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# MONOPULSE TRACKER OPERATING CHARACTERISTICS MEASUREMENT ERROR PERFORMANCE FOR AZIMUTH TRACKING

Scott Bolen

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Monopulse Tracker Operating Characteristics  
Measurement Error Performance for Azimuth Tracking

Scott Bolen  
July 1993

**Abstract:** This paper derives the operating characteristics for a monopulse radar tracker as a function of probability of detection and probability of false alarm. Measurement error performance is developed for azimuth tracking in an amplitude comparison monopulse system. A set of receiver operating characteristics is also presented to demonstrate how to determine the detection threshold in the signal processing system to minimize tracking errors for a given signal-to-noise ratio.

I. Introduction

The amplitude comparison monopulse system employs two overlapping antenna beams radiating simultaneously to obtain an angular error in one coordinate. The monopulse tracker is essentially a closed-loop control system that follows the change in position of a moving target. Target position is indicated by the relative amplitude of the receive signals from each of the two beams. Figure 1 shows the monopulse measurement process. In figure 1(a) the relative beam pattern is shown. There are two symmetric beams that can be considered to be offset by a distance of  $\pm x$  along an arbitrary axis X. The response of each beam can be defined as a function  $f(y)$  such that:

$f_1(y)$  = the response of the received signal  
from beam one

and

$f_2(y)$  = the response of the received signal  
from beam two.

In the monopulse system the receive signal from each beam is subtracted to form a difference response or error signal as shown in figure 1(b). This error signal is given by:

$$\text{error}(y) = f_1(y) - f_2(y). \quad (1)$$

The difference response forms the control signal for the closed-loop monopulse tracking system [1,2].

In the monopulse system a null is formed in the antenna pattern between the two radiated beams. The focus of this paper is to develop a set of operating characteristics for a tracking system that keeps the target within the null of the antenna pattern. This type of system is called a "null" tracker. The null used for tracking occurs in the center of the antenna or array. The amplitude of the error signal specifies the degree to which the antenna is centered on the target [1,2].

## II. Development of the Monopulse Tracker Operating Characteristics

The difference response represented in figure 1(a) shows that at the null of the antenna pattern

$$\text{error}(y) = 0.$$

In an unambiguous track condition, this response indicates that the target is situated in the center of the antenna between the two transmitted beams. A positive value for the error signal indicates that the target is in beam one and a negative value places the target in beam two.

The magnitude of the difference response is dependent on the received signal strength of the target and the location of the target with respect to the location of the transmitted beams (ie the tracking null). In practical radar systems, the antenna pattern developed by the transmission of the two beams will contain other nulls in addition to the desired tracking null. The presence of these unwanted nulls can cause ambiguities to occur in the tracking process. That is, when  $\text{error}(y) = 0$  this can indicate that: (a) the target is contained within the tracking null or (b) the target is in another null off angle from the primary tracking null. The standard approach to overcome this ambiguity is to form another response

$$\text{sum}(y) = f_1(y) + f_2(y). \quad (2)$$

This function is referred to as the sum response.  $\text{Sum}(y)$  is used for target detection and can also be used to avoid tracking ambiguities in azimuth measurements. A normalized difference response can be formed given by the function

$$\begin{aligned} \text{enorm}(y) &= \frac{\text{error}(y)}{\text{sum}(y)} & (3) \\ &= \frac{f_1(y) - f_2(y)}{f_1(y) + f_2(y)}. \end{aligned}$$

Ambiguities can be avoided if the  $\text{sum}(y)$  is set above a given detection threshold.

#### A. Measurement Error in Noise

In the difference pattern response shown in figure 1(b), the slope of the error response as it crosses through the zero point on the measurement axis is called the *difference slope* of the monopulse measurement [2]. The rate of change in the slope of the curve at this point indicates the relative measurement sensitivity of the system. A sharply rising slope indicates a highly sensitive system and a slow rising slope indicates a less sensitive system. In this case sensitivity refers to the dynamic response of the tracking system. Barton [2] defines a normalized difference slope as a differential function given by

$$k_m = \frac{d(\text{enorm}(y))}{d(\text{bmw}/\text{bmw}_3)}$$

where

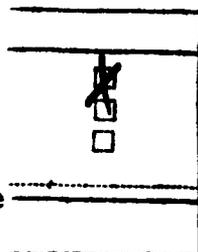
bmw = the antenna beamwidth,  
 bmw<sub>3</sub> = the half power antenna beamwidth.

