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IMPROVEMENT OF THE OPTIMAL CONTROL OF INERTIAL OBJECTS IN THE PRESENCE OF PERTURBATIONS

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IMPROVEMENT OF THE OPTIMAL CONTROL OF INERTIAL OBJECTS IN THE PRESENCE OF PERTURBATIONS.

Following is a translation of an article by V. V. Kazakevich and A. P. Yurkevich, entitled "Ob uluchshenii kachestva ekstremal'nogo regulirovaniya inertsionnykh ob"uektov pri nalichii vozmushcheniy" (English version above), in Doklady Akademii nauk SSSR (Reports of Academy of Sciences SSSR), Vol. 136, No. 4, 1961, pp. 783-786.

The solution of the problem of the optimal control of inertial objects in the presence of low frequency external perturbations demands the application of special systems [1,2]. In the cases of application of optimal controllers of the usual type, for example with a memory storage of the optimum, it is necessary to lower the speed of hunting because of the inertial effect of the object, which, in its turn, deteriorates the interference immunity of the system.

In [2] a method of optimal control is suggested and theoretically proven. It permits, in principle, to remove the harmful inertial effect of the object of any order, in the absence of dynamic links in the object, located before its optimal characteristic. The system is distinctive in that at the control input is applied a definite, generally non-linear, combination of signals in accordance with the derivatives of the controlled value and by means of a non-linear link there is also introduced an action in accordance with the change of the function.

The system has better characteristics with respect to the quality of the process of controlling inertial objects, for low-frequency perturbations even as compared with a system of the usual type with a non-inertial object, since the effect of the low-frequency perturbations to a certain extent is filtered out. However, in the presence of strong perturbations, the level of the optimum of the function changes very rapidly and at a not very rapid change of the executing organ, the losses in hunting may still be considerable.

Let us consider the frequently encountered case of optimal control of an object of the first order. Let the characteristic of the optimal element have the form

$$y_1 = -K_x x^2, \tag{1}$$

where x and y_1 are correspondingly the values of the input and output of the optimal element. Let also $|\dot{x}| = V_x = \text{const}$ and the relation of the output of the object y_2 with y_1 have a form

$$y_2 = \frac{k_0}{p\tau_0 + 1} y_1, \tag{2}$$

where k_0 and τ_0 are constants.

Using the method of control shown in ([2]), if the action of the non-linear link is not taken into consideration, the signal at the input of the optimal controller will be $z = K_д y_2$. Denoting $y_1 K_д K_0 / \tau = y$, we obtain

$$z = \frac{p\tau_0}{p\tau_0 + 1} y. \tag{3}$$

The transient conditions in the control system under consideration is described by a system of equations analogous to that which describes the transient conditions in a non-inertial object with the application of a dynamic converter of the input signal.

The dynamics of a system of optimal control with a dynamic converter in the form of eq. (3) is considered in [3,4], in which the determination of parameters of the transient conditions and limiting cycles, taking into account the external perturbations, is obtained by the method of construction of phase trajectories of the system on a multi-leaf surface [5] in coordinates

$$\alpha = x/V_x\tau_0, \quad \beta = \dot{z}/2K_x V_x^2 \tau_0.$$

We will apply this method here. Within the limits between reversals of the executing organ we have

$$\beta = (\beta_н + 1) e^{-|\Delta\alpha|} - 1,$$

where $\Delta\alpha = \alpha - \alpha_н$; $\beta_н$ and $\alpha_н$ are the coordinates which define the points for t = 0. The reversals of the operating organ occur each time after curve β intersects the abscissa axis and the change in the area between curve β and

