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A MODULAR APPROACH TO MODELING AN ISOLATED POWER SYSTEM ON A FINITE VOLTAGE BUS USING A DIFFERENTIAL ALGEBRAIC EQUATION SOLVING ROUTINE

by

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The major difficulty encountered in building a model which computes bus voltage at each time step is that bus voltage is a function of current and current derivative terms. Bus voltage is also an input to the state equations which produce the current and current derivatives. This creates an algebraic loop which is a form of implicit differential equation.

A routine has been developed by Linda Petzold of Lawrence Livermore Laboratory for solving these types of equations. The routine, called DASSL (Differential/Algebraic System Solver), has been implemented in a pre-release version of the software ACSL (Advanced Continuous Simulation Language) and has been made available to the Naval Postgraduate School on a trial basis. An isolated power system is modeled using this software and the DASSL routine. The system response to several dynamic situations is studied and the results are presented.

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I. INTRODUCTION

A. BACKGROUND

The modeling of synchronous generators has been primarily driven by the commercial power utility industry. These simulations most frequently make the assumption that the machine to be modeled is connected to a system bus in which voltage magnitude and frequency are fixed values. This so called "infinite bus" assumption provides good results for many studies, especially those involving huge power grids. However, in quite a few applications, such as in shipboard, aircraft and isolated emergency power systems this assumption is not valid.

For such systems, some loads may be a significant percentage of the generator capacity. When a large load is started on an isolated generator, neither voltage magnitude nor frequency remain constant. In these situations the voltage will dip appreciably and possibly cause other sensitive loads on the system bus to fail.

It is therefore important to be able to model accurately the behavior of a an isolated power system. The design engineer interested in building the smallest, least expensive machine that will do the job needs to know how the entire system will behave during dynamic loading.

B. THESIS OVERVIEW

The most common methods for studying power systems and the interaction between sources and loads involves use of the infinite bus model. The interaction between sources and loads is done by load (power) flow analysis. When a load is changed in such a simulation the power demand must be satisfied by increasing power supplied by a source.

This approach is fine, however, it still assumes each independent submodel is on an infinite bus. So for example, fluctuations in voltage magnitude and frequency occurring in one generator are considered negligible in the overall system.

The primary goal of this study is to develop a system model which allows sources and loads to be connected to a bus voltage model which accurately reflects the bus voltage behavior during transients and the effects of the transient bus voltage on the loads and sources. The approach taken is to develop an overall system model which consists of accepted accurate stand-alone source and load submodels. These submodels are then tied together by a bus voltage model. In final form, the system model allows the simple connection of multiple sources and loads.

Figure 1 represents the type of isolated power system studied in this thesis. The system consists of a gas turbine prime mover, synchronous generator, a system bus and system loads. Models for each element of the isolated system are presented independently then all are tied together with closed loop control to produce the system model.

1. Synchronous Generator Model

Much work has been done to develop accurate models for rotating electrical machinery. Krause [Ref. 1,pp 211-267] develops a state-space model for a synchronous machine using the well known Park's transformation [Ref. 2]. Circuit voltage equations are first developed in the three-phase *a-b-c* reference frame. The subsequent application of Park's transformation has the advantage of referencing all state variables to an orthogonal (*q-d-0*) reference frame. The reference frame transformation changes time varying winding inductance values into constants. The state-space model may be developed with either winding current or magnetic flux linkage as states. The model used in this presentation is formulated with current states.

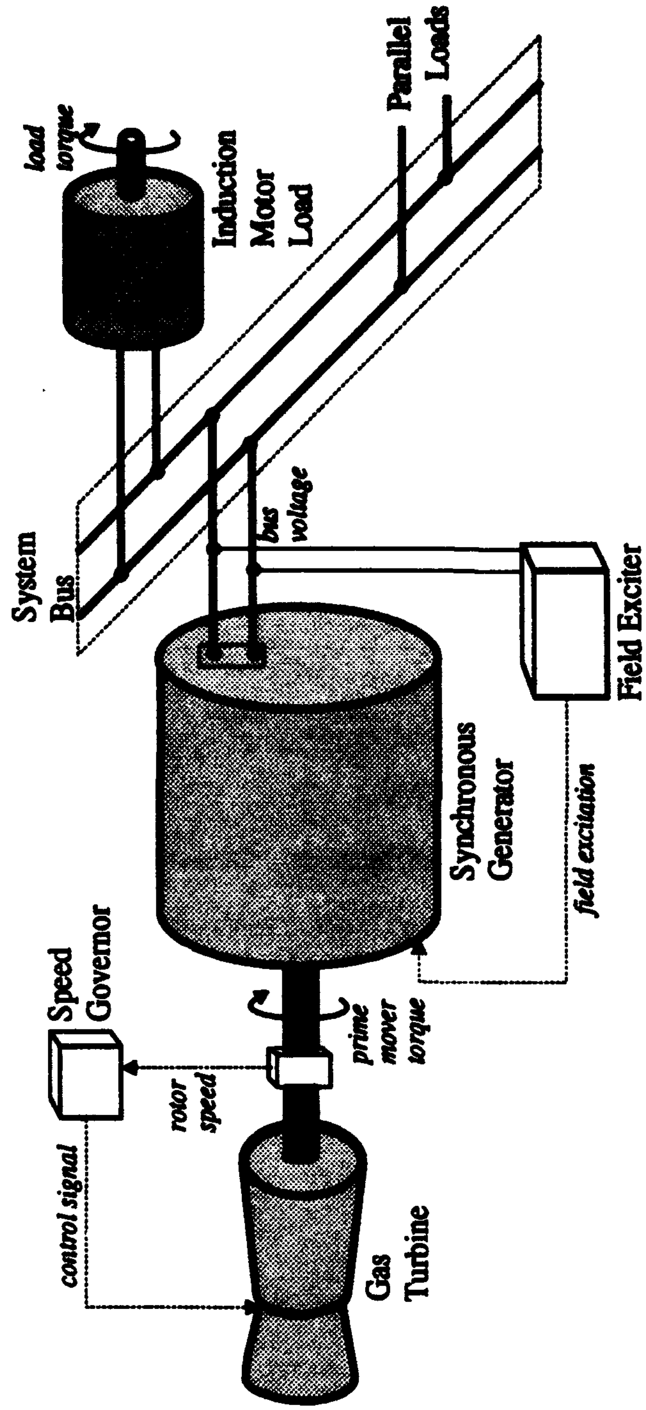

Figure 1. Isolated power system.

2. Load Models

Two load models are presented, a three-phase resistive-inductive (R-L) load and an induction motor load. These models are developed in a similar manner to the synchronous generator model. The equations describing load circuit behavior are transformed into the same orthogonal reference frame as that used for the synchronous generator. The load model state equations are presented with current states.

3. Bus Voltage Model

By formulating the submodel state equations in terms of current, a bus voltage model may be developed based on satisfying Kirchoff's Current Law (KCL) at the common node. Several possible models for the bus voltage are explored. Ultimately, a routine for solving implicit state equations and algebraic loops will be introduced. This routine allows a bus voltage model to be developed which supports the goals of modularity and expandability.

4. Generator Closed Loop Control

Output voltage magnitude and frequency must be controlled so that bus voltage will remain within specification. Voltage control is accomplished by a field exciter which senses generator output terminal voltage and adjusts the field winding excitation voltage to keep terminal voltage at the desired level. Frequency control is accomplished by driving the generator with a prime mover which is under the control of a speed regulating governor. Models for both regulation systems are presented.

5. Simulation Software

Speed, ease of use, quality of output and special capabilities were considered when choosing the simulation software. Work was done in the programs MATLAB and SIMULINK from MathWorks [Ref. 3] and in ACSL (Advanced Continuous Simulation Language) from Mitchell and Gauthier Associates [Ref. 4]. Both are excellent for modeling systems of linear and non-linear differential equations. However, ACSL was chosen for the power system simulation work presented here due to the special capabilities of this package.

A pre-release version of ACSL, level 10F, was provided to the Naval Postgraduate School on a trial basis. This version of ACSL contains an algorithm for solving differential algebraic equations (DAEs) which is described in Chapter IV. This algorithm allows systems of implicit differential equations to be solved. Specifically of use is the ability to solve implicit systems formed by a system of state equations subject to an algebraic constraint equation.

5. System Model Connection

The isolated power system simulation is modular in concept. It consists of several submodels. Each submodel is a stand-alone model which is tied into the system by the bus voltage submodel. The source and load models are well understood and have been validated extensively by others. The bus voltage model is presented and validated as an independent model in Chapter IV. After each piece of the system is presented, the total isolated power system is developed from the available building blocks. The ACSL code for the system model is described in detail.

6. System Model Response and Validation

After the system is connected and put under closed loop control, it is validated by comparing it with a finite bus system model developed at Purdue University by Mayer and Wasynczuk [Ref. 5]. This simulation scenario involves starting three induction motors on a system bus supplied by a single generator. Plots of model response are presented and discussed.

Additionally, the R-L model is modified for the case where the resistive part of the load is unbalanced. The system model is exercised by operating it with an unbalanced loading condition. Plots of the model response to this condition are presented and discussed.

7. **Conclusions and Future Work**

Finally, conclusions about the usefulness of the finite bus model are presented. Suggestions for expanding the system model to include winding saturation effects, more loads and a parallel generator are made. The need for more effort in validating the approach is also discussed along with some suggestions on how this could be accomplished.

II. DEVELOPMENT OF THE SYNCHRONOUS GENERATOR MODEL

The process used in developing the model for a synchronous generator is as follows. First the differential equations describing the circuit behavior of each winding in the machine are obtained. Unfortunately, because both the current and inductance terms in the equations vary with time, these equations are complicated. Using Park's transformation [Ref. 2], the equations describing the machine are changed to an orthogonal reference frame which has the advantage of making all inductance terms constant. This transformation from machine variables to reference-frame variables, along with the assumption of a linear relationship between current and flux linkages, allows the model state equations to be expressed with either current or flux linkages as the states. For a synchronous generator the orthogonal reference frame used will be the rotor reference frame.

A. SYNCHRONOUS GENERATOR EQUATIONS IN MACHINE VARIABLES

The equations used to develop the synchronous generator model are derived from the voltage equations for the windings of a three-phase machine. These equations, which relate voltage to current and magnetic flux, are not enough to completely describe the behavior of the machine. Additionally, an equation is needed which relates rotor rotational speed to torque where electrical torque is described as a function of current or flux linkage.

1. Machine Variable Voltage Equations

Figure 2 represents a two-pole, three-phase, salient-pole synchronous generator. The *as*, *bs*, and *cs* windings are on the stator and spaced 120° apart. These stator windings are identical, sinusoidally distributed and have N_s equivalent turns. On the rotor, *kq* and *kd* are damper windings while the *fd* winding is used for applying the field excitation. The rotor windings are situated in an orthogonal *q-d* reference frame. Rotor windings are also sinusoidally distributed but each may have a different number of equivalent turns; N_{kq}, N_{kd} and N_{fd} respectively. Note that the current direction is out of the stator windings (generator convention). The series resistive-inductive pair in each stator and rotor circuit represents the electrical characteristic of each winding. The rotor angular position and speed are represented by θ_r and ω_r respectively.

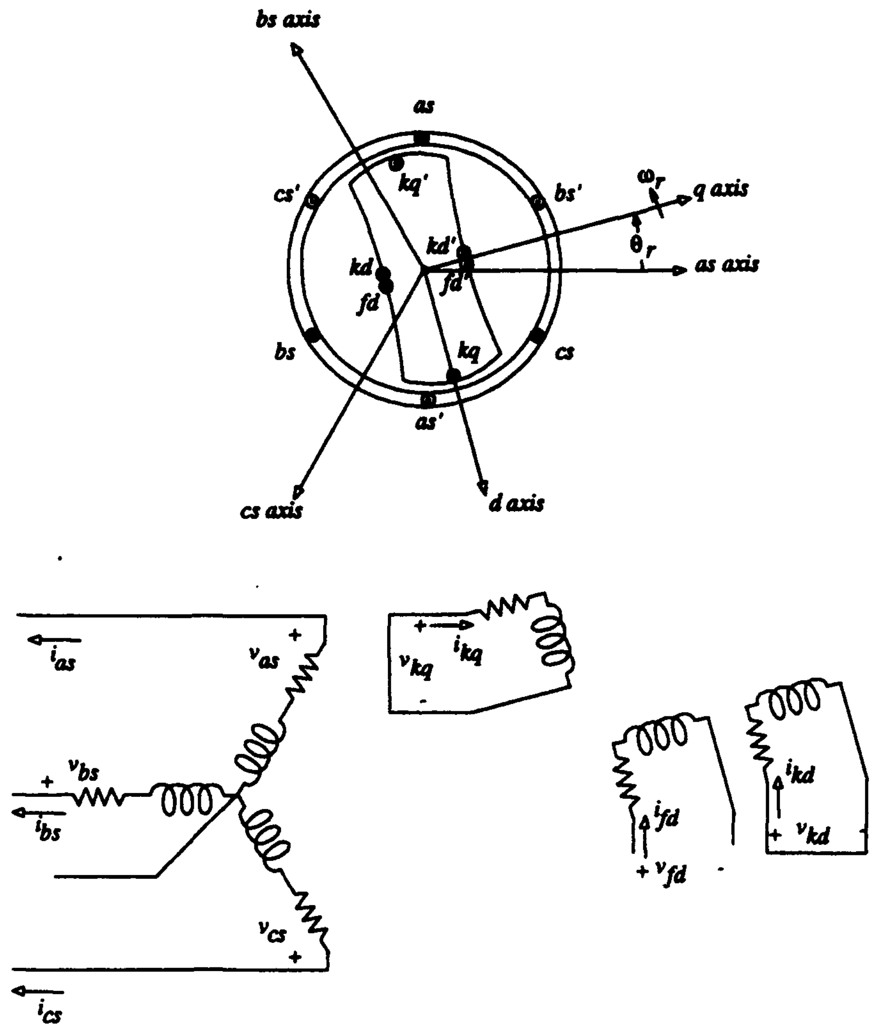

Figure 2. Two-pole, three-phase, salient-pole synchronous machine.

By summing voltages around each circuit loop, equations (1) through (6) are obtained. Equations (1), (2) and (3) represent the stator windings while (4), (5) and (6) model the rotor circuits. Voltage in the inductive elements is expressed by Faraday's law where the induced voltage equals the rate of change of the flux linkages. The damper windings are short circuited at the ends so that v_{kq} and v_{kd} equal zero.

$$v_{as} = -r_s i_{as} + \frac{d\lambda_{as}}{dt} \tag{1}$$

$$v_{bs} = -r_s i_{bs} + \frac{d\lambda_{bs}}{dt} \tag{2}$$

$$v_{cs} = -r_s i_{cs} + \frac{d\lambda_{cs}}{dt} \tag{3}$$

$$0 = r_{kq} i_{kq} + \frac{d\lambda_{kq}}{dt} \tag{4}$$

$$v_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt} \tag{5}$$

$$0 = r_{kd} i_{kd} + \frac{d\lambda_{kd}}{dt} \tag{6}$$

In order to use equations (1) through (6) to develop a state-space model in terms of current, the flux linkage derivative terms must be looked at in more detail. For a linear magnetic system the flux linkage may be related to current via the relationship

$$\underline{\lambda} = \mathbf{L}\underline{i} \tag{7}$$

where

$$\underline{\lambda} = \begin{bmatrix} \lambda_{as} & \lambda_{bs} & \lambda_{cs} & \lambda_{kq} & \lambda_{fd} & \lambda_{kd} \end{bmatrix}^T \tag{8}$$

$$\underline{i} = \begin{bmatrix} i_{as} & i_{bs} & i_{cs} & i_{kq} & i_{fd} & i_{kd} \end{bmatrix}^T \tag{9}$$

The terms of the matrix **L** represent the mutual and self inductance terms relating flux linkage to currents. In general L_{xz} would relate *x*-winding flux to *z*-winding current. So, for example, the expanded expression for the *as*-winding flux linkage is written as

$$\lambda_{as} = L_{aa}i_{as} + L_{ab}i_{bs} + L_{ac}i_{cs} + L_{akq}i_{kq} + L_{afd}i_{fd} + L_{akd}i_{kd} \tag{10}$$

where, in general, both inductance and current are functions of time. Using equation (7) the inductance-current product may be substituted for flux linkage in equations (1) through (6).

Because access to the rotor windings is difficult, the machine parameters (winding resistance, inductances, voltages etc.) are most commonly referred to the stator. Referring values from the rotor to the stator is done in a manner similar to referring variables from the primary to the secondary of a transformer (via the turns ratio). Krause [Ref. 1:pp. 167] uses a prime to denote referred variables. In this derivation all variables may be assumed to be referred to the stator. In particular the inductance matrix which follows as part of the compact voltage equations is written in referred quantities.

With flux linkage expressed in terms of currents and winding inductance, a compact vector-matrix form of the voltage equations may now be written as

$$\begin{bmatrix} \underline{v}_{abcs} \\ \underline{v}_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_s & 0 \\ 0 & \mathbf{r}_r \end{bmatrix} \begin{bmatrix} -\underline{i}_{abcs} \\ \underline{i}_{qdr} \end{bmatrix} + p \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ \frac{2}{3}(\mathbf{L}_{sr})^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} -\underline{i}_{abcs} \\ \underline{i}_{qdr} \end{bmatrix} \tag{11}$$

where the operator p represents the derivative with respect to time, d/dt. With the equations expressed in the machine reference frame the derivative operator must be applied to the inductance-current product, since both may vary with time. The constant, $\frac{2}{3}$, is a function of referring variables.

In equation (11) the inductance matrix is made up of four smaller matrices. The $\mathbf{L}_s$ matrix relates stator winding flux linkage to stator current. The $\mathbf{L}_r$ matrix relates rotor winding flux linkage to rotor current. The $\mathbf{L}_{sr}$ matrix relates rotor windings to stator windings. These matrices have the following form:

$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_A - L_B \cos 2\theta_r & -\frac{1}{2}L_A - L_B \cos 2(\theta_r - \frac{\pi}{3}) & -\frac{1}{2}L_A - L_B \cos 2(\theta_r + \frac{\pi}{3}) \\ -\frac{1}{2}L_A - L_B \cos 2(\theta_r - \frac{\pi}{3}) & L_{ls} + L_A - L_B \cos 2(\theta_r - \frac{2\pi}{3}) & -\frac{1}{2}L_A - L_B \cos 2(\theta_r + \pi) \\ -\frac{1}{2}L_A - L_B \cos 2(\theta_r + \frac{\pi}{3}) & -\frac{1}{2}L_A - L_B \cos 2(\theta_r + \pi) & L_{ls} + L_A - L_B \cos 2(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \tag{12}$$

$$\mathbf{L}_{sr} = \begin{bmatrix} L_{mq} \cos \theta_r & L_{md} \sin \theta_r & L_{md} \sin \theta_r \\ L_{mq} \cos(\theta_r - \frac{2\pi}{3}) & L_{md} \sin(\theta_r - \frac{2\pi}{3}) & L_{md} \sin(\theta_r - \frac{2\pi}{3}) \\ L_{mq} \cos(\theta_r + \frac{2\pi}{3}) & L_{md} \sin(\theta_r + \frac{2\pi}{3}) & L_{md} \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \tag{13}$$

$$\mathbf{L}_r = \begin{bmatrix} L_{lkq} + L_{mq} & 0 & 0 \\ 0 & L_{lfd} + L_{md} & L_{md} \\ 0 & L_{md} & L_{lkd} + L_{md} \end{bmatrix} \tag{14}$$

The inductance terms in matrices (12), (13) and (14) are either self or mutual inductances. Elements on the main diagonal of the $\mathbf{L}_s$ and $\mathbf{L}_r$ matrices are self inductances and are made up of leakage and magnetizing inductance parts ($L_{lx} + L_{mx}$). Off diagonal elements of $\mathbf{L}_s$ and $\mathbf{L}_r$ and all elements of $\mathbf{L}_{sr}$ represent mutual inductances and therefore are assumed to have no leakage inductance part. The variables L_{mq} and L_{md} represent the total magnetizing inductance in the *q* and *d* axes respectively.

Because the rotor windings are wound on an orthogonal set of axes, the terms of $\mathbf{L}_r$ are easily determined. Orthogonal magnetic lines of flux do not combine so orthogonal windings have zero mutual inductance. The mutual inductance for windings which share a common axis is computed as for a transformer which is the product of the number of turns divided by the common reluctance.

The situation is more complicated for the $\mathbf{L}_s$ and $\mathbf{L}_{sr}$ matrices The inductance terms depend on rotor position. Because it will be shown that these matrices are greatly simplified by the Park's transformation, a detailed explanation of these matrix elements will not be made here . Figure 3 demonstrates how, in the case of a salient-pole machine, rotor position (which changes the size of the air gap) will have an impact on the reluctance path and therefore on the inductance.

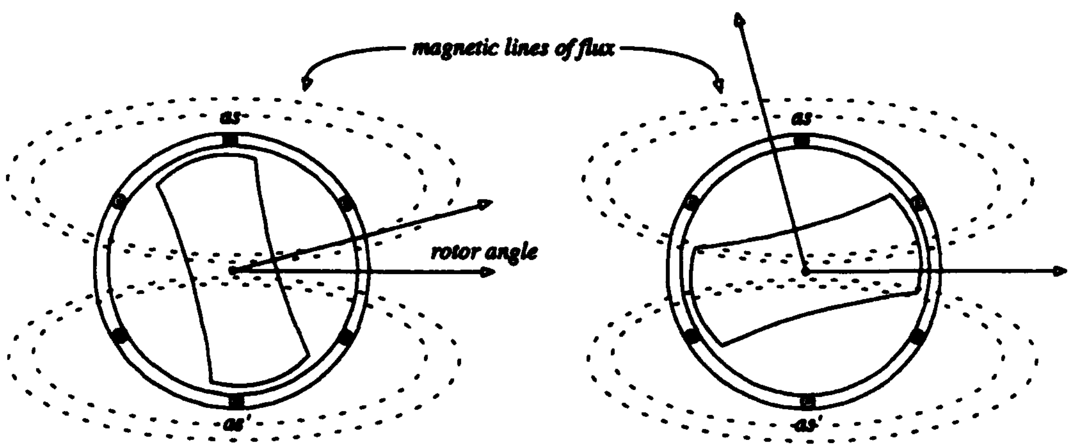

Figure 3. Rotor position influence on winding inductance.

For the *as* winding, minimum reluctance and maximum inductance ($L_A + L_B$) occurs when θ_r equals 90° and 270° while maximum reluctance and minimum inductance ($L_A - L_B$) is experienced at 0° and 180°. For a machine with a round rotor L_B is zero and the $\mathbf{L}_s$ terms are all constant. A complete description of the development of the θ_r dependent matrix terms may be found in Krause [Ref. 1:pp 211-227].