\*  $\text{bmw}/\text{bmw}_3$  is a normalized beamwidth parameter

In the presence of noise, spurious error signals can be generated that can corrupt the tracking process. It is assumed that the noise is a random variable with an even probability density function (pdf). For additive noise such that

$$\begin{array}{l} \text{Total} \\ \text{Received} \\ \text{Signal} \end{array} = \begin{array}{l} \text{Received} + \text{noise} \\ \text{Target} \\ \text{Signal} \end{array}$$

the mean of the noise pdf can be considered to be centered at the actual target position. For this case, the azimuth measurement position error will vary symmetrically about the actual target position on the measurement axis with an rms error given by [2]



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$$\sigma = \frac{bmw^3}{km[2(SNR)n]^{1/2}} \quad (4)$$

where SNR = signal-to-noise ratio  
 n = number of pulses integrated.

Equation (4) is the fundamental relationship that will be used throughout the paper to describe the rms position error of the monopulse estimate in a thermal noise environment.

#### B. Measurement Error Due to Target Dynamics

In classical control system theory, the lag error exhibited by a closed-loop tracking system, such as the monopulse tracker, can be described in terms of the dynamics of the system. Servomechanism control theory can be used to predict the lag error. In an established formulation, the lag error of the system is given by [2,3]

$$e = \frac{wa}{Kv} + \frac{d(wa)}{Ka} + \frac{d^2(wa)}{K3} + \dots$$

where Kv, Ka, and K3 are servo error coefficients.

In this paper, it is assumed that the target acceleration contributes significantly to the dynamics of system and that the other kinematic properties of the target are negligible. In this case, the system servo error (dynamic lag) can be defined simply as

$$ea = \frac{d(wa)}{Ka} \quad (6)$$

Barton [2] describes the servo error as

$$ea = \frac{a}{(2.5)RBn^2} \quad (7)$$

where a = target acceleration  
 R = range to target from the radar  
 Bn = servo bandwidth (eq noise bandwidth).

The variance of the azimuth measurement error specified as a function of thermal noise and dynamic lag can be determined by combining equation (4) and equation (7) (after Barton [2])

$$\text{var} = (\sigma)^2 + (ea)^2 \quad (8)$$

$$= \frac{(bmw3^2) Bn}{(km^2) (fr) (Bt) (SNR)} + \frac{a^2}{6.3 (R^2) (Bn^2)}$$

where fr = pulse repetition frequency  
Bt = time-bandwidth product.

### C. Measurement Error Expressed in Terms of Pd & Pf

The signal-to-noise ratio (SNR) refers to the power ratio of the detected signal to noise at the input of the radar receiver. Another parameter called the *detectability factor*, D, can be defined in the same way as the SNR except that we will let D be the required value of the power ratio calculated from the *probability of detection* (Pd) and *probability of false alarm* (Pf). For a Swerling Case 1 fluctuating target, the single pulse detectability factor is specified by [2]

$$D(1) = \frac{\ln(Pf)}{\ln(Pd)} - 1. \quad (9)$$

Substituting equation (9) into equation (8) yields

$$\begin{aligned} \text{var} = & \frac{(bmw3)^2 Bn}{(km)^2 (fr) (Bt) (\ln(Pf)/\ln(Pd) - 1)} \\ & + \frac{a^2}{6.3 (R)^2 (Bn)^2}. \end{aligned} \quad (10)$$

Equation (10) now defines the variance of the azimuth tracking error in terms of Pd and Pf. Simplifying and solving for Pd yields

$$\begin{aligned} Pd = & \exp\left(\text{var} - \frac{(a)^2}{6.3 (R)^2 (Bn)^2}\right) \\ & \left(\frac{(km)^2 (fr) (Bt) (\ln(Pf))}{(bmw3)^2 (Bn) + (\text{var}) (km)^2 (fr) (Bt) + (a)^2 (fr) (Bt)}\right). \end{aligned} \quad (11)$$

From equation (11) the Monopulse Tracker Operating Characteristics (MTOC) for single pulse detection and for a Swerling Case 1 target can be calculated. Contours of the measurement variance error can be calculated over the Pd-Pf plane.

