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TECHNICAL REPORT ARCCB-TR-94012

CALCULATIONS VIA SUCCESSIVE APPROXIMATIONS OF STRESS AND STRAIN DISTRIBUTION IN THICK-WALLED CONCENTRIC TUBES DUE TO A RADIAL TEMPERATURE GRADIENT

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MARCH 1994

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	March 1994	Final

4. TITLE AND SUBTITLE
CALCULATIONS VIA SUCCESSIVE APPROXIMATIONS OF STRESS AND STRAIN DISTRIBUTION IN THICK-WALLED CONCENTRIC TUBES DUE TO A RADIAL TEMPERATURE GRADIENT

5. FUNDING NUMBERS
AMCMS: 612624H180.000
PRON: M721F221M71A

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army ARDEC
Benét Laboratories, SMCAR-CCB-TL
Watervliet, NY 12189-4050

8. PERFORMING ORGANIZATION REPORT NUMBER
ARCCB-TR-94012

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army ARDEC
Close Combat Armaments Center
Picatinny Arsenal, NJ 07806-5000

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

When thick-walled tubes are subjected to radial temperature gradients, cooler portions of the tubes impose constraints on the thermal expansion of hotter segments. (For example, in an internally heated tube, outer portions of the tube contain the expansion of the inner layer.) Constraint of the thermal expansion results in triaxial stress and strain distributions. A sequence of successive approximations has been used to calculate such stress and strain fields, while accounting for the mutual effect between the stress and strain fields, as described by the constitutive equations. Equilibrium is satisfied in all three mutually orthogonal coordinate directions, and compatibility (or conservation and continuity of matter) requirements are being complied with. Temperature dependence of the modulus of elasticity of the material is accounted for. The Mises' numbers field is computed to detect potential yielding. The total strain (thermal and elastic), which at the boundaries (inner and outer diameter surfaces and the axial ends of the tube) represents the dilation of the tubes, is also calculated.

14. SUBJECT TERMS
Thick-Walled Tubes, Thermal Dilation, Temperature Gradient

15. NUMBER OF PAGES
18

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

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NOMENCLATURE

a - Tube's inner radius.

A - The fracture of unimpeded thermal expansion's differential which expands (at the tube's outer diameter) under the prevailing (through-wall) temperature gradient.

b - Tube's outer radius.

E - Material's modulus of elasticity.

M - Mises' number (the material yields plastically whenever M reaches the material's yield strength.

T - Temperature.

α - Material's thermal coefficient of expansion.

Δ - Difference in thermal expansion between the hot and the cold surfaces.

ϵ - Strain.

ν - Material's Poisson's factor.

<u>Subscripts</u>

i - Any coordinate (r, x, or θ).

i - Initial (i.e., T_i: initial temperature).

r - On the radial plane.

r - In the radial direction.

x - On the axial plane.

x - In the axial direction.

(a) - At a distance r = a (from the tube's axis of symmetry).

(b) - At a distance r = b (from the tube's axis of symmetry).

(r) - At any distance r (from the tube's axis of symmetry).

θ - On the tangential plane.

θ - In the tangential (hoop) direction.

Superscripts

E	-	Elastic.
t	-	Total.
T	-	Thermal.

INTRODUCTION

The solutions to most classical problems in elasticity commence with a prescribed traction (force distribution) on part of the body's external surface and a known displacement on the remainder of that surface. In the problem at hand--the effect of a radial temperature gradient in a thick-walled tube--there are no external forces acting on the body, and the displacements of the body's (radial and axial) external surfaces are being sought. Thus, this problem defies conventional approaches.

Temperature-dependent variations in the material's density further complicate compatibility considerations. The same applies to temperature-dependent variations in the material's modulus of elasticity. Yet, any solution should satisfy the constitutive relations between the stresses and the strains as well as satisfying the equations of equilibrium and comply with compatibility requirements (conservation and continuity of matter should be complied with). Due to the gradient in the material's density, standard compatibility equations are not applicable for the test of conservation and continuity of matter.

The constitutive equations, as given in Eqs. (1a), (1b), and (1c) of this report, describe the stress components σ_{xx}, $\sigma_{\theta\theta}$, and σ_{rr} in terms of the strain components, ϵ_{xx}, $\epsilon_{\theta\theta}$, and ϵ_{rr} at each point within the body, or conversely, as given in Eqs. (2a), (2b), and (2c) of this report, describe the strain components, ϵ_{xx}, $\epsilon_{\theta\theta}$, and ϵ_{rr}, as functions of the stress components, σ_{xx}, $\sigma_{\theta\theta}$, and σ_{rr}.

$$\sigma_{xx(r)} = \frac{E_{(r)}}{1 - \nu - 2\nu^2} \cdot \left[(1 - \nu)\, \epsilon^E_{xx(r)} + \nu \left(\epsilon^E_{\theta\theta(r)} + \epsilon^E_{rr(r)}\right)\right] \tag{1a}$$

$$\sigma_{\theta\theta(r)} = \frac{E_{(r)}}{1 - \nu - 2\nu^2} \cdot \left[(1 - \nu)\, \epsilon^E_{\theta\theta(r)} + \nu \left(\epsilon^E_{rr(r)} + \epsilon^E_{xx(r)}\right)\right] \tag{1b}$$

$$\sigma_{rr(r)} = \frac{E_{(r)}}{1 - \nu - 2\nu^2} \cdot \left[(1 - \nu)\, \epsilon^E_{rr(r)} + \nu \left(\epsilon^E_{xx(r)} + \epsilon^E_{\theta\theta(r)}\right)\right] \tag{1c}$$

$$\epsilon^E_{xx(r)} = \frac{1}{E_{(r)}} \cdot \left[\sigma_{xx(r)} - \nu\left(\sigma_{\theta\theta(r)} + \sigma_{rr(r)}\right)\right] \tag{2a}$$

$$\epsilon^E_{\theta\theta(r)} = \frac{1}{E_{(r)}} \cdot \left[\sigma_{\theta\theta(r)} - \nu\left(\sigma_{rr(r)} + \sigma_{xx(r)}\right)\right] \tag{2b}$$

$$\epsilon^E_{rr(r)} = \frac{1}{E_{(r)}} \cdot \left[\sigma_{rr(r)} - \nu\left(\sigma_{xx(r)} + \sigma_{\theta\theta(r)}\right)\right] \tag{2c}$$

The following are the three equations of equilibrium:

in the x direction:

$$\int_a^b \sigma_{xx(r)} \cdot r \cdot dr = 0 \tag{3}$$

in the θ direction:

$$\int_a^b \sigma_{\theta\theta(r)} \cdot dr = 0 \tag{4}$$

and in the r direction:

$$\frac{d\sigma_{rr(r)}}{dr} = \frac{\sigma_{\theta\theta(r)} - \sigma_{rr(r)}}{r} \tag{5}$$

In tubes of a significant length-to-wall thickness ratio, it is reasonable to assume that transverse planes normal to the tube axis remain planar and normal to the axis. Namely,

$$\epsilon^t_{xx} = constant \tag{6}$$

independent of the radius.

APPROACH

Without external forces and in the absence of a clear displacement function for the body's external surface, a sequence of successive approximations was chosen. These successive approximations narrow down on a numerical description of the surface displacements while satisfying equilibrium (Eqs. (3), (4), and (5)) and complying with the constitutive relations between strain and stress (Eqs. (1a), (1b), and (1c) or conversely (2a), (2b) and (2c)) throughout the entire body, while satisfying compatibility (conservation of matter).