2. Machine Variable Torque Equation

The torque equation is developed based on the assumption of linearity in the λ-i relationship of an electromagnetic system. This allows the energy stored (W) in the electromagnetic system to be expressed by

$$W = \frac{1}{2} L i^2 \tag{15}$$

and in matrix-vector form

$$W = \frac{1}{2} \underline{i}^T \mathbf{L} \underline{i} \tag{16}$$

Energy or work is also the product of force and displacement. Using this basic definition yields

$$\begin{aligned} W &= T\theta \\ T &= \frac{\partial W}{\partial \theta} \\ T &= \frac{\partial}{\partial \theta}\left(\frac{1}{2} \underline{i}^T \mathbf{L} \underline{i}\right) \end{aligned} \tag{17}$$

where T is torque and θ is angular displacement. From equation (17) Krause [Ref. 1:p 217] goes on to fully develop the electrical torque equation in the machine reference frame for a P-pole generator

$$T_e = \left(\frac{P}{2}\right)\left\{-\frac{1}{2}(i_{abcs})^T \frac{\partial[\mathbf{L}_s - L_{ls}\mathbf{I}]}{\partial\theta_r} i_{abcs} + (i_{abcs})^T \frac{\partial[\mathbf{L}_{sr}]}{\partial\theta_r} i_{qdr}\right\} \tag{18}$$

The electrical torque, T_e, is positive for generator action when the stator current flows out of the stator terminals.

One more differential equation is needed to develop a state-space model. Each voltage equation is a function of currents, current derivatives and rotor position. Rotor position must be related to the system states. The second derivative of rotor position may be related to torque which in turn is a function of the system states. The final differential equation is obtained by writing the relationship for the mechanical system with friction, windage and other mechanical losses neglected

$$p\omega_r = \left(\frac{P}{2J}\right)(T_I - T_e) \tag{19}$$

where J is the inertia, P is number of poles and T_I is the prime mover input torque. It can be seen that when input torque is greater than the produced electrical torque the rotor acceleration is positive and the machine speeds up. A large load will cause current to rise and from equation (18) electrical torque will also rise resulting in deceleration of the machine.

B. SYNCHRONOUS GENERATOR EQUATIONS TRANSFORMED

For the salient pole synchronous generator in the machine reference frame, most inductance terms are highly dependent on θ_r the rotor angle, which in turn is dependent on time. This makes the flux linkage derivative term a complicated chain rule expression (inductance and current are both time varying). If, however, the terms of the inductance matrix could be transformed into constants, the flux linkage derivative could be simply expressed as

$$p\underline{\lambda} = \mathbf{L}p\underline{i} \qquad (20)$$

The voltage equations may be rewritten in compact form by assuming that such a transformation is possible and making the substitution suggested by equation (20). This yields

$$\underline{v} = \mathbf{r}\underline{i} + \mathbf{L}p\underline{i} \qquad (21)$$

Equation (21) then becomes the basis for a set of state equations describing the behavior of the synchronous generator. The relatively simple form of equation (21) is not possible when the voltage equations are expressed in the machine reference frame, in other words, with stator voltages, currents and inductances expressed in the *a-b-c* reference frame.

Fortunately R. H. Park developed a transformation making equation (21) possible. The Park's transformation eliminates time varying inductance terms and introduces a reference frame speed term, ω, which may be chosen to be rotor speed. Thus the Park's equations put the voltage equations in the orthogonal *q-d-0* reference frame of the rotor.

1. Transformed Voltage Equations

The transformation changes variables from the *a-b-c* frame to the *q-d-0* frame of reference. For an arbitrary vector variable $\underline{f}$, representing voltage or current, the transformation matrix $\mathbf{K}_s^r$ operates as follows

$$\underline{f}_{qd0s} = \mathbf{K}_s^r \underline{f}_{abcs} \tag{22}$$

where the transformation matrix is

$$\mathbf{K}_s^r = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{23}$$

and rotor position is defined as

$$\theta_r = \int_0^t \omega_r(\xi)d\xi + \theta_r(0) \tag{24}$$

The transformation is now applied to the voltage equations (12) with the following result:

$$\begin{bmatrix} \underline{v}_{qd0s} \\ \underline{v}_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_s & 0 \\ 0 & \mathbf{r}_r \end{bmatrix}\begin{bmatrix} -\underline{i}_{qd0s} \\ \underline{i}_{qdr} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_s^r p\mathbf{L}_s(\mathbf{K}_s^r)^{-1} & \mathbf{K}_s^r p\mathbf{L}_{sr} \\ \frac{2}{3}p(\mathbf{L}_{sr})^T(\mathbf{K}_s^r)^{-1} & p\mathbf{L}_r \end{bmatrix}\begin{bmatrix} -\underline{i}_{qd0s} \\ \underline{i}_{qdr} \end{bmatrix} \tag{25}$$

where the transformation applied to $\mathbf{r}_s$ yields

$$\mathbf{K}_s^r \mathbf{r}_s (\mathbf{K}_r^s)^{-1} = \mathbf{K}_s^r r_s \mathbf{I} (\mathbf{K}_r^s)^{-1} = r_s \mathbf{K}_s^r \mathbf{I} (\mathbf{K}_r^s)^{-1} = r_s \mathbf{I} = \mathbf{r}_s \qquad (26)$$

Because it has equal values on the main diagonal, $\mathbf{r}_s$ is not altered.

Multiplying out the terms of the inductance matrix gives surprising results. By carefully following the rules of matrix multiplication, applying the derivative operator in the proper sequence and using the correct trigonometric identities the voltage equations may now be expressed as

$$v_{qs} = -(r_s + pL_q)i_{qs} - L_d\omega_r i_{ds} + pL_{mq} i_{kq} + L_{md}\omega_r i_{fd} + L_{md}\omega_r i_{kd} \qquad (27)$$

$$v_{ds} = L_q\omega_r i_{qs} - (r_s + pL_d)i_{ds} - L_{mq}\omega_r i_{kq} + pL_{md} i_{fd} + pL_{md} i_{kd} \qquad (28)$$

$$v_{0s} = -(r_s + pL_{ls})i_{0s} \qquad (29)$$

$$0 = -pL_{mq} i_{qs} + (r_{kq} + pL_{kq})i_{kq} \qquad (30)$$

$$v_{fd} = -p\frac{L_{md}^{\ 2}}{r_{fd}} i_{ds} + (L_{md} + pL_{md}\frac{L_{fd}}{r_{fd}})i_{fd} + p\frac{L_{md}^{\ 2}}{r_{fd}} i_{kd} \qquad (31)$$

$$0 = -pL_{md} i_{ds} + pL_{md} i_{fd} + (r_{kd} + pL_{kd})i_{kd} \qquad (32)$$

where all inductance terms are constants and the following definitions apply

$$L_q = L_{ls} + L_{mq} \qquad (33)$$

$$L_d = L_{ls} + L_{md} \qquad (34)$$

$$L_{kq} = L_{lkq} + L_{mq} \qquad (35)$$

$$L_{fd} = L_{lfd} + L_{md} \qquad (36)$$

$$L_{kd} = L_{lkd} + L_{md} \qquad (37)$$

The voltage equations may be used in the form of equations (27) through (32), but it is more common to see the inductances expressed as reactances. This is accomplished by using the relationship $\omega_b L = X$. The inductance is multiplied by a base angular frequency (often 60 Hz). Machine parameters are usually provided in ohms. Making this change and putting the state equations in matrix form gives

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \\ 0 \\ v_{fd} \\ 0 \end{bmatrix} = \begin{bmatrix} -r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & -r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & r_{kq} & 0 & 0 \\ 0 & 0 & 0 & 0 & X_{md} & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{kd} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i_{kq} \\ i_{fd} \\ i_{kd} \end{bmatrix} + \begin{bmatrix} 0 & \frac{-X_d}{\omega_b} & 0 & 0 & \frac{X_{md}}{\omega_b} & \frac{X_{md}}{\omega_b} \\ \frac{X_q}{\omega_b} & 0 & 0 & \frac{-X_{mq}}{\omega_b} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_r i_{qs} \\ \omega_r i_{ds} \\ \omega_r i_{0s} \\ \omega_r i_{kq} \\ \omega_r i_{fd} \\ \omega_r i_{kd} \end{bmatrix} + $$

$$\cdots \begin{bmatrix} \frac{-X_q}{\omega_b} & 0 & 0 & \frac{X_{mq}}{\omega_b} & 0 & 0 \\ 0 & \frac{-X_d}{\omega_b} & 0 & 0 & \frac{X_{md}}{\omega_b} & \frac{X_{md}}{\omega_b} \\ 0 & 0 & \frac{-X_{ls}}{\omega_b} & 0 & 0 & 0 \\ \frac{-X_{mq}}{\omega_b} & 0 & 0 & \frac{X_{kq}}{\omega_b} & 0 & 0 \\ 0 & \frac{-(X_{mq}{}^2)}{r_{fd}\omega_b} & 0 & 0 & \frac{X_{md}X_{fd}}{r_{fd}\omega_b} & \frac{X_{md}{}^2}{r_{fd}\omega_b} \\ 0 & \frac{-X_{md}}{\omega_b} & 0 & 0 & \frac{X_{md}}{\omega_b} & \frac{X_{kd}}{\omega_b} \end{bmatrix} \begin{bmatrix} pi_{qs} \\ pi_{ds} \\ pi_{0s} \\ pi_{kq} \\ pi_{fd} \\ pi_{kd} \end{bmatrix} \qquad (38)$$

The right hand side of equation (38) is divided into three parts

$$\underline{v} = \mathbf{A_L}\underline{i} + \mathbf{A_N}\omega_r\underline{i} + \mathbf{B}p\underline{i} \qquad (39)$$

a linear part ($\mathbf{A}_L i$), a nonlinear part ($\mathbf{A}_N \omega_r i$) and a current derivative part. This type of equation is known as an *implicit* differential equation. This is because a particular state equation, for example equation (27)

$$v_{qs} = -(r_s + pL_q)i_{qs} - L_d\omega_r i_{ds} + pL_{mq}i_{kq} + L_{md}\omega_r i_{fd} + L_{md}\omega_r i_{kd}$$

cannot be written *explicitly*. That is with one state derivative isolated on the left and a combination of states only on the right. This is a drawback to this development since most simulation software prefers the explicit form for the differential equations of the model.

2. Transformed Torque Equation

The complete form of the final state equation takes shape in the transformed reference frame. The dependence on angular displacement has been eliminated and replaced with a speed term ω_r. The torque-acceleration equation will provide this final equation. Krause [Ref. 1:p 227] substitutes the reference frame transformation into equation (18)

$$T_e = \left(\frac{P}{2}\right)[(\mathbf{K}_s^r)^{-1} \underline{i}_{qd0s}]^T \left\{-\frac{1}{2}\frac{\partial[\mathbf{L}_s - L_{ls}\mathbf{I}](\mathbf{K}_s^r)^{-1}}{\partial\theta_r}\underline{i}_{qd0s} + (\underline{i}_{abcs})^T \frac{\partial[\mathbf{L}_{sr}]}{\partial\theta_r}\underline{i}_{qdr}\right\} \tag{40}$$

and after considerable work arrives at

$$T_e = \frac{3P}{4\omega_b}[X_{md}(-i_{ds} + i_{fd} + i_{kd})i_{qs} - X_{mq}(-i_{qs} + i_{kq})i_{ds}] \tag{41}$$

Now when equation (19) is used for the final state equation, the speed derivative can be expressed as a function of the other states.

C. CHOICE OF STATES

Most developments of the Park's equations arrive at a set of state equations expressed in terms of magnetic flux linkage. Krause [Ref. 2:pp. 177], Anderson [Ref. 6:pp. 85-88] and many others express a preference for the flux linkage expressions over the current expressions because they have explicit form. The implicit set of equations requires that a matrix inversion be performed or that some sophisticated, and often slow, routine be used to solve the problem. The matrix inversion often involves a poorly conditioned matrix and methods such as LU decomposition may introduce significant error.

Sources and loads on a common system bus do not share flux linkage but do share currents (and therefore current derivatives). Because the goal here is to develop a model which allows an entire power system to be built up in a modular fashion, the system submodels will be expressed in terms of current. Also, intuitively, bus voltage must have some functional relationship to the bus current. Solution of the finite bus problem relies on choosing to express system state equations in terms of current.

In order to put the system in explicit form, equation (39) is manipulated to isolate the state derivative on the left hand side of the equation

$$p\underline{i} = \mathbf{V}(-\mathbf{A_L})\underline{i} + \mathbf{V}(-\mathbf{A_N})\omega_r\underline{i} + \mathbf{V}\underline{v} \tag{42}$$

where $\mathbf{V} = \mathbf{B}^{-1}$. This involves the matrix inversion mentioned above and therefore the possibility of error.

In order to minimize the possibility of error, the B matrix was inverted symbolically so that the matrix terms of equation (42) could be expressed as functions of the given machine parameters. MATHCAD 4.0 [Ref. 7] was used to perform the matrix inversion and surprisingly the terms were not terribly unwieldy. Appendix A contains the MATHCAD output showing the symbolically inverted matrix. These results were then used to write the final form of the state equations for the simulation. The explicit form of the system state equations may be seen in ACSL simulation code of Appendix B and are described in Chapter VI.

III. DEVELOPMENT OF LOAD MODELS

The next step in the process of modeling a finite bus power system involves modeling the system loads. In order to have a system simulation in which loads and sources can be put together in a modular fashion, the load model equations are developed with current states.

Two load models are looked at, a simple R-L load and an induction motor. The final simulation allows for either type load to be connected to the bus alone or for both type loads to be connected in parallel.

The choice of load model was motivated by the fact that even in isolated systems, such as a shipboard system, the system load can be looked at as a nearly constant power factor load most of the time. An R-L load model, allowing for the time varying of its resistive and reactive parts, adequately simulates many loading conditions.

The induction motor is a very common large load onboard ship. Fire pumps, hydraulic pumps and large ventilation fans are some of the uses for induction motors. For this reason an induction motor model was also chosen for a system load.

A. THE R-L LOAD MODEL

The three-phase R-L load is represented by Figure 4. The diagram may represent a balanced or unbalanced system. That is the resistance and inductance of each phase may or may not be equal. In practice considerable effort is made to balance the system load, however, power systems are frequently subjected to unbalanced loading conditions. The model will be developed for a balanced load. Methods to handle the unbalanced case will be discussed in Chapter VI.

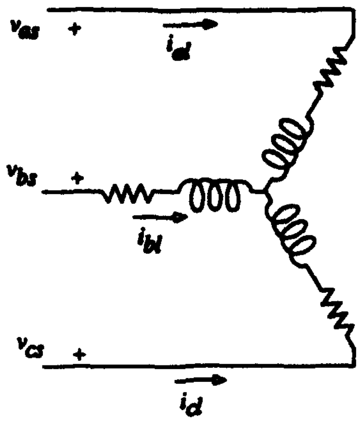

Figure 4. Three-phase R-L circuit.

The mathematical development for the load parallels the generator model development. First the voltage equations are written down as

$$v_{as} = r_l i_{al} + p\lambda_{al} \tag{43}$$

$$v_{bs} = r_l i_{bl} + p\lambda_{bl} \tag{44}$$

$$v_{cs} = r_l i_{cl} + p\lambda_{cl} \tag{45}$$

or in matrix form as

$$\underline{v}_{abcs} = \mathbf{r}_l \underline{i}_{abcl} + \mathbf{L}_l p \underline{i}_{abcl} \tag{46}$$

where the balanced resistance matrix is

$$\mathbf{r}_l = \begin{bmatrix} r_l & 0 & 0 \\ 0 & r_l & 0 \\ 0 & 0 & r_l \end{bmatrix} \tag{47}$$

and the balanced inductance matrix is

$$\mathbf{L}_l = \begin{bmatrix} L_l & 0 & 0 \\ 0 & L_l & 0 \\ 0 & 0 & L_l \end{bmatrix} \tag{48}$$

If the mutual inductance between phases is assumed negligible, the inductance matrix has terms only on the main diagonal. It is a fairly simple matter to include off-diagonal (mutual inductance) terms since the matrix still will have no time dependent terms. Note also that the derivative operator in (45) applies only to the current since there are no time varying terms in $\mathbf{L}_l$.

Next the load must be converted to the same reference frame (*q-d-0*) as the generator model. The same transformation used in equation (25) is applied to the R-L load equations. These equations may now be written as

$$\underline{v}_{qd0s} = \mathbf{r}_l \underline{i}_{qd0l} + \mathbf{K}_s^r[\mathbf{L}_l p(\mathbf{K}_s^r)^{-1} \underline{i}_{qd0l}] \tag{49}$$

Since both the transformation matrix and current vary with time the product rule for differentiation yields

$$\underline{v}_{qd0s} = \mathbf{K}_s^r \mathbf{r}_l (\mathbf{K}_s^r)^{-1} \underline{i}_{qd0l} + (\mathbf{K}_s^r \mathbf{L}_l p(\mathbf{K}_s^r)^{-1}) \underline{i}_{qd0l} + \mathbf{K}_s^r \mathbf{L}_l (\mathbf{K}_s^r)^{-1} p \underline{i}_{qd0l} \tag{50}$$

Using the same approach as (26), the first and third term on the right hand side are easily obtained. The second term illustrates how the speed term was introduced in equations

(27) through (32). The p operator is applied to the transformation matrix inverse with the following result

$$p(\mathbf{K}_s^r)^{-1} = p\begin{bmatrix} \cos\theta_r & \sin\theta_r & 1 \\ \cos(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r - \frac{2\pi}{3}) & 1 \\ \cos(\theta_r + \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) & 1 \end{bmatrix} = \omega_r\begin{bmatrix} -\sin\theta_r & \cos\theta_r & 0 \\ -\sin(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) & 0 \\ -\sin(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) & 0 \end{bmatrix}$$

(51)

then

$$\mathbf{K}_s^r\mathbf{L}_l\omega_r\begin{bmatrix} -\sin\theta_r & \cos\theta_r & 0 \\ -\sin(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) & 0 \\ -\sin(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) & 0 \end{bmatrix} = \mathbf{L}_l\omega_r\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (52)$$

Finally, after substituting reactance for inductance, the state equations for the load in *q-d-0* reference may be written in expanded form as

$$v_{qs} = r_l i_{ql} + \omega_r \frac{X_l}{\omega_b} i_{dl} + \frac{X_l}{\omega_b} p i_{ql} \qquad (53)$$

$$v_{ds} = -\omega_r \frac{X_l}{\omega_b} i_{ql} + r_l i_{dl} + \frac{X_l}{\omega_b} p i_{dl} \qquad (54)$$

$$v_{0s} = r_l i_{0l} + \frac{X_l}{\omega_b} p i_{0l} \qquad (55)$$

Equations (53) through (55) represent an explicit form of the load model state equations. These equations will easily "plug in" to the final system model.

B. THE INDUCTION MOTOR LOAD MODEL

Unlike the simple R-L load, the induction motor model does not result in a set of state equations which are explicit in current states. The induction motor model development is somewhat like the synchronous machine model. The development here will not be as detailed as the synchronous machine model was. However, key differences in the equations based on machine geometry and operating theory will be explained before the final set of equations is presented.

1. Voltage Equation Development

Figure 5 represents the two-pole, three-phase, induction motor load. The stator windings of this machine are identical, sinusoidally distributed and spaced 120° apart. As for the synchronous machine, these windings are designated *as*, *bs* and *cs* each having N_s equivalent turns. The rotor arrangement, however, is considerably different from the synchronous machine configuration.

For the model development, rotor windings are considered to be identical sinusoidally distributed windings spaced 120° apart on the rotor with N_r equivalent turns. The rotor windings are designated *ar*, *br* and *cr*. The rotor displacement and speed are represented by θ_{rm} and ω_{rm} respectively. The rotor windings are all shorted, although machines are available which allow the rotor windings to be excited externally.

The assumptions are not entirely valid because many, if not most, induction motors are of the squirrel-cage variety. In this type of machine the rotor windings consist of metal bars laid into the rotor which are shorted at the ends. Krause points out that in most cases uniformly distributed rotor bars are adequately described by the sinusoidal assumption [Ref. 1:p 167].

One obvious difference in the induction motor geometry from that of the synchronous machine is the shape of the rotor. Because the rotor is round, none of the terms of the stator-stator inductance matrix depend on θ_{rm}. The only rotor position dependence is seen in the inductance terms relating stator to rotor windings. Note that the subscript for rotor position and speed is *rm* to differentiate it from the synchronous machine rotor speed, ω_r.

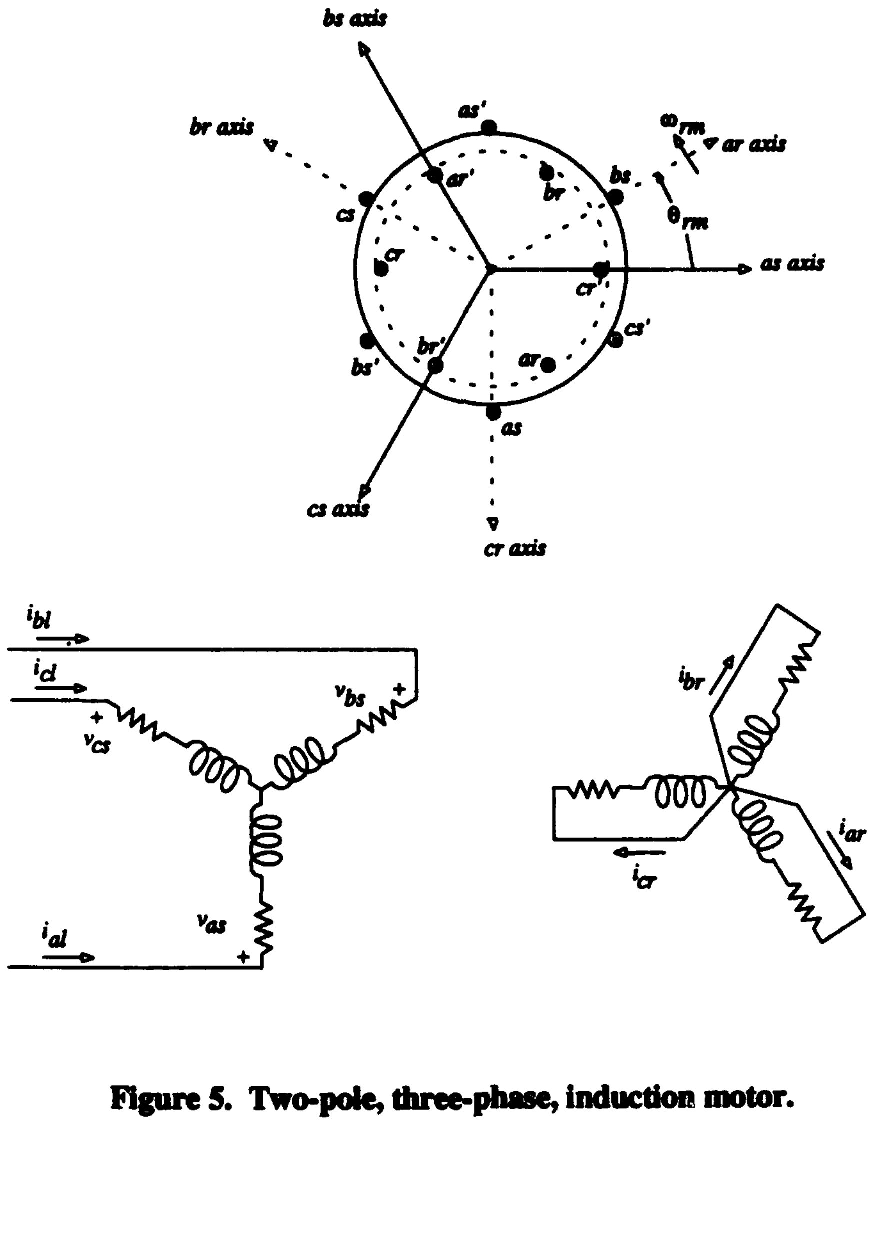

Figure 5. Two-pole, three-phase, induction motor.