Example - Computation of MTOC for a Given Set of Radar Parameters (see figure 2):

For the parameters:

km = 1.6  
fr = 1000.0 Hz  
Bt = 1.0  
R = 100.0 nmi  
Bn = 3.0 Hz  
a = 20.0 M/sec  
bmw3 = 0.02 radians.

Lines of constant azimuth tracking variance errors can be computed as a function of Pd and Pf. Figure 2 shows the monopulse azimuth variance errors or precision of the monopulse estimate expressed in decibels. Each line shown in the figure forms a contour for a specific error. The entire set of contours forms the MTOC for the example problem.

### III. Calculation of Receiver Operating Characteristic

The Receiver Operating Characteristic (ROC) can be calculated in terms of the Pd and Pf. There are several examples in the literature that describe the detection process and demonstrate how to determine the ROC [4]. In this paper it is assumed that the ROC is determined from a binary hypothesis test that forms a likelihood ratio such as the Neyman-Pearson decision test.

In the likelihood ratio test there are two hypotheses that are formed in the decision process. The observations for the two hypotheses are

$$H_0: s_i = n_i \quad i = 1, 2, \dots, N$$

(a target is not present)

and

$$H_1: s_i = m + n_i \quad i = 1, 2, \dots, N$$

(a target is present)

where  $s$  is the total signal taken at the receiver which is equal to the radar return signal plus noise;  $m$  = target signal,  $n_i$  = noise signal,  $N$  = number of samples taken.

Given the above hypothesis, the probability density of  $s_i$  can be described as a set of conditional probabilities for each hypothesis such that

$$P_{s_i/H1}(S_i/H1) = p_{ni}(S_i - m)$$

and

$$P_{s_i/H0}(S_i/H0) = p_{ni}(S_i).$$

For noise samples that are Gaussian with zero mean and variance of  $(\sigma)^2$ , the probability density function of  $s_i$  under each hypothesis can be described as

$$p_{ni}(S_i - m) = \frac{1}{\sqrt{2(\pi)(\sigma)^2}} \exp\left(-\frac{(S_i - m)^2}{2(\sigma)^2}\right)$$

and

$$p_{ni}(S_i) = \frac{1}{\sqrt{2(\pi)(\sigma)^2}} \exp\left(-\frac{(S_i)^2}{2(\sigma)^2}\right).$$

Because the  $n_i$  are statistically independent, the joint probability density of each  $s_i$  is the product of the individual probability densities such that

$$P_{\mathbf{s}/H1}(\mathbf{S}/H1) = \prod_{i=1}^N \frac{1}{\sqrt{2(\pi)(\sigma)^2}} \exp\left(-\frac{(S_i - m)^2}{2(\sigma)^2}\right)$$

and

$$P_{\mathbf{s}/H0}(\mathbf{S}/H0) = \prod_{i=1}^N \frac{1}{\sqrt{2(\pi)(\sigma)^2}} \exp\left(-\frac{(S_i)^2}{2(\sigma)^2}\right).$$

The likelihood ratio is given by

$$R(\mathbf{S}) = \frac{\frac{1}{\sqrt{2(\pi)(\sigma)^2}} \exp\left(-\frac{(S_i - m)^2}{2(\sigma)^2}\right)}{\frac{1}{\sqrt{2(\pi)(\sigma)^2}} \exp\left(-\frac{(S_i)^2}{2(\sigma)^2}\right)}.$$

So for a given threshold, the decision criteria is

$$R(\mathbf{S}) > \text{threshold} \quad \text{choose } H1$$

and

$R(S) < \text{threshold}$  choose  $H_0$ .

