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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The workshop on "Dynamical Systems and Oceanography" was held in Little Compton, RI on May 20-21,1993. The participants were from both the dyanmical systems and oceanography communities and were brought together with the goal of "brainstorming" on the future directions that the application of dynamical systems techniques to oceanography might take. The result of the conference was awhite paper that is appended to this report.

The workshop was organized into two separate days. On the first day, discussions revolved around the use of geometric techniques from dynamical systems in studying the mixing properties of the ocean. Main lectures were given by Wiggins (dynamical systems) and Samelson (oceanography). Fruitful discussions were held involving all participants on this topic. Short lectures were also givan by Kirwan, Brown, and Richardson. On the second day, focus was put on ths use of dynamical systems in understanding instabilities. A lecture was given by Pratt(oceanography) on the current problems of interest and also one by Jones (dynamical systems) on the available modern techniques.

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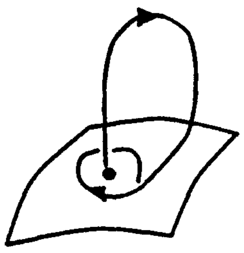

DYNAMICAL SYSTEMS AND OCEANOGRAPHY

A workshop on problems of oceanography that can be fruitfully addressed with the modern techniques of dynamical systems will be held in Little Compton, RI on May 20-21, 1993. The workshop will be funded by ONR. The format will involve a small number of main presentations supplemented by working discussion groups. The goal of the workshop is to explore the possibility of an Accelerated Research Initiative in this area to be sponsored by the Office of Naval Research.

The workshop will be held at the Stone House Club Inn in Little Compton, RI. Accommodation for 5/20 and meals will be provided at the Inn. The Inn is located in southern Rhode Island only a short walk from the ocean.

Participation is by invitation of the organizers. Any inquires should be addressed to: Prof. Christopher Jones, Division of Applied Mathematics, Brown University, Providence, RI 02912. Email: ckrtj@cfm.brown.edu. Telephone # (401) 863-3696.

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Dynamical Systems and Oceanography

Thursday, May 20

10:30	Welcome and Introduction: C. Jones and R. Malek-Madani.
11:00	Presentation: S. Wiggins, "Dynamical Systems Theory and Lagrangian Transport Problems in Fluid Mechanics".
12:15	Lunch
1:30	Presentation: R. Samelson, "Lagrangian Observations of Ocean Circulation".
2:30	Working groups.
4:30	Presentation: D. Kirwan, "Report Card '93 for Dynamical Systems in Oceanography 101".
5:00	Presentation: M. Brown, "Spatio-Temporal Chaos in the Ocean-Atmosphere System".
6:00	Break (Cash bar)
6:30	Dinner
8:00	Working groups

Friday, May 21

8:30	Working groups
9:30	Presentation: L. Pratt, "Stability and Instability in Geophysical Fluids".
10:00	Presentation: C. Jones, "Mathematical Techniques for Detecting Instabilities".
10:30	Break
11:00	Working groups
12:15	Lunch
1:30	Working groups

REIMBURSEMENT INFORMATION

In order to get reimbursed speedily, please fill out the following information and follow the instructions. Please fill out one and return to me by Thursday evening. Take the other with you and fill out any future expenses and return to me *with* originals.

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Prof. Christopher Jones
Division of Applied Mathematics
Brown University
182 George Street
Providence, RI 02912

Include original receipts as requested.

The workshop on Dynamical Systems and Oceanography was held in Little Compton, RI on May 20-21, 1993. The participants were from both the dynamical systems and the oceanography communities and were brought together with the goal of "brainstorming" on the future directions that the application of dynamical systems techniques to oceanography might take. The result of the conference was a white paper that is appended to this report.

The workshop was organized into two separate days. On the first day, discussions revolved around the use of geometric techniques from dynamical systems in studying the mixing properties of the ocean. Main lectures were given by Wiggins (dynamical systems) and Samelson (oceanography). Fruitful discussions were held involving all the particpants on this topic. Short lectures were also given by Kirwan, Brown and Richardson. On the second day, focus was put on the use of dynamical systems in understanding instabilities. A lecture was given by Pratt (oceanography) on the current problems of interest and also one by Jones (dynamical systems) on the available modern techniques.