Equilibrium and compatibility requirements in axial and tangential orthogonal coordinate directions are used to narrow down on the displacements at the axial and radial surfaces. Equilibrium alone is used to determine the radial component of the stress distribution. However, each of the normal stress and normal strain components (in each of the three orthogonal coordinates, x, θ, and r) is a function of the strain components in the other two directions (as well as of the strain component in its own direction), as described by the constitutive equations. Therefore, the distribution of stress and strain components, obtained for one direction (after satisfying equilibrium and compatibility in that direction) is being employed in the evaluation of the stress and strain components in the consecutive coordinate direction (to be evaluated). Due to the interdependency of the stress and strain components between all three orthogonal directions, the cycle of successive approximations is repeated in all three directions until it converges to within a satisfied tolerance.

PROCEDURE

Unconstrained thermal expansion is at the inner radial surface, r=a

$$\epsilon^{T}_{ii(a)} = \alpha \cdot \left(T_{(a)} - T_{i}\right) \tag{6a}$$

and at the outer radial surface, r=b

$$\epsilon^{T}_{ii(b)} = \alpha \cdot \left(T_{(b)} - T_{i}\right) \tag{6b}$$

where, ϵ_{ii}^{T} is the thermal strain (in any direction, i=x, θ, or r), $\alpha \equiv$ thermal coefficient of expansion, and T_a, T_b, and T_i are the temperatures at the inner surface, at the outer surface, and the original temperature, respectively.

For the purpose of the calculations presented here, the following logarithmic temperature distribution between the inner surface and the outer surface is considered

$$T_{(r)} = T_{(b)} + \left(T_{(a)} - T_{(b)}\right) \cdot \frac{\log\left(\frac{r}{b}\right)}{\log\left(\frac{a}{b}\right)} \tag{7}$$

as shown in Figure 1. The modulus of elasticity is assumed to depend on the temperature as given in Eq. (8).

$$E_{(r)} = \left(207 - 3.82 \cdot 10^{-2} T_{(r)} + 7.7 \cdot 10^{-5} \cdot T^{2}_{(r)} - 4.45 \cdot 10^{-7} \cdot T^{3}_{(r)}\right) \cdot 145 \tag{8}$$

where the temperature, T, is given in °C and the modulus of elasticity, E, in Kpsi, as shown in Figure 2. However, the procedure described here and the computer program designed to execute it are not restricted to the above temperature distribution nor are they restricted to the modulus of elasticity's dependency on the temperature described by Eq. (8). With the above given temperature distribution, the thermal expansion at any radius, $a \leq r \leq b$, within the body is computed as

$$\epsilon^{T}_{ii(r)} = \alpha \cdot \left(T_{(r)} - T_{i}\right) = \alpha \cdot \left[T_{(b)} + \left(T_{(a)} - T_{(b)}\right) \cdot \frac{\log\left(\frac{r}{b}\right)}{\log\left(\frac{a}{b}\right)} - T_{i}\right] \tag{6c}$$

The difference in thermal expansion,

$$\Delta = \epsilon^{T}_{ii(a)} - \epsilon^{T}_{ii(b)} = \alpha \cdot \left[\left(T_{(a)} - T_{l}\right) - \left(T_{(b)} - T_{i}\right)\right] = \alpha \cdot \left[T_{(a)} - T_{(b)}\right] \tag{9}$$

in the axial and tangential directions between that which prevails at the hotter inner surface and that which prevails at the cooler outer surface is assumed to be distributed as elastic strain, which varies throughout the tube's wall thickness.

AXIAL DIRECTION

It is assumed that the elastic strain, $\epsilon^E_{xx(b)}$, at the outer surface is a fraction, A_x, of the difference in the thermal strain, Δ, between the outer surface and the inner surface. Thus,

$$\epsilon^E_{xx(b)} = A_x \cdot \alpha \cdot \left[T_{(a)} - T_{(b)}\right] \tag{10}$$

where $0 \leq A_x \leq 1$. With the exception of the axial ends, invoking Saint Venant's principle, the total axial expansion

$$\epsilon^t_{xx(r)} = \epsilon^T_{ii(r)} + \epsilon^E_{xx(r)} = C_x \tag{11}$$

is a constant. This constant, then, is

$$C_x = \alpha \cdot \left[\left(T_{(b)} - T_i\right) + A_x \cdot \left(T_{(a)} - T_{(b)}\right)\right] \tag{12}$$

Thus

$$\epsilon^E_{xx(r)} = C_x - \epsilon^T_{ii(r)} \tag{13}$$

where $\epsilon^T_{ii(r)}$ is computed by Eq. (6c).

Thus, from Eqs. (12), (13), and (6c)

$$\epsilon^E_{xx(r)} = \alpha \cdot \left(T_{(a)} - T_{(b)}\right) \cdot \left[A_x - \frac{\log\left(\frac{r}{b}\right)}{\log\left(\frac{a}{b}\right)}\right] \tag{14}$$

Applying the elastic strain from Eq. (14) and the modulus of elasticity from Eq. (8) into Eq. (1a) yields the distribution of the axial stress component as a function of the radius, r. In the first round, the strain components in the tangential direction and in the radial direction are assumed to be zero, while in any consecutive cycle, one applies the values that were obtained in the prior cycle.

The total axial force is the integral of the approximated axial stress component times its distance from the tube's center, r. Thus,

$$F_x = 2\,\pi \cdot \int_a^b \sigma_{xx(r)} \cdot r \cdot dr \tag{15}$$

Successive corrections of the value of A_x leads to A_x(optimal), which corresponds to $F_x=0$. The axial elastic strain and elastic stress, $\epsilon^E_{xx(r)}$ and $\sigma_{xx(r)}$, respectively, that correspond to the above optimal A_x are considered in the computation of the elastic strain and the elastic stress components in the tangential direction.

TANGENTIAL DIRECTION

As in the computation of the strain and stress components in the axial direction, it is assumed that the elastic strain, $\epsilon^E_{\theta\theta(b)}$, at the outer surface is a fraction, A_θ, of the difference in the thermal expansion, Δ, between the outer surface and the inner surface. Thus,

$$\epsilon^E_{\theta\theta(b)} = A_\theta \cdot \alpha \cdot \left(T_{(a)} - T_{(b)}\right) \tag{16}$$

where $0 \leq A_\theta \leq 1$. It can be shown that compatibility requires that

$$b \cdot \epsilon^t_{\theta\theta(b)} - r \cdot \epsilon^t_{\theta\theta(r)} = \int_r^b \epsilon^t_{rr(r)} \cdot dr \tag{17}$$

where ϵ^t_{ii} is the sum of the thermal expansion, ϵ^T_{ii}, plus the elastic strain, ϵ^E_{ii}, in each of the three orthogonal coordinate directions (x, θ, and r). Rearranging Eq. (17) yields

$$\epsilon^t_{\theta\theta(r)} = \frac{1}{r} \cdot \left[b \cdot \epsilon^t_{\theta\theta(b)} - \int_r^b \epsilon^t_{rr(r)} \cdot dr\right] \tag{17a}$$

As shown in the Appendix, complying with Eq. (17a) is equivalent to lack of shear in the r-θ plane.

Equilibrium requires that

$$\int_a^b \sigma_{\theta\theta(r)} \cdot dr = 0 \tag{18}$$

from Eq. (4), where $\sigma_{\theta\theta(r)}$ is computed by Eq. (1b). Through a sequence of successive approximations, an optimal value of A_θ, which results in the integral in Eq. (4) approaching zero as close as desired, is determined.

RADIAL DIRECTION

In the radial direction, the equation of equilibrium (in that direction), Eq. (5), is invoked and solved by using the Runge-Kutta method. The intermediate values of $\sigma_{\theta\theta(r)}$ for points between those for which it is computed in the above cycle, is interpolated linearly. Equation (2c) is employed for the calculation of the strain in the r direction.