Figure 3 shows the density functions for calculating the probability of detection and false alarm given a Gaussian distribution. The decision threshold is marked on the figure and is used to calculate the values of  $P_d$  and  $P_f$  where [4]

$$P_f = 1 - \text{erf}\left(\frac{\ln(\text{threshold})}{d} + \frac{d}{2}\right)$$

and

$$P_d = 1 - \text{erf}\left(\frac{\ln(\text{threshold})}{d} - \frac{d}{2}\right)$$

where erf denotes the error function and  $d$  is the distance between the means of the two distributions.

Figure 4 shows the ROC as a function of  $P_d$  and  $P_f$ . Each line forms a contour of the signal-to-noise ratio present at the receiver for a given  $P_d$  and  $P_f$ . The entire set of contours forms the ROC for the given hypothesis test.

#### IV. Summary - Characterizing System Performance

System performance can be completely described by the probability of detection and false alarm. From the example problems, the system performance can be characterized by overlaying the ROC contours onto the MTOC contours as shown in figure 5. The intersection of the contours defines the operating point of the system. For a given  $P_d$  and  $P_f$ , the receiver signal-to-noise ratio can be found and the subsequent tracker error can be determined from the graph. Figure 5 can be used to either predict track performance or to specify receiver performance based on a required track performance.

Also, since  $P_d$  and  $P_f$  are a function of the detection threshold, a particular receiver operating point can be determined by specifying a detection threshold. This threshold can be determined by trading off performance in terms of  $P_d$  and  $P_f$  which also defines the track error. Hence, system performance can be completely defined by the value of the detection threshold level.

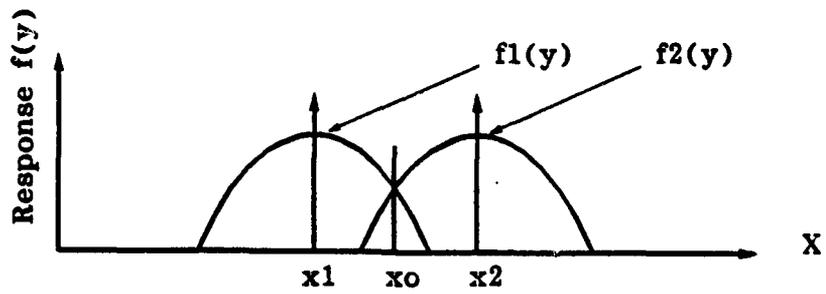
REFERENCE:

[1] Skolnik, M. I., Introduction to Radar Systems. McGraw-Hill Book Company, New York, 1980, pp 160-183.

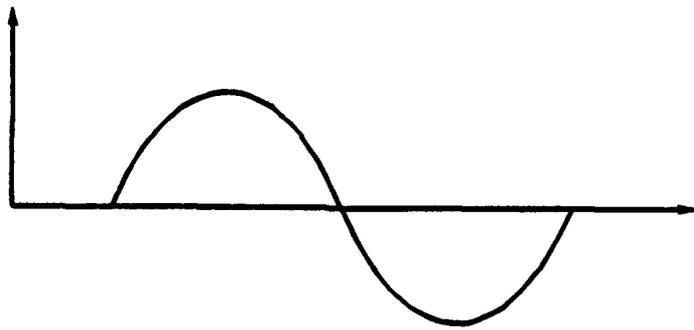
[2] Barton, D. K., Modern Radar System Analysis. Artech House, Boston, 1988, pp 57-89, 377-472.

[3] DiStefano, J. J., Stubberud, A. R., Williams, I. J., Schaum's Outline Series for Feedback and Control Systems, second edition. McGraw-Hill, Inc, New York, 1990, pp 214-219.

[4] Van Trees, H. L., Detection, Estimation, and Modulation Theory - Part I. John Wiley & Sons, New York, 1968, pp 19-39.



(a)



(b)

Figure 1: Monopulse measurement process: (a) monopulse response, (b) difference response.

