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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Research is reported on the dynamic behavior of viscoelastic structures, with emphasis on the interaction between passive damping and active feedback control mechanisms. A main focus is the approximation of optimal compensators for simple mechanical systems involving viscoelastic elements by realizable compensators. In the frequency domain, this corresponds to the mathematical problem of approximating a transcendental function by a rational one, under suitable stability constraints and performance criteria. The work involves the analytic study of partial product approximations to transfer functions for rods and beams, as well as a mainly numerical study concerned with replacing the complex modulus for the viscoelastic material by a rational function that reflects the most significant properties of the material. A second focus of the research is a new formula that makes clear the relation between initial data and the smoothness and decay rates of solutions of the equations for a viscoelastic system with stabilizing boundary feedback.

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# FINAL TECHNICAL REPORT



on

# ENERGY DECAY AND CONTROL FOR ELASTIC AND VISCOELASTIC DISTRIBUTED PARAMETER SYSTEMS

**AFOSR GRANT AFOSR-91-0083**

for the period

15 November 1990 - 14 April 1994

by

K. B. Hannsgen and R. L. Wheeler

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Blacksburg, Virginia 24061-0123



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# I. SUMMARY

The subject grant supported research on dynamic behavior of viscoelastic structures, with emphasis on the interaction between passive viscoelastic damping and active feedback control. An approximation procedure was developed in which theoretically optimal control mechanisms for viscoelastic systems can be replaced by simpler, physically realizable ones that approximate their performance while retaining their stability properties. Errors were analyzed through a combination of numerical and analytic methods, involving mathematically the approximation of a transcendental function by a rational one under appropriate constraints.

For boundary feedback problems involving viscoelastic systems, a new representation formula was developed to make clear the relation between initial data and the smoothness and decay rates of solutions. In the frequency domain, the solution appears as a linear combination of solutions of simpler open-loop problems, with coefficients that depend on the frequency. When these coefficients can be realized as Laplace transforms of scalar functions, the behavior of the solution can be deduced.

Additional results concerned precise energy decay rates in systems with given strain histories on an infinite interval and local existence of solutions of a semilinear integral equation in a Banach space.

## II. INTRODUCTION

This report contains a summary of the results partially supported under the Air Force Office of Scientific Research Grant AFOSR-91-0083 during the period 15 November 1990 to 14 April 1994. This project was concerned with the interaction between mechanical (viscoelastic) dissipation and active feedback control mechanisms in various bodies and structures. The principal investigators were Professors Kenneth B. Hannsgen and Robert L. Wheeler.

During the 41 month period, two research papers were published; four additional papers are in preparation. Also, two graduate students were partially supported by the grant; one of these students has received his Ph. D. degree and the remaining student is expected to complete his work in the near future.

We have investigated several specific problems for viscoelastic systems. Specifically, we

1.) established a finite dimensional approximation procedure for a class of feedback problems with irrational transfer functions and carried out detailed error estimates for the case of torsional vibration in a viscoelastic rod [1, 4, 5] as well as for a viscoelastic Euler-Bernoulli beam [6],

2.) found a new setting for existence and decay results for boundary feedback problems, corresponding to frequency-dependent linear combinations of open loop semigroups [7],

3.) investigated decay rates for energy in simple systems where the decay rate is algebraic and the strain history is prescribed for all $t < 0$, as well as the interaction of thermal and viscoelastic damping [3],

4.) established local existence of mild solutions for a class of semilinear integrodifferential equations in Banach space, with scalar kernels satisfying natural conditions for memory kernels of viscoelastic materials [2].

## III. HIGHLIGHTS OF RESEARCH

In this section we discuss some of the work performed under the grant. Reprints of the papers will be forwarded to AFOSR as soon as they become available.

### (1) Approximation of optimal compensators

The main focus of research was the approximation of optimal compensators for viscoelastic systems by finite-dimensional, physically realizable ones that preserve the desired performance and stability properties. This work has been carried out jointly with Dr. Olof Staffans of the Helsinki University of Technology. Partial results were reported in [1], and a more complete version will appear in [4].

