	15	8.701	0.722	0.08	-0.32	1.96	123	0.02	116	-2.60	0.98	181	0.02	45
207	15	9.509	0.661	0.08	-0.05	2.04	112	0.03	85	-2.60	1.26	160	0.04	349
214	15	10.939	0.574	0.09	-0.07	2.32	103	0.05	39	-2.48	1.72	127	0.06	344
208	15	12.419	0.506	0.08	0.18	2.67	95	0.10	11	-2.40	1.88	105	0.06	330
209	15	16.436	0.382	0.10	0.18	3.45	65	0.19	234	-2.30	1.86	63	0.20	261
210	15	19.674	0.319	0.09	0.34	3.57	42	0.35	203	-2.18	1.70	40	0.36	215
211	15	24.559	0.256	0.09	0.26	3.05	29	0.60	172	-2.20	1.36	31	0.52	170
212	15	32.387	0.194	0.09	0.18	2.77	22	0.92	106	-2.26	1.24	27	0.58	109
213	15	48.627	0.129	0.09	0.16	2.82	16	0.70	47	-2.32	1.26	21	0.40	55
194	20	2.474	2.540	0.19	0.39	0.13	152	0.00	207	-6.04	0.04	216	0.00	358
195	20	4.875	1.289	0.21	0.46	0.53	160	0.00	255	-6.00	0.08	135	0.00	317
197	20	6.403	0.981	0.18	0.44	1.29	130	0.03	353	-5.96	0.46	180	0.02	207
198	20	7.652	0.821	0.18	0.02	1.38	93	0.13	289	-6.02	0.90	153	0.02	334
199	20	9.372	0.670	0.18	-0.40	0.91	78	0.20	211	-6.18	1.06	119	0.06	293
196	20	9.533	0.659	0.20	0.12	0.97	87	0.13	221	-6.02	0.90	99	0.10	285
201	20	10.807	0.581	0.19	-0.32	0.02	340	0.00	355	-6.16	0.02	272	0.00	30
201	20	12.296	0.511	0.24	-0.32	1.09	90	0.17	143	-6.14	0.94	77	0.16	221
216	20	16.372	0.384	-0.85	-0.28	1.74	0	0.27	141	-6.02	0.38	74	0.40	355
200	20	16.392	0.383	0.19	-0.49	1.54	89	0.24	210	-6.10	0.80	60	0.16	135
202	20	22.805	0.276	0.06	-0.81	1.70	84	0.39	168	-6.30	0.50	58	0.12	145
203	20	32.152	0.195	0.18	-1.03	1.64	86	0.42	144	-6.38	0.24	95	0.16	143