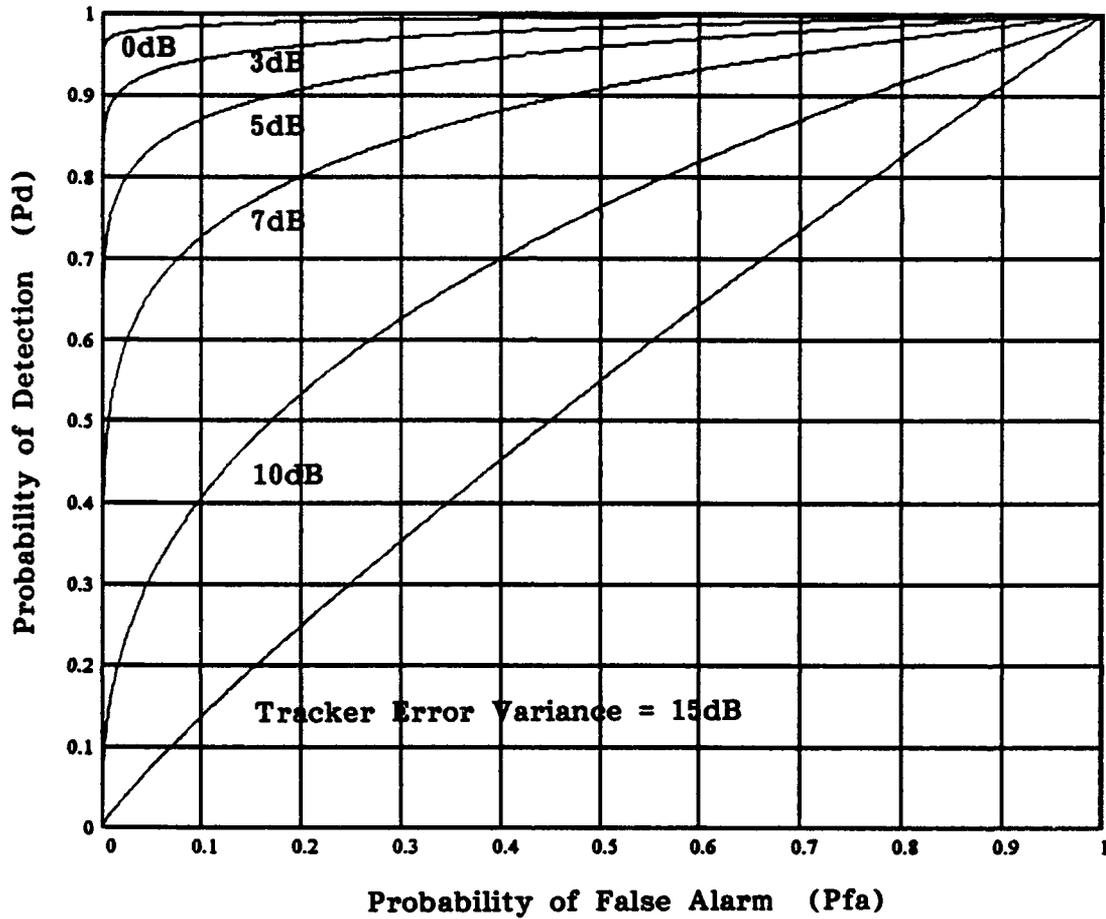


Figure 2: Monopulse Tracker Operating Characteristic (MTOC).  
 Variance of tracking error in azimuth due to dynamic lag  
 and thermal noise.