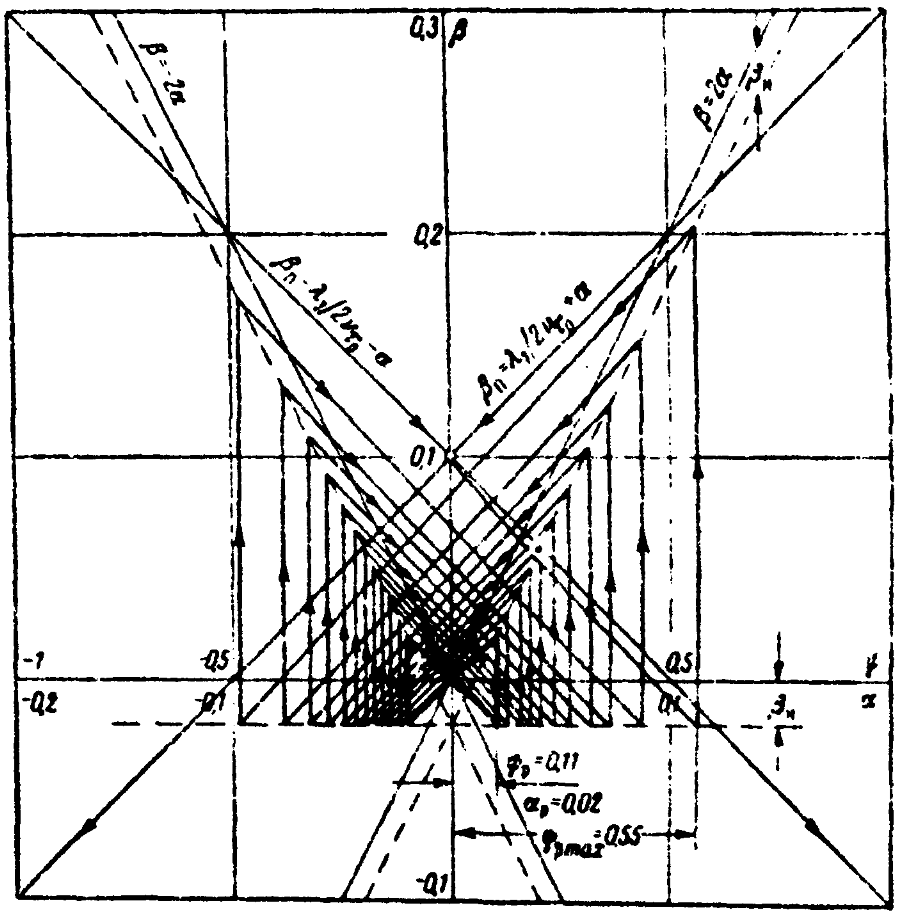


Figure 1

abscissa axis reaches the value $S_н = \delta_н / 2\eta v_{\tau_0}^2$,
where $\delta_н = z_н / y_э$, $z_н$ is the zone of insensitivity of the controller, $y_э$ is the value of the controlled quantity at the optimal point, $\eta = K_x x_{max}^2 / y_э$, $v_{\tau_0} = \tau_0 V_x / x_{max}$. At the reversal points β changes by a jump by $\pm 2\alpha$ depending on the value of $\dot{\alpha}$.

We will assume that starting at some moment of time t = 0 for x = 0 and y = 0, the action of the external perturbation is expressed by a linear time function. Then instead of (1) we have

$$y_1 = -K_x x^2 + K_1 t, \tag{4}$$

and at the initial moment (for t = 0) $\beta = \beta_0 = K_1 / 2K_x V_x^2 \tau_0$. The process of control corresponding to the given case, in terms of projection on the phase surface α, β for system parameters $\delta_н = 0,5\%$, $\lambda_1 = K_1 / K_x V_x X_{max} = 1$, $v_{\tau_0} = 5$, is shown in Fig. 1. It may be seen that there is an oscillation about the optimum of a diminishing amplitude which contracts to the limiting cycle with an amplitude $\alpha = \alpha_p$. For the given parameters the variation α in the process of oscillation is equal to 0.11 (to which corresponds $\psi = x / x_{max}$ = 0.55) and the timing is equal to $\approx 3\tau_0$. The introduction of a switch circuit which periodically reverses the system every period t_k, decreases the shown variations to the value $\psi_k = V_x t_k / x_{max}$.

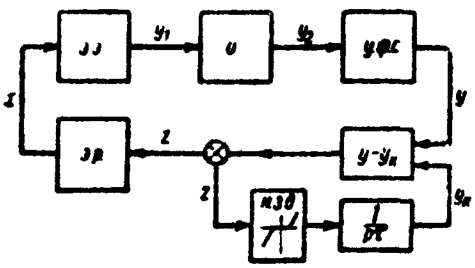
In the presence of a strong perturbation the effect of which appears to be a quadratic function of time giving $y_1 = -K_x x^2 + K_n t^2$, the deviation of the system from the optimum in the limiting cycle, in the absence in the controller of an insensitivity zone, can be described by a simplified formula

$$\psi_p = -\frac{v_{\tau_0}}{1.15} \ln(1 - \lambda_n), \qquad (5)$$

where $\lambda_n = K_n / K_x V_x^2$. These deviations for a large value of τ_0 and a limited velocity V_x can be appreciable. For example, for $\lambda_n = 0.2$ and $v_{\tau_0} = 5$ $\psi_p = 1$. For a reduction in the change time of the executing organ, there is a reduction of the time of the transient condition and of the amplitude Δa for the case of either a linear or a quadratic perturbation.

The quality of the optimal control of inertial objects in the presence of perturbations can be improved by application of a combination system for conversion of the input signal as shown in Figure 2.

Into the structural diagram of the system enter the following links: ye. ye. - optimal element with a characteristic $y_1 = -K_x x^2$; o., 0. - dynamic part of the object which is described by the differential equation $y_2^n + \psi_1(y_2', y_2'', \ldots \ldots y^{n-1}) + \psi_2(y_2) = k_0 y_1$; u. f. s. - the generating device which develops the sig-



ЭЭ = ye. ye
УФС = u. f. s.
Э.Р. = ye. r
Н.З.Д. = n. z. d.