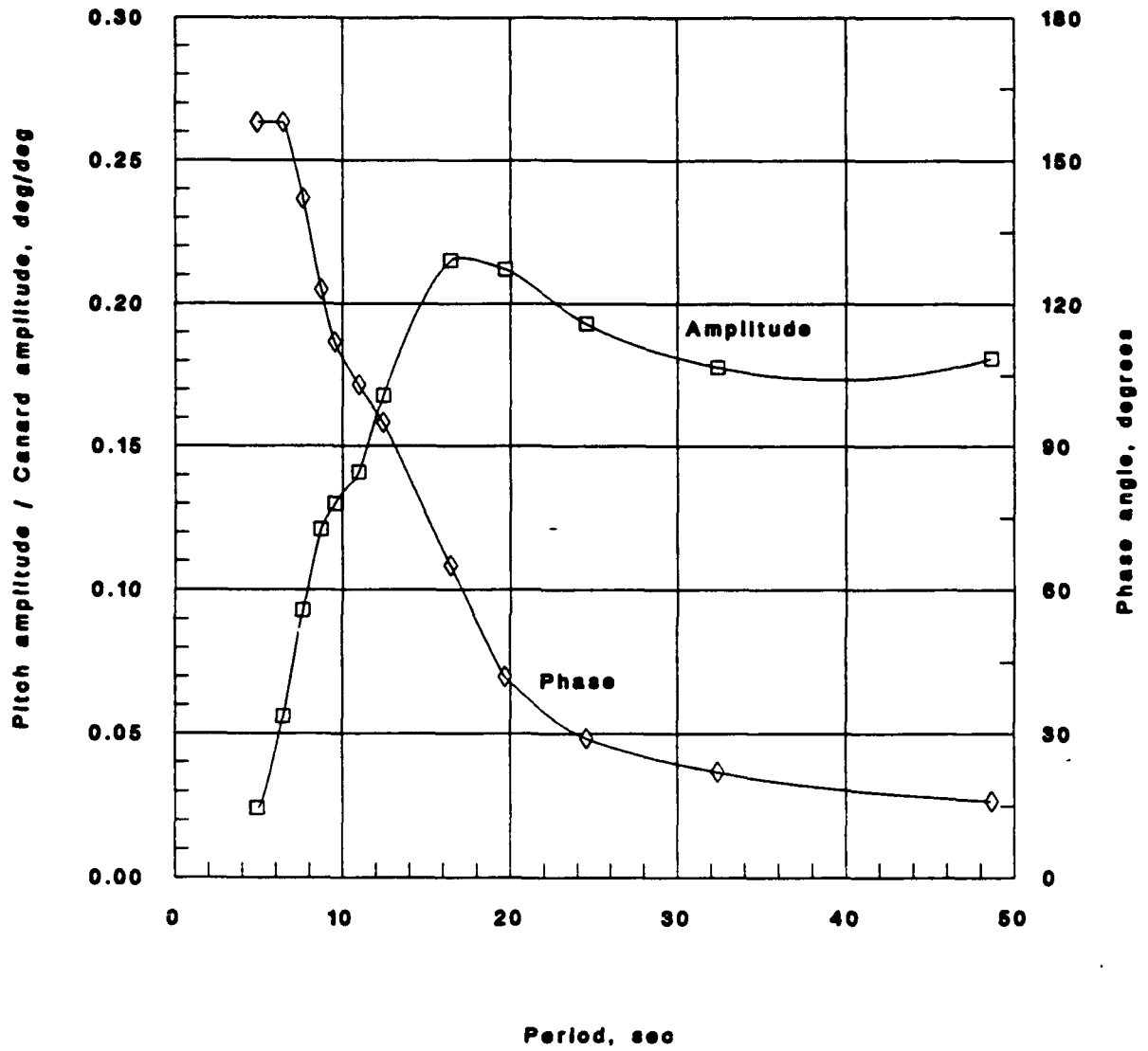


FIGURE C1 PITCH RESPONSE IN FORCED OSCILLATION TEST, V=15 KNOTS

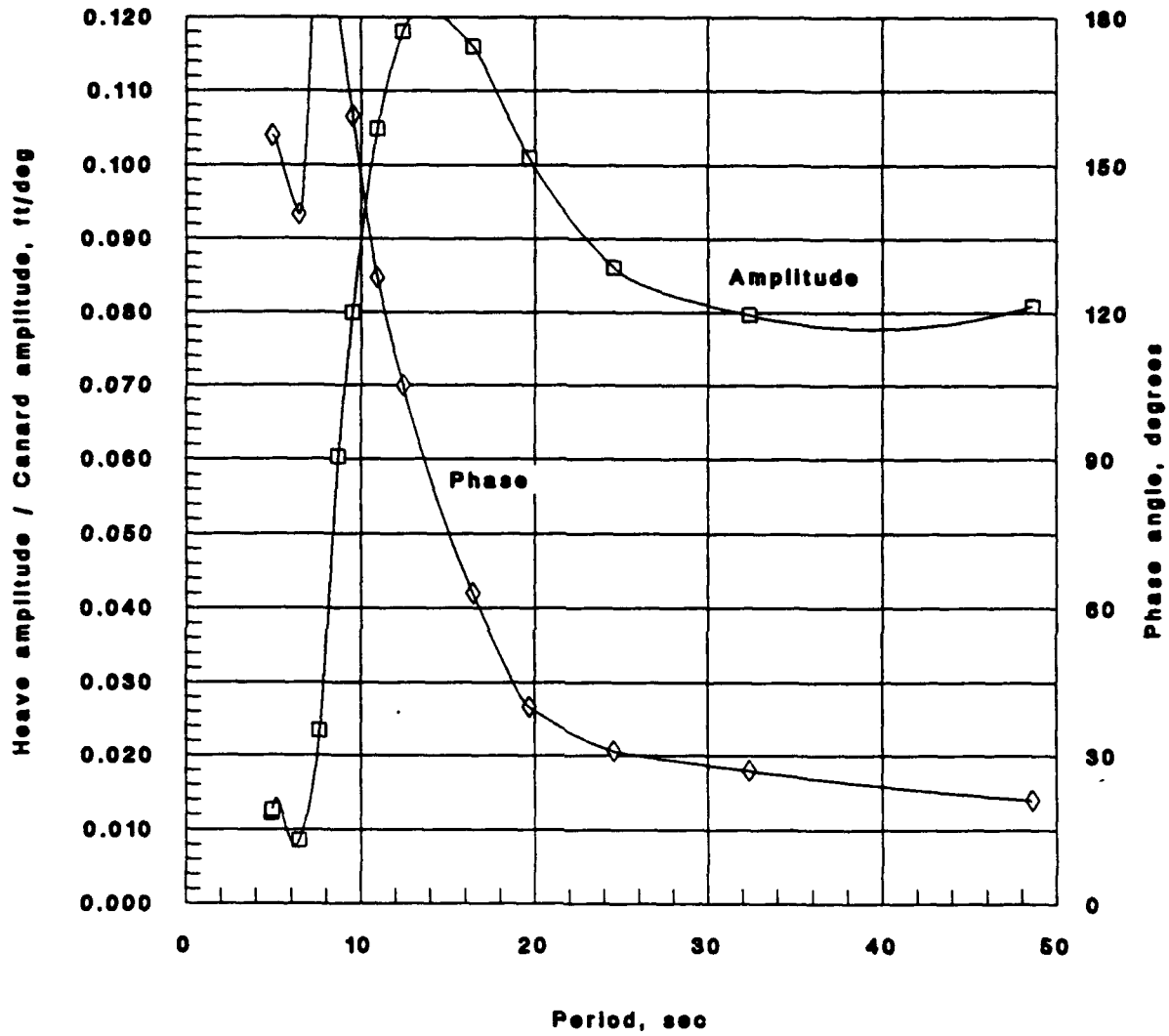


FIGURE C2 HEAVE RESPONSE IN FORCED OSCILLATION TEST, V=15 KNOTS

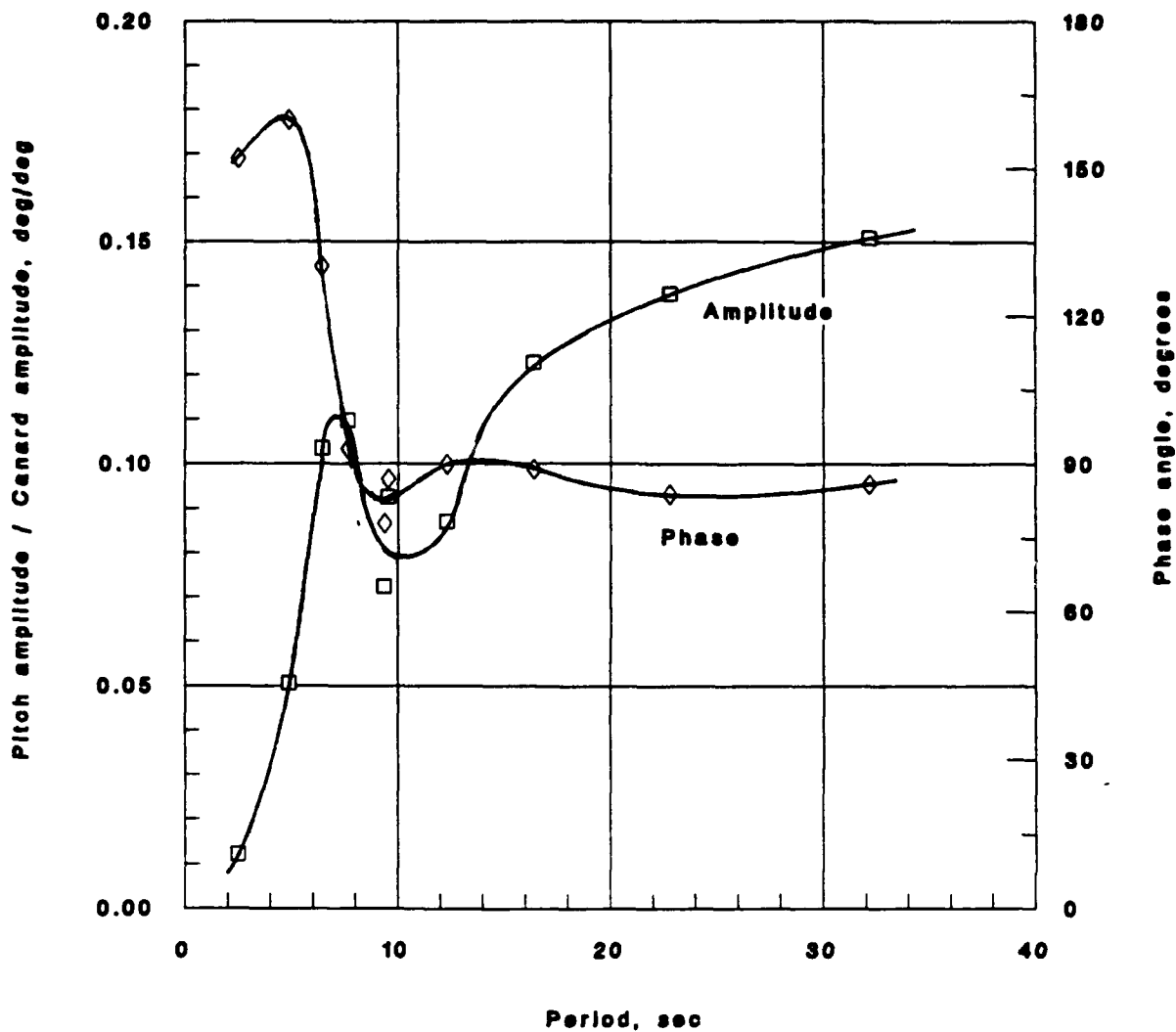


FIGURE C3 PITCH RESPONSE IN FORCED OSCILLATION TEST, V=20 KNOTS

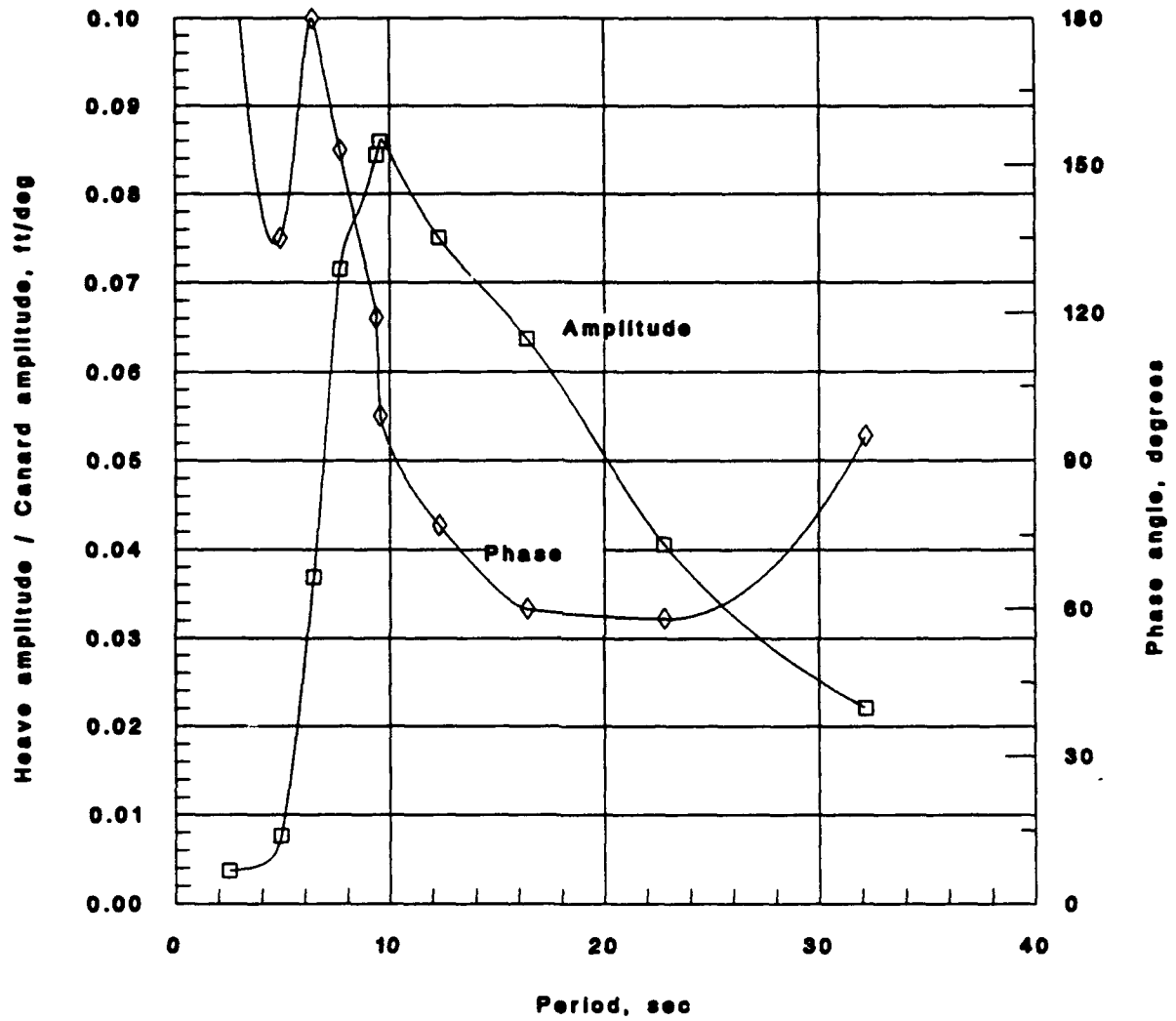


FIGURE C4 HEAVE RESPONSE IN FORCED OSCILLATION TESTS, V=20 KNOTS

APPENDIX D

CALIBRATION OF FIN BALANCE

To measure lift and drag on a canard, a two-component balance was designed which would fit inside of a lower hull of a SWATH model (Figure 4). The balance consisted of a strain-gaged lift spring used in the previous tests (Reference 1) and a specially modified drag balance.

The balance was calibrated on a tankside calibration stand by application of known weights at the approximate center of pressure location of the fins. The apparatus was rotated so that lift, drag, and combinations thereof could be applied. The digitized voltage readings were expressed as linear functions of both lift and drag:

$$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = [C] \begin{Bmatrix} L \\ D \end{Bmatrix}$$

The coefficients in the matrix [C] were determined by means of a multivariate least squares fit. Inversion of this matrix gives the calibration rates R_{ij} :

$$\begin{Bmatrix} L \\ D \end{Bmatrix} = [R] \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} ; \quad [R] = [C]^{-1} ,$$