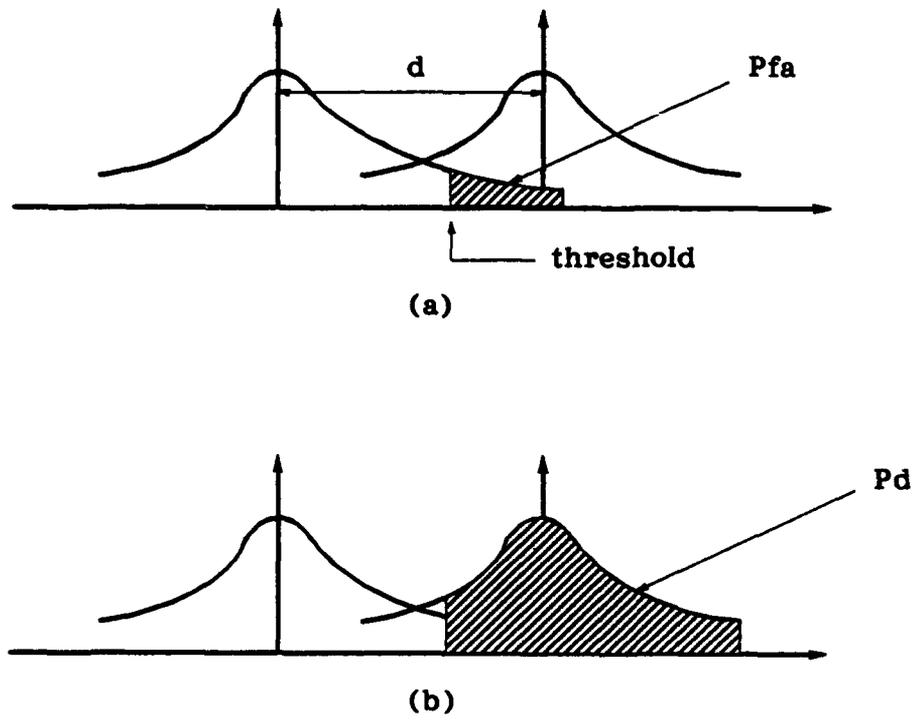


Figure 3: Error probabilities, Gaussian distribution: (a)  $P_{fa}$  calculation, (b)  $P_d$  calculation.