In the transform domain, the open and closed loop behavior of single-input single-output systems is represented by transfer functions that are analytic in a half plane. Vibration problems of interest here can be cast in the form of the sensitivity optimization problem of $H^\infty$ theory. Because the sensor and actuator are collocated in the cases addressed to date, the optimal compensator can be expressed as the product of a rational function (in $s$, the independent variable in the complex frequency domain) and the reciprocal of $P(s)$, the (generally transcendental) open-loop transfer function. One is led, then, to approximate $P^{-1}$ by rational functions of $s$ in a weighted $H^\infty$ norm.

In [4] a procedure is analyzed in detail for approximating the transfer function in the case of torsional motion in a homogeneous or nonhomogeneous cylindrical viscoelastic rod with circular cross section, for the case where a sensor and actuator are located at one end. Using the Hermite-Biehler Theorem of classical function theory, one obtains an infinite product expansion

$$P(s) = \frac{1}{\rho_0 s^2} \prod_{k=1}^{\infty} \frac{1 + (\beta/\xi_k)^2}{1 + (\beta/\eta_k)^2}. \tag{1}$$

where $\beta = \beta(s)$ is a frequency-dependent complex parameter that characterizes the viscoelastic material. Here the poles and zeros occur, respectively, at points $\beta = \pm\eta_k i$, $\beta = \pm\xi_k i$, with

$$\eta_k = \frac{k\pi}{\rho_1} + O(\frac{1}{k}), \quad \xi_k = \frac{(2k-1)\pi}{2\rho_1} + O(\frac{1}{k}) \qquad k \to \infty$$

where the "big $O$" terms vanish if the density is uniform along the rod, and $\rho_0$ and $\rho_1$ are constants which depend on the density.

The approximation requires three basic steps. First, the infinite product (1) is truncated; the "tail" is replaced by a new irrational convergence factor $F_N(s)$. This part of the procedure is the same for all materials, and depends only on the geometry of the problem; good convergence estimates (which do, however, depend on the viscoelastic properties represented analytically by $\beta(s)$) are proved. Second, one replaces the fractional linear factors in the remaining finite partial product with rational factors; a combination of numerical calculations and analytic estimates (valid when the viscoelastic damping is relatively small) justifies this step. Finally a very simple rational function is used to replace $F_N$; here the evidence for accuracy is all largely numerical, and it depends strongly on the fact that the approximation is being carried out in an $H^\infty$ norm with weights that vanish at infinity.

Parallel studies of bending vibrations in a simple viscoelastic Euler-Bernoulli beam have been carried out but involve enough differences from the rod to warrant treatment in a separate paper [6]. Also, we are preparing a short expository paper [5] that outlines how our procedure may be applied in the simplified cases of a damped wave equation and a heat equation, where the complications that arise in the second step due to the viscoelastic memory kernel do not arise.

There are several ways in which the approximation method developed so far needs to be improved and extended. The first truncation step depends crucially on the infinite product structure of $P$. Preliminary work shows that this structure arises and can be exploited in a number of related situations (such as a rod with actuator at one end and certain distributed sensors that could model a fiber optic sensor mounted along the rod), but is inapplicable in other cases such as, for instance, a rod with sensor and actuator located at opposite ends of the rod. Replacing the irrational fractional linear factors with rational ones is the source of the greatest error in the case considered in [4].

### (2) A semigroup setting for boundary feedback problems

In joint work with Professor Wolfgang Desch, Hannsgen and Wheeler have generalized partially the representation formula of [REF 1] for boundary feedback problems in viscoelasticity, in order to find a precise relation between initial data and the smoothness and decay rate of the solution. For elastic problems, the corresponding results can be formulated in a state space determined by the feedback relating, say, applied boundary forces to velocities. In viscoelastic bodies this relationship depends, typically, on frequencies and thus does not correspond to a fixed function space. The work of [REF 1], which pertained to torsional vibrations in a rod, showed that the solution can be expressed (in the frequency domain) as a linear combination of functions in two fixed spaces. The linear coefficients, though frequency dependent, can be realized as Laplace transforms of integrable scalar functions, and the properties of smoothness and exponential decay are easily read off. (The rates correspond, as expected, to poles of the transfer function.)