CONCLUSION

During the first cycle, the stress components in the θ and r directions are assumed to be zero. In each successive cycle, the values obtained in the prior cycle are employed and a new and refined optimal A_r and optimal A_θ are computed. Furthermore, with all three orthogonal stress components, Eq. (19) is used to compute the Mises' stress as a function of the radial distance, r, where $a \le r \le b$.

$$M_{(r)} = \sqrt{\frac{1}{2}\left[\left(\sigma_{xx(r)} - \sigma_{\theta\theta(r)}\right)^2 + \left(\sigma_{\theta\theta(r)} - \sigma_{rr(r)}\right)^2 + \left(\sigma_{rr(r)} - \sigma_{xx(r)}\right)^2\right]} \qquad (19)$$

If and when the material's yield strength as a function of its temperature is known, the Mises' stress, $M_{(r)}$, indicates whether there is a potential yielding. Yielding under hydrostatic tension signals a potential for structural damage or even failure. Allowing for yielding in tension and/or under hydrostatic compression requires readjustments of the stress and strain fields, as determined by the above calculations. However, such adjustments are beyond the scope of this work. Nevertheless, it should be pointed out that since any such yielding is constrained by the elastic portion of the body, strains in the plastically deformed region are anticipated to be the same order of magnitude as a fully elastic tube.

Tables 1 and 2 and the corresponding Figures 3, 4a, and 4b represent the stress distribution with the Mises' numbers and the strain distribution (elastic in Figure 4a and the total strain in Figure 4b), respectively. These tables and figures represent the results obtained for a steel tube of wall ratio b/a = 2.8, heated from T_i = 22°C to T_a = 650°C at r = a and to T_b = 220°C at r = b.

Table 1. Radial Distribution of Temperature, Modulus of Elasticity, and Strain Components

	r	$T_{(r)}$	$E_{(r)}$	$\epsilon^T_{ii(r)}$	$\epsilon^E_{xx(r)}$	$\epsilon^t_{xx(r)}$	$\epsilon^E_{\theta\theta(r)}$	$\epsilon^t_{\theta\theta(r)}$	$\epsilon^E_{rr(r)}$	$\epsilon^t_{rr(r)}$
1	1.25000	0.6500E+03	0.1404E+08	0.7536E-02	-0.3395E-02	0.4141E-02	-0.3752E-02	0.3784E-02	0.1311E-02	0.8847E-02
2	1.29500	0.6352E+03	0.1507E+08	0.7359E-02	-0.3218E-02	0.4141E-02	-0.3349E-02	0.4009E-02	0.1211E-02	0.8570E-02
3	1.34000	0.6210E+03	0.1603E+08	0.7188E-02	-0.3047E-02	0.4141E-02	-0.2990E-02	0.4198E-02	0.1108E-02	0.8296E-02
4	1.38500	0.6072E+03	0.1691E+08	0.7022E-02	-0.2881E-02	0.4141E-02	-0.2666E-02	0.4356E-02	0.1005E-02	0.8027E-02
5	1.43000	0.5938E+03	0.1772E+08	0.6862E-02	-0.2721E-02	0.4141E-02	-0.2374E-02	0.4488E-02	0.9035E-03	0.7765E-02
6	1.47500	0.5809E+03	0.1847E+08	0.6707E-02	-0.2566E-02	0.4141E-02	-0.2107E-02	0.4599E-02	0.8037E-03	0.7510E-02
7	1.52000	0.5683E+03	0.1917E+08	0.6556E-02	-0.2415E-02	0.4141E-02	-0.1864E-02	0.4692E-02	0.7065E-03	0.7262E-02
8	1.56500	0.5561E+03	0.1982E+08	0.6410E-02	-0.2269E-02	0.4141E-02	-0.1641E-02	0.4768E-02	0.6124E-03	0.7022E-02
9	1.61000	0.5443E+03	0.2042E+08	0.6268E-02	-0.2127E-02	0.4141E-02	-0.1436E-02	0.4831E-02	0.5216E-03	0.6789E-02
10	1.65500	0.5328E+03	0.2098E+08	0.6129E-02	-0.1989E-02	0.4141E-02	-0.1247E-02	0.4882E-02	0.4341E-03	0.6564E-02
11	1.70000	0.5216E+03	0.2150E+08	0.5995E-02	-0.1854E-02	0.4141E-02	-0.1072E-02	0.4923E-02	0.3501E-03	0.6345E-02
12	1.74500	0.5107E+03	0.2199E+08	0.5864E-02	-0.1723E-02	0.4141E-02	-0.9095E-03	0.4955E-02	0.2694E-03	0.6133E-02
13	1.79000	0.5000E+03	0.2244E+08	0.5736E-02	-0.1596E-02	0.4141E-02	-0.7582E-03	0.4978E-02	0.1920E-03	0.5928E-02
14	1.83500	0.4897E+03	0.2287E+08	0.5612E-02	-0.1471E-02	0.4141E-02	-0.6170E-03	0.4995E-02	0.1177E-03	0.5730E-02
15	1.88000	0.4796E+03	0.2326E+08	0.5491E-02	-0.1350E-02	0.4141E-02	-0.4850E-03	0.5006E-02	0.4655E-04	0.5537E-02
16	1.92500	0.4697E+03	0.2364E+08	0.5372E-02	-0.1231E-02	0.4141E-02	-0.3612E-03	0.5011E-02	-0.2171E-04	0.5350E-02