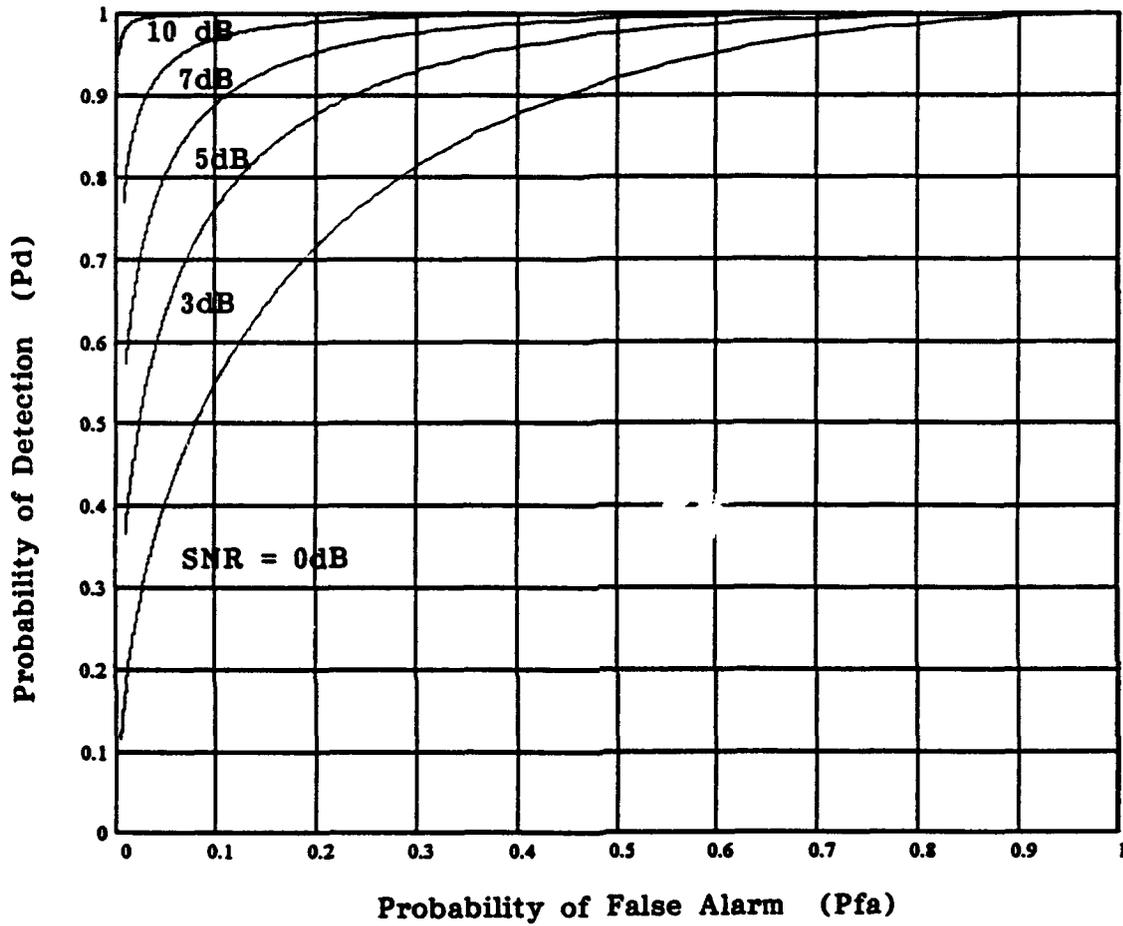


Figure 4: Receiver Operating Characteristic (ROC) for a known signal in white Gaussian noise.

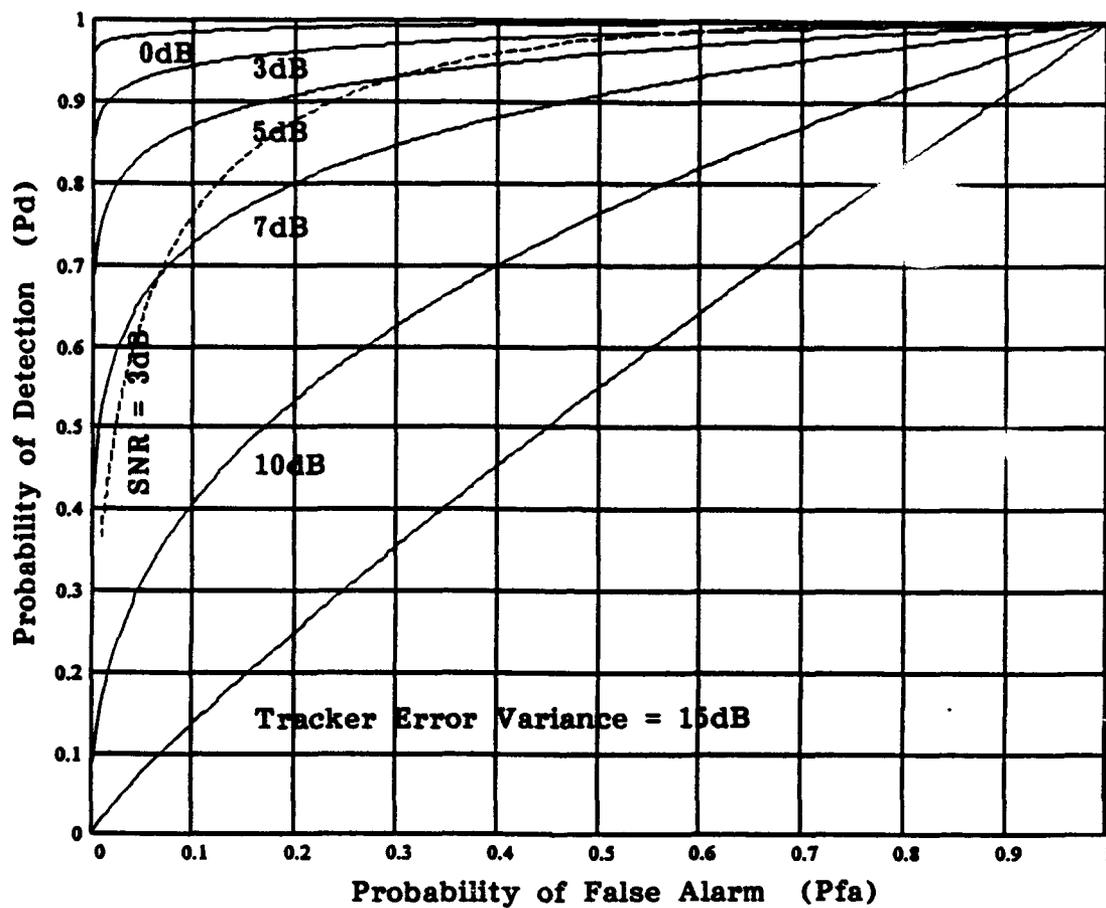


Figure 5: Monopulse tracker system performance for a given receiver operating point.

— Tracker Contours  
 - - - Receiver Performance

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