The white paper, appended, was written by Haller, Jones and Wiggins based on the outcome of these discussions.

Participants: Brown, Haller, Herron, Jones, Kirwan, Pedlosky, Pratt, Richardson,Samelson, Wiggins and students from Brown U. and Woods Hole. Malek-Madani was present represnting the Office of Naval Research.

1 Lagrangian Ocean Dynamics

Fluid mechanics from the Lagrangian perspective has played a central role in our understanding of the dynamics of the oceans. An important tool for understanding the ocean current structure and dynamics is quasi-Lagrangian current-following floats and drifters, see Davis [1991] for a recent review. Float observations have revealed many features of the current patterns in the oceans; as examples we give the following.

1. They have demonstrated the dominance of mesoscale activity in the general circulation pattern.
2. They have shown patterns of vertical motion at different phases of meanders in the Gulf Stream.
3. They have provided the data that show how meanders amplify into loops and subsequently form rings in the Gulf Stream.

These observations have been important for the construction of a variety of models that, to varying degrees, have been able to explain such phenomena. Nevertheless, oceanography is a very young science and, to quote Davis [1991], "new observational tools often lead directly to discovery or even to revamping of concepts". Thus it should be very important to develop analytical tools which are adapted to the Lagrangian approach and which can be used in both the analysis of Lagrangian data and in the development of theoretical models that can be used to study transport, mixing, and stirring properties of ocean currents.

Over the past several years there have been developments in the framework of the modern theory of nonlinear dynamical systems that provide a new viewpoint and techniques for studying Lagrangian issues in fluid mechanics in general and oceanography in particular. We give some brief background. Let us consider the special situation of a two-dimensional, incompressible time-periodic fluid flow. The equations for fluid particle paths are given by

$$\begin{aligned} \dot{x} &= \frac{\partial \psi}{\partial y}(x,y,t), \\ \dot{y} &= -\frac{\partial \psi}{\partial x}(x,y,t), \end{aligned}$$

where $\psi(x, y, t)$ is the stream function. From the dynamical systems viewpoint, these are Hamilton's equations where $\psi(x, y, t)$ is the Hamiltonian function and the phase space of this dynamical system is actually the physical space where the fluid flows. Through time periodicity the study of these equations can be reduced to the study of a two-dimensional symplectic Poincaré map and once the problem has been cast in this setting a variety of techniques and ideas from dynamical systems theory can be applied for the purpose of studying fluid transport and mixing issues. For example, KAM tori represent barriers to fluid transport and mixing, chaotic dynamics should act to enhance mixing, and invariant manifolds, such as the stable and unstable manifolds of hyperbolic periodic points, are manifested as "organized structures" in the fluid flow. See Ottino [1989] and volume 3 (1991), number 5 of the Physics of Fluids A for recent reviews. The mathematical framework of dynamical systems theory is remarkably similar to the experimental framework of modern flow visualization (from, e.g. quasi-Lagrangian current-following floats and drifters,) in that each is concerned with the motion in time over large regions of space and the role that lower dimensional organized "structures" in the space play in governing the motion. Consequently, this type of mathematical approach should prove ideal for analyzing transport and mixing processes associated with the large scale, organized motions observed in geophysical flows.

In the following we describe several important areas of oceanography where dynamical systems theory can play an important role in Lagrangian trajectory studies.

1.1 Lagrangian studies of weakly 3-dimensional flows

A primitive model of ocean circulation, dating back to Stommel, involves two cells, corresponding, say, to the sub-polar and sub-tropical gyres. Of fundamental interest in these studies is the nature of the cross-gyre transport of fluid. Recently Spall [19XX] has studied this problem using a steady model of the joint sub-tropical, sub-polar oceanic circulation. The issue of cross-gyre transport is important because fluid that breaks free from the sub-tropical gyre and enters the sub-polar gyre is subsequently involved in an air-sea interaction that leads to the climatically important process of deep water formation. Spall's work is highly suggestive of some type of "separatrix" that controls the cross-gyre transport. Moreover, his work also clearly shows that the flow is "weakly" three-dimensional. Time-dependent versions of this problem can also be studied using the model of Cox and

Bryan [1984].