where off-diagonal elements R_{12} , R_{21} account for cross-coupling.

Results of the calibration are summarized below:

Lift applied	Lift computed	Difference	Drag applied	Drag computed	Difference
0.000	0.003	0.003	0.063	0.064	0.001
0.000	0.002	0.002	0.125	0.126	0.001
0.000	-0.005	-0.005	0.250	0.252	0.002
0.000	-0.003	-0.003	0.500	0.497	-0.003
0.000	0.000	0.000	0.750	0.748	-0.002
0.000	0.001	0.001	1.000	0.998	-0.002
0.000	0.004	0.004	1.250	1.249	-0.001
0.985	0.989	0.004	0.174	0.176	0.002
1.970	1.972	0.002	0.347	0.352	0.005
0.940	0.933	-0.007	0.342	0.348	0.006
-0.940	-0.948	-0.008	0.342	0.347	0.005
0.500	0.507	0.007	0.000	-0.001	-0.001
1.000	1.000	0.000	0.000	-0.003	-0.003
3.000	2.997	-0.003	0.000	-0.001	-0.001
2.000	1.998	-0.002	0.000	-0.003	-0.003

The calibration rates are:

$$\text{Lift} = -0.0070998 V_1 - 0.0000739 V_2$$

$$\text{Drag} = 0.0000141 V_1 - 0.0016039 V_2$$

where V_1 and V_2 are the digitized voltage readings from the lift and drag channels, respectively. The calibrations are plotted on Figure D1.

Dynamic Calibration

Because the balance was to be used to measure unsteady lift and drag in waves, a dynamic calibration was performed in addition to the static calibration described above. This was accomplished by mounting the balance on a scotch yoke which could oscillate sinusoidally in a horizontal plane with adjustable frequency and amplitude. Weights of various denominations were fastened to the balance at the approximate center of pressure