Performing a Kirchoff's Voltage Law (KVL) sum around each loop allows the voltage equations to be written in compact form as

$$\begin{bmatrix} \underline{v}_{abcs} \\ \underline{0} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{r}_r \end{bmatrix} \begin{bmatrix} \underline{i}_{abcs} \\ \underline{i}_{abcr} \end{bmatrix} + p \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ (\mathbf{L}_{sr})^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \underline{i}_{abcs} \\ \underline{i}_{abcr} \end{bmatrix} \tag{56}$$

where all variables are referred to the stator via the turns ratio. The zero elements of the voltage vector are due to the fact that the rotor windings are shorted on the ends.

The inductance matrix is made up of smaller matrices. In the expressions below, the leakage inductances are L_{ls} and L_{lr} for the stator and rotor windings respectively. With all variables referred to the stator windings, the only magnetizing inductance term appearing in the matrices is L_{ms} (the stator magnetizing inductance). The matrices consist of

$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \tag{57}$$

$$\mathbf{L}_r = \begin{bmatrix} L_{lr} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{lr} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{lr} + L_{ms} \end{bmatrix} \tag{58}$$

$$\mathbf{L}_{sr} = L_{ms}\begin{bmatrix} \cos\theta_{rm} & \cos(\theta_{rm} + \frac{2\pi}{3}) & \cos(\theta_{rm} - \frac{2\pi}{3}) \\ \cos(\theta_{rm} - \frac{2\pi}{3}) & \cos\theta_{rm} & \cos(\theta_{rm} + \frac{2\pi}{3}) \\ \cos(\theta_{rm} + \frac{2\pi}{3}) & \cos(\theta_{rm} - \frac{2\pi}{3}) & \cos\theta_{rm} \end{bmatrix} \tag{59}$$

The voltage equations must be transformed to the orthogonal reference frame. Applying the transformation matrix, $\mathbf{K}_s^r$, results in

$$\begin{bmatrix} \underline{v}_{qd0s} \\ \underline{0} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_s & 0 \\ 0 & \mathbf{r}_r \end{bmatrix}\begin{bmatrix} \underline{i}_{qd0s} \\ \underline{i}_{qd0r} \end{bmatrix} + p\begin{bmatrix} \mathbf{K}_s^r\mathbf{L}_s(\mathbf{K}_s^r)^{-1} & \mathbf{K}_s^r\mathbf{L}_{sr}(\mathbf{K}_s^r)^{-1} \\ \mathbf{K}_s^r(\mathbf{L}_{sr})^T(\mathbf{K}_s^r)^{-1} & \mathbf{K}_s^r\mathbf{L}_r(\mathbf{K}_s^r)^{-1} \end{bmatrix}\begin{bmatrix} \underline{i}_{qd0s} \\ \underline{i}_{qd0r} \end{bmatrix} \tag{60}$$

and after performing the multiplication and making the reactance for inductance substitution, the equations, in the separated form of equation (38) become

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & r_r & 0 & 0 \\ 0 & 0 & 0 & 0 & r_r & 0 \\ 0 & 0 & 0 & 0 & 0 & r_r \end{bmatrix}\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i_{qr} \\ i_{dr} \\ i_{0r} \end{bmatrix} + \begin{bmatrix} 0 & \frac{\omega_r X_{ss}}{\omega_b} & 0 & 0 & \frac{\omega_r X_M}{\omega_b} & 0 \\ -\frac{\omega_r X_{ss}}{\omega_b} & 0 & 0 & -\frac{\omega_r X_M}{\omega_b} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\omega_\delta X_M}{\omega_b} & 0 & 0 & \frac{\omega_\delta X_{rr}}{\omega_b} & 0 \\ -\frac{\omega_\delta X_M}{\omega_b} & 0 & 0 & -\frac{\omega_\delta X_{rr}}{\omega_b} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i_{qr} \\ i_{dr} \\ i_{0r} \end{bmatrix} +$$

$$\cdots\begin{bmatrix} \frac{X_{ss}}{\omega_b} & 0 & 0 & \frac{X_M}{\omega_b} & 0 & 0 \\ 0 & \frac{X_{ss}}{\omega_b} & 0 & 0 & \frac{X_M}{\omega_b} & 0 \\ 0 & 0 & \frac{X_{ls}}{\omega_b} & 0 & 0 & 0 \\ \frac{X_M}{\omega_b} & 0 & 0 & \frac{X_{rr}}{\omega_b} & 0 & 0 \\ 0 & \frac{X_M}{\omega_b} & 0 & 0 & \frac{X_{rr}}{\omega_b} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{X_{lr}}{\omega_b} \end{bmatrix}\begin{bmatrix} pi_{qs} \\ pi_{ds} \\ pi_{0s} \\ pi_{qr} \\ pi_{dr} \\ pi_{0r} \end{bmatrix} \tag{61}$$

where the following definitions apply

$$X_M = \omega_b L_{ms} \tag{62}$$

$$X_{ss} = \omega_b (L_{ls} + L_{ms}) \tag{63}$$

$$X_{rr} = \omega_b (L_{lr} + L_{ms}) \tag{64}$$

$$\omega_\delta = (\omega_r - \omega_{rm}) \tag{65}$$

It is interesting to note that the speed term developed when the transformation is applied to the L_{sr} matrix (ω_δ), is the difference between the reference frame speed (ω_r)and the motor's rotor speed(ω_{rm}). The reference frame speed usually chosen for induction motor simulation is the bus voltage electrical angular frequency. For the system model being developed here, electrical frequency is defined by the rotor speed of the synchronous generator. This speed difference term, known as slip speed, is basic to the operation of the induction motor. The rotor currents will be zero at steady state unless the reference speed and rotor speed are unequal. Rotor current will be shown necessary to produce torque in the next section.

One more equation is needed to complete the state-space model. As was the case for the synchronous machine model, the speed term is related to the other states via the torque equation.

2. Torque Equation Development

Using the argument based on energy stored in an electromagnetic system the electrical torque developed by an induction motor can be shown to be

$$T_e = \left(\frac{P}{2}\right)(i_{abcs})^T \frac{\partial(\mathbf{L}_{sr})}{\partial\theta_{rm}} i_{abcr} \tag{66}$$

for a P-pole motor [Ref. 1:pp 169-170]. Equation (66) does not involve $\mathbf{L}_s$ or $\mathbf{L}_r$ because only the $\mathbf{L}_{sr}$ matrix is dependent on θ_{rm}. This equation shows that torque will be zero when rotor current is zero.

Application of the transformation matrix to (66) yields

$$T_e = \left(\frac{P}{2}\right)[(\mathbf{K}_s^r)^{-1} i_{qd0s}] \frac{\partial(\mathbf{L}_{sr}\mathbf{K}_r^r)}{\partial\theta_{rm}} i_{qd0r} \tag{67}$$

which in terms of currents may be expressed as

$$T_e = \left(\frac{3P}{4}\right)\frac{X_M}{\omega_b}(i_{qs}i_{dr} - i_{ds}i_{qr}) \tag{68}$$

with T_e positive for motor action. With an expression for torque as a function of current states the final state equation may now be written.

The friction and windage losses are once again neglected and the differential equation for the mechanical system is written down. In the case of a motor, electrical torque will accelerate the rotor while applied load torque (T_l) will slow the rotor down. The equation describing this is

$$p\omega_{rm} = \left(\frac{P}{2J_m}\right)(T_e - T_l) \tag{69}$$

where J_m is the inertia of the motor's rotor and P is the number of poles.

3. Explicit Form of the Induction Motor Model

The voltage equations (61) may be manipulated in the same manner as those for the synchronous machine in order to put them in explicit form. MATHCAD was again used to symbolically invert the matrix associated with the state derivatives. The result of the matrix inversion is contained in Appendix A and the full form of the state equation may be seen in the ACSL code of Appendix B and are described in Chapter VI.

IV. THE BUS VOLTAGE MODEL

The next step in building a total system model based on Figure 1 involves connecting the source model with the load model (or load models in parallel). The physical entity which joins load and source is the system bus. Figure 6 is a block diagram of the system to be modeled without the field excitation or speed regulator loops closed.

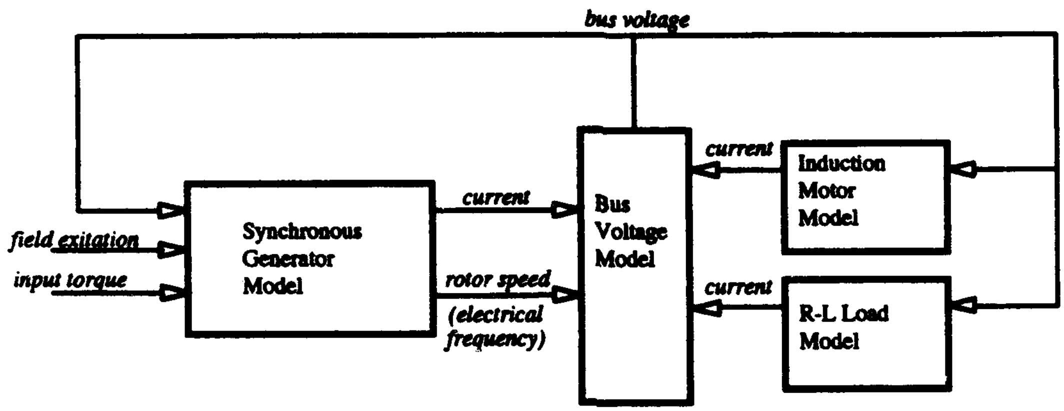

Figure 6. Isolated power system block diagram.

The source and load models have been discussed in some detail, now a model for the system bus voltage must be developed. The simplest approach, and the approach most frequently taken, is to assume the bus voltage is of fixed magnitude and frequency. This is the so called infinite bus assumption.

Another method of modeling the bus voltage is to attempt to develop a dynamic mathematical expression for bus voltage. The difficulty with this approach is that the previously mentioned algebraic loop problem must be avoided. As a practical matter, the simulation code cannot use bus voltage to solve for the current derivative terms if bus voltage is described as a function of those current derivatives.

Two methods are presented for dealing with the algebraic loop. The first uses algebraic manipulation to eliminate current derivative terms from the equation describing bus voltage. The second method involves treating the total system as a large implicit model by using the DASSL algorithm [Ref. 8].

A. INFINITE BUS MODEL

Use of the infinite bus model greatly simplifies the power system simulation. Anderson [Ref. 2:p. 26] notes, "A major bus of a power system of a very large capacity compared to the rating of the machine under consideration is approximately an infinite bus". Simulations of this type have been done extensively and are well understood. The generator and load models presented in Chapter II and III have been validated as infinite bus models. Park [Ref. 2], Krause [Ref. 1], Anderson [Ref. 6] and many others have demonstrated the validity of these models with both flux linkage and current states. However, for reasons previously mentioned, the finite bus model will not be used for the isolated power system.

B. PARALLEL LARGE RESISTANCE MODEL

Another method of modeling bus voltage so that it may be used as a varying input to all the system submodels is to connect a large parallel resistance on the bus. Figure 7 shows this conceptually. The use of a very large resistance allows source current and load current to be approximately equal. The bus voltage may be computed using Ohm's law and then fed back as an input to the load and source models. This approach eliminates voltage dependence on the current derivative but does not accurately model the system.

Additionally, before any other computational errors are accounted for, some accuracy is sacrificed because a small current is bled off by the resistor. While this small current may not be significant in many applications, this solution method is not as satisfying as the methods which follow.

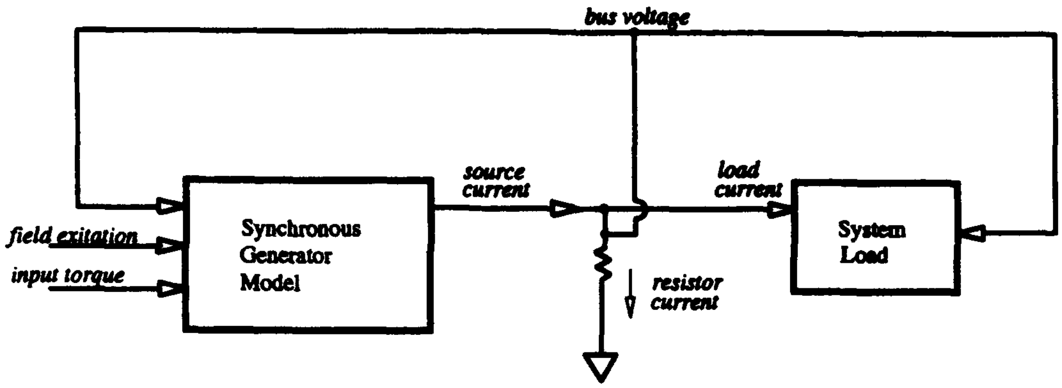

Figure 7. Parallel resistor bus voltage model.

C. MATHEMATICAL EXPRESSION FOR BUS VOLTAGE

One satisfying method of solving the bus voltage model dilemma is to develop a mathematical expression based on well understood and accepted theory. At the node connecting load and source the sum of the currents is zero by Kirchoff's Current Law (KCL). It also follows, because differentiation is a linear operation, that the sum of current derivatives is zero.

The algebraic loop difficulty is introduced when current derivative terms show up in the mathematical expression for bus voltage. This is because bus voltage is an input to the equations from which current derivatives are computed. Most simulation software will fail when algebraic loops are encountered. This problem can be addressed by finding a way to eliminate the current derivatives from the bus voltage equation.

In order to demonstrate this approach, a simplified system will be examined. Figure 8 represents a circuit containing a voltage source, V_s, and passive elements L_1, L_2, R_1 and R_2. A single current, i_s, flows in the circuit and the differential equation describing the system is

$$V_s = (L_1 + L_2)\frac{di_s}{dt} + (R_1 + R_2)i_s \tag{70}$$

Later in the chapter this circuit will be placed under closed loop control. For control system work, the s-domain transfer function form of the system equation is

$$\frac{I_s(s)}{V_s(s)} = \frac{1}{(L_1 + L_2)s + (R_1 + R_2)} \tag{71}$$

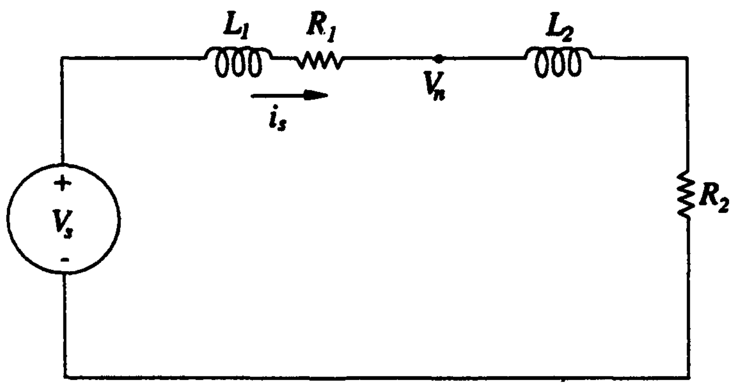

Figure 8. Simple circuit to demonstrate bus voltage equation representation.

Once the current is known it is possible to compute the dynamic voltage behavior at the node. This voltage, V_n, is the voltage appearing across the elements L_2 and R_2. The differential equation for this quantity is

$$V_n = L_2 \frac{di_s}{dt} + R_2 i_s \tag{72}$$

The transfer function from current to node voltage is

$$\frac{V_n(s)}{I_s(s)} = L_2 s + R_2 \tag{73}$$

and the transfer function relating source voltage to node voltage may now be written as the product of (71) and (73)

$$\frac{V_n(s)}{I_s(s)} \frac{I_s(s)}{V_s(s)} = \frac{V_n(s)}{V_s(s)} = \frac{L_2 s + R_2}{(L_1 + L_2)s + (R_1 + R_2)} \tag{74}$$

The transfer function representation of the system for node voltage given source voltage has no delay (the order of the numerator is not smaller than the order of the denominator). This is the way the algebraic loop phenomenon manifests itself in the s-domain. Node voltage can not be properly fed back to contribute to the source voltage without adding a controller delay. As an open loop transfer function, equation (74) may be simulated in many software packages. The system current and node voltage may also be computed using equations (70) and (72).

The ultimate goal of the system simulation being developed here is to be able to take independent submodels and tie them together with a bus voltage model. This modular approach has the advantage of not requiring the entire model to be redeveloped if one of the sources or loads changes. It is not desirable in complicated systems to develop a new system model each time a submodel changes. Figure 9 shows how the basic circuit of

Figure 8 may be broken in two pieces, source and load, to demonstrate the modular approach.

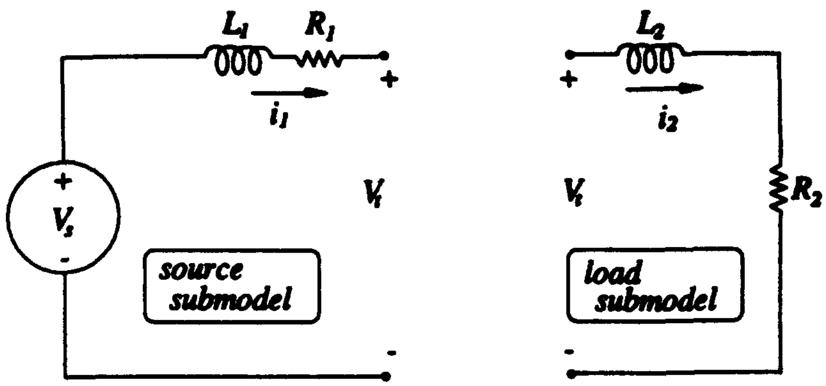

Figure 9. Dividing the system into submodels.

The equations describing each submodel may then be obtained independently. For the source submodel

$$pi_1 = -\frac{R_1}{L_1} i_1 + \frac{(V_s - V_t)}{L_1} \tag{75}$$

and for the load submodel

$$pi_2 = -\frac{R_2}{L_2} i_2 + \frac{V_t}{L_2} \tag{76}$$

These submodels are accurate and it is easy to see that if terminal (bus) voltage, V_t, is a fixed value (infinite bus) both equations are easily solved. However, a solution which models the source-load *interaction* and the terminal voltage dynamic behavior is desired. In order to do this, terminal voltage must be solved at each time step of the simulation.

The terminal voltage is then fed without delay to the set of equations describing source and load. The previously derived expression for node voltage, equation (72), can not be used. A current derivative term appears in this expression which will cause an algebraic loop error.

One possible solution is to develop an expression which eliminates the derivative terms. Based on KCL at the connecting node the current derivatives may be set equal. The resulting equation for terminal voltage has no derivative terms

$$V_t = \left(\frac{L_1 L_2}{L_1 + L_2}\right)\left(-\frac{R_1}{L_1} i_1 + \frac{R_2}{L_2} i_2 + \frac{V_s}{L_1}\right) \tag{77}$$

This allows V_t to be solved at each time step of the simulation and then fed into the submodel equations. The system of Figures 8 and 9 was modeled in ACSL. The model was formulated using both the single system approach and the modular system approach. Appendix B contains the ACSL code used for this simulation. The model parameters used were

$$R_1 = 1.0\Omega \qquad R_2 = 5.0\Omega$$
$$L_1 = 0.6\text{H} \qquad L_2 = 0.2\text{H}$$

and the source voltage was a .5 duty cycle square wave pulse train with magnitude equal to 1.0 and a period of 2.5 seconds.

Figure 10 compares the circuit response using the single loop model with the response obtained from the submodel approach. The single loop solution for voltage, VNODE, is computed from equation (72) after the differential equation for current (70) is solved. The plot labeled VT is the solution for the terminal voltage using the

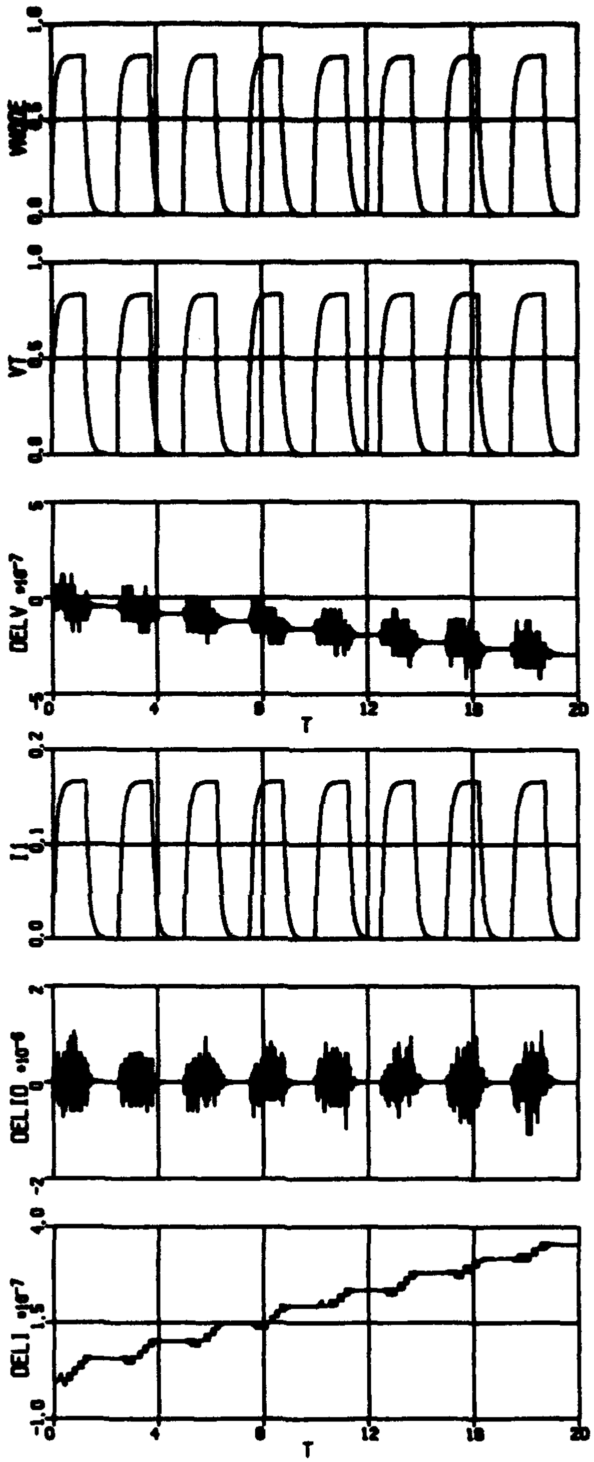

Figure 10. Comparison of *single loop* solution with solution obtained using the *bus voltage equation* submodel approach.

submodel approach and equation (77). A voltage is computed at each time step then fed as an input to the source and load submodel equations (75) and (76). The difference between the two solution methods is DELV.