There has been some recent work in dynamical sytems theory that provides some techniques to treat this problem. Mezić and Wiggins [1993] have studied three-dimensional *volume preserving* vector fields that are invariant under the action of a one-parameter symmetry group whose infinitesimal generator is autonomous and volume preserving. They showed that there exists a coordinate system in which the vector field assumes a simple form. In particular, the evolution of two of the coordinates is governed by a time-dependent, one-degree-of-freedom Hamiltonian system with the evolution of the remaining coordinate being governed by a first order differential equation that depends only on the other two coordinates and time. The new coordinates depend only on the symmetry group of the vector field. Therefore they are *field independent*. The coordinate transformation is constructive. If the vector field is time independent, then it posseses an integral of motion. Moreover, they showed that the system can be further reduced to *action-angle-angle* coordinates. These are analogous to the familiar action-angle variables from Hamiltonian mechanics and should be quite useful for perturbative studies of the class of sytems we consider. In fact, they have shown that their coordinate transformation allows one to apply recent extensions of the KAM theorem for three-dimensional, volume-preserving maps as well as three-dimensional versions of Melnikov's method. These methods provide beginning tools for understanding the nature of float trajectories in this problem and in particular the relationship of the cross-gyre flow and the initial position of the floats.

1.2 Particle dynamics in localized structures and disturbances

Often oceanographic phenomena can be modelled by various localized structures, such as coherent distributions of vorticity, or disturbances such as internal or surface waves. Below we list a variety of problems in this setting where a dynamical systems approach to Lagrangian trajectory analysis would be insightful.

- Finite amplitude saturation of baroclinic and barotropic instability often leads to a statistically steady state dominated by coherent vortices. One example is the triangular jet, which breaks up into a vortex street (Bell and Pratt [1992]). By modelling these eddies as point vortices one may sometimes arrive at a qualitative understanding of the saturated state. The point vortex model leads very naturally to a dynamical system. Possible applications of this model include the analysis of the finite amplitude meandering states in ocean jets such as the Gulf Stream.

- Lagrangian trajectory studies in wave disturbances such gravity waves, capillary waves etc. Some work in this area has recently been done by Feng and Wiggins [1993]. They studied particle motions in nearly square containers due to gravity and capillary waves generated by vertical, periodic oscillation of the container. The method of second order partial averaging was used to decompose the particle motions into periodic oscillations and a slow Stokes drift. This provided a mathemtically rigorous way of dealing with the realtionship between spatial and temporal variation of the velocity field which had not previously been recognized. In the case of gravity waves, it was shown that long distance (several wave length) particle transport is possible. In the case of capillary waves, it was shown that, in agreement with experimental observations of Ramshankar *et al.* [1990], particle trajectories can be chaotic even when the wave pattern is regular so long as the pattern is spatially modulated.

- One example of a localized vorticity distribution often occurs when an open ocean warm core ring interacts with topography such as the continental slope. This interaction can produce daughter cold core eddies which interact with the parent ring. This interaction often results in a steady rotation of the complete vortex system. Such a system may be responsible for much of the cross-shelf transport of material in the mid-Atlantic Bight or Texas-Mexico shelf.

  Mied et al. [1992] have developed a modon model to describe this system. This is a quasi-geostrophic model in which the potential vorticity is piecewise continuous. Although there is an analytic solution the flow pattern resulting from this model is very complicated. Generally there are regions where fluid parcels are trapped or transported across the structure as well as apparent chaotic regions. This system has not yet been analyzed by dynamical systems methods. Such an analysis would be very useful as many characteristics of the flow patterns are seen in satellite images.

- Highly idealized flow models which produce complicated and chaotic Lagrangian particle trajectories seem to tell us that Lagrangian chaos and mixing are spatially localized. A crucial problem is understanding the nature of this localization in terms of properties of the potential vorticity field. For example, it would be important to know if chaos is largely confined to regions where the potential vorticity has already been homogenized.