location of the fins, and time histories of lift and drag were recorded for several frequencies of oscillation in the range of encounter frequencies expected in the tests. Readings were also taken with no weights fastened to the balance to evaluate the "tare mass" of the balance acting on the springs. Time histories of displacement and acceleration were also measured. A photograph of the apparatus is included as Figure D2.

Time histories of the signals were fit to a harmonic series as described in Data Processing. The amplitude of the dynamic force on the balance is given by

$$F = (W + w) a/g$$

where W is the applied weight, w is the tare weight, and a is the acceleration of the scotch yoke. Results are tabulated below.

Lift Calibration

Added Weight lb	Radian Frequency rps	Accel ft/sec ²	Tare lb	Applied Force lb	Measured Force lb	ω_n rps	Ω
0	11.38	14.35	-		0.05		
0	13.87	21.32	-		0.08		
0	15.46	26.49	-		0.10		
0.56	8.20	7.45	0.03	0.16	0.16	196.2	.042
0.56	11.26	14.05	0.05	0.30	0.30	196.2	.057
0.56	13.78	21.05	0.08	0.45	0.46	196.2	.070
0.56	15.19	25.57	0.09	0.54	0.56	196.2	.077
1.06	5.91	3.87	0.01	0.14	0.15	132.9	.045
1.06	7.84	6.81	0.02	0.25	0.26	132.9	.059
1.06	11.04	13.51	0.05	0.49	0.51	132.9	.083
1.06	13.80	21.11	0.08	0.78	0.80	132.9	.104
1.06	18.12	36.39	0.13	1.33	1.40	132.9	.136
2.07	8.53	8.06	0.03	0.54	0.56	88.4	.097
2.07	11.62	14.97	0.05	1.01	1.06	88.4	.132
2.07	13.90	21.41	0.08	1.46	1.53	88.4	.157
2.07	15.25	25.78	0.09	1.75	1.85	88.4	.173