17	1.97000	0.4600E+03	0.2398E+08	0.5256E-02	-0.1115E-02	0.4141E-02	-0.2450E-03	0.5011E-02	-0.8719E-04	0.5169E-02
18	2.01500	0.4506E+03	0.2431E+08	0.5143E-02	-0.1002E-02	0.4141E-02	-0.1355E-03	0.5008E-02	-0.1500E-03	0.4993E-02
19	2.06000	0.4414E+03	0.2461E+08	0.5032E-02	-0.8916E-03	0.4141E-02	-0.3235E-04	0.5000E-02	-0.2104E-03	0.4822E-02
20	2.10500	0.4323E+03	0.2490E+08	0.4924E-02	-0.7833E-03	0.4141E-02	0.6513E-04	0.4989E-02	-0.2685E-03	0.4656E-02
21	2.15000	0.4235E+03	0.2517E+08	0.4818E-02	-0.6773E-03	0.4141E-02	0.1574E-03	0.4975E-02	-0.3244E-03	0.4494E-02
22	2.19500	0.4149E+03	0.2542E+08	0.4714E-02	-0.5735E-03	0.4141E-02	0.2448E-03	0.4959E-02	-0.3782E-03	0.4336E-02
23	2.24000	0.4064E+03	0.2566E+08	0.4613E-02	-0.4718E-03	0.4141E-02	0.3278E-03	0.4940E-02	-0.4302E-03	0.4182E-02
24	2.28500	0.3981E+03	0.2588E+08	0.4513E-02	-0.3721E-03	0.4141E-02	0.4068E-03	0.4920E-02	-0.4805E-03	0.4032E-02
25	2.33000	0.3899E+03	0.2609E+08	0.4415E-02	-0.2744E-03	0.4141E-02	0.4819E-03	0.4897E-02	-0.5292E-03	0.3886E-02
26	2.37500	0.3819E+03	0.2629E+08	0.4319E-02	-0.1785E-03	0.4141E-02	0.5536E-03	0.4873E-02	-0.5764E-03	0.3743E-02
27	2.42000	0.3741E+03	0.2648E+08	0.4225E-02	-0.8445E-04	0.4141E-02	0.6220E-03	0.4847E-02	-0.6224E-03	0.3603E-02
28	2.46500	0.3664E+03	0.2665E+08	0.4133E-02	0.7887E-05	0.4141E-02	0.6874E-03	0.4820E-02	-0.6672E-03	0.3466E-02
29	2.51000	0.3589E+03	0.2682E+08	0.4042E-02	0.9855E-04	0.4141E-02	0.7501E-03	0.4792E-02	-0.7111E-03	0.3331E-02
30	2.55500	0.3514E+03	0.2698E+08	0.3953E-02	0.1876E-03	0.4141E-02	0.8101E-03	0.4763E-02	-0.7541E-03	0.3199E-02
31	2.60000	0.3441E+03	0.2712E+08	0.3866E-02	0.2751E-03	0.4141E-02	0.8676E-03	0.4733E-02	-0.7964E-03	0.3069E-02
32	2.64500	0.3370E+03	0.2726E+08	0.3780E-02	0.3611E-03	0.4141E-02	0.9229E-03	0.4703E-02	-0.8382E-03	0.2941E-02
33	2.69000	0.3299E+03	0.2739E+08	0.3695E-02	0.4456E-03	0.4141E-02	0.9761E-03	0.4671E-02	-0.8797E-03	0.2815E-02
34	2.73500	0.3230E+03	0.2752E+08	0.3612E-02	0.5288E-03	0.4141E-02	0.1027E-02	0.4639E-02	-0.9210E-03	0.2691E-02
35	2.78000	0.3162E+03	0.2763E+08	0.3530E-02	0.6106E-03	0.4141E-02	0.1077E-02	0.4607E-02	-0.9625E-03	0.2568E-02
36	2.82500	0.3095E+03	0.2775E+08	0.3450E-02	0.6910E-03	0.4141E-02	0.1124E-02	0.4574E-02	-0.1004E-02	0.2446E-02
37	2.87000	0.3029E+03	0.2785E+08	0.3371E-02	0.7702E-03	0.4141E-02	0.1170E-02	0.4541E-02	-0.1046E-02	0.2324E-02
38	2.91500	0.2964E+03	0.2795E+08	0.3293E-02	0.8482E-03	0.4141E-02	0.1214E-02	0.4507E-02	-0.1089E-02	0.2203E-02
39	2.96000	0.2900E+03	0.2804E+08	0.3216E-02	0.9250E-03	0.4141E-02	0.1257E-02	0.4473E-02	-0.1133E-02	0.2082E-02
40	3.00500	0.2837E+03	0.2813E+08	0.3140E-02	0.1001E-02	0.4141E-02	0.1299E-02	0.4439E-02	-0.1179E-02	0.1961E-02
41	3.05000	0.2775E+03	0.2822E+08	0.3066E-02	0.1075E-02	0.4141E-02	0.1339E-02	0.4405E-02	-0.1226E-02	0.1839E-02
42	3.09500	0.2714E+03	0.2830E+08	0.2992E-02	0.1148E-02	0.4141E-02	0.1379E-02	0.4371E-02	-0.1276E-02	0.1716E-02
43	3.14000	0.2653E+03	0.2837E+08	0.2920E-02	0.1221E-02	0.4141E-02	0.1417E-02	0.4336E-02	-0.1329E-02	0.1591E-02
44	3.18500	0.2594E+03	0.2844E+08	0.2849E-02	0.1292E-02	0.4141E-02	0.1454E-02	0.4302E-02	-0.1384E-02	0.1464E-02
45	3.23000	0.2535E+03	0.2851E+08	0.2778E-02	0.1362E-02	0.4141E-02	0.1490E-02	0.4268E-02	-0.1445E-02	0.1334E-02
46	3.27500	0.2477E+03	0.2858E+08	0.2709E-02	0.1432E-02	0.4141E-02	0.1525E-02	0.4234E-02	-0.1510E-02	0.1199E-02
47	3.32000	0.2421E+03	0.2864E+08	0.2641E-02	0.1500E-02	0.4141E-02	0.1559E-02	0.4200E-02	-0.1581E-02	0.1060E-02
48	3.36500	0.2364E+03	0.2869E+08	0.2573E-02	0.1568E-02	0.4141E-02	0.1593E-02	0.4166E-02	-0.1659E-02	0.9136E-03
49	3.41000	0.2309E+03	0.2875E+08	0.2507E-02	0.1634E-02	0.4141E-02	0.1625E-02	0.4132E-02	-0.1746E-02	0.7601E-03
50	3.45500	0.2254E+03	0.2880E+08	0.2441E-02	0.1700E-02	0.4141E-02	0.165[illegible]E-02	0.4099E-02	-0.1643E-02	0.5978E-03
51	3.50000	0.2200E+03	0.2885E+08	0.2376E-02	0.1765E-02	0.4141E-02	0.1690E-02	0.4066E-02	-0.1375E-02	0.1001E-02