To extend the work of [REF 1] to more general problems, such as Timoshenko beams and three-dimensional motion in solids, a far more general solution formula is needed, and the linear coefficients must be realized as transforms. The first of these problems has been solved for a fairly wide class of problems, through semigroup methods and a linear algebra argument. The general system is

$$v'(t) = D\sigma(t), \quad v(0) = v_0$$

$$\sigma(t) = \sigma_0 + A * \tilde{D}v(t),$$

with boundary feedback

$$P\sigma(t) = -\tilde{P}v(t).$$

Here $v$ and $\sigma$ are Hilbert space valued generalizations of velocity and stress, respectively,

$D$ and $\tilde{D}$ are unbounded operators and $P$ and $\tilde{P}$ are finite rank operators generalizing evaluation at the boundary, and the adjoint relationship

$$< \tilde{D}v, \sigma > + < v, D\sigma > = < \tilde{P}v, P\sigma >$$

holds. $A(t)$ is a memory kernel for the material, equal to a bounded linear operator for each $t$, and $*$ denotes convolution on $(0, \infty)$. Thus, if $A(t)$ were a scalar constant, we would have a generalized wave equation. In fact, the scheme covers a fairly wide range of problems, the main restriction being that sensors and actuators (which correspond to $\tilde{P}$ and $P$) must be finite dimensional; for example, we could not have pointwise velocity-stress feedback distributed over an infinite set of points on the boundary.

The restriction on distributed feedback is perhaps not as serious as the lack of a general realization for the frequency-dependent coefficients that relate the transfer function for the feedback problem to those for certain open-loop problems (for which good semigroup formulations are known). Currently, we must analyze the representation formula for each problem separately in order to deduce existence and exponential decay. These results on the semigroup setting for boundary feedback problems will appear in [7].

### (3) Energy decay

Scott Inch, in his Ph. D. thesis [3] completed under Wheeler's direction, investigated the precise quantitative nature of energy decay in simple viscoelastic systems in the case where the stress relaxation modulus decays to equilibrium at an algebraic rate (in which case the creep behavior dominates). He shows that if one assumes *a priori* knowledge of the kinetic component of the energy, then the rate of energy decay in a viscoelastic rod with suitable strain history prescribed for all $t < 0$, and conservative boundary conditions, is no slower than that recently obtained by other authors in the presence of dissipative boundary velocity feedback. In the second part of his thesis, Inch exhibits an interesting example showing that the addition of thermal damping in a simple thermo-viscoelastic rod can lead to a rate of decay that is slower than in the case where there is no thermal damping.

### (4) A semilinear problem

Finally, we mention a result [2] obtained jointly by Hannsgen and Professor Sergiu Aizicovici of Ohio University. Local existence is proved for mild solutions of the problem

$$x'(t) = \int_0^t a(t-s)Lx(s)\,ds + (Fx)(t), \quad x(0) = x_0,$$

where $L$ is an unbounded linear operator in a Banach space $X$, and $F$ is a continuous hereditary operator defined on continuous $X$-valued functions of $t$. It is assumed that $L$ has a compact resolvent and that $a(t)$ has properties that are natural for memory kernels of viscoelastic materials.

## ADDITIONAL REFERENCES

REF.1. K. B. Hannsgen and R. L. Wheeler, A representation formula for one-dimensional viscoelasticity with stabilizing boundary feedback, in *Estimation and Control of Distributed Parameter Systems*, W. Desch, F. Kappel and K. Kunish, eds., International Series on Numerical Mathematics, Vol. 100, Birkäuser (1991), pp. 139-147. (Partially supported by AFOSR-89-0268.)

## IV. CUMULATIVE LIST OF PUBLICATIONS

AFOSR Grant AFOSR-91-0083


1. K. B. Hannsgen, O. J. Staffans and R. L. Wheeler, Rational approximation of the transfer function of a viscoelastic rod, in *Analysis and Optimization of Systems: State and Frequency Domain Approaches for Infinite-Dimensional Systems, Proc. 10th International Conference, Sophia-Antipolis, France, 1992*, R. Curtain, ed., Lecture Notes in Control and Information Sciences, Vol. 185, Springer-Verlag (1993), pp. 551-562.

2. S. Aizicovici and K. B. Hannsgen, Local existence for abstract semilinear Volterra integrodifferential equations, *J. Integral Equations and Applications*, 5 (1993), pp. 299-313.

3. S. E. Inch, Precise energy decay rates for some viscoelastic and thermo-viscoelastic rods, Ph. D. Thesis, Virginia Polytechnic Institute and State University, September, 1992.

