AD-A281 416

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

(1)

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	June 1994	Final 1 Jan 92-31 May 93

4. TITLE AND SUBTITLE
Workshop on Adaptive Methods for Partial Differential Equations

5. FUNDING NUMBERS
DAAL03-92-G-0009

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARO 29079.1-MA-CF

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

During this workshop, approximately 100 participants (whose names are appended) from industry, academia, Department of Defense, and other national laboratories heard talks on the theory and practice of adaptive approaches in several mathematical areas and physical disciplines. Also, for the first time in this series of workshops, a full day tutorial was held on May 17, covering some of the more germane issues in adaptivity. This tutorial, conducted by J. Tinsley Oden and two of the meeting co-organizers, Joseph E. Flaherty and Mark Shephard, discussed topics ranging from the underlying principles of a priori error estimation, to adaptive methods for transient problems, to computational geometric approaches for automatic three-dimensional finite element mesh generation.

14. SUBJECT TERMS
Workshop, Adaptive Methods, Partial Differential Equations, Adaptivity, Priori Error Estimation

15. NUMBER OF PAGES
7

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

FINAL REPORT

U.S. Army Research Office Contract DAAL03-92-G-0009

Period: 1 January 1992 - 30 June 1994

Title of Research: Workshop on Adaptive Methods for Partial Differential Equations

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Interest and progress on the development of reliable, robust, and efficient software for the automatic numerical solution of partial differential equations continues to grow. The U.S. Army Research Office (ARO) sponsored the initial workshop in this area at the University of Maryland in 1983. A second ARO-sponsored workshop was held at the Rensselaer Polytechnic Institute in 1988. Funding of this project supported the *Third ARO Workshop on Adaptive Methods for Partial Differential Equations* which was also held at Rensselaer, 18-22 May, 1992. During this workshop, approximately 100 participants (whose names are appended) from industry, academia, Department of Defense, and other national laboratories heard talks on the theory and practice of adaptive approaches in several mathematical areas and physical disciplines. Also, for the first time in this series of workshops, a full day tutorial was held on May 17, covering some of the more germane issues in adaptivity. This tutorial, conducted by J. Tinsley Oden and two of the meeting co-organizers, Joseph E. Flaherty and Mark Shephard, discussed topics ranging from the underlying principles of a priori error estimation, to adaptive methods for transient problems, to computational geometric approaches for automatic three-dimensional finite element mesh generation.

Written proceedings of the invited and some contributed lectures at the workshop were published as a special issue (Volume 14, Numbers 1-3, April 1994) of *Applied Numerical Mathematics* which was edited by Kenneth Clark of ARO and Flaherty and Shephard of Rensselaer. The 18 papers in this volume spanned 365 pages and covered topics involving h-, p-, and r-refinement strategies for transient and steady problems; hierarchical solution and modeling techniques; a posteriori error estimation; parallel solution techniques; mesh generation; and applications to problems in elasticity, fluid mechanics, and biology.

Hierarchical strategies were the dominant theme at the workshop and in these proceedings. The papers by Fish et al. and McCormick and Rüde describe hierarchical h-refinement strategies where solutions on finer meshes are regarded as corrections to those on coarser ones. With a composite-grid formulation, McCormick and Rüde utilize multigrid solution techniques to enhance solution convergence. Biswas et al. describe a spatially discontinuous hierarchical hp-refinement strategy for hyperbolic systems of conservation laws. Oden et al. describes a method for obtaining a posteriori error estimates of adaptive hp-refinement processes that may be useful on a broad spectrum of problems.

Turning to a relatively new direction, papers by Babuska et al., Shephard and Wentorf, and Noor et al. relate hierarchical solution techniques to the assumptions used in formulating the mathematical model. They discusses the importance of specifying computational accuracy in relation to the idealizations of the mathematical model. Error estimates include both discretization errors, as usual in adaptive computation, and modeling errors, which arise when a more exact formulation is replaced by a simpler one. Typical situations involve the relationship of a plate or shell model to, a more exact, three-dimensional elastic formulation or a homogenized model of the behavior of a composite media. Continuing in this vein, Shephard and Wentorf describe the structure of a framework for automating such modeling decisions. We believe that these innovations will become more widespread in the future.

Adaptive solution techniques for transient systems continues to grow. Local refinement strategies where space and time are locally enriched are represented by the papers of Berger and Saltzman and Ewing and Lazarov. Techniques for steady and unsteady fluid flows are described in papers by Berger and Saltzman; Biswas et al.; Grove; Lottati and Eidleman; Powell; and Ramakrishnan. Grove utilizes sophisticated front-tracking methods to avoid spurious effects near solution irregularities, while most of the other authors use artificial dissipation and solution limiting.

The papers of Berger and Saltzman and Biswas et al. discuss parallel adaptive procedures, which we view as another aspect of the field that will become more prevalent in future symposia. The goal and the challenge here are to develop strategies that simultaneously minimize both the computational cost and the redistribution cost that is incurred during adaptive enrichment.

Several problems in mechanics have been mentioned; however, Johnson and MacLeod describes a new application of adaptive methods to a problem in medical imaging. Enhanced derivative recovery through least squares techniques is the subject of Belytschko and Blacker's paper while Dougherty and Hyman and Simpson describe mesh-generation strategies. Finally, Kozlovsky describes a programming environment for developing adaptive solution strategies.

It may be interesting to trace the growth of adaptive methods over the ten-year period of the U. S. Army-sponsored workshops. None of the papers at the 1983 workshop involved three-dimensional computations whereas at least four contributions in these proceedings (those by Berger and Saltzman, Ewing and Lazarov, Johnson and MacLeod, Oden et al., and Shephard and Wentorf) involve difficult three-dimensional problems. At the time of the first workshop, the state of the art of adaptive techniques for steady problems was further advanced then it was for transient problems. The papers in this volume would suggest that research on transient problems has closed the gap. Many of the papers dealing with transient phenomena now address two- and three-dimensional problems while those in the proceedings of the first workshop concentrated on one-dimensional problems. As yet, however, no research on hierarchical techniques in both space and time is represented.

While parallel solution techniques have grown with the availability of hardware at, e.g., national computer centers, their use with adaptive techniques continues to be limited. The challenges are substantial, since adaptivity and parallelism are at odds. The most successful parallel solution strategies have employed simple algorithms and uniform structures while the most successful adaptive techniques utilize complex logic, sophisticated solution strategies involving mesh and order variation, and nonuniform structures. Nevertheless, these difficulties must be overcome if adaptive methods are to be used to address the most difficult three-dimensional transient and steady problems that arise in modern science and engineering.

Some shortcomings cited in the proceedings of the first two workshops continue to be apparent. Suitable benchmark calculations illustrating the effectiveness of an approach with respect to more or less clearly formulated aims and performance measures have yet to be defined. Notions of adaptivity are common in fields such as biology, optimal control, and artificial intelligence. Our aim was to present related ideas of adaptivity used in some of these fields at the workshop and to stimulate a discussion with comparisons and synergism. Most adaptive techniques are still being applied to problems in mechanics. We would hope to see more varied usage and, in this respect, find Johnson and MacLeod's application to a problem in medical imaging refreshing. We will endeavor to have applications in other disciplines represented at future workshops. Once again, the synergy provided by individuals conducting similar activities in different fields can only be beneficial.

The workshop and published proceedings represented, in our opinion, a realistic picture of today's state of the art. The area of adaptive computational methods for partial differential equations is highly promising and offers many challenging research problems. The field is still young but is having a profound impact on computational strategies in several disciplines.

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