Table 2. Radial Distribution of Temperature, Modulus of Elasticity, Mises' Numbers, Elastic Strain Components, and Elastic Stress Components

	r	$T_{(r)}$	$E_{(r)}$	$M_{(r)}$	$\epsilon^E_{xx}(r)$	$\sigma_{xx}(r)$	$\epsilon^E_{\theta\theta}(r)$	$\sigma_{\theta\theta}(r)$	$\epsilon^E_{rr}(r)$	$\sigma_{rr}(r)$
1	1.25000	0.6500E+03	0.1404E+08	0.7550E+05	-0.3395E-02	-0.7649E+05	-0.3752E-02	-0.7434E+05	0.1311E-02	0.6650E+02
2	1.29500	0.6352E+03	0.1507E+08	0.7223E+05	-0.3218E-02	-0.7698E+05	-0.3349E-02	-0.7222E+05	0.1211E-02	-0.2484E+04
3	1.34000	0.6210E+03	0.1603E+08	0.6867E+05	-0.3047E-02	-0.7675E+05	-0.2990E-02	-0.6959E+05	0.1108E-02	-0.4783E+04
4	1.38500	0.6072E+03	0.1691E+08	0.6493E+05	-0.2881E-02	-0.7593E+05	-0.2666E-02	-0.6660E+05	0.1005E-02	-0.6842E+04
5	1.43000	0.5938E+03	0.1772E+08	0.6109E+05	-0.2721E-02	-0.7462E+05	-0.2374E-02	-0.6334E+05	0.9035E-03	-0.8672E+04
6	1.47500	0.5809E+03	0.1847E+08	0.5723E+05	-0.2566E-02	-0.7291E+05	-0.2107E-02	-0.5990E+05	0.8037E-03	-0.1029E+05
7	1.52000	0.5683E+03	0.1917E+08	0.5340E+05	-0.2415E-02	-0.7085E+05	-0.1864E-02	-0.5635E+05	0.7065E-03	-0.1171E+05
8	1.56500	0.5561E+03	0.1982E+08	0.4961E+05	-0.2269E-02	-0.6852E+05	-0.1641E-02	-0.5273E+05	0.6124E-03	-0.1294E+05
9	1.61000	0.5443E+03	0.2042E+08	0.4591E+05	-0.2127E-02	-0.6596E+05	-0.1436E-02	-0.4908E+05	0.5216E-03	-0.1400E+05
10	1.65500	0.5328E+03	0.2098E+08	0.4231E+05	-0.1989E-02	-0.6320E+05	-0.1247E-02	-0.4541E+05	0.4341E-03	-0.1490E+05
11	1.70000	0.5216E+03	0.2150E+08	0.3883E+05	-0.1854E-02	-0.6029E+05	-0.1072E-02	-0.4176E+05	0.3501E-03	-0.1566E+05
12	1.74500	0.5107E+03	0.2199E+08	0.3549E+05	-0.1723E-02	-0.5724E+05	-0.9095E-03	-0.3814E+05	0.2694E-03	-0.1629E+05
13	1.79000	0.5000E+03	0.2244E+08	0.3231E+05	-0.1596E-02	-0.5409E+05	-0.7582E-03	-0.3455E+05	0.1920E-03	-0.1680E+05
14	1.83500	0.4897E+03	0.2287E+08	0.2931E+05	-0.1471E-02	-0.5085E+05	-0.6170E-03	-0.3101E+05	0.1177E-03	-0.1719E+05
15	1.88000	0.4796E+03	0.2326E+08	0.2652E+05	-0.1350E-02	-0.4755E+05	-0.4850E-03	-0.2753E+05	0.4655E-04	-0.1748E+05
16	1.92500	0.4697E+03	0.2364E+08	0.2396E+05	-0.1231E-02	-0.4419E+05	-0.3612E-03	-0.2410E+05	-0.2171E-04	-0.1767E+05
17	1.97000	0.4600E+03	0.2398E+08	0.2169E+05	-0.1115E-02	-0.4080E+05	-0.2450E-03	-0.2073E+05	-0.8719E-04	-0.1778E+05
18	2.01500	0.4506E+03	0.2431E+08	0.1976E+05	-0.1002E-02	-0.3737E+05	-0.1355E-03	-0.1743E+05	-0.1500E-03	-0.1781E+05
19	2.06000	0.4414E+03	0.2461E+08	0.1822E+05	-0.8916E-03	-0.3393E+05	-0.3235E-04	-0.1419E+05	-0.2104E-03	-0.1777E+05
20	2.10500	0.4323E+03	0.2490E+08	0.1715E+05	-0.7833E-03	-0.3048E+05	0.6513E-04	-0.1101E+05	-0.2685E-03	-0.1766E+05
21	2.15000	0.4235E+03	0.2517E+08	0.1657E+05	-0.6773E-03	-0.2704E+05	0.1574E-03	-0.7901E+04	-0.3244E-03	-0.1748E+05
22	2.19500	0.4149E+03	0.2542E+08	0.1651E+05	-0.5735E-03	-0.2359E+05	0.2448E-03	-0.4857E+04	-0.3782E-03	-0.1726E+05
23	2.24000	0.4064E+03	0.2566E+08	0.1692E+05	-0.4718E-03	-0.2016E+05	0.3278E-03	-0.1879E+04	-0.4302E-03	-0.1698E+05
24	2.28500	0.3981E+03	0.2588E+08	0.1773E+05	-0.3721E-03	-0.1675E+05	0.4068E-03	0.1034E+04	-0.4805E-03	-0.1665E+05
25	2.33000	0.3899E+03	0.2609E+08	0.1887E+05	-0.2744E-03	-0.1336E+05	0.4819E-03	0.3881E+04	-0.5292E-03	-0.1628E+05
26	2.37500	0.3819E+03	0.2629E+08	0.2025E+05	-0.1785E-03	-0.9996E+04	0.5536E-03	0.6664E+04	-0.5764E-03	-0.1587E+05
27	2.42000	0.3741E+03	0.2648E+08	0.2179E+05	-0.8445E-04	-0.6664E+04	0.6220E-03	0.9383E+04	-0.6224E-03	-0.1543E+05
28	2.46500	0.3664E+03	0.2665E+08	0.2345E+05	0.7887E-05	-0.3365E+04	0.6874E-03	0.1204E+05	-0.6672E-03	-0.1495E+05
29	2.51000	0.3589E+03	0.2682E+08	0.2518E+05	0.9855E-04	-0.1042E+03	0.7501E-03	0.1463E+05	-0.7111E-03	-0.1445E+05
30	2.55500	0.3514E+03	0.2698E+08	0.2695E+05	0.1876E-03	0.3116E+04	0.8101E-03	0.1716E+05	-0.7541E-03	-0.1391E+05
31	2.60000	0.3441E+03	0.2712E+08	0.2873E+05	0.2751E-03	0.6293E+04	0.8676E-03	0.1962E+05	-0.7964E-03	-0.1335E+05
32	2.64500	0.3370E+03	0.2726E+08	0.3051E+05	0.3611E-03	0.9424E+04	0.9229E-03	0.2202E+05	-0.8382E-03	-0.1277E+05
33	2.69000	0.3299E+03	0.2739E+08	0.3227E+05	0.4456E-03	0.1251E+05	0.9761E-03	0.2436E+05	-0.8797E-03	-0.1217E+05
34	2.73500	0.3230E+03	0.2752E+08	0.3401E+05	0.5288E-03	0.1553E+05	0.1027E-02	0.2663E+05	-0.9210E-03	-0.1155E+05
35	2.78000	0.3162E+03	0.2763E+08	0.3572E+05	0.6106E-03	0.1851E+05	0.1077E-02	0.2883E+05	-0.9625E-03	-0.1091E+05
36	2.82500	0.3095E+03	0.2775E+08	0.3738E+05	0.6910E-03	0.2142E+05	0.1124E-02	0.3097E+05	-0.1004E-02	-0.1026E+05
37	2.87000	0.3029E+03	0.2785E+08	0.3900E+05	0.7702E-03	0.2427E+05	0.1170E-02	0.3304E+05	-0.1046E-02	-0.9631E+04
38	2.91500	0.2964E+03	0.2795E+08	0.4056E+05	0.8482E-03	0.2706E+05	0.1214E-02	0.3503E+05	-0.1089E-02	-0.8927E+04
39	2.96000	0.2900E+03	0.2804E+08	0.4207E+05	0.9250E-03	0.2977E+05	0.1257E-02	0.3695E+05	-0.1133E-02	-0.8244E+04
40	3.00500	0.2837E+03	0.2813E+08	0.4351E+05	0.1001E-02	0.3240E+05	0.1299E-02	0.3879E+05	-0.1179E-02	-0.7553E+04
41	3.05000	0.2775E+03	0.2822E+08	0.4487E+05	0.1075E-02	0.3495E+05	0.1339E-02	0.4055E+05	-0.1226E-02	-0.6857E+04
42	3.09500	0.2714E+03	0.2830E+08	0.4615E+05	0.1148E-02	0.3741E+05	0.1379E-02	0.4221E+05	-0.1276E-02	-0.6155E+04
43	3.14000	0.2653E+03	0.2837E+08	0.4734E+05	0.1221E-02	0.3976E+05	0.1417E-02	0.4376E+05	-0.1329E-02	-0.5451E+04
44	3.18500	0.2594E+03	0.2844E+08	0.4842E+05	0.1292E-02	0.4199E+05	0.1454E-02	0.4520E+05	-0.1384E-02	-0.4746E+04
45	3.23000	0.2535E+03	0.2851E+08	0.4938E+05	0.1362E-02	0.4408E+05	0.1490E-02	0.4651E+05	-0.1445E-02	-0.4041E+04
46	3.27500	0.2477E+03	0.2858E+08	0.5019E+05	0.1432E-02	0.4601E+05	0.1525E-02	0.4766E+05	-0.1510E-02	-0.3338E+04
47	3.32000	0.2421E+03	0.2864E+08	0.5083E+05	0.1500E-02	0.4775E+05	0.1559E-02	0.4862E+05	-0.1581E-02	-0.2640E+04
48	3.36500	0.2364E+03	0.2869E+08	0.5125E+05	0.1568E-02	0.4924E+05	0.1593E-02	0.4935E+05	-0.1659E-02	-0.1950E+04
49	3.41000	0.2309E+03	0.2875E+08	0.5139E+05	0.1634E-02	0.5044E+05	0.1625E-02	0.4979E+05	-0.1746E-02	-0.1270E+04
50	3.45500	0.2254E+03	0.2880E+08	0.5116E+05	0.1700E-02	0.5123E+05	0.1658E-02	0.4984E+05	-0.1843E-02	-0.6045E+03
51	3.50000	0.2200E+03	0.2885E+08	0.5738E+05	0.1765E-02	0.6516E+05	0.1690E-02	0.4297E+05	-0.1375E-02	0.0000E+00