4. K. B. Hannsgen, O. J. Staffans and R. L. Wheeler, Rational approximations of transfer functions of some viscoelastic rods, with applications to robust control, in preparation.

5. K. B. Hannsgen, O. J. Staffans and R. L. Wheeler, Square root rational approximations of the transfer function of the wave and heat equations, in preparation.

6. K. B. Hannsgen, O. J. Staffans and R. L. Wheeler, Rational approximation of the transfer functions for viscoelastic Euler-Bernoulli beams with collocated sensor and actuator, in preparation.

7. W. Desch, K. B. Hannsgen and R. L. Wheeler, A semigroup representation for viscoelastic dynamics with boundary feedback, in preparation.

## V. INVESTIGATORS PARTIALLY SUPPORTED UNDER THIS GRANT

K. B. Hannsgen - Principal Investigator

R. L. Wheeler - Principal Investigator

Scott Inch - Graduate Student (Earned Ph.D. September 1992, Thesis: "Precise Energy Decay Rates for some Viscoelastic and Thermo-viscoelastic Rods", Advisor: Robert Wheeler. Currently, Assistant Professor of Mathematics, Bloomsburg University of Pennsylvania.)

Daniel Eno - Graduate Student (Working on Ph.D.)

## VI. INTERACTIONS

### (i) Presentations at professional meetings:

Hannsgen

1. "Rational approximation of the transfer functions for a viscoelastic rod" at SIAM Conference on Control and its Applications, Minneapolis, Minnesota, September, 1992, (contributed talk).
2. "Approximation of transfer functions for viscoelastic systems" at Special Session on Control, 99th Annual Meeting of Amer. Math. Soc. , San Antonio, Texas, January, 1993, (invited talk).
3. "Mild solutions of abstract semilinear Volterra integrodifferential equations" at International Conference on Mathematical Problems in Viscoelasticity, Oberwolfach, Germany, May, 1993, (invited talk).
4. "Frequency domain analysis and finite dimensional robust control design for a viscoelastic rod" at International Conference on Control and Estimation of Distributed Parameter Systems, Vorau, Austria, July, 1993, (invited talk).

Wheeler

1. "Finite dimensional robust control of a viscoelastic rod" at Special Session on Nonlinear Volterra Equations, World Congress of Nonlinear Analysts, Tampa Florida, August, 1992, (invited talk).
2. "Rational suboptimal compensators for vibration damping in a viscoelastic rod" at SIAM Conference on Control and its Applications, Minneapolis, Minnesota. September, 1992, (contributed talk).
3. "Rational approximation of compensators for some control problems for viscoelastic systems" at International Conference on Mathematical Problems in Viscoelasticity, Oberwolfach, Germany, May, 1993, (invited talk).

Other

1. "Rational approximation of the transfer function of a viscoelastic rod, by K. B. Hannsgen, O. J. Staffans and R. L. Wheeler (presented by Staffans), at 10th International Conference on Analysis and Optimization of Systems, INRIA-Sophia. France, June, 1992 (invited talk).

### (ii) Other off campus activities

Contact with Lt. Col. Ronald Bagley of the Air Force Institute of Technology at Wright-Patterson Air Force Base continued with a visit by Hannsgen and Wheeler to AFIT on April 19, 1991, and a visit by Lt. Col. Bagley to Virginia Tech on September 7, 1991, which included a seminar talk concerning his work on fractional derivative models for viscoelastic materials.

On August 7, 1991, Hannsgen and Wheeler visited Phillips Laboratory, Edwards AFB, California, to discuss current Air Force interests in the areas of control of advanced composites and large space structures. The primary discussions were with Lt. Steven Griffen, OLAC PL/STSS, whose interests in control and damping questions most closely matched those of the Principal Investigators. The visit included technical discussions and an extended tour of the laboratory work underway at Phillips Lab.

Wheeler attended the Robust Control Theory Workshop, September 21-25, 1992, and the Distributed Parameter Control Workshop, November 9-14, 1992, held at the Institute for Mathematics and its Applications, University of Minnesota.

Hannsgen and Wheeler attended the AFOSR grantees/contractors 1993 Workshop on Dynamics and Control, Ann Arbor, Michigan, May 24-25, 1993.