Comparison of the current solution in each submodel is also significant. Ideally, the two currents will be equal since the submodels share a common node. DELI is the difference between i_1 and i_2 and DELID is the variation between pi_1 and pi_2. Also plotted is i_1, representing current flowing in the system.

Figure 10 demonstrates that the bus voltage equation submodel yields good results. The magnitude of error in voltage and current is extremely small and this approach achieves the desired modularity of the system model. There are, however, some disadvantages to this approach.

The current error variation (DELI), although small, is still growing at the end of the run. This is due to the formulation of the bus voltage equation. The current derivatives are set equal but nothing in the equation forces the currents to stay together.

Another problem is that the bus voltage equation is not simple for complex systems. The equations for the synchronous generator with an R-L load are quite complicated and with an induction motor load are more so. This approach also requires that the bus voltage model be redeveloped for different source and load submodels. The addition of submodels in parallel on the bus creates difficulties for the same reasons as just mentioned for complex systems.

D. DASSL BUS VOLTAGE MODEL

In order to have a system model which is truly modular, a better solution than the bus voltage equation must be found. The requirement to find a new and often complicated expression for the bus voltage each time a submodel is altered becomes extremely unwieldy and inconvenient. The previously mentioned DASSL algorithm provides a more satisfactory solution to the problem.

1. How DASSL Works

The basic idea of the DASSL routine is to replace the derivative in the implicit equation with a difference approximation and then solve, using Newton's method, for the derivative at the current time step. It has been implemented in the commercially available simulation package ACSL. The class of problem DASSL is designed to solve are known as differential algebraic equations (DAE). DAEs include systems of implicit differential equations and systems of equations containing algebraic loops.

To show how the routine works the load-source-bus problem will be set up as a DAE or implicit equation. One way to write the general DAE [Ref. 9:p. 41] which applies to the case of the load, source and bus voltage models is to represent the load and source model equations as a system of differential equations of the form

$$p\underline{i} = f_1(\underline{i}, \underline{v}, t) \tag{78}$$

coupled to an algebraic constraint

$$\underline{0} = g_1(\underline{i}, \underline{v}, t) \tag{79}$$

If it is then assumed that voltage is some unknown function of the state and state derivative vectors

$$\underline{v} = h(p\underline{i}, \underline{i}, t) \tag{80}$$

the system equations may be represented in a fully implicit form by substituting for voltage in equations (78) and (79). Implicit equations of this type have been solved most successfully using backward differentiation formulas (BDF) [Ref. 9:p.42].

The simplest BDF method involves replacing the derivative with a first order backwards difference. DASSL extends the idea of the BDF. Rather than using the first order difference, the derivative is approximated by the k^{th} order difference and k ranges from one to five. Order and step size are automatically selected based on the behavior of the solution. Because of the flexibility of the routine, DASSL has been shown to be highly stable and robust [Ref. 9:pp. 115-116].

Two criteria must be met to successfully solve a DAE system. The system must be *solvable* and *implementable* [Ref. 9:p. 16]. *Solvable* means that the states are differentiable over the time interval of interest. The solution must be smooth enough to make this possible. *Implementable* refers to the solution technique. The method used to solve the nonlinear system of equations must provide a solution at each time step. In the case at hand, as is often done, the solvability will be assumed. The current states are relatively smooth functions. The implementability of the DASSL routine is also assumed.

The routine as implemented in ACSL may be used to solve implicit differential equations, algebraic loops and systems of differential equations with an algebraic constraint. Conceptually the bus voltage problem may be looked at as a system of state equations with an algebraic constraint based on Kirchoff's Current Law. In order to solve for a node voltage in ACSL using the implicit equation solver the constraint equation must first be defined. Based on KCL at a connecting node, a residual, r, may be defined

$$r = \sum_k p i_k + \sum_k i_k \tag{81}$$

where it is desired to keep the residual at or near zero. The node voltage, which is assumed implicitly related to this KCL equation, may then be found by using the DASSL implicit solver. The operator in ACSL is IMPLC, and the syntax is

$$V_{node} = \text{IMPLC}(r, V_{node,ic}) \tag{82}$$

where $V_{node,ic}$ is the node voltage initial condition [Ref. 10:pp. 1-4].

2. The Advantage of the DASSL Bus Voltage Model

The fact that the constraint equation (81) for the system is simple regardless of the complexity of subsystems connected at the node of interest gives the DASSL bus voltage model a great advantage over the mathematical model presented in the previous section. It is a simple matter to add or remove loads and sources from the larger system model. No reformation of the bus voltage model is required. Additionally, the constraint equation keeps the error between submodel currents close to zero. Even if a transient occurs which makes the error grow momentarily, the implicit solver adjusts voltage to move the current error back towards zero.

The simple system of figure 9 was simulated using the implicit feature of ACSL. The same model parameters were used as in the simulation results presented in figure 10. The code used may be seen in Appendix B. Figure 11 contains the simulation results using the DASSL routine. As with the results of the bus voltage equation submodel, the DASSL bus voltage submodel is in excellent agreement with the single loop solution. Notice the improvement in current derivative error and current error, DELID and DELI respectively.

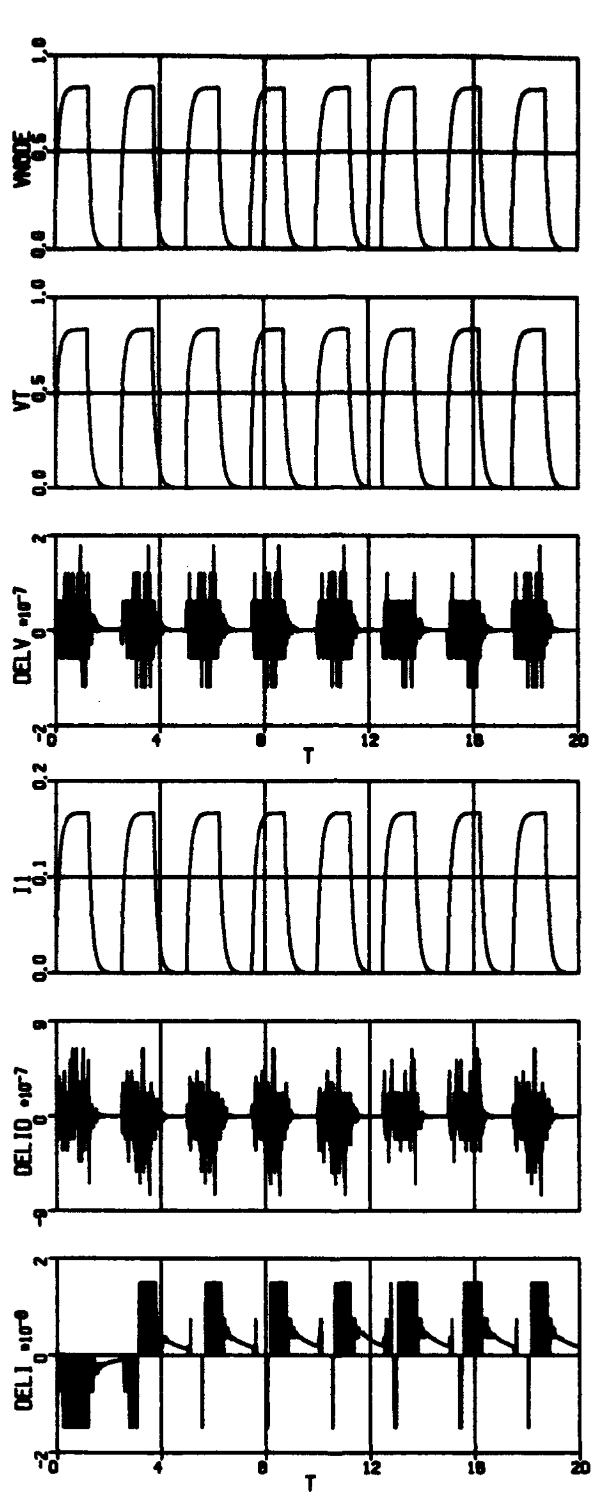

Figure 11. Comparison of *single loop* solution with solution obtained using the *DASSL bus voltage* submodel approach

E. THE BUS VOLTAGE MODEL AND CLOSED LOOP CONTROL

One final characteristic required of the bus voltage model is that its output be able to be fed to a bus voltage control circuit. The power system model which is being developed here requires that field excitation for the generator be controlled in order to maintain the bus voltage at or near specification. To accomplish this, the bus voltage submodel must accurately track the terminal voltage behavior during transients so that the control circuit submodel responds correctly.

In order to demonstrate the behavior of the bus voltage submodel, the source-load system of Figure 9 will again be used. The transfer function relating source voltage to terminal voltage was developed and given as equation (74). Using this transfer function form of the system model a control system may be developed. In this case a cascade controller was designed using the root locus technique which produced a highly damped response with a fast settling time and small steady state error. The transfer function design is shown in Figure 12.

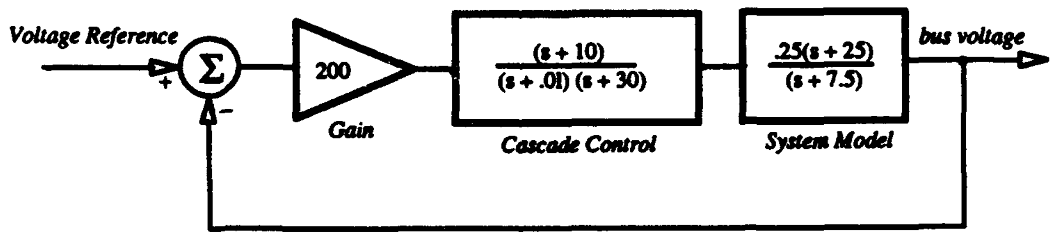

Figure 12. Transfer function form of the simple source-load system with bus voltage control.

The closed loop response was modeled three ways using ACSL. The transfer function form of the system model was compared with a closed loop form using both the bus voltage equation and the DASSL routine. In the simulation runs, system parameters were initially set to the values given on page 39. The voltage reference was then step changed from zero to one. All forms of the system model produced a similar step response. Next the load parameters were step changed to $R_2 = 0.01$ and $L_2 = 0.001$. The two submodel forms of the system exhibited similar behavior, a large increase in load current and a large initial dip in bus voltage. This transient was followed by bus voltage recovery to the commanded value.

Figure 13 shows the response of the closed loop system to a step change in the voltage reference from zero to one. Results for both bus voltage submodels are shown. The plot labeled DELV is the difference between the transfer function solution and the indicated submodel. Both bus voltage submodels provide very good results and are in excellent agreement with the transfer function model. Note the plot of DELI which is moving away from zero in the bus voltage equation submodel.

Figure 14 is the response of the system to the step load change. The DASSL results are in very good agreement with the bus voltage equation submodel. The difference to note is again the DELI plot, Figure 14 (b). The simulation was allowed to continue to 100 seconds. The DASSL routine, while allowing some error between load and source current, keeps forcing the error back toward zero. The bus voltage equation allows the DELI error to grow 50 to 100 times larger than the DASSL routine allows.

F. CHOICE OF BUS VOLTAGE MODEL

While the methods presented here are by no means exhaustive, several reasonable possibilities for a bus voltage model have been looked at. Some advantages and disadvantages of each have been discussed. Remember that modularity and simplicity are desired characteristics of the total system model.

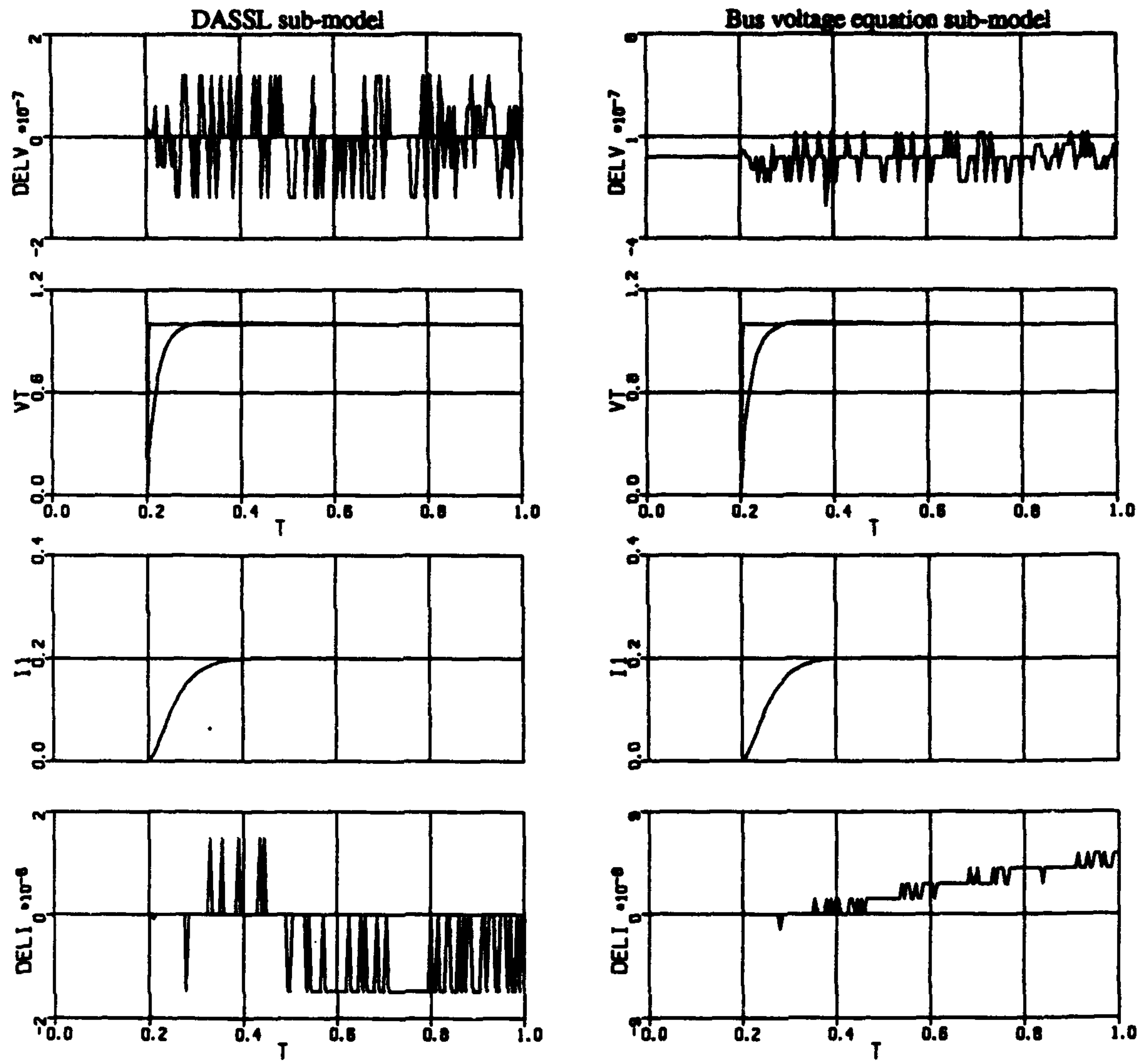

Figure 13. System response of both voltage submodels to step change in reference voltage. DELV compares model response to a transfer function form of the system.

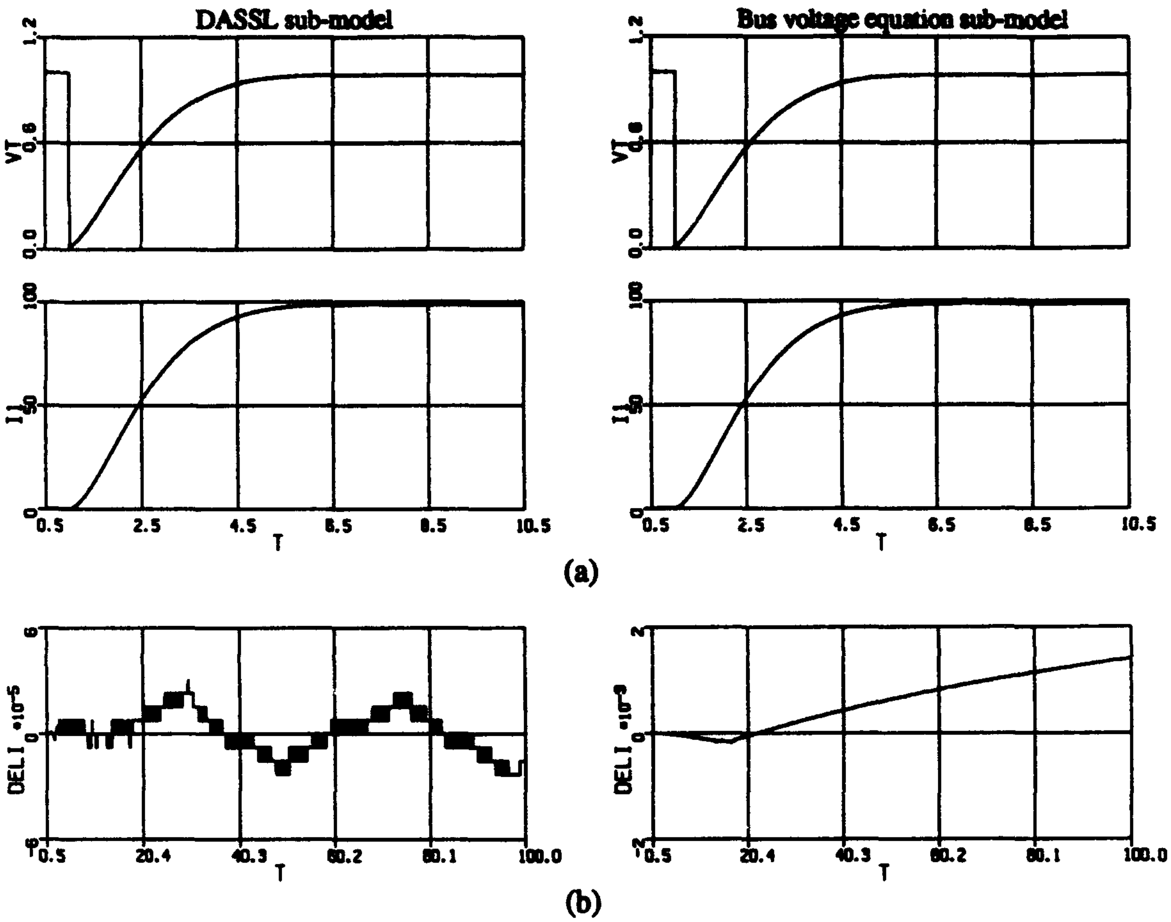

Figure 14. Step change in load for both submodels; (a) terminal voltage and current response, (b) source and load current difference to 100 seconds.

The only choice presented which allows both modularity and simplicity in implementation as the total system model grows is the DASSL bus voltage model. The routine accurately solves for bus voltage and then feeds that solution to all connected submodels. Additional submodels may be included by simply adding the current and current derivative terms from the new submodel into the KCL constraint equation. The DASSL routine is easy to use, fast, accurate and robust.

V. SYSTEM MODEL DESCRIPTION

Most of the pieces required to build an isolated power system operating on a finite bus have been developed and presented. Validated submodels for source, loads and bus voltage are available to be used as building blocks for a larger model. This section puts the system model together by providing a means for closed loop control, a description of the per-unit (pu) system and a detailed description of the ACSL code used in the final system simulation.

A. SYSTEM CLOSED LOOP CONTROL

In order to complete the system model and put it under closed loop control, two more submodels must be developed. These are the field excitation system and the prime mover and speed governor system. The field excitation system is a voltage regulator and exciter which senses the magnitude of the bus voltage and then adjusts the voltage applied to the field winding as necessary to maintain the bus voltage at or near the commanded reference level. The speed governor senses the mechanical speed of the rotor and applies a control signal to the prime mover which maintains rotor speed at or near the commanded reference level.

1. Field Excitation System Model

There are many types of field excitation systems used for synchronous generator voltage control. The IEEE Type 2 representation [Ref. 10] presented here is typical for power systems used aboard US Navy ships. It is also the type of field excitation system in the model which is used for comparison in the next chapter.

Figure 15 is a block diagram of the IEEE Type 2 regulator and exciter. The first section, from the summing junction to V_{re}, is the regulator. From the second summing junction to the output is the exciter section. There are two saturation functions, one associated with each part of the system. The details of implementing this model in the simulation are discussed in the section describing the ACSL code.