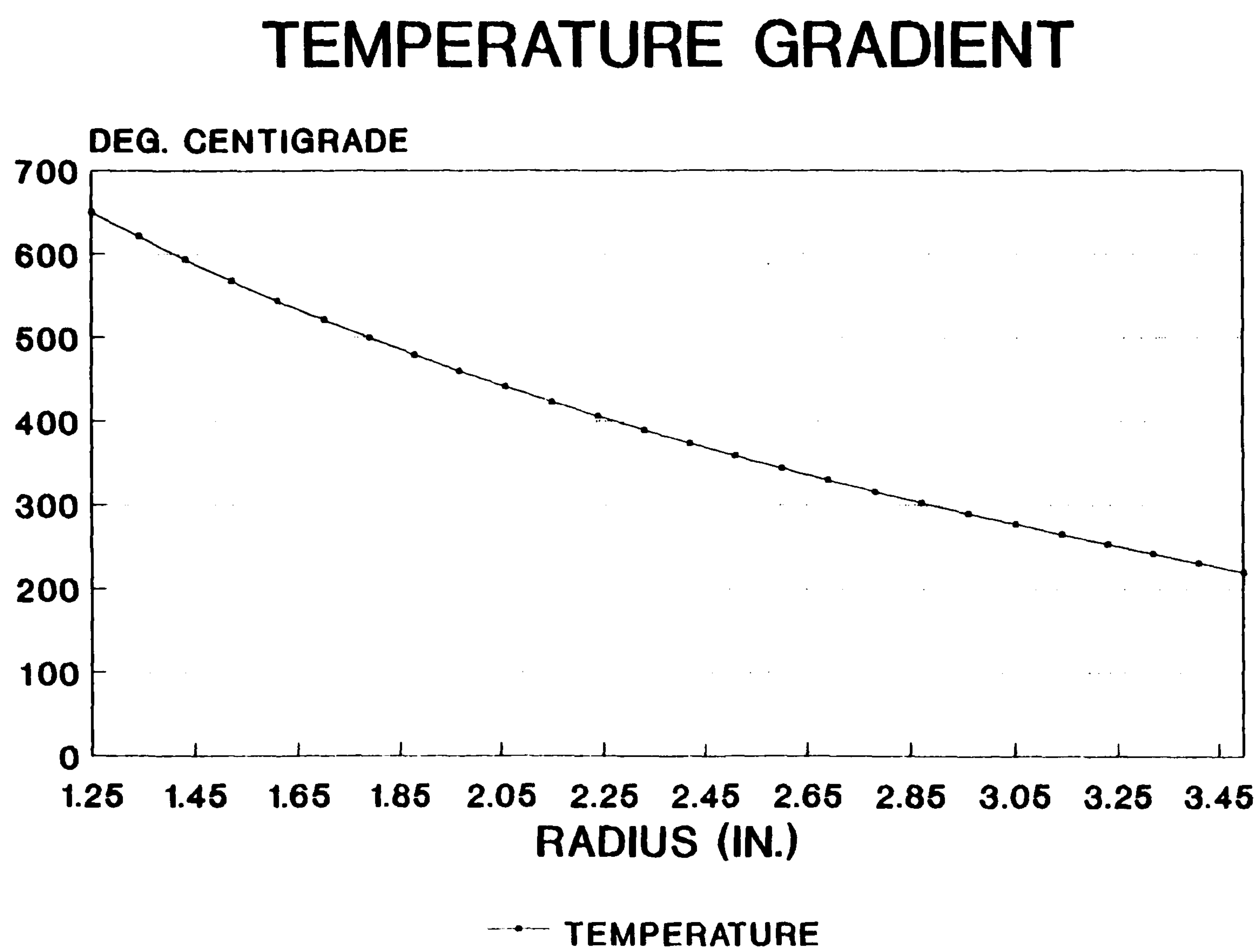

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Figure 1. Radial temperature gradient.

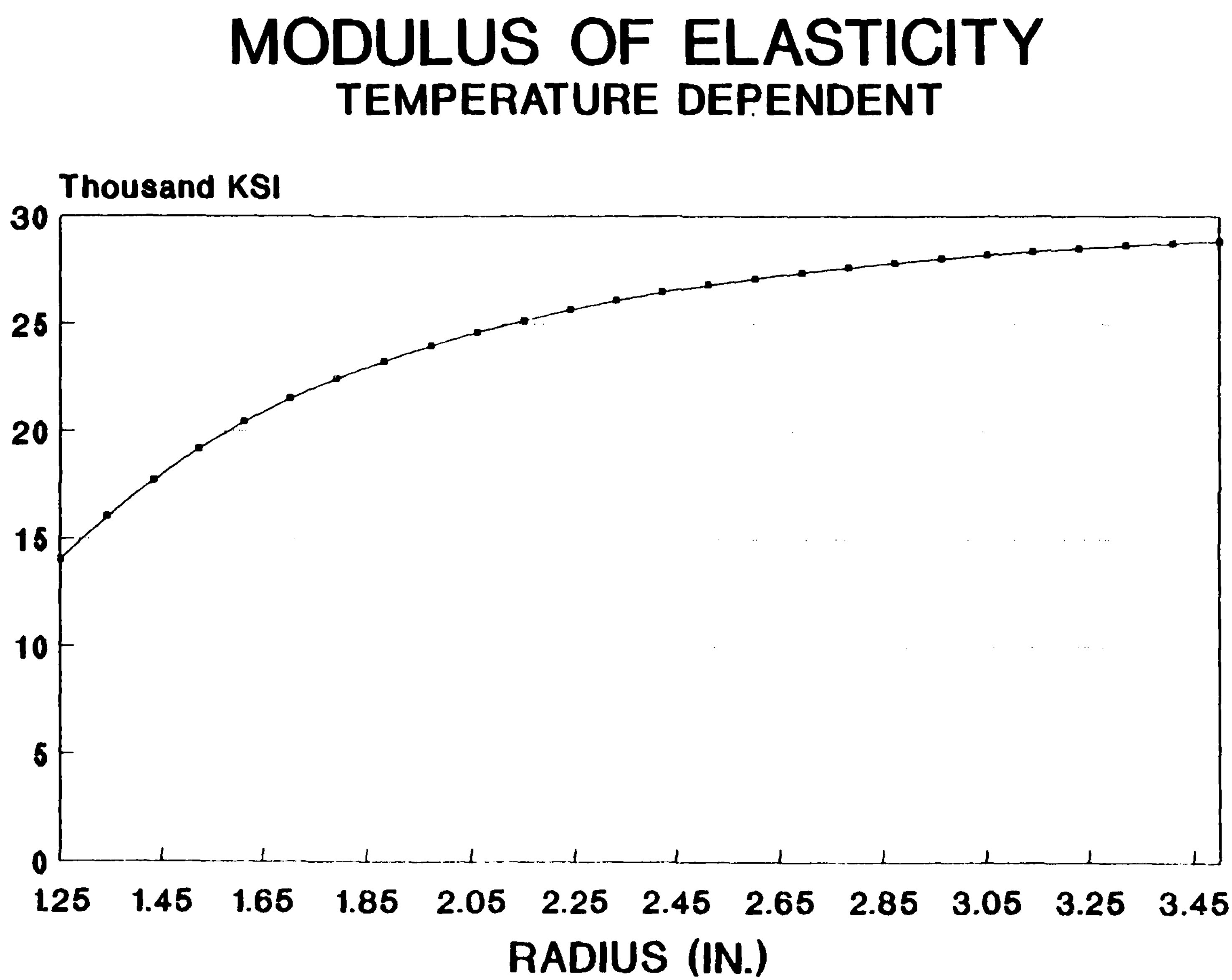

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Figure 2. Radial distribution of modulus of elasticity.