Drag Calibration

Added Weight lb	Radian Frequency rps	Accel ft/sec ²	Tare lb	Applied Force lb	Measured Force lb
0	7.57	6.24	-		0.16
0	10.31	11.62	-		0.30
0	13.22	19.08	-		0.50
0	14.76	23.80	-		0.62
0	16.31	29.06	-		0.75
0	16.87	31.14	-		0.80
0	18.18	36.32	-		0.94
0	20.40	43.33	-		1.18
0.24	9.19	9.21	0.24	0.31	0.31
0.24	11.39	14.14	0.37	0.47	0.47
0.24	14.09	21.68	0.56	0.72	0.72
0.54	8.29	7.50	0.20	0.32	0.32
0.54	11.37	14.10	0.37	0.60	0.60
0.54	14.47	22.87	0.60	0.98	0.97
1.04	7.68	6.44	0.17	0.38	0.37
1.04	11.65	14.80	0.39	0.86	0.85
1.04	14.35	22.46	0.58	1.31	1.29

The natural frequency, ω_n , of the balance varies with the applied mass. Natural frequencies of the lift spring were determined from oscillograph records taken while "ringing" the balance. The results are included in the table above, along with the ratio $\Omega = \omega/\omega_n$.

The frequency response of the lift balance is shown on Figure D3, where gain (ratio of measured to applied force) is plotted against the frequency ratio. The natural frequency of the balance with the fin as used in the tests was estimated using Figure D4, which is a plot of natural frequency against weight on the springs. Using the known weight of the fin (with shaft) and an estimated added mass, the effective weight on the springs during the tests was found to be about 0.40 lb. According to Figure D4 the corresponding natural frequency is 280 rps, or 44.6 Hz. The maximum encounter frequency expected in the tests was about 2 Hz, so that the maximum frequency ratio would be 0.045. Figure D3 shows that the response of the lift balance is essentially flat in this range.

Results for the drag balance show a flat response for the entire range of weights and frequencies used in the calibration.

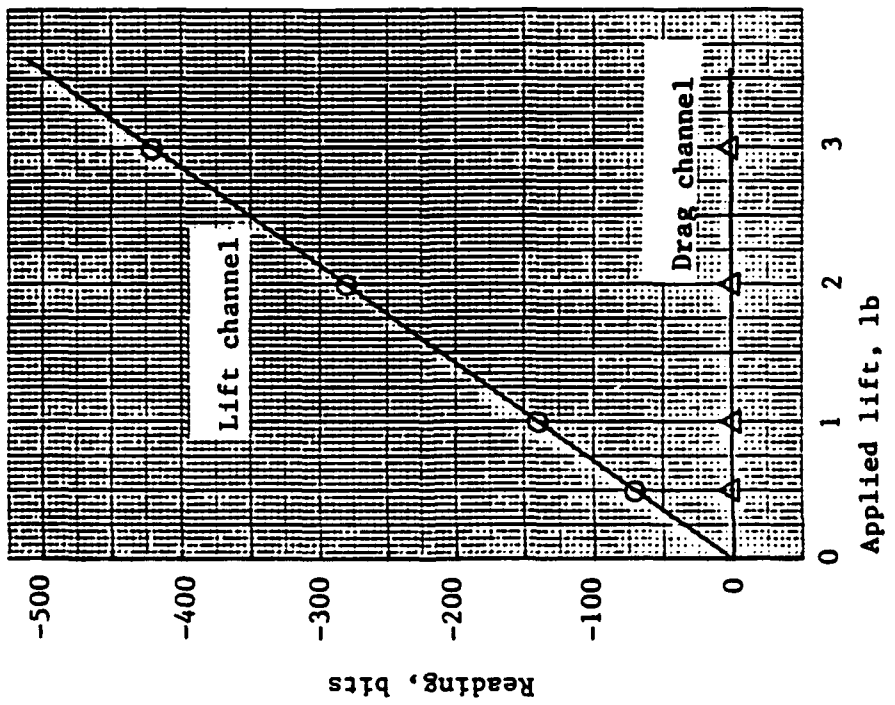
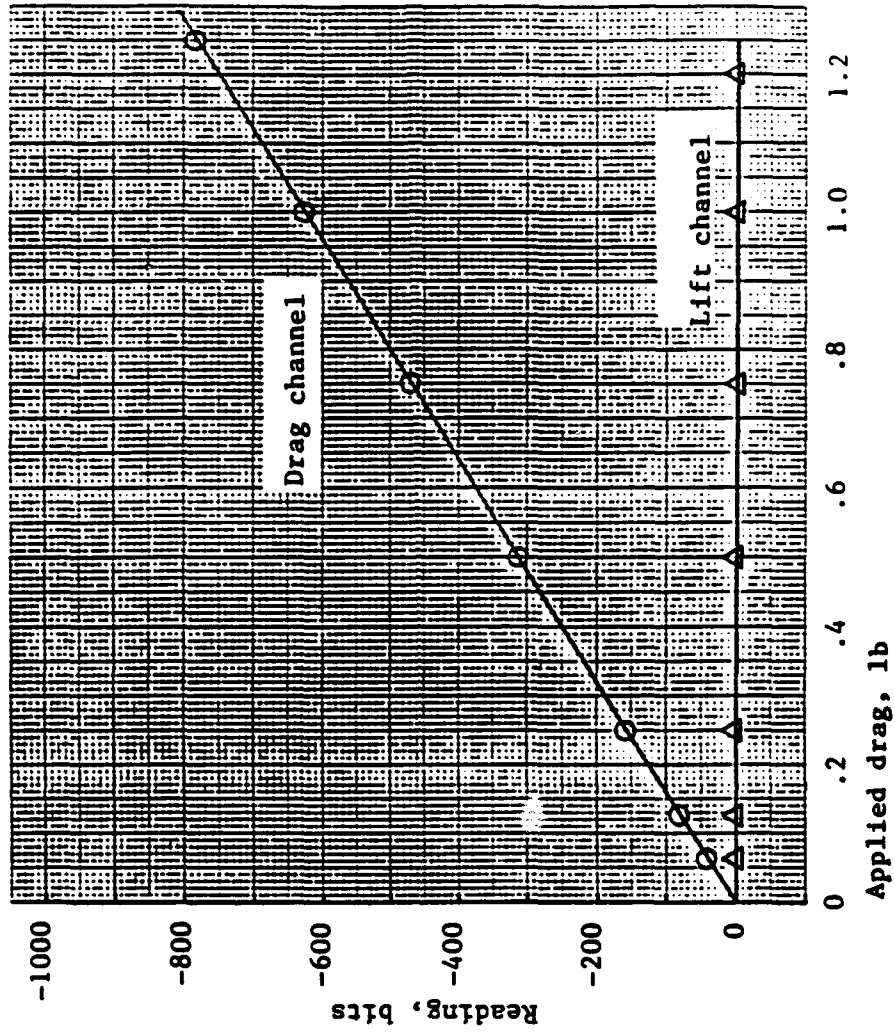