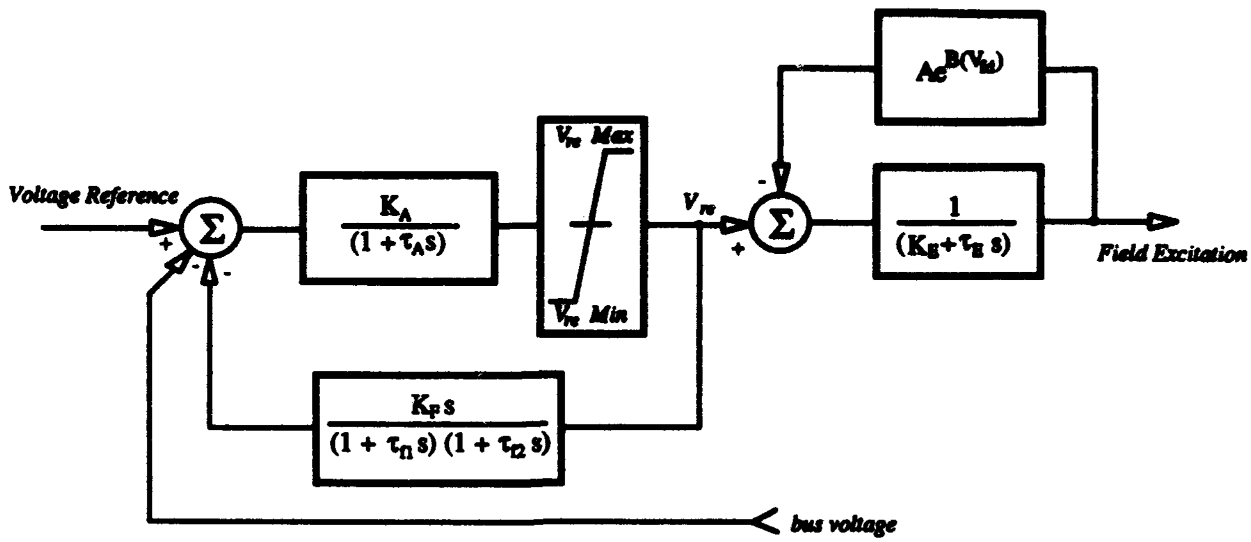

Figure 15. IEEE Type 2 regulator and exciter system based of Ref. 10.

2. Prime Mover and Speed Governor Model

As for the excitation system, many models are available for prime mover and speed governor. Steam turbines, hydro-turbines, diesel engines and gas turbines are a few examples of the types of system used to drive synchronous generators. A simple transfer function form for one of these systems might consist of two simple first order delays, one for the speed governor and one for the prime mover. Choice of the prime mover and governor model was driven by the desire to compare results of this system simulation with other work

Mayer and Wasynczuk [Ref. 5] provide a model for an Allison 501 gas turbine engine and governor. Figure 16 is the s-domain representation of this model. The model is a per-unit model. A per-unit model references all model variables to some base value.

Per-unit models are very common in power system simulation work and will be described in more detail in the next section.

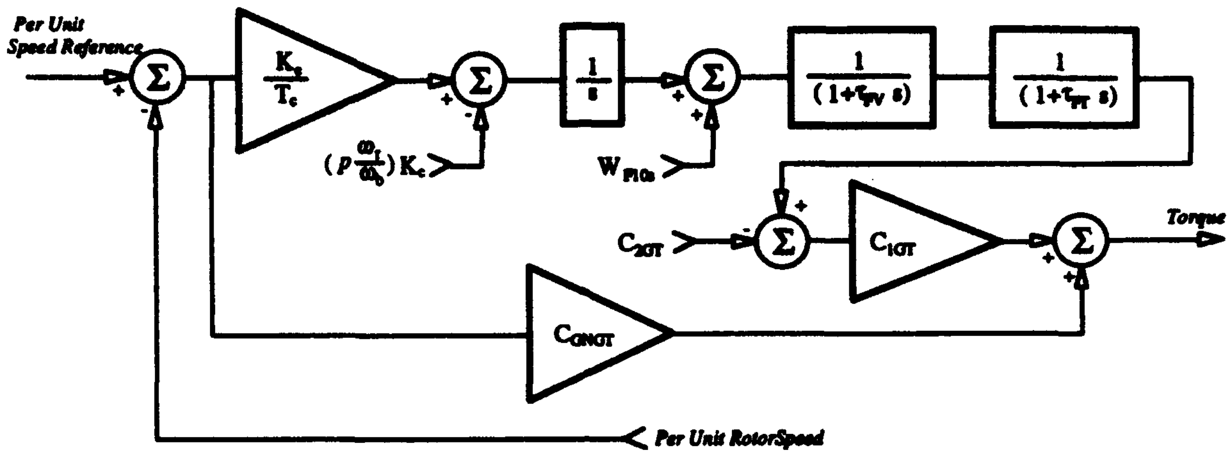

Figure 16. Gas turbine and speed governor model based on Ref. 5.

B. THE PER-UNIT SYSTEM

The per-unit (pu) system was developed to simplify the calculation and interpretation of results in power system simulation work. A pu quantity is defined as an actual quantity (voltage, current, power etc.) divided by a base or reference quantity. The base values are selected according to known characteristics of the machine being studied. The final system simulation presented here is in the per-unit system.

The pu system is based on machine rated bus voltage, power and synchronous speed. Machine parameters are usually supplied by the manufacturer in pu. The base quantities used here are

V_{base} : rated operating voltage

P_{base} : rated volt-amperes for synchronous generator,

rated horsepower times 746 for induction motor

ω_{base} : rated operating electrical frequency

With these defined, impedance and torque base quantities may be derived

$$\begin{aligned} Z_{base} &= \frac{V_{base}^2}{P_{base}} \\ T_{base} &= \frac{P_{base}}{(2/P)\omega_{base}} \end{aligned} \tag{83}$$

Use of the pu system slightly changes the state equations for the synchronous machine and induction machine. In the pu system the inertia (J) is replaced by the inertia constant (H) which has units of seconds. The two inertia terms are related by

$$H = J\left(\frac{\omega_{base}}{PT_{base}}\right) \tag{84}$$

The electrical torque equations, (41) and (68), are altered in the pu system. For the synchronous machine they are written as

$$\begin{aligned} T_e &= X_{md}(-i_{ds} + i_{fd} + i_{kd})i_{qs} - X_{mq}(-i_{qs} + i_{kq})i_{ds} \\ p\omega_r &= \left(\frac{\omega_b}{2H}\right)(T_l - T_e) \end{aligned} \tag{85}$$

and for the induction machine as

$$\begin{aligned} T_e &= X_M(i_{qs}i_{dr} - i_{ds}i_{qr}) \\ p\omega_{rm} &= \left(\frac{\omega_b}{2H_m}\right)(T_e - T_l) \end{aligned} \tag{86}$$

Use of the pu system introduces a compatibility problem for the system model. Since it is desired to build a model where sources and loads are interconnected, all submodels must be referenced to the same base values. The generator and motor models share a common base voltage and electrical frequency; however, base power is different in each submodel. For the work presented here, the synchronous machine drives the selection of base quantities. This requires that the other submodels be referenced to the synchronous machine. By manipulation of (83) and (84), the machine parameters of the induction motor may be put in the synchronous generator base as follows:

$$\begin{aligned} Z_{pu(sync\_mach)} &= \left(\frac{P_{base(sync\_mach)}}{P_{base(ind\_mach)}}\right) Z_{pu(ind\_mach)} \\ H_{pu(sync\_mach)} &= \left(\frac{P_{base(ind\_mach)}}{P_{base(sync\_mach)}}\right) H_{pu(ind\_mach)} \end{aligned} \tag{87}$$

C. SYSTEM MODEL IMPLEMENTATION IN ACSL

The ACSL program code is contained in Appendix B. The description of system model implementation follows the layout of the program code. In order to make the ACSL program easier to follow, throughout this section the variables will be referred to by the same name used in the ACSL code. The block diagram of Figure 17 represents the system simulation as implemented in ACSL. The blocks labeled Synchronous Generator Model (SGM) and Load 1 Model (L1M) through Load n Model (LnM) represent the stand-alone state-space models developed in Chapters II and III. The models are

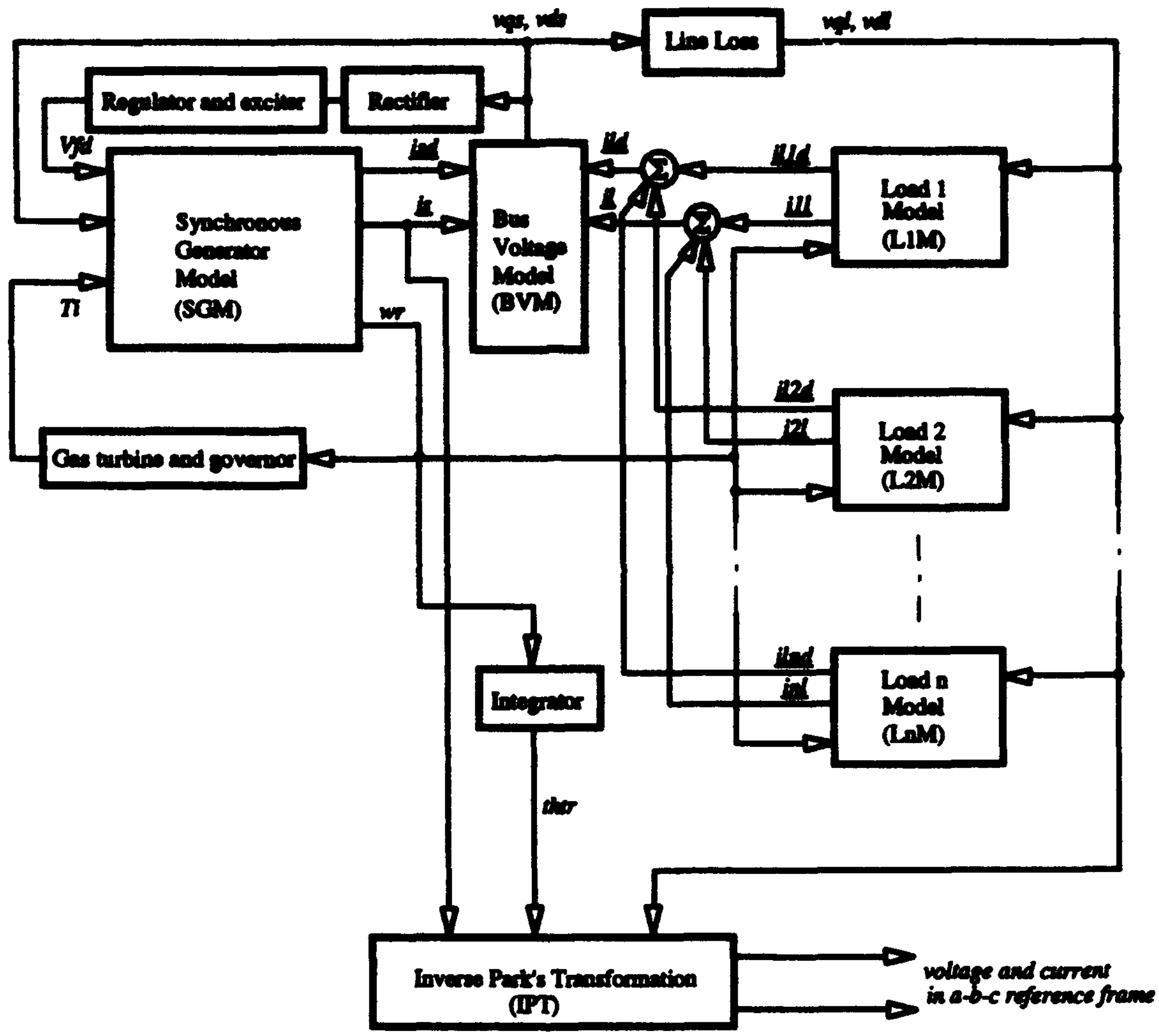

Figure 17. Total system simulation block diagram as implemented in ACSL.

implemented with current states. At each time step of the simulation, vectors representing the model currents (*is* and *il*) and current derivatives (*isd* and *ild*) are computed. Each of these models requires bus voltage as an input. The bus voltage to the load may experience a line loss.

The orthogonal bus voltage values, *vqs* and *vds*, are generated by the Bus Voltage Model (BVM). For the system simulation presented here, the DASSL bus voltage model is used. In order to set up the implicit solution, the BVM requires all *q* and *d* axis source and load current and current derivative terms as inputs. The model forms a residual from these inputs based on KCL. The bus voltage is then computed, using the DASSL routine, at each step of the simulation. The DASSL routine drives the residual close to zero.

The SGM requires, in addition to bus voltage, an input torque (*Ti*) and field winding voltage (*Vfd*). The SGM outputs are currents (*is*) current derivatives (*isd*) and rotor speed (*wr*). Speed control for the SGM is provided by the gas turbine and governor which provides the input torque based on the behavior of *wr*. The voltage control is accomplished by the regulator/exciter which uses the rectified magnitude of the bus voltage to provide the appropriate level of *Vfd*.

L1M through LnM receive bus voltage inputs *vql* and *vdl*. These voltage quantities are the BVM outputs modified by accounting for transmission line resistance and reactance. Additionally, these load models require an input representing the electrical frequency. For this system model, the electrical frequency is the *wr* output from the SGM.

The block labeled Inverse Park's Transformation (IPT) allows the system voltages and currents, which are expressed in the orthogonal reference frame, to be changed to the *a-b-c* reference. The IPT uses *thtr*, the variable representing rotor position, to perform the transformation. This variable is obtained by integrating rotor speed with respect to time.

1. Program and Initial Sections

The first few lines of the program set simulation parameters. It is here that things like integration algorithm, integration step size and communication interval for the output file may be specified. The default integration algorithm is a Runge-Kutta fourth order routine. The step size is selected, based on the Nyquist criteria, so that the dynamic response transients can be computed and printed. Following the setting of simulation parameters the machine parameters needed for each state-space submodel are entered.

The *initial* section of the code computes all coefficients needed for the source and load submodels using the machine parameters. Starting with the synchronous generator, the elements of the inverse **B** matrix are computed. The expressions for these elements are obtained from the MATHCAD output of Appendix A. After the inverse matrix elements are computed, the terms for the linear and nonlinear matrices are calculated. The procedure is repeated for each induction motor model. Finally initial conditions for each integration in the simulation are entered. These are obtained by doing steady state calculations or by running the model with initial conditions set equal to zero until the model reaches steady state.

2. Dynamic Section

The *dynamic* section of the ACSL code is the heart of the simulation. This section contains all the differential equations describing the models. It is this section of the code which is executed at each time step of the simulation. The speed of execution is a function of the number of integrations which must be performed, the integration algorithm selected, integration step size and the total time which must be simulated. Execution time is also affected by the DASSL routine which slows the simulation down at each time step by varying amounts in order to drive the bus voltage to a value that satisfies the constraint equation.

The constants *vref* and *wref* are the commanded inputs for voltage magnitude and speed. The constants *mot_onX* and *cb1* represent circuit breakers for the loads. If these are set to zero, no voltage is applied to the load. The induction motors each have a speed square law load applied to them. This type of load is typical for a pump, a common application for induction motors on board ship. The R-L load parameters are initially set to large values so that, unless they are changed, the load will be very small even with bus voltage applied (when *cb1* is set to one).

Next some quantities are derived so that they may be output. A ripple term is added to the voltage magnitude to simulate the output of a rectifier. To make the output more standard, torque quantities for the three motors are changed from the generator torque base to the torque base for each induction motor.

The exciter and prime mover/governor models come from the s-domain models of Figures 19 and 20. The following technique was used to convert the transfer function form of the models to the form seen in the program:

$$\begin{aligned} B &= \frac{AK_a}{1+\tau_a s} \\ B + B\tau_a s &= AK_a \\ \dot{B}\tau_a &= AK_a - B \end{aligned} \tag{88}$$

where A is the input, B is the output and $\dot{B} = Bs$. Then this piece of the larger system may be represented in ACSL as

$$\begin{aligned} \dot{B} &= \frac{AK_a - B}{\tau_a} \\ B &= \left[\int \dot{B}\right] + B_{ic} \end{aligned} \tag{89}$$

This form, unlike the transfer function form, allows initial conditions to be input for each integration if desired.

One of the states for the synchronous machine is *wr*, the rotor speed. This quantity is also the basis for the reference frame chosen for the models. In order to use the inverse Park's transformation to convert *q-d-0* quantities to *a-b-c* quantities, the rotor position (*thtr*) must be derived. This is done by integrating rotor speed with respect to time.

Although it has less meaning in the case of an isolated generator than in an infinite bus or multiple generator study, rotor angle (*del*) is computed. Figure 18 is a partial phasor diagram which shows how these quantities are related. The rotor angle is sometimes referred to as the torque angle to avoid confusion with *thtr*. The physical position of the rotor, *thtr*, is constantly changing as the prime mover drives the generator; however, for a given load on the machine *del* is a constant.

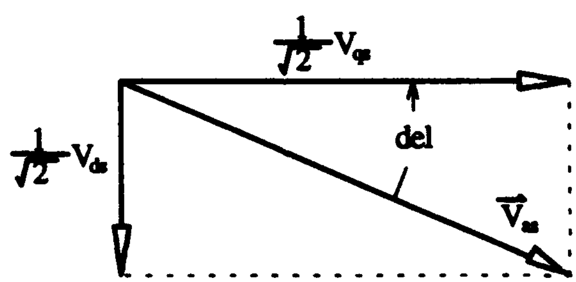

Figure 18. Relationship between *Vqs*, *Vds*, *Vas* and *del*.

The state equation section of th code contains the ACSL implementation of the synchronous machine and induction machine equations in explicit form. The coefficients on the right-hand side of the state equations are named so that they may be easily identified. Coefficients beginning with *L* multiply with *linear* state terms Those with an *N* multiply with ***nonlinear*** terms (those involving a current-speed product). The number appended to the name gives the position of the coefficient in its parent matrix. In the induction motor equations there are two additional elements to the naming conventions just described. Because there are multiple motor models, a third digit is added to the end of the coefficient name to indicate the motor number. A distinction is also made between the nonlinear coefficients. Those involving the electrical speed only begin with NE, and those involving the slip speed or difference between electrical speed and motor rotor speed begin with ND.

The DASSL bus voltage model is simply an expanded version of the model used in the example of Chapter IV. A residual of the currents and current derivatives is formed for each *q-d-0* axis . Then the voltage value is obtained using the implicit system solver. A line loss model based on the R-L model of Chapter III modifies the value of line voltage applied to the motor loads.

The final few lines of code convert the voltage and current results to the *a-b-c* reference frame for output and set a simulation stop time. The ACSL system compiles the model in FORTRAN for execution. The executable FORTRAN form of the model runs faster than models produced in some other simulation software. The main model may be linked to one or more command files which allow the user to more easily exercise the model under a variety of conditions and then obtain the desired output. An command file, which was used with this simulation, follows the system model code in Appendix B.

VI. SYSTEM RESPONSE AND MODEL VALIDATION

The model for the isolated power system has now been fully developed. The system simulation is extremely modular and relies on accepted models for the source and load submodels. These accepted models for a synchronous generator, induction motor and resistive-inductive load have been extensively validated.

Other submodels were needed to complete the system. The DASSL submodel for the bus voltage was demonstrated in Chapter IV. Chapter V added accepted field exciter and prime mover/governor models to the system.

Although the separate pieces of the system have been validated to one degree or another, the whole system must be tested against some standard for validation. Mayer and Wasynczuk [Ref. 5.] of Purdue University presented a simulation of a portion of the USS Arleigh Burke (DDG-51) power distribution system which will be used as the standard for comparison. This model will be referred to as the Purdue model.

A. DESCRIPTION OF THE PURDUE MODEL

The Purdue model is a systematic method for taking the differential algebraic equations describing a power system and using them to establish a conventional state-space model. On each bus of the system, one machine is designated as the root machine and any other connected machine is a nonroot machine. A root machine has current and current derivatives as its inputs and stator voltages as its outputs. The root machine inputs are formed by summing the currents and current derivatives from all connected nonroot models. After establishing forms for the root and nonroot models, an interconnection procedure is established based on the KCL constraint.

The interconnection procedure is conceptually similar to the bus voltage equation development presented in Chapter IV. Based on the linearity of the derivative operator, expressions for the current derivatives for the root and nonroot models are set equal, thus eliminating the derivative terms. Then, after complicated matrix algebraic manipulation, an equation for determining stator voltages from the states only is produced.

Mayer and Wasynczuk validate their model by comparing it with the output produced by the Power System Simulator at Purdue University. This facility has been used extensively in system design work and provides detailed three-phase output based on state-of-the-art representations of the power system components.

B. VALIDATION BY COMPARISON WITH THE PURDUE MODEL

Mayer and Wasynczuk describe the scenario and provide all the model parameters for the results presented in their paper. The model parameters are contained in Table 1. The generator and induction motor parameters are all per-unit values with a 450V rms, line-to-line, base voltage. The base power for the generator is 3125 KVA, and for the motors is determined by the horsepower rating.

For the comparison simulation, the generator is initially in a steady state unloaded condition. It is under closed loop regulation, operating at rated voltage and speed with stator currents zero. At an arbitrary time, a circuit breaker is closed energizing all three induction motors. The three motors draw large start-up currents which cause bus voltage to dip initially before the voltage regulator and exciter circuit can react and return the bus voltage to the commanded magnitude. The initial large currents also produce a large electrical torque in the generator which tends to slow rotor speed. The prime mover/governor reacts to the speed change by applying more input torque to return the system to commanded speed. Most of the system transient behavior is complete in three seconds. Figures 19 and 20 compare the results of the DASSL based simulation with the results obtained by the Purdue model.