Figure 2

nal y, the dynamic converter and the optimal controller ye. r. Thus, the system contains the device described in ([2]), generating a signal in accordance with the derivatives and changes of function y_2, which removes the effect of the inertia of the controlled object and besides it has a dynamic converter, representing a proportional link, encompassing an integrating feedback with an insensitive zone z_0. The dynamic converter at its optimal value for a constant time τ in a number of cases practically eliminates or lowers the unfavorable effect of low-frequency perturbations. The introduction into the feedback of the insensitive zone z_0 increases the precision of control on the sloped portions of the optimal characteric near the optimum for the action of monotonic external perturbations, which change x and at slow speed of the executing organ. The given system permits a great increase in the sensitivity of the controller (6).

For determination of parameters of transient conditions and limiting cycles of the system also in this case it is convenient to apply the method of construction of its phase trajectories in the coordinates α and β on a multi-leaf surface.

In the ideal case, with the absence of instrument distortions in operation of the installation, the excluding effect of the inertia of the object can be taken as $y = k_{\text{д}1} y_1$.

Conversion of signal y into $\dot{z} = u$ is determined by equations

$$u = \frac{p^2\tau}{p\tau + 1} y \quad \text{at} \quad -z_0 > z > z_0; \tag{6}$$

$$u = py \quad \text{at} \quad -z_0 < z < z_0. \tag{7}$$

The transition from eq. (6) to eq. (7) when the external perturbations are absent can be accomplished by $\dot{y}_\text{к} = 0$, thereby $\dot{z} = \dot{y}$, to which corresponds $\beta = \beta_n = x$.

In the presence of an external perturbation which is a linear function of time

$$\beta_n = \lambda_1 / 2v_\tau - x \operatorname{sign} \dot{x},$$

where $\quad v_\tau = \tau V_x / x_{max}$.

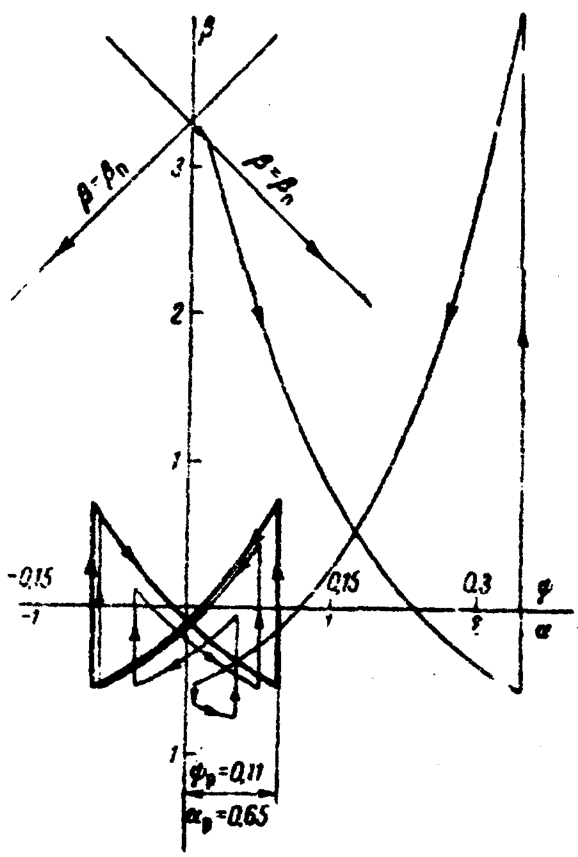


Figure 3

The operation of the system within the limits of the insensitivity zone of the feedback z_0 corresponds to some change β_n, whereby the area between the portion of the straight line β_n and its projection on the abscissa axis is equal to $S_0 = \delta_0/2\eta v_\tau^2$, where $\delta_0 = z_0/y_2$. The phase trajectories corresponding to equation (6) have the same form as for equation (3).

On Fig. 3 are given the projections on the phase plane of the phase trajectories of the optimal control for $v_\tau = \; =0.15$ and the same external conditions as in the case corresponding to Fig. 1. As can be seen on Fig. 3 deviations of the executing organ from the position corresponding to the optimum of the function diminished and the process of the setting up the limiting cycle was considerable speeded up; under this condition the transient period ψ does not exceed 0.33.

In the case of reaction of external perturbations, which are a quadratic function of time, the precision of control is also determined by eq. (5), but v_τ is determined by relation $\tau V_x/x_{max}$ which can be many times smaller than $\tau_0 V_x/x_{max}$. The value ψ_p will be correspondingly smaller.

The introduction into the control system of a switching circuit permits to reduce greatly the indicated momentary deviations of the systems.

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