FIGURE D1 CALIBRATION OF FIN BALANCE

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FIGURE D2 SETUP FOR DYNAMIC CALIBRATION OF FIN BALANCE

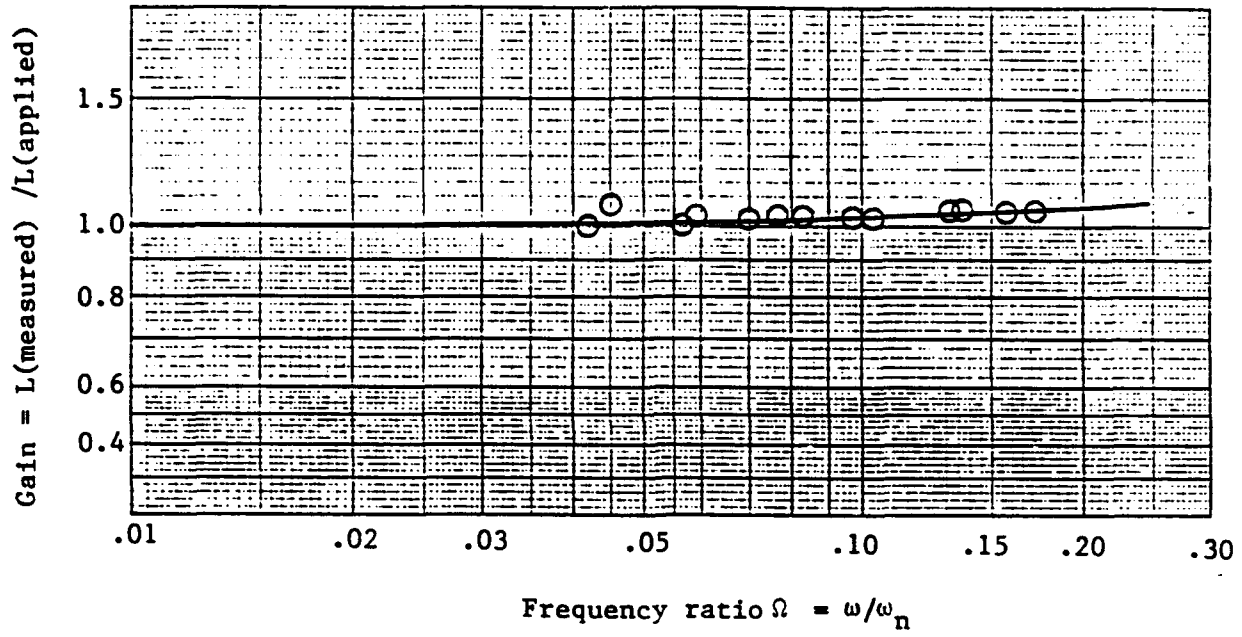


FIGURE D3 FREQUENCY RESPONSE OF LIFT BALANCE

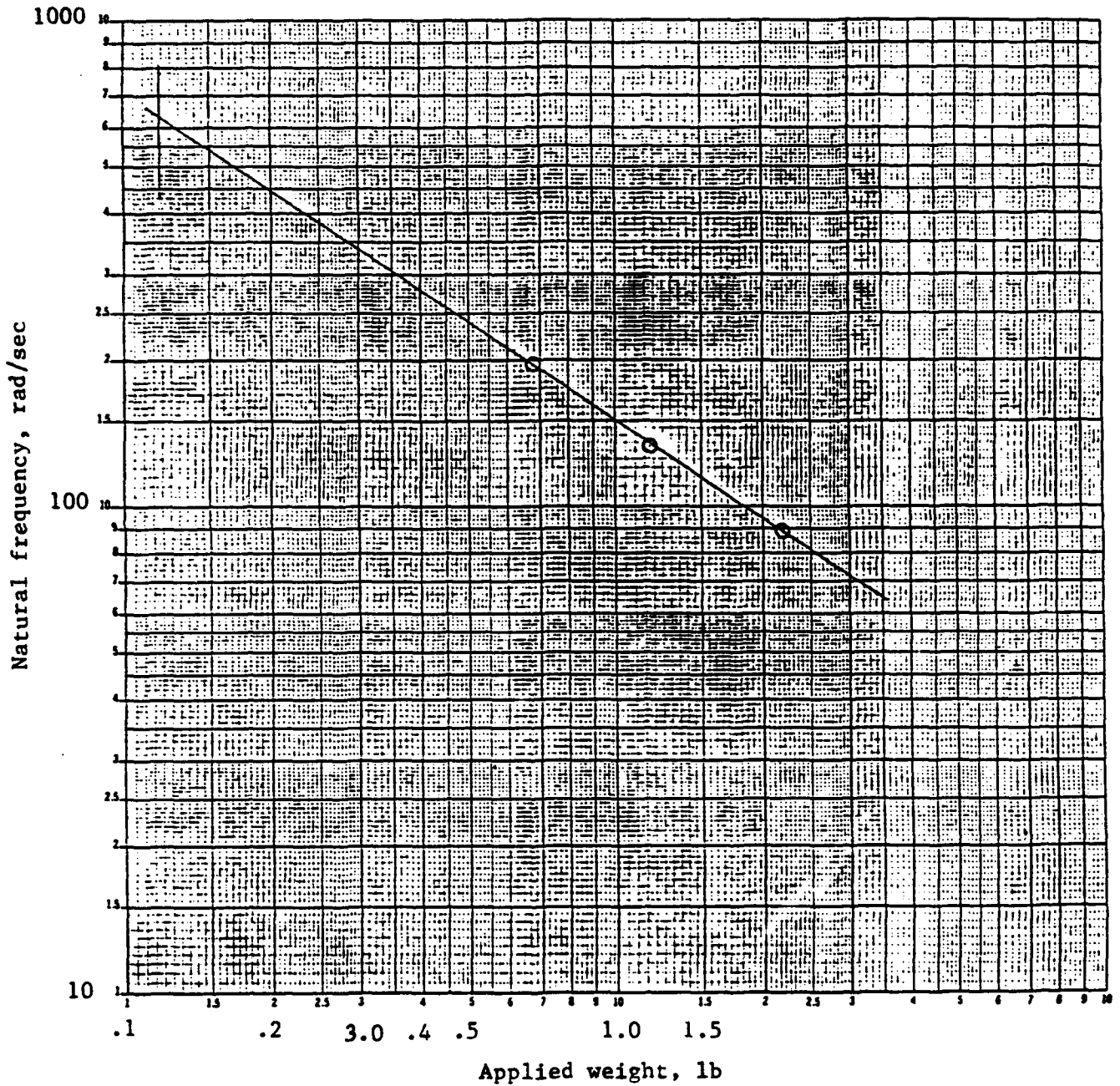


FIGURE D4 NATURAL FREQUENCY OF LIFT BALANCE

APPENDIX E

Tabulation of Water Temperatures

DATE (1987)	RUNS	TEMPERATURE (°F)
Phase 1: Tests of Unappended Model in Calm Water		
1/29	12-36	73.0
1/30	37-95	73.0
2/2	96-103	73.2
Phase 3: Tests with Instrumented Canard		
4/28	10-25	72.3
4/29	34-60	72.3
4/30	65-83	72.3
5/1	88-120	72.1
5/4	127-155	72.3
5/5	163-188	72.5
Phase 4: Tests with Pitch Control System		
8/18	45-71	74.9
8/19	72-88	75.0
8/20	89-113	75.3
8/21	116-145	75.2
8/24	146-177	74.9
8/25	178-206	74.6
8/26	207-220	74.3

APPENDIX F

COMPUTATION OF ANGLE OF ATTACK OF CANARDS INDUCED BY
REGULAR WAVES

The angle of attack of a canard, under the assumption that the canard and model do not disturb the fluid, can be expressed as

$$\alpha = \arctan (v/(u+V))$$

in head seas, where u and v are the wave-induced particle velocities in the horizontal and vertical directions, respectively. If the wave profile is written as

$$\eta = a \cos (kx - \omega t)$$

where x is increasing in the direction of wave advance and k is the wavenumber, it may be shown (see Reference F1) that the particle velocities are

$$u = u_0 \cos \omega t \quad v = -v_0 \sin \omega t$$

at $x=0$ (for example), where

$$\begin{aligned} u_0 &= a\omega \cosh k(y+h) / \sinh kh \\ v_0 &= a\omega \sinh k(y+h) / \sinh kh \end{aligned}$$

and a is the wave amplitude, y is the vertical coordinate of the point of interest (the canard centerline) relative to the calm water surface, and h is the water depth. Thus

$$\begin{aligned} \alpha &= \arctan [-v_0 \sin \omega t / (V + u_0 \cos \omega t)] \\ &= \arctan [-(v_0/V) \sin \omega t / (1 + (u_0/V) \cos \omega t)] \end{aligned}$$