TABLE 1. SYSTEM MODEL PARAMETERS

Prime Mover/Governor

$K_c = 22.5$	$T_c = 0.55$	$T_{FV} = 0.01$	$T_{FT} = 0.05$
$W_{F10s} = 0.23$	$C_{2GT} = 0.251$	$C_{1GT} = 1.3523$	$C_{GNGT} = 0.5$

Regulator/Exciter

$K_A = 400.0$	$T_A = 0.01$	V_{re} Max = 8.4	V_{re} Min = 0
$K_F = 0.01$	$T_{F1} = 0.15$	$T_{F2} = 0.06$	$K_E = 1.0$
$T_E = 0.1$	$A = 0.1$	$B = 0.3$	

Synchronous Generator

$r_s = 0.00515$	$r_{kq} = 0.0613$	$r_{fd} = 0.00111$	$r_{kd} = 0.02397$
$X_{ls} = 0.08$	$X_{lkq} = 0.3298$	$X_{lfd} = 0.13683$	$X_{lkd} = 0.33383$
$X_{mq} = 1.0$	$X_{md} = 1.768$	$H = 2.137$	

Induction Motors

	IM1	IM2	IM3
Hp	200	150	40
r_s	0.01	0.0051	0.005
X_{ls}	0.0655	0.0553	0.0587
X_m	3.225	2.678	2.952
X_{lr}	0.0655	0.0553	0.0587
r_r	0.0261	0.0165	0.0165
H	0.922	1.524	1.054

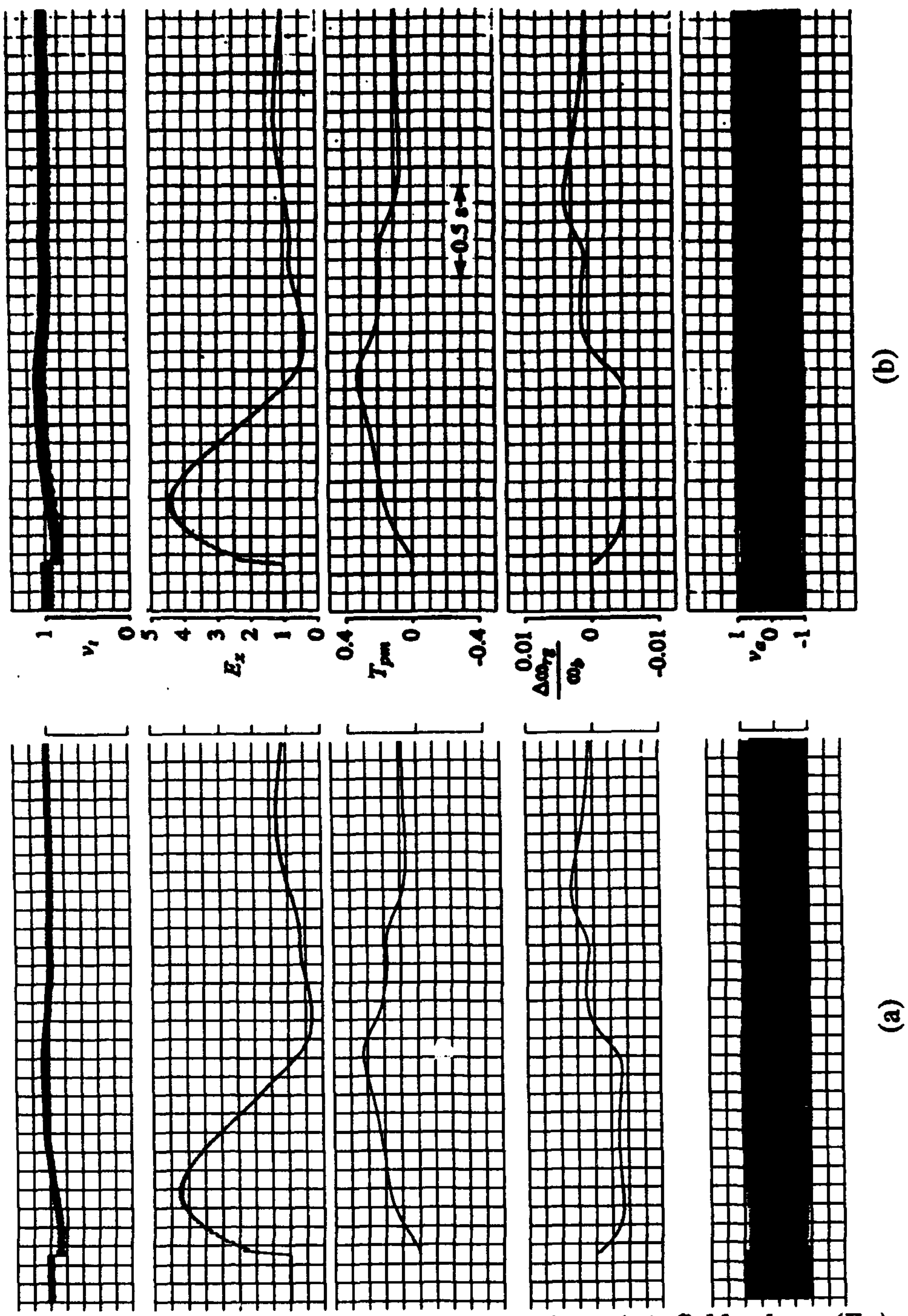

Figure 19. Comparison of results: ***terminal voltage (v_t), field voltage (E_x), prime mover torque (T_{pm}), rotor speed (ω_r), phase A voltage (v_a);*** **(a) DASSL model, (b)Purdue model.**

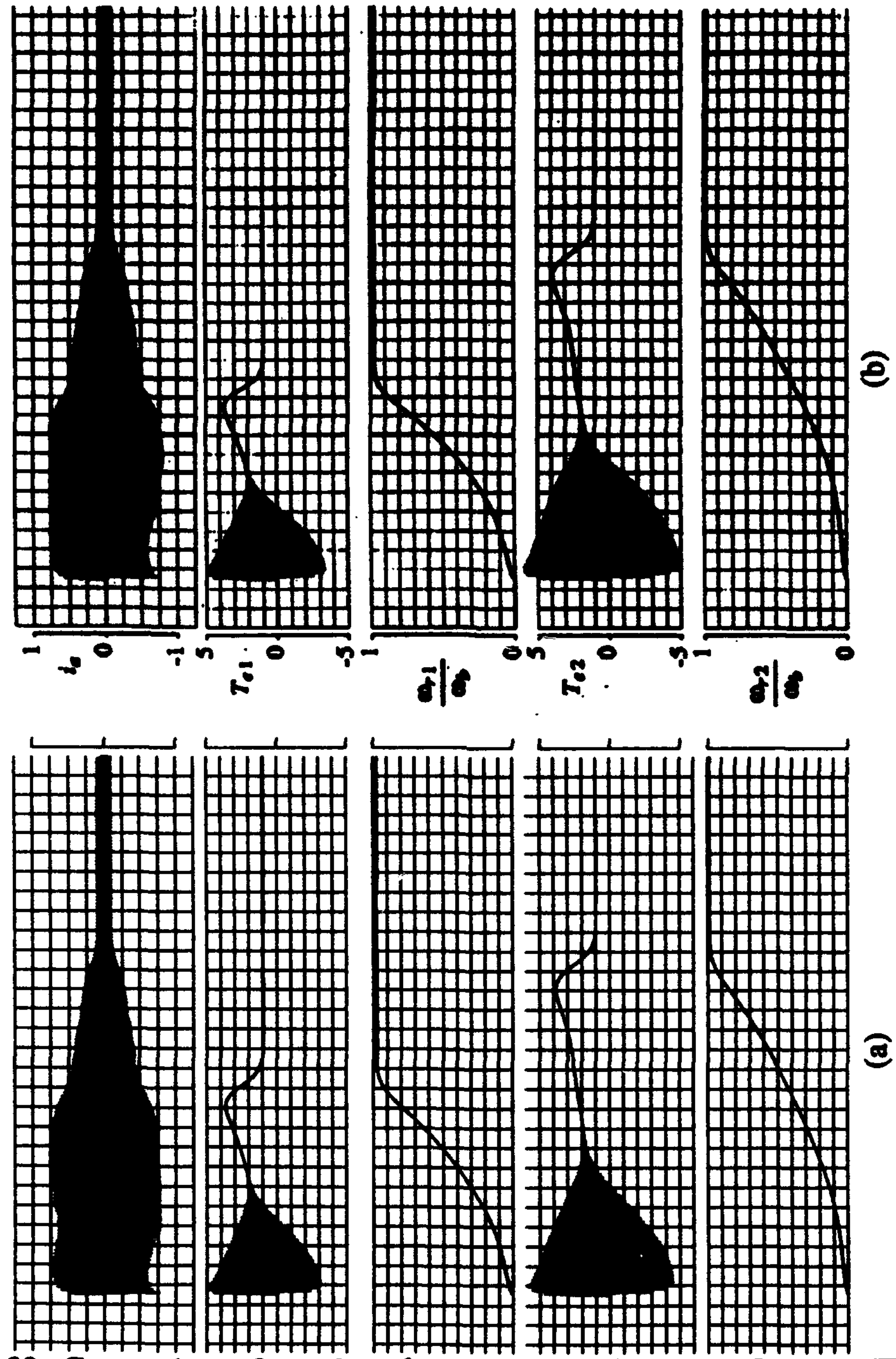

Figure 20. Comparison of results: ***phase A current (i_a), motor 1 torque (T_{e1}), motor 1 speed (ω_{r1}),motor 2 torque (T_{e2}), motor 2 speed (ω_{r2})*****; (a) DASSL model, (b)Purdue model.**

The DASSL based model is in excellent agreement with the Purdue model results. Additionally it agrees well with the expected behavior of such a system. Bus voltage transient behavior is tracked during dynamic loading of the system. During the simulation the difference between generator current and the sum of the load currents stayed bounded in the micro amp range.

Both simulations were implemented in ACSL using a fourth-order, Runge-Kutta integration algorithm. Using an integration step size of 1.0 millisecond, the DASSL based model required 5.4 seconds of cpu time on a Sun SPARC 10 Model 41 workstation. For comparison, the generator and three induction motor simulation using an infinite bus voltage model used 2.1 seconds of cpu time on the same workstation.

C. DASSL MODEL RESPONSE WITH UNBALANCED LOAD

In three-phase power systems every effort is made by the system designers to keep the phase loads equal. In practice this is impossible. All three phase systems experience some degree of unbalanced loading. On board a ship this is a common problem, particularly on older ships. Partial grounds, lighting alterations and equipment updates are some factors that contribute to the problem. Behavior of the system presented in the previous section to an unbalanced loading condition will be investigated.

An unbalanced R-L load in parallel with the three induction machines simulates a situation which frequently occurs on board ship. Single phase lighting loads are served by tapping a lighting circuit transformer primary into one phase of the three phase power system. This often results in different loading conditions on the three phases.

1. The Unbalanced Load

Unbalanced loading introduces significantly more complexity to the state equations in the *q-d-0* reference frame. In the R-L load development of Chapter III, balanced loading was assumed. If the phase resistance values are allowed to be unequal the resistance matrix becomes

$$\mathbf{r_l} = \begin{bmatrix} r_{al} & 0 & 0 \\ 0 & r_{bl} & 0 \\ 0 & 0 & r_{cl} \end{bmatrix}$$

The transformation to the orthogonal reference frame results in the $\mathbf{K_s r_l}(\mathbf{K_s}^{-1})$ matrix taking the form

$$\frac{2}{3}\begin{bmatrix} \begin{matrix} r_{al}\cos^2 A + r_{bl}\cos^2 B \\ \cdots + r_{bl}\cos^2 C \end{matrix} & \begin{matrix} r_{al}\cos A \sin A + r_{bl}\cos B \sin B \\ \cdots + r_{bl}\cos C \sin C \end{matrix} & \begin{matrix} r_{al}\cos A + r_{bl}\cos B \\ \cdots + r_{bl}\cos C \end{matrix} \\ \begin{matrix} r_{al}\cos A \sin A + r_{bl}\cos B \sin B \\ \cdots + r_{bl}\cos C \sin C \end{matrix} & \begin{matrix} r_{al}\sin^2 A + r_{bl}\sin^2 B \\ \cdots + r_{bl}\sin^2 C \end{matrix} & \begin{matrix} r_{al}\sin A + r_{bl}\sin B \\ \cdots + r_{bl}\sin C \end{matrix} \\ \begin{matrix} \frac{1}{2}(r_{al}\cos A + r_{bl}\cos B \\ \cdots + r_{bl}\cos C) \end{matrix} & \begin{matrix} \frac{1}{2}(r_{al}\sin A + r_{bl}\sin B \\ \cdots + r_{bl}\sin C) \end{matrix} & \frac{1}{2}(r_{al} + r_{bl} + r_{cl}) \end{bmatrix} \tag{90}$$

where

$$A = \theta_r$$

$$B = \theta_r - \frac{2\pi}{3}$$

$$C = \theta_r + \frac{2\pi}{3}$$

This significantly complicates the state equations. All three states now appear in each of the three equations describing the R-L load in the *q-d-0* reference frame. Note that an unbalanced inductance matrix will cause all three state derivative terms to appear in all state equations (an implicit form). Putting such a system of equations in explicit form will not be dealt with here.

2. Simulation Results with Unbalanced Loading

For the unbalanced load study, the model is started and loaded in an identical manner to the simulation of Figures 19 and 20. Once the system is in steady state, a circuit breaker is closed which connects an unbalanced R-L load in parallel with the motor loads. The per-unit unbalanced load parameters used are

$$r_{al} = 5.0$$
$$r_{bl} = 30.0$$
$$r_{cl} = 5.0$$
$$X_l = 3.0$$

After application of the unbalanced load, the system is again allowed to come to steady state. The effect of this load on the phase currents may be seen in Figure 21. The phase currents are visibly unequal. The magnitude of i_{bs} is about half the magnitude of the other phase currents. Of more interest is the manner in which other variables are affected.

Figure 22 is a plot of several variables affected by the unbalanced load. The unbalanced load causes oscillations to occur in the generator electrical torque. These variations in turn cause the rotor to oscillate. The field winding current also exhibits this oscillatory behavior due to the motion of the rotor. Not only is the generator affected, but the plot of torque produced by induction motor two shows oscillations. Such oscillations can be potentially damaging to equipment.

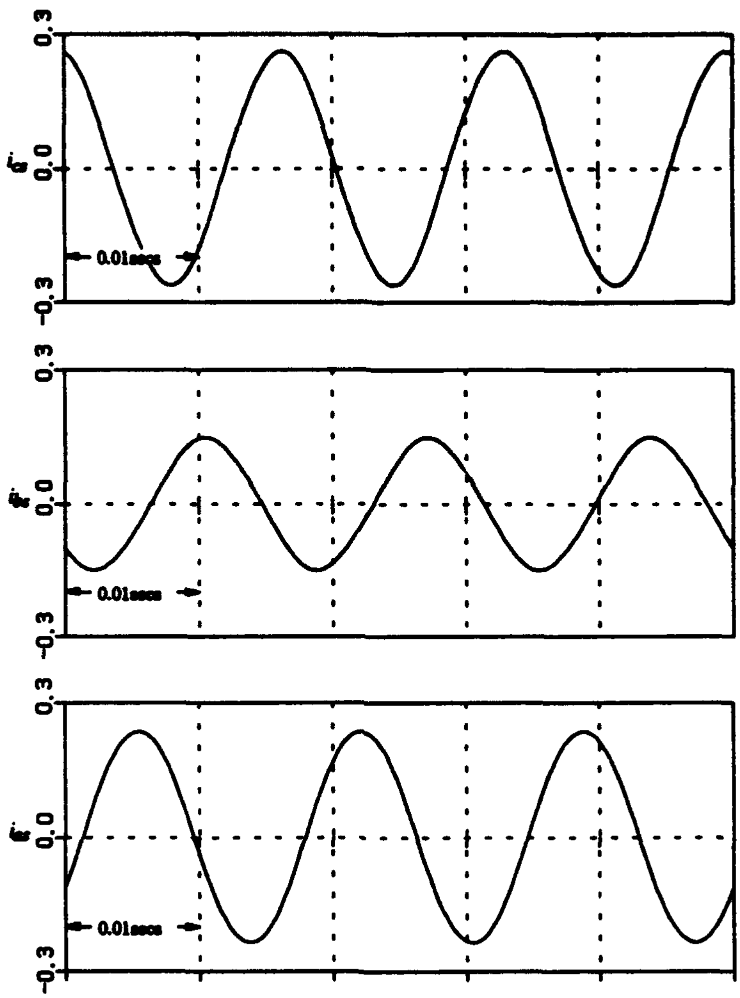

Figure 21. System phase currents i_{as}, i_{bs} and i_{cs} with unbalanced load.

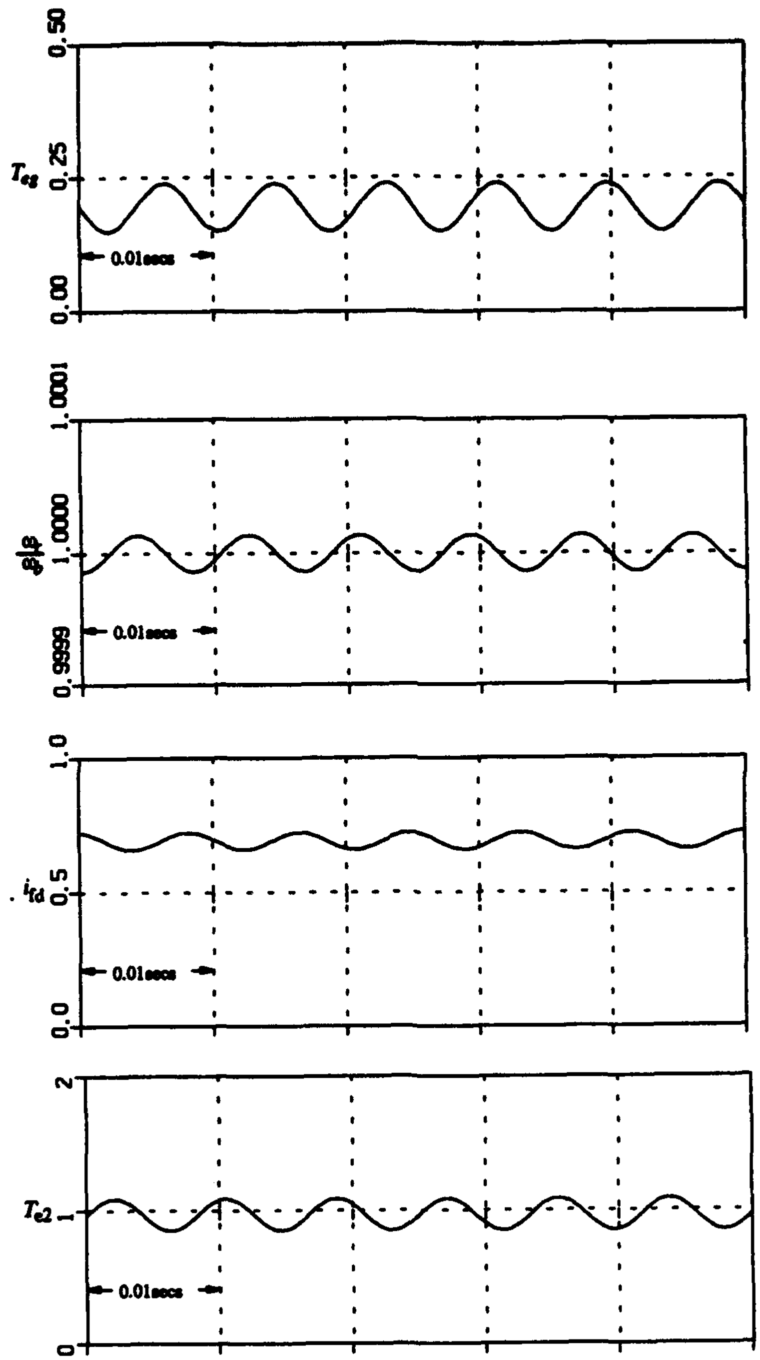

Figure 22. System response to unbalanced load; generator electrical torque (T_{eg}), generator rotor speed (ω_r/ω_b), field current (i_{fd}) and motor 2 torque (T_{e2}).

D. DASSL BASED MODEL FLEXIBILITY

Many other types of studies may be done with the DASSL based model. The system is extremely flexible. For example, the load on one or more of the induction motors could be suddenly changed. The full transient effect of this change could then be observed throughout the system. The response of the generator, generator control systems and other loads could all be studied. This type of capability is extremely valuable to the designers of isolated power systems.

VII. CONCLUSIONS AND FUTURE WORK

There is a need for power system simulations which do not rely on the infinite bus voltage assumption. In the recent past, the limited capabilities of computer hardware and software made the modeling of isolated power systems difficult. Designers relied on questionable equations to approximate terminal voltage dip under various loading conditions. Reduced order modeling was done which provided some data at the expense of losing transient behavior results. It has been demonstrated that by treating the equations for the power system as a set of differential algebraic equations and using a proven DAE solver, excellent results can be achieved.

A. ADVANTAGES OF THE DASSL MODEL

The approach presented in this thesis has several advantages over methods that have been used in the past. Some of the advantages are:

- the system model is highly modular in design (submodels)
- the DASSL bus voltage submodel constraint equation makes the model simple to expand
- the model provides transient data
- the submodels are standard, well validated state-space models
- simulation speed is excellent
- the model uses a highly regarded, commercially available DAE solving routine

B. DISADVANTAGES

There are some problems with the system model which still must be overcome. Some of the possible disadvantages and limitation of the model are:

- the source and load submodels need to be developed with current states and in the *q-d-0* reference frame (not always the most convenient form)
- the total system model has not been validated against hardware results
- the generator and motor models do not include saturation effects
- the generator and motor models assume sinusoidally distributed windings

The last disadvantage is really a disadvantage of the standard development of the synchronous and induction machine models. The fact that the windings are actually not perfectly distributed introduces harmonics on the system bus. Generally these effects are minor and may be neglected; however, some types of loads (solid state power converters for example) may be highly sensitive to these neglected harmonics.

C. FUTURE WORK

In order to realize a benefit from the DASSL model approach significant work remains to be done. One of the most important tasks that could be accomplished is a hardware validation of the model. Figure 23 is a block diagram of a possible hardware configuration for accomplishing validation.

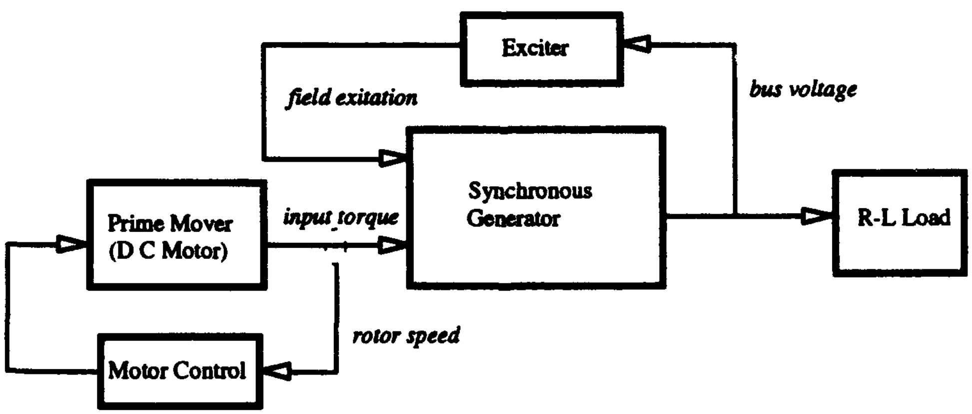

Figure 23. Possible hardware configuration for model validation.

This configuration has the advantage of being complex enough to validate the DASSL model and simple enough to build and test in the lab. Some other suggestions for future work with this basic model are:

- addition of one or more parallel generators
- development of other types of load models
- include winding saturation effects
- try other DAE solution methods which offer the promise of improved efficiency (for example, Halin [Ref. 12] reports a significant improvement over DASSL with his method)

This list is by no means conclusive and much work remains to be done in this area.

The process of engineering design involves trade-offs. The DASSL model, because it involves multiple iterations at each simulation time step, requires more than double the cpu time than that used by the infinite bus model. At one time this *cost* may have been considered too great for the *benefit* derived. However, today's simulation capabilities make the DASSL based finite bus model a practical design tool, worthy of continued investigation.

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APPENDIX A: CONVERTING STATE EQUATIONS TO EXPLICIT FORM

In Chapters II and III the state equations for the synchronous machine and the induction machine were developed as implicit equations with the currents as states. Those equations had the form

$$\underline{v} = \mathbf{A_L}\underline{i} + \mathbf{A_N}\omega\underline{i} + \mathbf{B}p\underline{i}$$

where $\mathbf{A_L}$ is the linear terms matrix and $\mathbf{A_N}$ is the nonlinear terms matrix.