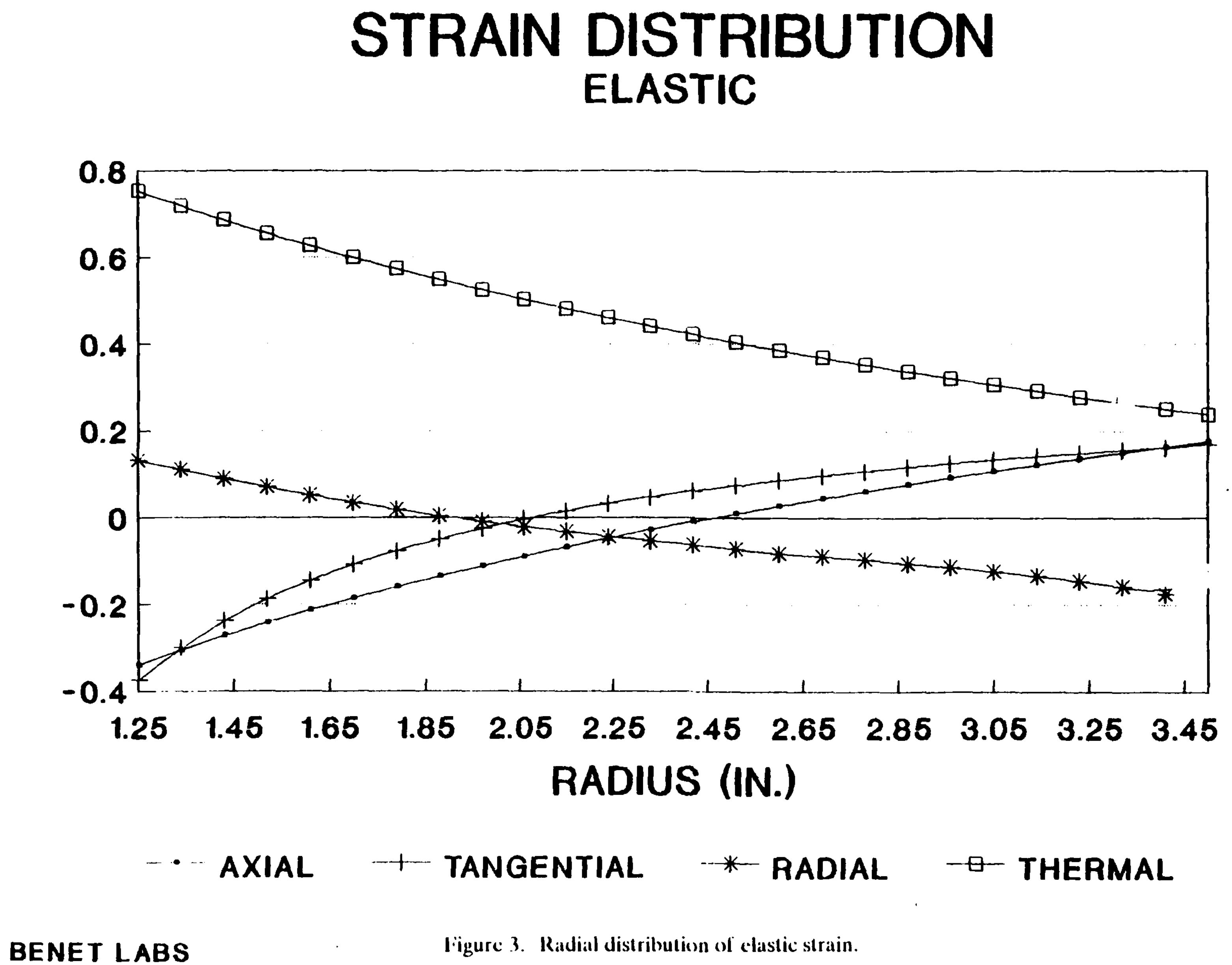

Figure 3. Radial distribution of elastic strain.

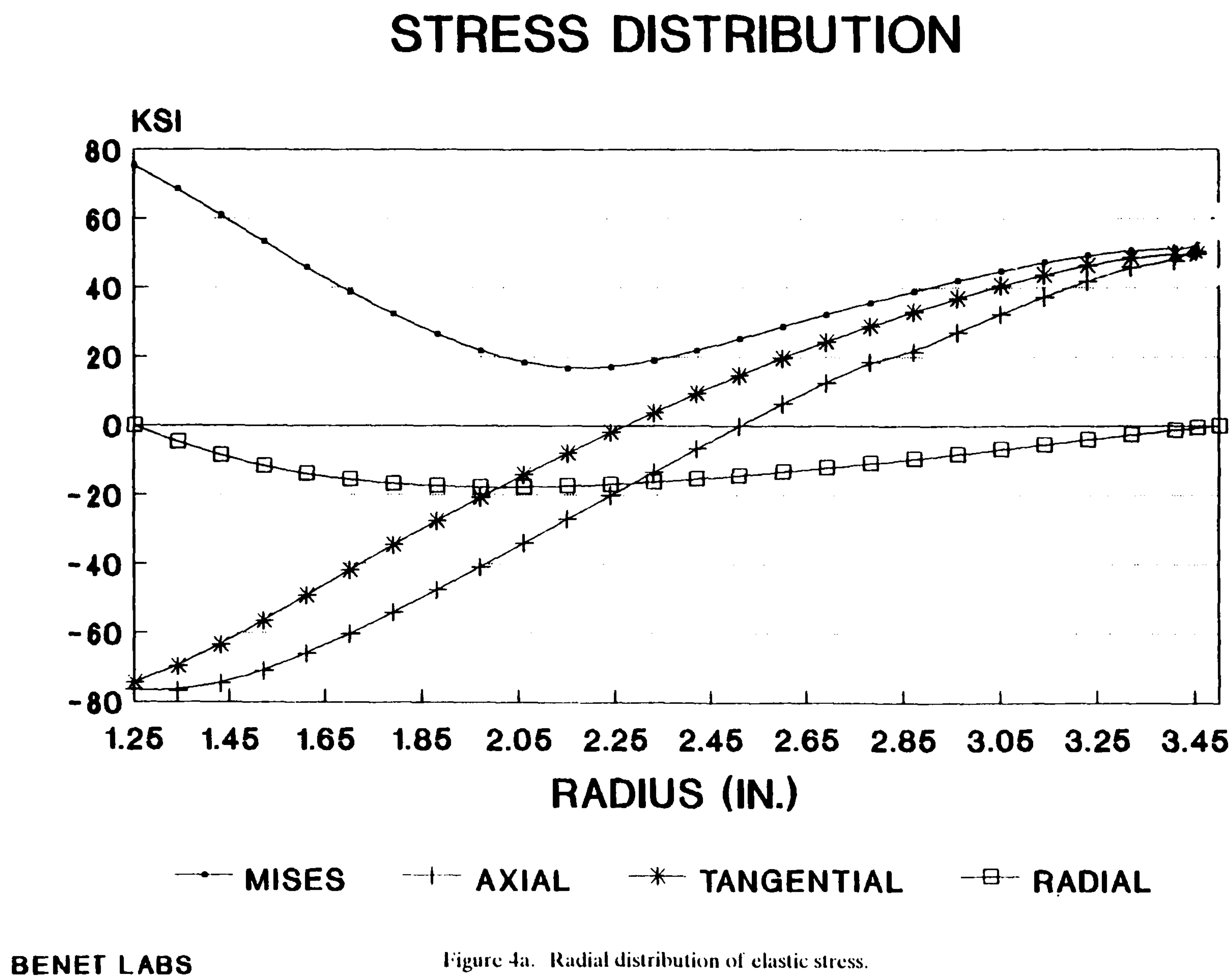

Figure 4a. Radial distribution of elastic stress.