Let

$$\sin \omega t / [1 + (u_0/V) \cos \omega t] = F$$

be represented by a Fourier series

$$F = A_0 + A_1 \cos \omega t + B_1 \sin \omega t + A_2 \cos 2\omega t + B_2 \sin 2\omega t + \dots$$

As shown in Appendix G, the coefficients in the series have the following form:

$$\begin{aligned} A_i &= 0 \\ B_i &= -2(\sqrt{1 - \beta^2} - 1)^i / \beta^{i+1} \end{aligned}$$

where $\beta = u_0/V$. For small β , the coefficients have the following asymptotic form (see Appendix G):

$$B_i = (-\beta/2)^{i-1}$$

so that

$$B_1 = 1$$

$$B_2 = -\beta/2$$

$$B_3 = \beta^2/4$$

Thus if the particle velocity is sufficiently small relative to the ship speed to justify neglecting B_2 and further terms, one obtains

$$\alpha = \arctan [-B_1 (v_0/V) \sin \omega t]$$

or

$$\tan \alpha = (B_1 v_0/V) \cos(\omega t + \pi/2)$$

so that the angle (and thus the lift force in a "quasi-steady" theory) leads the wave crest by 90° (or, equivalently, lags the wave crest by 270°) which is in agreement with the test results. In addition, in the tests the second and higher harmonics of the measured fin lift were insignificant, which would justify neglecting B_2 and higher terms in the Fourier series in the present case.

In following seas the ship has velocity $-V$ in the direction of wave advance. Thus $-V$ is to be substituted for V in the formulas above for following seas. The only effect of this change is to reverse the sign of the angle of attack. Thus in following seas if the ship is overtaking the waves, the angle of attack lags the wave crest by 90° rather than by 270° . If the waves are overtaking the ship, a negative phase angle is interpreted as a phase lead; so in overtaking waves the fin angle of attack leads the wave crest by 90° (or lags the wave crest by 270°) as in head seas.