In order to convert the equations to implicit form the **B** matrix must be inverted so that the state derivative vector may be isolated on the left-hand side of the equation. Since numerical matrix inversion can often lead to problems in the case of poorly conditioned matrices, the matrix inversion for the two models was done using the MAPLE symbolic engine in MATHCAD 4.0.

THE SYNCHRONOUS MACHINE

$$\mathbf{B}^{-1} = \begin{bmatrix} -h & 0 & 0 & a & 0 & 0 \\ 0 & -k & 0 & 0 & b & b \\ 0 & 0 & -c & 0 & 0 & 0 \\ -a & 0 & 0 & g & 0 & 0 \\ 0 & -\frac{b^2}{d} & 0 & 0 & \frac{e}{d}\cdot b & \frac{b^2}{d} \\ 0 & -b & 0 & 0 & b & f \end{bmatrix}^{-1}$$

where the following definitions apply:

$a = X_{mq}; b = X_{md}; c = X_{ls}$

$d = R_{fd}; e = X_{fd}; f = X_{kd}$

$g = X_{kq}; h = X_q; k = X_d$

$\mathbf{B}^{-1} =$

$$
\begin{bmatrix}
\frac{g}{(-h\cdot g+a^2)} & 0 & 0 & \frac{-a}{(-h\cdot g+a^2)} & 0 & 0 \\
0 & \frac{(-e\cdot f+b^2)}{(k\cdot e\cdot f-k\cdot b^2-b^2\cdot f+2\cdot b^3-b^2\cdot e)} & 0 & 0 & -d\cdot\frac{(-f+b)}{(k\cdot e\cdot f-k\cdot b^2-b^2\cdot f+2\cdot b^3-b^2\cdot e)} & -b\cdot\frac{(b-e)}{(k\cdot e\cdot f-k\cdot b^2-b^2\cdot f+2\cdot b^3-b^2\cdot e)} \\
0 & 0 & \frac{-1}{c} & 0 & 0 & 0 \\
\frac{a}{(-h\cdot g+a^2)} & 0 & 0 & \frac{-h}{(-h\cdot g+a^2)} & 0 & 0 \\
0 & b\cdot\frac{(-f+b)}{(k\cdot e\cdot f-k\cdot b^2-b^2\cdot f+2\cdot b^3-b^2\cdot e)} & 0 & 0 & -d\cdot\frac{(-k\cdot f+b^2)}{[(k\cdot e\cdot f-k\cdot b^2-b^2\cdot f+2\cdot b^3-b^2\cdot e)\cdot b]} & b\cdot\frac{(-k+b)}{(k\cdot e\cdot f-k\cdot b^2-b^2\cdot f+2\cdot b^3-b^2\cdot e)} \\
0 & b\cdot\frac{(b-e)}{(k\cdot e\cdot f-k\cdot b^2-b^2\cdot f+2\cdot b^3-b^2\cdot e)} & 0 & 0 & d\cdot\frac{(-k+b)}{(k\cdot e\cdot f-k\cdot b^2-b^2\cdot f+2\cdot b^3-b^2\cdot e)} & \frac{(k\cdot e-b^2)}{(k\cdot e\cdot f-k\cdot b^2-b^2\cdot f+2\cdot b^3-b^2\cdot e)}
\end{bmatrix}
$$

THE INDUCTION MACHINE

$$
\mathbf{B}^{-1} = \begin{bmatrix}
X_{ss} & 0 & 0 & X_M & 0 & 0 \\
0 & X_{ss} & 0 & 0 & X_M & 0 \\
0 & 0 & X_{ls} & 0 & 0 & 0 \\
X_M & 0 & 0 & X_{rr} & 0 & 0 \\
0 & X_M & 0 & 0 & X_{rr} & 0 \\
0 & 0 & 0 & 0 & 0 & X_{lr}
\end{bmatrix}^{-1}
$$

$\mathbf{B}^{-1} =$

$$
\begin{bmatrix}
\frac{-X_{rr}}{[-X_{ss}\cdot X_{rr}+(X_M)^2]} & 0 & 0 & \frac{X_M}{[-X_{ss}\cdot X_{rr}+(X_M)^2]} & 0 & 0 \\
0 & \frac{-X_{rr}}{[-X_{ss}\cdot X_{rr}+(X_M)^2]} & 0 & 0 & \frac{X_M}{[-X_{ss}\cdot X_{rr}+(X_M)^2]} & 0 \\
0 & 0 & \frac{1}{X_{ls}} & 0 & 0 & 0 \\
\frac{X_M}{[-X_{ss}\cdot X_{rr}+(\lambda_M)^2]} & 0 & 0 & \frac{-X_{ss}}{[-X_{ss}\cdot X_{rr}+(X_M)^2]} & 0 & 0 \\
0 & \frac{X_M}{[-X_{ss}\cdot X_{rr}+(X_M)^2]} & 0 & 0 & \frac{-X_{ss}}{[-X_{ss}\cdot X_{rr}+(X_M)^2]} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{X_{lr}}
\end{bmatrix}
$$

THE STATE EQUATIONS FORMED EXPLICITLY FOR ACSL

The final form of the state equations isolates the current derivative vector on the left-hand side. In the ACSL program code an equation is written for each current derivative. In order to develop an equation for each state derivative, the matrix equations must be multiplied out. This development will be demonstrated for the synchronous machine, the induction machine is treated in an identical manner. Equation (42)describes the procedure in compact from.

$$\begin{aligned} p\underline{i} &= \mathbf{L}\underline{i} + \mathbf{N}\omega_r\underline{i} + \mathbf{V}\underline{v} \\ \mathbf{V} &= \mathbf{B}^{-1} \\ \mathbf{L} &= \mathbf{V}(-\mathbf{A}_L) \\ \mathbf{N} &= \mathbf{V}(-\mathbf{A}_N) \end{aligned}$$

Using MATHCAD again, the state equations may be developed as seen in the ACSL code of Appendix B. First the **L** and **N** matrices must be computed

$$\mathbf{L} = \begin{bmatrix} V11 & 0 & 0 & V14 & 0 & 0 \\ 0 & V22 & 0 & 0 & V25 & V26 \\ 0 & 0 & V33 & 0 & 0 & 0 \\ V41 & 0 & 0 & V44 & 0 & 0 \\ 0 & V52 & 0 & 0 & V55 & V56 \\ 0 & V62 & 0 & 0 & V65 & V66 \end{bmatrix} \cdot \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & -r_{kq} & 0 & 0 \\ 0 & 0 & 0 & 0 & -X_{md} & 0 \\ 0 & 0 & 0 & 0 & 0 & -r_{kd} \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} V11 \cdot r_s & 0 & 0 & -V14 \cdot r_{kq} & 0 & 0 \\ 0 & V22 \cdot r_s & 0 & 0 & -V25 \cdot X_{md} & -V26 \cdot r_{kd} \\ 0 & 0 & V33 \cdot r_s & 0 & 0 & 0 \\ V41 \cdot r_s & 0 & 0 & -V44 \cdot r_{kq} & 0 & 0 \\ 0 & V52 \cdot r_s & 0 & 0 & -V55 \cdot X_{md} & -V56 \cdot r_{kd} \\ 0 & V62 \cdot r_s & 0 & 0 & -V65 \cdot X_{md} & -V66 \cdot r_{kd} \end{bmatrix}$$

$$\mathbf{N}=\begin{bmatrix} V11 & 0 & 0 & V14 & 0 & 0 \\ 0 & V22 & 0 & 0 & V25 & V26 \\ 0 & 0 & V33 & 0 & 0 & 0 \\ V41 & 0 & 0 & V44 & 0 & 0 \\ 0 & V52 & 0 & 0 & V55 & V56 \\ 0 & V62 & 0 & 0 & V65 & V66 \end{bmatrix}\cdot\begin{bmatrix} 0 & \frac{X_d}{\omega_b} & 0 & 0 & \frac{X_{md}}{\omega_b} & \frac{X_{md}}{\omega_b} \\ \frac{X_q}{\omega_b} & 0 & 0 & \frac{X_{mq}}{\omega_b} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{N}=\begin{bmatrix} 0 & V11\cdot\frac{X_d}{\omega_b} & 0 & 0 & -V11\cdot\frac{X_{md}}{\omega_b} & -V11\cdot\frac{X_{md}}{\omega_b} \\ -V22\cdot\frac{X_q}{\omega_b} & 0 & 0 & V22\cdot\frac{X_{mq}}{\omega_b} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & V41\cdot\frac{X_d}{\omega_b} & 0 & 0 & -V41\cdot\frac{X_{md}}{\omega_b} & -V41\cdot\frac{X_{md}}{\omega_b} \\ -V52\cdot\frac{X_q}{\omega_b} & 0 & 0 & V52\cdot\frac{X_{mq}}{\omega_b} & 0 & 0 \\ -V62\cdot\frac{X_q}{\omega_b} & 0 & 0 & V62\cdot\frac{X_{mq}}{\omega_b} & 0 & 0 \end{bmatrix}$$

then the terms of the equation are *multiplied out*

$$\mathbf{L}\underline{i}=\begin{bmatrix} L11 & 0 & 0 & L14 & 0 & 0 \\ 0 & L22 & 0 & 0 & L25 & L26 \\ 0 & 0 & L33 & 0 & 0 & 0 \\ L41 & 0 & 0 & L44 & 0 & 0 \\ 0 & L52 & 0 & 0 & L55 & L56 \\ 0 & L62 & 0 & 0 & L65 & L66 \end{bmatrix}\cdot\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i_{kq} \\ i_{fd} \\ i_{kd} \end{bmatrix} = \begin{bmatrix} L11\cdot i_{qs}+L14\cdot i_{kq} \\ L22\cdot i_{ds}+L25\cdot i_{fd}+L26\cdot i_{kd} \\ L33\cdot i_{0s} \\ L41\cdot i_{qs}+L44\cdot i_{kq} \\ L52\cdot i_{ds}+L55\cdot i_{fd}+L56\cdot i_{kd} \\ L62\cdot i_{ds}+L65\cdot i_{fd}+L66\cdot i_{kd} \end{bmatrix}$$

$$\mathbf{N}\omega_r\underline{i}=\begin{bmatrix} 0 & N12 & 0 & 0 & N15 & N16 \\ N21 & 0 & 0 & N24 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N42 & 0 & 0 & N45 & N46 \\ N51 & 0 & 0 & N54 & 0 & 0 \\ N61 & 0 & 0 & N64 & 0 & 0 \end{bmatrix}\cdot\omega_r\cdot\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i_{kq} \\ i_{fd} \\ i_{kd} \end{bmatrix} = \begin{bmatrix} N12\cdot\omega_r\cdot i_{ds}+N15\cdot\omega_r\cdot i_{fd}+N16\cdot\omega_r\cdot i_{kd} \\ N21\cdot\omega_r\cdot i_{qs}+N24\cdot\omega_r\cdot i_{kq} \\ 0 \\ N42\cdot\omega_r\cdot i_{ds}+N45\cdot\omega_r\cdot i_{fd}+N46\cdot\omega_r\cdot i_{kd} \\ N51\cdot\omega_r\cdot i_{qs}+N54\cdot\omega_r\cdot i_{kq} \\ N61\cdot\omega_r\cdot i_{qs}+N64\cdot\omega_r\cdot i_{kq} \end{bmatrix}$$

$$
\mathbf{V}\underline{v} = \begin{bmatrix} V11 & 0 & 0 & V14 & 0 & 0 \\ 0 & V22 & 0 & 0 & V25 & V26 \\ 0 & 0 & V33 & 0 & 0 & 0 \\ V41 & 0 & 0 & V44 & 0 & 0 \\ 0 & V52 & 0 & 0 & V55 & V56 \\ 0 & V62 & 0 & 0 & V65 & V66 \end{bmatrix} \cdot \begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \\ 0 \\ v_{fd} \\ 0 \end{bmatrix} = \begin{bmatrix} V11 \cdot v_{qs} \\ V22 \cdot v_{ds} + V25 \cdot v_{fd} \\ V33 \cdot v_{0s} \\ V41 \cdot v_{qs} \\ V52 \cdot v_{ds} + V55 \cdot v_{fd} \\ V62 \cdot v_{ds} + V65 \cdot v_{fd} \end{bmatrix}
$$

finally, the expression for the state derivative vector may be written as the sum of the three terms

$$
p\begin{vmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i_{kq} \\ i_{fd} \\ i_{kd} \end{vmatrix} = \begin{bmatrix} N12 \cdot \omega_r \cdot i_{ds} + N15 \cdot \omega_r \cdot i_{fd} + N16 \cdot \omega_r \cdot i_{kd} + L11 \cdot i_{qs} + L14 \cdot i_{kq} + V11 \cdot v_{qs} \\ N21 \cdot \omega_r \cdot i_{qs} + N24 \cdot \omega_r \cdot i_{kq} + L22 \cdot i_{ds} + L25 \cdot i_{fd} + L26 \cdot i_{kd} + V22 \cdot v_{ds} + V25 \cdot v_{fd} \\ L33 \cdot i_{0s} + V33 \cdot v_{0s} \\ N42 \cdot \omega_r \cdot i_{ds} + N45 \cdot \omega_r \cdot i_{fd} + N46 \cdot \omega_r \cdot i_{kd} + L41 \cdot i_{qs} + L44 \cdot i_{kq} + V41 \cdot v_{qs} \\ N51 \cdot \omega_r \cdot i_{qs} + N54 \cdot \omega_r \cdot i_{kq} + L52 \cdot i_{ds} + L55 \cdot i_{fd} + L56 \cdot i_{kd} + V52 \cdot v_{ds} + V55 \cdot v_{fd} \\ N61 \cdot \omega_r \cdot i_{qs} + N64 \cdot \omega_r \cdot i_{kq} + L62 \cdot i_{ds} + L65 \cdot i_{fd} + L66 \cdot i_{kd} + V62 \cdot v_{ds} + V65 \cdot v_{fd} \end{bmatrix}
$$

APPENDIX B: ACSL CODE

A. BUS VOLTAGE EQUATION MODEL

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!  !!
!!                                                                                 !!
!!        Mark Kipps                                                NPS Monterey   !!
!!                                                                                 !!
!!        Program to demonstrate validity of bus voltage equation model          . !!
!!        Example circuit is solved using two methods and the results              !!
!!        are compared.                                                            !!
!!                                                                                 !!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!  !!

PROGRAM
        NSTEPS          nstp = 1
        CINTERVAL       cint = 1e-2
        MAXTERVAL       maxt = 1e-3

DYNAMIC

    DERIVATIVE

        !--Circuit parameters
        CONSTANT    R1=1.0
        CONSTANT    R2=5.0
        CONSTANT    L1=0.6
        CONSTANT    L2=0.2

        !--Source voltage

        Vs      =PULSE(0.0, 2.5, 1.25)

        !--State equations for one loop solution

        id      = -(R1+R2)/(L1+L2)*i + Vs/(L1+L2)
        i       = INTEG(id,0.0)

        Vnode   = L2*id + R2*i

        !--State equations for the two sub-model solution

        i1d     = -R1/L1*i1 + Vs/L1 - Vt/L1
        i1      = INTEG(i1d,0.0)

        i2d     = -R2/L2*i2 + Vt/L2
        i2      = INTEG(i2d,0.0)
```

```
      !--Bus voltage equation for sub-models

      Vt        =(L1*L2)*(-R1*i1/L1 + R2*i2/L2 + Ve/L1)/(L1 + L2)

      !--Differences for output

      delv      = Vt - vnode
      deli      = i1 - i2
      delid     = i1d - i2d

      END  ! of derivative

   CONSTANT  tstop = 20.
   TERMT(t .GE. tstop)

END   ! of dynamic
END   ! of program
```

B. DASSL BUS VOLTAGE MODEL

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!  !!
!!                                                                    !!
!!      Mark Kippe                                     NPS Monterey   !!
!!                                                                    !!
!!      Program to demonstrate validity of DASSL bus voltage model.  !!
!!      Example circuit is solved using two methods and the results   !!
!!      are compared.                                                 !!
!!                                                                    !!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!  !!

PROGRAM
        NSTEPS          nstp = 1
        CINTERVAL       cint = 1e-2
        MAXTERVAL       maxt = 1e-3

DYNAMIC

   DERIVATIVE

        !--Circuit parameters
        CONSTANT    R1=1.0
        CONSTANT    R2=5.0
        CONSTANT    L1=0.6
        CONSTANT    L2=0.2

        !--Source voltage

        Vs      =PULSE(0.0, 2.5, 1.25)

        !--State equations for one loop solution

        id      = -(R1+R2)/(L1+L2)*i + Vs/(L1+L2)
        i       = INTEG(id,0.0)

        Vnode   = L2*id + R2*i

        !--State equations for the two sub-model solution

        i1d     = -R1/L1*i1 + Vs/L1 - Vt/L1
        i1      = INTEG(i1d,0.0)

        i2d     = -R2/L2*i2 + Vt/L2
        i2      = INTEG(i2d,0.0)

        !--DASSL bus voltage model sums current at the node

        resi    = i1d + i1 - i2d - i2
        Vt      = IMPLC(resi,0.0)
```

```
        !--Differences for output

        delv    = Vt - vnode
        deli    = i1 - i2
        delid   = i1d - i2d

        END  ! of derivative

    CONSTANT  tstop = 20.
    TERMT(t .GE. tstop)

END  ! of dynamic
END  ! of program
```

C. BUS VOLTAGE EQUATION MODEL UNDER CONTROL

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! !!
!!                                                                      !!
!!      Mark Kipps                                      NPS Monterey    !!
!!                                                                      !!
!!      Program to demonstrate validity of bus voltage equation model.  !!
!!      Example circuit is solved using two methods and the results     !!
!!      are compared. System under cascade voltage control.             !!
!!                                                                      !!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! !!

PROGRAM
        NSTEPS      nstp = 1
        CINTERVAL   cint = 5e-3
        MAXTERVAL   maxt = 1e-3

DYNAMIC

    DERIVATIVE

        !--Circuit parameters
        CONSTANT    R1=1.0
        CONSTANT    R2=5.0
        CONSTANT    L1=0.6
        CONSTANT    L2=0.2

        Vref    =STEP(0.2)

        !--Cascade voltage controller from root-locus design

        verr    =Vref-Vt
        v01d    =200*verr - 30*v01
        v01     =INTEG(v01d,0.0)
        Vsd     =v01d + 10*v01 - .01*Vs
        Vs      =INTEG(Vsd,0.0)


        !--State equations for two sub-model solution

        i1d     = -R1/L1*i1 + Vs/L1 - Vt/L1
        i1      = INTEG(i1d,0.0)

        i2d     = -R2/L2*i2 + Vt/L2
        i2      = INTEG(i2d,0.0)

        !--Bus voltage equation

        resi    = i1d + i1 - i2d - i2
        Vt      =(L1*L2)*(-R1*i1/L1 + R2*i2/L2 + Vs/L1)/(L1 + L2)
```

```
        !--Transfer function form of the system

        ve              =Vref - Vb
        DIMENSION       p(3), q(4)
        CONSTANT        p= 1.0, 35.0, 250.0, q= 1.0, 37.51, 225.375, 2.25
        Vb              =TRAN(2,3,p,q,50.0*ve)

        !--Differences for output

        deli     = i1 - i2
        delid    = i1d - i2d
        delv     = Vt - Vb

        END   ! of derivative

    CONSTANT  tstop = 1.0
    TERMT(t .GE. tstop)

END   ! of dynamic
END   ! of program
```

D. DASSL BUS VOLTAGE MODEL UNDER CONTROL

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!  !!
!!                                                                                         !!
!!      Mark Kipps                                                        NPS Monterey    !!
!!                                                                                         !!
!!      Program to demonstrate validity of DASSL bus voltage model.   !!
!!      Example circuit is solved using two methods and the results       !!
!!      are compared. System under cascade voltage control.               !!
!!                                                                                         !!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!  !!

PROGRAM
        NSTEPS        nstp = 1
        CINTERVAL     cint = 5e-3
        MAXTERVAL     maxt = 1e-3

DYNAMIC

    DERIVATIVE

        !--Circuit parameters
        CONSTANT    R1=1.0
        CONSTANT    R2=5.0
        CONSTANT    L1=0.6
        CONSTANT    L2=0.2


        Vref    = STEP(0.2)

        !--Cascade voltage controller from root-locus design

        verr    =Vref-Vt
        v01d    =200*verr - 30*v01
        v01     =INTEG(v01d,0.0)
        Vsd     =v01d + 10*v01 - .01*Vs
        Vs      =INTEG(Vsd,0.0)


        !--State equations for two sub-model solution

        i1d     = -R1/L1*i1 + Vs/L1 - Vt/L1
        i1      = INTEG(i1d,0.0)

        i2d     = -R2/L2*i2 + Vt/L2
        i2      = INTEG(i2d,0.0)

        !--DASSL bus voltage model

        resi    = i1d + i1 - i2d - i2
```

```
        Vt      = IMPLC(resi,0.0)

        !--Transfer function form of the system for comparison

        ve              =Vref - Vb
        DIMENSION       p(3), q(4)
        CONSTANT        p= 1.0, 35.0, 250.0, q= 1.0, 37.51, 225.375, 2.25
        Vb              =TRAN(2,3,p,q,50.0*ve)

        !--Differences for output

        deli    = i1 - i2
        delid   = i1d - i2d
        delv    = Vt - Vb

        END  ! of derivative

    CONSTANT  tstop = 1.0
    TERMT(t .GE. tstop)

END  ! of dynamic
END  ! of program
```

E. TOTAL SYSTEM MODEL

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!    !!
!!                                                                              !!
!!      Mark Kipps                                          NPS Monterey        !!
!!                                                                              !!
!!      Program to simulate a synchronous generator and loads on a finite    !!
!!      system bus.  Bus voltage is computed using the implicit equation    !!
!!      solving routine DASSL.                                                  !!
!!                                                                              !!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!    !!