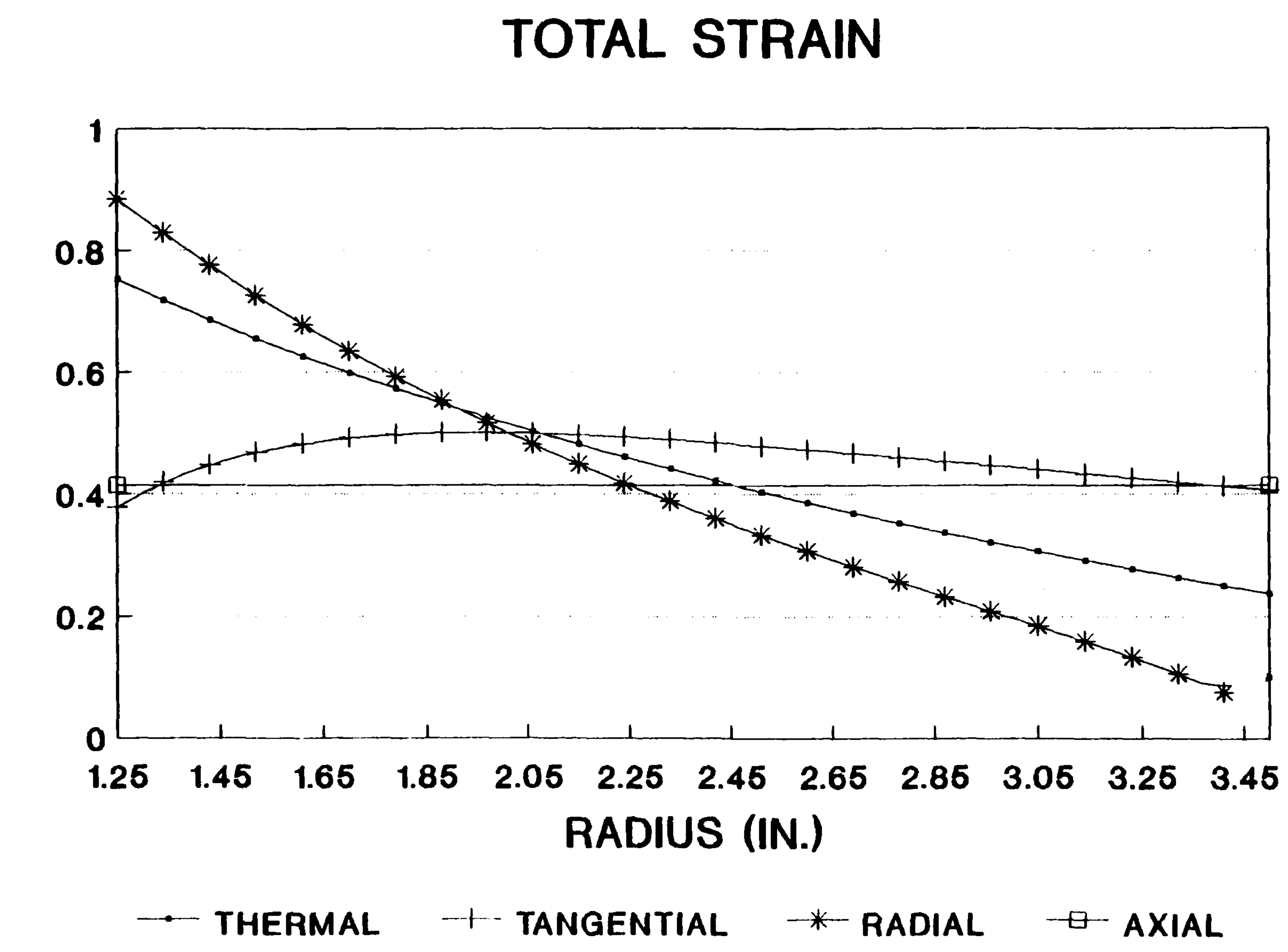

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Figure 4b. Radial distribution of the total strain (elastic plus thermal).

APPENDIX

Conservation of matter (or compatibility) can also be expressed as

$$r \cdot \epsilon^t_{\theta\theta(r)} - a \cdot \epsilon^t_{\theta\theta(a)} = \int_a^r \epsilon^t_{rr(r)} \cdot dr \tag{A-1}$$

or

$$\epsilon^t_{\theta\theta(r)} = \frac{1}{r}\left[\int_a^r \epsilon_{rr(r)}\, dr + a \cdot \epsilon^t_{\theta\theta(a)}\right] \tag{A-1a}$$

instead of Eqs. (17) and (17a) in the text.

If, however, one implies from the axisymmetry of the tube that a straight line through the center remains straight and that any angle, θ, between any two such lines remains constant, then

$$a^T = a_o\left[1 + \epsilon^t_{\theta\theta(a)}\right] \tag{A-2}$$

when the tube is being heated, and the length of any arc

$$l_{o(a)} = a_o \cdot \theta \tag{A-3}$$

will become

$$l^T_{(a)} = a^T \cdot \theta = a_o \cdot \left[1 + \epsilon^t_{\theta\theta(a)}\right] \cdot \theta \tag{A-4}$$

An equivalent arc, $l_{o(r)}$, at a distance r_o from the center of the tube is

$$l_{o(r)} = r_o \cdot \theta \tag{A-5}$$

whereas

$$\begin{aligned} r^T &= a_o\left(1 + \epsilon^t_{\theta\theta(a)}\right) + \int_a^r \left(1 + \epsilon^t_{rr}(r)\right) \cdot dr = \\ &= a_o \cdot \epsilon^t_{\theta\theta(a)} + r_o + \int_a^r \epsilon_{rr(r)} \cdot dr \end{aligned} \tag{A-6}$$

and the arc, $l_{o(r)}$, will become

$$l^T_{(r)} = \left[a_o \cdot \epsilon^t_{\theta\theta(a)} + r_o + \int_a^r \epsilon^t_{rr(r)} \cdot dr\right] \cdot \theta \tag{A-7}$$

Thus,

$$\epsilon^t_{\theta\theta(r)} = \frac{l_{(r)} - l_{o(r)}}{l_{o(r)}} =$$

$$= \frac{\left[a_o \cdot \epsilon^t_{\theta\theta(a)} + r_o + \int_a^r \epsilon^t_{rr(r)} \cdot dr\right] \theta - r_o \cdot \theta}{r_o \cdot \theta} = \tag{A-8}$$

$$= \frac{1}{r_o}\left[\int_a^r \epsilon^t_{rr(r)} \cdot dr + a_o \cdot \epsilon^t_{\theta\theta(a)}\right]$$

which is the same as Eq. (A-1a) above.

Thus, complying with compatibility as expressed by Eq. (17a) or Eq. (A-1a) is equivalent to the lack of shear, $\epsilon_{r\theta} = \epsilon_{\theta r}$, in the cylindrical plane r-θ.

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