The canard lift coefficient is predicted by multiplying the angle of attack from the expressions above by the steady-flow lift curve slope of the fin (including the "groundboard effect" of the hull as discussed in the main text). This is referred to as a "quasi-steady" theory, because effects of unsteadiness on the angle of attack ("memory effects") are neglected.

Figures 10 and 11 show that this theory consistently underpredicts the amplitude of the canard lift coefficient by about 15% at a ship speed of 20 knots; the ratio of particle velocity to ship speed was between .05 and .15 in these tests. Predicted phase angles are in accord with the observations, indicating that there is no appreciable time lag in the development of lift in the range of observed conditions.

REFERENCE

- F1. Newman, J.N., *Marine Hydrodynamics*, MIT Press, 1977.

APPENDIX G

EVALUATION OF AN EXPRESSION WHICH OCCURS IN THE ANGLE OF ATTACK
COMPUTATION

1 Problem Statement

Consider the periodic function

$$F(t) = \sin \omega t / [1 + \beta \cos \omega t] \quad (1)$$

and find its Fourier series of the form

$$F(t) = A_0 + A_1 \cos \omega t + B_1 \sin \omega t + A_2 \cos 2\omega t + B_2 \sin 2\omega t + \dots \quad (2)$$

2 Derivation

The coefficients of the Fourier expansion of $F(t)$ are given by [1]

$$A_0 = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} F(t) dt \quad (3)$$

$$A_i = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} F(t) \cos(i\omega t) dt \quad (4)$$

$$B_i = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} F(t) \sin(i\omega t) dt \quad (5)$$

for $i = 1, \dots, \infty$

Now, by definition of the function $F(t)$, it follows immediately that

$$F(-t) = -F(t) \quad (6)$$

and thus that $F(t)$ is an odd function. Since the integration interval is symmetric, it follows immediately that all the integrals involving the term $\cos(i\omega t)$ must vanish because $\cos(i\omega t)$ is an even function, and therefore

$$A_i = 0, \quad (7)$$

for $i = 0, 1, 2, \dots, \infty$.

To find a form for the remaining Fourier coefficients, use the product formula for sines [2]

$$2 \sin(z_1) \sin(z_2) = \cos(z_1 - z_2) - \cos(z_1 + z_2) \quad (8)$$

to rewrite the expression for B_i into the equivalent form

$$B_i = C_{i-1} - C_{i+1} \quad (9)$$

where the quantity C_i is defined by the integral

$$C_i = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} \cos(i\omega t)/(1 + \beta \cos(\omega t)) dt \quad (10)$$

Using the fact that the integrand is even and the integration interval is symmetric, then it can be rewritten as

$$C_i = \frac{\omega}{\pi} \int_0^{\pi/\omega} \cos(i\omega t)/(1 + \beta \cos(\omega t)) dt \quad (11)$$

and then a change of variables to $x = \omega t$ yields

$$C_i = \frac{1}{\pi} \int_0^{\pi} \cos(ix)/(1 + \beta \cos(x)) dx \quad (12)$$

which is an integral in standard form and can be looked up in an integral table.

From [3] 3.613.1, provided that $\beta^2 < 1$, then

$$C_i = \frac{1}{\sqrt{1-\beta^2}} ((\sqrt{1-\beta^2} - 1)/\beta)^i \quad (13)$$

for $i = 0, 1, \dots, \infty$. As examples, note that

$$\begin{aligned} C_0 &= 1/\sqrt{1-\beta^2} \\ C_1 &= (\sqrt{1-\beta^2} - 1)/(\beta\sqrt{1-\beta^2}) \\ C_2 &= (2 - \beta^2 - 2\sqrt{1-\beta^2})/(\beta^2\sqrt{1-\beta^2}) \\ C_3 &= (-4 + 3\beta^2 + (4 - \beta^2)\sqrt{1-\beta^2})/(\beta^3\sqrt{1-\beta^2}) \\ C_4 &= (7 - 8\beta^2 + \beta^4 - 4(2 - \beta^2)\sqrt{1-\beta^2})/(\beta^4\sqrt{1-\beta^2}) \end{aligned} \quad (14)$$

Using these results in the expression above for B_i yields explicit values for the non-zero coefficients in the Fourier expansion, namely

$$B_i = C_{i-1} - C_{i+1}$$

$$\begin{aligned}
&= [(\sqrt{1-\beta^2}-1)/\beta]^{i-1}/\sqrt{1-\beta^2} - [(\sqrt{1-\beta^2}-1)/\beta]^{i+1}/\sqrt{1-\beta^2} \\
&= [(\sqrt{1-\beta^2}-1)/\beta]^{i-1}/\sqrt{1-\beta^2} \\
&\quad \times [\beta^2 - (1-\beta^2 + 1 - 2\sqrt{1-\beta^2})]/\beta^2 \\
&= -2(\sqrt{1-\beta^2}-1)^i/\beta^{i+1} \tag{15}
\end{aligned}$$

This finishes the calculation of the Fourier expansion.

As a final issue, consider the behavior of the coefficients as β goes to zero, then

$$\begin{aligned}
B_i &= -2(\sqrt{1-\beta^2}-1)^i/\beta^{i+1} \\
&\approx -2(-\beta^2/2)^i/\beta^{i+1} \\
&\approx (-\beta/2)^{i-1} \tag{16}
\end{aligned}$$

which gives the asymptotic behavior of the coefficients in the physically interesting case when β becomes small.

References

- [1] M. R. Spiegel *Theory and Problems of Advanced Calculus*, Schaum's Outline Series, McGraw-Hill Book Company, New York, 1963, Chapter 14.
- [2] M. Abramowitz and I. A. Stegun *Handbook of Mathematical Functions*, Dover Publications, Inc., New York, 1972 4.3.31
- [3] I. S. Gradshteyn and I. M. Ryzhik *Table of Integrals, Series, and Products* Academic Press, New York, 1980