PROGRAM
        NSTEPS          nstp = 1
        CINTERVAL       cint = 1e-3
        MAXTERVAL       maxt = 1e-3

INITIAL

        pi      = 4.0*atan(1.0)
        wb      = 120.*pi

!!!!!!!!---Synchronous Machine Parameters------------------------

        H       = 2.137
        smpb    = 3125.
        zb      = 1.0
        Xs      = .08*zb
        Xmq     = 1.0*zb
        Xmd     = 1.768*zb
        Xlkd    = .33383*zb
        Xlfd    = .13683*zb
        Xlkq    = .3298*zb
        Rfd     = .00111*zb
        Xkd     = Xlkd+Xmd
        Xfd     = Xlfd+Xmd
        Xkq     = Xlkq+Xmq
        Rs      = .00515*zb
        Rkd     = .02397*zb
        Rkq     = .0613*zb
        Xd      = Xs+Xmd
        Xq      = Xs+Xmq


!!!!!!!!---Induction Motor Parameters 200hp machine--------------

        impb1   = 149.14
```

```
zb_im1 = smpb/impb1
Rs_im1 = 0.01*zb_im1
Xls_im1 = 0.0655*zb_im1
Xm_im1          = 3.225*zb_im1
Xlr_im1 = 0.0655*zb_im1
Rr_im1  = 0.0261*zb_im1
H_im1   = 0.992*149.14/3125.
Xss_im1 = Xls_im1+Xm_im1
Xrr_im1 = Xlr_im1+Xm_im1

!!!!!!!!!---Induction Motor Parameters 150hp machine-------------

impb2   = 111.85
zb_im2 = smpb/impb2
Rs_im2 = 0.0051*zb_im2
Xls_im2 = 0.0553*zb_im2
Xm_im2          = 2.678*zb_im2
Xlr_im2 = 0.0553*zb_im2
Rr_im2  = 0.0165*zb_im2
H_im2   = 1.524*111.85/3125.
Xss_im2 = Xls_im2+Xm_im2
Xrr_im2 = Xlr_im2+Xm_im2

!!!!!!!!!---Induction Motor Parameters 40hp machine-----------------

impb3   = 29.83
zb_im3 = smpb/impb3
Rs_im3 = 0.005*zb_im3
Xls_im3 = 0.0587*zb_im3
Xm_im3          = 2.952*zb_im3
Xlr_im3 = 0.0587*zb_im3
Rr_im3 = 0.0165*zb_im3
H_im3   = 1.054*29.83/3125.
Xss_im3 = Xls_im3+Xm_im3
Xrr_im3 = Xlr_im3+Xm_im3

!!!!!!!!!---Field Exciter Parameters------------------------------

Kaa = 400.
Tae = 0.01
Kfe = 0.01
Tfe1 = 0.15
Tfe2 = 0.06
Kee = 1.0
Tee = 0.1

!!!!!!!!!---Prime Mover and Speed Governor Parameters---------------

Kc = 22.5
Tc = 0.55
Tfv = 0.01
```

```
        Tft = 0.05
        Wf10s = 0.23
        C2gt = 0.251
        C1gt = 1.3523
        Cgngt = 0.5

!!!!!!!!!---Synchronous Machine model coefficients-----------------
        !---Inverse matrix (obtained from MATHCAD)

        V11 = wb*Xkq/(Xmq**2 - Xq*Xkq)
        V14 = -wb*Xmq/(Xmq**2 - Xq*Xkq)
        V22 = wb*(Xmd**2 - Xfd*Xkd)/ &
                (Xd*Xfd*Xkd + (2*Xmd - Xkd - Xd - Xfd)*Xmd**2)
        V25 = wb*Rfd*(Xkd - Xmd)/ &
                (Xd*Xfd*Xkd + (2*Xmd - Xkd - Xd - Xfd)*Xmd**2)
        V26 = wb*Xmd*(Xfd - Xmd)/ &
                (Xd*Xfd*Xkd + (2*Xmd - Xkd - Xd - Xfd)*Xmd**2)
        V33 = -wb/Xs
        V41 = wb*Xmq/(Xmq**2 - Xq*Xkq)
        V44 = -wb*Xq/(Xmq**2 - Xq*Xkq)
        V52 = wb*Xmd*(Xmd - Xkd)/ &
                (Xd*Xfd*Xkd + (2*Xmd - Xkd - Xd - Xfd)*Xmd**2)
        V55 = wb*Rfd*(Xd*Xkd - Xmd**2)/ &
                (Xd*Xfd*Xkd + (2*Xmd - Xkd - Xd - Xfd)*Xmd**2)/Xmd
        V56 = wb*Xmd*(Xmd - Xd)/ &
                (Xd*Xfd*Xkd + (2*Xmd - Xkd - Xd - Xfd)*Xmd**2)
        V62 = wb*Xmd*(Xmd - Xfd)/ &
                (Xd*Xfd*Xkd + (2*Xmd - Xkd - Xd - Xfd)*Xmd**2)
        V65 = wb*Rfd*(Xmd - Xd)/ &
                (Xd*Xfd*Xkd + (2*Xmd - Xkd - Xd - Xfd)*Xmd**2)
        V66 = wb*(Xfd*Xd - Xmd**2)/ &
                (Xd*Xfd*Xkd + (2*Xmd - Xkd - Xd - Xfd)*Xmd**2)

        !---Linear terms matrix

        L11 = V11*Rs
        L14 = -V14*Rkq
        L22 = V22*Rs
        L25 = -V25*Xmd
        L26 = -V26*Rkd
        L33 = V33*Rs
        L41 = V41*Rs
        L44 = -V44*Rkq
        L52 = V52*Rs
        L55 = -V55*Xmd
        L56 = -V56*Rkd
        L62 = V62*Rs
        L65 = -V65*Xmd
        L66 = -V66*Rkd

        !---Nonlinear terms matrix
```

```
        N12 = V11*Xd/wb
        N15 = -V11*Xmd/wb
        N16 = -V11*Xmd/wb
        N21 = -V22*Xq/wb
        N24 = V22*Xmq/wb
        N42 = V41*Xd/wb
        N45 = -V41*Xmd/wb
        N46 = -V41*Xmd/wb
        N51 = -V52*Xq/wb
        N54 = V52*Xmq/wb
        N61 = -V62*Xq/wb
        N64 = V62*Xmq/wb

!!!!!!!!---Coefficients for Induction Motor Loads------------------


        MACRO imcoef (x)
                !---Inverse matrix

                B11&x = Xrr_im&x/(Xss_im&x*Xrr_im&x - Xm_im&x**2)*wb
                B14&x = -Xm_im&x/(Xss_im&x*Xrr_im&x - Xm_im&x**2)*wb
                B22&x = Xrr_im&x/(Xss_im&x*Xrr_im&x - Xm_im&x**2)*wb
                B25&x = -Xm_im&x/(Xss_im&x*Xrr_im&x - Xm_im&x**2)*wb
                B33&x = 1/Xls_im&x*wb
                B41&x = -Xm_im&x/(Xss_im&x*Xrr_im&x - Xm_im&x**2)*wb
                B44&x = Xss_im&x/(Xss_im&x*Xrr_im&x - Xm_im&x**2)*wb
                B52&x = -Xm_im&x/(Xss_im&x*Xrr_im&x - Xm_im&x**2)*wb
                B55&x = Xss_im&x/(Xss_im&x*Xrr_im&x - Xm_im&x**2)*wb
                B66&x = 1/Xlr_im&x*wb

                LM11&x = -B11&x*Rs_im&x
                LM14&x = -B14&x*Rr_im&x
                LM22&x = -B22&x*Rs_im&x
                LM25&x = -B25&x*Rr_im&x
                LM33&x = -B33&x*Rs_im&x
                LM41&x = -B41&x*Rs_im&x
                LM44&x = -B44&x*Rr_im&x
                LM52&x = -B52&x*Rs_im&x
                LM55&x = -B55&x*Rr_im&x
                LM66&x = -B66&x*Rr_im&x

                NE12&x = -B11&x*Xss_im&x/wb
                NE15&x = -B11&x*Xm_im&x/wb
                NE21&x =  B22&x*Xss_im&x/wb
                NE24&x =  B22&x*Xm_im&x/wb
                NE42&x = -B41&x*Xss_im&x/wb
                NE45&x = -B41&x*Xm_im&x/wb
                NE51&x =  B52&x*Xss_im&x/wb
                NE54&x =  B52&x*Xm_im&x/wb
```

```
            ND12&x = -B14&x*Xm_im&x/wb
            ND15&x = -B14&x*Xr_im&x/wb
            ND21&x =  B25&x*Xm_im&x/wb
            ND24&x =  B25&x*Xr_im&x/wb
            ND42&x = -B44&x*Xm_im&x/wb
            ND45&x = -B44&x*Xr_im&x/wb
            ND51&x =  B55&x*Xm_im&x/wb
            ND54&x =  B55&x*Xr_im&x/wb

      MACRO END  !of imcoef

      imcoef ("1")
      imcoef ("2")
      imcoef ("3")

!!!!!!!!!---Initial conditions------------------------------

      !---sources and loads
      iqsic    = 0.0
      idsic    = 0.0
      iosic    = 0.0
      ikqic    = 0.0
      ifdic    = 1/Xmd
      ikdic    = 0.0
      wric     = 376.991
      iqlpic   = iqsic
      idlpic   = idsic
      iolpic   = iosic
      vqsic    = 1.0
      vdsic    = 0.0
      vosic    = 0.0
      thtric   = 0.0
      !---exciter------
      vfdic    = 1.
      vreic    = vfdic + .1*exp(.3*vfdic)
      vo1ic    = 0.0113
      vfbic    = 0.00008
      !---prime mover-----
      Tiic     = 0.0
      wftic    = (Tiic/C1gt) + C2gt
      wfvic    = wftic
      werr3ic = wfvic - Wf10s

END  ! of INITIAL

DYNAMIC

      DERIVATIVE

            CONSTANT   vref    = 1.
            CONSTANT   wref    = 376.991
```

```
!---Load Parameters------------------------------

CONSTANT  mot_on1=0.0 , mot_on2=0.0 , mot_on3=0.0
CONSTANT  cb1=0.0

!---Square law pump loads------------------------

tl1     = (wr1)**2
tl2     = (wr2)**2
tl3     = (wr3)**2

!---R-L load parameters--------------------------

CONSTANT     rl = 5., Xl = 2.

!---Derive quantities for output-----------------

zero  = 0.0*t
pelec = 3*vas * ias
pmech = 0.5*wr*Tl/wb
pdevl = 0.5*wr*Te
deliq = iqs - iqlp - iql
delid = ids - idlp - idl
iamag = sqrt(iqs**2 + ids**2 + ios**2)
IF (ABS(iqs) .LT. 0.0001) THEN
        iaphs = 0.0
ELSE
        iaphs = atan(ids/iqs)*180.0/pi
END IF
vrip    = ABS(.07*cos(wr*t)) - .035  !to simulate rectifier
vamag = sqrt((vqs**2 + vds**2 + vos**2)) + vrip
frq   = (wr - wb)/wb
!---Convert to ind motor base--------------------
t1      = te_im1*smpb/impb1
t2      = te_im2*smpb/impb2
t3      = te_im3*smpb/impb3
wr1     = wr_im1/wb
wr2     = wr_im2/wb
wr3     = wr_im3/wb

!!!!!!!!!---exiter model--------------------------

verr    = vref - vamag - vfb
vred    = (verr*Kae - vre)/Tae
vre     = LIMINT(vred,vreic,0.0,8.5)
sat     = vre - .1*exp(.3*vfd)
vfdd    = (sat - vfd*Kee)/Tee
vfd     = INTEG(vfdd,vfdic)
vo1d    = (vre*Kfe - vo1)/Tfe1
vo1     = INTEG(vo1d,vo1ic)
```

```
        vfbd    = (vo1d - vfb)/Tfe2
        vfb     = INTEG(vfbd,vfbic)

!!!!!!!!---prime mover/governor model-----------------

        CONSTANT Gp = 1.
        werr1   = Gp*(wref - wr)/wb
        werr2   = Kc*werr1/Tc - Kc*wrd/wb
        werr3   = INTEG(werr2,werr3ic)
        werr4   = werr3 + Wf10s
        Wfvd    = (werr4 - wfv)/Tfv
        Wfv     = INTEG(Wfvd,Wfvic)
        Wftd    = (Wfv - Wft)/Tft
        Wft     = INTEG(Wftd,Wftic)
        Wft2    = (Wft - C2gt)*C1gt
        Ti      = Wft2 + Cgngt*werr1

        !---derive angles-------------------------

        thtrd   = wr
        thtr    = INTEG(thtrd,thtric)

        del     = atan(vds/vqs)*180.0/pi

!!!!!!!!---state equations for synchronous machine----

        iqsd    = L11*iqs + N12*wr*ids + L14*ikq + N15*wr*ifd &
                + N16*wr*ikd + V11*vqs
        iqs     = INTEG(iqsd,iqsic)

        idsd    = N21*wr*iqs + L22*ids + N24*wr*ikq + L25*ifd &
                + L26*ikd + V22*vds + V25*vfd
        ids     = INTEG(idsd,idsic)

        iosd    = L33*ios + V33*vos
        ios     = INTEG(iosd,iosic)

        ikqd    = L41*iqs + N42*wr*ids + L44*ikq + N45*wr*ifd &
                + N46*wr*ikd + V41*vqs
        ikq     = INTEG(ikqd,ikqic)

        ifdd    = N51*wr*iqs + L52*ids + N54*wr*ikq + L55*ifd &
                + L56*ikd + V52*vds + V55*vfd
        ifd     = INTEG(ifdd,ifdic)

        ikdd    = N61*wr*iqs + L62*ids + N64*wr*ikq + L65*ifd &
                + L66*ikd + V62*vds + V65*vfd
        ikd     = INTEG(ikdd,ikdic)

        !---GENERATOR electrical torque equation in terms of currents
```

```
        Te      = (Xmd*(-ids+ild+ikd)*iqs)-(Xmq*(-iqs+ikq)*ids)

        !---final state equation

        wrd     = wb*(Tl-Te)/2.0/H
        wr      = INTEG(wrd,wric)

        !---Speed of generator rotor determines electrical frequency

        we      = wr

!!!!!!!!!---state equation macro for the induction motor loads

        MACRO imstateqn (x)

                iqd_&x = LM11&x*iq_im&x + NE12&x*we*id_im&x + &
                        ND12&x*wd_im&x*id_im&x + LM14&x*iqr_im&x &
                        + NE15&x*we*idr_im&x + ND15&x*wd_im&x*idr_im&x &
                        + B11&x*(vqs+vqll)*mot_on&x
                iq_im&x = INTEG(iqd_&x,0.0)

                idd_&x = NE21&x*we*iq_im&x + ND21&x*wd_im&x*iq_im&x &
                        + LM22&x*id_im&x + NE24&x*we*iqr_im&x + &
                        ND24&x*wd_im&x*iqr_im&x + LM25&x*idr_im&x &
                        + B22&x*(vds+vdll)*mot_on&x
                id_im&x  = INTEG(idd_&x,0.0)

                iod_&x = LM33&x*io_im&x + B33&x*(vos+voll)*mot_on&x
                io_im&x = INTEG(iod_&x,0.0)

                iqrd_&x = LM41&x*iq_im&x + NE42&x*we*id_im&x + &
                        ND42&x*wd_im&x*id_im&x + LM44&x*iqr_im&x + &
                        NE45&x*we*idr_im&x + ND45&x*wd_im&x*idr_im&x &
                        + B41&x*(vqs+vqll)*mot_on&x
                iqr_im&x = INTEG(iqrd_&x,0.0)

                idrd_&x = NE51&x*we*iq_im&x + ND51&x*wd_im&x*iq_im&x &
                        + LM52&x*id_im&x + NE54&x*we*iqr_im&x + &
                        ND54&x*wd_im&x*iqr_im&x + LM55&x*idr_im&x &
                        + B52&x*(vds+vdll)*mot_on&x
                idr_im&x = INTEG(idrd_&x,0.0)

                iord_&x = LM66&x*ior_im&x
                ior_im&x = INTEG(iord_&x,0.0)

                Te_im&x  = Xm_im&x*(iq_im&x*idr_im&x - id_im&x*iqr_im&x)

                wrd_im&x = wb*(Te_im&x - (Tl&x*impb&x/smpb))/2.0/H_im&x
                wr_im&x  = INTEG(wrd_im&x,0.0)
                wd_im&x  = we - wr_im&x
```

```
        MACRO END  !of imstateqn

!!!!!!!!---call induction motor model macro

        imstateqn ("1")
        imstateqn ("2")
        imstateqn ("3")

!!!!!!!!!---sum of motor currents and current derivatives for
        !----constraint equation

        iqld    = (iqd_1 + iqd_2 + iqd_3)
        iql     = (iq_im1 + iq_im2 + iq_im3)

        idld    = (idd_1 + idd_2 + idd_3)
        idl     = (id_im1 + id_im2 + id_im3)

        iold    = (iod_1 + iod_2 + iod_3)
        iol     = (io_im1 + io_im2 + io_im3 )

!!!!!!!!!---state equations for the parallel R-L load-------------

        iqlpd   = (-wb*rl/Xl)*iqlp - wr*idlp + (wb/Xl)*(vqs+vqll)*cb1
        iqlp    = INTEG(iqlpd,iqlpic)

        idlpd   = wr*iqlp - (wb*rl/Xl)*idlp + (wb/Xl)*(vds+vdll)*cb1
        idlp    = INTEG(idlpd,idlpic)

        iolpd   = (-wb*rl/Xl)*iolp + (wb/Xl)*vos*cb1
        iolp    = INTEG(iolpd,iolpic)

!!!!!!!!!---DASSL bus voltage model based on implict relation-------

        resiq   = iqsd + iqs - iqlpd - iqlp - iqld - iql
        vqs     = IMPLC(resiq,vqsic)

        resid   = idsd + ids - idlpd - idlp - idld - idl
        vds     = IMPLC(resid,vdsic)

        resio   = iosd + ios - iolpd - iolp - iold - iol
        vos     = IMPLC(resio,vosic)

!!!!!!!!!---line loss model-----------------------------------

        CONSTANT rll = .005
        CONSTANT xll = .001
        vqll    =(- rll*iqs - xll*iqsd/wb - wr*xll*iqs/wb)
        vdll    =(- rll*ids - xll*idsd/wb + wr*xll*iqs/wb)
        voll    =(- rll*ios - xll*iosd/wb)
```

```
!!!!!!!!!---convert currents to a-b-c reference for output------------
            ias     = iqs*cos(thtr)                + ids*sin(thtr)                + ios
            ibs     = iqs*cos(thtr-2.0*pi/3.0)  + ids*sin(thtr-2.0*pi/3.0)  + ios
            ics     = iqs*cos(thtr+2.0*pi/3.0) + ids*sin(thtr+2.0*pi/3.0) + ios

            ial     = iql*cos(thtr)                + idl*sin(thtr)                + iol
            ibl     = iql*cos(thtr-2.0*pi/3.0)  + idl*sin(thtr-2.0*pi/3.0)  + iol
            icl     = iql*cos(thtr+2.0*pi/3.0) + idl*sin(thtr+2.0*pi/3.0) + iol

!!!!!!!!!---convert voltages to a-b-c reference for output------------
            vas     = vqs*cos(thtr)                + vds*sin(thtr)                + vos
            vbs     = vqs*cos(thtr-2.0*pi/3.0)  + vds*sin(thtr-2.0*pi/3.0)  + vos
            vcs     = vqs*cos(thtr+2.0*pi/3.0) + vds*sin(thtr+2.0*pi/3.0) + vos

        END  ! of DERIVATIVE
        CONSTANT     tstop = 6.25
        TERMT(t .GE. tstop)

END  ! of DYNAMIC
END  ! of PROGRAM
```

```
!!!!! ---COMMAND FILE FOR DASSL SYSTEM MODEL--- !!!!!!!

prepare t,iqs,iq_im1,vqs,vds,vfd,Te,wr,del,ias,vfb,vre,verr,werr1,ti,frq &
        deliq,delid,iqsd,iqld,zero,vamag,iamag,iaphs,vas,t1 &
        iq_im2,iq_im3,iqlp,t2,wr1,wr2,pelec,pmech,vql,vdl
set calplt=.f.,strplt=.t.,alcplt=.f.
set  nrwitg=.t.

proced mot
        start
        s mot_on1=1. mot_on2=1. mot_on3=1. tstop=9.0
        cont
        show2
end

proced mot2
        s mot_on1=1. mot_on2=1. mot_on3=1. tstop=3.0
        start
end

proced stpld
        start
        s mot_on1=1. mot_on2=1. mot_on3=1. tstop=9.0
        cont
```

```
        s rl=2. xql=2. xdl=2. xol=2. cb1=1. tstop=12.
        cont
        show2
end

proced short
        s rl=.0001 xdl=.0001 xql=.0001 xol=.0001 cb1=1. tstop=6.5
        cont
        s rl=15. xdl=10. xql=10. xol=10. cb1=0. tstop=8.0
        cont
        plot ias deliq vas vamag/xlo=6.2/xhi=tstop
        pause
        plot frq ti vfd
end

proced show
        plot wr2 t2/lo=-5/hi=5  wr1 t1/lo=-5/hi=5/xlo=6.0/xhi=tstop
        pause
        plot ias/lo=-1.5/hi=1.5 deliq vas vamag
        pause
        plot frq/lo=-.01/hi=.01  ti/lo=-.2/hi=.4 vfd/lo=0.0
end

proced savplt
        s xtispl=.16667 ytispl=.2 grdspl=.t. satspl=.t.
        s devplt=4 plt=11
        plot wr2/hi=1./xlo=6.0/xhi=tstop
        s plt=12
        s ytispl=.16667
        plot t2/lo=-6/hi=6
        s plt=13
        s ytispl=.2
        plot wr1/hi=1.
        s plt=14
        plot t1/lo=-5/hi=5
        s plt=15
        s ytispl=.16667 yinspl=1.6667
        plot ias/lo=-1.25/hi=1.25
        s plt=16
        s ytispl=.25 yinspl=2.
        plot vas/lo=-2./hi=2.
        s plt=17
        plot frq/lo=-.01/hi=.01
        s plt=18
        s ytispl=.2
        plot ti/lo=-.5/hi=.5
        s plt=19
        plot vfd/lo=0.0
        s plt=20
        s ytispl=.2 yinspl=1.4
        plot vamag/hi=1.4/lo=0.0
```

```
        s yinspl=2.
end

proced show2
        s xtispl=.16667 ytispl=.2 grdspl=.t. satspl=.t.
        s devplt=6
        plot wr2/hi=1./lo=0./xlo=6.0/xhi=tstop
        pause
        s ytispl=.16667
        plot t2/lo=-6/hi=6
        pause
        s ytispl=.2
        plot wr1/hi=1.
        pause
        plot t1/lo=-5/hi=5
        pause
        s ytispl=.25 yinspl=2.5
        plot ias/lo=-1.25/hi=1.25
        pause
        s ytispl=.25 yinspl=2.
        plot vas/lo=-2./hi=2.
        pause
        plot frq/lo=-.01/hi=.01
        pause
        s ytispl=.2
        plot ti/lo=-.5/hi=.5
        pause
        plot vfd/lo=0.0
        s ytispl=.2 yinspl=1.4
        pause
        plot vamag/hi=1.4/lo=0.0
        s yinspl=2.
end
```

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